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MEMORANDUM

No 13/2008

Waiting To Merge*

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Department of Economics
University of Oslo
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Waiting To Merge*

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Abstract

We set up a sequential merger game to study a firm’s incentives to pass up on an opportunity to merge with another firm. We find that such incentives may exist when there are efficiency gains from a merger, firms are of different sizes, there is an antitrust authority present to approve mergers, and there is a sufficient alignment of interests between the antitrust authority and the firms. We point out three distinct motives for not merging: the external-effect motive, the bargaining-power motive, and the pill-sweetening motive.

JEL classification: L11; L13; L41; G34.

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1 Introduction

What reasons may a firm have for passing up on an opportunity to merge with a rival? One obvious answer is that a merger may simply be unprofitable. In this paper, we explore some less obvious answers, which all are based on the notion that the firm is forward-looking: a firm may pass up on an opportunity to merge because, without that merger, the industry will develop in a way that is better for the firm than what would follow the merger. In order to do this, we construct a model of sequential mergers in which the antitrust authority takes active part. We delineate three possible motivations for a firm, in addition to a mere lack of profitability, for passing up on a merger.

First, the firm may abstain from merger in order to free-ride on other firms’ mergers. This possibility is particularly lucrative when the antitrust authority would not be interested in allowing too many mergers. A merger typically raises the equilibrium price in the industry and thus benefits also non-merging firms. If the firm is small relative to the other firms in the industry and, say, only a single merger would be allowed, then it may be better for the firm to pass up on its own opportunity to merge and be on the outside of a big entity with a considerable price rise, than to merge itself and be on the inside of a smaller merger with a meager price rise. We call this the external-effect motive for not merging.

Secondly, the antitrust authority may allow several mergers to take place, but the firm would improve its bargaining position vis-à-vis its merging partners by arriving late at the bargaining table. The reason for this is related to the external-effect motive just discussed: a firm on the outside of a big merged unit has a large outside option because of the external effect and therefore has a strong bargaining position when entering negotiations to join the big unit. And so it may pay for a firm to pass up on an early opportunity to merge in order to strengthen its bargaining position before eventually merging later. We call this the bargaining-power motive for not merging.

Thirdly, there are intermediate cases where the antitrust authority may or may not allow several mergers, and where the firm, by postponing its own merger, may get more mergers through the antitrust authority than it would if the firm itself merges immediately. This happens in cases where the firm is small relative to the other firms in the industry. If the firm were to merge, the industry would become rather balanced and the antitrust authority would be content with that situation and not willing to allow further mergers. If, on the other hand, the firm does not merge and instead other firms in the industry do, then the industry would be so unbalanced that the antitrust authority would be more inclined to allow further mergers, in order to gain balance or, when large efficiency gains are present, even to allow complete monopoly. We call this the pill-sweetening motive for not merging.

We present a two-period model with three firms and an antitrust authority. In the first period, the firms and the antitrust authority play a sequential merger game, to be described in more detail shortly. In the second period, the firms left after any consolidations in the first period compete in quantities in the
product market. In line with our focus on a firm’s incentives to pass up on an opportunity to merge, the merger game is such that one firm has the opportunity at the initial stage to find a merger partner and propose a merger. If a merger is proposed - more generally, after any merger proposal - the antitrust authority makes a decision whether or not to approve the merger. Thus, the only mergers that are actually carried out are those that pass the scrutiny of the antitrust authority. The agency is assumed to be forward-looking, but unable to commit to any future action.

If the first firm decides not to merge, or if it does but the merger is not approved, then the two other firms decide whether to merge with each other. If they do, and their proposal is approved, then the first firm is allowed a second chance at merging, since now the situation has changed from the previous stage when a decision not to merge was made. This opens up for the industry to end up in complete monopoly even when the game starts out with a decision by the first firm not to merge.

