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The BMW model: a new framework for teaching monetary macroeconomics in closed and open economies

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The BMW model: a new framework for teaching monetary macroeconomics in closed and open economies*

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---Abstract---

While the IS/LM-AS/AD model is still the central tool of macroeconomic teaching it has been criticised by several economists. The model is unable to deal with a monetary policy that uses the interest rate as its operating target (Romer [2000]). Walsh [2002] has criticised that it is not suited for an analysis of inflation targeting. We present the BMW model as an alternative framework, which develops the Romer approach into a simple macroeconomic model. It can deal with issues like inflation targeting, monetary policy rules, and central bank credibility. Our open-economy version is a powerful alternative to the IS/LM-based Mundell-Fleming (MF) model. The main advantage of the open-economy BMW model is its ability to discuss the role of inflation and the determination of flexible exchange rates while the MF model is based on fixed prices and constant exchange rates.

JEL classification: A 20, E 10, E 50, F 41

Keywords: monetary policy, inflation targeting, optimal interest rate rules, simple rules, IS/LM, Mundell-Fleming

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1 Introduction

While the IS/LM-AS/AD model is still the central tool of macroeconomic teaching in most macroeconomic textbooks, it has been criticised by several economists. Colander [1995] has demonstrated that the framework is logically inconsistent, Romer [2000] has shown that it is unable to deal with a monetary policy that uses the interest rate as its operating target, Walsh [2002] has criticised that it is not well suited for an analysis of inflation targeting. In our paper we start with a short discussion of the main flaws of the IS/LM-AS/AD model. In section 3 we present the BMW model as an alternative framework, which develops the Romer approach into a very simple, but comprehensive macroeconomic model. In spite of its simplicity it can deal with issues like inflation targeting, monetary policy rules, and central bank credibility.

2 Four main flaws of the IS/LM-AS/AD model

The standard version of the IS/LM-AS/AD model suffers from several serious flaws, which we will discuss in the following.

- As Colander [1995] has shown, it suffers from an inconsistent explanation of aggregate supply.
- The model is designed for a monetary policy that targets the money supply. Thus, as emphasised by Romer [2000], it is not well suited to cope with real world monetary policy, which is conducted in the form of interest rate targeting.
- As pointed out by Walsh [2002], the model has problems in modelling the inflation rate. As a consequence, the expectations augmented Phillips curve is not an integral part of the model. Furthermore, the model cannot easily deal with modern concepts such as inflation targeting, credibility, monetary policy rules and loss functions.

The first flaw has been formulated by Colander [1995] as follows:

“Given that the Keynesian model includes assumptions about supply, one cannot logically add another supply analysis to the model unless that other supply analysis is consistent with the Keynesian model assumption about supply. The AS curve used in the standard AS/AD model is not; thus the model is logically inconsistent.” (ibid., p. 176)
This argument can be demonstrated with the help of a simple graphical analysis. It starts with a slightly different approach to the IS curve. As this curve is the locus where aggregate demand and aggregate supply are identical, we can make both curves explicit. First, we draw an aggregate demand curve that depends negatively on the nominal interest rate (upper panel of Chart 1). The aggregate supply can be derived under the assumption that there is a full employment output which is determined in a neoclassical labour market ($Y_F$). From the logic of Keynesian economics the aggregate supply is determined by aggregate demand as long as it is below $Y_F$. In the lower panel we have depicted the AS/AD model i.e. a classical aggregate supply curve – which has been derived in the same way as $Y_F$ – and a typical aggregate demand curve.

The inconsistency is obvious for negative demand shocks, which shift both aggregate demand curves to the left. The main message of the IS/LM-AS/AD model is now that this shock leads to a fall of the price level, which is caused by an excess supply ($Y_F > Y_1$) at the old price level $P_0$.

**Chart 1: The classical IS/LM-AS/AD model**
But this effect on the price level is only possible if the firms are actually supplying the full employment output ($Y_F$). According to the logic of the IS curve they would simply adjust their supply to the given demand $Y_1$ so that the price level would remain constant. Thus, the whole explanation of the price level provided by the IS/LM-AS/AD model rests on inconsistency between a Keynesian determination of demand in the IS/LM plane and a neoclassical determination in the AS/AD plane.

The second main flaw of the IS/LM-AS/AD model concerns its approach to the implementation of monetary policy. As Romer [2000] has shown the LM curve is derived under the assumption that the central bank uses the monetary base as its operating target. With a constant multiplier this is automatically translated into a targeting of the money supply. This approach is not compatible with actual practice of central banks using a short-term interest rate (or a set of short-term rates) as operating target. An additional short-coming of the standard derivation of the LM curve is the fact that the money supply process is discussed in a completely mechanistic way which does not take into account the relevant interest rates (central bank refinancing rate and loan rate of banks); a price-theoretic approach is presented in Bofinger [2001]. For teaching purposes the LM curve has the main disadvantage that it is not well suited to model changes in the official interest rates (the Federal Funds Rate or the ECB’s Repo Rate) on the economy. In addition, representing monetary policy by the LM curve requires that one uses a nominal interest rate. While the *nominal* interest rate is the relevant opportunity cost of holding non-interest bearing money aggregate demand depends on the *real* interest rate. In order to make the two rates compatible the IS/LM-model has to assume that the inflation rate is zero and hence, that prices are constant.\(^1\)

As mentioned by Romer [2000], a decline in the price level, which is the consequence of a negative demand shock in the IS/LM-AS/AD model has been rarely encountered in the post-war period. However, a decline in the inflation rate is something very common. As a consequence of its focus on the price level, the IS/LM-AS/AD model is also not able to integrate the standard expectations-augmented Phillips curve. Thus, the common textbook procedure is the presentation of an aggregate supply curve based on price levels and one or several chapters later a separate presentation of the Phillips curve based on the inflation rate.\(^2\) We will also see that this approach is responsible for the inconsistent derivation of aggregate supply. Because of its inability to include the inflation rate, the IS/LM-AS/AD model is unable to discuss new concepts
such as inflation targeting, monetary policy rules, and loss functions which are all based on the inflation rate.

3 The BMW model

3.1 Its main building blocs

The BMW model consists of four building blocs:
- an aggregate demand equation,
- a Phillips curve equation,
- the assumption that output is demand determined in the short run, and
- an interest rate equation.

Aggregate demand, which is presented, in the form of the output gap \( y \) depends on autonomous demand components \( a \), negatively on the real interest rate and a demand shock \( \varepsilon_1 \):

\[
y^D = a - br + \varepsilon_1.
\]

As Chart 2 shows, this approach is very much in line with Romer [2000].

Chart 2 The aggregate demand curve

We assume for our short-run analysis that aggregate supply is determined by aggregate demand and that there are no capacity constraints:
Therefore, for the sake of simplicity we do not differentiate between $y^S$ and $y^D$ in the following. The fourth building bloc is the expectations-augmented *Phillips curve* (Chart 3) which we model in a similar way as Walsh [2002]:

\[(3) \quad \pi = \pi^e + dy + \varepsilon_2.\]

The inflation rate is determined by inflation expectations, the output gap, and a supply shock. In the most simple version one can assume that the central bank is *credible*, i.e. that private inflation expectations are identical with the central bank’s inflation target ($\pi_0$). Thus, the Phillips curve becomes

\[(4) \quad \pi = \pi_0 + dy + \varepsilon_2.\]

It is important to note that this curve is not a short-term supply curve but simply a device for calculating the inflation rate that is associated with an output gap, which is determined in Chart 3.

