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Female wage profiles: An Additive Mixed Model approach to employment breaks due to childcare

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Female wage profiles: An Additive Mixed Model approach to employment breaks due to childcare

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Abstract

The paper investigates female wage profiles in West-Germany between 1984 and 2008 using data from the German Socio Economic Panel. The empirical study focuses on the short-run wage loss due to childcare and the long-run wage-profile in post-birth employment, respectively. This is compared with wage profiles from females who are not mothers.

As statistical analysis tool Additive Mixed Models are employed and estimated separately for different levels of educational achievements. The models are dynamic in that main covariate effects are allowed to vary smoothly with working experience. The intention of the paper is to demonstrate with state of the art statistical models how wages are affected by labour market experience, employment interruptions and other covariates. The educational level of the mother and the time off the job influence the amount of wage loss and the wage profile afterwards. Labour market experience, as one major determinant of human capital, influences wages heavily and follows a dynamic pattern.

*JEL classification:* C14, C23, J13, J24, J31

*Keywords:* Additive Mixed Models, Dynamic Effects, Maternity Leave, Panel Data, Employment Interruption, Wage Profiles, Female Labour Supply
1 Introduction

The rising labour market participation of females in western countries in the last century as described by Fitzenberger and Wunderlich (2004) and Rubery et al. (1999) forces females to combine family-related responsibilities with employment and to allocate their time between the labour market and the household. As a result, this issue is still not only discussed in the economic literature but is also a main political topic in many countries with different legislation for maternity leave rights.

Besides the more country-specific legal frameworks of parental leave policies, the wages females earn are the most valid economic value research focusses on when analyzing female labour force participation. Females’ wage profiles and their labour supply have been discussed thoroughly in the economic literature over the last years. Corcoran et al. (1983) and Bloemen and Kalwij (2001) focus on female labour supply taking family decisions into account. The gender wage gap has been under investigation by Lundberg and Rose (2000), Kunze (2002) and more recently by Munasinghe et al. (2008). As a result, females have to face both, a wage penalty when interrupting their career due to child care (or other family related responsibilities) as well as consequences for their working career after having returned to the labour market.

In this paper we analyse the effect of motherhood on wages. More precisely we investigate how temporary withdrawal from the labour market due to childcare and family reasons influences both, (a) the wage of a mother in the short-run when returning into work and (b) the ongoing wage profile in the long-run. Although the legal framework for parental leave has changed over the last decades in most countries, mothers are still faced with severe
economic consequences after an employment interruption due to childbirth. We therefore restrict our analysis to women.

In this article we focus on three questions: First, how does an employment break due to childcare effect the wage of mothers in the short- and the long-run, dependent on their achieved educational level. Secondly, we analyse the dynamic behaviour of covariates, like the labour force experience and other exogeneous variables which are likely to directly influence the wage when reentering the labour market. Finally we look at a possible catch up of mothers with their wages compared to females not having children. The aspired analysis is carried out with longitudinal data from the German Socio Economic Panel (GSOEP, www.diw.de/soep).

The analysis of female wage profiles which are affected by employment breaks has been carried out for instance by Lundberg and Rose (2000) and Munasinghe, Reif, and Henriques (2008) who explain wage reductions by human capital theory, referring to Becker (1993). Beblo and Wolf (2000) investigate how periods of non-employment and part-time work effect the gross hourly wage rate of females based on German data. The common consens when analysing employment breaks is empirical evidence about net-depreciated firm-specific and transferable human capital resulting in wage penalties when mothers return to work. These findings are independent from the reason of withdrawal, which are usually motherhood or unemployment. Kunze (2002) differentiates between reasons for employment breaks and concludes that females can lose more than 10 percent of wage per year due to maternity leave compared to the wage earned before leaving the labour market temporarily. Classical human capital theory, as described by Becker (1993), allows to in-
interpret wage losses due to employment-breaks as both: depreciation of the human capital accumulated prior to the interruption as well as lost rates of return due to failed human capital investigation when being off the job. The latter is likely to effect the wages in the long-run while net-depreciation of human capital is assumed to result in short-run losses just when the female reenters the labour market. To capture the effect of accumulated human capital and interrupted employment on personal income or wages, the majority of authors, like Kunze (2002), Francesconi (2002), Ondrich et al. (2003) and Kreyenfeld et al. (2007) follow Mincer (1974) and Mincer and Ofek (1982). Thereby, many authors differentiate between firm-specific and transferable human capital when analyzing wages. The firm-specific human capital focuses on the experience an employee has earned while working in the current job due to on-the-job-training and the adaptation of job-specific skills, which only fit for the current job. In contrast, general or transferable human capital intends to capture the skills of the employee, which can be used in other jobs as well. The latter definition of human capital refers to the years of schooling, the age and the full-time work-experience in the past. This type of accumulated human capital is assumed to be transferable between different jobs in the labour market. More years of schooling, higher achieved educational level as well as long full-time work experience are considered to have positive effects on the wage. In contrast, periods of unemployment and other employment interruptions result in wage losses due to human capital net-depreciation and lost rates of return.

