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Working Paper
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Tübinger Diskussionsbeiträge, No. 220

Provided in Cooperation with:
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Endogenous Skilled-Biased Technological Change and Matching Unemployment

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Tübinger Diskussionsbeitrag Nr. 220
September 2001

Wirtschaftswissenschaftliches Seminar
Mohlstraße 36, 72074 Tübingen
Endogenous Skilled-Biased Technological Change and Matching Unemployment

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Abstract

This paper presents a general-equilibrium model of endogenous skilled-biased technological change and matching unemployment in a disaggregated economy. We simultaneously endogenise both the direction and pace of technological change as well as the unemployment rates. We show that an increase in the supply of high-skilled labour can explain skilled-biased technological change, a reduction in high-skilled unemployment and a rise in the high-skilled wage differential. In accordance with convincing empirical evidence, the high-skilled suffer from shorter and fewer spells of unemployment.

JEL classification: J21, J41, J64, O31, O41
1 Introduction

It is by now a well-documented fact that starting in the 1980s, wage differentials between high- and low-skilled workers have risen and simultaneously there has been a sharp decline in demand for low-skilled workers (see Katz 2000, Deardoff, Hakura 1994 and Johnson 1997 for overviews and Fitzenberger 1999 for a detailed analysis for West Germany). Somewhat surprisingly, this rise in relative high-skilled wages was accompanied by a large increase in the supply of skilled labour (see OECD 1993, OECD 2000, Katz, Murphy 1992). Two possible causes for the increased wage dispersion most commonly stated are increased trade with low-skill labour abundant countries and skilled-biased technological change whereby new technologies and high-skilled labour are complements. Although most empirical tests tend to favour the skilled-biased technological change hypothesis and tend to dismiss the trade hypothesis (see, for example, Desjondqueres et al. 1999 and Fitzenberger 1999 for empirical evidence), as Wood (1998) points out, the two different explanations are not mutually exclusive. Further, Aghion et al. (1999) show that by dropping the assumption that increased international trade is only in final goods and instead analysing increased trade in intermediate goods can account for a much larger share of the increase in wage inequality than the conventional empirical tests. In addition, as shown for example in Acemoglu (1999), Kremer, Maskin (1996) and Lindbeck, Snower (1996), there has been a substantial amount of organisational change within firms in recent years and that this change has increased the productivity gap (and therefore wages) between workers with different skill levels. However, as shown in Aghion et al. (1999), both the increased trade in intermediate goods as well as the organisational change within firms can be attributed to skilled-biased technological change. It is for this reason that the paper focuses on skilled-biased technological change and how it influences the unemployment rates of the high- and low-skilled respectively.

Even most modern growth models either treat labour as homogeneous in which case there cannot be any skill-bias, or, even if heterogeneous labour is taken into account, assume neutral technological change (see, for example, Şener 2001, Li 2001 or Dinopoulos, Thompson 1999). Models that do explicitly incorporate skilled-biased technological change (see, for example, Albrecht, Vroman 2001,
Acemoglu 1999, Mortensen, Pissarides 1999, Gregg, Manning 1997, Eicher 1996, Mincer 1995, Bound, Johnson 1995, 1992, Katz, Murphy 1992 or Juhn et al. 1993) assume that it is exogenous. Notable exceptions are the models of Kiley (1999), who analyses an expanding variety model, and Acemoglu (1998) who concentrates on rising quality. However, both assume perfectly competitive labour markets, so that they cannot analyse the effects of skilled-biased technological change on unemployment. The aim of the present paper is to extend Acemoglu’s quality-ladder model by allowing for matching frictions in the labour market for both low- and high-skilled workers to not only explain the rise in relative high-skilled wages but also in long-term unemployment, especially amongst the low-skilled.

As in Acemoglu (1998), whether research firms develop new components to be used by high- or low-skilled workers depends on two counteracting forces. On the one hand, a higher supply of skilled labour implies that there are more workers available who are able to use the high-skill complementary components. Therefore, research firms invent new components for a larger market so that the flow profits from these inventions increase. This is called the directed technology effect. On the other hand, by the standard substitution effect, a higher supply of skilled labour reduces the high-skilled wage. Thus, the high-skilled good commands a lower price which works as a disincentive to innovate for this sector. Obviously, an analogous argumentation holds for low-skilled workers. In a steady-state equilibrium, profits from research targeted at high- and low-skill complementary components must be equal. We find that an increase in the fraction of high-skilled workers can lead both to a lower high-skilled unemployment rate as well as to a larger wage differential between the low- and high-skilled. In addition, compared to perfectly competitive labour markets, the directed technology effect is smaller and innovative firms require time to fill their newly established vacancies and start their production, so that the steady-state innovation rate is lower.

