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Immigration and Two–Component Unemployment

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Abstract

We analyze the employment effects of immigration within a model that accounts for several stylized facts of the German labor market. The co-existence of positive wage spans and unemployment is explained by wage rigidities that are simultaneously caused by efficiency-wage setting \textit{and} minimum wages. The observed positive relation between wage spans and minimum wages results from employment shifts from low-wage to high-wage sectors. Employment effects of immigration are opposite to those of a rise in the minimum wage. For plausible parameter values, immigration raises employment of the home labor force even if all immigrants find employment.

Keywords: immigration, unemployment, efficiency wage, wage drift, wage span

JEL classifications: F11,F22,J31,J64
1 Introduction

Immigration of labor is widely considered as a threat for the labor–market prospects of the home labor force. Native workers directly competing with immigrants typically fear an erosion of their wage incomes or even a loss of their jobs. Traditionally, writers on immigration dismissed the issue as purely distributional, emphasizing instead the collective income gains of the native population resulting from immigration (for an overview, see Borjas 1999).

More recent contributors take a different stance. They argue that the traditional assumption of perfectly flexible wage rates is clearly contrary to fact, esp. for European labor markets. With unemployment caused by union wage setting, there are again potential efficiency gains from immigration, but of a very different kind. When native workers are replaced by immigrants, they either become unemployed (Schmidt, Stilz & Zimmermann 1994; Bauer & Zimmermann 1997) or are forced to apply for low–wage jobs in a secondary labor market (Fuest & Thum 2000, 2001). This replacement effect, so goes the argument, induces the unions, who represent the native workers, to reduce their wage demands, thus mitigating the inefficiency created by high wages.

The present paper argues that it is premature to conclude that immigration disciplines the unions. There is more to be said about the labor market. In the present paper, we take additional stylized facts of the German labor market into account. The resulting analysis of immigration cast doubts on the replacement effect.

We analyze the effects of immigration on wages and employment in a simplified model of the German economy, viewing Germany as a small open economy. We assume that unemployment is mainly caused by downward inflexibility of wages due to minimum wages.\footnote{The institutional differences between minimum wages and German “Tariflöhne” are of no importance in our context.} However, this cannot be the whole story. Although there is unemployment, German wages are in general higher than the legal minimum; (relative) wage spans \((w - w_{\text{min}})/w_{\text{min}}\), with \(w\) as the actual wage and \(w_{\text{min}}\) as the minimum wage, are positive. As has been recognized by Gahlen & Ramser (1987) and Schlicht (1992), positive
wage spans can be explained by assuming that the minimum wage influences the standard of fairness in the Akerlof–Yellen efficiency–wage model (Akerlof 1982; Akerlof & Yellen 1990).

The efficiency–wage approach of Gahlen and Ramser and Schlicht implies that wage spans fall if the minimum wage rises. However, as these authors have already noted themselves, aggregate data show that the average wage span rises with the minimum wage. Using slightly different data,\(^2\) we find qualitatively the same result. With quarterly data from the beginning of 1970 to the end of 1995 (104 observations), a linear regression of the average wage \(\hat{w}\) on an index of minimum wages \(w_{\text{min}}\) yields the following result:

\[
\ln \hat{w} = -15.805 + 1.0176 \ln w_{\text{min}}
\]

\(t = 175.6) \quad (t = 476.2)

\[R^2 = 0.9996 \quad F = 226984\]

Thus, the elasticity of the average wage w.r.t. minimum wage is 1.0176 > 1, which means that the average wage span \((\hat{w} - w_{\text{min}})/w_{\text{min}}\) rises with \(w_{\text{min}}\). This phenomenon is sometimes called wage drift.

Schlicht (1992) attributes wage drift, which is inconsistent with the results from his one-sector model, to the effects of aggregation. This is a reasonable explanation. It is known that there persist intersectoral wage differentials. If a rise of the minimum wage shifts employment to high-wage sectors, the average wage may increase relative to the minimum wage even if wage spans fall in every sector. If such an explanation is correct, however, there follow further, more surprising consequences.