In this paper we investigate empirically if and how these rather static and fixed effects of depreciation are appropriate to describe the economic conse-
quences for females around labour market transitions related to childbirth. We therefore use longitudinal panel data to obtain a reliable database of individual female wage losses and covariates affecting the ongoing wage profiles after employment breaks. The data at hand trace from the German Socio Economic Panel (GSOEP). The GSOEP provides suitable data and allows us to empirically explore the wage on a microlevel. We analyse wages from 1984 to 2008 for 3998 females contributing to 23445 observations in the study. For details about the GSOEP we refer to Wagner, Frick, and Schupp (2007) and Haisken-DeNew and Frick (2005).

The statistical models employed in this article are built upon the classical regression model for longitudinal data (see Diggle et al. (2002).) Instead of a restrictive linear structure we allow for smooth functional effects which can capture potential non-linearities in the data without the challenge of specifying the structure of the model a priori. Note that non-linearities are likely so that the functional approach pursued in this paper seems appropriate. For example, the experience a woman has earned while working full-time is supposed to have an increasing, but concave-running effect on the wage as her working career continues. Models with smooth, functional covariate effects have been coined ‘varying coefficient models’ by Hastie and Tibshirani (1993). The models allow for flexible fitting even in the presence of complex interaction terms. We refer to Ruppert, Wand, and Carroll (2003) for a readily introduction. For fitting we make use of penalized splines to estimate the smooth functional covariate effects as described by Wood (2006). Besides the functional estimation approach we have to take the unbalanced panel structure of the data into account also yielding unobserved heterogeneity. We
do this by including unobserved individual (random) effects to the models. This yields a mixed model described in the statistical literature for instance in Wood (2006), Jiang (2007) and Zuur et al. (2008).

As a result of our data analysis we can graphically investigate the dynamics of the main covariates affecting wages and wage losses. Looking at the estimation results we underline that the effect of full-time work experience in females’ biographies follows the assumed non-linear, concave shape and matches therefore with economic theory given for instance in Becker (1993).

In addition, the duration of the employment interruption due to childcare affects the wage differently, depending on the levels of educational achievement. Most effects of the considered covariates reveal different labour market characteristics for mothers and non-mothers, modifying results of previous studies. Our models allow for an advanced analysis and interpretation compared to traditional parametric models which are likely to misspecify several coefficients and therefore lead to questionable conclusions concerning the underlying economic theory.

The paper is organized in five Sections: in Section 2 we give details about the data used in this paper and show some descriptive statistics. Section 3 introduces penalized spline smoothing within additive mixed models and explains the resulting statistical models. Section 4 gives the data analysis resulting from the models before we conclude in Section 5.
In this paper we analyse female wages from three different aspects, as sketched in Figure 1: First, we model the wage a female receives in her working-biography if she has not given birth to a child (yet). This wage profile is drawn as solid line in Figure 1 and is defined as the reference model referring to the population of the childless women. Secondly, we consider females pausing their professional activity at timepoint $t_{\text{leave}}$ due to maternity leave.
and possible subsequent child care. We model the wageloss a woman has often to accept when she stops working at $t_{\text{leave}}$ and finally consequently starts working again at $t_{\text{return}}$. We define the wageloss as the (log) ratio of the wage earned when returning into a job in relation to the wage received before leaving the labour force temporarily at $t_{\text{leave}}$. We refer to this as the wageloss model. Finally, we capture the post-birth wage profile of mothers with restauration of human capital resulting in a possible catch-up to the wage level of the childless population. It is worth noting that $t_{\text{return}}$ is defined as the timepoint in females work-experience of final return into the labour market with an assumed completed family planning. Due to this definition, $t_{\text{leave}}$ can equal $t_{\text{return}}$ if the mother stays out off the labour market for exclusive childcare and returns without additional gains in labour force experience. In contrast, taking multiple maternity leave periods off the job with multiple reentries into employment leads to $t_{\text{return}} > t_{\text{leave}}$, due to additional labour force experience gained since $t_{\text{leave}}$. We abstract from analyzing the working-behaviour between $t_{\text{leave}}$ and $t_{\text{return}}$ as being sketched in the gray box in Figure 1 in detail. Instead, we make use of the additionally accumulated labour force experience by modelling the post-birth wage profile for the population of mothers if $t_{\text{return}} > t_{\text{leave}}$, named the restauration model throughout the paper.

The analysis is based on data collected annually by the GSOEP, covering the years from 1984 to 2008. If available, we make use of the provided monthly spell data on individual labour force status, too. As response variable for modelling wage profiles and wagelosses we calculate the hourly gross wages of females earned at the labour market. We use the contract-based weekly
working hours to receive reliable values of the wages. This restriction excludes females being self-employed in the current year. As second aspect of the wage, we use the monthly gross labour income in euros. The income is inflation-adjusted by the German consumer price-index to the year 1995 and is provided by the GSOEP in euros.

As covariates we consider variables which are assumed to influence the individual wage in the context of human capital theory directly, as used by Mincer and Ofek (1982) and Becker (1993). We therefore abstract from variables which are assumed to influence the wage indirectly or rather focus on the reentering-decision, like the income of the spouse or the availability of childcare facilities. For a modelling of duration time of maternity leave and the point of time reentering the labour market we refer to Kuhlenkasper and Kauermann (2010).