The paper is organised as follows: Section 2 describes the dynamic time path of households’ expenditures as a result of intertemporal optimisation decisions. In Section 3 the production side of the economy is introduced. A final consumption good is produced using two intermediates as inputs. These intermediates use a variety of components whose qualities are upgraded by a sequence of innovations due to intentional R&D activity carried out in a separate research sector. Section 4
analyses the pricing behaviour of firms in the component sector. Section 5 focuses on R&D competition. The labour markets for high- and low-skilled workers are characterised by matching frictions outlined in Section 6. Section 7 solves the model and identifies the determinants of both technological change as well as unemployment. Finally, Section 8 concludes.

2 Households’ Spending Behaviour

In the household sector, we assume that individuals share identical preferences according to the time-separable discounted utility function

\[ U(C) = \int_0^\infty e^{-\rho t} \frac{C^{1-\gamma} - 1}{1 - \gamma} \, dt \]  

where \( C \) is the consumption level, \( \rho \) is the common rate of time preference, \( t \) is the time index, and \( \gamma > 0 \) is the elasticity of marginal utility.\(^1\)

Households which are composed of high- and low-skilled workers as well as unemployed individuals, maximise the utility function subject to their dynamic budget constraints

\[ \dot{G} = rG + I_w - C \]

where the price of the final consumption good is normalised to one, \( r \) is the nominal (and real) interest rate, \( G \) denotes the value of household assets, and \( I_w \) the average wage income of each household. Solving this intertemporal optimisation problem yields the \text{KEYNES-RAMSEY} rule

\[ \frac{\dot{C}}{C} = \frac{1}{\gamma} (r - \rho) \]  

Because of the homothetic preferences, the growth rate (2) applies not only to each household, but also to the whole economy when \( C \) denotes the aggregated consumption level.

\(^1\) For the special case of \( \gamma = 1 \), the utility function approximates to \( U(C) = \int_0^\infty e^{-\rho t} \ln C \, dt \).
3 Input Decisions of Competitive Firms in the High- and Low-Skilled Sectors

The production technology setup is similar to that in Acemoglu (1998, 2001a,b). The homogeneous consumption good $Y$ is produced using low- and high-skilled labour according to the aggregated CES production function:

$$ Y = \left[ \delta (A_l N_l^\alpha)^{\frac{\sigma-1}{\sigma}} + (1 - \delta) (A_h N_h^\alpha)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (3) $$

where $\delta \in (0,1)$ is a distribution factor which measures the relative importance of the input factors, $A_s$, $s \in \{l, h\}$ are the technology parameters of type $s$ labour where the index $l$ denotes low-skilled and $h$ high-skilled labour. The terms $N_l$ and $N_h$ denote the number of (employed) low- and high-skilled workers respectively. They are determined by $N_h = (1 - u_h)\psi N$ and $N_l = (1 - u_l)(1 - \psi)N$ where $u_s$ are the respective unemployment rates to be endogenised below and $\psi$ is the exogenously given fraction of the workforce which is skilled. The size of the workforce is assumed to be exogenously given by $N$. The output elasticity of labour is given by $\alpha \in (0,1)$. The parameter $\sigma > 0$ denotes the constant elasticity of substitution between low- and high-skilled workers. With the production technology as specified in equation (3), the two types of labour are gross substitutes if $\sigma > 1$. Many empirical studies have estimated the value of $\sigma$ (see, for example, BOUND, JOHNSON 1992 and KATZ, MURPHY 1992). Although there is a large variation in the estimation results, most studies come to a value greater than one so it is this case that we will concentrate on.

As can be seen by comparing the relative marginal products of high- and low-skilled labour

$$ \frac{\partial Y}{\partial N_h} - \frac{\partial Y}{\partial N_l} = \frac{1 - \delta}{\delta} \left( \frac{A_h}{A_l} \right)^{\frac{\sigma-1}{\sigma}} \left( \frac{N_h}{N_l} \right)^{\frac{\alpha(\sigma-1)-\sigma}{\sigma}} \quad (4) $$

for the considered case of $\sigma > 1$, skilled-biased technological change occurs due to a relative increase in the high-skilled technology $A_h$ which raises the relative marginal productivity of the high-skilled.\(^2\) That is, when the two factors are

\(^2\) Imposing $\left( \frac{A_h}{A_l} \right)^{\frac{\sigma-1}{\sigma}} > \left( \frac{N_h}{N_l} \right)^{\frac{\alpha(\sigma-1)-\sigma}{\sigma}}$ ensures that high-skilled marginal productivity is higher than that of the low-skilled. The same result can also be achieved by assuming that the high-skilled are more productive than the low-skilled when using unskilled technology $A_l$.\(^2\)
gross substitutes, an increase in the high-skilled (low-skilled) productivity and
the resulting rise (fall) in the relative wage of high- to low-skilled workers leads
to a more than proportionate increase in low-skilled (high-skilled) labour demand,
so that the relative marginal productivity of the high-skilled will be higher (lower)
than before.