Albert & Meckl (2001a) demonstrate that efficiency–wage setting in a multisectoral model leads to counterintuitive employment effects. Combining efficiency–wage setting with minimum wages in the way envisaged by Schlicht (1992) preserves these counterintuitive effects. Using a simplified model with constant wage spans, Albert & Meckl (2001b) show that the combination of efficiency wages with a minimum wage generates two additive components

\(^2\)In contrast to Schlicht (1992), we have used the average wage (series 2154013 of the Statistische Bundesamt) since the minimum wage (series 2557016) is a general index. However, using Schlicht’s data makes no important difference. Note that the series cannot be prolonged beyond 1995, because then East Germany is included.
of unemployment, minimum-wage unemployment and efficiency-wage unemployment, where the latter is again a potential source of surprises because it depends on the sectoral structure of the economy. Specifically, it is shown that efficiency-wage unemployment increases with a shift of employment to high-wage sectors.

The present paper extends the methodology of Albert and Meckl (2001b) to the case of variable wage spans, presenting a model that is capable of explaining wage drift. Again, there exist the same two components of unemployment. Wage drift results if and only if a rising minimum wage leads to relatively more employment in the high-wage sectors. This, however, means that both minimum-wage unemployment and efficiency-wage unemployment increase.

While this is bad news in connection with rising minimum wages, it turns out to be good news in connection with immigration. For plausible values of the relevant parameters, immigration raises employment of the home labor force even if, as required by German immigration laws, all immigrants find employment. Hence, as in a model with flexible wages, immigration leads to efficiency gains. However, these efficiency gains involve more employment and lower wages. Although we do not model union behavior in this paper, the implications for such an extended model are clear: there exists an incentive for unions to appropriate at least some of the efficiency gains by raising the minimum wage, thus offsetting the positive effects of immigration at least to some part. If efficiency gains remain, it is despite, and not because of, the unions' reaction to immigration.

Section 2 introduces the model. Section 3 analyzes the effects of a change in minimum wages and of immigration. Section 4 concludes.

2 The Model

We consider a small open economy using labor and $m$ other primary factors to produce $n \leq m$ goods under conditions of perfect competition. The production functions $f_j, j = 1, \ldots, n$ are linearly homogeneous.

The prices of the $m$ factors other than labor are determined on perfectly competitive national factor markets. The fixed supplies of these flex-price
factors are denoted by the vector $\mathbf{v}$; their prices are denoted by $\mathbf{r}$. For the vector product we write $\mathbf{r} \cdot \mathbf{v}$.

2.1 Efficiency–Wage Setting

There is a fixed number $L$ of workers, each supplying one unit of labor. One cause of involuntary unemployment is a minimum wage, which is determined by some centralized wage-setting process. We incorporate efficiency wages as a second cause of unemployment in order to make the model consistent with two important stylized facts: the persistence of intersectoral wage differentials over time, and the existence of a positive span between minimum wages and effective wages (wage span). Our efficiency–wage approach is summarized in the following assumptions.

Assumption 1 The sectoral labor input in efficiency units is $g_j(w_j/\ell)L_j$, where $L_j$ is employment in sector $j$ and $\ell$ is a reference wage against which workers measure the wage offer $w_j$ of sector $j$’s representative firm.

Assumption 2 The function $g_j$ is strictly increasing and strictly concave with $g_j(1) = 0$ and $\lim_{x \to \infty} g_j'(x) = 0$.

Assumption 3 The reference wage $\ell$ is a nonnegative, linearly homogenous, increasing and strictly concave function of the minimum wage $w^{\text{min}}$ and of average labor income $\bar{w} \overset{\text{def}}{=} \sum_{j=1}^{n} w_j L_j / L$ satisfying $\ell(1, 1) = 1$.