Human capital is generally accumulated by the educational achievements or the years of schooling and the labour force experience. See Ben-Porath (1967) and Mincer (1974). As a proxy for years of schooling and vocational training we divide the population into three different strata by looking at the achieved educational level of a female in the current year. The educational attainment of a woman is measured with the „International Standard Classification of Education (ISCED)“ which is consistently available in the version of 1997 for the GSOEP data and used for our analysis, see UNESCO Institute for Statistics (2006) and Projectgroup SOEP - DIW (2009) for details and definitions. Our analysis is carried out separating the females into three different groups of ISCED-levels: a lower group consisting of levels 1 and 2, a medium group consisting of levels 3 and 4 and finally a group with levels 5 and 6 represent-
ing high educated females\(^1\). The separation into three different strata is due to assumed major structural differences appearing in both, the population and the labour market depending on the different educational attainments. Taking this differentiation into account we will estimate the models for the three strata of the corresponding population separately. In addition to the education, the experience at the labour market and the experience within the current firm a female has collected are postulated to be major determinants when analysing wages with respect to general and firm-specific human capital. The past labour market experience is separated into full- and part-time experience of the individual up to the current year. The variables \textit{fulltime} and \textit{parttime} give the entire time period of full- and part-time employment in the females’ career up to the annual time point of the interview, respectively. To capture the stock of firm-specific human capital we include \textit{firm-time} comprehending the years an employee has been working in the current firm. To contrast the withdrawal from the labour market due to childcare to unemployment spells of the female, we include the variable \textit{unemployment} expressing the years a female has been registered unemployed and hence is off the labour market involuntarily. By taking individual unemployment into account, the model can also capture negative stigma-effects on wages related to unemployment, discussed in Belzil (1995). In all models being employed, we use the experience-variables \textit{fulltime}, \textit{parttime}, \textit{unemployment} and \textit{firm-time} for the entire recorded biography of females. Therefore we can capture possible temporarily reentries to the labour market between \(t_{\text{leave}}\) and \(t_{\text{return}}\)

\(^1\)We exclude females with ISCED-level 0, which represent young females still being in school and therefore not available for the labour market full-time.
without going in detail for that timeperiod. In addition we therefore allow for restoration of previously accumulated human capital with possible catch-up in wages.

While the above introduced exogenous variables are common in economic research to analyze wages, the inclusion of the weekly working hours as an explanatory variable is heavily discussed. Mincer and Polachek (1974) and Beblo and Wolf (2002) control for the working hours and find evidence about its importance when analyzing wages. However, the amount of working hours can either be interpreted as an endogenous or as an exogenous variable. According to classical economic theory, females allocate their time between labour work, work at home and leisure. As Polachek and Siebert (1993) point out, an increase of wages can lead to an increase in supply of labour work due to high opportunity costs of staying at home. When analyzing male working-supplies in contrast, substitution effects are revealed between labour work and leisure time. Females however often face the responsibility for the work at home as a third usage of time, which results in wage elasticities depending on how the work at home is substitutable to other time usage. In addition to the substitution effect of time-patterns, a causal relationship from the wage to the working hours can also be due to income effects: females earning low wages can be forced to work more hours to achieve a satisfactory monthly income. As a result, Mincer and Polachek (1974) estimated a negative relationship of wages on the working hours. While the income effects can be observed primarily for single-females, in households containing married couples, the substitution effect dominates. The progressive tax-system in Germany can be an additional reason of low
incentives for married women to increase their amount of labour work. In Germany, as the Federal Bureau of Statistics found out, the time supply of labour work is heavily dependent on the availability and the opening-hours of childcare facilities. As a result, the proportion of mothers working full-time or at least with increased working hours starts rising again when the oldest child living in the household has reached the entering age of public schools. In Germany therefore, it is reasonable to treat the amount of working hours as an exogenous variable effecting the wage of females and not vice versa, see also Wolf (2002) and Statistisches Bundesamt (2007). The resulting covariate $w_{hours}$ can also be interpreted as the degree of labour-market attachment: the more hours a female works, the more attached to labour market is she assumed to be. A higher labour market attachment might result in more frequent offerings to participate in further professional trainings, since the employer can expect higher returns on his investment due to more working hours of the female within the firm. To link the change in working hours with the change in earned wages at $t_{\text{return}}$, we include the ratio of post-birth working hours to pre-birth working hours ($\text{ratio}_{w_{hours}}$) when modelling the wageloss. A low ratio indicates therefore a severe reduction in working hours when returning to a job with possible consequences for the wage earned. Due to the postulated time-dependency of absolute human-capital depreciation, we include the length of maternity leave as covariate ($\text{duration}_{off}$) when modelling the wageloss, too. In this paper we interpret maternity leave as voluntarily and temporarily withdrawal from the labour market. As maternity leave we therefore define the period off the job due to pregnancy in the last weeks prior to the birth plus the time subsequently after childbirth.
staying at home. With this definition maternity leave is not restricted to a fixed job-protection-period granted by law in Germany and also includes unpaid maternity leave due to childcare where the mother stays voluntarily at home. For a discussion concerning the german legal framework we refer to Gutierrez-Domenech (2005) and Buchner and Becker (2008). According to the GSOEP-questionaires and the provided monthly spell data we distinguish between maternity leave, housewife and registered unemployment. A time period being a housewife following immediately after being in maternity leave is interpreted as a voluntarily elongation of maternity leave while being registered unemployed is treated as an involuntarily change of the labour force status. Besides the individual covariates introduced so far, the personal economic situation is likely to be affected by general economic performance. We therefore include the year of observation as an exogenous variable to capture effects of business-cycles between 1984 and 2008. All covariates introduced above are metrical and will be included in each of the models in a functional form, allowing for smooth and dynamic effects if they are existing. In addition to the above introduced metrical covariates, we create dummy-variables, leading to semi-parametric mixed models. Besides firmtime capturing firm-specific human capital, we estimate fixed effects for the size of the company the female is currently working in, with small.firm and large.firm indicate whether the female is either employed in a company with less than 20 employees or at least with 2000 employees, respectively. While controlling for business-cycle effects due to the year of the observation, we add the dummy-variable south.germany to the models, indicating if the female is living in either Bavaria, Baden-Württemberg or Hessen. These three southern states
are assumed to perform economically stronger than others states in Germany and might therefore honour human capital differently. For the population of the mothers we also created a dummy-variable indicating whether the mother has given birth to more than one child before finally returning into work.