**Chart 3: The expectations augmented Phillips curve**

\[\pi \quad \quad \quad \quad \pi^e=\pi_1<\pi_0 \quad \quad \pi^e=\pi_0<\pi_1\]

As a fourth building bloc we assume that the central bank is able to determine a *real interest rate*. In the most simplest version we assume that the central bank decides on interest rates on the basis of the observable demand ($\varepsilon_1$) and supply shock ($\varepsilon_2$).
Additionally we will present two alternative notions according to which the central bank implements its policy. Firstly, optimal monetary policy under inflation targeting: The central bank sets its instrument in order to minimize a loss function depicting its preferences. Secondly, simple rules: The most prominent simple rule is a Taylor rule. The central bank changes its instrument only in response to a subset of variables of the model.

As the central bank controls the nominal interest rate on the money market, it determines the required nominal rate by adding the (endogenously determined) inflation rate to the real interest rate:

\[ (6) \quad \bar{r} = \bar{r} + \pi. \]

### 3.2 An very simple approach to an analysis of supply and demand shocks

With these building blocs we can already describe fundamental principles of monetary policy in a very simple and at the same time very comprehensive way.

- Starting with a negative demand shock \( \varepsilon_1 < 0 \) we can see in Chart 4 that the demand curve is shifted downwards so that at constant real interest rates a negative output gap emerges. In the lower panel this is translated into an inflation rate \( \pi_1 \) that is below the central bank’s target rate. If the central bank lowers the real rate from \( r_0 \) to \( r_1 \) the output gap is closed and the inflation rate is brought back on its target level. Thus, the model shows that there is no trade-off between output and inflation in the situation of a demand shock.
If the economy is confronted with a positive supply shock ($\varepsilon_2 > 0$), Chart 5 shows in the lower panel that the Phillips curve is shifted upwards. If the central bank decides to remain passive we can see from the upper panel that at a constant real interest rate the output gap remains zero. The inflation rate rises from $\pi_0$ to $\pi_1$. It is important to note that this requires an equivalent increase in the nominal rate since the inflation rate has gone up. Alternatively the central bank can increase the real rate in order to keep inflation on track. In this case, it has to accept a negative output gap (point A). Of course, the central bank can also decide to target intermediate combinations of $y$ and $\pi$ which lie on the Phillips curve between point A and B.
Thus, the basic version of the BMW model helps us to understand the underlying principles of monetary policy in a very simple way. It is important to note that in this basic version no aggregate demand curve is required in the $\pi/y$ diagram. In this respect our approach differs from the model presented by Romer [2000]. Compared with the IS/LM-AS/AD model this basic version of the model is at the same time

- simpler since it does not require readjustments of the LM curve that take place if the price level changes the real money supply,
- more powerful since it allows to explain changes in the inflation rate,
- more comprehensive since it includes the expectations-augmented Phillips curve,
- more consistent since, it avoids an inconsistent determination of aggregate supply.

The model is also well suited for discussing inflationary processes where inflation expectations shift the Phillips curve upwards and where the central bank has to accept a negative output gap if it wants to keep inflation close to the target level.
In our view for an introductory textbook this version of the BMW model would be completely sufficient. It goes without saying that the standard presentation of fiscal policy can be easily included in the model. It would also be possible to present aggregate demand in terms of output levels instead of output gaps. In this case equation (1) can be stated as

\[ Y^D = a - br + \varepsilon_1. \]

As a benchmark we would then have to define a full employment output \( (Y_F) \), which can be derived from a neoclassical labour market. The Phillips curve would be formulated as:

\[ \pi = \pi^e + d \left( \frac{Y - Y_F}{Y_F} \right) + \varepsilon_2. \]

For an introductory text one could also omit the shock terms \( \varepsilon_1 \) and \( \varepsilon_2 \).

### 3.3 Inflation Targeting

The BMW model allows a simple discussion of the increasingly popular strategy of inflation targeting within a simple macroeconomic framework. For that purpose one has to introduce a loss function for the central bank. It is typically defined as:

\[ L = (\pi - \pi_0)^2 + \lambda y^2 \text{ with } \lambda \geq 0. \]

If \( \lambda > 0 \) such preferences are defined as a policy of “flexible inflation targeting”, if \( \lambda = 0 \) we speak of “strict inflation targeting” or an “inflation nutter” (Svensson [1999])).

Algebraically we derive the optimal interest rate in response to the exogenous variables \( \varepsilon_1 \) (demand shock) and \( \varepsilon_2 \) (supply shock) by inserting the Phillips curve (4) into the loss function (7) and solving it for \( y \). This gives the optimum output gap:

\[ y = -\frac{d}{d^2 + \lambda} \varepsilon_2. \]
If we insert equation (8) into the Phillips curve (4) we get the following reduced form expression for the deviation of the inflation rate from the inflation target.

\[(\pi - \pi_0) = \frac{\lambda}{d^2 + \lambda} e_2\]  

The optimal interest rate can be represented in its reduced form as a function of the exogenous variables \(e_1\) and \(e_2\) by inserting equation (8) in (1).

\[r^{\text{opt}} = \frac{a}{b} + \frac{1}{b} e_1 + \frac{d}{b(d^2 + \lambda)} e_2.\]  

This optimal monetary policy rule is characterised by the following features:

- The optimal response to demand shocks \(e_1\) does not depend on preferences \(\lambda\). Therefore each preference type \(\lambda\) adjusts the real interest rate according to \(\Delta r^{\text{opt}} = (1/b) e_1\) which closes the initial output gap.

- The reaction of the central bank to supply shocks depends on preferences \(\lambda\). A central bank that only cares about inflation (\(\lambda=0\)), requires a strong real rate increase and accordingly a high output loss. With an increasing \(\lambda\) the interest rate response weakens and accordingly the output loss decreases whereas the inflation loss increases.

- In the long run equilibrium the real interest rate will be given by \(r^{\text{opt}} = a / b\). In line with Blinder [1998], p. 31 this rate can be regarded as a neutral real short-term interest rate.

We can represent these results graphically as follows:

In the \(\pi/y\)-space loss functions can be represented by circles around a “bliss point” (see Chart 6). We can derive the normal form of a circle by transforming the loss function (7) according to\(^1\):

\[1 = \left(\frac{\pi - \pi^*_0}{\sqrt{L^{\text{opt}}}}\right)^2 + \left(\frac{y}{\sqrt{L^{\text{opt}}}}\right)^2\] \(\big|_{L=1}\)

\(^1\) For the more general case of an ellipse see Bofinger et al. [2002].
where \((0; \pi_0)\) is the centre of the circle and the radius is given by \(\sqrt{L^{\text{opt}}}\). The bliss point is defined by an inflation rate, which is equal to the inflation target and an output gap of zero. It represents the optimal outcome. In the case of a demand shock we can see from Chart 6 that monetary policy is able to maintain the “bliss combination” of \(\pi\) and \(y\) at the centre of the circle.