In order to keep the analysis simple, we reinterpret the production function as
given by (3) and assume that the consumption good $Y$ is produced from two
intermediates, each produced in a separate sector using only one type of labour.\(^3\)
In this case

$$Y = \left[ \delta Y_l^{\frac{1}{\sigma}} + (1 - \delta) Y_h^{\frac{1}{\sigma}} \right]^{\frac{1}{\sigma - 1}}$$

where $Y_l$ and $Y_h$ are intermediate goods produced using only low- and high-skilled labour respectively, according to the production functions

$$Y_s = A_s N_s^\alpha, \quad s \in \{l, h\}$$

Denoting with $p_s$ the price of the two intermediate goods, means that their relative price can be expressed as:

$$\frac{p_h}{p_l} = \frac{1 - \delta}{\delta} \left( \frac{Y_l}{Y_h} \right)^{\frac{1}{\sigma}}$$

As each type of labour is only employed in the production of one of these two intermediates, $s \in \{l, h\}$ can also be interpreted as a sector index. Within each sector, production uses sector-specific labour and a continuum $j_s \in [0, 1]$ of different components. The assumption that these components are also sector-specific is the means by which the technology differs for high- and low-skilled labour. The highest component quality currently available is denoted by $q_s(j)$. The demand for each component $j$ used by firm $i$ in sector $s$ is denoted by $x_s(i, j)$ and productivity is given by:

$$A_s(i) = \frac{1}{1 - \alpha} \int_0^1 q_s(j) x_s(i, j)^{1 - \alpha} dj$$

\(^3\) Alternatively, the original production function can be interpreted either as there being only one good which is produced using low- and high-skilled workers as imperfect substitutes or a combination of the above two possibilities with the economy being comprised of various sectors each producing goods which are imperfect substitutes for another and in which all sectors employ both types of labour.
which means that production of \( Y_l \) and \( Y_h \) takes place at constant returns to scale. Flow profits of firm \( i \) purchasing components of quality \( q_s(j) \) are determined by

\[
\pi_s(i, j) = p_s A_s(i)n_s(i)^\alpha - \int_0^1 \chi_s(j)x_s(i, j) dj - w_s n_s(i) \tag{8}
\]

where \( n_s(i) \) is the amount of labour of type \( s \) employed by firm \( i \), with total labour demand in each sector given by \( \int_{0}^{\bar{i}_s} n_s(i) di = N_s \) where \( \bar{i}_s \) is the number of firms in either sector, and \( \chi_s(j) \) is the price of a component with quality \( q_s(j) \) in the respective sector. Note, however, that due to the constant returns to scale production technology, labour demand in each sector is independent of the number of firms.

Using the profit function, optimal aggregate demand \( X_s(j) \) for component \( j \) in sector \( s \) is

\[
X_s(j) = \left( \frac{p_s q_s(j) N_s^\alpha}{\chi_s(j)} \right)^{\frac{1}{\alpha}} \tag{9}
\]

This concludes the description of the final and intermediate goods sectors. The next section analyses the behaviour of firms producing the components used in the intermediate sectors.

4 Pricing Behaviour of Incumbent Firms

In all industries in either sector, \( q_s(j) \) units of the final good are needed to manufacture one unit of the state-of-the-art component \( j \). Thus, the production costs increase with the components’ quality. An industry leader whose technology is assumed to be perfectly protected by an infinitely lived patent, will maximise his profit function

\[
\Pi_s(j) = \chi_s(j)x_s(j) - q_s(j)x_s(j) \tag{10}
\]

with respect to its price \( \chi_s(j) \). This yields constant markup pricing

\[
\chi_s(j) = \frac{q_s(j)}{1 - \alpha} \tag{11}
\]
Each innovation carried out in the research sector improves the quality of a component by the exogenously given factor $\lambda > 1$. The size of each quality improvement is the same for all components in both sectors. Imposing $\lambda > (1 - \alpha)^{-1}\lambda$, i.e. assuming drastic innovations, ensures that firms producing the intermediate goods $Y_l$ and $Y_h$ will prefer buying the highest quality components even if lower quality ones are sold at marginal costs.

With the component prices as given by equation (11), each firm in either sector buys $x_s(i,j)$ components so that $X_s(j) = [(1 - \alpha)p_sN_s]^\frac{1}{\alpha}$ and by equation (7), equilibrium productivity in sector $s$ is given by

$$A_s = (1 - \alpha)^{\frac{1 - \alpha}{\alpha}Q_s[p_sN_s]^\frac{1}{\alpha}}$$

where $Q_s \equiv \int_0^1 q_s(j)\text{d}j$ is the average quality of components used in sector $s$.

