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**Würzburg Economic Papers**

Nr. 99-7

**CAPACITY CONSTRAINED PRICE COMPETITION AND  
ENTRY DETERRENCE IN HETEROGENEOUS PRODUCT  
MARKETS**

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This version 03.09.99

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## Abstract

In this paper three issues are pursued. First, a model of capacity constrained price competition is suggested. The basic feature of this model is that a pure strategy equilibrium exists for all price subgames. Second, this permits Cournot outcomes in heterogeneous markets to be interpreted as the unique subgame perfect equilibrium of a two stage game where firms simultaneously set capacities first and then prices. Third, the capacity constrained price competition game can be used to extend the entry deterrence models of the Dixit-Stackelberg type in order to analyze the effect of heterogeneity and development of demand. The results support the view that entry deterrence should be a rare event for growing dynamic markets with ample opportunities of product differentiation.

# CAPACITY CONSTRAINED PRICE COMPETITION AND ENTRY DETERRENCE IN HETEROGENEOUS PRODUCT MARKETS

The contribution of Kreps and Scheinkman (1983) has given a significantly superior foundation to the Cournot model. It can now be understood as a reduced form of a capacity constrained price game with no need of some auctioneer. Unfortunately the price stage of this game has only mixed strategy equilibria for a certain range of capacities. This is quite annoying, if one wants to employ the model structure not only to simultaneous choices of capacity but also in a sequential context of some sort as for example in contexts of entry deterrence. In this paper we abandon the assumption of a homogeneous product market as analyzed in Kreps and Scheinkman. This allows us to have pure strategy equilibria in the price stage. In a setting with simultaneous capacity choices the unique equilibrium is again identical to the Cournot outcome in a heterogeneous product market. Moreover the model can be used to reconsider the possibility of entry deterrence in such a market. The respective results support the view that a decreasing degree of substitutability among the commodities under consideration hampers the possibility of profitable entry deterrence. They are thus in line with the perspective that entry deterrence as modelled by e.g. Dixit (1980) is expected to be a rare event.

It should be noted that Yin, X. and Yew-Kwang Ng (1997) have analyzed a similar but not identical model suggesting that the Kreps/Scheinkman result can be extended to heterogeneous markets. These authors assume that consumers are fully aware of the capacities and take them into account when formulating demand. In this form this implies a strong informational assumption on the part of customers which appears unreasonable in many circumstances. Alternatively one could view their approach as one where customers first formulate their unrestricted demand and then visit firms sequentially. If the first firm visited is capacity constrained given their demand they are rationed and will adjust the demand at the second firm. If this is the underlying story it is assumed implicitly that going from one firm to the other is costless. One may object that such costs can be taken into account when formulating unrestricted demand. This is certainly true if customers are not rationed by capacities. If they are rationed a customer may visit a firm first where capacities do not bind. When she finds that she is rationed at the second firm she has to go back to the first firm because her demand for the first firm's commodity will have changed

because of rationing at the second firm. Therefore there will typically be additional transactions costs if customers are not aware of capacity constraints. Either one must assume that such costs are negligible or some expectation of rationing must be formed. To be sure the assumption of zero transactions costs of this kind has a long tradition in economics for good reasons. Therefore the preceding comments are not intended to criticize these authors in this respect. They are rather intended to prepare the ground for the view that is taken in this paper.