Assumptions 1-3 capture the essentials of Schlicht’s (1992) modification of the fair-wage approach of Akerlof (1982) and Akerlof and Yellen (1990). When deciding about their effort, workers respect a fairness norm. The effort required by this norm is assumed to depend on the employer’s wage offer $w_j$ and a reference wage $\ell$. Effort actually supplied by a worker is then an increasing function of the relative wage $w_j/\ell$. Following a suggestion by Layard, Nickell & Jackmann (1994: 37), we assume that the relation between

\footnote{Sector-specific minimum wages can also be accommodated as long as minimum wages for all sectors always change by the same percentage. This is the case if, e.g., sector-specific minimum wages grow by the same rate as national productivity.}
the productivity of labor and effort—just like that of any other production function—is sector-specific.

The assumptions on the shape of $\ell(w^{\text{min}}, w)$ are taken from Schlicht (1992). They reflect the idea that the reference is a weighted average of the minimum wage and average labor income.  

The technical assumptions on the shape of $g_j$ are standard and give rise to proposition 1.

**Proposition 1** Sectoral wages are uniquely determined by a fixed and sector-specific markup $q_j > 0$ on the reference wage: $w_j = (1 + q_j)\ell$.

**Proof.** A competitive firm facing a given minimum wage $w_L$ and given prices for other factors of production chooses a wage offer $w_j$ minimizing the costs $w_j/g_j(w_j/\ell)$ of labor in efficiency units. It is necessary and sufficient for a solution that the elasticity of the function $g_j$ is equal to 1 (Solow 1979). In view of assumption 2, this is true at some unique value $w_j/\ell > 1$. Hence, the cost-minimizing wage offer is $w_j = (1 + q_j)\ell$ for some $q_j > 0$.  

The basis of the chosen wage rate $w_j = (1 + q_j)\ell$ and corresponding productivity of labor $\bar{g}_j \overset{\text{def}}{=} g_j(1 + q_j)$, firms solve the standard cost-minimization problem, treating the reference wage $\ell$ as a parameter:

$$b_j(\ell, \mathbf{r}) \overset{\text{def}}{=} \min_{L_j, \mathbf{v}^j \geq 0} \{ (1 + q_j)\ell L_j + \mathbf{r} \mathbf{v}^j : f_j(\bar{g}_j L_j, \mathbf{v}^j) \geq 1 \}$$

(2)

This unit-cost function has all the standard properties. The envelope theorem implies

$$\begin{align*}
(a) \quad & \frac{\partial b_j}{\partial \ell} = (1 + q_j)a_{L_j} \\
(b) \quad & \frac{\partial b_j}{\partial r_h} = a_{hj}, \quad h = 1, \ldots, m,
\end{align*}$$

(3)

where $a_{L_j}$ is the input coefficients of labor and $a_{hj}$ is the input coefficient of flex-price factor $h$.

A major simplification of the presentation results from a change of variable. Mainly for want of a better word, we have opted for a name that has, at least, some mnemonic value.

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4If we were to consider changes in goods prices, we would have to deflate the reference wage by an appropriate index of consumer prices. For present purposes, we can just assume this index to be equal to 1.

5
Definition 1  The variable $N_j \overset{\text{def}}{=} (1 + q_j)L_j$ is called the (labor) absorption of sector $j$. The variable $N \overset{\text{def}}{=} \sum_j N_j$ is called aggregate (labor) absorption.

We define production functions using the new variable:
\[ \tilde{f}_j(N_j, \nu^j) \overset{\text{def}}{=} f_j \left[ \bar{g}_j N_j / (1 + q_j), \nu^j \right] \]  
This definition just hides the constants in $f_j$ and can be used to re-write (2):
\[ b_j(\ell, r) \equiv \min_{\nu_j, \nu_j \geq 0} \{ \ell N_j + r \nu^j : \tilde{f}_j(N_j, \nu^j) \geq 1 \} \]  
Thus, the reference $\ell$ is the price of sectoral labor absorption, and absorption enters the cost minimization problem in the same way as employment does in the standard case. The envelope theorem works as before (see (3)), with the difference that we now interpret
\[ a_{N_j} \overset{\text{def}}{=} \frac{\partial b_j}{\partial \ell} = (1 + q_j) a_{L_j} \]  
as the input coefficient of labor absorption.

