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Monetary Policy under Uncertain Planning Horizon

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Abstract

This note focuses on the role of the planning horizon in monetary policy games. We analyze the case of an uncertain horizon of the monetary policy game that has not been considered, yet. In addition to that we will - different from the basic Barro-Gordon-model - assume a quadratic loss function for the policy maker.

Keywords: monetary policy, time consistency
JEL-Classification: E50

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1 Introduction

The traditional view on optimal monetary policy was guided by the intuition that monetary instrument variables have to be chosen to solve a dynamic optimization problem. The insightful paper by Kydland and Prescott (1977) on time inconsistency initiated a reversion of this view. A policy appears to be time inconsistent if it sets time paths for the control variables at time zero that are no longer optimal at a later time $t$. Barro and Gordon (1983a,b) transferred these considerations to monetary policy and underlined that monetary policy is not a game against nature but against rational economic agents. They argue that monetary authorities face a time inconsistency problem when they try to minimize a social loss function that includes both: an ambitious output target (i.e. an output target that exceeds natural output) and price stability. While the monetary authority is assumed to determine inflation by appropriate use of monetary instruments, private agents (respectively trade unions) have to build inflationary expectations to decide on nominal-wage-claims optimally. Given that output is determined by a Lucas-supply-curve and private agents have rational expectations the situation ends up in an inflationary bias, i.e. an excessive inflation rate without real effects.

The following expositions aim at analyzing the crucial role of the time horizon in the above described monetary policy game. While the cases of a finite and an infinite horizon are well examined, there has been no analysis of the case of an uncertain horizon of the interaction process - possibly due to the fact, that this case has not been seen as relevant, yet. It is shown that the case of an uncertain horizon can be analyzed analogously to the infinitely repeated case. Besides that we show the basic results of the Barro-Gordon model to hold true under the more realistic assumption of a quadratic loss function of the monetary authority.
2 Model, First- and Second-Best-Solution

The following expositions draw on a nonstochastic economy that is described by the fundamental relation:

\[ Y_t = \omega P_t - \omega^e_P \]  

where \( Y_t \) denotes (the log of) current output, \( \omega P_t \) inflation and \( \omega^e_P \) the trade unions’ inflationary expectations. This equation refers to the well-known Lucas-supply-function pointing out that expectation-errors with regard to inflation are causing real effects, i.e. generate deviations of the current output (employment) from the natural rate of output (employment). For convenience we define the natural rate of output to be zero.

There are two actors in the standard monetary policy game. On the one hand we have the monetary authority that is assumed to be able to manipulate the inflation rate directly.\(^1\) On the other hand we deal with trade unions that form inflationary expectations to be able to decide on the labor supply optimally.

The monetary authority is supposed to be interested in realizing a social optimal inflation rate \( \omega^*_p \) and an ambitious output target \( Y^* > 0 \). Different from Barro and Gordon (1983b) we assume the monetary authority’s loss function to be quadratic in both, the inflation and the output objective.\(^2\)

The monetary authority’s one-period loss function is given by:

\[ l_t = \frac{1}{2} \cdot (\omega P_t - \omega^*_p)^2 + \frac{1}{2} \cdot \beta \cdot (Y_t - Y^*)^2 \quad \text{with} \quad Y^* > 0, \]  

where \( \beta \) denotes the relative importance of the output objective. It is assumed that the monetary authority’s objectives confirm with society’s preferences. Monetary authority minimizes the present value \( L \) of expected present and future losses:

\[ L = E \left[ \sum_{i=0}^{\infty} \left( \frac{1}{1 + r} \right)^i \cdot l_{i+1} \right], \]

\(^1\)This unrealistic assumption can be relaxed without substantial effect on the argument.

\(^2\)This is due to the fact, that the loss function used by Kydland/Prescott (1977) and Barro/Gordon (1983b) is in general inconsistent with the welfare criterion of pareto optimality usually employed in models with optimizing agents. Compare Sargent/Wallace (1976, S.181) for a more detailed discussion of this aspect.
where $r$ is a discounting factor.

Trade unions try to prevent to be fooled by the monetary authority. When forming their expectations they make use of all relevant and available information, i.e. they have rational expectations.

The sequential structure of the game is as follows. At the beginning of every single period the monetary authority announces a certain rate of inflation $w^p_t$. Afterwards trade unions form their inflationary expectations and anticipate them during the wage negotiations. Finally the monetary authority determines the rate of inflation in order to minimize its expected intertemporal loss.

