Zierahn, Ulrich

Working Paper
Agglomeration, congestion, and regional unemployment disparities

HWWI Research Paper, No. 108

Provided in Cooperation with:
Hamburg Institute of International Economics (HWWI)

Suggested Citation: Zierahn, Ulrich (2011) : Agglomeration, congestion, and regional unemployment disparities, HWWI Research Paper, No. 108, HWWI Institute of International Economics, Hamburg

This Version is available at:
http://hdl.handle.net/10419/48272

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Agglomeration, Congestion, and Regional Unemployment Disparities

Ulrich Zierahn
Agglomeration, Congestion, and Regional Unemployment Disparities*

Ulrich Zierahn

Hamburg Institute of International Economics (HWWI) and University of Kassel
July 12, 2011

Abstract

Regional labor markets are characterized by huge disparities between unemployment rates. Models of the New Economic Geography explain how disparities between regional goods markets endogenously arise but usually assume full employment. This paper discusses regional unemployment disparities by introducing a wage curve based on efficiency wages into the New Economic Geography. The model shows how disparities between regional goods and labor markets endogenously arise through the interplay of increasing returns to scale, transport costs, congestion costs, and migration. The level and stability of regional labor market disparities depends on the extent of labor market frictions.

JEL Classification: J64, R12, R23

Keywords: regional unemployment, New Economic Geography, core-periphery, wage curve, labor migration

*Helpful comments and suggestions from Jochen Michaelis, Michael Pflüger, and the participants of the HWWI PhD Seminar, GfR Winterseminar, MAGKS PhD Seminar, and IAB-Colloquium are gratefully acknowledged.
1 Introduction

Regional labor markets are characterized by huge disparities. This is illustrated by figure 1 for regional unemployment rates and regional wages in German planning regions. Whereas the unemployment rate was 2.6% in Ingolstadt, it amounted to up to 16.3% in Mecklenburgische Seenplatte in 2008. Similar patterns hold true for other countries as well. Empirical studies such as Blanchard and Katz (1992), Decressin and Fatás (1995), and Overman and Puga (2002) further show that such disparities between regional unemployment rates are persistent.

Disparities in regional unemployment rates are closely linked to disparities in regional wages: In regions where the unemployment rate is comparatively low, wages are comparatively high (figure 1). This is known as the wage curve. Following the seminal work of Blanchflower and Oswald (1994) an extensive literature has emerged, providing evidence for the existence of wage curves for a huge number of countries.¹

The wage curve, however, only presents an empirical phenomena and several questions remain, such as: (1) Why are some regions located at the upper tail of the wage curve with low unemployment and high wages whereas other regions are located at the lower tail with high unemployment and low wages? (2) Why do these regional disparities remain? According to neoclassical arguments people living in regions that are located at the lower tail of the wage curve are expected to migrate towards regions that are located at the upper tail of the wage curve. Due to decreasing returns to scale this should be accompanied by diminishing disparities between the regional unemployment rates and wages. (3) How does the shape of the wage curve, which is formed by labor market frictions, influence the disparities between the regional labor markets? Where do regions move when the wage curve moves due to changes in labor market frictions; do they converge or disperse?

The economic literature has already presented several models to discuss regional labor markets. According to Elhorst (2003), the regional labor market model that is probably the most encompassing is the one presented in the seminal work of Blanchard and Katz (1992). They use a wage-setting price-setting model to discuss how regional labor markets adjust to shocks in labor demand through migration, participation, and unemployment and how disparities in these variables evolve over time as a consequence of such shocks. They include a wage curve to cover labor market frictions. In their model however, migration leads to diminishing disparities in regional unemployment rates and wages. The wage curve can only persist because people in regions at the lower tail of the wage curve are compensated for the poorer labor market situation by local

amendities. Only these amenities prevent an equalization of regional disparities in unemployment and wages and hence prevent the wage curve from collapsing into a point.

Due to their focus on shocks in labor demand, Blanchard and Katz (1992) do not discuss how disparities between regional labor markets arise endogenously. Nevertheless, Blanchard and Katz (1992) highlight the importance of labor demand for regional labor market disparities. Overman and Puga (2002) present empirical evidence for European regions, showing that labor demand is indeed responsible for regional labor market disparities. When labor demand causes regional labor market disparities, then regional goods markets play a crucial role, because labor demand is deducted from the goods market. Disparities between regional goods markets in turn are discussed by the New Economic Geography following the pioneering work of Krugman (1991).
The New Economic Geography discusses how disparities between regional goods markets endogenously arise through the interplay of centrifugal and centripetal forces. However, models of the New Economic Geography usually assume perfect labor markets and hence do not usually cover unemployment. The present paper therefore introduces labor market frictions into the New Economic Geography to discuss how disparities between regional labor markets endogenously arise and how a wage curve emerges and persists within this framework. It is further used to discuss how labor market frictions influence regional labor market disparities.

To design a regional wage curve, the present model relies on efficiency wages because according to Blien (2001) the efficiency wage approach is superior for modeling a wage curve on the regional level, when compared to the alternatives. Wage bargaining models might be equally suitable when unions bargain on the local or firm level. Here, however, efficiency wages are applied instead so that the models results also hold true for countries where bargaining takes place on the sectoral level as, for example, in Germany.

There already exist different models combining labor market frictions and the New Economic Geography.\(^2\) Epifani and Gancia (2005) for example, introduce job-matching into the New Economic Geography. They show how disparities between regional economies endogenously arise and how these result in disparities between regional labor markets. They especially discuss how this results in disparities between regional unemployment rates. Francis (2009) extends their model to cover endogenous job-destruction. Vom Berge (2011) uses a similar approach but in his model unemployment might be lower or higher in the agglomeration than in the periphery. Still, Epifani and Gancia (2005) cover only frictions in job-matching and conclude that further research is necessary to discuss frictions in wage setting within the New Economic Geography. In contrast to these job-matching approaches the present paper directly introduces the – well studied – empirical phenomena of the wage curve into an New Economic Geography model by relying on wage setting frictions through efficiency wages.

Südekum (2005) in turn introduces frictions in wage setting based on efficiency wages into an agglomeration model. Nevertheless, since he focuses exclusively on centripetal forces, full agglomeration is prevented only by relying on a home bias for regional migration. He therefore does not refer to his model as a New Economic Geography model, due to the omission of centrifugal forces. Since migration is not explicitly included, his model actually explains why

\(^2\)Only models focusing on the regional level are presented here, i.e. models that cover migration. There exist models introducing labor market frictions into the New Economic Geography or the closely related New Trade Theory which do not include migration and hence focus on the international level. Examples are Chen and Zhao (2009), Helpman and Itskhoki (2010), Helpman et al. (2010), Méjean and Patureau (2010), Monford and Ottaviano (2002), Picard and Toulemonde (2006), or Strauss-Kahn (2005). Other models focus on differences in labor market institutions and hence are more suitable for the international than for the regional level. Examples are Peeters and Garretsen (2000) and Pfäger (2004).
regions of different size are characterized by different productivity levels and hence labor market disparities. He only verbally describes how migration would lead to endogenous disparities in his model.

Egger and Seidel (2008) combine a wage curve based on a fair wage approach with the New Economic Geography. In their model the work effort of the low qualified is influenced by the fairness of their wages. This leads to a link between wages and unemployment. However, only the low qualified may become unemployed. Furthermore, the low qualified are (in contrast to the highly qualified) inter-regionally immobile. In addition, the wage of the low qualified is fixed to one in both regions. Thus a wage curve exists only in the sense that the unemployment rate of the low qualified is linked to the wage of the highly qualified.

