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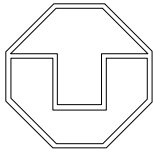
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TECHNISCHE UNIVERSITÄT DRESDEN

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**National vs. International Welfare Effects
of Horizontal Mergers**

by

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National vs. International Welfare Effects of Horizontal Mergers

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Abstract

Empirical evidence suggests that many mergers do not increase profits of the participating firms and decrease welfare. Due to the globalization of markets we should take an international view on mergers and their welfare effects. This paper develops a Bertrand-model of an international heterogeneous market. It shows that there are neither positive incentives to merge nor positive international welfare effects from a merger if there are no positive synergy effects. Furthermore, it shows that national welfare effects are bigger than international ones when there are many domestic firms relative to domestic demand.

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1 Introduction

Mergers of enterprises always played a very important role in the development of the different economic sectors. While markets get more and more international, this role becomes even more relevant. However, it is not clear from the outset in what way this increasing internationality of markets influences economic policy judgments.

There are two basic effects resulting from the globalization of markets: first, international competitors of the merging enterprises have to be considered in addition to national ones. Second, the decisions of an enterprise affect the international consumers as well as national ones.

But to what extent are these effects reflected by policy decisions? On the one hand, a small country like Switzerland might have less difficulties in agreeing with a merger of two chemical giants like Ciba and Sandoz (1996) than a larger country. With most of the consumers living abroad, negative effects of a merger on the consumer surplus were not be very dominant.

On the other hand, a generally very liberal country like the US might have incentives to be comparatively restrictive against enterprises like American Airlines or Microsoft. In both cases the share of the domestic demand is much larger than in the Switzerland example. Before we can investigate these effects more precisely, we have to identify the incentives to merge. Salant, Switzer and Reynolds (1983) show on basis of a Cournot-model under which circumstances firms profits decline after a merger. The merging companies lower their output to rise the product price. Their competitors are motivated by the rising product price to rise their output which lowers the product price again. As a consequence the resulting price might be too small to increase the profit of the merging firms.

Deneckere and Davidson (1985) develop a model of a heterogeneous market with Bertrand-competition. They show that mergers in this market lead to higher profits for the merging firms but their competitors' profits rise even more. However, the authors do not consider potential synergy-effects of mergers.

Mergers often lead to social costs. Besides dismissals caused by a merger the increasing product price and the drop in consumer surplus plays an important role.¹ Braid (1999) shows that merging enterprises in a heterogeneous market with Bertrand-competition rise their price. The increase is smaller if the heterogeneity is not only one- but two-dimensional.

Farrell and Shapiro (1990) explicitly consider synergy-effects in their examination of a merger in a Cournot-competition. They conclude, that all mergers with no synergy-effects lead to higher product prices. The authors point out that beside the consumer surplus the producer surplus should be considered when a merger has to be judged. A shortfall of this work is the unrealistic assumption of a Cournot-competition.

¹See Hay and Werden (1993), p. 173.

This work gets rid of this shortfall. Section 2 develops a model of a merger in a heterogeneous market with Bertrand-competition. Section 3 examines under which conditions an enterprise increases its profit by merging and under which conditions this increase is higher than the one of its competitors.

In section 4 the welfare effects of a merger are examined. It will be distinguished between the development of the national and the international welfare. It will be shown that under some conditions a national merger department would support too many mergers and under some conditions it will be too restrictive.

In section 5 the results are simulated for different hypothetical markets and section 6 concludes the basic results of the work.

2 The Model

2.1 The Pre-Merger Market

This section develops the model of a heterogeneous market. There are N firms in a Bertrand-competition, producing with the identical constant marginal- and average-costs of c . Following the work of Deneckere and Davidson (1985) each firm is facing the following demand:

$$q_i = V - p_i - \gamma \left(p_i - \frac{1}{N} \sum_{j=1}^N p_j \right), \quad i = 1, \dots, N \quad (1)$$

Therefore, the demand q_i of company i depends negative on its own price p_i and the price difference between its own price and the average price of the market weighted with $\gamma \geq 0$. If γ is zero, the products of the different firms are not substitutable and the firms act like monopolists. If γ is infinite the products are totally homogeneous² and the market-result would correspond to the Bertrand-Paradox.

If all prices are identical each consumer buys the product that serves him best. At a market-price of zero every firm can sell the quantity V and at a market-price of V the demand is zero. Therefore, the market only exists if $V > c$, which is assumed here.

The firms maximize the following profit-function by their prices:

$$\pi_i = (p_i - c)q_i \quad (2)$$

$$\Rightarrow p_i^* = \frac{V + [1 + \gamma(1 - \frac{1}{N})]}{2 + \gamma(2 - \frac{2}{N})} + \frac{\gamma \frac{1}{N} \sum_{j \neq i} p_j}{2 + \gamma(2 - \frac{2}{N})} \quad (3)$$

In the pre-merger market all firms have identical maximizing problems and all firms have identical optimal results. This leads to the following market-price, which is set by each of

²See Deneckere and Davidson 1985, p. 475.

the N firms:

$$p^* = \frac{V + c \left[1 + \gamma \left(1 - \frac{1}{N}\right)\right]}{2 + \gamma \left(1 - \frac{1}{N}\right)} \quad (4)$$

This price depends negative on γ and N . This is no surprise since γ represents the competition pressure on the enterprises and $N - 1$ is equivalent to the number of their competitors.

The corresponding quantities and profits are:

$$q^* = \frac{V - c}{2 + \gamma \left(1 - \frac{1}{N}\right)} \left[1 + \gamma \left(1 - \frac{1}{N}\right)\right] \quad (5)$$

$$\pi^* = \left[\frac{V - c}{2 + \gamma \left(1 - \frac{1}{N}\right)} \right]^2 \left[1 + \gamma \left(1 - \frac{1}{N}\right)\right] \quad (6)$$

Hence, the demand depends positive on γ and N . This is obvious since the demand depends negative on the price. Profits depend negative on γ and N . Here the same arguments apply as for the price.

2.2 The Merger

This subsection will examine in which way the prices, quantities, and profits of the market are affected by a merger of two firms. After two firms have merged their maximizing problems are changing. In contrast to the pre-merger situation they maximize not only their own but their joint profit. In addition to this some synergy-effects resulting from the merger will need to be considered. These synergy-effects may be positive but also negative.³

In the following it is assumed that the merging firms produce with the constant marginal- and average-costs $\frac{c}{k}$ after the merger, where $k > 0$. If $k > 1$ the synergy-effects are positive, if $k < 1$ they are negative, and if $k = 1$ there are no synergy-effects. The profit-function of each of the merging firms is therefore:

$$\pi_i = (p_i - \frac{c}{k})q_i \quad (7)$$

The demand-function does not change for the merging firms. The only difference results from the fact that they consider the price of the merging partner as a parameter for action. The merging partner is in the same situation, hence, both of the merging firms will set the same price p^m .