In the present paper the view is taken that it is reasonable to assume that customers do not have full knowledge of the firms capacities. Indeed it is assumed that customers expect that their demands are met when formulating their demand. This expectation will never be falsified in equilibrium. In contrast to the Yin/Ng paper the demand structure can be understood as resulting from a discrete choice situation. Customers have an outside option yielding some reservation utility level. They know the prices and products on offer because of advertising or other easily available information. They buy one and only one unit from one of the firms or stick to their outside option. The increase in utility when buying a unit from one of the firms rather than using their outside option covers only the cost to visit one firm. Suppose for example there is an offer of some commodity which is locally available (the outside option) but there are also the offers from the two firms at least one of which is a better deal than the outside option. The two firms are located far apart such that because of e.g. time constraints only the firm with the better deal is worthwhile visiting. Then the demand system used in the present paper can be seen as the aggregate demand facing the two firms. For example utility functions of the type  $xq$  and some uniform distribution of income can generate such a system. Here  $x$  denotes the quantity of numeraire consumption and  $q$  denotes the perceived quality of the commodity under consideration. Such formulation of utility functions are common in models of vertical product differentiation (e.g. Shaked and Sutton (1982)). The proof that such a discrete choice model can generate the demand system presented in the next section is available on request from the author. It is not included here because it is quite obvious.

To relate this view to the one in Yin/Ng the two distinguishing assumptions in this paper are: Customers do not know the capacities of the firms and they incur substantial costs when visiting one firm, while visiting two is prohibitive. Such a setup seems as reasonable as assuming that there are no transactions costs at all. It implies that demand decisions are not revised if rationing occurs. A rationed customer at one firm does not transfer his demand to the competitor but rather sticks to his outside option. This excludes the

possibility that a firm can set a high price in order to profit from the competitor's capacity constraint.

In the framework used here customers never face rationing, as the firms always set prices which render demand compatible with their capacities. Indeed firms never have an incentive to set a price which is so low as to generate more demand than available capacity permits. This would only decrease their revenue per unit without being able to recoup that loss through increased sales which are after all restricted by available capacities. Thus a belief on the side of consumers of never facing a rationing constraint and the belief on the side of firms that consumers are not aware of potential capacity constraints is never falsified in this game. Both expectations are mutually consistent and have a flavor of self fulfilling prophecies. It therefore is compatible with rational behavior. The assumption of heterogeneous products together with the lacking incentive to generate demand exceeding capacity has obviously the additional advantage of being able to avoid any more or less ad hoc specification of some rationing scheme as criticized e.g. by Davidson and Deneckere (1986).

If contrasted with the paper by Yin/Yew-Kwang Ng this assumption buys also a lot of tractability of the model. It should be mentioned at this point that there is a gap in the proof contained in that paper. The resulting best response functions for some price subgames are not continuous any more and more importantly price equilibria in pure strategies fail to exist for some of these price subgames. This is elaborated in Schulz (1999). In our view both the compatibility with rational behavior and the increased tractability speak for the proposed modeling approach. It should be noted, however, that the approach of Yin/Ng is closer in spirit to the Kreps/Scheinkman paper. The model used in the present paper is not intended to be a straight generalization of the Kreps/Scheinkman results to differentiated products.

The paper is organized as follows. The next section presents the model and the analysis of the price stage subgames. The following section considers simultaneous choices of capacities and presents the Cournot outcomes. Then we reconsider the possibilities of first mover advantages of an incumbent including the possibility of entry deterrence in a fashion similar to Dixit's classical analysis.

## The model and the price stage

Consumer demand is modelled in a linear symmetric way. The system of inverse demand is given by ( $0 \leq \theta < 1$ ):

$$p_1 = a - x_1 - \theta x_2$$

$$p_2 = a - x_2 - \theta x_1$$

Equivalently the demand system is given by

$$x_1 = \frac{a}{1+\theta} - \frac{p_1}{1-\theta^2} + \frac{\theta p_2}{1-\theta^2}$$

$$x_2 = \frac{a}{1+\theta} - \frac{p_2}{1-\theta^2} + \frac{\theta p_1}{1-\theta^2}.$$

In this stage the capacities of the two firms are given by  $K_1, K_2$ . Variable production costs are assumed to be linear. As is well known we do not lose generality by setting these marginal costs equal to zero. Firms are supposed to maximize profit under their capacity constraint:

$$\max p_i \left( \frac{a}{1+\theta} - \frac{p_i}{1-\theta^2} + \frac{\theta p_j}{1-\theta^2} \right) \text{ s.t. } \left( \frac{a}{1+\theta} - \frac{p_i}{1-\theta^2} + \frac{\theta p_j}{1-\theta^2} \right) \leq K_i$$

This modelling approach implies that customers are not aware of potential capacity constraints and that firms do not consider the possibility that a high price may render their competitor's capacity constraint binding which could then change the demand for their own commodities. I will comment on this assumption at the end of this section.