The present model instead introduces a wage curve based on efficiency wages into the New Economic Geography in order to discuss frictions in wage setting and consequences for regional labor market disparities. In contrast to Egger and Seidel (2008) the present model shows a link between wages and the unemployment rate of the same labor market group, and the unemployed migrate between the regions.

In addition, the present model focuses on the influence of labor market frictions on the extent of regional labor market disparities: Changes in labor market frictions shift the wage curve, which has a stronger effect on regions at the lower tail of the wage curve and influences labor market disparities. This in turn has an effect on the centripetal forces and therefore further influences labor market disparities. As a result, increasing labor market frictions lead to an increase of regional disparities.

The rest of the paper is organized as follows: Section 2 presents the basic setup of the model. The equilibrium and its dependency on migration are discussed in Section 3. Section 4 deals with the stability of the equilibria and regional disparities. The influence of labor market frictions on regional disparities are discussed in section 5. Section 6 compares the results to existing models. Conclusions are drawn in the final chapter.

2 Basic Model

The present paper develops a New Economic Geography model with labor market rigidities based on efficiency wages (and thus a wage curve). The model is constructed to discuss how disparities between regional labor markets endogenously arise and how this is influenced by labor market rigidities. The New Economic Geography part of this model is based on that of Fujita et al. (1999). Their household model is extended to the disutility of work effort, which is essential to modeling efficiency wages. Efficiency wages are based on the approach of Shapiro and Stiglitz.
(1984). The goods market in turn is based on Fujita et al. (1999). However, the assumption of full employment is dropped, and unemployment arises as a consequence of efficiency wages.

There exist two regions, \( r \) and \( s \), as well as two sectors, the agricultural sector \( A \) and manufacturing \( M \). Agriculture is characterized by perfect competition on both the goods and the labor market. Manufacturing in contrast is characterized by monopolistic competition on the goods market and efficiency wages in the labor market. Labor is inter-sectoral immobile, but labor in the manufacturing sector is interregionally mobile. Labor in agriculture is considered to be interregionally immobile.

\[ 2.1 \text{ Households} \]

Households live in regions \( r \) and \( s \). They receive utility \( \mathcal{U} \) through the consumption of agricultural goods \( C_A \) and through the consumption of manufacturing goods. \( C_M \) represents a composite index of manufacturing goods. Utility \( \mathcal{U} \) is lowered by work effort \( e \). Furthermore, congestion costs \( H \) influence utility.

\[
\mathcal{U} = H \left( C_M^{\mu} C_A^{1-\mu} - e \right)
\]

The congestion costs are modeled similar to Ricci (1999). That is, given the share of the manufacturing labor force \( \lambda \) in region \( r \), the congestion cost \( H \) for regions \( r \) and \( s \) are:

\[
H_r = h^{1 - \frac{\lambda}{1 - \lambda}},
\]

\[
H_s = h^{1 - \frac{1 - \lambda}{\lambda}}.
\]

Here, \( h \) is a parameter of congestion costs. For \( h > 1 \) increasing agglomeration in one region decreases utility of households in the agglomeration, but increases utility of households in the other region, since agglomeration occurs only through the division \( \lambda \) of the fixed manufacturing labor force between the regions. Congestion costs are introduced in an analogy to the iceberg transportation costs: Both are constructed in a rather simple fashion in order to keep track of how the agglomeration pattern changes when transport costs and congestion costs change. The congestion costs can be interpreted as a local fixed supply of goods, such as land or housing with positive and decreasing marginal utility.\(^3\)

The composite index of manufacturing goods is a CES utility function:

\(^3\)By replacing equations 2 and 3 through housing supply, a housing-market can be explicitly modeled within the same framework, see e.g. Helpman (1998) and Pflüger and Südekum (2004).
The number of firms is given by $m$ and the elasticity of substitution between the varieties of the manufacturing goods is $\theta > 1$. Households maximize their utility in two stages. They decide upon the optimum division of their income on agricultural and manufacturing goods. In addition they decide on the optimum composition of the varieties of the manufacturing good. The budget constraint of household $j$ is:

$$GC_M^j + P_A C_A^j = I_j.$$ (5)

Household $j$ uses all of its income $I_j$ for the consumption of agricultural goods $C_A$ at price $P_A = 1$ and for the consumption of the composite index of manufacturing goods $C_M$ at price index $G$. Due to the standardization $P_A = 1$, the prices of the manufacturing goods (and all wages) are measured relative to agricultural prices. Inserting the budget constraint into the utility function produces:

$$U_j = H \left( C_M^j (I_j - GC_M^j)^{1-\mu} - e \right).$$ (6)

Utility maximization ($\partial U_j / \partial C_M^j = 0$) leads to the consumption expenditure shares of agricultural and manufacturing goods in income. Note that the result of utility maximization is independent of congestion costs and work effort:

$$C_M^j = \mu \frac{I_j}{G},$$ (7)

$$C_A^j = (1 - \mu) I_j.$$ (8)

The optimum division of expenditures for manufacturing goods among individual varieties results from utility maximization over the varieties. This is equal to minimizing the expenditures for the varieties (Shepards Lemma):

$$\min \int_0^m P_i C_i d_i \text{ s.t. } \left[ \int_0^m C_i^{\frac{\theta - 1}{\theta}} d_i \right]^{\frac{\theta}{\theta - 1}} = C_M.$$ (9)

This leads to the CES manufacturing price index $G$:

$$G = \left[ \int_0^m P_i^{1-\theta} d_i \right]^{\frac{1}{1-\theta}}.$$ (10)
In the two-region case with identical firms and iceberg transport costs $\tau \geq 1$, this leads to the manufacturing goods price index $G_r$ in region $r$:

$$G_r = \left[ m_r P_r^{1-\theta} + m_s (\tau P_s)^{1-\theta} \right]^{1/\tau - \theta},$$

(11)

whereas $m_r$ and $m_s$ represent the number of firms (=varieties) in the corresponding region. The demand for variety $i$ by household $j$ follows from minimizing expenditures for the varieties:

$$C_{ij} = \left( \frac{P_i}{G} \right)^{-\theta} C_{Mj} = \left( \frac{P_i}{G} \right)^{-\theta} \mu I_j P.$$

(12)

### 2.2 Labor Market

The labor market is modeled within the efficiency wage framework of Shapiro and Stiglitz (1984), similar to Zenou and Smith (1995). Zenou and Smith (1995) construct a two city model with intra- and inter-city migration including efficiency wages based on Shapiro and Stiglitz (1984). For the purpose of the present model, the inter-city migration is adopted to the two region case. Only the derivation of the wage curve for region $r$ is presented. The wage curve for region $s$ follows analogously. Employees (which are equal to households) receive utility $v(w_r)$ through the wage $w_r$ in region $r$, which is equal to the household income $I_j$. The income of unemployed households is zero. Households suffer from disutility of work effort $e$ and their utility is lowered by congestion cost factor $H_r$, analogous to the utility function (1):

$$v(w_r) = H_r C_M \mu C_A^{1-\mu}. $$

(13)

Thus, the term $v(w_r)$ is only a means to abbreviate the derivation of the wage curve. Due to the disutility of work effort, employees have an incentive to shirk and hence to avoid work effort. The expected life-time utilities of employees who do not shirk, those who do, and the unemployed, are (Shapiro and Stiglitz; 1984):

$$\rho V_r^{ns} = v(w_r) - H_r e - \psi (V_r^{ns} - V_r^u), $$

(14)

$$\rho V_r^s = v(w_r) - (\psi + 1 - \gamma) (V_r^s - V_r^u), $$

(15)

$$\rho V_r^u = \delta_r (V_r^{ns} - V_r^u). $$

(16)