**Chart 6: Demand shock and optimal monetary policy**

In the case of a supply shock the loss function helps us to identify the optimum combination of \(\pi\) and \(y\). Here the Phillips curve serves as a restriction under which the loss function has to be minimised. The optimum combination is graphically given by the locus on the Phillips curve that is tangent to an isoquant of the loss function (see Chart 7).
Eliminating the supply shock $\varepsilon_2$ from equation (8) and equation (9) we arrive at the following relationship between the inflation rate and the output gap:

\[(12) \quad \pi = \pi_0 - \frac{\lambda}{d} y.\]

It represents the reaction function of the central bank. For any given value of private expectations and thus any given location of the Phillips curve it shows the inflation rate which produces a minimum loss for the central bank. The optimal outcome is thus described by the intersection of the Phillips curve $PC_1$ with the reaction function of the central bank. Equation (12) shows that an increasing $\lambda$ (“increasing weight on output stabilization”) leads to a steepening of the reaction function $RF(\lambda, y)$ and accordingly the point of intersection moves along the new Phillips curve $PC_1$ from point A to point B.
3.4 The Taylor rule: monetary policy guided by a simple rule

Given the prominence of simple rules in the conduct of monetary policy the BMW model has the advantage that it can also analyse them in a relatively easy way. The most prominent version of a simple rule is the Taylor rule (Taylor [1993]) according to which the real interest rate is determined by a neutral real rate ($r_0$) and the weighted inflation gap and output gap.

\begin{equation}
    r = r_0 + e(\pi - \pi_0) + fy \text{ with } e, f > 0
\end{equation}

Graphically the Taylor rule gives an upward-sloping interest rate line (MP) in the $r/y$ diagram (Chart 8). Hence we assume that the central bank reacts to variables which it can control directly.

**Chart 8: Taylor rule**

The MP line is shifted upwards if the inflation rate increases. In the $\pi/y$ diagram the Taylor rule leads to a downward-sloping aggregate demand function which is derived in complete analogy to the aggregate demand curve within the IS/LM-AS/AD framework.

Graphically the $y^{d}(\pi)$ curve can be derived as follows. Initially the inflation rate is equal to the inflation target $\pi_0$. The MP line corresponding to this inflation rate is $MP(\pi_0)$ which is associated with $y=0$ (Point A, lower panel). If the inflation rate increases to $\pi_1$ the MP line is shifted upwards to $MP(\pi_1)$. This leads to an output decline corresponding to a negative output gap ($y_1$). In the $\pi/y$ diagram this combination of inflation and output leads to point B which together with point A allows us to draw a downward-sloping $y(\pi)$ line.
With a Taylor rule we get different outcomes compared to optimal monetary policy if the economy is subject to shocks. Starting with a negative demand shock we get a downward shift of the \( y(r) \)-line in the upper panel of Chart 10. In response to the decrease of the output gap from 0 to \( y' \) the central bank lowers – by moving along the \( MP(\pi_0) \)-line - real interest rates from \( r_0 \) to \( r' \). In the lower panel the \( y(\pi) \)-line shifts from \( y^d_0(\pi) \) to \( y^d_1(\pi) \). As the inflation rate initially remains unchanged the new \( y^d_1(\pi) \) curve has to go through a point, which is a combination of the new output gap \( y' \) and an unchanged inflation rate \( \pi_0 \). However, with a negative output gap the Phillips curve which is the inflation determining relationship tells us that the inflation rate will start to fall. The new equilibrium is the intersection of the shifted \( y^d_1(\pi) \) line with the unchanged Phillips curve. It is characterised by a somewhat dampened output decline that is due to fact that the central bank reduces real rates because of the lower inflation rate. Thus, we also get a downward shift of the MP line so that it intersects with the \( y^d_i(r) \) line at the same output level as the intersection of the \( y^d_i(\pi) \) line with the Phillips curve. This sounds rather difficult but
the mechanics of the shifts are identical with the shifts in the IS/LM-AS/AD model in the case of
the same shock which leads to a downward shift of the IS-curve:

- In our model the increase in *inflation* has a restrictive monetary impulse since it increases
  the *real* interest rate.
- In the IS/LM-AS/AD model the increase in the *price level* reduces the real money stock,
  which also has a restrictive effect since it increases the *nominal* interest rate.

**Chart 10: Taylor rule and demand shocks**

For the discussion of a *supply shock* we start the graphical analysis in the lower panel of Chart
11. The Phillips curve is shifted upwards which increases inflation rate. Because of the higher
inflation rate the Taylor rule line in the upper panel is also shifted upwards from $\text{MP}(\pi_0)$ to
$\text{MP}(\pi_1)$. The increase in real interest rates from $r_0$ to $r_1$ produces a negative output gap ($y_1$) which
corresponds to the inflation rate $\pi_0$. 
Algebraically we can derive this function by inserting the Taylor rule (13) into the \( y^d(r) \)-curve.

(14) \[ \pi = \frac{(a - br_0) + be\pi_0 + \varepsilon_i}{be} - \frac{1 + bf}{be} y \]

The \( y^d(\pi) \)-curve is characterised by the following features:

- In sharp contrast to optimal monetary policy the demand shock \( \varepsilon_i \) has an impact on the \( y^d(\pi) \)-curve due to the sub optimal monetary policy reaction.

- An increasing weight on output gap stabilization \( f \) leads to a steppening of the \( y^d(\pi) \) curve and accordingly to smaller output gaps. An increasing weight on stabilizing the inflation rate leads to a flattening of the \( y^d(\pi) \)-curve and hence to smaller inflation gaps \( (\pi - \pi_0) \) (see Chart 12).
3.5 Inflation Targeting versus Taylor Rules

In the following we compare simple and optimal monetary policy rules algebraically and graphically. We showed that if the shocks are observable the central bank is able to pursue an optimal monetary policy:

\[
    r^{\text{opt}} = \frac{a}{b} + \frac{1}{b} \varepsilon_1 + \frac{d}{b(d^2 + \lambda)} \varepsilon_2 .
\]

As alternative monetary policy we introduced simple rules. Although the central bank reacts to a weighted average of the inflation gap and the output gap, one can evaluate the implicit reaction of the central bank to demand and supply shocks. The Taylor rule in terms of shocks can be computed as:

\[
    r^{\text{Taylor}} = r_0 + \frac{ed + f}{1 + bf + bed} \varepsilon_1 + \frac{e}{1 + bf + dbf} \varepsilon_2 .
\]

Simple versus optimal rules: demand shocks

Obviously optimal and simple monetary policy rules are only identical if and only if the coefficients in front of \( \varepsilon_i \) in equations (15) and (16) are identical. Hence in particular it has to hold that:
rearranging yields:

\[ \text{(18)} \quad \text{bed} + b\text{f} = 1 + b\text{f} + \text{bed} \]

Obviously this equality can never hold. Assuming reasonable parameter values \((b=0.4; d=0.34)^3\) and a Taylor rule \((e=f=0.5)\) the interest rate reaction will move in the right direction but it will be two weak to restore the bliss point (see Chart 13). Hence it is important to note that compared with inflation targeting (monetary policy guided by a loss function) a simple rule leads to a sub-optimal outcome. It produces a change of the real rate that leads into the right direction but the decline of the real rate is too weak to restore the “bliss point”. This outcome is not implausible since it demonstrates that a simple rule cannot be identical with a policy that has perfect information about the shocks \(\varepsilon_1\) and \(\varepsilon_2\). Another justification for simple rules which we do not consider here is model uncertainty (see Levin et al. [1999]).