The other firms in the market (outsiders) are in a different situation. First, they consider only their own prices when maximizing their profits and second, they still produce with the costs c . All outsiders set the price p^o .

³See Perry and Porter (1985), p. 220.

The profit maximizing price of the merging firms could be expressed as a function of the outsider-price p^o :⁴

$$p^m = \frac{V + \frac{c}{k} [1 + \gamma (1 - \frac{2}{N})]}{2 + \gamma (2 - \frac{4}{N})} + \frac{\gamma (1 - \frac{2}{N})}{2 + \gamma (2 - \frac{4}{N})} p^o \quad (8)$$

The price of the merging firms therefore depends positive on the price of the outsiders. The outsiders in the market still maximize their profits like in the pre-merger situation stated in section 2.1. Their profit maximizing prices p^o could be expressed as a function of the merging-firms' price p^m :

$$p^o = \frac{V + c [1 + \gamma (1 - \frac{1}{N})]}{2 + \gamma (1 + \frac{1}{N})} + \frac{\gamma \frac{2}{N}}{2 + \gamma (1 + \frac{1}{N})} p^m \quad (9)$$

The price of the outsiders depends positive on the price of the merging firms. From equations (8) and (9) we get the following prices of the merging firms and the outsiders:

$$p^m = \frac{V [2 + \gamma (2 - \frac{1}{N})] + \frac{c}{k} [2 + \gamma (3 - \frac{3}{N}) + \gamma^2 (1 - \frac{1}{N} - \frac{2}{N^2})]}{4 + \gamma (6 - \frac{6}{N}) + \gamma^2 (2 - \frac{4}{N})} + \frac{c [\gamma (1 - \frac{2}{N}) + \gamma^2 (1 - \frac{3}{N} + \frac{2}{N^2})]}{4 + \gamma (6 - \frac{6}{N}) + \gamma^2 (2 - \frac{4}{N})} \quad (10)$$

$$p^o = \frac{V [2 + \gamma (2 - \frac{2}{N})] + c [2 + \gamma (4 - \frac{6}{N}) + \gamma^2 (2 - \frac{6}{N} + \frac{4}{N^2})]}{4 + \gamma (6 - \frac{6}{N}) + \gamma^2 (2 - \frac{4}{N})} + \frac{\frac{c}{k} [\gamma \frac{2}{N} + \gamma^2 (\frac{2}{N} - \frac{4}{N^2})]}{4 + \gamma (6 - \frac{6}{N}) + \gamma^2 (2 - \frac{4}{N})} \quad (11)$$

The price of the merging firms as well as the price of the outsiders depends negative on the synergy-parameter k . The quantities and the profits of the merging firms and the outsiders could be derived from these prices.

$$q^m = \left[\frac{V [2 + \gamma (2 - \frac{1}{N})] - \frac{c}{k} [2 + \gamma (3 - \frac{3}{N}) + \gamma^2 (1 - \frac{3}{N} + \frac{2}{N^2})]}{4 + \gamma (6 - \frac{6}{N}) + \gamma^2 (2 - \frac{4}{N})} + \frac{c [\gamma (1 - \frac{2}{N}) + \gamma^2 (1 - \frac{3}{N} + \frac{2}{N^2})]}{4 + \gamma (6 - \frac{6}{N}) + \gamma^2 (2 - \frac{4}{N})} \right] \left[1 + \gamma \left(1 - \frac{2}{N} \right) \right] \quad (12)$$

$$q^o = \left[\frac{V [2 + \gamma (2 - \frac{2}{N})] - c [2 + \gamma (4 - \frac{6}{N}) + \gamma^2 (2 - \frac{6}{N} + \frac{4}{N^2})]}{4 + \gamma (6 - \frac{6}{N}) + \gamma^2 (2 - \frac{4}{N})} + \frac{\frac{c}{k} [\gamma \frac{2}{N} + \gamma^2 (\frac{2}{N} - \frac{4}{N^2})]}{4 + \gamma (6 - \frac{6}{N}) + \gamma^2 (2 - \frac{4}{N})} \right] \left[1 + \gamma \left(1 - \frac{1}{N} \right) \right] \quad (13)$$

⁴A more detailed derivation is located in the appendix A.

$$\pi^m = \left[\frac{V \left[2 + \gamma \left(2 - \frac{1}{N} \right) \right] - \frac{c}{k} \left[2 + \gamma \left(3 - \frac{3}{N} \right) + \gamma^2 \left(1 - \frac{3}{N} + \frac{2}{N^2} \right) \right]}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} + \frac{c \left[\gamma \left(1 - \frac{2}{N} \right) + \gamma^2 \left(1 - \frac{3}{N} + \frac{2}{N^2} \right) \right]}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} \right]^2 \left[1 + \gamma \left(1 - \frac{2}{N} \right) \right] \quad (14)$$

$$\pi^o = \left[\frac{V \left[2 + \gamma \left(2 - \frac{2}{N} \right) \right] - c \left[2 + \gamma 2 + \gamma^2 \left(\frac{2}{N} - \frac{4}{N^2} \right) \right]}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} + \frac{\frac{c}{k} \left[\gamma \frac{2}{N} + \gamma^2 \left(\frac{2}{N} - \frac{4}{N^2} \right) \right]}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} \right]^2 \left[1 + \gamma \left(1 - \frac{1}{N} \right) \right] \quad (15)$$

The quantities and the profits of the merging firms depend positive and the quantities and profits of the outsiders negative on k . The following presents a more detailed examination of the effects on the prices, quantities, and profits of the different enterprises following the merger. After that the incentives to merge can be identified.

3 Incentives to Merge

In the last section the optimal prices, quantities, and profits of the different firms before and after the merger were derived. This section examines the necessary conditions for positive incentives to merge.

The first condition for a positive incentive to merge is that the participating firms achieve higher profits after the merger. The second condition is that the potential merging firms have higher profits than the outsiders. We can imagine a situation where a merger would rise the profits of every firm but no firm would like to merge because it would achieve a lower profit than an outsider.

The following conditions have to be fulfilled for the existence of a positive incentive to merge:

$$\pi^m \geq \pi^* \quad \wedge \quad \pi^m \geq \pi^o \quad (16)$$

To examine these conditions, we may consider the case where the merging firms do not change their price or $p^m = p^*$. The corresponding k to this price is k' . Inserting this price p^* into the price-function (9) of the outsiders, we get the same optimal price for the outsiders ($p^o = p^m = p^*$).