In our analysis of this model, we find four features that are crucial for the occurrence of the three motives for the first firm not to merge. The first such feature is the presence of the antitrust authority. Clearly, there cannot be any pill-sweetening motive for not merging without the antitrust authority around. But also the external-effect motive hinges on the agency being present. This motive comes about because of the presence of the antitrust authority who will only allow a limited number of mergers. The bargaining-power motive, on the other hand, shows up even when the antitrust authority disappears. In fact, the interesting result is rather of the opposite flavour: despite the antitrust authority’s presence, cases exist where merger to monopoly takes place at the same time as it pays for the first firm to delay its own entry into the sequence of mergers.

Secondly, asymmetry among firms is crucial in producing the pill-sweetening motive for not merging: with all firms of equal size there is no way the number of approved mergers can be affected by the sequence they arrive in. We model firm asymmetry in the simplest way possible: we let the first firm differ from the other two, which on the other hand are of equal size.

Thirdly, there must be some efficiency gains associated with a merger. Without such gains, the antitrust authority would not see any reason to allow any merger, and we would not be able to see the interplay between what the agency would allow and what the firm would like to see happen as we have described above. The presence of efficiency gains also creates an incentive for firms to merge in the first place. We apply a simple version of Perry and Porter’s (1985) model of merger, making a merger interesting because an input factor is in total fixed supply and available only inside the industry, and because a merger reduces costs for the firms taking part in it. The crucial industry-specific factor can be thought of as human capital: knowledge about doing business in this industry is available inside the industry only, and the more you have of it, the more efficiently you can run your firm.

Finally, there must be sufficient alignment between the firm’s interests and those of the antitrust authority. In our model, the antitrust authority is more
interested in allowing mergers the smaller the market, since a small market means there are too many firms in the industry. But if the antitrust authority’s aim is to maximize consumer surplus, then mergers will only be allowed for very small markets. In particular, the first firm’s incentives to pass up on the initial opportunity to merge only show up for market sizes for which the consumer-surplus maximizing antitrust authority does not allow any mergers. With more weight put on firms’ profits in the antitrust authority’s objective function, on the other hand, the scope for allowing mergers increases, and there is eventually an overlap between combinations of market size and firm asymmetry for which on the one hand the antitrust authority is interested in allowing one or more mergers and on the other hand the first firm is interested in waiting to merge.

The present model belongs to a growing literature on endogenous mergers. According to Horn and Persson (2001a), a model of endogenous merger is one where more than one merger is possible, and they delineate three distinct approaches to such models. One of them is atemporal, based on cooperative game theory, and exemplified by Horn and Persson themselves. The second approach is pioneered by Kamien and Zang (1990), with the owner of each firm, in each round of the merger game, setting a bid price for each of the other firms and an ask price for her own firm. Finally, there is the bargaining approach suggested by Ray and Vohra (1999), with bargaining taking place according to a fixed protocol, where a protocol is a sequence of proposers and, for each proposer, a sequence of respondents. Our model contains such a fixed protocol, although a very simple one, since we restrict attention to pairwise mergers, so that, for each proposer, there is a single respondent. We differ from Ray and Vohra, however, in letting merged units stay in the game so that they can take part in further mergers, whereas their merged units leave the game. In this respect, we are related to the game of Macho-Stadler, et al. (2006), who unlike us, however, have a random protocol.

In many analyses of endogenous mergers, it is assumed that only one merger can happen, and so the interest centers on which one; see, e.g., Fridolfsson and Stennek (2005a). The interest in the literature in the study of sequential mergers, where one merger decision is followed by one or more others, starts with Caves (1991). Kamien and Zang (1993) extend their 1990 paper to a situation where a sequence of mergers is allowed. Nilssen and Sørgard (1998) analyze sequential merger decisions made by disjoint sets of firms, while Fauli-Oller (2000) and Neary (2007) analyze sequential mergers in situations where acquirers and targets belong to disjoint sets. Particularly pertinent with respect to our focus on firm asymmetries is Salvo’s (2007) observation that symmetry is crucial for the merger-wave like equilibrium outcomes in models like Nilssen and Sørgard (1998) and Fauli-Oller (2000).