Whereas $\rho$ is the discount rate of utility, $\delta_r$ is the endogenous job generation rate, $\psi$ is the exogenous job destruction rate and $1 - \gamma$ is the probability of detecting shirking. The labor
<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_A$</td>
</tr>
<tr>
<td>$C_M$</td>
</tr>
<tr>
<td>$C_i$</td>
</tr>
<tr>
<td>$e$</td>
</tr>
<tr>
<td>$G$</td>
</tr>
<tr>
<td>$H$</td>
</tr>
<tr>
<td>$h$</td>
</tr>
<tr>
<td>$I$</td>
</tr>
<tr>
<td>$L_{A,M}$</td>
</tr>
<tr>
<td>$M$</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$N_{A,M}$</td>
</tr>
<tr>
<td>$q_i$</td>
</tr>
<tr>
<td>$()_{r,s}$</td>
</tr>
<tr>
<td>$s$</td>
</tr>
<tr>
<td>$\Omega_j$</td>
</tr>
<tr>
<td>$u_{r,s}$</td>
</tr>
<tr>
<td>$v(w)$</td>
</tr>
<tr>
<td>$V^{ns}, V^s, V^u$</td>
</tr>
<tr>
<td>$w$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>$1 - \gamma$</td>
</tr>
<tr>
<td>$\delta$</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$\pi$</td>
</tr>
<tr>
<td>$\rho$</td>
</tr>
<tr>
<td>$\tau$</td>
</tr>
<tr>
<td>$\phi$</td>
</tr>
<tr>
<td>$\psi$</td>
</tr>
</tbody>
</table>

Table 1: List of Variables and Parameters
Market equilibrium is given by two conditions. First, employers pay efficiency wages to prevent shirking at the margin. Therefore, the wages are set at the level required to equalize utilities of shirking and non-shirking employees \( V^{ns}_r = V^s_r \):

\[
v(w_r) = H_\gamma e \left( 1 + \frac{\rho + \psi + \delta_r}{1 - \gamma} \right). \tag{17}
\]

Second, in equilibrium the inflow to unemployment \( \psi L_{Mr} \) is equal to the outflow of unemployment \( \delta_r (N_{Mr} - L_{Mr}) \) (where \( L_{Mr} \) is the number of manufacturing employees and \( N_{Mr} \) is the manufacturing labor force in region \( r \)). Therefore, the endogenous rate of job creation is given by:

\[
\delta_r = \psi L_{Mr} / (N_{Mr} - L_{Mr}). \tag{18}
\]

Taking both the definition for \( v(w_r) \) and the definition of the unemployment rate \( u_r = L_{Mr} / (N_{Mr} - L_{Mr}) \) into account provides the wage curve for region \( r \) (the wage curve for region \( s \) is constructed in the same way).

\[
w_r = \frac{G^\mu_r}{K} e \left[ 1 + \frac{\rho}{1 - \gamma} + \frac{\psi}{(1 - \gamma) u_r} \right] \text{ where } K = \left( \frac{1 - \mu}{\mu} \right)^{1-\mu} \tag{19}
\]

Equation 19 directly links the wage to the unemployment rate and represents the wage curve resulting from efficiency wages. It represents the wage firms pay in order to prevent shirking at the margin.

Now, migration takes place. Individuals who migrate are initially unemployed in their region of destination at first. An individual decides to migrate when her expected lifetime utility as an unemployed is larger abroad than in the current status at home. However, to monitor whether migration takes place, it is sufficient to compare expected lifetime utilities among the unemployed in both regions. This is due to the fact that the expected lifetime utility of employees is always larger than that of the unemployed: \( V^{ns}_r > V^u_r \). Therefore, migration takes place when \( V^u_r < V^u_s \).

This is true as long as the focus of this work remains on whether someone migrates instead of who (employees or unemployed) migrates. In order to observe migration we therefore compare the expected lifetime utilities of the unemployed in both regions.

Taking the definition for \( v(w_r) \) into account, the expected lifetime utility of an unemployed worker in region \( r \) is given by:

\[
\rho V^u_r = \frac{\delta_r}{\rho + \psi + \delta_r} H_\gamma \left[ w_r \frac{K}{G^\mu_r} - e \right]. \tag{20}
\]
Emigration and immigration takes place when the expected lifetime utility of the unemployed in the neighboring region is larger (lower) than the expected lifetime utility of the unemployed in the home region.

2.3 Goods Market

The goods market is based on the core-periphery model of Fujita et al. (1999) and is separated into agriculture and manufacturing sectors. The agricultural sector produces a homogeneous good under perfect competition, and trade between regions is free and costless, i.e. a single price results. The labor market of the agricultural sector is characterized by perfect competition as well, leading to full employment. Labor input $L_A$ and output in agriculture $C_A$ are linked through the production function $C_A = L_A$. Due to marginal productivity payments in the agricultural labor market, the price of agricultural goods is equal to 1: $P_A = \frac{\partial C_A}{\partial L_A} = w_A = 1$. Prices and wages in agriculture are fixed to 1 and serve as a reference for prices and wages in manufacturing.

Firms in manufacturing instead produce under increasing returns to scale and monopolistic competition. There is trade in manufacturing goods at iceberg transport costs $\tau$. The production function in manufacturing is:

$$L_{Mi} = \beta + \phi q_i + s_i. \quad (21)$$

For production, firm $i$ needs a fixed labor input $\beta$, a variable labor input $\phi$ per unit of output $q_i$, and an additional labor input $s_i$ due to shirking employees. Labor demand $L_{Mi}$ of firm $i$ is the sum of these three components. Due to efficiency wages there is no shirking and hence no additional labor input is needed: $s_i = 0$. Since variety is desired, no combination of firms exists, which produce the same variety. The yield of firm $i$ is given by:

$$\pi_i = P_i q_i - w_r (\beta + \phi q_i). \quad (22)$$

Firms maximize their profit through prices and ignore their influence on the price index $G$. This leads to the price setting rule for the regional price $P_r$, which is identical for all firms of a region (due to identical wages within a region and due to identical firms):

$$\frac{\partial \pi_i}{\partial P_i} = 0 \Rightarrow \frac{P_r}{\theta - 1} w_r \phi. \quad (23)$$

The number of firms is endogenous. New firms enter the market until the profits decrease to zero (zero-profit condition):
\[ \pi_i = \frac{\theta}{\theta - 1} w_r \phi q_i - w_r (\frac{\theta - 1}{\theta} \phi q_i) = 0 \Rightarrow 0 = w_r \left( \frac{\phi q_i}{\theta - 1} - \beta \right). \]  

Then production and employment of a firm in equilibrium are given by:

\[ q_i = \frac{\beta (\theta - 1)}{\phi}, \]  
\[ L_{Mi} = \beta + \phi \frac{\beta (\theta - 1)}{\phi} = \beta \theta. \]  

Production and employment per firm in equilibrium are constant and equal for all firms irrespective of their region. This leads to the number of firms in a region:

\[ m_r = \frac{L_{Mr}}{L_{Mi}} = \frac{L_{Mr}}{(\beta \theta)}. \]  

The labor input per unit of output is standardized to \( \phi \equiv (\theta - 1)/\theta \), so that price and production reduce to:

\[ P_r = w_r, \]  
\[ q_i = \theta \beta = L_{Mi}. \]  

The fix labor input is standardized to \( \beta \equiv \mu/\theta \). Thus the number of firms (=varieties) in a region as well as the production of a firm are given by:

\[ m_r = \frac{L_{Mr}}{\mu}, \]  
\[ q_i = L_{Mi} = \mu. \]  