The first best solution $w_{P_t} = w^*_p$, $Y_t = Y^*$ of the above described game can be calculated by minimizing the one period loss from expression 2 with respect to $w_{P_t}$ and $Y_t$. Because the realization of the first best solution would require a systematic expectation error by the trade unions and the trade unions are assumed to form rational expectations it is clear that first best can not be reached. The second best solution, i.e. the best solution that can be realized under the trade unions’ rational expectations, requires

$$w_{P_t} = w^e_{P_t} = w^*_p$$

and results in the one-period-loss:

$$l^b_t = \frac{1}{2} \cdot \beta \cdot Y^* \cdot Y^*$$

3 Nash-equilibria of finite-horizon-games

We will now turn to the Nash equilibria of finite horizon games. In the one-shot game monetary authority announces the second best inflation rate from expression 4. After the trade unions have formed their expectations and wage negotiations took place, the optimization problem has changed since the monetary authority can now treat inflationary expectations as given. Thus ex post the monetary authority has a strong incentive to perform surprise inflation in order to push current output above the natural level. Since the trade unions form rational expectations they are aware of that incentive. The game ends up in the well known inflationary bias

$$w_{P_t} = w^e_{P_t} = w^*_p + \beta \cdot Y^*$$
and the one-period-loss
\[ l_t^d = \frac{1}{2} \cdot \beta^2 \cdot Y^*^2 + \frac{1}{2} \cdot \beta \cdot Y^*^2. \] (7)

The same solution applies to each finitely repeated game. This is due to the fact that the solution of the one-shot game can be transferred from the last period to all preceding periods by backward induction.

4 Nash-equilibria of infinite-horizon-games

Since there is no last period in infinite horizon games, such games can not be solved by backward induction. Thus, building up a reputation in favour of an anti-inflationary policy is possible. As Barro and Gordon (1983b) have shown we have to compare the gains and losses from violating the announced rate of inflation to be able to judge whether a certain announced rate of inflation is credible.

The temptation of violation \( TOV_t \) can be calculated as difference between the one period loss under sticking to the announcement \( l_t^{st} \) and the one when cheating \( l_t^{ch} \)

\[ TOV_t = l_t^{st} - l_t^{ch}. \] (8)

When sticking to the announcement, i.e. \( w_{Pt} = w_{Pt} = w_{Pt}^a \), the monetary authority realizes the loss

\[ l_t^{st} = \frac{1}{2} \cdot (w_{Pt}^a - w_P)^2 + \frac{1}{2} \cdot \beta \cdot Y^*^2 \] (9)

whereas cheating, i.e. \( w_{Pt}^{ch} = w_P + \beta \cdot Y^* \) and \( w_{Pt}^{ch} = w_{Pt}^a \), ends up in the loss

\[ l_t^{ch} = \frac{1}{2} \cdot \beta^2 \cdot Y^*^2 + \frac{1}{2} \cdot \beta \cdot (w_P^a + \beta Y^* - w_P^a - Y^*)^2. \] (10)

The cost of violating an announced rate of inflation \( COV_t \) depends on the trade unions’ reactions on a successful deception by the monetary authority. In the following we will assume the same trigger mechanism as in Barro and Gordon (1983b):

\[ w_{Pt}^{e} = \begin{cases} w_P^a & \text{; if } w_{Pt-1}^a = w_{Pt-1} \\ w_P + \beta \cdot Y^* & \text{; else} \end{cases}. \] (11)
i.e. trade unions penalize cheating in period $t$ by uncooperative behaviour in the following period $t+1$. In period $t+2$ we face the same problem as in period $t$ because reputation is restored at that time. The cost of violation can be calculated as discounted difference between the loss under the discretionary solution $w_P^t = w_P^c = w_P^* + \beta \cdot Y^*$ and the one when sticking to the announced rate of inflation:

$$COV_t = \frac{1}{1+r} \cdot (l_t^d - l_t^s)$$

In figure 1 we graph the temptation and the cost of violation versus the announced rate of inflation. The resulting curves are similar to the case of an asymmetric objective function since $TOV$ is a convex and $COV$ a concave function of the announced inflation rate.$^3$ Since cheating appears to be reasonable for the monetary authority if

$$TOV_t > COV_t,$$

the $TOV$- and the $COV$-curve determine a range of credible inflation rate announcements with the lower bound $w_P^{a,min}$ and the upper bound $w_P^{a,max}$.