**Simple versus optimal rules: supply shocks**

In the case of a supply shock it is well possible that simple and optimal monetary policy yield identical results. This can be shown by equating the respective coefficients in front of the supply shock \(\varepsilon_2\). It has to hold that:

\[ \text{(19)} \quad \frac{d}{b(d^2 + \lambda)} = \frac{e}{1 + b\text{f} + d\text{be}} \]

Rearranging the equality yields:

\[ \text{(20)} \quad \lambda = \frac{d(1 + b\text{f})}{eb} \]

Equation (20) tells us that if the equality holds the following is true:

- ceteris paribus an increasing weight on the output gap \(f\) (Taylor rule) corresponds in line with intuition to increasing values of \(\lambda\) (optimal policy).
• ceteris paribus an increasing weight on stabilizing the inflation rate around the inflation target \( e \) (Taylor rule) corresponds to decreasing values of \( \lambda \) (optimal policy).

We can illustrate equation (20) with the help of a simple example. Let us assume for instance that monetary policy is conducted according to a Taylor rule, hence \( e = f = 0.5 \). Additionally we calibrate \( d = 0.34 \) and \( b = 0.4 \). Then as can be seen from equation (20) optimal and simple monetary policy rules yield identical results in the case of supply shocks if and only if \( \lambda = 2.04 \).

**Chart 13 Inflation Targeting versus Taylor Rules**

Note, that again compared with inflation targeting (monetary policy guided by a loss function) a simple rule leads to a sub-optimal outcome. Unless equation (20) holds with equality. Optimal monetary policy would choose the tangency point of the inner ellipse with the Phillips curve. As monetary policy is conducted according to a Taylor rule the final outcome will be the intersection of the Phillips curve \( PC_1 \) and the aggregate demand curve \( y_0^d(e,f,\pi) \). The loss attached to this outcome is given by the outer circle of the loss function. The distance between the two circles is the welfare loss implied by sticking to a simple rule. We can see that in this...
case the Taylor rule outcome closely matches with the optimal solution under inflation targeting (see Chart 13).

3.6 The inflation bias in monetary policy

So far we have assumed that the central bank follows a loss function which is compatible with an output gap of zero. In this context, we have discussed the Taylor rule as a “heuristic” or “rule of thumb” which allows fast and successful decision making even if the central bank is confronted with diverse kinds of uncertainties like non observable demand and supply shocks or Brainard uncertainty, model uncertainty or data uncertainty (see Levin et al. [1999])

However, in much of the literature the term “rule” is also used with a somewhat different meaning. Based on the seminal model by Barro and Gordon [1983]), the purpose of a “rule” is not facilitating the decision making of a central bank under pure discretion but rather to limit this discretion in order to avoid the problem of an inflation bias. For a discussion of these issues we have to modify our loss function as follows:

\( L = (\pi - \pi_0)^2 + \lambda (y - k)^2 \) with \( k > 0 \)

By introducing the parameter \( k \), the central bank targets an output gap that is above zero. This could be justified by monopolistic distortions in goods and labour markets which keep potential output below an efficient level. Compared with the loss function that we have used so far, the bliss point \((k; \pi_0)\) has moved to the right.

In line with the Barro/Gordon model the game between the private sector and the central bank can be modelled as follows. The private sector builds its inflation expectations which enter the goods and labour market contracts. Observing private expectations the central bank chooses an inflation rate that minimizes its loss function.

3.6.1 Reaction function of the central bank

If we insert the Phillips curve in the loss function, we get the following optimization problem for the central bank:
Minimizing the loss function $L$ with respect to the inflation rate yields:

\[
\pi^{\text{opt}}(\pi^e) = \frac{\lambda}{\lambda + d^2} \pi^e + \frac{d^2}{\lambda + d^2} \pi_0 + \frac{\lambda d}{\lambda + d^2} k
\]

Thus, the optimum inflation rate depends on inflation expectations, the inflation target, and the parameter $k$. Inserting the optimal inflation rate into the Phillips curve relationship leads to the following expression for the output gap:

\[
y^{\text{opt}}(\pi^e) = -\frac{d}{\lambda + d^2}(\pi^e - \pi_0) + \frac{\lambda}{\lambda + d^2} k
\]

If we solve the Phillips curve for $\pi^e$ and insert it in equation (24) we can derive a relationship between the output gap ($y$) and the optimum inflation rate $\pi^{\text{opt}}$ of the central bank. This rate is identical with the actual inflation rate since the central bank can control inflation perfectly.

\[
\pi = \pi_0 + \frac{\lambda}{d} k - \frac{\lambda}{d} y
\]

In contrast to equation (12) the new reaction function is shifted to the right.

### 3.6.2 Surprise inflation, rational expectations and commitment

If we want to make predictions on the final monetary policy outcome we have to specify the way in which the private sector builds its expectations. Following Barro/Gordon we can now distinguish between three different outcomes:

- Discretion and surprise inflation: $\pi^e = \pi_0 < \pi$
- Discretion and rational expectations: $\pi^e = \pi^{\text{opt}} = \pi$
- Commitment solution: $\pi^e = \pi_0 = \pi$
**Surprise Inflation**

As a starting point we assume that the central bank announces an inflation target of \( \pi_0 \) and that the public believes in the announcement. Thus, expectations of the private sector are given by: \( \pi^e = \pi_0 \). Based on these expectations the central bank chooses the optimal inflation rate \( \pi^s \) according to its reaction function (25) as follows:

\[
\pi^s = \pi_0 + \frac{\lambda d}{\lambda + d^2} k
\]

It is obvious that this rate exceeds the announced inflation target \( \pi_0 \). The second term on the right hand side of equation (26) denotes the inflation bias under surprise inflation. The output gap is:

\[
y^s = \frac{\lambda}{\lambda + d^2} k
\]

Due to the surprise inflation it is positive. shows this combination of the output gap \( y^s \) and inflation \( \pi^s \) and the corresponding loss circle.

**Chart 14 The Bliss Point and Optimal Monetary Policy**

![Diagram showing the bliss point and optimal monetary policy](chart)

**Discretion and rational expectations**

With discretion of the central bank the outcome of surprise inflation is not very realistic. Let us now assume that the private sector forms its expectations rationally. This means that the optimal
value \( \pi^{\text{opt}}(\pi^e) \) is used for forming expectations on \( \pi^e \) and that the private sector minimizes the following loss function:

\[
L = \left( \pi(\pi^e) - \pi^e \right)^2
\]

The first order condition is given by:

\[
\pi^{\text{opt}}(\pi^e) = \pi^e
\]

Inserting the respective expressions yields:

\[
\pi^e = \frac{\lambda}{\lambda + d^2} \pi^e + \frac{d^2}{\lambda + d^2} \pi_0 + \frac{\lambda d}{\lambda + d^2} k
\]

Solving for \( \pi^e \) we obtain the following rational expectations equilibrium for the inflation rate.

\[
\pi^{\text{opt}} = \pi_0 + \frac{\lambda}{d} k
\]

This rate lies again above the inflation target \( \pi_0 \) and also above inflation under surprise inflation \( \pi^s \) as given by equation (26). As \( \pi^e \) equals \( \pi \), the Phillips curve shows that the output gap is zero. Chart 15 depicts the rational expectations solution. Compared to surprise inflation this solution is clearly inferior since it leads to a higher inflation rate without a positive gain in output. The loss circle lies outside the loss circle attached to the solution with surprise inflation.