Consider first capacities that are not binding for both firms. In this case the best response functions are

$$p_i^B(p_j) = \frac{(1-\theta)a + \theta p_j}{2}$$

and the equilibrium prices are the usual (Bertrand) prices for heterogeneous markets:

$$p_i^B = \frac{1-\theta}{2-\theta} a$$

Inserting these values into the demand functions yields

$$x_i(p_i^B, p_j^B) = \frac{a}{(1+\theta)(2-\theta)}.$$

As the profit functions are concave in  $p_i$  the Bertrand prices are equilibrium prices for

$$\min(K_1, K_2) \geq \frac{a}{(1+\theta)(2-\theta)}.$$

Let us consider now the general case. The capacity constraint can be rewritten as

$$p_i \geq (1-\hat{\epsilon})a - (1-\hat{\epsilon}^2)K_i + \hat{\epsilon}p_j =: p_i^K(p_j, K_i).$$

It is straightforward to verify that the best response functions - taking the capacity constraint into account - is

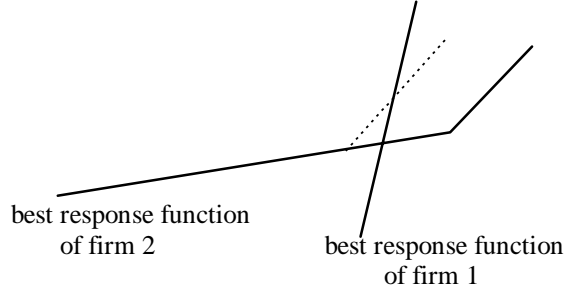
$$p_i(p_j, K_i) = \max(p_i^B(p_j), p_i^K(p_j, K_i)).$$

Apparently these best response functions are continuous. From the representation of  $p_i^B$  and  $p_i^K$  it is obvious that  $a > p_i(p_j, K_i) \geq (1-\theta)a/2$ . Hence Brouwer's fixed point theorem guarantees the existence of a pure strategy equilibrium in these subgames. By inspection of  $p_i^B$  and  $p_i^K$  the slope of the best response function of firm 2 is strictly less than the slope for the best response function of firm 1 for any  $\theta < 1$ . This implies that this equilibrium is unique.

The rest of this section provides a closed form solution for the unique price equilibria of the subgames characterized by different capacity choices. To this purpose consider first the case  $K_1 = K_2 = K$ . For  $K \geq a / ((1+\theta)(2-\theta))$ , we have the Bertrand equilibrium  $p_i^B$  set out above. For the  $K < a / ((1+\theta)(2-\theta))$ , we have equilibrium prices  $p = a - K - \theta K$ , as is easily verified.

Let us turn now to the case  $K_1 < K_2$ . If  $K_1 \geq a / ((1+\theta)(2-\theta))$  both capacities are large enough to have an intersection point in the  $p_i^B$  - part of both best response functions. Hence equilibrium coincides again with the Bertrand solution  $p_i^B$ . The interesting case is thus  $K_1 < a / ((1+\theta)(2-\theta))$ . This implies that the intersection point of both best response functions is on capacity constrained part of firm 1's function. It remains to be clarified, whether the intersection point is on the capacity constrained part of firm 2 or on its unconstrained part. For this purpose it is easiest to use a geometric argument. In the following figure both response functions are exhibited. If  $K_2$  is large enough the solid line represents a best response function of firm 2 such that the intersection point is on its unconstrained part. For

smaller choices of  $K_2$  the dashed line is part of the best response function of firm 2. In the exhibited case, the intersection point is on the capacity constrained part. This yields a simple characterization of the cases with an intersection point on the constrained part: If the intersection point of  $p_1^K$  with  $p_2^K$  is above the intersection point of  $p_1^K$  with  $p_2^B$ , then the intersection point of the best response functions is on the constrained part of  $p_2$ . Otherwise it is on the unconstrained part.