Our analysis is restricted to females living in West-Germany without having a migrational background. Although we could include females living in East-Germany and/or having a migrational background in our analysis, it is not the intention of this article to estimate interesting but well known negative effects of the corresponding covariates indicating wage penalties. As already mentioned above, self-employed females lack reliable and valid values for the current contract-based weekly working hours and can not be included. We also exclude females who have been already mothers when joining the GSOEP and therefore lack data on their last maternity leave and the related covariates. Observations from females being younger than 18 years of age or older then 55 years are excluded, too. This ensures a reliable comparison between working mothers and working non-mothers. Observations are excluded from our analysis if a woman has been on her last maternity leave for more than 6 years. At this time children have reached the entering age of public schools in Germany and voluntarily intensive childcare has no longer to be provided by the mother. This restriction excludes especially several low educated females from the dataset. Nevertheless, the exclusion is inevitable since coming back to the labour market after more than six years reveals a completely changing preference towards work, impeding to draw valid conclusions for labour-attached females from the results in section 4.
2.2 Wage Losses and Female Work Behaviour

Looking at the dataset we find descriptive evidence about a wageloss due to child related employment interruptions: while highly educated females have to face the highest absolute wage-losses due to a higher average wage-level, the percentage of losses is almost constant through the ISCED-strata: females having high, middle and low educational attainments loose on average 12%, 13% or 9% of their pre-birth wage, respectively. In contrast, the covariates assumed to affect these losses are different for the subsets as analyzed in Section 4. Looking at the labour force status of the females, we find that childless women work with an average of 36.6 hours per week with only a standard deviation of 6 hours. With 35.7 hours, 36.7 hours and 37.3 hours for high, middle and low educated females, respectively, full-time employment is dominant for females who are not mothers. In contrast, the average working hours decrease after childbirth to an average of 23.9 with standard deviation of 9.2. With 23.4 hours, 24 hours and 23.7 hours, respectively, working hours do not change significantly with the ISCED-strata.

When analyzing female working behaviour, the timing of births has to be considered as well. We refer to Wetzels (2001) and Bloemen and Kalwij (2001) for a discussion. Females in our dataset give birth to their first child with an average age of 28.2 years after having worked fulltime for on average 5.6 years. However, the timing is likely to differ between the employed strata: Highly educated women become mothers on average being 30.7 years old after having spended 5.8 years in fulltime-employment. Women belonging to the middle ISCED-strata give their first birth on average being 28 years old and having worked fulltime for 6.1 years. Finally, low educated females
become mothers being only 24.8 years old after fulltime experience of 3.7 years on average. Differences can also be found by looking at the timepoint of final return into the labour market, corresponding to $t_{\text{return}}$. On average, mothers return finally into employment being 30.5 years old with 5.9 years of fulltime workexperience. However, highly educated mothers return with 6.2 years of fulltime experience being 33 years old on average. Mothers belonging to the middle strata return on average being only 30.1 years old but having accumulated 6.3 years of fulltime labour force experience. Those low educated females returning to the labour market finally are on average 27.7 years old with only 4.1 years of experience gained when working fulltime. In the remaining of the paper we will model the wagelosses and the wages using functional approaches to capture non-linear covariate effects.

3 Theoretical Background

3.1 Additive Mixed Models and Spline Smoothing

Classical regression models assume that a response or endogeneous variable $y$, respectively, depends on some covariates $x_1, \ldots, x_p$ in a linear fashion

$$y = \beta_0 + x_1\beta_1 + \cdots + x_p\beta_p + \epsilon,$$

where $\epsilon$ is a random noise error usually assumed to be normally distributed. While the linear approach is simple it is certainly too simplistic for our covariates at hand. Instead, letting $x_1, \ldots, x_q$ with $q < p$ denote metrically scaled covariates (like full-time work experience) we replace the linear structure by
a functional form
\[ y = \beta_0 + f_1(x_1) + \ldots + f_q(x_q) + x_{q+1}\beta_{q+1} + \ldots + x_p\beta_p + \epsilon. \] (1)

Here \( f_j(x_j) \) are smooth but otherwise undetermined functions to be estimated from the data. Models of class (1) have been coined Generalized Additive Models by Hastie and Tibshirani (1990) and are extensively discussed in Wood (2006), see also Ruppert, Wand & Carroll (2003, 2009). Apparently, model (1) itself is not identifiable since the offset can go in any function. One therefore needs the further constraint that \( f_j(x_j) \) integrates out to zero with respect to the (empirical) distribution function of \( x_j \). Fitting of model (1) will be carried out with penalized spline smoothing. The idea is thereby to replace function \( f_j(x_j) \) by some high dimensional basis representation

\[ f_j(x_j) = B_j(x_j)b_j, \]

where \( B(.) \) can be taken as cubic smoothing spline, see Wahba (1978). To reduce the computational burden the use of so called “low rank smoothers” has proven to be reliable and stable, which explains the dominance of the routine in available software. Note that since basis \( B_j(.) \) is high dimensional the resulting fit will be poor unless we impose a penalty in coefficient vector \( b_j \). The common choice is to work with quadratic penalties of the form \( \lambda_j b_j^T D_j b_j \) with \( D_j \) as penalty matrix (see Wood (2006) for more details) and \( \lambda_j \) as penalty parameter. Using cubic smoothing splines it can be shown that the quadratic form penalizes the integrated squared second order derivative of function \( f_j(.) \).