**Commitment to a rule**

So far we have seen that an inflationary bias is inherently nested in the rational expectations solution under discretion. Even if the central bank announces an inflation target, rational market participants will realise that it has a strong incentive to renege on its announcement. In order to avoid the high negative social loss under discretion, a mechanism is required that credibly commits the central bank to a socially optimal inflation target. We assume that such a rule can be
designed and that the private sector expects now always the inflation target \( \pi^e = \pi_0 \) which by assumption becomes the actual inflation rate that equals the inflation target:

\[
(32) \quad \pi^e = \pi_0 = \pi
\]

The output gap is again zero.

**Comparison of the three solutions**

As we can see from Chart 15 the first-best outcome is surprise inflation. Discretion turns out to be the worst solution as the public anticipates the higher inflation rate without generating a positive output effect. The commitment solution is second best since it allows to reach the inflation target but monetary policy is unable to come closer to its bliss point combination. These results are also shown by the concrete values of the social loss under the three different scenarios. It becomes also obvious that the whole problem of the inflation bias is due to the rather arbitrary assumption of \( k > 0 \). With \( k = 0 \) the central bank has no incentive to deviate from an announced inflation target and the social loss is always zero.

**Discretion: Surprise Inflation** \( \pi^e = \pi_0 < \pi \)

\[
(33) \quad L^S = \lambda k^2 \cdot \frac{d^2}{\lambda + d^2}
\]

**Discretion: Rational Expectations** \( \pi^e = \pi^{opt} = \pi \)

\[
(34) \quad L^{rat} = \lambda k^2 + \frac{\lambda}{d^2} k^2
\]

**Commitment Solution** \( \pi^e = \pi_0 = \pi \)

\[
(35) \quad L^C = \lambda k^2
\]
3.6.3 The Barro/Gordon model in the BMW framework

Thus, the BMW framework can be easily extended for an analysis of the issues that are related to the Barro/Gordon model. While have not made explicit the adjustment of real interest rates that is required to generate the specific values of inflation and output gap, our approach has the advantage that it discusses surprise inflation with a framework that also includes the demand side of the economy. As a consequence, one could show that a rule does not prevent an optimum reaction of the central to demand shocks. As demonstrated in Chart 15, the central bank is still able to cope with such a disturbance while remaining on an unchanged position on the Phillips curve. One could also discuss supply shocks within our framework. They would show that – depending on the size of the shock – the outcome under discretion with rational expectations could be better than a commitment to a rule which requires a constant inflation rate.

4 The BMW model for an analysis of monetary policy in an open economy

4.1 Modifications in the basic building blocs

For a discussion of monetary policy in an open economy we have to include the effects of the real exchange rate on aggregate demand. Thus equation (1) becomes:

\[ y = a - b r + c \Delta q + \varepsilon, \]
where $\Delta q$ is the change of the real exchange rate and $a$, $b$, and $c$ are positive structural parameters of the open economy. For the determination of the inflation rate we will differentiate between two polar cases. In the first case which represents a *long-term perspective* especially for a small economy the domestic inflation rate is completely determined by the foreign rate of inflation expressed in domestic currency terms ($\pi^f$), and hence by purchasing power parity (PPP):

\[(37) \quad \pi = \pi^f = \pi^* + \Delta s .\]

Because of the long-term perspective we do not include a shock term. Thus, the domestic inflation rate equals the foreign inflation rate ($\pi^*$) plus the depreciation of the domestic currency ($\Delta s$). In other words, we assume that the real exchange rate $\Delta q = \Delta s + \pi^* - \pi$ remains constant.

In the second case we adopt a *short-term perspective*. We assume that companies follow the strategy of *pricing-to-market* so that they leave prices unchanged in each local market even if the nominal exchange rate changes. As a consequence, changes in the exchange rate affect mainly the profits of enterprises. One can regard this as an *open-economy balance-sheet channel* where changes in profitability are the main lever by which the exchange rate affects aggregate demand. In this case the Phillips curve is identical with the domestic version (see Chapter 3.1):

\[(38) \quad \pi = \pi_0 + dy + \varepsilon_2 .\]

Of course, it would be interesting to discuss an intermediate case where the real exchange has an impact on the inflation rate. But using an equation like
would make the presentation very difficult, above all the graphical analysis. As a further ingredient of open economy macro models we have to take into account the behaviour of international financial markets’ participants which is in general described by the uncovered interest parity condition (UIP):

\[ \Delta s + \alpha = i - i^*. \]

According to equation (40) the differential between domestic (i) and foreign (i*) nominal interest rates have to equal the rate of nominal depreciation (\( \Delta s \)) and a stochastic risk premium (\( \alpha \)).

This basic model can now be used to assess monetary policy under the options of independently floating (4.2) and absolutely fixed (4.3) exchange rates.

### 4.2 Three variants of independently floating exchange rates

For a discussion of monetary policy under independently floating exchange rates it is important to decide how a flexible exchange rate is determined. In the following we discuss three different variants:

- PPP and UIP hold simultaneously (4.2.1),
- UIP holds, but deviations from PPP are possible (4.2.2),
- the exchange rate is a pure random variable (4.2.3).
4.2.1 Monetary policy under flexible rates if PPP and UIP hold simultaneously (long-term scenario)

As it is well known that PPP does not hold in the short-term, the first case can mainly be regarded as a long-term perspective. PPP implies that the real exchange remains constant by definition:

\[(41) \quad \Delta q = \Delta s + \pi^* - \pi = 0.\]

For the sake of simplicity we assume a UIP condition that is perfectly fulfilled and thus, without a risk premium:

\[(42) \quad \Delta s = i - i^*,\]

which can be transformed with the help of the Fisher equation for the domestic interest rate

\[(43) \quad i = r + \pi\]

and the foreign interest rate

\[(44) \quad i^* = r^* + \pi^*,\]

and equation (37) into

\[(45) \quad r = r^*.\]
Thus, one can see that in a world where PPP and UIP hold simultaneously there is no room for an independent real interest rate policy, even under independently floating rates. As the domestic real interest rate has to equal the real interest rate of the foreign (world) economy, the central bank cannot target aggregate demand by means of the real rate.

This does not imply that monetary policy is completely powerless. As equations (6) and (43) show, the central bank can achieve a given real rate (which is determined according to equation (45) by the foreign real interest rate) with different nominal interest rates. Changing nominal interest rates in turn go along with varying rates of nominal depreciation or appreciation of the domestic currency $\Delta s$, for a given nominal foreign interest rate (see equation (42)). If $i^*$ and $r^*$ are exogenous, then $\pi^*$ is exogenous as well, and the chosen (long-run) nominal interest rate finally determines via the related $\Delta s$ and the PPP equation (41) the (long-run) domestic inflation rate $\pi$.

In sum, the long-term scenario with valid UIP and valid PPP leads to the conclusion that monetary policy has

- no real interest rate autonomy for targeting aggregate demand, but
- a nominal interest rate autonomy for targeting the inflation rate.

This comes rather close to the vision of the proponents of flexible rates in the 1960s who argued that this arrangement would allow each country an autonomous choice of its inflation rate (see Johnson [1972]). It can be regarded as an open-economy version of the classical dichotomy according to which monetary policy can affect nominal variables only without having an impact on real variables.
4.2.2 Monetary policy under flexible rates if UIP holds but not PPP (short-run scenario)

In our second scenario for flexible rates we assume that the domestic inflation is not affected by the exchange rate. This assumption corresponds with empirical observation that in the short-run the real exchange is rather unstable and mainly determined by the nominal exchange rate (see Chart 16).