In a situation where the merging firms as well as the outsiders set the price p^* the demand is the same as in the pre-merger phase: each firm can sell the quantity q^* . Therefore, the profits of the merging firms and the outsiders are:

$$\pi^m = \left(p^* - \frac{c}{k'} \right) q^* \quad (17)$$

$$\pi^o = (p^* - c)q^* = \pi^* \quad (18)$$

Hence, in a situation where each firm sets its price at p^* the profits of the outsiders are as high as in the pre-merger phase. Whether the profits of the merging firms after their merger are higher or lower than before the merger depends on the parameter k' which leads to a price $p^m = p^*$. If $k' > 1$ the merger profits rise and if $k' < 1$ the profits fall. To examine this, we should consider the price of the merging firms in a situation where $k = 1$:

$$p^m(k = 1) = \frac{V \left[2 + \gamma \left(2 - \frac{1}{N} \right) \right] + c \left[2 + \gamma \left(4 - \frac{5}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right) \right]}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} \quad (19)$$

Comparing this price with the price before the merger from equation (4) we can see that $p^m(k = 1)$ is larger than p^* . As already mentioned in the last section p^m depends negative on k . Therefore, k' must be larger than 1. Consequently the merging firms have higher profits than before the merger and than the outsiders when $k = k'$.

The profits of the merging firms depend positive on k , hence, in a situation where π^m is equal to π^* ($k = k^*$) or is equal to π^o ($k = k^o$), k must be smaller than k' .

Following the last section, the profits of the outsiders depend negative on k . Hence, with any k that is smaller than k' the outsiders have higher profits than in the pre-merger phase.

As it was shown, every profit of a merging firm that is higher than the profit of an outsider, is automatically higher than the pre-merger profit. Therefore, only the second condition for a positive incentive to merge is important. That is $\pi^m \geq \pi^o$.

Now we may consider the case where there are no synergy-effects from a merger, or $k = 1$. The profits of the merging firms and the outsiders from equation (14) and (15) are as follows:

$$\pi^m(k = 1) = \left[\frac{(V - c) \left[2 + \gamma \left(2 - \frac{1}{N} \right) \right]}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} \right]^2 \left[1 + \gamma \left(1 - \frac{2}{N} \right) \right] \quad (20)$$

$$\pi^o(k = 1) = \left[\frac{(V - c) \left[2 + \gamma \left(2 - \frac{2}{N} \right) \right]}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} \right]^2 \left[1 + \gamma \left(1 - \frac{1}{N} \right) \right] \quad (21)$$

Comparing the profit $\pi^m(k = 1)$ with the profit of the outsiders $\pi^o(k = 1)$ and the profit before the merger π^* , we can see that in a situation where $k = 1$ the profit of a merging firm is higher than before the merger, but smaller than the profit of an outsider.⁵ Therefore, it is sufficient that the synergy-effects are only a little negative ($k^* < 1$) to enable the merging firms to rise their profits. Nonetheless in a situation where $k = 1$ no firm wants to merge because its profits would be higher if it remains an outsider ($k^o > 1$).

⁵This result is equal to the study of Deneckere and Davidson (1985).

How high the synergy-effects must be to serve as positive incentives to merge depends on the parameters V , c , N , and γ . It was already shown that k^o must be between 1 and k' . In this range the following applies for the prices and quantities:

$$p^m > p^o > p^* \quad \wedge \quad (22)$$

$$q^m < q^* < q^o \quad (23)$$

In this section it was shown, which conditions are necessary and sufficient for positive incentives to merge. In reality, for a merger it is not only necessary that there are positive incentives to merge but that some kind of merger department allows the merger. Under which conditions this happens will be examined in the next section.

4 Welfare Effects

As it was shown in the last sections a merger has two different effects on prices and quantities in a market. On the one hand the increasing concentration in the market leads to higher prices and lower quantities. But on the other hand positive synergy-effects have a negative effect on the prices and a positive effect on the quantities. Therefore, it is necessary to compare these two effects to see the overall effect.⁶

In reality it is extremely difficult for a merger department to estimate the synergy-effects of a merger à priori. Therefore, they often make their decisions only on the basis of the concentration-effects of a merger. Farrell and Shapiro (1990, p. 108) write:

”Implicitly, the guidelines assume a reliable (inverse) relationship between market concentration and market performance. ...But if the competing firms are not equally efficient, or if there are economies of scale, there is no reason to expect that concentration and welfare will move in opposite directions in response to a merger.”

It is assumed that the merger department makes its decision on basis of the total welfare-effect of a merger. It is further assumed that the market is an international one. A market is international if either not all producers or not all demander are domestic ones or both conditions are fulfilled.

When judging a merger the result can depend on whether one cares about the national or the international welfare effects. These different views are examined in the following.

⁶See Jacquemin (1990), p. 542.

4.1 International Welfare Effects

An international merger department would favor a merger if, as a result, the international welfare did not decline. The welfare consists of the consumer and the producer surplus.⁷ Because of the international view all firms and the hole demand have to be taken into account.

The international producer surplus (Π^i) consists of the profits of all firms. Hence, the change in producer surplus ($\Delta\Pi^i$) is given by:

$$\Delta\Pi^i = 2(\pi^m - \pi^*) + (N - 2)(\pi^o - \pi^*) \quad (24)$$

It is equal to the increase of the profits of the two merging firms plus the increase of the profits of the $N - 2$ outsiders. For synergy-effects between 1 and k' the change in producer surplus is positive in any case, because in that range both the profits of the merging firms and the profits of the outsiders are higher than in the pre-merger phase.

To see how the change in producer surplus depends on the synergy-effect we should consider the following derivation:

$$\begin{aligned} \frac{d\Delta\Pi^i}{dk} = & 4 \frac{c}{k^2} \frac{2 + \gamma \left(3 - \frac{3}{N}\right) + \gamma^2 \left(1 - \frac{3}{N} + \frac{2}{N}\right)}{4 + \gamma \left(6 - \frac{6}{N}\right) + \gamma^2 \left(2 - \frac{4}{N}\right)} q^m \\ & - 2(N - 2) \frac{c}{k^2} \frac{\gamma \frac{2}{N} + \gamma^2 \left(\frac{2}{N} - \frac{4}{N}\right)}{4 + \gamma \left(6 - \frac{6}{N}\right) + \gamma^2 \left(2 - \frac{4}{N}\right)} q^o \end{aligned} \quad (25)$$

This derivation is positive for $1 \leq k \leq k'$.⁸ In this range the growth of producer surplus increases with k and is always positive.