The model implies positive wage spans and persistent intersectoral wage differentials.

Proposition 2  The wage span of sector $j$ is $(w_j - w_{\min})/w_{\min} = (1 + q_j)\ell/w_{\min} - 1$. The bilateral wage differential between sectors $j$ and $i$ is $(w_j - w_i)/w_j = (q_j - q_i)/q_i$. With employment being $L \overset{\text{def}}{=} \sum_j L_j$, the average wage $\bar{w}$ is $\bar{w} \overset{\text{def}}{=} \sum_j w_j L_j/L = \ell N/L$. Central wage differentials are given by $(w_j - \bar{w})/\bar{w} = [(1 + q_j)L - N] / N$.

Proof.  Follows from proposition 1 and definition 1. \hfill \blacksquare

Thus, bilateral wage differentials are fixed by the technology and worker preferences and, therefore, not affected by market conditions. Central wage differentials, depending on employment and its sectoral structure as reflected in labor absorption $N$, are variable but will also persist over time. The reason for this persistence is that firms prefer not to employ workers at lower wages because the reduction in wage payments is not worth the loss of worker efficiency.
2.2 Equilibrium Conditions

Since firm behavior can be described by unit-cost functions with standard properties, the equilibrium allocation can be described with the help of standard techniques, the only difference being that absorption takes the place of employment.

The subsequent analysis assumes that there exists an equilibrium where firms can, in fact, behave as described in the last subsection without encountering further restrictions.

**Assumption 4** An equilibrium exists where employment \( L \) is not higher than the labor supply \( \tilde{L} \), and where the efficiency wages \( (1 + q_j)\ell \) are not lower than the minimum wage.

If the first condition were violated, rationing of labor would have to be considered. If the second condition were violated, at least the firms in the sector with the lowest wage would have to pay the minimum wage instead of the efficiency wage, the sectoral wage span would be zero, and the efficiency of workers would no longer be constant. Both regimes can consistently be analyzed but are of no interest in the context of the present paper.

Let \( \mathbf{p} \) be the vector of the \( n \) exogenously given output prices \( p_j \). The equilibrium allocation can be described with the help of the GDP function (Dixit & Norman 1980: 44):

\[
y(\mathbf{p}, N, \mathbf{v}) \overset{\text{def}}{=} \min_{z, \mathbf{r} \geq 0} \{ zN + \mathbf{r} \cdot \mathbf{v} : b_j(z, \mathbf{v}) \geq p_j \forall j \} \tag{7}
\]

The function \( y(\mathbf{p}, N, \mathbf{v}) \) yields the GDP (total factor income). It is non-decreasing, convex and linearly homogeneous in output prices, and non-decreasing, concave and linearly homogeneous in factor endowments. The derivatives w.r.t. output prices are the equilibrium outputs; the derivatives w.r.t. the factor endowments are the equilibrium factor prices. The derivative of \( y \) w.r.t. \( N \), denoted by \( y_N \), is equal to the shadow price of \( N \). We assume that the GDP function is twice differentiable w.r.t. labor absorption \( N \), which is unproblematic since we do not have more goods than flex-price factors.