Chart 16: Nominal and real exchange rate of the euro area

As a starting point we derive in a general way the optimum interest rate for our model on the basis of equation (38). If we insert it in the loss function (7), we can derive the optimum output gap:

\[
y = -\frac{d}{d^2 + \lambda} \cdot \varepsilon_2.
\]

Using equation (36) we calculate the optimum real interest rate:
\[
(47) \quad r^{\text{opt}} = \frac{a}{b} + \frac{1}{b} \varepsilon_1 + \frac{d}{b(d^2 + \lambda)} \varepsilon_2 + \frac{c}{b} \Deltaq.
\]

It shows that the central has to react to demand and supply shocks and to changes in the real exchange rate.

For the case of flexible rates where UIP holds the real exchange rate in (47) can be substituted as follows. The UIP condition with a risk premium

\[
(48) \quad \Delta s + \alpha = i - i^*.
\]

can be transformed – by subtracting \(\pi - \pi^*\) on both sides – into a real UIP equation

\[
(49) \quad \Deltaq = r - r^* - \alpha.
\]

Inserting (49) into (47) and solving for \(r\) (assuming that \(r = r^{\text{opt}}\)) yields:

\[
(50) \quad r^{\text{opt}} = \frac{a}{b-c} + \frac{1}{b-c} \varepsilon_1 + \frac{d}{(b-c)(d^2 + \lambda)} \varepsilon_2 - \frac{c}{b-c} (r^* + \alpha).
\]

Equation (50) provides the optimal real interest rate for a central bank in a system of floating rates where UIP holds (with the possibility of risk premium shocks) while PPP does not hold. It shows that real interest rate has to respond to the following types of shocks:

- domestic shocks: supply and demand shocks,
international shocks: the shock of a change in the foreign real interest rate and the shock of a change in the risk premium.

Since \( r^{opt} \) can be set autonomously, the central bank can target the real interest rate in the open economy in the same way as in a closed economy. The functioning of the model can also be demonstrated graphically.

For this purpose we have to construct the \( y^d(r) \)-curve in a way that it is not directly affected by the real exchange rate. This can be achieved by replacing \( \Delta q \) in equation (36) by the value for \( \Delta q \) in equation (49). This leads to a demand curve for the open economy, which is only determined by the domestic real interest rate:

\[
(51) \quad y^d(r) = a - (b - c) r - c (r^* + \alpha) + \varepsilon_i .
\]

This curve is characterised by two features:

- the slope of the \( y^d(r) \)-curve is negative as long as \( b > c \), i.e. the interest rate channel of aggregate demand prevails over the exchange rate channel; we refer to this as the “normal” case;
- the slope of the \( y^d(r) \)-curve is steeper in an open economy compared to a closed economy: \( 1/(b - c) > 1/b \). That implies that an identical change of the real interest rate has a stronger effect on aggregate demand in closed economy than in the open economy since in the latter interest rate changes are accompanied by counteracting real exchange rate changes.
We begin with, which illustrates the interest rate reaction of the central bank in the presence of a negative shock affecting the demand side of the economy. From equation (51) we can see that such shocks have their origin either in the behaviour of domestic actors such as the government or consumers \( \varepsilon_1 < 0 \), or in the international environment in the form of an increase of the foreign real interest rate or the risk premium. The latter group of shocks affects domestic demand via the real exchange rate. Thus, the aggregate demand curve can now be shifted by domestic and foreign shocks. In the case of a negative shock it shifts to the left, resulting in a negative output gap \( y_1 \) and a decrease of the inflation rate \( \pi_1 \). As a consequence, the central bank lowers the real interest rate from \( r_0 \) to \( r_1 \) so that the output gap disappears, and hence, the deviation of the inflation rate from its target. One can see that the graphical solution for the open economy is fully identical with the closed economy case.

**Chart 17: Interest rate policy in the case of shocks, which affect the demand side**
The same applies to the discussion of a *supply shock*. Chart 18 shows that the central bank is again confronted with a trade-off between output and inflation stabilisation. A positive supply shock \((\varepsilon_2 > 0)\) shifts the Phillips curve to the left. If there is no monetary policy reaction (the real interest rate remains at \(r_0\)), the output gap is unaffected, but the inflation rate rises to \(\pi_1\) (point B). If, on the other hand, the central bank tightens monetary policy by raising the real interest rate to \(r_1\), the output gap becomes negative, thereby lowering the inflation rate to \(\pi_0\) (point A). As in the closed economy case, the optimum combination of \(y\) and \(\pi\) depends on the preferences \(\lambda\) of the central bank. If \(\pi\) and \(y\) are equally weighted in the loss function, the iso-loss locus is a circle, and PC₁ touches the circle at \((\pi_2, y_2)\).

**Chart 18: Interest rate policy in the case of a supply shock**
4.2.3 Monetary policy under exchange rates that behave like a random walk

One of the main empirical findings on the determinants of the exchange rate is that in a system of independently floating exchange rates no macroeconomic variable is able to explain exchange rate movements (especially in the short and medium run which is the only relevant time horizon for monetary policy) and that a simple random walk out-performs the predictions of the existing models of exchange rate determination (Messe and Rogoff [1983]). In a very simple way such random walk behaviour can be described by

\[ \Delta q = \eta \]

where \( \eta \) is a random white noise variable. Inserting equation (52) into (47) yields the following optimum interest rate:

\[ r^{\text{opt}} = \frac{a}{b} + \frac{1}{b} \varepsilon_i + \frac{d}{b(d^2 + \lambda)} \varepsilon_{z} + \frac{c}{b} \eta. \]

Random exchange rate movements constitute now an additional shock to which the central bank has to respond with its interest rate policy. At first sight, even under this scenario monetary policy autonomy is still preserved. However, there are obvious limitations, which depend on

- the size and the persistence of such shocks, and
- the impact of real exchange rate changes on aggregate demand, which is determined by the coefficient \( c \) in equation (36).

Empirical evidence shows that the variance of real exchange rates exceeds the variance of underlying economic variables such as money and output by far. This so-called “excess volatility
puzzle” of the exchange rate is excellently documented in the studies of Baxter and Stockman [1989] and Flood and Rose [1995]. Based on these results we assume that \( \text{Var}[\eta] \gg \text{Var}[\varepsilon_1] \).

Thus if a central bank would try to compensate the demand shocks created by changes in the real exchange rate, it could generate highly unstable real interest rates. While this causes no problems in our purely macroeconomic framework, there is no doubt that most central banks try to avoid an excessive instability of short-term interest rates (“interest rate smoothing”) in order to maintain sound conditions in domestic financial markets. If this has the consequence that the central bank does not sufficiently react to a real exchange rate shock, the economy is confronted with a sub-optimal outcome for the final targets \( y \) and \( \pi \).