It is straightforward to calculate the two intersection points of interest. The intersection of  $p_1^K$  with  $p_2^K$  is

$$p_1(K_1, K_2) = a - K_1 - \theta K_2$$

$$p_2(K_1, K_2) = a - K_2 - \theta K_1.$$

The intersection point of  $p_1^K$  with  $p_2^B$  is

$$p_1(K_1, K_2) = \frac{(2 + \theta)(1 - \theta)a - 2(1 - \theta^2)K_1}{2 - \theta^2}$$

$$p_2(K_1, K_2) = \frac{(1 - \theta^2)a - \theta(1 - \theta^2)K_1}{2 - \theta^2}.$$

Hence the first intersection point is relevant iff

$$a - K_2 - \theta K_1 \geq \frac{(1 - \theta^2)a - \theta(1 - \theta^2)K_1}{2 - \theta^2},$$

which is equivalent to

$$a \geq (2 - \theta^2)K_2 + \theta K_1.$$

Putting these pieces together for  $K_1 \leq K_2$ , we have the following price equilibria:

If

$$(I) \ a \geq (2 - \theta^2)K_2 + \theta K_1 \text{ and } K_1 \leq \frac{a}{(1 + \theta)(2 - \theta)}$$

the equilibrium is



$$p_1(K_1, K_2) = a - K_1 - \theta K_2$$

$$p_2(K_1, K_2) = a - K_2 - \theta K_1.$$

If

$$(II) \ a \leq (2 - \theta^2)K_2 + \theta K_1 \text{ and } K_1 \leq \frac{a}{(1 + \theta)(2 - \theta)}$$

the equilibrium is

$$p_1(K_1, K_2) = \frac{(2 + \theta)(1 - \theta)a - 2(1 - \theta^2)K_1}{2 - \theta^2}$$

$$p_2(K_1, K_2) = \frac{(1 - \theta^2)a - \theta(1 - \theta^2)K_1}{2 - \theta^2}.$$

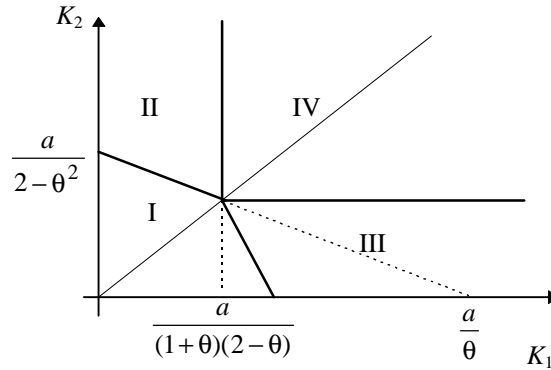
If

$$(IV) \ K_1 \geq \frac{a}{(1 + \theta)(2 - \theta)} \text{ and } K_2 \geq \frac{a}{(1 + \theta)(2 - \theta)}$$

the equilibrium is

$$p_i^B = \frac{1 - \theta}{2 - \theta} a, \ i = 1, 2.$$

The case  $K_1 \geq K_2$  is analogous. The type of equilibrium can be summarized conveniently in the following figure:



For further reference we note here the resulting profits. In case I, we have the following profits:

$$\Pi_1^I(K_1, K_2) = (a - K_1 - \theta K_2)K_1$$

$$\Pi_2^I(K_1, K_2) = (a - K_2 - \theta K_1)K_2.$$

In case II, we have the following profits:

$$\Pi_1^{\text{II}}(K_1, K_2) = \frac{(2 + \theta)(1 - \theta)a - 2(1 - \theta^2)K_1}{2 - \theta^2} K_1$$

$$\Pi_2^{\text{II}}(K_1, K_2) = \frac{(1 - \theta^2)a - \theta(1 - \theta^2)K_1}{2 - \theta^2} \frac{a - \theta K_1}{2 - \theta^2}$$

In case III, which is case II with the roles of firms 1 and 2 exchanged, we have analogous profits and for case IV, we have the following profits:

$$\Pi_i^{\text{IV}}(K_1, K_2) = \frac{1 - \theta}{(1 + \theta)(2 - \theta)^2} a^2 \text{ for } i = 1, 2.$$

This concludes the determination of the equilibria of all subgames.