Equilibrium is described by the condition that the shadow price of labor absorption \( N \) is equal to the reference wage \( \ell \):

\[
y_N(\mathbf{p}, N, \mathbf{v}) = \ell. \tag{8}
\]
Proposition 3. The (unique and stable) equilibrium is reached when demand equals supply, in other words, when the shadow price of labor absorption is equal to its reservation price.

\[ y^N(x, N; L, w) = H(N/L; w) \]

(11)

We summarize the results in the following proposition.

\[ \ell = \frac{H(N/L; w)}{\ell} \]

(10)

where \( H(x) = \frac{1}{x - h(x)} \) is increasing with \( H(1) = 1 \). Moreover, as \( h(x) \) can easily be shown, the elasticity of \( h \) at the equilibrium point \( N = L/w \)

\[ \theta = \frac{\ell N_m}{w} \]

(9)

In order to solve (9), note that \( \frac{\ell N_m}{w} = \frac{H(w, w)}{w} \) for some nonnegative, increasing and strictly concave function \( h \) satisfying \( h(1) = 1 \). Hence, \( \frac{\ell}{\ell} \) is equivalent to \( \frac{H(N/w)}{H(w)} \) or \( H(w, w) / H(1) \). Substituting into \( \ell = \frac{H(N/L; w)}{\ell} \), we find as the second equilibrium condition

\[ \theta = \frac{\ell N_m}{w} \]

However, (8) is only one of two equilibrium conditions since \( \ell = \frac{\ell N_m}{w} \).
This condition determines aggregate labor absorption \( N \) and its price \( \ell = y_N(p, N, v) \). Condition (11) implies that, ceteris paribus, \( N \) rises when the minimum wage \( w^{\text{min}} \) falls or when the labor supply \( \tilde{L} \) rises, as with immigration.

Proof. The equilibrium condition follows from putting (8) and (10) together. Existence is assured by assumption 4. Uniqueness and stability then follow from the fact that the RHS of (11) falls with \( N \) while the LHS rises. This also implies the comparative-static result.

\[ \Box \]

2.3 Two Components of Unemployment

In the present model, it is possible to analytically separate two components of unemployment, a minimum-wage and an efficiency-wage component. The key to understanding this idea is the following proposition.

Proposition 4 If \( N \) is interpreted as employment, and \( \ell \) is interpreted as a minimum wage, condition (8) describes the standard multisectoral minimum-wage model.

Proof. The production functions \( \tilde{J}_j \) defined by (4) have all the properties of neoclassical production functions. Therefore, the unit-cost functions (5) and the GDP function (7) have all the standard properties, \( N \) behaves like employment in a neoclassical model of production.

\[ \Box \]

Since labor absorption \( N \) behaves like employment but is actually always greater than employment, there is no reason why \( N \) should not be greater than \( \tilde{L} \). An equilibrium with \( N > \tilde{L} \) can be analyzed without further problems. However, since such a case is of no special interest in our context and complicates statements of results, we rule it out by an assumption similar to the assumption of a binding minimum wage.

Assumption 5 Equilibrium is characterized by \( N < \tilde{L} \).
According to proposition 4, there is an analytically separable part of total unemployment, namely, \( \bar{L} - N \), that behaves like minimum-wage unemployment caused by \( \ell \). If we forget all the complications of efficiency-wage setting, we can analyze equilibrium condition (8) like the equilibrium condition of a standard model with a binding minimum wage. If, additionally, we take (10) into account, we can analyze the model like a model with an endogenous but binding minimum wage, where \( \bar{L} - N \) is minimum-wage unemployment.

In line with this intuitive separation of components of unemployment, we introduce the following definition.

**Definition 2** Sector \( j \)'s contribution to efficiency-wage unemployment is \( N_j - L_j \). Aggregate efficiency-wage unemployment is \( \sum_j (N_j - L_j) = N - L \). Aggregate minimum-wage unemployment is total unemployment minus aggregate efficiency-wage unemployment, \( (\bar{L} - L) - (N - L) = \bar{L} - N \).

The following proposition summarizes the implications of these definitions and the relevant assumptions.