5 R&D Competition of Potential Entrants

The quality of components can be upgraded by a sequence of innovations, each of which builds upon its predecessors. To produce a higher quality component, a blueprint is needed, which is developed by innovative firms in a separate R&D sector. The lure of innovation rents drives potential entrants to engage in risky R&D projects to search for the blueprint of a higher quality component. The price for an innovation is the profit flow (10) that will last until the next innovation success is achieved within this industry. There is free entry into each innovation race and the potential entrepreneur can target his research efforts at any of the continuum of state-of-the-art components in either sector. Any research firm undertaking R&D at intensity $h_s(j)$ for a time interval of length $dt$ will succeed in taking the next step up the quality ladder for the targeted component with probability $h_s(j)$. This implies that the number of realised innovations in each industry follows a Poisson process with the industry-specific arrival rate $h_s(j)$, which is given by

$$h_s(j) = z_s(j)\phi(z_s(j))$$

where $z_s(j)$ is R&D input in terms of the final good and it is assumed that $\phi'(z_s(j)) < 0$ and $h'_s(z_s(j)) = \phi(z_s(j)) + z_s(j)\phi'(z_s(j)) \geq 0$ to account for decreasing returns to R&D effort. With research productivity given by $1/\mu$, at a flow
cost of $\mu q_s(j)z_s(j)dt$ over the time interval $dt$, each research firm participating
in the innovation race in sector $s$ can attain the stock value $J_s(j)$ of a successful
entrepreneur with the leading technology in industry $j$ with probability $h_s(j)$. Thus, free entry into any innovation race leads to the zero-profit conditions

$$\phi(z_s(j)) J_s(j) = \mu q_s(j)$$  \hspace{1cm} (14)$$

which hold for all industries in either sector.

Each firm participating in a R&D race has no internal funds to finance its R&D
activities and must therefore issue equity claims. These claims pay nothing if the
firms’ R&D efforts fail, but yield the profit stream (10), being paid out continu-
ously as dividends, if the firm succeeds in winning the race and last as long as the
firm keeps the industry leadership. With probability $h_s(j)$, one of the tar-
geted innovation efforts will succeed and a new entrepreneur will take over the
leadership so that equity owners of the incumbent firm will suffer a total capital
loss of $J_s(j)$. Before being able to implement the new technology, the innovating
firm must first open up new vacancies to find suitable workers to operate the new
technology embodied in the new component.

Due to matching frictions in the labour market, there is an expected delay of
$d_s = 1/f(\theta_s)$ to fill vacancies, where $f_s$ is the rate at which vacancies are occupied
and $\theta_s$ is an indicator of market tightness. To keep the analysis tractable, we
follow Aghion, Howitt (1994) in assuming that by the law of large numbers,
the time it takes to fill each vacancy is deterministic. Therefore, during the time
span $d_s$, the current components will still be in use. This means that the total
time a component of a particular vintage is in operation is independent of the
time needed to fill a vacancy, as the search process delays the starting and the
end point by the same amount. Once the vacancies have been filled, the demand
for all previous vintages drops to zero and the workers using these now obsolete
components are dismissed. Letting $\tau$ denote the random time interval between two
innovations in an industry in a specific sector, then due to the Poisson process,
$\tau$ is exponentially distributed over an infinite time horizon with parameter $h_s(j)$. Therefore, the value $J_s(j)$ of a research firm owning the leading technology $q_s(j)$
in sector $s$ is given by

$$J_s(j) = e^{-rd_s} \left\{ \int_0^\infty h_s(j)e^{-h_s(j)\tau} \left[ \int_0^{\tau} \Pi_s(j)e^{-rt} dt \right] d\tau \right\}$$
\[ e^{-rd_s} \frac{\Pi_s(j)}{r + h_s(j)} \]  

(15)

where it will be shown below that in the steady-state equilibrium the interest rate \( r \) is constant.

6 Matching Technology in the Labour Markets

Vacancies posted by firms are skill-specific. It is assumed that the cost \( c \) of posting a vacancy (measured in terms of final output) is irrespective of skill type. A low-skilled worker cannot apply for a high-skilled job because he is assumed to be unable to operate the components used in the high-skilled sector. The high-skilled, on the other hand, can perform the tasks of the low-skilled workers. However, the returns to search are always higher when applying to high-skilled vacancies as will be shown below. This means that a high-skilled worker has no incentive to apply for a low-skilled job.

The matching function is assumed to be the same for both skill-types and takes on the standard form

\[ m_s = m(u_s, v_s) \]  

(16)

where \( m_s \) denotes the number of matches and \( v_s \) the vacancy rate for type \( s \) workers. The matching function is assumed to be concave, increasing in both arguments and homogeneous of degree one.\(^4\)

The (sector-specific) rates \( f_s \) at which vacancies are filled are given by:

\[ f(\theta_s) = \frac{m_s}{v_s}, \quad f'(\theta_s) < 0 \]  

(17)

where \( \theta_s \equiv \frac{v_s}{u_s} \) is an indicator of labour-market tightness with increasing values of \( \theta_s \) implying that firms will need longer to find suitable workers.

\(^4\) See Petrongolo, Pissarides (2001) for a survey of the theoretical and empirical literature on matching functions. In this survey many articles are cited confirming that a matching function which is homogeneous of degree one is consistent with the overwhelming empirical evidence.
Using this equation, the rate at which the unemployed find a job in sector $s$ is

$$\frac{m_s}{u_s} = f (\theta_s) \theta_s$$  

(18)

By the properties of the matching function (16), $f (\theta_s)$ has an elasticity in the interval $(-1,0)$. Therefore, the job-finding-rate is an increasing function in $\theta_s$, implying that the higher the ratio of vacancies to job-seekers is, the easier (and faster) will workers find a new job.