Following Wahba (1978), Wong and Kohn (1996) or Wood () we can interpret the quadratic penalty as prior on the spline coefficients in the form \( b_j \sim \)
\( N(0, \lambda_j^{-1}D_j^{-1}) \), which replaces the additive model (1) by

\[
y|b_1, \ldots, b_j \sim N \left( \beta_0 + \sum_{j=1}^{q} B_j(x_j)b_j + \sum_{j=q+1}^{p} x_j \beta_j, \sigma^2_\epsilon \right)
\]

\[ b_j \sim N(0, \lambda_j^{-1}D_j^{-1}), \ j = 1, \ldots, q. \]  

(2)

The Bayesian formulation resulting from (2) is well established under the phrase Linear Mixed Model in statistics, see e.g. Searle et al. (1992) or McCulloch and Searle (2001) and estimation can be easily carried out with maximum likelihood theory. In fact, integrating out \( b_j \) in (2) gives the likelihood and we can comprehend \( \sigma^2_\epsilon, \lambda_j, j = 1, \ldots, 1 \) as well as \( \beta_j, j = q + 1, \ldots, p \) as parameters. This is implemented in available software, where we make use of R, see Pinheiro and Bates (2000) and R Development Core Team (2010)

For our data analysis where we have multiple observations per individual we supplement model (2) by introducing an individual specific random effect. This takes on the one hand unobserved heterogeneity in the data into account and secondly controls for serial correlation. To be more specific we replace model (2) by

\[
y_{it}|b_1, \ldots, b_j \sim N \left( \beta_0 + \sum_{j=1}^{q} B_j(x_j) b_j + \sum_{j=q+1}^{p} x_{jit} \beta_j + \gamma_{i0}, \sigma^2_\epsilon \right)
\]

\[ b_j \sim N(0, \lambda_j^{-1}D_j^{-1}), \ j = 1, \ldots, q \]

\[ \gamma_{i0} \sim N(0, \tau^2_0), \]  

(3)

where indices \( it \) refer to the \( t \)-th observation drawn from the \( i \)-th individual. Here \( \gamma_{i0} \) is the latest individual effect. Though model (3) is a conceptually serious extension of model (2) it is again a Linear Mixed Model and hence fitting is done in the same fashion and with the same software.
3.2 Statistical Models

To model and to estimate female wages and wage losses we now employ the above introduced (Generalized) Additive Mixed Models. Economically the models trace back to Mincer (1974) and Mincer and Ofek (1982). Models (4) to (6) are each estimated based on data with respect to the achieved ISCED group with index $i$ referring to the individual in the data set and index $t$ referring to the timepoint of observation. Model 4 shows the assumed relationship for the reference model.

$$
\log(wage_{it}) = f_1(\text{fulltime}_{it}) + f_2(\text{parttime}_{it}) + f_3(\text{unemployed}_{it}) \\
+ f_4(\text{firmtime}_{it}) + f_5(\text{whours}_{it}) + f_6(\text{Year}_{it}) \\
+ \beta_0 + \beta_1 \text{small.firm}_{it} + \beta_2 \text{large.firm}_{it} \\
+ \beta_3 \text{south.germany}_{it} + \gamma_{i0} + \epsilon_{it} \quad (4)
$$

with $\gamma_{i0} \sim N(0, \tau_0^2)$ and $\epsilon_{it} \sim N(0, \sigma^2)$. Here $f_1(.)$ is the smooth but otherwise unspecified effect of full-time work experience and according definitions for the remaining functions.

In contrast, we model the wage loss due to childbirth related employment interruptions by

$$
\log(\text{ratio.wages}_{it_r}) = f_1(\text{fulltime}_{it_r}) + f_2(\text{parttime}_{it_r}) + f_3(\text{unemployed}_{it_r}) \\
+ f_4(\text{firmtime}_{it_r}) + f_5(\text{ratio.whours}_{it_r}) \\
+ f_6(\text{duration.of f}_{it_r}) + f_7(\text{Year}_{it_r}) \\
+ \beta_0 + \beta_1 \text{small.firm}_{it_r} + \beta_2 \text{large.firm}_{it_r} \\
+ \beta_3 \text{south.germany}_{it_r} \\
+ \beta_4 \text{more.than.one.Kid}_{it_r} + \epsilon_{it} \quad (5)
$$
with \( \text{ratio.wages}_{it} = \text{wage}_{it}/\text{wage}_{it} \) and \( \text{ratio.whours}_{it} = \text{whours}_{it}/\text{whours}_{it} \) and \( \epsilon_{it} \sim N(0, \sigma^2) \). Here, \( t_r \) and \( t_l \) define the year of returning to the labour market and the year of leaving the job due to childbirth, respectively, see Figure 1. In the wageloss model (5) we do not have to control for unobserved heterogeneity since females enter the model with only one observation each when they entered the labour market finally again.