The second part of the international welfare (W^i) is given by the international consumer surplus (U^i). In the pre-merger phase each firm sets its price equal to p^* . Following the equation (1) in this situation each consumer buys the product that suits him most. The consumer surplus (U^*) could be derived directly from the demand-function:

$$U^* = N \frac{1}{2} (V - p^*) q^* \quad (26)$$

To analyze how this utility or consumer surplus changes due to a merger, it is assumed for the present that all firms set their prices at p^m . Thus the consumers of each firm would have a change in utility (Δu_i^m) as follows:

$$\Delta u_i^m = \frac{1}{2} (p^* - p^m) (q^* + q^{m'}) \quad (27)$$

Here $q^{m'}$ is the demand for the products of one firm, in the case where all firms set their prices equal to p^m . Therefore, $q^{m'} = V - p^m$. In figure 1 the hatched area is equal to the loss of consumer surplus when the market price is p^m .

⁷See e.g. Farrell and Shapiro (1990), p. 108.

⁸For a proof see appendix B.

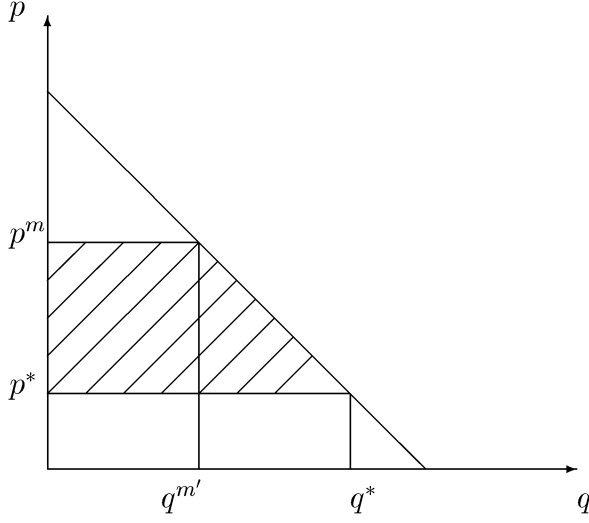


Figure 1: Loss of consumer surplus with p^m .

But not all firms set their price equal to p^m . The outsiders choose the price p^o . Due to this the consumers who prefer the product of an outsider when the prices are all equal, have a change in utility (Δu_i^o) given by:

$$\Delta u_i^o = \frac{1}{2}(p^* - p^o)(q^* + q^{o'}) \quad (28)$$

Here $q^{o'}$ is the demand for the products of one firm, in the case where all firms set their prices equal to p^o . Therefore, $q^{o'} = V - p^o$. When adding up the changes in utility Δu_i^m and Δu_i^o over all corresponding firms, we get the change in consumer surplus in the case when no consumer changes his supplier.

Given $\gamma > 0$ and $p^m \neq p^o$, there are some consumers who change their supplier. A merging firm wins $q^m - (V - p^m)$ and an outsider wins $q^o - (V - p^o)$, where

$$2[(V - p^m) - q^m] = (N - 2)[q^o - (V - p^o)] \quad (29)$$

For all $k < k'$ the price of the merging firms is higher than the price of the outsiders. The consumers who take advantage of this price difference ($p^m - p^o$) and buy from an outsider instead of a merging firm, have an increase in utility (ΔU^{om}) given by:

$$\Delta U^{om} = \frac{1}{2}(N - 2)(p^m - p^o)[q^o - (V - p^o)] \quad (30)$$

Figure 2 shows the change in consumer surplus for consumers buying from an outsider. The hatched area with the positive slope is equal to the loss of consumer surplus of the old customers of the outsider. The second hatched area is equal to the gain of consumer surplus for the consumers that switched to the outsider.

Adding up all the changes in consumer surplus we get the total change in international consumer surplus (ΔU^i):

$$\Delta U^i = \frac{1}{2} [2(p^* - p^m)(q^* + V - p^m) + (N - 2)(p^* - p^o)(q^* + V - p^o) + (N - 2)(p^m - p^o)(q^o - V + p^o)] \quad (31)$$

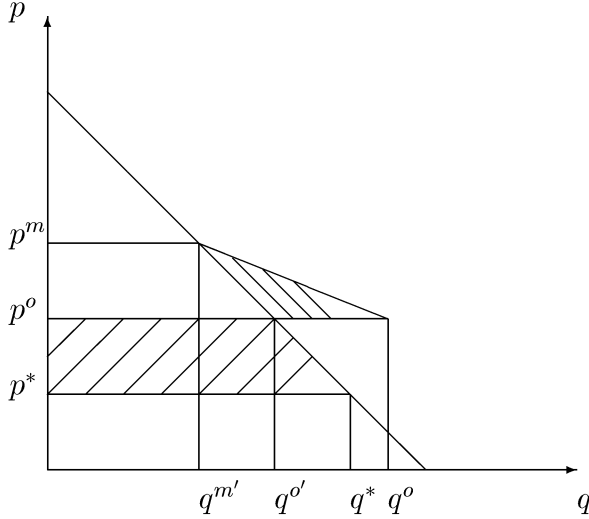


Figure 2: Development of the consumer surplus at an outsider.

After some transformations we get the shorter expression:

$$\Delta U^i = \frac{1}{2} [2(p^* - p^m)(q^m + q^*) + (N - 2)(p^* - p^o)(q^o + q^*)] \quad (32)$$

We can easily see that change in consumer surplus is zero if $p^m = p^o = p^*$ respectively $k = k'$. For all prices p^m and p^o that are lower than the initial price p^* the consumer surplus rises and it falls if they are higher than the initial price. Therefore, the change in consumer surplus is negative if $1 \leq k \leq k'$.

Next, we consider the derivative of ΔU^i with respect to k :

$$\begin{aligned} \frac{d\Delta U^i}{dk} &= -\frac{dp^m}{dk}(q^m + q^*) + \frac{dq^m}{dk}(p^* - p^m) \\ &\quad - \frac{1}{2}(N - 2) \left[\frac{dp^o}{dk}(q^o + q^*) - \frac{dq^o}{dk}(p^* - p^o) \right] \end{aligned} \quad (33)$$

This derivation is positive in the relevant range ($1 \leq k \leq k'$).⁹

Relevant for an international merger department is the movement of the total international

⁹A proof can be found in the appendix C.

welfare. That is given by the addition of the movement of the producer surplus from equation (24) and the consumer surplus from equation (32).

$$\begin{aligned}\Delta W^i &= 2(\pi^m - \pi^*) + (N - 2)(\pi^o - \pi^*) \\ &\quad + \frac{1}{2} [2(p^* - p^m)(q^m + q^*) + (N - 2)(p^* - p^o)(q^o + q^*)]\end{aligned}\quad (34)$$

Following the equations (25) and (33) within the range $1 \leq k \leq k'$ both the derivative of the change in the producer surplus and the derivative of the change in the consumer surplus with respect to k is positive. Therefore, in the same range the derivative of the change in total welfare with respect to k must be positive.