**Proposition 5** Equilibrium is characterized by the condition \( L < N < \bar{L} \), where efficiency-wage unemployment \( N - L > 0 \) and minimum-wage unemployment \( \bar{L} - N \) add to total unemployment \( \bar{L} - L \).

**Proof.** Assumption 4 ensures \( L < \bar{L} \). Since \( N_j - L_j = q_j L_j \), \( N < L \). Assumption 5 ensures \( N < \bar{L} \). ■

It is, of course, possible to ignore definition 2 and analyze the model without using the terms minimum-wage unemployment and efficiency-wage unemployment. Definitions do not carry content, after all, and no substantive conclusions change if we decide to avoid certain words. We believe, however, that our terminology makes it easier to follow the subsequent argument.

### 2.4 The Equilibrium Factor Allocation

In order to determine the factor allocation, we evaluate the GDP function in the equilibrium determined by (11). The prices \( r \) of the non-labor factors
and the output quantities $x$ are given by the appropriate partial derivatives of $y$. Factor prices determine input coefficients, and the product of input coefficients with output quantities yields sectoral inputs. Specifically, we need sectoral labor absorptions:

\[(a) \quad N_j = a_{Nj} \frac{\partial y}{\partial p_j} \quad (b) \quad a_{Nj} = \frac{\partial b(\ell, r)}{\partial \ell} \quad (c) \quad r_h = \frac{\partial y}{\partial v_h}, \tag{12}\]

Total employment is

\[L = \sum_{j=1}^{n} L_j = \sum_{j=1}^{n} \frac{N_j}{1 + q_j}, \tag{13}\]

which implies that efficiency-wage unemployment is

\[N - L = \sum_{j=1}^{n} q_j N_j, \tag{14}\]

and the total employment rate is

\[1 - u = \frac{L}{L} = \frac{N}{L} \sum_{j=1}^{n} \frac{1}{1 + q_j} \frac{N_j}{N}, \tag{15}\]

where $u$ is the rate of total unemployment.

3 Comparative–Static Results

3.1 Minimum Wage and Employment

While the behavior of minimum-wage unemployment can be predicted by standard arguments, the behavior of efficiency-wage unemployment is tied to the sectoral structure of the economy, which means that even the direction of change cannot be predicted without further information.

**Proposition 6** While minimum-wage unemployment $N - L$ rises with the minimum-wage, efficiency-wage unemployment $N - L$ can rise or fall. The effect on total employment is ambiguous.

**Proof.** A priori, there is no presumption concerning the behavior of the $N_j$. Depending on elasticities of substitution, small changes of exogenous variables
can lead to large reallocations. Therefore, the two components of unemployment can behave quite differently. Even if \( N \) falls and minimum-wage unemployment \( \tilde{L} - N \) rises, efficiency-wage unemployment \( N - L = \sum_{j=1}^{n} \frac{q_j N_j}{1 + q_j} \) can rise or fall, depending on structural effects not derivable from the aggregate model described by (11). The overall effect on the employment rate depends on the behavior of \( N/\tilde{L} \), which is unambiguous, and the behavior of the sectoral absorption shares, which again can show large changes in any direction.

Of course, specific results can always be derived by assuming a certain production structure as, e.g., it is done in the Heckscher–Ohlin model. Alternatively, empirical data can be used to find the empirically relevant regime. We proceed according to the latter alternative. We use the empirical results given in the introduction to fix the signs of comparative-static results concerning changes of the minimum wage, which, together with some plausible assumptions, will enable us to sign the results of comparative-static results concerning immigration.

### 3.2 Minimum Wage and Wage Drift

If the minimum wage rises, the reference wage \( \ell \) must also rise. Since, however, aggregate labor absorption \( N \) falls, (10) implies that \( \ell \) rises by a smaller percentage than \( u_{\min} \), and the sectoral wage spans \( w_j/u_{\min} - 1 = (1 + q_j)\ell/w_{\min} - 1 \) fall. The average wage span can nevertheless rise (wage drift).