In equilibrium, the production of a firm is equal to the sum of regional demand for the variety of the firm and import demand of the neighboring region for the firms variety (taking into account iceberg transport costs \( \tau \)).  

\[ q_i = \mu I_r P_r^{-\theta} G_r^{\theta - 1} + \mu I_s (\tau P_r)^{-\theta} G_s^{\theta - 1} \tau \]  

---

4If one unit of the manufacturing good is transfered to the neighboring region, only \( 1/\tau \) units arrive. Therefore \( \tau \) units have to be sent when one unit shall arrive.
\[ w_r = \left[ I_r G_r^{\theta-1} + I_s G_s^{\theta-1} r^{1-\theta} \right]^\frac{1}{\theta} \] (33)

The latter equation represents the goods market equilibrium in the form of a price setting function. It represents the wage at which the condition of zero profits is fulfilled and no firms enter or leave the market. For lower wages, the profit of an additional firm is greater than zero so that new firms enter the market. This results in increasing employment, decreasing unemployment and increasing shirking. To prevent shirking, firms increase wages (wage curve). This process continues until the wage fulfilling the zero-profit condition is equal to the wage preventing shirking.

3 Equilibrium and Migration

The simultaneous equilibrium in both regions is defined by the price indexes, price setting functions, incomes and wage curves of both regions (only equations for region \( r \) are presented, equations for region \( s \) are constructed analogously):

\[ G_r = \left[ \frac{1}{\mu} \left( L_{Mr} w_r^{1-\theta} + L_{Ms} (\tau w_s)^{1-\theta} \right) \right]^{1/\gamma}, \] (34)

\[ w_r = \left[ I_r G_r^{\theta-1} + I_s G_s^{\theta-1} r^{1-\theta} \right]^\frac{1}{\theta}, \] (35)

\[ I_r = w_r L_{Mr} + L_{Ar}, \] (36)

\[ w_r = \frac{G_r^{\alpha} \rho + \psi + \delta_r + 1 - \gamma}{1 - \gamma}. \] (37)

From 34, it follows that the region with the larger number of manufacturing employees has a lower price index. This is because a larger number of manufacturing employees results in a larger number of varieties produced, increasing competition. Then the demand for any individual variety is lower, its price and corresponding revenues decrease, leading to a lower price index. Furthermore, transport costs are lower in the agglomeration, which further reduces the price index in the agglomeration.

The price setting equation (35)\(^5\) represents the wage (or price) at which firms reach their break-even point (i.e. where profits are zero). The higher the incomes and prices and the lower transport costs are, the higher this wage is. Regions with a higher income have greater purchasing power and the break-even point of firms lies at a higher wage. An increase of income in a region leads to a lower or higher increase in employment, depending on the wage elasticity of labor supply. When the increase in employment is larger, centripetal forces dominate: A region that

\(^5\)This equation is labeled the “wage equation” by Fujita et al. (1999).
once manages to gain higher income will be able to use this advantage to attract new firms, income and demand, enforcing an agglomeration process. This process endogenously leads to agglomeration and regional disparities.

The region with the larger number of manufacturing employees thus has higher nominal wages (backward linkage) so that this region is more attractive for firms due to its higher purchasing power. This region is further characterized by a larger number of varieties and thus a lower price index and is therefore more attractive for immigration (forward linkage). These forward and backward linkages establish the centripetal forces leading to endogenous agglomeration. These are opposed to centrifugal forces resulting from the demand by agricultural employees.

Equation 36 defines the income in region $r$ and 37 represents the wage curve, which is a key extension of this paper to the core-periphery model. The wage curve is the link between employment and wages, leading to unemployment. It represents the wage set by firms to prevent shirking.

For a compact illustration of the model, the labor force (as a sum of agricultural and manufacturing labor force) is standardized to one. This labor force is separated into agriculture ($N_A$) and manufacturing ($N_M$) according to the expenditure shares of agricultural and manufacturing goods in income ($\mu$). The agricultural labor force is equal in both regions whereas the labor force in manufacturing is divided between the regions according to $\lambda$. Due to full employment in agriculture, the corresponding labor force is equal to employment in both regions ($N_{Ar} = L_{Ar}$ and $N_{As} = L_{As}$).

$$L_{Ar} = \frac{1 - \mu}{2}$$
$$L_{As} = \frac{1 - \mu}{2}$$
$$N_{Mr} = \mu \lambda$$
$$N_{Ms} = \mu (1 - \lambda)$$

The simultaneous equilibrium in the short term depends on the parameters disutility of work effort ($e$), probability to observe shirking ($1 - \gamma$), job destruction rate ($\psi$), share of expenditures for manufacturing ($\mu$), elasticity of substitution between the varieties of the manufacturing goods ($\theta$) and discount rate ($\rho$). The model is intractable and results are derived by simulation, which is standard practice in New Economic Geography.

In the long term, the unemployed are allowed to migrate between regions. Unemployed compare their expected life-time utilities in both regions and decide to migrate when their utility
is higher in the neighboring region. Their utility depends on their real wages, chances to find employment, and congestion costs in both regions. The migration behavior is thus given by the difference in expected lifetime utility of the unemployed between both regions ($M$) (based on equation 20):

$$
M = \frac{\delta_r}{\rho + \psi + \delta_r} H_r \left( w_r \frac{K}{G^r} - e \right) - \frac{\delta_s}{\rho + \psi + \delta_s} H_s \left( w_s \frac{K}{G^s} - e \right),
$$

(42)

$$
\dot{\lambda} > 0 \text{ for } M > 0, \\
\dot{\lambda} = 0 \text{ for } M = 0, \\
\dot{\lambda} < 0 \text{ for } M < 0.
$$

(43)

In case of symmetry ($\lambda = 0.5$) there is no migration since the endogenous variables are equal in both regions. When there is no symmetry ($\lambda \neq 0.5$), the endogenous variables can differ between both regions and migration might occur depending on these differences. For any given $0 < \lambda < 1$, a short term equilibrium exists. If the utility of the unemployed differs between the regions in the short term equilibrium, unemployed workers migrate until a long-term equilibrium is reached where there is no incentive to migrate. In the long term equilibrium the expected lifetime utility of the unemployed is equal in both regions and therefore there is no incentive to migrate.

Figure 2 displays the difference between the expected lifetime utility of the unemployed in region $r$ minus the expected lifetime utility of the unemployed in region $s$ ($M$) for different constellations of the parameters (the parameter constellations of all figures are summarized in table 2). Qualitatively, three different situations can be compared. In situation A, the expected lifetime utility of the unemployed is always lower in the larger region. The unemployed migrate back to the smaller region until the symmetrical simultaneous equilibrium in both regions is reached at $\lambda = 0.5$. Then, the symmetry is the only stable equilibrium. In contrast, in situation B the expected lifetime utility of the unemployed is larger in the larger region for intermediate levels of $\lambda$ and is smaller in the larger region for very small or very large $\lambda$. That is, once a region becomes

---

6The real wage in region $r$ is $w_r \frac{K}{G^r}$.
7The chance to find employment depends on the endogenous job creation rate which is directly linked to the unemployment rate.
8This definition of migrant behavior is motivated by optimal migration decisions based on static expectations on the differences in real wages, unemployment, and congestion costs between both regions (Baldwin et al., 2003, Appendix 2.B.4). It further extends the underlying logic of the basic efficiency wage model to the migration-case: In the basic model, the equilibrium is reached when the expected lifetime utilities of shirking and non-shirking employees are equal. Analogously, long-term equilibrium is reached when the expected lifetime utilities of the unemployed in both regions are equal.
larger than another region, an advantage results for the former, and an agglomeration process sets in. Nevertheless, the agglomeration does not attract all labor since large agglomerations suffer from congestion costs. There are therefore two stable equilibria in this situation, both of which are agglomerations. The symmetry is instable. Finally, three stable equilibria exist in situation C: Symmetry and agglomeration both may result, depending on the initial value of $\lambda$.