Considering the case where $k = k'$ we can see that welfare increases by $2(\pi^m - \pi^*)$ after the merger. In the case where $k = 1$ the change in the international welfare is negative. Therefore, the critical value of k must be between 1 and k' . Whether this critical value is higher or lower than the critical value for positive incentives to merge depends on the parameters V , c , γ , and N . In section 5 this question is answered for different cases.

4.2 National Welfare Effects

In this subsection it is examined how the national welfare develops compared to the international one. It is assumed that the merger is a merger between two national firms.¹⁰

Further it is assumed that the number of national firms is n , where $2 \leq n \leq N$. The fraction of the total demand requested by national consumers is α , where α is exogenous and $0 \leq \alpha \leq 1$.¹¹

Therefore, the development of the national welfare (ΔW^n) is given by:

$$\begin{aligned}\Delta W^n &= 2(\pi^m - \pi^*) + (n - 2)(\pi^o - \pi^*) \\ &\quad + \frac{1}{2}\alpha [2(p^* - p^m)(q^m + q^*) + (N - 2)(p^* - p^o)(q^o + q^*)]\end{aligned}\quad (35)$$

For $n = N$ and $\alpha = 1$ the national welfare is equal to the international one. In this case we are only looking at a national market. More interesting is a case when it is an international market, i.e. at least one of the above conditions is not fulfilled.

Following the last subsection 4.1 in the range $1 \leq k \leq k'$ both the derivative of the development of the producer surplus and the derivative of the development of the consumer surplus with respect to k is positive. Hence, in this range the development of the national

¹⁰The results change only unessential when it is an international merger. In this case the development of the national producer surplus depends on the development of the profits of one merging firm and $n - 1$ outsiders.

¹¹It is assumed that the national consumers do not differ from the international ones. This also applies for the price level.

welfare depends positive on k .

Considering the case that $k = k'$ we can easily see that the national welfare is equal to the international one for all values of n and α . Here the change in national welfare is $2(\pi^m - \pi^*)$.

For every k below k' the profits of the outsiders are higher after the merger than before it ($\pi^o > \pi^*$). Hence, in this range the change in the national welfare depends positive on n . If $\alpha = 1$ and $n < N$ the critical value for an acceptance of a merger is higher for a national merger department than for an international one. Nevertheless, the critical value also would be between 1 and k' .

For a k below k' the consumer surplus is negative. In this range the change in national welfare depends negative on α . If $n = N$ and $\alpha < 1$ the critical value for an acceptance of a merger is lower for a national merger department than for an international one. This critical value can also be below the critical value for an increasing profit of the merging firms (k^*).

The critical value of the synergy-effects depends negative on the number of national firms n and positive on the demand fraction α . Whether the total effect of n and α on this critical value is positive or negative depends on the parameters V , c , γ , and N .

Regarding the international welfare as relevant, a national merger department could have the tendency to allow too many mergers if it is a small country with relatively many domestic firms. The opposite applies for a large country with only few domestic firms. Section 5 presents different cases and their corresponding proportion of national to international welfare.

5 Simulation

As shown in the last sections the different results quantitatively and qualitatively depend on the exogenous parameters. It is not offhand possible to compute the different critical values exactly. Therefore, the different investigations are represented in this section for various hypothetic markets. Thereby, different values for the parameters V , c , γ , N , n , and α are assumed. For each case the dependence of the merger incentives and the development of the welfare from the synergy-effects k is displayed in diagrams.

In this section the following values of V , c , and N are assumed:

$$V = 10 \quad c = 1 \quad N = 10 \tag{36}$$

Figure 3 shows the dependence of the profit differences $\pi^m - \pi^*$ and $\pi^m - \pi^o$ and the welfare effects ΔW^n and ΔW^i on k . Thereby, it is assumed that $\gamma = 1$, $n = 8$, and $\alpha = 0.5$. As we can see both the profit differences and the welfare effects depend positive on k . At $p^m = p^*$ the profit difference $\pi^m - \pi^*$ is equal to the profit difference $\pi^m - \pi^o$ and the national welfare-effect is equal to the international one. The corresponding k is

about 1.209.

Further we can see that the profits of the merging firms (π^m) are equal to their pre-merger

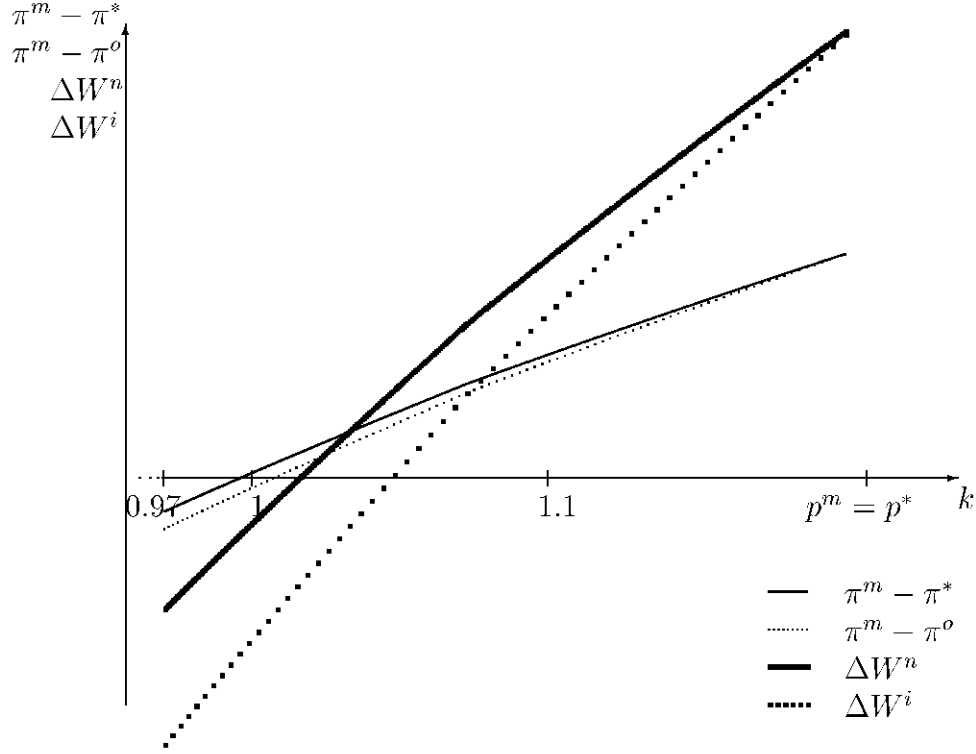


Figure 3: $\gamma = 1$, $n = 8$ und $\alpha = 0.5$

profits (π^*) at a $k < 1$. In the present case this k is about 0.996. In contrast the critical value of k for positive incentives to merge ($\pi^m > \pi^o$) is above one. In the present case this critical value is about 1.007.