The following sections are devoted to the capacity choices. First the simultaneous choice is considered and the Cournot outcome generated. Then a form of sequential choices of capacities is analysed.

### **Cournot outcomes**

Considering the choice of capacity we assume that capacity costs are linear with marginal costs equal to  $r$ . The profit functions relevant for this context are

$$\Pi_i^k(K_1, K_2) - rK_i, \quad i = 1, 2, \quad k = \text{I, II, III, IV},$$

where the  $\Pi_i$  are those derived in the preceding section. Let us first consider the best response functions of firm 1. If we analyze the profit function in each of cases I to IV separately, it becomes apparent by inspection of  $\Pi_i$  that the global maxima within areas III and IV are at the left boundary of these areas. In case I, it is straightforward to verify that within this area the best response function is

$$K_1(K_2) = \max\left(0, \min\left(\frac{a - r - \theta K_2}{2}, \frac{a - (2 - \theta^2)K_2}{\theta}\right)\right),$$

where the second expression in the min-term reflects the upper boundary line of case I. This implies that no pair  $(K_1, K_2)$  in the area of case III is part of the best response function. For further reference it should be noted that both expressions in the parenthesis above coincide at

$$K_1 = \frac{(2+\theta)(1-\theta)a - (2-\theta^2)r}{4(1-\theta^2) + \theta^2},$$

which is always strictly less than  $a/((1+\theta)(2-\theta))$ . It is obviously positive for

$$\frac{a}{r} > \frac{2-\theta^2}{(2+\theta)(1-\theta)}.$$

For  $\theta$  close to 1 this inequality cannot hold. Hence in this case the best response function in the area of I is

$$K_1(K_2) = \max\left(0, \frac{a-r-\theta K_2}{2}\right).$$

Finally we have to study the profit maxima in the area of case II. Maximizing the relevant profit function gives

$$K_1(K_2) = \max\left(0, \frac{(2+\theta)(1-\theta)a - (2-\theta^2)r}{4(1-\theta^2)}\right).$$

This expression is always strictly less than  $a/((1+\theta)(2-\theta))$ . Hence no pair  $(K_1, K_2)$  in the area of case IV is part of the best response function.

Putting this information together we have the best response function

$$K_1(K_2) = \max\left(0, \frac{a-r-\theta K_2}{2}\right),$$

if

$$\frac{a}{r} \leq \frac{2-\theta^2}{(2+\theta)(1-\theta)}.$$

If this inequality does not hold, the best response function jumps approximately at the boundary line between the areas relevant for cases I and II. More precisely, there exists a  $\hat{K}_2$  in the nondegenerate interval

$$\left(\frac{(1-\theta)a + \theta r}{2(1-\theta^2)}, \frac{(2-\theta)a + \theta r}{4(1-\theta^2) + \theta^2}\right),$$

such that

$$K_1(K_2) = \frac{a-r-\theta K_2}{2} \text{ for } K_2 \leq \hat{K}_2$$

$$K_1(K_2) = \max\left(0, \frac{(2+\theta)(1-\theta)a - (2-\theta^2)r}{4(1-\theta^2)}\right) \text{ for } K_2 \geq \hat{K}_2.$$