4 Stability

Multiple equilibria exist and three basic situations arise. The stability characteristics of the system – illustrated by the above situations – depend on the transport costs $\tau$ and the congestion costs $h$. As in Fujita et al. (1999) the stability characteristics of the system can be described by the break- and sustain-points. However, since these points now depend on both, transport costs $\tau$ and congestion costs $h$, the stability characteristics are more complex. As illustrated by Figure 2, both symmetry and agglomeration might be stable or instable, depending on transport
Table 2: Parameter constellations of the figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>(\mu)</th>
<th>(\theta)</th>
<th>(\tau)</th>
<th>(e)</th>
<th>(\rho)</th>
<th>(\psi)</th>
<th>(\gamma)</th>
<th>(h)</th>
<th>(\lambda)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 A</td>
<td>0.6</td>
<td>4</td>
<td>3.2</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>1.12</td>
<td>/</td>
</tr>
<tr>
<td>2 B</td>
<td>0.6</td>
<td>4</td>
<td>2.5</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>1.04</td>
<td>/</td>
</tr>
<tr>
<td>2 C</td>
<td>0.6</td>
<td>4</td>
<td>3.2</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>1.04</td>
<td>/</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>4</td>
<td>/</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>1.04</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>4</td>
<td>2</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>/</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>6</td>
<td>/</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>4</td>
<td>/</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>7</td>
<td>0.6</td>
<td>4</td>
<td>/</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>8</td>
<td>0.6</td>
<td>4</td>
<td>/</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>9,10 A</td>
<td>0.6</td>
<td>4</td>
<td>/</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>9,10 B</td>
<td>0.6</td>
<td>4</td>
<td>/</td>
<td>0.1</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>9,10 C</td>
<td>0.6</td>
<td>4</td>
<td>/</td>
<td>0.3</td>
<td>0.05</td>
<td>0.3</td>
<td>0.2</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

costs and congestion costs. The break-point then describes the point where symmetry changes from being instable to being stable, whereas the sustain-point describes the point where the agglomeration transitions from being instable to being stable, both with regard to changes in transport costs and congestion costs. Subsequently both points are deduced, the agglomeration pattern is described and consequences for labor market disparities are discussed.

4.1 Break-Point

The break-point describes the situation, in which a change in transport or congestion costs leads to the symmetric equilibrium changing from instable to stable (or vice versa). To illustrate this point, consider the following: If a marginal deviation from symmetry (i.e. a marginal increase/decrease of \(\lambda\)) leads to a larger expected lifetime utility of unemployed in the marginally larger region, then symmetry is instable and an agglomeration endogenously arises. Hence the derivative of the difference in utility \(M\) against \(\lambda\) is larger than zero. In contrast, if this derivative is smaller than zero, symmetry is stable. In this case, the break-point lies exactly at that symmetrical equilibrium where the derivative of the differences in utilities of unemployed is zero \((dM = 0)\).

To calculate this point, \(\lambda\) is set to 0.5 and we can utilize the fact that all endogenous variables are equal in both regions. Furthermore, the change of one variable in a region is equal to the negative change of the same variable in the other region. Therefore the system can be expressed in units of region \(r\) and the index for regions is dropped. Consequently, the break-point is expressed by the system of equations 44 to 56:
\[ G = \left[ \frac{1 + \tau^{1-\theta}}{\mu} \left( L_M w^{1-\theta} \right) \right]^{1/(1-\theta)}, \quad (44) \]
\[ w = \left[ (1 + \tau^{1-\theta}) I G^\theta - 1 \right]^{1/\theta}, \quad (45) \]
\[ I = wL_M + L_A, \quad (46) \]
\[ w = \frac{G^\theta}{K} \rho + \psi + \delta + 1 - \gamma, \quad (47) \]
\[ \delta = \frac{\psi L_M}{\mu \lambda - L_M}, \quad (48) \]
\[ H = h^{1-\frac{1}{\lambda}}, \quad (49) \]
\[ dG = \frac{G^\theta}{\mu} (1 - \tau^{1-\theta}) \left[ \frac{w^{1-\theta}}{1-\theta} dL_M + L_M w^{-\theta} \right], \quad (50) \]
\[ dw = \frac{w^{1-\theta} G^{\theta-1}(1 - \tau^{1-\theta})}{\theta} \left[ dI + (\theta - 1) I \frac{dG}{G} \right], \quad (51) \]
\[ dI = wL_M + L_M dw, \quad (52) \]
\[ dw = \mu \frac{w}{G} dG + \frac{G^\mu}{K} \frac{e}{1-\gamma} d\delta, \quad (53) \]
\[ d\delta = \frac{\mu \psi}{(\mu \lambda - L_M)^2} (\lambda dL_M - L_M d\lambda), \quad (54) \]
\[ \frac{dM}{2} = 0 = \frac{\delta}{\rho + \psi + \delta} \left[ \frac{K}{G^\mu} - e \right] dH + \frac{HK G^\mu}{G^{\mu+1} dG} - \frac{\mu H w K}{G^{\mu+1} dG}, \quad (55) \]
\[ dH = -\frac{1}{(1-\lambda)^2 H} \ln(h) d\lambda. \quad (56) \]

By finding the solution to this system we can identify all combinations of \( \tau \) and \( h \) where a break-point exists. Assume the congestion cost factor \( h \) is given. Then figure 3 illustrates the behavior of the symmetry for changes in \( \tau \). When \( h > 1 \), we see that \( dM < 0 \) for very small \( \tau \). That is, a marginal deviation of the system from symmetry goes along with incentives to migrate back into the symmetry and symmetry is stable. For further increases in \( \tau \), this situation flips and emigration out of the symmetry leads to a self-reinforcing agglomeration process since migration is accompanied by gains in utility for the migrants. For small transport costs, a marginal deviation from symmetry leads to a higher nominal wage and a smaller price index in the larger region. Therefore, the real wage (unemployment rate) is higher (smaller) in the larger region and immigration into the larger region sets in. Symmetry is then instable. However,
this is only true if the agglomeration’s advantages (higher real wages, lower unemployment) over-compensate the higher congestion costs in the agglomeration.

For large $\tau$ in turn, the symmetry again becomes stable. When transport costs are large, a marginal deviation from symmetry does not allow the marginally larger region to export its production due to high transport costs. The effect of the larger manufacturing employment in the larger region on income in that region cannot offset the negative effects of higher congestion costs in the agglomeration and the negative effects of decreased manufacturing employment on income and import demand in the smaller region. The utility of the unemployed is therefore smaller in the agglomeration and re-immigration to the smaller region sets in, resulting in symmetry. Thus, there exist two break-points at very low and at large transport costs for any given level of $h > 1$.

If we instead focus on the congestion costs $h$ and assume fixed transport costs $\tau$, there only exists one break-point. Figure 4 illustrates the behavior of the symmetry for changes in $h$. We see that for very low congestion costs $h$, emigration to the marginally larger region is accompanied by a gain in utility. Then the symmetry is instable. However, when congestion costs increase
further, symmetry becomes stable since the agglomeration advantages are offset by congestion costs.

Figure 6 combines the information gathered so far: For all transport costs $\tau$ and congestion costs $h$ the break-point line illustrates where the break-points lie. Outside the field defined by this curve and the line $h = 1$, symmetry is stable – and accordingly symmetry is instable inside this field.