The welfare effects are getting positive at a higher value of k than the merger incentives. Hence, not all wanted mergers would be permitted under the chosen parameter values. Additional we can see that the national welfare effects are getting positive for a smaller value of k than the international ones. For a national merger department the critical value of k is about 1.015 and for an international one about 1.047.

Figure 4 shows the same market for $n = 5$ and $\alpha = 0.8$. Nothing changes here for the firms and the international welfare-effect but for the national welfare-effect. Regarding the new parameter constellation, the national welfare-effect gets positive at a higher k than the international one. The critical value of k is now about 1.051.

This result matches with the above investigations. In the relevant range the national welfare-effect depends positive on the number of national firms and negative on the demand fraction α .

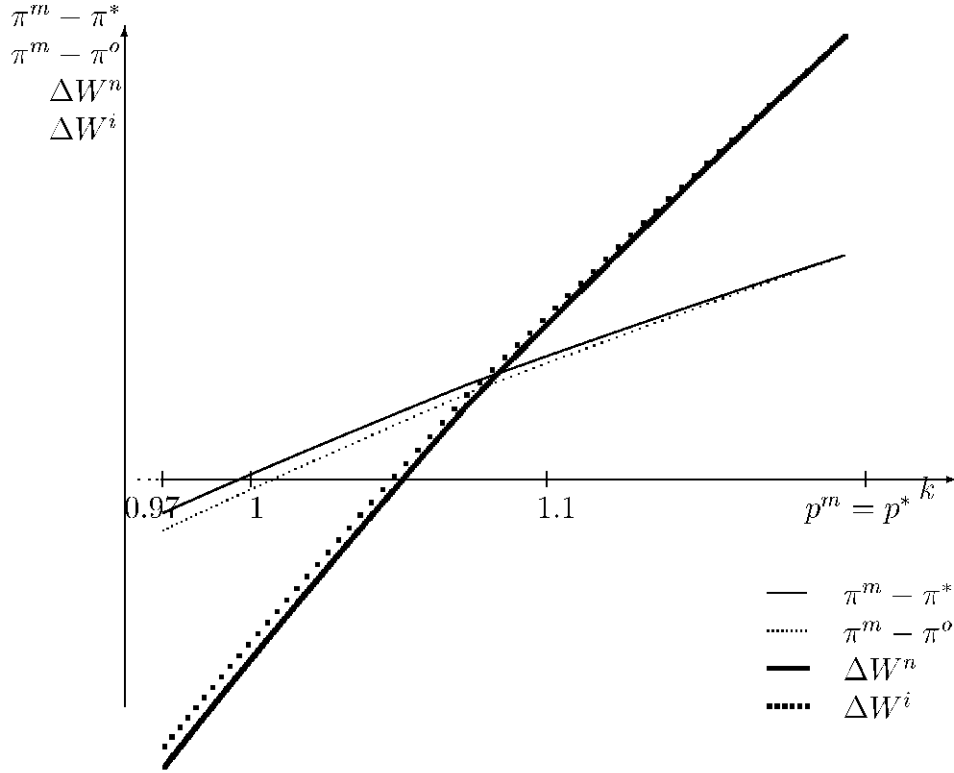


Figure 4: $\gamma = 1$, $n = 5$ und $\alpha = 0.8$

Next, we may consider the case where $n = 8$ and $\alpha = 0.5$ again. Figure 5 represents the merger incentives and the welfare effects for $\gamma = 18$. Comparing this diagram with figure 3 we can see that the k corresponding to $p^m = p^*$ is lower now. It is about 1.062.

The incentives to merge and the welfare effects have moved also. The critical values of k are lower for each effect. The critical value of k for an increase in profits of the merging firms ($\pi^m = \pi^*$) is now about 0.994 and the critical value of k for positive incentives to merge ($\pi^m = \pi^o$) is about 1.005.

In this new case the critical value of k for a positive international welfare-effect is below the critical value for positive incentives to merge. It is now about 1.004. The critical value of k for a positive national welfare-effect is even below the critical value for increasing profits of the merging firms. It is now about 0.985.

In the case where $\gamma = 18$, $n = 8$, and $\alpha = 0.5$ a national merger department would even favor a merger that leads to decreasing profits of the merging firms. Also an international merger department would favor a merger when there are no positive incentives to merge. In both cases there could be a welfare increasing merger when the merger department initiates that the outsiders compensate the merging firms in a way that they have at least equal profits. Nevertheless these profits would be higher for all firms than in the pre-

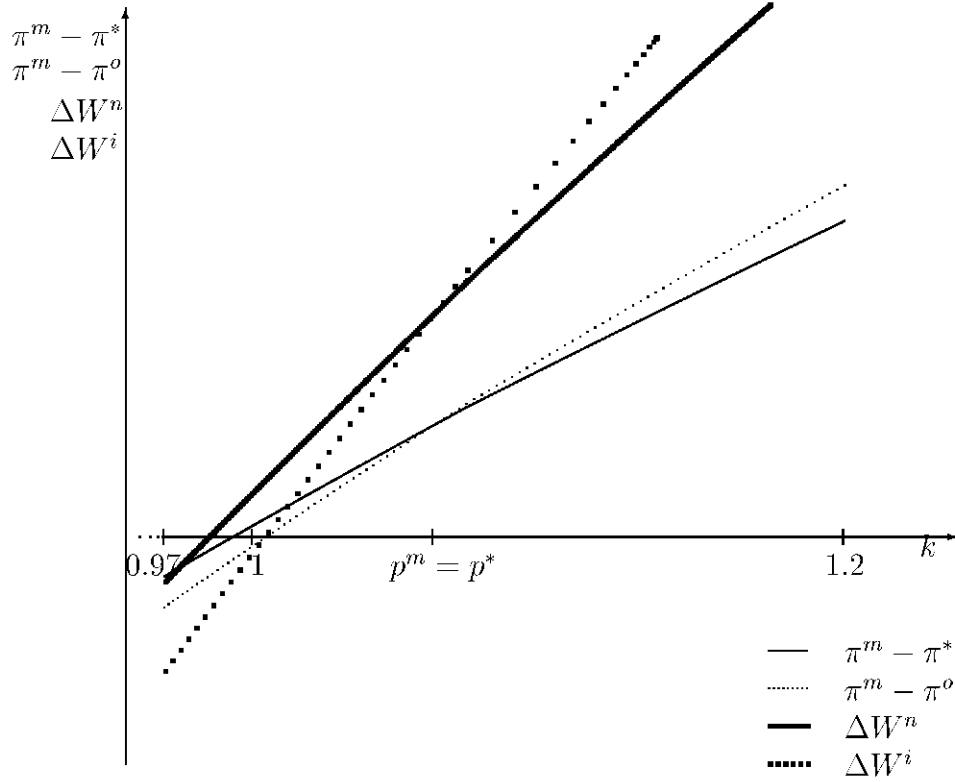


Figure 5: $\gamma = 18$, $n = 8$ und $\alpha = 0.5$

merger phase.