Finally, modelling the wage profile for the population of the mothers (restauration model) differs in (6) by the amended dummy-variable indicating multiple births. To be specific

\[
\log(\text{wage}_{it}) = f_1(\text{fulltime}_{it}) + f_2(\text{parttime}_{it}) + f_3(\text{unemployed}_{it}) \\
+ f_4(\text{firmtime}_{it}) + f_5(\text{whours}_{it}) + f_6(\text{Year}_{it}) \\
+ \beta_0 + \beta_1 \text{small.firm}_{it} + \beta_2 \text{large.firm}_{it} \\
+ \beta_3 \text{south.germany}_{it} + \beta_4 \text{more.than.one.Kid}_{it} \\
+ \gamma_{i0} + \epsilon_{it} \tag{6}
\]

with \( \gamma_{i0} \sim N(0, \tau_0^2) \) and \( \epsilon_{it} \sim N(0, \sigma^2) \) where again \( f_j(.) \) are smooth functions.

The resulting smooth effects will be displayed on the scale of the linear predictor with pointwise 2-standard-error confidence lines for each estimation. The estimation of the models 0 to 2 is carried out in R using the package \texttt{gamm4}, which is based on the packages \texttt{mgcv} and \texttt{lme4}. See Wood (2010a), Bates and Maechler (2010) and Wood (2010b) for the packages, respectively.
4 Data analysis

The aim of this section is to analyse the effects of the metrical and \( \{0,1\} \)-covariates on the wages for mothers and non-mothers and on the wage-loss for mothers just having reentered the labour market after child related employment interruptions. Figures 2 to 4 show the empirical results of the estimated functional effects for the employed models, while Table 1 to reftab:mod2 give the estimation results for the parameters \( \beta_j \) and \( \tau^2_0 \). For all estimated models in Figures 2 to 4, we added the estimated intercept \( \hat{\beta}_0 \) to the major smooth effect of the experience gained working full-time (\( \hat{f}_1(\text{fulltime}) \)). This allows interpreting the smooth estimated effects below the first rows in the figures as deviation from this effect and additionally we can identify the different wage-levels for the three ISCED strata.

**reference model:**

While the effect of the full-time work experience indicates a clear increasing, but concave-running shape for all ISCED-levels, it is not surprising to see higher wages for higher educated women. Although females belonging to this strata earn higher average wages, the increase of the effect is less sharp in the first six years. In contrast, working part-time in the past seems to have weaker, but negative effects on the wage, strengthening with the length of experience gained in working with reduced weekly working hours. Time spented in unemployment starts to have negative effects after an accumulated duration of more than one year. Females having achieved middle and low educational attainments can profit from a short period of being unemployed, but they are penalized for longer durations, too. While almost every woman has to face negative effects on her wage when being employed for
only up to five years in the current company, staying longer than 20 year
in the same job is not rewarded to low-educated females, resulting in severe
wage penalties as the duration exceeds 20 years of company affiliation. Sur-
prisingly, working part-time currently with contract-based working hours less
than 35 is rewarded by positive effects on the wage. This stays in contrast to
the effects of the above analyzed duration of past part-time periods. Besides
this, the general economic performance, captured by the effect of the current
year in the last row, postulates positive and even rising effects on the wage
up to the year 1999, followed by a sharp decline until 2004 with recovery on a
low level thereafter. In Table 1 we find evidence about honouring individual
human capital by large firms with more than 2000 employees and by working
in one of the three southern german states. Wage penalties have therfore to
be taken into account when working in companies with less than 20 employees
and/or in northern West-Germany. These fixed estimated effects are almost
identical for all three ISCED-subsets. The reference model is estimated for
964 females with high educational achievements contributing 4203 observa-
tions, for 2450 females belonging to the middle ISCED-subset with 10542
observations and for 1230 low educated females with 3532 observations.

wageloss model:

Figure 3 and Table 2 give the results for modelling the wageloss. Therefore,
high and average educated mothers can attenuate their (relative) wageloss
due to child related employment-breaks by placing births at the beginning
of their (full-time) work career. For low educated females, this conclusion
can not be drawn. The part-time experience gained at the labour market
seems to have no major effect on the relation of post-wages to pre-wages.
Having not been unemployed is rewarded when reentering the labour market consistently over all three ISCED-subsets. In contrast to the past full- and part-time experience, giving birth to a child when only being employed in the current firm for a short period of time leads to wage penalties, while the ratio of the working hours does not seem to have significant effects on the wageloss. High and middle educated females can attenuate their losses by returning to the labour market within one year after leaving the job due to childcare. Note that this time period includes the weeks anteceding giving birth due to laws on maternity protection. Like the wage of non-mothers in the reference model, the year of return seems to influence the wage-ratio of mothers with decreasing and even negative effects from 1999 to 2004. In contrast, low educated mothers can not profit from a possible recovery after 2004. Highly educated females can attenuate their wageloss by returning into a large company while low educated mothers face heavy wage penalties when being employed in a large company. In contrast to model 0, giving birth to a child in southern germany is likely to aggravate the loss for high and average educated females. However, bearing more than one child while being off the labour market temporarily has severe negative effects on the wage only for high and low educated mothers. The wageloss model is estimated for 507 females having high educational attainments, for 900 females belonging to the middle ISCED-strata and for 185 low educated mothers, contributing one observation each.