For the graphical solution the \( y^d(r) \)-curve is simply derived by inserting equation (52) into (36) which eliminates \( \Delta q \):

\[
(54) \quad y^d(r) = a - b r + c\eta + \varepsilon_1.
\]

Exchange rate shocks \( \eta \) lead to a shift of the \( y^d(r) \)-curve, similar to what happens in the case of a demand shock. In Chart 19 we introduced a smoothing band that limits the room of manoeuvre of the central bank’s interest rate policy. In order to avoid undue fluctuations of the interest rate, the central bank refrains from a full and optimal interest rate reaction in response to a random real appreciation (\( \eta < 0 \)) that shifts the \( y^d(r) \)-curve to the left. As a result, the shock is only partially compensated so that the output gap and the inflation rate remain below their target levels.
4.3 Monetary policy under fixed exchange rates

With fixed exchange rates a central bank completely loses its leeway for a domestically oriented interest rate policy. In order to avoid destabilising short-term capital inflows or outflows, the central bank has to follow UIP in a very strict way. For $\Delta s = 0$ the UIP condition becomes:

\[(55) \quad i = i^* + \alpha\]

Inserting equation (55) into (43) shows how the real interest rate is determined under fixed exchange rates:
4.3.1 Fixed exchange rates as a destabilising policy rule

As the real interest rate is only determined by foreign variables and as it depends negatively on the domestic inflation rate, the central bank can no longer pursue an autonomous real interest rate policy. In principle, equation (56) can be interpreted as a special case of a “simple rule”. This becomes obvious if equation (56) is transformed into

\[ r = i^* + \alpha - \pi. \]

\[ r = (i^* + \alpha - \pi_0) + (-1)(\pi - \pi_0) + 0 \cdot y. \]

Compared to a Taylor rule (equation (13)) this rule has the negative feature that the inflation rate enters with a negative sign (instead with a coefficient of greater than zero which is required for a stabilising Taylor rule, see our extended discussion of the model in Bofinger et al. [2002]) and that a weight of zero is given to the output gap. Thus, if a positive demand shocks drives the inflation rate up, fixed exchange rates require a decline in the real interest rate. In other words, monetary policy tends to destabilise the economy under fixed exchange rates.

This can also be shown with a graphical analysis which follows the presentation for a Taylor rule in Chapter 3.4. In the (r,y)-space we can derive a horizontal monetary-policy line (MP) from the interest rate equation (56) since the real rate is not affected by the domestic output gap. As in the second case of floating rates (Chapter 4.2.2) we can insert equation (49) into (36) which helps to eliminate \( \Delta q \) in the \( y^d(r) \)-curve:

\[ y^d (r) = a - (b - c) r - c (r^* + \alpha) + \epsilon_i. \]
The corresponding $y^d(\pi)$-curve in the $(\pi,y)$-space is derived in a similar way as in the case of a Taylor rule for the closed economy. By inserting the interest rate equation (56) in equation (58) we get:

\begin{equation}
(59) \quad y^d(\pi) = a - b\left(r^* + \alpha\right) - (b - c)\pi^* + (b - c)\pi + \varepsilon_i.
\end{equation}

Again, we assume that $(b - c)$ is positive. This implies that the $y^d(r)$-curve has a negative slope whereas the slope of the $y^d(\pi)$-curve is positive. Compared with the negative slope of the $y^d(\pi)$-curve under a Taylor rule (see Chapter 3.4), the positive slope of the $y^d(\pi)$-curve shows again the destabilising property of the interest rate “rule” generated by fixed exchange rates. For the graphical analysis it is important to see that

- the slopes of the $y^d(r)$-curve and the $y^d(\pi)$-curve have the same absolute value, but the opposite sign,
- the slope of the $y^d(\pi)$-curve is $1/(b - c)$ which exceeds one if $c < b < 1$. Thus, the $y^d(\pi)$-curve is steeper than the slope of the Phillips curve with a slope of $d$ for which we also assume that is positive and smaller than one.

Monetary policy under fixed exchange rates can now be discussed graphically using Chart 20. While the Phillips curve and the $y^d(r)$-curve are identical with the curves we used under flexible exchange rates, we have now an additional $y^d(\pi)$-curve, which is generated by the “policy-rule” of fixed exchange rates.
4.3.2 The impact of demand and supply shocks

In Chart 20 we use this framework to discuss the consequences of a negative shock that affects the demand side of the domestic economy. According to equation (58) the source for such a shift in the demand curve can originate either from a domestic demand shock ($\varepsilon_1$) or from an increase in the foreign real interest rate ($r^*$) or the risk premium ($\alpha$). The result is a shift of the $y^d(r)$-curve to the left. Without repercussions on the real interest rate the output gap would fall to $y'$ and the inflation rate to $\pi'$. However, in a system of fixed exchange rates the initial fall in $\pi$ increases the domestic real interest rates since the nominal interest rates are kept unchanged on the level of the foreign nominal interest rates. Thus, in a first step, we use the new output gap ($y'$) and an unchanged inflation rate ($\pi_0$) to construct the new location of the $y^d(\pi)$-curve in the ($\pi/y$)-diagram. It also shifts to the left to $y^d_1(\pi)$. This finally leads to the new equilibrium combination ($\pi_1, y_1$) which is the intersection between the Phillips curve and the new $y^d(\pi)$-curve. This equilibrium goes along with a rise of the real interest rate from $r_0$ to $r_1$, which is equal to the fall of the inflation rate from $\pi_0$ to $\pi_1$. It is obvious from Chart 20 that the monetary policy reaction in a system of fixed exchange rates is destabilising since $\pi_1<\pi'$ and $y_1<y'$. 
In Chart 21 we discuss the effects of a supply shock. Initially, the shock shifts the Phillips curve to the left, which leads to a higher inflation rate ($\pi'$) with unchanged output gap. Since the rise in inflation lowers the real interest rate, a positive output gap emerges which leads to a further rise of $\pi$. The final equilibrium is the combination ($\pi_1, y_1$). Again, one can see that the “policy rule” of fixed exchange rate has a destabilising effect. It causes an increase of the inflation rate, which is even higher than under a completely passive real interest rate policy in a closed economy.
Chart 21: Fixed exchange rates and supply shocks

Chart 22 shows that this combination is also sub-optimal compared with the outcome a central bank chooses under optimal policy behaviour in a system of independently floating exchange rates (see Chart 18). Assuming again that the central bank equally weights π and y in its loss function, the grey circle ($\pi_{if}^*, y_{if}^*$) depicts the loss under independently floating exchange rates. If the central bank had followed a policy of constant real interest rates (that is absence of any policy reaction) the dotted circle would have been realised with ($\pi',0$). Under fixed exchange rates, however, the iso-loss circle expands significantly, and the final outcome in terms of the final targets is ($\pi_1, y_1$).
4.3.3 Fixed rates can also be stabilising

From this analysis one could come to the conclusion that a system of fixed exchange rates is always a bad thing. However, this result is difficult to reconcile with the empirical fact that countries like the Netherlands, Austria, and Estonia could follow a very successful macroeconomic policy under almost absolutely fixed exchange rates.

There are two possible explanations for this observation. First, our analysis leaves it open of how the foreign real exchange rate is determined. A stabilising movement of the domestic real interest rate can be generated if the foreign central bank is confronted with and reacts to the same demand shocks as the domestic economy. This was certainly the case in the Netherlands and Austria, which pegged their currency to the D-Mark until 1998. The economies in both countries are very similar to the German economy. Thus, in the literature on optimum currency areas the correlation of real shocks plays a very important role (see e.g. Bayoumi and Eichengreen [1992]; we also simulated a two-country case with a varying degree of correlation between demand shocks in Bofinger et al. [2002]).
A second explanation refers to the *size of countries*, which follow a policy of fixed exchange rates. In general, one can say that the preference for fixed rates is high in very small and open economies. In this case, one can assume that in equation (36) the impact of the real exchange rate (c) on domestic demand is higher than the impact of real interest rates (b). Thus, the slope of the $y^d(\pi)$-curve in Chart 20 and Chart 21 would become negative. The intuition runs as follows: As mentioned above, the negative demand shock reduces the inflation rate. But now the negative impact of the higher real interest rate is overcompensated by the positive effects of the real depreciation, which is caused by a decline of the domestic inflation rate. As a consequence, the fixed rate regime would have similar – stabilising – properties as a Taylor rule in a closed economy.