In this section different cases were discussed exemplary. Of course further cases would lead to further results but the chosen cases already cover all extremes.

Hence, in some cases a national merger department would be more likely to agree to a merger than an international department and vice versa. Further we can think of parameter constellations where an international merger department would even favor a merger when there are no positive incentives to merge and of constellations where it would not agree with every merger. There are even cases where a national merger department would favor a merger that leads to decreasing profits of the merging firms.

6 Conclusions

In the present work a heterogeneous Bertrand-model was developed and it was investigated which incentives to merge exists and how a merger affects the welfare. Thereby, potential synergy-effects of mergers were considered explicitly and a differentiation was

made between national and international welfare.

It was shown that mergers with no synergy-effects lead to increasing profits for the merging firms, but these profits are not as high as the profits of the outsiders. Further it was shown that each merger that leads to higher profits of the merging firms than the profits of the outsiders is leading to an increasing profit for a merging firm at the same time.

Following the investigation of the international welfare effects of a merger they are only positive if there are positive synergy-effects but the critical value of the synergy-effects for a positive international welfare-effect could be below the critical value for positive incentives to merge.

The national welfare effects of a merger depend positive on the number of domestic firms and negative on the domestic fraction of the total demand. Thereby, it depends on the proportion of these two parameters whether a national merger department would have the tendency to favor more or less mergers than an international merger department.

Appendix

A Price setting of the merging firms

The merging firms maximize their joint profit by their prices:

$$\begin{aligned} Max_{(p_i, p_j)} & \quad \left(p_i - \frac{c}{k} \right) \left[V - p_i - \gamma \left[p_i - \frac{1}{N} \left(p_i + p_j + \sum_{k \neq (i, j)} p_k \right) \right] \right] \\ & + \left(p_j - \frac{c}{k} \right) \left[V - p_j - \gamma \left[p_j - \frac{1}{N} \left(p_j + p_i + \sum_{k \neq (i, j)} p_k \right) \right] \right] \end{aligned}$$

First order conditions:

$$V - 2p_i \left(1 + \gamma \frac{N-1}{N} \right) + 2\gamma \frac{1}{N} p_j + \gamma \frac{N-2}{N} p^o + \frac{c}{k} \left(1 + \gamma \frac{N-2}{N} \right) = 0 \quad (37)$$

$$V - 2p_j \left(1 + \gamma \frac{N-1}{N} \right) + 2\gamma \frac{1}{N} p_i + \gamma \frac{N-2}{N} p^o + \frac{c}{k} \left(1 + \gamma \frac{N-2}{N} \right) = 0 \quad (38)$$

From (38) follows:

$$p_i = -\frac{N}{2\gamma} V + p_j \left(\frac{N}{\gamma} + N - 1 \right) - \frac{N-2}{2} p^o - \frac{c}{k} \left(\frac{N}{2\gamma} + \frac{N-2}{2} \right) \quad (39)$$

By inserting this into (37) we get:

$$V \left(\frac{N}{\gamma} + N \right) - 2 \left(\frac{N}{\gamma} + 2(N-1) + \gamma(N-2) \right) p_j + (N-2)(1+\gamma)p^o + \frac{c}{k} \left(\frac{N}{\gamma} + 2(N-1) + \gamma(N-2) \right) = 0 \quad (40)$$

Hence, the optimal price p_j is:

$$p_j = \frac{V + \frac{c}{k} \left(1 + \gamma \frac{N-2}{N} \right) + \gamma \frac{N-2}{N} p^o}{2 \left(1 + \gamma \frac{N-2}{N} \right)} \quad (41)$$

After inserting this price into (39) we get:

$$p_i = \frac{V + \frac{c}{k} \left(1 + \gamma \frac{N-2}{N} \right) + \gamma \frac{N-2}{N} p^o}{2 \left(1 + \gamma \frac{N-2}{N} \right)} \quad (42)$$

Therefore, the prices are identical and equal to the price p^m in equation (8).

B Dependence of the producer surplus development on k

The derivatives of π^m , π^o and π^* with respect to k are:

$$\frac{d\pi^m}{dk} = 2 \frac{c}{k^2} \frac{2 + \gamma \left(3 - \frac{3}{N} \right) + \gamma^2 \left(1 - \frac{3}{N} + \frac{2}{N^2} \right)}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} q^m \quad (43)$$

$$\frac{d\pi^o}{dk} = -2 \frac{c}{k^2} \frac{\gamma \frac{2}{N} + \gamma^2 \left(\frac{2}{N} + \frac{4}{N^2} \right)}{4 + \gamma \left(6 - \frac{6}{N} \right) + \gamma^2 \left(2 - \frac{4}{N} \right)} q^o \quad (44)$$

$$\frac{d\pi^*}{dk} = 0 \quad (45)$$

Hence, the derivative of the change in producer surplus with respect to k is equal to equation (25). For 25 to be positive the following condition has to be fulfilled:

$$2 \left[2 + \gamma \left(3 - \frac{3}{N} \right) + \gamma^2 \left(1 - \frac{3}{N} + \frac{2}{N^2} \right) \right] q^m > (N-2) \left[\gamma \frac{2}{N} + \gamma^2 \left(\frac{2}{N} + \frac{4}{N^2} \right) \right] q^o \quad (46)$$

\Rightarrow

$$\left[2 + \gamma \left(3 - \frac{3}{N} \right) + \gamma^2 \left(1 - \frac{3}{N} + \frac{2}{N^2} \right) \right] q^m > \left[\gamma \left(1 - \frac{2}{N} \right) + \gamma^2 \left(1 - \frac{4}{N} + \frac{4}{N^2} \right) \right] q^o \quad (47)$$

The relevant range of k is in between 1 and k' . Since the left side of the inequation depends positively on k and the right side depends negatively on k (see section 2.2) the problem can be reduced to the case where $k = 1$:

$$\begin{aligned} & [2 + \gamma (3 - \frac{3}{N}) + \gamma^2 (1 - \frac{3}{N} + \frac{2}{N^2})] \\ & (V - c) [2 + \gamma (2 - \frac{1}{N})] [1 + \gamma (1 - \frac{2}{N})] \\ & > [\gamma (1 - \frac{2}{N}) + \gamma^2 (1 - \frac{4}{N} + \frac{4}{N^2})] \\ & (V - c) [2 + \gamma (2 - \frac{2}{N})] [1 + \gamma (1 - \frac{1}{N})] \end{aligned}$$