This jump indicates that a proof of existence might be difficult, if more general demand systems or cost structures are analyzed. In particular it is not sufficient to note that the areas where some capacities are not fully employed (here cases II, III, and IV) are not relevant for an equilibrium outcome and then to proceed by studying exclusively those parts of best response functions which are relevant for fully employed capacities. This is of course sufficient to characterize an equilibrium outcome but it provides no proof of existence. In the present linear context the jump does not pose any problems. From the arguments above it can easily be verified that the best response functions have a unique intersection point at the usual Cournot outcomes which in turn are such that the potential jump of the best response functions do not matter. Summarizing:

*Proposition 1:* In a model with linear demand and costs and a two stage game, where two identical firms simultaneously set capacities first and then prices as set out in the first section, the unique subgame perfect equilibrium generates the Cournot outcome for heterogeneous products:

$$K_i = \frac{a-r}{2+\theta}, i = 1, 2.$$

### **Sequential capacity choice and entry deterrence**

The fact that all price subgames have pure strategy equilibria makes it possible to easily extend the usual analysis of entry deterrence to heterogeneous markets. As indicated in the introduction our focus will be on the facilitating or hampering factors for entry deterrence due to increased heterogeneity in the product market. A natural starting point is therefore the case of homogeneous products. We will thus concentrate on values of  $\theta$  close to 1. This implies that we do not have to bother about discontinuous best response functions.

We consider a three stage game. In the first stage an incumbent can choose a capacity which is assumed to be associated with fully sunk costs and free of deterioration. In addition she can choose a price. In the second stage another firm may enter the market. If this

happens both firms simultaneously choose their capacities. The incumbent cannot decrease her capacity below the level chosen in the first stage. The third stage consists in the capacity constrained price game of both firms. We consider the subgame perfect equilibrium of this game.

For the third stage nothing remains to be analyzed as for all capacity choices the first section contains all price equilibria. As for the second stage the resulting best response function of the entering firm are those analyzed in the preceding section. For the analysis of the best response function of the incumbent firm in the second stage denote the capacity chosen by the incumbent in the first stage by  $K_1^I$ . Obviously the incumbent's best response function in the second stage is

$$K_2^I(K_1^I, K^E) = \max\left(K_1^I, \frac{a-r-\theta K^E}{2}\right).$$

From this the unique equilibrium of the second stage follows immediately:

For  $K_1^I < (a-r)/(2+\theta)$  the equilibrium is

$$K_2^I = \frac{a-r}{2+\theta} = K^E > K_1^I \text{ and the incumbent's profit in this stage is}$$

$$\Pi_2^I(K_1^I) = \left(\frac{a-r}{2+\theta}\right)^2 + rK_1^I.$$

For  $(a-r)/(2+\theta) \leq K_1^I < (a-r)/\theta$  the equilibrium is

$$K_2^I = K_1^I, K^E = \frac{a-r-\theta K_1^I}{2} \text{ and the incumbent's profit in this stage is}$$

$$\Pi_2^I(K_1^I) = \frac{1}{2}\left((2-\theta)a + \theta r - (2-\theta^2)K_1^I\right)K_1^I.$$

For  $(a-r)/\theta \leq K_1^I$  the equilibrium is

$$K_2^I = K_1^I, K^E = 0 \text{ and the incumbent's profit in this stage is}$$

$$\Pi_2^I(K_1^I) = (a - K_1^I)K_1^I.$$

This sets the stage for the analysis of the first period. In this stage we assume, that demand may be smaller than in the following stages. We model this by considering a profit function of the following type:

$$\Pi_1^I(K_1^I, p) = \tau p(a - p) + \Pi_2^I(K_1^I) - rK_1^I$$

Here  $p$  denotes the price of the incumbent charged in the first stage.  $\tau$  is a number between 0 and 1. If  $\tau$  is smaller than 1 this models a growing market. If  $\tau$  is equal to 1, we have a stagnating market and as will become clear later on this case gives also the relevant information for declining markets. In the following we will concentrate on the extreme cases  $\tau = 0$  and  $\tau = 1$ . The equilibrium of the first stage is determined by maximizing this profit function under the constraint  $\tau(a - p) \leq K_1^I$ .