restauration model:
After having returned to the labour market, highly educated mothers do not catch up with their wages quickly, as the slow increasing effect of the
newly gained full-time and part-time experiences in Figure 4 indicates. While the ongoing full-time experience influences post-birth wages positively for all three ISCED-groups, newly gained part-time experience is not rewarded to middle and low educated mothers. However, having been employed with reduced weekly working hours in the past can lead to wage increases for highly educated mothers if this experience exceeds five years. Past unemployment results in severe wage-penalties for high and middle educated females. The firm affiliation and therefore the firm-specific human capital seems to loose importance in post-birth employments as the almost non-significant effects of the covariate \textit{firtime} indicate. Interestingly, highly educated females face a weak, but decreasing effect as the time in the current firm continues. In contrast, wages from mothers coming from the middle and low ISCED-strata are affected about the same as the childless women by the time in the firm. The amount of contract-based working hours seems to influence the wages of highly educated females strongly: working more than 25 hours a week (abstracting from overtime) is penalized, while these negative effects are visible for middle and low-educated females only if they are employed for working more than 35 hours and therefore working almost fulltime. As a result, highly educated mothers can profit from working less than 25 hours according to their wages. This holds for past and current parttime employment. The business-indicator in the last row is almost identical to the findings in the reference model. In addition, Table 3 displays negative wage effects when working in small firms. Living in southern germany leads to a wage plus only to highly and average educated females. Interestingly, giving birth to more than one child is only penalized when having achieved a high ISCED-level.
\( \hat{\beta}_4 \) can also be interpreted as a proxy effect of multiple interruptions between \( t_{\text{leave}} \) and \( t_{\text{return}} \) since only a small minority of mothers give birth to more than one child within one maternity leave period. The restauration model is estimated with 881 observations coming from 201 highly educated mothers, with 2181 observations contributed by 429 average educated females and with 439 observations due to 106 mothers with low educational attainment.

5 Conclusion

The fitted smooth and parametric effects of the employed covariates reveal a multi-dimensional framework for analyzing females’ wages and the consequences resulting from employment interruptions due to childcare. The different pattern of covariate effects as displayed in Figures 2 to 4 are likely to have their origin in structural differences in the labour market for more or less educated females. These findings support the use of strata referring to the educational achievements of the women. The theoretical assumptions postulating an increasing, but concave-running effect of the labour market experience is met when focussing on full-time labour experience for the population of childless females. However, this does not hold for mothers with average or low educational achievements who reveal an increasing but rather linear effect of full-time experience. Highly educated mothers can not thereby restaurate their human capital accumulated prior to birth quickly, as indicated by a less sharp increase of the effect after giving birth to a child. The most interesting effects can be found by comparing past and current part-time employment. While non-mothers are likely to be penalized having worked
part-time in their working biography, current part-time employment can lead to wage increase, even for low educated females who are assumed to work in jobs with a rather lousy image. In contrast, highly educated mothers can profit from working part-time currently and by having worked with reduced working hours in their work biography. The return of these ”high potentials” might be an important goal of employers willing to avoid high costs due to searching for new females and make use of the firm-specific human capital the woman has built up before withdrawal. This demand of highly-educated mothers is valued by higher wages, even if they only want to work part-time. In contrast, mothers without high educational attainments seem to be more substitutable in their firm and perform poorer when reentering the labour market, indicated by less valued work-experience for both, full- and part-time work. Again, periods of unemployment after leaving the labour due to childcare result in wage penalties. A negative stigma-effect on unemployment is likely to force wages down for mothers and non-mothers. For mothers however, being unemployed in addition to voluntarily withdrawal from the labour market due to childcare might signal low labour market attachment to the employer. Surprinsingly, the time working in a company as a proxy for specific human capital yields lower rates of return, compared to the rates of return gained from accumulated general human capital. The all over weak effect of time worked in the current company is almost vanishing for mothers trying to gain a foothold in the labour market again. However, the size of the firm seems to influence the wages since large firms pay higher wages. Highly educated females can loose this advantage of being employed in large companies if they decide to have more than one children. The per-
sonal human capital seems to determine the individual wages significantly but the time-period a woman works in seems to influence her wage, too. The sharp decline from 1999 to 2004 is prominent, no matter what educational attainment the woman has. Interestingly, no catch-up in wages can be found for low-educated females after 2004.

While the reference and the restauration models are likely to describe the wages of females quite well, underlining the findings of Mincer (1974) and Becker (1993), trying to model the wage-loss is a more difficult task. This underlines the assumption of interpreting wage-losses as a net-depreciation of the individual stock of human capital rather than being influenced by several covariates. However, the wage-loss model reveals some factors which can influence the loss. The discussion of the timing of births in females’ biographies, as carried out for example in Bloemen and Kalwij (2001), Bernardi et al. (2008) and Kunze (2002), often reveals a delay in giving birth to children for higher educated women. With our findings however, the average relative wage-loss can be attenuated by timing the pregnancy in the first two years of full-time work. However, highly educated mothers can profit from early maternity leave periods in their career by working part-time when returning into the labour market. Note, that this conclusion on wage-losses are based upon the relative rather than the absolute wage-loss. When focussing on absolute earnings, full-time employment is likely to be dominant by single mothers with the sole responsibility for their income. In the wage-loss model, the attachment of the mother to the firm being employed in seems to play a more important role compared to the effects in the profile models. Bearing a child after having worked for at least five years in the current company
can attenuate the relative wage loss in all three ISCED-groups. The above mentioned demand for highly educated females after maternity leave by their employers is underlined in the wage loss model. Low educated females can not make profit from quickly returning to the labour market. This might explain the rather long duration times of maternity leave for this subset and the low proportion of these women ever returning finally to the labour market. The common hypothesis of increasing wage penalties as the duration of leave holds on, can not be supported by our findings, since the effect vanishes as the duration exceeds two years.