Since the wage sum is \( \ell N \), (10) implies that the average wage is

\[
\bar{w} \triangleq \frac{\ell N}{(1 - u)\tilde{L}} = H(N/\tilde{L}) \frac{u_{\min} N/\tilde{L}}{(1 - u)}.
\]

Using (15), we find

\[
\frac{\bar{w}}{u_{\min}} - 1 = \frac{H(N/\tilde{L}) N/\tilde{L}}{(1 - u)} - 1 = \frac{H(N/\tilde{L})}{\sum_{j=1}^{n} \frac{1}{1 + q_{j}/N}} - 1.
\]
While $N/\bar{L}$ and, therefore, $H(N/\bar{L})$ certainly falls when the minimum wage rises, there is no a priori restriction to the change in sectoral labor absorption shares $N_j/N$. For the average wage span to rise, a fall of $H(N/\bar{L})$ must be overcompensated by a fall in the denominator of (17). This means that labor absorption must shift to the high-wage sectors where $q_j$ is high and, therefore, $1/(1 + q_j)$ is low. Since high-wage sectors contribute more to efficiency-wage unemployment than low-wage sectors, this means that a rise in efficiency-wage unemployment adds to the rise in minimum-wage unemployment caused by $w^\text{min}$. We summarize this in a proposition.

**Proposition 7** Wage drift occurs iff a rise in the minimum wage leads to a sufficiently high increase in efficiency-wage unemployment.

Subsequently, we assume that the parameters of the model imply wage drift.

**Assumption 6** Wage drift occurs, i.e. the average wage span $\frac{\dot{w}}{w^\text{min}} - 1$ rises with the minimum wage $w^\text{min}$.

Formally, assumption 6 means that the total differential of (17) must be positive. This yields the condition

$$\frac{d(1 - u)}{du} \frac{w^\text{min}}{1 - u} < \frac{1}{\Theta} \frac{dN}{du} \frac{w^\text{min}}{N}.$$  \hspace{1cm} (18)

The following assumption, which is quite reasonable for Germany, can be used to further restrict the range of possible effects.

**Assumption 7** The aggregate wage share $tN/y$ is technologically fixed at $Q = 0.6$.

**Proposition 8** There exists a threshold for the elasticity of the employment rate w.r.t. the minimum wage:

$$\frac{d(1 - u)}{du} \frac{w^\text{min}}{1 - u} < -\frac{1}{1 - \Theta Q} \in (-2.5, -1).$$  \hspace{1cm} (19)
Proof. At constant output prices $p$ and with given non–labor resources $v$, the GDP function can then be written as $AN^Q$, where $A$ is a constant. Thus, we can write (11) as

$$QAN^{Q-1} = H(N/\bar{L})w_{\min}.$$  \hspace{1cm} (20)

From this, we find

$$\frac{dN}{dN} \frac{w_{\min}}{N} = -\frac{\Theta}{1 - \Theta Q} < 0.$$  \hspace{1cm} (21)

The threshold (19) then results from (18) and (21).

3.3 The Effects of Immigration

Immigration has of course the opposite effect on unemployment as a rise in the minimum wage. There is also a threshold corresponding to (19).

**Proposition 9** Immigration raises employment. There exists a positive lower threshold for the elasticity of the employment rate w.r.t. immigration:

$$\eta \overset{\text{def}}{=} \frac{d(1 - u)}{dL} \frac{\bar{L}}{1 - u} > \frac{1 - \Theta}{1 - \Theta Q} \frac{1}{1 - \Theta Q} > 0.$$  \hspace{1cm} (22)

Proof. From (20), we find that the effect of an immigration of magnitude $d\bar{L}/\bar{L}$ is the same as the effect of a decrease of the minimum wage by

$$\frac{d(w_{\min})}{w_{\min}} = -\frac{1 - \Theta}{\Theta} \frac{d\bar{L}}{\bar{L}}.$$  \hspace{1cm} (23)

The threshold (22) results from substituting (23) into (19).