The value of a vacancy to a firm in sector $s$ is denoted by $W^V_s$ as long as it is vacant and by $W^F_s$ once a successful applicant has been found so that the corresponding Bellman equation can be derived as

$$rW^V_s = f (\theta_s) (W^F_s - W^V_s) - c$$

(19)

Thus, the return on a vacancy is equal to the expected gain from finding a suitable applicant minus the costs of keeping the vacancy open. Since there is free market entry, job creation will occur as long as there are positive rents so that in equilibrium $W^V_s = 0$ must hold. From equation (19), this implies that in a steady-state

$$W^F_s = \frac{c}{f (\theta_s)}$$  

(20)

i.e. that the value of filling a position is equal to the expected search costs. Therefore, a tighter labour market leading to a longer search time, implies that the value of any newly opened vacancies must rise accordingly.

The value of a job from the perspective of an employed worker is denoted by $V^E_s$ and that of unemployment by $V^U_s$. This means that the total surplus $S_s$ from a match is:

$$S_s = W^F_s + V^E_s - V^U_s$$

(21)

Assuming that this surplus is divided amongst firms and workers according to a bargaining process where $\beta_s$ denotes worker (or union) bargaining power leads to:

$$W^F_s = (1 - \beta_s) S_s$$

(22)
so that

\[ V_s^E - V_s^U = \beta_s S_s \]  

(23)

Since firms have the option of closing the job at any point in time and workers can always quit a job, a filled position continues in operation as long as the value of the surplus is positive. With \( w_s \) denoting the respective wage rates, the three Bellman equations for \( W_s^F, V_s^E \) and \( V_s^U \) can be written as

\[ rW_s^F = y_s - w_s + h_s(W_s^V - W_s^F) \]  

(24)

\[ rV_s^E = w_s + h_s(V_s^U - V_s^E) \]  

(25)

\[ rV_s^U = f(\theta_s) \theta_y y_s \]  

(26)

where from above, \( h_s \) is the arrival rate of new innovations and \( y_s \equiv p_s \partial Y_s / \partial N_s \) is the value of marginal output of a worker in sector \( s \).

Using equations (22) to (26) we can express the wage rates \( w_s \) as

\[ w_s = \beta_s y_s + (1 - \beta_s)rV_s^U \]  

(27)

where

\[ rV_s^U = \frac{r + h_s + \beta_s f(\theta_s) \theta_y y_s}{r + h_s + \beta_s f(\theta_s) \theta_y} \]  

(28)

Finally, using (19) and (24) one obtains

\[ c = \frac{f(\theta_s) (1 - \beta_s)(y_s - rV_s^U)}{r + h_s} \]  

(29)

Using equations (20) to (28), enables us to solve for the value of a total match yields

\[ (r + h_s)S_s = y_s - \frac{\beta_s c}{1 - \beta_s} \theta_s \]

As noted above, as soon as a zero-surplus \( S_s \) is reached, a job will be destroyed. Therefore, the critical productivity level is given by

\[ y_s = \frac{\beta_s c}{1 - \beta_s} \theta_s \]  

(30)
where the r.h.s. is the expected return to searching for a job.

Assuming that low-skilled workers are more likely to be organised in a union and thus have a higher bargaining power $\beta_l$ than do high-skilled workers, means that for a given productivity level, the high-skilled labour market will be tighter. The returns to searching for a job are the opportunity costs of being employed and correspond to the minimum wage that is needed to prevent a worker from quitting. As long as these opportunity costs of employment is not too large, it will outweigh the costs of terminating the job and opening up a new vacancy.

As can be seen from equation (30), an increase in $y_s$ means that jobs with a lower job-specific productivity are still profitable so that more vacancies will be created as a result, i.e. labour-market tightness $\theta_s$ increases. Secondly, a higher bargaining position $\beta_s$ will lead to a higher ratio of unemployed workers to vacancies (i.e. a reduction in $\theta_s$) as each job becomes less profitable for firms. Finally, solving (28) and (29) for $c$ shows that factors leading to an increase in the innovation rates $h_s$ result in a lower labour-market tightness $\theta_s$. This can be explained by the fact that higher innovation rates increase the rates at which jobs are destroyed leading to a larger number of unemployed relative to the number of vacancies.

In order to endogenously determine $u_s$ and $v_s$ a flow-equilibrium condition is needed which is characterised by

$$m(u_s, v_s)dt = (1 - u_s)h_sd t$$

(31)

Using equations (17) and (31), we derive an innovation-based BEVERIDGE-Curve as

$$u_s = \frac{h_s}{h_s + \theta_s f(\theta_s)}$$

(32)

This enables us to determine the steady-state equilibrium of the model and allows us to derive some comparative statics in the next section.