### 4.2 Sustain-Point

Similar to the break-point, the sustain-point describes the situation, where changes in transport or congestion costs result in a situation in which the agglomeration goes from being instable to stable (or vice versa). To illustrate this point, consider the following: If a marginal deviation from agglomeration (i.e. a marginal increase/decrease of $\lambda$) leads to a larger expected lifetime utility among the unemployed in the periphery, then the agglomeration is instable and collapses.
Then the derivative of the differences in utility \( dM \) against \( \lambda \) is smaller than zero (assuming that \( r \) is the agglomeration and thus \( \lambda > 0.5 \)). In contrast, if this derivative is larger than zero, agglomeration is stable. The sustain-point then lies exactly at that agglomeration where the derivative of the differences in expected lifetime utilities of the unemployed is zero. Consider figure 2: In situation C there is a local maximum at \( \lambda > 0.5 \) and to the right of this maximum there is a stable agglomeration. The sustain-point then is the situation where this maximum is tangent to the line \( M = 0 \).

It is apparent that in contrast to the break-point the situation is now more complex since \( \lambda \) is not given in advance (by \( \lambda = 0.5 \)), but is instead endogenous. The sustain-point is defined by the situation where the derivative of the difference in utilities \( dM \) against \( \lambda \) is zero and where at the same time the difference in utilities \( M \) is zero, too. These two conditions are always fulfilled at the break-point. However, these two conditions are fulfilled for \( \lambda \neq 0.5 \) for some constellations of \( \tau \) and \( h \) (we will discuss the interpretation of this in more detail in the subsequent chapter).

A mathematical definition of this point is attached to the appendix.

To understand the behavior of the system around the sustain-point, it is helpful to consider the case of \( h = 1 \). When \( h = 1 \), no congestion costs exist so that any agglomeration is a full agglomeration (i.e. \( \lambda = 0 \) or \( \lambda = 1 \)). In this case it is sufficient to monitor the difference in utility \( M \) for different \( \tau \), which is done in figure 5.

When there is full agglomeration, increasing transport costs lead to an increase of the price index in the periphery and the periphery becomes less attractive (the agglomeration becomes more stable). At the same time, however, increasing transport costs lead to a decrease in the level of the wage at which firms reach their break-even point in the periphery. Thus, the periphery becomes more attractive for firms when transport costs increase (the agglomeration pattern becomes more instable). For low transport costs the first effect dominates and the agglomeration becomes more stable. For high transport costs, the second effect dominates and the agglomeration becomes instable. The net effect is illustrated in figure 5 by showing the development of the difference in utility \( M \) against transport costs. Thus, agglomeration forces are strongest for intermediate transport costs and are low both for large and for low transport costs. When congestion costs are larger than zero \( (h > 1) \), then \( \lambda \) is endogenous and the figure cannot be computed, nor can the difference in expected lifetime utilities against congestion costs be computed, for the same reason. However, the characteristics of the agglomeration’s forces remain the same with regard to transport costs: \( h \) only influences the level of these forces (since it influences the migration decision, but not the goods market equilibrium, similar to the case of the break-point).

Since we know the conditions for the sustain-point, we can calculate all combinations of \( \tau \) and
Figure 5: Migration at full agglomeration for different $\tau$

$h$, where there is a sustain point and arrange them on a map. This is done in figure 6. Inside the field defined by the sustain-point curve and the line $h = 1$, the agglomeration is stable – and instable outside, accordingly.

### 4.3 Agglomeration Pattern

Combining the information on the break- and sustain-point delivers an illustration of the systems behaviour. This is done in figure 6. For any level of $h$, one can depict as in figure 6 at which levels of $\tau$ there are sustain- and break-points.

From figure 6 it becomes obvious that when $\tau$ is large and decreases, then there is a level of $h$ at which sustain- and break-points coincide. To understand what happens here we must go back to figure 2. Qualitatively there are three different situations. In situation A, only symmetry is stable. In figure 6 this is the region outside the fields marked by the two curves and the $h = 1$ line. In situation B, only the agglomeration is stable. This is the region inside the break-point
curve and the $h = 1$ line in figure 6. Finally, in situation C both break- and sustain-point are stable. This is the field marked by the sustain-point-line and the $h = 1$ line minus the field marked by the break-point line and the $h = 1$ line.

In situation C there is a local maximum in figure 6, which is above the $(M = 0)$-line and thus enables both, agglomeration and symmetry to be stable. This situation arises when $h$ is small and $\tau$ is large (but not too large). At this level of transport costs (and congestion costs) agglomeration forces are strong enough to reinforce the agglomeration only when $\lambda$ is large (or small) enough. When $\lambda$ is close to 0.5, then agglomeration forces are not strong enough and symmetry is stable.

At the margin, this maximum is tangential to the $M = 0$ line, representing the sustain-point. Now, imagine how the transport costs decrease. To remain on the sustain-point line, congestion costs need to increase (since for decreasing transport costs, agglomeration forces increase). However, this means that the sustain-point in figure 2, which is the maximum of the curve of figure C as a tangent to the $M = 0$ line, moves towards the symmetrical equilibrium
at $\lambda = 0.5$. This means that the sustain-point approaches the break-point and the sustain-point becomes the break-point at the margin.

To understand why this happens recall figures 3 and 5. When there are no transport costs ($\tau = 1$), then an increase in the transport costs in figure 3 destabilizes the symmetry and at the same time stabilizes the agglomeration in figure 5 (since agglomeration forces increase). The slope of the M-curve in figure 2 is then monotonous for zero congestion costs. The stability of the system now depends on congestion costs only. An increase in the congestion costs leads to a monotonous increase in the agglomeration’s disadvantages (instead of a non-monotonous decrease as in the case of transport costs). Therefore the slope of the M-curve in figure 2 remains monotonous, but the sign of the slope depends on congestion costs. Since the slope is monotonous, only symmetry or agglomeration can be stable.

However, when transport costs increase further the agglomeration advantages decrease in figure 3, but agglomeration remains stable for even larger values of $\tau$ in figure 5. That is, the agglomeration forces are not monotonous in transportation costs anymore for sufficient large levels of $\tau$. In this case the strength of agglomeration forces depends on the level of $\lambda$ – agglomeration forces are only strong enough to reinforce agglomeration when the agglomeration is large enough. Then the sustain-point is different from the break-point and the sustain- and break-point lines in figure 6 divide from each other.

### 4.4 Bifurcations

By introducing unemployment and congestion costs into the model of Fujita et al. (1999), the agglomeration pattern changes considerably and conclusions for regional labor market disparities can be drawn since unemployment disparities arise. This section deals with the implications of the model for regional labor market disparities by using bifurcation diagrams.

With figure 6 in mind we can qualitatively distinguish three different agglomeration patterns for different levels of the congestion costs parameter $h$. (1) For low congestion costs an agglomeration pattern arises where agglomeration and symmetry might be stable simultaneously. (2) For intermediate congestion costs, either agglomeration or symmetry is stable, but not both simultaneously. (3) Finally, for high congestion costs symmetry is always stable and agglomeration always instable. The third case is trivial. Hence, the first and second case are of special interest here. Both cases are visualized as bifurcation diagrams for the share of manufacturing employees and for the difference in unemployment rates of both regions in figures 7 and 8.