However, proposition 9 does not imply that the home labor force profits in terms of employment. Regulations like the German “green card” require that immigrants have a job before they are allowed to enter. Hence, the home labor force can only profit if more jobs are created by immigration than those occupied by immigrants. We need a further empirical assumption in order to derive at a conclusion concerning the likelihood of such a result.
Assumption 8 The rate of unemployment in the initial equilibrium is \( u = 0.09 \).

Proposition 10 For a wide range of values of \( \Theta \) (approx. \( 0 < \Theta < 0.96 \)), which allows for a relatively small sensitivity of the reference wage \( \ell \) w.r.t. changes in the average wage \( \hat{w} \), immigration raises employment among the home labor force.

Proof. Employment increases by more than the number of immigrants if the elasticity \( \eta \) is greater than \( u/(1 - u) \):

\[
\eta \equiv \frac{d(1 - u)}{dL} \frac{L}{1 - u} > \frac{u}{1 - u}.
\]

(24)

Using (22), we find that it is sufficient for (24) that

\[
\frac{1 - \Theta}{\Theta} \frac{1}{1 - \Theta Q} > \frac{u}{1 - u}.
\]

(25)

The RHS of (25) is approx. 0.1. Given this approximation and \( Q = 0.6 \), there results a quadratic inequality for \( \Theta \): \( 0.06\Theta^2 - 1.1\Theta + 1 > 0 \). Since \( \Theta \) is between 0 and 1, the smaller solution (which is approximately 0.96) for the corresponding equation is an upper bound for \( \Theta \). Note that \( \Theta \) is the elasticity of \( \ell \) w.r.t. \( w^{\text{min}} \), because \( \ell \) is a linearly homogeneous function of \( w^{\text{min}} \) and \( \hat{w} \), \( 1 - \Theta \) is the elasticity of \( \ell \) w.r.t. \( \hat{w} \).

With \( \Theta = 1 \) there would be no effect of average labor income on the reference wage; therefore, immigration would no longer have any effect since lowering average labor income is the only possible effect of immigration at a given level of employment. Yet, as our robustness analysis shows, even a small influence of average labor income on the reference wage is sufficient to generate our results.

4 Conclusion

Intuitively, the mechanism explaining our results runs as follows. A rise in the minimum wage increases the reference wage, but by a smaller percentage.
Hence, minimum-wage unemployment goes up and sectoral wage spans fall. If, nevertheless, the aggregate wage span rises (wage drift), this must be because the change shifts employment to high-wage sectors. This is in line with the usual assumption that a rise in minimum wages mainly endangers low-wage jobs. However, high-wage sectors contribute most to efficiency-wage unemployment. Therefore, the wage-drift phenomenon indicates that there is a double effect on unemployment. Not only minimum-wage unemployment, but also efficiency-wage unemployment rises when the the minimum wage is increased.

Immigration works in the other direction. It puts a downward pressure on average labor income, which lowers the reference wage of workers. Hence, minimum-wage unemployment falls. In a one-sector model, such a change would never be sufficient to create enough employment to employ the immigrants, not to speak of employment for the home labor force. In a multisectoral model, however, there exists an additional effect, since efficiency-wage unemployment depends on the sectoral structure of employment. Observation of wage drift implies that efficiency-wage unemployment moves in the same direction as minimum-wage unemployment. Moreover, this effect must be quite pronounced. Under quite plausible assumptions, the effect is large enough to create more employment than necessary to employ the immigrants.

The depression of wage spans by immigration as well as the positive effect on home employment imply that immigration does not work as a disciplining device for unions. On the contrary, immigration creates an incentive for unions to raise the minimum wage and thus to appropriate some of the gains from immigration. An extension of the model covering union behavior and endogenizing the minimum wage is needed in order to find out how strong this effect might be. This extension, alas, is beyond the scope of the present paper.

References