### 7 Equilibrium and Comparative Statics

In a steady-state equilibrium, the value of firms owning the leading technologies will be constant. Combing this with equations (9), (10), (11) and (14) means that
equation (15) becomes:

$$\alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} p^\frac{1}{\alpha} e^{-rd_s} N_s = \mu \left( \frac{r + z_s(j)\phi(z_s(j))}{\phi(z_s(j))} \right)$$

(33)

for all $j \in [0, 1]$ and $s = \{l, h\}$. The l.h.s of (33) denotes the flow profits for innovating firms from component sales to the intermediate goods producers. If these profits increase, the research effort $z_s(j)$ aimed at component $j$ will also rise. The profits will be higher if the product price is higher or more workers use the new component so that their demand is higher. Given symmetric firms, it can be seen from (33) that $z_s(j) = z_s$ within both sectors, so that the same amount of research effort is aimed at all components in either sector. Seeing as the r.h.s of (33) is increasing in research effort $z_s$, it follows that the relative research effort $z_h/z_l$ is increasing in $p_h/p_l(e^{-rd_h}N_h/e^{-rd_l}N_l)^{\alpha}$. This means that if $p_h$ is high relative to $p_l$, it is more profitable to invent high-skill-complementary components because their output commands a higher price. From (5), (6) and (12) we obtain

$$\frac{p_h}{p_l} = \left( \frac{1 - \delta}{\delta} \right)^{\frac{\alpha}{\alpha}} \left( \frac{Q_h}{Q_l} \right)^{\frac{-\alpha}{\alpha}} \left( \frac{N_h}{N_l} \right)^{\frac{\alpha}{\alpha}} \left( \frac{e^{-rd_h}}{e^{-rd_l}} \right)^{\frac{\alpha}{\alpha}}$$

(34)

Equation (34) shows the expected price-effect: Since $\sigma > 1$ is assumed, an increase in relative high-skilled labour employment $N_h/N_l$ will lower the relative price of the high-skilled product so that researching for low-skill complementary technologies becomes more attractive. However, this argument does not take the effect on average technology due to an increase in relative high-skilled labour demand into account.

In a steady state, $Q_h/Q_l$ must be constant so that the respective growth rates of the technologies must be equal which only occurs if research effort is the same for both sectors, i.e. $z_h = z_l$. Inserting this result into (33) yields

$$\frac{p_h}{p_l} = \left( \frac{e^{-rd_h}}{e^{-rd_l}} \right)^{-\alpha}$$

(35)

Combining this equation with (34) leads to

$$\frac{Q_h}{Q_l} = \left( \frac{1 - \delta}{\delta} \right)^{\sigma} \left( \frac{e^{-rd_h}}{e^{-rd_l}} \right)^{1 + \alpha(\sigma - 1)} \left( \frac{N_h}{N_l} \right)^{\alpha(\sigma - 1)}$$

(36)
As can be seen from equation (36), relative average (endogenous) technology is an increasing function of relative labour supply, i.e. if there are more workers using components designed for the high-skilled, it will become increasingly profitable to develop components that can be used by this worker group.

Assuming perfect competition for the consumption good $Y$ leads to

$$1 = \delta^\sigma p^1\sigma + (1 - \delta)^\sigma p_h^1\sigma$$ (37)

Combining equations (35) and (37) means that (33) can be rewritten as

$$\alpha(1 - \alpha)^{1 - \alpha} \left[ \delta^\sigma (e^{-rd} N_i)^{\alpha(\sigma - 1)} + (1 - \delta)^\sigma (e^{-rd} N_h)^{\alpha(\sigma - 1)} \right]^{1 \alpha(\sigma - 1)} = \mu \left( r + z^* \phi(z^*) \right) \phi(z^*)$$ (38)

where $z_h = z_i = z^*$ is the equilibrium research effort targeted at any component in either sector. In the case of perfect labour markets with no frictions and no unemployment, the term in square brackets would be larger. Thus, it can immediately be seen from (38) that the profits from technology sales are lower than they are in the case of perfectly competitive labour markets. These lower profits reduce the incentives to research. As shown below, the interest rate is an increasing function of the growth rate. Therefore, in order for (38) to hold with equality, the interest and growth rates must also be lower than in the case where all markets are fully competitive. There are three reasons for this lower growth rate. Firstly, with unemployment there are fewer employees in each sector which lowers innovation incentives. Secondly, the lower number of workers reduces the amount of final output and thus the available research resources. These are pure scale effects. Thirdly, with matching frictions, all successful innovators have to delay the introduction of their technology by the time interval $d$, further reducing the net value of the innovation.

With this equilibrium research intensity $z^*$, the economy’s growth rate $g^*$ is determined in terms of the exogenous innovation size $\lambda$ and the steady state arrival rate $h^* = z^* \phi(z^*)$, i.e.

$$g^* = g^*(\lambda, h^*), \quad g^*(\lambda) > 0, \quad g^*(h^*) > 0$$ (39)

\cite{Acemoglu1998} argues that the growth rate $g^*$ can be expressed by $g^* = (\lambda - 1)h^*$. However, this is not the correct expression which, normalising initial qualities of all components to one,
This implies that in equilibrium equation (2) becomes

\[ r = \rho + \gamma g^* \]  

Finally, equations (30), (32) and (38) yield three equations (for each sector) in the three remaining unknowns, \( u_s, v_s \) and \( z^* \).