In the first case there are low congestion costs (here: $h = 1.04$). When transport costs are high, then only the symmetry is stable and there are no differences between the regions.
However, when transport costs start to decline, a catastrophic agglomeration pattern arises: For a certain range of $\tau$ both – agglomeration and symmetry – are stable. Whether symmetry or agglomeration results depends on the initial distribution of manufacturing employees. The agglomeration is marked by a smaller unemployment rate. Further decreases in transportation costs lead to an even lower unemployment rate in the agglomeration compared to the periphery, and at a certain level of $\tau$, symmetry becomes unstable. Agglomeration then is the only stable equilibrium. That is, for intermediate levels of transportation cost the previously described centripetal forces endogenously lead to agglomeration and result in unemployment disparities. When transportation costs decline further centripetal forces decline. Nevertheless, this does not lead to a catastrophic change in the agglomeration pattern, but instead the agglomeration becomes smaller until both regions are equally large and symmetry results. This is because the break- and sustain-points coincide. Simultaneously the unemployment rates converge.

In the second case there are intermediate congestion costs (here: $h = 1.12$). When transport costs are high, then only the symmetry is stable and there are no differences between the
regions. However, when transport costs start to decline, symmetry becomes unstable and agglomeration results. This process is smooth, i.e. one of the regions becomes successively larger when transport costs decrease. This is accompanied by a decrease in the unemployment rate in the agglomeration relative to the unemployment rate in the periphery. There is no catastrophic change from symmetry to agglomeration since break- and sustain-point coincide. In a similar fashion, further decreasing transport costs lead to a decline of the centrifugal forces and the agglomeration again approaches symmetry in a smooth process, both in terms of manufacturing employees and unemployment rates.

5 Labor Market Frictions and Disparities

The present model shows how regional labor market disparities endogenously arise through agglomeration in the presence of labor market frictions. However, these labor market frictions influence the extent and presence of regional labor market disparities through their interaction
with the agglomeration forces. The subsequent section deals with this interaction.

Due to the shape of the wage curve and due to the characteristics of agglomeration forces, increasing labor market frictions accompany increasing labor market disparities: The agglomeration forces are expressed by the break-even point, represented by the real wage in equation 35. In the agglomeration this real wage is higher, meaning that firms yield positive profits up to higher real wages than in the periphery and hence the unemployment rate is lower in the agglomeration since new firms and jobs are created at even higher wages in the agglomeration compared to the periphery. Furthermore the absolute slope of the wage curve decreases with increasing unemployment.

Changes in labor market frictions at first only affect the wage curve, since the relevant variables only appear here. Furthermore the break-even point, which is unaffected by changes in labor market friction at first, defines the real wage. Thus in the first round, changes in labor market frictions affect the regional unemployment rate but not the real wage (i.e. break-even point). Since the absolute slope of the wage curve decreases with increasing unemployment, the effect of increasing labor market frictions on unemployment (holding the wage / break-even point constant) is higher, the higher the unemployment rate is. Therefore the effect of increasing labor market frictions is higher for the periphery (where the unemployment rate is higher) than in the agglomeration, and hence labor market frictions increase (compare the appendix for a formal proof). That is, in the first round increasing labor market frictions lead to an increase of regional labor market disparities.

However, when labor market disparities increase, this leads to an adjustment through migration: Labor market disparities increase relative to the congestion costs, so that more people move to the more attractive region, i.e. the agglomeration. Hence, secondary to the shift of the wage curve, increasing labor market frictions intensify agglomeration forces and further exacerbate labor market disparities. This is illustrated in figure 9, where the difference in the unemployment rates of regions $r$ and $s$ is plotted against the transport costs $\tau$ (only the stable agglomeration equilibria are plotted, not the stable symmetry equilibria). In situation $A$ the disutility of work effort $e$ is 0.2; it is 0.1 in Situation $B$ and 0.3 in situation $C$. The figure highlights, that disparities increase when frictions increase. Similar patterns arise, when labor market frictions vary in form of the discount rate of utility $\rho$, the job destruction rate $\psi$ or the probability of not detecting shirking $\gamma$.

The figure further implies that the stability characteristics change when frictions change: The range of transportation costs at which the agglomeration is stable increases with increasing frictions. This is further illustrated by figure 10, where all break- and sustain-points are plotted against transport costs $\tau$ and congestion costs $h$. In situation $A$ the disutility of work effort $e$ is
0.2, in situation $B$ it is 0.1, and in situation $C$ it is 0.3. The higher $\epsilon$ is, meaning the stronger the labor market frictions are, the larger the area is where agglomeration is stable and the smaller the area, where symmetry is stable. Similar pattern arise for changes in $\rho$, $\psi$ and $\gamma$.

6 Comparison to Existing Results

The model shows how regional labor market disparities arise endogenously through agglomeration in the presence of labor market frictions. Its results are comparable to those of Epifani and Gancia (2005). They also base their regional labor market model on the model of Fujita et al. (1999). Disparities in wages and unemployment endogenously arise through agglomeration in a similar way. However, they focus on labor market frictions in job matching processes and assume flexible wages, whereas this paper focuses on frictions in wage setting. Thus, their approach and the model presented here complement each other by discussing different labor market frictions in the framework of the New Economic Geography. The present model further shows how the
agglomeration pattern changes when congestion costs arise. Both a catastrophic and a smooth agglomeration pattern might result depending on congestion costs. Changes in congestion costs lead to a smooth transition between both agglomeration patterns.

The results are further comparable to Südekum (2005). He discusses a wage curve based on efficiency wages within the framework of a regional goods market model. In contrast to the present paper, he exclusively focuses on centripetal forces to be able to solve the model analytically. In his model, agglomeration patterns lead to higher wages and lower unemployment in the core compared to the periphery. Nevertheless, without any additional assumptions on migration, full agglomeration necessarily results due to the lack of centrifugal forces. The present model instead discusses disparities between regional labor markets within the interplay of centrifugal and centripetal forces, and is therefore able to distinguish under which conditions disparities arise (or not) and how the agglomeration pattern depends on congestion and transport costs.

Another key result of the present model is that agglomeration becomes more likely and dis-
parities become more pronounced, as labor market rigidities increase:\(^9\) Regions on the lower tail of the wage curve react more strongly to increasing labor market frictions, leading to more pronounced disparities. These in turn spur further immigration into the agglomeration, exacerbating regional disparities. Hence, policies that reduce labor market frictions in the form presented here, also reduce regional labor market disparities.

7 Conclusions

Disparities between regional labor markets are a key characteristic in many countries. In the literature on the wage curve it is argued that there exists a negative relationship between wages and unemployment on the regional level (Blanchflower and Oswald; 1994). However, this literature cannot explain how disparities in these variables endogenously arise. The New Economic Geography in turn explains how disparities between regional economies endogenously arise, but usually assumes full employment. The present model hence introduces efficiency wages into the New Economic Geography in order to explain how disparities between regional labor markets arise endogenously.

Using New Economic Geography’s arguments, the present model discusses how disparities between regional goods markets arise endogenously. Depending on the level of transport costs and congestion costs, an agglomeration might arise endogenously through the interplay of increasing returns to scale, monopolistic competition, and migration. This results in disparities between regional goods markets. The level of these disparities as well as the agglomeration behavior (in particular whether a catastrophic pattern arises or not) varies considerably, depending on the strength of congestion costs.