To determine the global maximum of this function recall that  $\Pi_1^I$  has no single analytical representation. Rather - as exhibited above - there are three cases to consider : (a)  $K_1^I < (a - r)/(2+\theta)$ , (b)  $(a - r)/(2+\theta) \leq K_1^I < (a - r)/\theta$ , and (c)  $(a - r)/\theta \leq K_1^I$ .

In case (a)  $K_1^I = (a - r)/(2+\theta)$  is always a global maximum within the reach of this case. If demand at the sales maximizing price  $a/2$  exceeds the largest capacity of this case  $(a - r)/(2+\theta)$ , the profit function has a unique global maximum at this largest capacity. This is obviously the case with  $\tau = 1$ . Otherwise the profit function will be constant between the demand at  $a/2$  and the upper boundary of this case. Hence for  $\tau = 0$ , the profit function is constant in capacities.

In case (b) note first that the optimal capacity is always larger than  $(a - r)/(2+\theta)$ . Therefore case (a) can be deleted from further consideration. It is a tedious exercise to check that the optimal capacity is nondecreasing in  $\tau$ . This is also very plausible: If  $\tau$  increases, cost can be spread over an increased volume of sales, giving an incentive to increase capacity. For  $\tau = 0$ , the optimal capacity (within this case) turns out be

$$K_1^I = \frac{2 - \theta}{2(2 - \theta^2)}(a - r)$$

which is larger than  $(a - r)/(2+\theta)$ . It is also smaller than  $(a - r)/\theta$ . Hence for all parameter constellations the profit maximizing capacity is at least as large as this capacity.

For  $\tau = 1$  the optimal capacity (again within this case) is

$$K_1^I = \frac{(4-\theta)a - (2-\theta)r}{2(4-\theta^2)}.$$

This expression is smaller than the upper boundary of this case  $(a-r)/\theta$  iff

$$(8-4\theta-\theta^2)(a-r) - 2\theta r > 0.$$

The left hand side decreases in  $\theta$ . For  $\theta = 1$  this condition reads  $3a > 5r$ . Hence for sufficiently profitable markets there is always an interior optimal capacity for case (b). Otherwise ( $3a < 5r$ ) the optimal capacity will be at  $(a-r)/\theta$  (where  $K^E$  becomes zero) for sufficiently large  $\theta$ .

Finally, we have to consider case (c). Not surprisingly, the optimal capacity is nondecreasing in  $\tau$ . All questions of interest can therefore be restricted to the two extreme values of  $\tau$ . For  $\tau = 0$  the lower boundary of the case is the unique optimal capacity of case (c). For  $\tau = 1$ , the first order conditions for profit maximization (ignoring the bounds of the case) yield

$$K_1^I = \frac{2a-r}{4}.$$

This is larger than  $(a-r)/\theta$  iff

$$(8-4\theta)(a-r) - 2\theta r < 0.$$

From this definition we can deduce the following results:

*Proposition 2:* For  $\tau = 0$ , the subgame perfect capacity is  $K_1^I = \frac{2-\theta}{2(2-\theta^2)}(a-r)$ .

This conforms with the standard result for homogeneous markets and the Dixit-Stackelberg solution. As in the standard results, entry deterrence is not profitable without some fixed costs on the part of the entrant. We return to this issue in a moment.

*Proposition 3:* For  $\tau = 1$ , the subgame perfect capacity is

$$(i) \quad K_1^I = \frac{(4-\theta)a - (2-\theta)r}{2(4-\theta^2)} \quad \text{for} \quad \frac{a-r}{r} > \frac{2\theta}{8-4\theta-\theta^2}$$

$$(ii) \quad K_1^I = \frac{a-r}{\theta} \quad \text{for} \quad \frac{2\theta}{8-4\theta} \leq \frac{a-r}{r} \leq \frac{2\theta}{8-4\theta-\theta^2}$$

$$(iii) \quad K_1^I = \frac{2a-r}{4} \quad \text{for } \frac{a-r}{r} < \frac{2\theta}{8-4\theta}.$$

Note that case (i) obtains for  $\theta = 1$  and  $a > 5r/3$ . This case reflects accommodated entry. As argued above, if this case is relevant for some  $\underline{\theta}$  then it is relevant for all smaller  $\theta$ . In other words: if entry is accommodated for some degree of substitutability then it is accommodated as well for any smaller degree of substitutability.

