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COMMENT ON YIN, XIANGKANG AND YEW-KWANG NG:

QUANTITY PRECOMMITMENT AND BERTRAND COMPETITION YIELD
COURNOT OUTCOMES: A CASE WITH PRODUCT DIFFERENTIATION

Norbert Schulz

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University of Wuerzburg
Lehrstuhl für Volkswirtschaftslehre,
insbesondere Industrieökonomik
Sanderring 2, D-97070 Würzburg
norbert.schulz@mail.uni-wuerzburg.de

In a recent paper Yin, X. and Yew-Kwang Ng (1997) analyze a two stage game where firms simultaneously set capacity first and then prices. In contrast to the celebrated paper by Kreps and Scheinkman (1983) they consider a heterogeneous market. The authors argue correctly that in this case there is no need to bother with the type of rationing procedure in such a framework, if consumers are aware of the capacity choices. Therefore the critical assessment of e.g. Davidson and Deneckere (1986) which stresses the importance of the choice of rationing scheme are not relevant with equal force in their context. Their result would support the original contribution of Kreps and Scheinkman in a more robust situation. Unfortunately their analysis is not complete. More specifically their Lemma 1, stating the continuity of the best response functions in the price game, is not correct. This note concentrates on pointing out this defect. Some remedies are also proposed.

It turns out to be very helpful to start with a description of the demand behavior of consumers as modeled by the authors. They assume a utility function of the form

$$u(x_0, x_1, x_2) = x_0 + \alpha(x_1 + x_2) - (x_1^2 + 2\gamma x_1 x_2 + x_2^2) / 2$$

where x_0 denotes the quantity of some numeraire commodity and x_i , $i = 1, 2$ denotes the quantity of the consumed commodity supplied by firm i . Consumers are aware of the capacities of the two firms K_i . Therefore their demand behavior is derived by maximizing the utility function above under the following constraints:

$$I \geq x_0 + p_1 x_1 + p_2 x_2, \quad K_1 \geq x_1 \geq 0, \quad K_2 \geq x_2 \geq 0$$

The result of this optimization exercise can be conveniently summarized in the following figure:

figure 1 should be inserted here

In the diamond shaped area we have interior solutions:

$$x_1 = \frac{\alpha}{1+\gamma} - \frac{p_1}{1-\gamma^2} + \frac{\gamma p_2}{1-\gamma^2}$$

$$x_2 = \frac{\alpha}{1+\gamma} - \frac{p_2}{1-\gamma^2} + \frac{\gamma p_1}{1-\gamma^2}$$

Let us turn next to the best response functions of firm 1. Consider first prices

$$p_2 \leq \alpha - K_2 - \gamma K_1.$$

As long as $p_1 \leq \alpha - K_1 - \gamma K_2$ profits are increasing. Therefore $p_1 = \alpha - K_1 - \gamma K_2$ will be the global maximand within the rectangle containing the origin. Within the neighboring trapezoid the profit maximizing choice is $(\alpha - \gamma K_2)/2$, if this turns out to be larger than $\alpha - K_1 - \gamma K_2$. If not, the maximizing price is $\alpha - K_1 - \gamma K_2$. Summarizing, the best response function is given by

$$p_1(p_2) = \max(\alpha - K_1 - \gamma K_2, (\alpha - \gamma K_2) / 2)$$

for all prices p_2 satisfying the restriction above. Note that

$$p_1(p_2) = \alpha - K_1 - \gamma K_2$$

for $2K_1 + \gamma K_2 \leq \alpha$.

Next consider prices p_2 satisfying

$$\alpha - K_2 - \gamma K_1 \leq p_2 \leq \alpha - K_2.$$

For small prices p_1 - hence for pairs of prices in the left trapezoid - profits again are increasing. Within this area the maximizing prices are therefore determined by the boundary of this area:

$$p_1 = (1 - \gamma)\alpha + \gamma p_2 - (1 - \gamma^2)K_1$$

For higher prices such that the pair of prices are within the diamond shaped area the maximizing price is

$$p_1 = \frac{(1 - \gamma)\alpha + \gamma p_2}{2},$$

if the pair of prices defined by this equation is contained in the diamond shaped area. Note that this pair of prices would lie in the trapezoid to the left for all prices p_2 satisfying the restriction above, if $(2 - \gamma^2) K_1 + \gamma K_2 \leq \alpha$. Together with our result for low prices p_2 this implies that the best response function coincides with the boundary between the left trapezoid and the diamond, if $2K_1 + \gamma K_2 \leq \alpha$.