As it is assumed that the high-skilled have a higher marginal productivity and a lower bargaining power \( \beta_h \) means that the value of their output \( y_h \) will be higher. Therefore, as can be seen from equation (30), the labour market for the high-skilled is tighter. Inserting this result together with the fact that the research intensity is identical in both sectors into the unemployment equation (32) shows that the unemployment rate is lower for the high-skilled than for the low-skilled. Further, the higher labour-market tightness for the high-skilled also means that they need a shorter time period to find a new job.

Turning to the wage differential between the two skill groups, it can be seen from equation (27) that wages for the two skill groups in terms of the final goods are proportional to their respective values of marginal output, \( y_s \). In relative terms, this value is

\[ \frac{y_h}{y_l} = \frac{p_h A_h}{p_l A_l} \left( \frac{N_h}{N_l} \right)^{-(1-\alpha)} \]

which, by inserting equations (12), (35) and (36), becomes

\[ \frac{y_h}{y_l} = \left( \frac{1 - \delta}{\delta} \right) \left( \frac{e^{-rd_h}}{e^{-rd_l}} \right)^{\alpha(\sigma-1)} \left( \frac{N_h}{N_l} \right)^{\alpha(\sigma-1)-1} \]  

which means from the wage equation (27) that the wage differential is proportional to

\[ \frac{w_h}{w_l} \propto \frac{\beta_h}{\beta_l} \left( \frac{1 - \delta}{\delta} \right)^{\alpha(\sigma-1)} \left( \frac{N_h}{N_l} \right)^{\alpha(\sigma-1)-1} \]

is in fact given by

\[ g = \frac{d(f_0^1 \lambda^b d j)/dt}{\int_0^1 \lambda^b d j} \]

where \( b_j \) are the number of quality improvements of component \( j \).
The relationship given by (42) shows that a higher value of \( \psi \) which c.p. leads to a rise in the supply of high-skilled labour \( N_h \), can lead to an increase in the wage differential between high- and low-skill labour. There are two effects of a higher supply of high-skilled labour which counteract each other. Firstly, there is the standard substitution effect which decreases the wage differential. Secondly, there is the directed technology effect, whereby a larger number of high-skilled workers increases the demand for components complementary to these workers and so alters the direction of technological change leading to an increase in the wage differential. The second effect is more likely to dominate the larger \( \sigma \) is, i.e. the closer substitutes the two intermediates are, or the higher \( \alpha \) is, i.e. the smaller are the decreasing returns to labour within each sector. Further, it can be seen from the threshold productivity level equation (30), that for the case that the directed technology effect outweighs the substitution effect, firms employing high-skilled labour will create more vacancies so that the high-skilled labour market becomes tighter, i.e. \( \theta_h \) increases. This means that firms in this sector will require longer to fill their vacancies, which reduces the incentives to innovate for this sector. However, whether the R&D intensity rises or falls depends on whether the total returns to innovative activities, as given by the l.h.s of equation (38), decrease or increase. This cannot be unambiguously determined, as the increase in the high-skilled labour supply is counteracted by the decrease in the supply of the low-skilled. However, for the case that the increased high-skilled labour-market tightness and the decrease in the supply of the low-skilled dominate, the research intensity \( z^* \) and therefore the innovation arrival rate \( h^* \) will decline. In this case, it can be seen from the BEVERIDGE curve (32), that the high-skilled unemployment rate decreases. Seeing as the value of marginal output by the low-skilled will decrease for large values of \( \alpha \) or \( \sigma \), their labour-market tightness will decline. In this case, even if the job-destruction rate is lower due to the lower research intensity, fewer jobs will also be created so that the low-skilled unemployment rate may rise or fall depending on which effect is stronger.

Finally, (42) also shows that the wage differential depends on the relative bargaining power of the two types of labour. Assuming that low-skilled workers are more likely to be unionised than their high-skilled counterparts and therefore have a higher bargaining power \( \beta_l \) and that this bargaining power is particularly strong in most European countries, will lead to wage differentials in these
countries being more compressed than in countries with weaker union power. This result corresponds to empirical comparisons of relatively high wage differentials in Anglo-Saxon countries with little union power, and much lower wage differentials in most European countries.

A decrease in the distribution parameter $\delta$, i.e. a shift away from the low-skilled towards high-skilled workers leads directly, as can be seen from (41), to a higher relative value of the high-skilled marginal output $y_h$ and therefore, by equation (42), to an increase in the wage differential. As above, this leads to a rise in the market tightness for the high-skilled and also to a decrease in the returns to innovation. Further, as $\delta > 0.5$, this reduction in the distribution parameter lowers the profits from research. Both effects lead to a reduction in the innovation arrival rate $h^*$ and thereby to a reduced job-destruction rate, so that the high-skilled unemployment rate falls. At the same time, the low-skilled labour market will become less tight as $\delta$ decreases, so that it is not unambiguously clear whether the unemployment rate will be higher or lower than before.