Case (iii) is only feasible, if  $a < 3r/2$ . This case reflects blockaded entry. The respective capacity is the one which would be chosen, if the incumbent were not threatened by entry. In line with conventional wisdom a lower degree of substitutability makes blockaded entry less likely.

Case (ii) reflects deterred entry. An unchallenged incumbent would install a smaller capacity. But this would invite entry. Here again, it is clear from studying the boundaries of the case that entry deterrence is less likely for lower values of substitutability. The picture is thus very clear. The lower the degree of substitutability the more costly is entry deterrence. This has two parts: a lower degree of substitutability enhances the profitability of the market, if entry is permitted. And at the same time it takes a larger capacity to deter entry. This reduces the incentive for entry deterrence.

Note again that for the case  $\tau = 1$  we did not assume any fixed costs to make entry deterrence profitable. Comparing with the case  $\tau = 0$ , where entry deterrence is never profitable without fixed costs, leads us to conclude that entry deterrence is easier for  $\tau = 1$ . Moreover the monotonic dependence of the optimal capacity on the value of  $\tau$  suggests that entry deterrence is easier with a high value of  $\tau$ . As  $\tau$  smaller than 1 represents growing markets while  $\tau$  larger than 1 represents declining markets, we have here another supporting argument for the common view that entry deterrence is quite difficult in growing markets but easier in stagnating or declining markets.

As a final point let us consider fixed costs as usually introduced in order to study entry deterrence. In homogeneous markets the most frequently model uses  $\tau = 0$ . Therefore we concentrate on this case. Are any of our findings above modified significantly by this possibility? To summarize the following arguments: the answer is no. Again lower degrees of substitutability renders entry deterrence less likely for the same reasons as above:

*Lemma 1:* For  $\tau = 0$  the profit at accommodated entry is



$$\frac{1}{8} \frac{(2-\theta)^2}{2-\theta^2} (a-r)^2$$

and this expression decreases in  $\theta$ .

In other words the more homogeneous the products the less attractive is entry accomodation.

*Lemma 2:* For  $\tau = 0$  and some fixed costs of the entrant  $F$ , the profit at the entry deterring capacity is

$$(a-r-2\sqrt{F})(2\sqrt{F}-(1-\theta)(a-r))\theta^{-2}.$$

and this expression increases in  $\theta$  if  $a-r > 4\sqrt{F}$ .

The inequality at the end of the lemma reflects the case that entry is not blockaded for homogeneous markets. Taking these two lemmata together we have the announced result: The incentive to deter entry decreases when the degree of substitutability becomes lower.

Summarizing the results of this section: The classical Dixit-Stackelberg treatment of sequential capacity choice can easily be extended to the context of heterogeneous markets in the context of a capacity constrained price game. The results support the intuition that entry is more difficult to deter in growing and heterogeneous markets. An entrant into a growing market can evade entry deterring measures if she can offer a sufficiently different product. This also supports the view that entry deterrence in the sense of this model can be expected to be a rare event.

### **Concluding remarks**

In this paper three issues have been pursued. First a model of capacity constrained price competition was suggested. The basic feature of this model is that a pure strategy equilibrium exists for all price subgames as analyzed in the first section. As the following section has revealed this permits Cournot outcomes in heterogeneous markets to be interpreted as the unique subgame perfect equilibrium of a two stage game where firms simultaneously set capacities first and then prices. Thirdly, the capacity constrained price competition game can be used to extend the entry deterrence models of the Dixit-Stackelberg type.

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