### 4.4 Summary and comparison with the results of the Mundell-Fleming (MF) model

For a summary of the open-economy version of the BMW-model it seems useful to compare it with the main result of the MF model (see also Table 1).

For *fixed exchange rates* the MF model comes to the conclusion that

- *monetary policy* is completely ineffective, while
- *fiscal policy* is more effective than in a closed-economy setting.

The BMW model shows that *monetary policy* is not only ineffective but rather has a destabilising effect on the domestic economy. Compared with the MF model the sources of demand shocks can be made more explicit (above all the foreign real interest rate and the risk premium) and it becomes also possible to analyse supply shocks. It is important to note that the BMW model can also show that for small economies and in the case of very similar economies fixed rates can also have a stabilising effect. As far as the effects of *fiscal policy* are concerned the BMW model also
comes to the conclusion that it is an effective policy tool and that it is more effective than in a closed economy. If we treat a restrictive fiscal policy as a negative demand shock we can use the results of Chart 20. We see immediately that the initial effect on the output gap is magnified by the destabilising feature of fixed exchange rates. In the case of a very small economy the opposite is the case.

For *independently floating exchange rates* the MF models provides two main results:

- *monetary policy* is more effective than in a closed-economy setting, while
- *fiscal policy* becomes completely ineffective.

It is important to note that the MF model implicitly assumes that neither UIP nor PPP hold. As far as UIP is concerned, the MF model assumes that a reduction of the domestic interest rate is associated with a depreciation of the domestic currency (because of capital outflows). For PPP the MF model must assume that it is always violated if the nominal exchange rate changes since the MF model assumes absolutely fixed prices.

For the three versions of flexible rates the BMW models comes to results that are partly compatible and partly incompatible with the MF model.

For a world where PPP and UIP (*long-term perspective*) hold the BMW model produces the contradictory result that there is no *monetary policy* autonomy with regard to the real interest rate. Thus, the central bank is unable to cope with demand shocks. However, because of its control over the nominal interest rate it can target the inflation rate and thus react to supply shocks. For fiscal policy the BMW model also differs from the MF model. As it assumes an exogenously determined real interest rate, i.e. a horizontal monetary policy line, fiscal policy has
the same effects as in a closed economy. By shifting the $y^d(r)$-curve it can perfectly control the output-gap and indirectly also the inflation rate.

Under a short-term perspective (UIP holds, PPP does not hold) the results of the BMW model are identical with regard to monetary policy as far as the signs are concerned. The central bank can control aggregate demand and the inflation rate by the real interest rate. However, because of the UIP condition a change in the real interest rate (i.e. a decline) is always accompanied by an opposite change in the real exchange rate (i.e. a real appreciation), the effects of changes in the real interest rate are smaller in the open economy than in the closed economy. Fiscal policy is again effective and if one assumes that the central bank does not react to actions of fiscal policy (constant real rate) it is as effective as in a closed economy.

In the third and most realistic scenario for flexible exchange rates (random walk) the results of the BMW model are in principle identical with those of the short-term perspective. However, the ability of monetary policy to react to exchange rate shocks can be limited by the need to follow a policy of interest rate smoothing. Thus, there can be clear limits to the promise of monetary policy autonomy made by the MF model. Again fiscal policy remains fully effective.

In sum, the BMW model shows that for flexible rates a much more differentiated approach is needed than under the MF model. Above all, the results of the MF model concerning fiscal policy are no longer valid if monetary policy is conducted in the form of interest rate policy instead of a monetary targeting on which the MF model is based. In the BMW model fiscal policy remains a powerful policy tool in all three version of floating.
Table 1: Summary of the results in an open economy

<table>
<thead>
<tr>
<th></th>
<th>Monetary policy</th>
<th>Fiscal policy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MF model</strong></td>
<td>Ineffective</td>
<td>More effective than in a closed economy</td>
</tr>
<tr>
<td><strong>BMW model</strong></td>
<td>b&gt;c: Destabilising</td>
<td>b&gt;c: More effective than in a closed economy</td>
</tr>
<tr>
<td></td>
<td>b&lt;c: Stabilising</td>
<td>b&lt;c: Less effective than in a closed economy</td>
</tr>
<tr>
<td><strong>MF model</strong></td>
<td>More effective than in a closed economy</td>
<td>Ineffective</td>
</tr>
<tr>
<td><strong>BMW model</strong></td>
<td>Real interest rate: Ineffective</td>
<td>Effective as in closed economy</td>
</tr>
<tr>
<td><strong>(PPP and UIP)</strong></td>
<td>Nominal interest rate: effective</td>
<td></td>
</tr>
<tr>
<td><strong>BMW model</strong></td>
<td>Effective as in closed economy, but with b&gt;c</td>
<td>Effective as in closed economy</td>
</tr>
<tr>
<td><strong>(UIP only)</strong></td>
<td>real interest rate changes are less effective</td>
<td></td>
</tr>
<tr>
<td><strong>BMW model</strong></td>
<td>Effective as in closed economy, but with b&gt;c</td>
<td>Effective as in closed economy</td>
</tr>
<tr>
<td><strong>(random walk)</strong></td>
<td>real interest rate changes are less effective.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Limits by the need of interest rate smoothing</td>
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</tr>
</tbody>
</table>

5 Summary

In sum, the BMW model provides obvious advantages over the IS/LM-AS/AD model. As far as the closed-economy set-up is concerned, the BMW model is in most basic version more simple and at the same time more powerful than the IS/LM-AS/AD model. In its more complex versions it can analyse important concepts such as loss functions and monetary policy rules without getting more difficult than the IS/LM-AS/AD model. Thus, the basic version of the BMW model helps us to understand the underlying principles of monetary policy in a very simple way. Additionally it serves as an easy to use vehicle to compare optimal and simple monetary policy
rules. With respect to the open economy version of the BMW model the degree of complexity is more or less similar to that of the MF model. As the BMW model assumes full capital mobility, it can avoid a discussion of the balance of payments adjustment process that requires an intensive discussion in the MF model. The BMW model is somewhat more complicated as far as the determination of the flexible exchange rate is concerned. However, this makes it much more powerful than the MF model.
REFERENCES
Notes

1 McCallum [1989] presented a model which tries to deal with these two approaches under a positive inflation rate. However, it is much too complicated for introductory purposes.

2 See for instance Blanchard [2000], Abel and Bernanke [2001].

3 See Orphanides, Athanasios and Volker Wieland [1999].

4 As is usually done in the literature, $\Delta q > 0$ is a real depreciation of the domestic currency.

5 According to (39) the overall inflation rate would be calculated as a weighted (by the factor e) average of domestic inflation $\pi^d$ (determined by (38)) and imported inflation $\pi^f$ (determined by (37)).

6 However, most models, as the one presented here, fail to integrate the variance of interest rates and its consequences into a macroeconomic context.

7 In fact, the described shift of the $y^d(\pi)$-curve is only true in the case of $\varepsilon_1$-shocks which affect the $y^d(\pi)$-curve and the $y^d(r)$-curve by exactly the same extent (see equations (58) and (59)). If, however, the economy is hit by a $\alpha$-shock or a $r^*$-shock, the $y^d(\pi)$-curve shifts by a larger amount than the $y^d(r)$-curve as $b > c$. 