The need for a forward-looking merger policy in situations with sequential merger decisions is pointed out by Nilssen and Sørgard (1998), observing that the safe-harbor criterion of Farrell and Shapiro (1990) may accept too many mergers when used myopically. A similar point is made by Pesendorfer (2005). Brito (2005) discusses how the endogenous-merger perspective can provide the antitrust authority with a revealed-preference argument for putting an upper
limit on a proposed merger’s efficiency gains: a proposed merger must be more profitable than those not proposed. Selleslachts, et al. (2007) study empirically whether merger prohibitions have a deterrence effect on future merger proposals.

Incorporating the antitrust authority’s decisions into the analysis of sequential mergers, with each decision by firms to merge followed by a decision by the antitrust authority whether or not to accept, has been done only very recently. In fact, several of the earlier studies explicitly have to restrict firms from merging to monopoly in order to counter-balance the absence of the antitrust authority from their models. Lyons (2003) does incorporate the antitrust authority and focuses on how varying the agency’s objective affects equilibrium outcomes, a theme that also shows up in the present work. The model closest to the present one, though, is by Motta and Vasconcelos (2005). However, they assume that firms are symmetric and limit the analysis to the case of an antitrust authority maximizing consumer surplus. In a different variation of the Motta-Vasconcelos analysis than ours, Fumagalli and Vasconcelos (2008) discuss a model of sequential mergers with multiple antitrust authorities, two national ones and one supranational, and also discuss the effect of varying the antitrust authorities’ objectives. Like Motta and Vasconcelos, Nocke and Whinston (2007) limit the discussion to a consumer-surplus maximizing antitrust authority and find conditions under which it is optimal for the agency to evaluate mergers completely myopically.

In the present work, we make the point that firm asymmetry is crucial for our results. The importance of firm asymmetry for outcomes of merger games is also stressed by Barros (1998). Using a different cost structure, and thus a different kind of firm asymmetry, and excluding merger to monopoly by assumption, Barros finds in a three-firm oligopoly that a big asymmetry leads to a merger between the two most efficient firms while a medium-sized asymmetry leads to a merger between the most and the least efficient firm. Tombak (2002) extends the analysis of Kamien and Zang (1993) to the case of asymmetric firms and finds that asymmetry increases the scope for merger to monopoly. Qiu and Zhou (2007) find that firm heterogeneity is crucial for the creation of a merger wave. Fridolfsson (2007) extends the analysis of Fridolfsson and Stennek (2005a) to the case of asymmetric firms and stresses the firms’ incentives to pursue mergers that are anti-competitive rather than pro-competitive. Catalão-Lopes (2007) discusses the merits of a merger policy based on the Herfindahl index in an industry with asymmetric firms.

The role of firm asymmetry has also been highlighted in other aspects of competition policy. In particular, discussions of firms’ incentives to collude by Compte, et al. (2002) and others show that collusion is less likely when firms are asymmetric. Vasconcelos (2005) uses this insight to discuss merger policy in asymmetric industries when a merger would increase the symmetry in the industry and thus facilitate collusion, while Ganslandt, et al. (2008)...

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1In Lommerud, et al. (2006), there are no antitrust authorities involved, but their discussion of the role of trade unions in sequential international mergers resembles that of Fumagalli and Vasconcelos on the role of antitrust authorities, except of course that the objectives of trade unions and antitrust authorities differ.
introduce indivisible costs of running a cartel, making an asymmetric industry more conducive to collusion than a symmetric one, and discuss implications of this for merger policy.

Our focus presently is on firms’ incentives not to carry out a merger even in cases where it, seen in isolation, would be profitable. The opposite concern, namely, firms’ incentives to merge even in cases where the merger, seen in isolation, is unprofitable, has been highlighted by several authors. The first to discuss the preemptive motive for merger were Nilssen and Sørgard (1998): A group of firms may choose to merge in order to stop another merger from taking place. This preemptive motive to merge also shows up in the work of Horn and Persson (2001b), Brito (2003), Fridolfsson and Stennek (2005a), Pesendorfer (2005), and Macho-Stadler, et al. (2006).