If we restrict our attention to such capacities it is easy to see that the best response function coincides for all prices p_2 with the right boundary of the area with $x_1 = K_1$. Of course, this response function is continuous indeed. The problem arises with higher capacities.

To illustrate this point, let us consider a case where $2K_2 + \gamma K_1 < \alpha$ such that the best response function of firm 2 coincides with the upper boundary of the area with $x_2 = K_2$. At the same time let $(2 - \gamma^2) K_1 + \gamma K_2 < \alpha < 2K_1 + \gamma K_2$. According to the arguments above the best response function will be $(\alpha - \gamma K_2)/2$ for $p_2 \leq \alpha - K_2 - \gamma K_1$. For $\alpha - K_2 - \gamma K_1 \leq p_2 \leq \alpha - K_2$ the situation is more complex. Recall that under this restriction the maximizing price within the left trapezoid and the diamond together is determined by the common boundary of these two areas. But for relatively small prices p_2 (prices slightly above $\alpha - K_2 - \gamma K_1$) there is another local maximum at $(\alpha - \gamma K_2)/2$, which is strictly greater than the boundary point. It may be helpful to summarize this situation in the following figure:

figure 2 should be inserted here

Under the parameter restrictions above the fat lines correspond to local maxima of firm 1's profit function for all $p_2 \leq \alpha - K_2$. Clearly for p_2 above the upper dashed line the global maximum coincides with the common boundary of the left trapezoid and the diamond, and for p_2 below the lower dashed line the global maximum is $(\alpha - \gamma K_2)/2$. By continuity of the profit function along the curves of local maxima it must be true that the best response function jumps strictly between the two dashed lines.

This proves that the best response function is not continuous in this case contradicting Lemma 1 of the authors' paper. Note that this lack of continuity will arise whenever $\alpha < 2K_1 + \gamma K_2$. Hence the problem arises under quite general circumstances.

In general, discontinuous best response functions do not necessarily harm the existence of a Nash equilibrium. However, as noted above, under the parameter restriction considered here the best response function of firm 2 is the upper boundary of the area with $x_2 = K_2$. And this means that the two best response functions do not intersect. Hence there does not exist a pure strategy equilibrium in the price stage of the game.

What remedies might be proposed? The most straightforward suggestion is, of course, to explicitly analyze equilibria in mixed strategies. This would probably leave the main result of the authors intact. I probed only very partially into this matter. But it seems that the mixed strategy equilibria will at least partly involve pure strategies where capacities are not fully employed. Hence as in the Kreps/Scheinkman paper, mixed strategy equilibria are not expected to be the outcome of the entire game when capacities are chosen at their equilibrium levels. If this remedy is employed it will be accompanied by a quite tedious analysis of a multitude of cases.

A second suggestion is to circumvent these problems by setting an upper bound on capacities like $K_i < \alpha/(2+\gamma)$. This enforces continuous best response functions and leaves the analysis of the authors intact. While such an assumption could be rationalized by prohibitive costs of capacities above such a bound, it would be very ad hoc and thus unsatisfactory.

A third alternative might be to drop the assumption that consumers are aware of the capacities chosen by the firms. This reintroduces the problem of rationing.. If however consumers face large transaction costs when visiting both firms then firms will abstain from charging high prices in order to profit from capacity constraints of the competing firms. In this case rationing is not profitable. If this approach is taken, the necessity of considering mixed strategy equilibria in prices does not exist. For an analysis along these lines see Schulz (1999): It is basically for this reason that the remedy of an analysis of mixed strategy equilibria was not pursued in any depth in this note, although it is of some interest to validate the hypothesis that the results of the authors can be amended in this way.

REFERENCES:

- Davidson, C. and R. Deneckere (1986) "Long-Run Competition in Capacity, Short-Run Competition in Prices, and the Cournot Model" *Rand Journal of Economics*, vol 17, pp. 404-415
- Kreps, D. and J. Scheinkman (1983) "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes" *Bell Journal of Economics*, vol 14, pp. 326-337
- Schulz, N. (1999) "Capacity Constrained Price Competition and Entry Deterrence in Differentiated Product Markets" *Würzburg Economic Papers*, 99-7
- Yin, Xiangkang and Yew-Kwang Ng (1997) "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes: A Case with Product Differentiation" *Australian Economic Papers*, vol 36, pp. 14-22