\Rightarrow

$$\begin{aligned} & [2 + \gamma (3 - \frac{3}{N}) + \gamma^2 (1 - \frac{3}{N} + \frac{2}{N^2})] [2 + \gamma (4 - \frac{5}{N}) + \gamma^2 (2 - \frac{5}{N} + \frac{2}{N^2})] \\ & > [\gamma (1 - \frac{2}{N}) + \gamma^2 (1 - \frac{4}{N} + \frac{4}{N^2})] [2 + \gamma (4 - \frac{4}{N}) + \gamma^2 (2 - \frac{4}{N} + \frac{2}{N^2})] \end{aligned}$$

\Rightarrow

$$\begin{aligned} & 4 + \gamma \left(12 - \frac{12}{N}\right) + \gamma^2 \left(12 - \frac{13}{N} + \frac{7}{N^2}\right) + \gamma^3 \left(4 - \frac{10}{N} + \frac{2}{N^2} + \frac{4}{N^3}\right) \\ & + \gamma^4 \left(\frac{1}{N} - \frac{5}{N^2} + \frac{8}{N^3} - \frac{4}{N^4}\right) > 0 \end{aligned}$$

For $N \geq 3$ this condition is fulfilled. Therefore, in the relevant range ($1 \leq k \leq k'$) the producer surplus depends positively on k .

C Dependence of the consumer surplus development on k

Following section 2.2 we have:

$$\frac{dp^m}{dk}, \frac{dp^o}{dk}, \frac{dq^o}{dk} < 0 \quad \text{and} \quad \frac{dq^m}{dk} > 0 \quad (48)$$

Following section 3 $p^m \leq p^o \leq p^*$ is given in the relevant range of k . Thereby:

$$-\frac{1}{2}(N-2)\frac{dp^o}{dk}(q^o + q^*) > 0 \quad (49)$$

$$\frac{1}{2}(N-2)\frac{dk^o}{dk}(p^* - p^o) > 0 \quad (50)$$

$$-\frac{dp^m}{dk}(q^m + q^*) > 0 \quad (51)$$

but:

$$\frac{dq^m}{dk}(p^* - p^m) < 0 \quad (52)$$

Therefore, it is sufficient for derivation (33) to be positive that:

$$-\frac{dp^m}{dk}(q^m + q^*) + \frac{dq^m}{dk}(p^* - p^m) > 0 \quad (53)$$

The derivatives of the prices and quantities of the merging firms with respect to k are:

$$\frac{dp^m}{dk} = -\frac{c}{k^2} \frac{2 + \gamma \left(3 - \frac{3}{N}\right) + \gamma^2 \left(1 - \frac{1}{N} - \frac{2}{N^2}\right)}{4 + \gamma \left(6 - \frac{6}{N}\right) + \gamma^2 \left(2 - \frac{4}{N}\right)} \quad (54)$$

$$\frac{dq^m}{dk} = \frac{c}{k^2} \frac{2 + \gamma \left(3 - \frac{3}{N}\right) + \gamma^2 \left(1 - \frac{3}{N} + \frac{2}{N^2}\right)}{4 + \gamma \left(6 - \frac{6}{N}\right) + \gamma^2 \left(2 - \frac{4}{N}\right)} \left[1 + \gamma \left(1 - \frac{2}{N}\right)\right] \quad (55)$$

The second derivatives are:

$$\frac{d^2 p^m}{dk^2} = -\frac{2}{k} \frac{dp^m}{dk} \quad (56)$$

$$\frac{d^2 q^m}{dk^2} = -\frac{2}{k} \frac{dq^m}{dk} \quad (57)$$

The derivative of the left side of the inequation (53) (signed by L) with respect to k is:

$$\frac{dL}{dk} = -\frac{d^2 p^m}{dk^2}(q^m + q^*) + \frac{d^2 q^m}{dk^2}(p^* - p^m) - 2 \frac{dp^m}{dk} \frac{dq^m}{dk} \quad (58)$$

\Rightarrow

$$\frac{dL}{dk} = -\frac{2}{k} L - 2 \frac{dp^m}{dk} \frac{dq^m}{dk} \quad (59)$$

As we can see this equation is positive if $L = 0$. The function $L(k)$ could only change from the negative to the positive and not vice versa. Therefore, starting from a positive value of L the function could not get negative again.

The inequation (53) with $k = 1$ is:

$$-\frac{dp^m}{dk}(k=1)(q^m(k=1) + q^*) - \frac{dq^m}{dk}(k=1) \frac{(V-c)\gamma \frac{1}{N} \left[2 + \gamma \left(1 + \frac{1}{N}\right)\right]}{\left[4 + \gamma \left(6 - \frac{6}{N}\right) + \gamma^2 \left(2 - \frac{4}{N}\right)\right] \left[2 + \gamma \left(1 - \frac{1}{N}\right)\right]} > 0$$

This inequation is fulfilled if it holds true for $q^* = 0$:

$$-\frac{dp^m}{dk}(k=1) \frac{(V-c) \left[2 + \gamma \left(2 - \frac{1}{N}\right)\right] \left[1 + \gamma \left(1 - \frac{2}{N}\right)\right]}{4 + \gamma \left(6 - \frac{6}{N}\right) + \gamma^2 \left(2 - \frac{4}{N}\right)} - \frac{dq^m}{dk}(k=1) \frac{(V-c)\gamma \frac{1}{N} \left[2 + \gamma \left(1 + \frac{1}{N}\right)\right]}{\left[4 + \gamma \left(6 - \frac{6}{N}\right) + \gamma^2 \left(2 - \frac{4}{N}\right)\right] \left[2 + \gamma \left(1 - \frac{1}{N}\right)\right]} > 0$$

\Rightarrow

$$-\frac{dp^m}{dk}(k=1)(V-c) \left[2 + \gamma \left(2 - \frac{1}{N}\right)\right] \left[1 + \gamma \left(1 - \frac{2}{N}\right)\right] \left[2 + \gamma \left(1 - \frac{1}{N}\right)\right] - \frac{dq^m}{dk}(k=1)(V-c)\gamma \frac{1}{N} \left[2 + \gamma \left(1 + \frac{1}{N}\right)\right] > 0$$

\Rightarrow

$$\begin{aligned} & \left[2 + \gamma \left(3 - \frac{3}{N} \right) + \gamma^2 \left(1 - \frac{1}{N} - \frac{2}{N^2} \right) \right] \left[2 + \gamma \left(2 - \frac{1}{N} \right) \right] \left[2 + \gamma \left(1 - \frac{1}{N} \right) \right] \\ & - \left[2 + \gamma \left(3 - \frac{3}{N} \right) + \gamma^2 \left(1 - \frac{3}{N} + \frac{2}{N^2} \right) \right] \gamma \frac{1}{N} \left[2 + \gamma \left(1 + \frac{1}{N} \right) \right] > 0 \end{aligned}$$

For $N \geq 3$ this inequation is fulfilled. Therefore, in the relevant range the consumer surplus depends positively on k .

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