The findings of Mincer (1974) and Becker (1993) are verified in general by our modelling exercise. Although many effects fit to the widely accepted coherence of determining wage profiles, our analysis reveals both, some deviations characterising the German labour market for females and the flexibility and capacity of penalized spline smoothing as estimation routine for functional data. In addition, our results show that the wages of German females are not solely dependend on the individual human capital but are also influenced for example by the business cycle and even by the location of living in Germany. The functional approach pursued in this article lays open some non-linearities which are hard to anticipate a priori as well as some vanishing effects in the higher domain of some exogenous variables. The changing role allocation of males and females and their converging time patterns when taking care of a child is left to further research. We refer to Gutierrez-Domenech (2005) and Burgess et al. (2008) for a discussion. We recommend however using a functional approach to get interesting insights of families, not only in Germany.
<table>
<thead>
<tr>
<th>effect</th>
<th>high ISCED</th>
<th>middle ISCED</th>
<th>low ISCED</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>2.93 (&lt; 0.01)</td>
<td>2.61 (&lt; 0.01)</td>
<td>1.99 (&lt; 0.01)</td>
</tr>
<tr>
<td>small.firm</td>
<td>-0.04 (&lt; 0.01)</td>
<td>-0.07 (&lt; 0.01)</td>
<td>-0.13 (&lt; 0.01)</td>
</tr>
<tr>
<td>large.firm</td>
<td>0.04 (&lt; 0.01)</td>
<td>0.07 (&lt; 0.01)</td>
<td>0.13 (&lt; 0.01)</td>
</tr>
<tr>
<td>south.germany</td>
<td>0.06 (&lt; 0.01)</td>
<td>0.04 (&lt; 0.01)</td>
<td>0.10 (&lt; 0.01)</td>
</tr>
<tr>
<td>Var(γ_i0)</td>
<td>0.12</td>
<td>0.10</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 1: parametric estimation results from the reference model

<table>
<thead>
<tr>
<th>effect</th>
<th>high ISCED</th>
<th>middle ISCED</th>
<th>low ISCED</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-0.09 (0.09)</td>
<td>-0.27 (&lt; 0.01)</td>
<td>0.01 (0.94)</td>
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<tr>
<td>small.firm</td>
<td>-0.01 (0.92)</td>
<td>-0.09 (0.1)</td>
<td>-0.27 (0.02)</td>
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<tr>
<td>large.firm</td>
<td>0.16 (0.03)</td>
<td>-0.03 (0.62)</td>
<td>-0.43 (&lt; 0.01)</td>
</tr>
<tr>
<td>south.germany</td>
<td>-0.19 (0.02)</td>
<td>-0.1 (0.05)</td>
<td>-0.02 (0.84)</td>
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<tr>
<td>more.than.oneKid</td>
<td>-0.22 (&lt; 0.01)</td>
<td>0.02 (0.74)</td>
<td>-0.19 (0.12)</td>
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</tbody>
</table>

Table 2: parametric estimation results from the wageloss model
<table>
<thead>
<tr>
<th>effect</th>
<th>high ISCED</th>
<th>middle ISCED</th>
<th>low ISCED</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>2.89 (&lt; 0.01)</td>
<td>2.56 (&lt; 0.01)</td>
<td>2.44 (&lt; 0.01)</td>
</tr>
<tr>
<td>small.firm</td>
<td>-0.11 (&lt; 0.01)</td>
<td>-0.12 (&lt; 0.01)</td>
<td>-0.33 (&lt; 0.01)</td>
</tr>
<tr>
<td>large.firm</td>
<td>0.11 (&lt; 0.01)</td>
<td>0.02 (0.17)</td>
<td>-0.03 (0.45)</td>
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<tr>
<td>south.germany</td>
<td>0.07 (&lt; 0.01)</td>
<td>0.05 (&lt; 0.01)</td>
<td>0.01 (0.75)</td>
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<tr>
<td>more.than.one.Kid</td>
<td>-0.22 (&lt; 0.01)</td>
<td>-0.06 (&lt; 0.01)</td>
<td>-0.03 (0.47)</td>
</tr>
<tr>
<td>Var(\gamma_0)</td>
<td>0.18</td>
<td>0.17</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 3: parametric estimation results from the restauration model

References

Bates, D. and M. Maechler (2010). *lme4: Linear mixed-effects models using S4 classes*. R package version 0.999375-34.


Wood, S. Thin plate regression splines.


Figure 2: Fitted dynamic effects of the reference model for high, average and low educated females (first, second and third column respectively.)
Figure 3: Fitted dynamic effects of the wageloss model for high, average and low educated mothers (first, second and third column respectively.)
Figure 4: Fitted dynamic effects of the restauration model for high, average and low educated mothers (first, second and third column respectively.)
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