A rise in either the rate of time preference $\rho$ or the elasticity of marginal utility $\gamma$ both lead to a higher interest rate $r$. On the one hand, this higher interest rate leads to an increase in the r.h.s of the innovation intensity equation (38) and on the other hand, the returns to innovating, i.e. the l.h.s of (38), decreases. In the new equilibrium, this decrease in the returns can only be compensated by firms closing vacancies so that the labour markets become less tight. This however means that unemployed individuals need longer to find a new job so that the unemployment rates rise. Further, the lower returns to innovation will also lead to a lower research intensity which works to both counteract the initial increases in the interest rate as well as the rise in the unemployment rates. As all of these adjustments affect both sectors equally, the direction of technological change will not be altered, so that the wage differential will also remain unchanged.

The same qualitative results occur if the size of technological improvements $\lambda$ increases which leads to a higher growth rate. In this case, as can be seen from equation (40), the interest rate $r$ will rise and the same adjustment mechanism as described above will occur.

The parameter $\mu$ is a measure of the marginal research costs associated with inventing a new higher quality component. Therefore, an increase in this parameter
will mean that temporarily, the costs of inventing a new component, as given by the r.h.s of equation (38), will be higher than the returns. This means that the research intensity $\dot{z}^*$ and thereby the innovation arrival rate $\dot{h}^*$ must fall. This means that the job-destruction rate also falls, so that by equation (32), the respective unemployment rates will also decline. This in turn increases the demand for components, thereby increasing the returns from innovation. As these costs equally influence the costs of new components in both sectors, no skill-bias will result, so that the relative productivities of the two labour types and thus the wage differential will remain constant.

There are two direct effects of higher values of either the substitution elasticity $\sigma$ or the elasticity of output with respect to labour $\alpha$. Firstly, as can be seen from equation (41), the relative value of high-skilled marginal output will rise so that the wage-differential between the two groups also increases. Secondly, the returns to innovating will increase as innovations now yield a higher flow profit. These direct changes lead to the labour market for the high-skilled becoming relatively tighter. Although this means that firms will now need longer to fill their vacancies so that the return to a new innovation will decrease as it will not be implemented for a longer time period, the overall effect is that research intensity rises. This leads to a higher job-destruction rate so that the overall effect on high-skilled unemployment is ambiguous. On the one hand the higher labour market tightness will decrease high-skilled unemployment, on the other hand the higher innovation arrival rate will increase it. However, the low-skilled unemployment rate will unambiguously rise as due to their lower value of marginal output there will be fewer vacancies as well as a higher job-destruction rate.

The final comparative static effect to be analysed is an increase in the costs of posting a vacancy $c$. As can be seen from equation (30) which defines the threshold productivity level, higher vacancy costs imply that the critical productivity level must also adapt and rise accordingly. At the same time, with higher vacancy costs, some vacancies will now no longer yield a positive surplus so that labour-market tightness must fall. This lower value of $\theta_*$ means that firms will need a shorter time period to hire new workers once a new component has been innovated. This increases the returns to innovation so that research intensity $\dot{z}^*$ increases. With both fewer vacancies and a higher innovation arrival rate $\dot{h}^*$, the unemployment rates for both labour types must increase. Seeing as these higher costs effect both
sectors equally, the direction of technological change remains unchanged so that the wage differential will also remain unchanged.

8 Conclusion

Since the 1980s, the supply of high-skilled labour has increased in all developed economies. At the same time, technological change has been (high-)skilled-biased, the wage differential between high- and low-skilled workers has increased and unemployment rates have remained roughly constant for the high-skilled whereas they have increased sharply for the low-skilled. The model presented here is able to simultaneously explain these empirical facts as the direction of technological change is endogenously determined. This means that an increase in high-skilled labour will lead to two counteracting effects. On the one hand, a higher supply will lead to a lower wage for this skill-group causing firms to create more positions for these workers. At the same time, researchers will increase their research efforts aimed at components used by high-skilled workers so that technological change will be high-skill biased. Although this bias increases the rate of job destruction, under plausible conditions the job-creation effect will dominate so that the unemployment rate of these workers will fall. At the same time, the increased rate of technological change aimed at high-skilled workers will increase their productivity so that the wage differential also rises. Finally, we find that the innovation rate of an economy characterised by imperfect labour markets is lower than in a perfectly competitive economy. There are three reasons for this result. Firstly, with unemployment there are fewer employees in each sector which lowers innovation incentives. Secondly, the lower number of workers reduces the amount of final output and thus the available research resources. These are pure scale effects. Thirdly, with matching frictions, all successful innovators have to delay the introduction date of their technology, further reducing the net value of innovations.
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