By introducing wage setting frictions based on efficiency wages to the model, these disparities are accompanied by disparities between regional labor markets. The model is thus able to explain how disparities between regional labor markets arise endogenously. The results are comparable to Epifani and Gancia (2005). Whereas they introduce job-matching-frictions into the New Economic Geography, however, the present model rests upon frictions in wage-setting based of efficiency wages to cover unemployment in the New Economic Geography-framework. In contrast to this job-matching approach, the present model directly introduces the – intensively studied – empirical phenomenon of the wage curve through efficiency wages into the New Economic Geography. Similar to the model of Epifani and Gancia we observe higher wages and lower

\(^9\)Helpman and Itskhoki (2010) and Helpman et al. (2010) also discuss the relation between labor market rigidities and unemployment (disparities). However, they focus on the international level and do not take migration into account.
unemployment in the agglomeration compared to the periphery. However, the agglomeration pattern is only similar to their model when congestion costs are set at a low level. The model additionally shows how unemployment disparities arise in different agglomeration patterns and how changes in the congestion costs lead to a smooth transition between the agglomeration pattern.

The models’ results are further comparable to Südekum (2005): The unemployment rate is lower in the agglomeration compared to the periphery whereas the opposite is true for the wage, so that a stable wage curve emerges. But whereas Südekum only covers centripetal forces, however, the present model additionally includes centrifugal forces. It therefore represents a full New Economic Geography model and is hence able to explain how regional labor market disparities arise endogenously, leading to a stable wage curve.

A special feature of the present model is that the disparities between regional goods markets do not simply translate into disparities between regional labor markets through labor market frictions. Instead labor market frictions play a role for the level of these disparities and for the agglomeration characteristics: Increasing labor market frictions shift the wage curve, which results in a stronger increase of unemployment in the periphery than in the agglomeration and hence raises regional labor market disparities. Subsequently, the relative attractiveness of the regions changes leading to an adjustment of migration, changing the agglomeration’s behavior. Thus, increasing labor market frictions not only exacerbate regional labor market disparities, they further support the chance that agglomeration occurs.
A Appendix

A.1 Definition of the Sustain-Point

The sustain-point is defined by the equilibrium conditions 34 to 37 for region \( r \) and for region \( s \) accordingly, the definition of the congestion costs 2 and 3, the no-migration-condition 42, and the derivative of the system against \( \lambda \) (only the derivative for region \( r \) is presented here, the equations for region \( s \) are calculated in the same way):

\[
dG_r = \frac{G_r^\mu}{\mu} \left[ w_r^{1-\theta} dL_{Mr} + L_{Mr} w_r^{-\theta} dw_r + \frac{(w_s \tau)^{1-\theta}}{1-\theta} dL_{Ms} + L_{Ms} \frac{\tau^{1-\theta}}{w_s^{\theta}} dw_s \right] 
\]

(57)

\[
dw_r = \frac{w_r}{G_r} \left[ \frac{1}{1-\theta} \left[ dI_r + (\theta - 1) \frac{I_r}{G_r} dG_r \right] + \frac{(w_r}{G_s})^{1-\theta} \frac{\tau^{1-\theta}}{\theta} \left[ dI_s + (\theta - 1) \frac{I_s}{G_s} dG_s \right] \right] 
\]

(58)

\[
dI_r = w_r dL_{Mr} + L_{Mr} dw_r 
\]

(59)

\[
dw_r = \frac{w_r}{G_r} dG_r + \frac{G_r^\mu}{K} \frac{e}{1-\gamma} d\delta_r 
\]

(60)

\[
dH_r = -\frac{1}{(1-\lambda)^2} \ln(h) H_r d\lambda 
\]

(61)

\[
d\delta_r = \frac{\mu \psi}{(\mu \lambda - L_{Mr})^2} (\lambda dL_{Mr} - L_{Mr} d\lambda) 
\]

(62)

\[
dM = \frac{\delta_r}{\rho + \psi + \delta_r} \left[ w_r \frac{K}{G_r} - e \right] dH_r + H_r \frac{K}{G_r^\mu} dG_r - \mu H_r \frac{w_r K}{G_r^{1+\mu}} dG_r 
\]

\[
+ \frac{\rho \psi}{(\rho + \psi + \delta_r)^2} H_r \left[ w_r \frac{K}{G_r} - e \right] d\delta_r 
\]

\[
- \frac{\delta_s}{\rho + \psi + \delta_s} \left[ w_s \frac{K}{G_s} - e \right] dH_s + H_s \frac{K}{G_s^\mu} dG_s - \mu H_s \frac{w_s K}{G_s^{1+\mu}} dG_s 
\]

\[
- \frac{\rho \psi}{(\rho + \psi + \delta_s)^2} H_s \left[ w_s \frac{K}{G_s} - e \right] d\delta_s 
\]

(63)

A.2 Frictions and Disparities

Labor market frictions are characterized by the parameters \( e, \rho, \gamma \) and \( \psi \). When these parameters increase, frictions increase. These parameters are only included in the wage curve, whereas they do not appear in the equations for the goods market equilibrium, hence they influence the goods market only indirectly through their effects on the wage curve. From the goods market the wage results, at which firms reach their break-even point. Until this point is reached, new firms enter the market. Therefore unemployment adjusts, until the break-even point (and its corresponding wage) is reached. Therefore the primary influence of labor market frictions is only
on the unemployment rate, whereas the break-even point and its corresponding wage changes in reaction to the changes of the unemployment rate. Thus the real wage is fix if we only look at the wage curve. The same is true for migration. Thus we reformulate the wage curve:

\[
\frac{v(w_r)}{H_r} = e \left( 1 + \frac{\rho + \psi/u_r}{1 - \gamma} \right)
\]

The same holds true for region \( s \). We further know from the arguments above that:

\[
d \left( \frac{v(w_r)}{H_r} \right) = 0 \quad \text{and} \quad d \left( \frac{v(w_s)}{H_s} \right) = 0
\]

Then we can show for changes in the labor market frictions that changes in the unemployment rate are higher for regions, where the unemployment rate is higher:

\[
du_r = \left( \frac{u_r^2 \rho}{e \psi} + \frac{u_r}{e} + \frac{u_r^2 (1 - \gamma)}{e \psi} \right) de
\]

\[
du_r = d\rho u_r^2
\]

\[
du_r = d\psi u_r
\]

\[
du_r = d\gamma \left( \frac{\rho + \psi u_r^2}{(1 - \gamma) \psi u_r^2} \right)
\]

Hence, the primary effect of increasing frictions (which occur through the wage curve) is that they enforce disparities. The increasing disparities then spur migration from the periphery to the agglomeration which further enforces disparities through its effect on the goods market.
References


107. Efficient Redistribution: Comparing Basic Income with Unemployment Benefit
   Felix FitzRoy, Jim Jin, Hamburg, March 2011

106. The Resource Curse Revisited: Governance and Natural Resources
   Matthias Busse, Steffen Gröning, Hamburg, March 2011

105. Regional Unemployment and New Economic Geography
   Ulrich Zierahn, Hamburg, March 2011

104. The Taxation-Growth-Nexus Revisited
   K. P. Arin, M. Berlemann, F. Koray, T. Kuhlenkasper, Hamburg, January 2011
The **Hamburg Institute of International Economics (HWWI)** is an independent economic research institute, based on a non-profit public-private partnership, which was founded in 2005. The University of Hamburg and the Hamburg Chamber of Commerce are shareholders in the Institute.

The HWWI's main goals are to:
- Promote economic sciences in research and teaching;
- Conduct high-quality economic research;
- Transfer and disseminate economic knowledge to policy makers, stakeholders and the general public.

The HWWI carries out interdisciplinary research activities in the context of the following research areas:
- Economic Trends and Global Markets,
- Regional Economics and Urban Development,
- Maritime Economics and Aviation Business,
- Institutions and Institutional Change,
- Energy and Raw Material Markets,
- Environment and Climate,
- Migration and Integration,
- Labour and Family Economics,
- Health and Sports Economics,
- Family Owned Business and SME,
- Real Estate and Asset Markets.