For convenience we define $w_P^* = 0$. In this case the interceptions of the $COV_T$- and the $TOV_t$-curve can be calculated as:

$$w_P^{a,min} = \frac{\beta \cdot (1 - \beta) \cdot Y^* - \beta Y^*}{1 - \beta + \frac{1}{1+r}}$$

$$w_P^{a,max} = \frac{\beta \cdot (1 - \beta) \cdot Y^* + \beta Y^*}{1 - \beta + \frac{1}{1+r}}.$$  

For increasing discount rates $r$ the solution converges against the discretionary inflation rate:$^4$

$$\lim_{r \to \infty} \frac{\beta \cdot (1 - \beta) \cdot Y^* \pm \beta Y^*}{1 - \beta + \frac{1}{1+r}} = \beta \cdot Y^*.$$ 

Altogether we can conclude that the results of Barro and Gordon’s analysis (1983b) hold true in the case of a symmetric objective function of the monetary authority, too. 

$^3$Compare Barro/Gordon (1983b), figure 1, p. 112.

$^4$Compare equation 6.
5 Nash-equilibria of games with uncertain horizons

We will now consider the case of an uncertain horizon of the monetary policy game. We will show that the assertion, a monetary policy game with stochastically determined end would end up in the same solution as the infinitely repeated game,\(^5\) is not true. Therefore we assume the game to end with probability \(h\) in the end of each period \(t\) and this information to be common knowledge.

The case of an uncertain planning horizon can be analyzed analogously to the above case of an infinite horizon game. The temptation to violate an announced inflation rate does not alter compared to the infinitely repeated case. However, the cost of violation decreases owing to the fact that the loss of reputation does not embody a potential threat to the monetary authority when the game ends in the next period. Thus, we have

\[
COV_t = \frac{1 - h}{1 + r} \cdot (l^d_t - l^s_t)
\]  

(17)

as the cost of violation in games with uncertain horizon. For the lowest (respectively the highest) credible announced inflation rate we get

\[
w_{P, min}^a = \frac{\beta \cdot (1 - \beta) \cdot Y^* - (1 - h) \cdot \beta Y^*}{1 - \beta + (1 - h) \cdot \frac{1}{1+r}}
\]  

(18)

\[
w_{P, max}^a = \frac{\beta \cdot (1 - \beta) \cdot Y^* + (1 - h) \cdot \beta Y^*}{1 - \beta + (1 - h) \cdot \frac{1}{1+r}}
\]  

(19)

To be able to compare the two situations under infinite and uncertain planning horizon, both cases are shown in figure 1. The introduction of uncertainty about the planning horizon affects the \(COV\)-curve only. The higher the probability of game-interruption, the less steep the slope of the \(COV\)-curve gets.\(^6\) Thus the lower bound of the range of credible announcements seems to grow with increasing probability of game interruption. Analytically

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\(^6\)In figure 1 we have shown the two cases \(h = 0, 2\) and \(h = 0, 7\).
Figure 1: Credible versus incredible announcements under certain and uncertain planning horizon

this can be shown by calculating the lower bound of credible announcements from equation 18 for given discount rates \( r \) and rising probability of game-interruption \( h \). It is easy to see that \( w_{P}^{a,\text{min}} \) is increasing in \( h \). For \( h = 1 \), i.e. the game will end after period \( t \) with certainty, we get the discretionary solution \( w_{P}^{a,\text{min}} = \beta \cdot Y^{*} \).

To summarize, we can state that the lowest credible inflation rate announcement under uncertain planning horizon is positively related to the probability of game interruption.

6 Conclusions

There are two things we can learn from the above expositions.
Firstly we have shown that the basic results of the Barro-Gordon-model hold true under the realistic assumption of a quadratic loss function for the monetary authority. This has often been claimed\textsuperscript{7} but never been shown for the infinitely repeated case.

Secondly it was demonstrated that the planning horizon of economic agents in monetary policy games is a crucial factor for the resulting inflationary bias. The case of an uncertain planning horizon of the game can be analyzed analogously to infinite horizon games but is not identical to the infinitely repeated case.

There are several situations the case of an uncertain planning horizon can be applied to. A historical example is the Maastricht Treaty that created a situation of uncertainty for the EU member states by fixing criteria for the beginning of European Monetary Union and thus making the further responsibility for monetary policy of most European national central banks somewhat uncertain. If the EU member states would have agreed on an unconditional starting date for monetary union, the inflation rates in the EU member countries during the transitional phase possibly would have been significantly higher. The same problem could arise in the countries applying for a future membership in EMU. With increasing probability to be admitted for monetary union at a certain future date the central bank’s ability to make use of its reputation will erode. Thus these countries should choose adequate strategies to be able to resist the incentive to produce inflationary surprises during the last periods of their responsibility for monetary policy. According to Rogoff’s (1985) proposal a successful strategy might be to grant the central bank a high degree of independence and to install a weight-conservative central banker.

References