The external-effect motive for not merging was first noted by Stigler (1950) and was later given analytical treatments in Salant, et al. (1983) and Kamien and Zang (1990). It is called the hold-up motive for not merging by Fridolfsson and Stennek (2005b) and the inducement mechanism by Fridolfsson (2007). Lindqvist and Stennek (2005), calling it the insiders’ dilemma, find evidence of this motive in an experimental analysis. These works differ from ours in that we incorporate the external-effect motive in a sequential-merger model with an antitrust authority present. The models cited all restrict attention to situations where, by assumption, only a single merger can happen. In our model, the external-effect motive occurs in cases where the antitrust authority does not allow more than one merger, and so the restriction to a single merger here is endogenous.

The other two motives for not merging that we highlight here, the bargaining-power motive and the pill-sweetening motive, we have not seen discussed earlier. We record, though, the observation made by Toxvaerd (2008) that there may be an option value to not merging, a motive which is not covered in our analysis: In a model of sequential mergers with disjoint sets of acquirers and targets where each acquirer is restricted from doing more than one merger and mergers are irreversible, he finds that an acquirer may choose to wait before carrying out the option to merge. It seems that his prediction of a merger wave hinges on his restriction of one merger per acquirer.

Our highlighting of the importance of a certain alignment in the firms’ and the antitrust authorities’ interests for the occurrence of firms’ not merging is related to recent discussions in the literature of what is and what should be the antitrust authority’s objective. When it comes to what it should be, there is a nice overview of the issues and most of the literature on this topic by Farrell and Katz (2006), showing there are several arguments in favour of either a consumer-welfare standard or something close to it, even when government maximizes total welfare.

In terms of the other question, what the antitrust authority’s objective actually is, it seems to be commonly agreed that current policy in both the EU and the US is strongly consumer biased. However, a recent report by the US Antitrust Modernization Commission contains recommendations to move the weight considerably in the direction of a total welfare standard; see AMC (2007).
and Carlton (2007). Moreover, outside the EU and the US, the picture is mixed. For example, Ross and Winter (2005) argue that Canadian merger policy, after a clarifying court decision in 2003, now is close to the total welfare standard. The merger policy of Norway, which is outside the EU, was, until a revision of the country’s competition law in 2004, explicitly at the total welfare standard. Now the antitrust authority is told to put particular weight on consumer welfare, although there have not been any cases so far that can tell exactly how consumer biased the implementation of this instruction will end up being. As discussed above, the scope for firms passing up on merger opportunities is greatest in countries that are close to the total welfare standard, such as Canada and Norway - and perhaps the US in the future.

The paper is organized as follows. In the next section, we present the model and the social optimum. In Section 3, we present the equilibrium outcome, the three motives not to merge, and how these results hinge on some crucial assumptions made. In Section 4, we extend the analysis to the case of four firms. In Section 5, we discuss some possible alterations of our model, while Section 6 concludes. The Appendix contains the formal analysis of the model.

2 The model

We model a game consisting of two parts. The first part is a merger game, a sequence of pairwise mergers starting from a status-quo situation of an industry with three independent firms. Following any merger proposal is a decision by the antitrust authority (henceforth, AA) whether to approve the proposed merger or not. The merger game stops when there are no more mergers to form, either because they are not profitable, because they would not be approved, or because the industry has reached complete monopoly. The second part is a product-market competition game among the entities that are present after the merger game. We are looking for the subgame-perfect equilibrium of the whole game.

The product market we analyze has an inverse demand given by

\[ p(X) = a - X, \]

where \( a \) is a parameter describing the size of the market, and \( X \) is total supply from the firms in the industry. Firms compete by setting quantities, i.e., by playing a Cournot game. At the outset, i.e., before any mergers, the supply side consists of three firms, belonging to the set \( S := \{1, 2, 3\} \) of firms. Following Perry and Porter (1985), we assume that each firm has a cost function given by:

\[ C_i(x_i) = \frac{x_i}{k_i}, \quad i \in S, \]

where \( x_i \) is the production quantity of firm \( i \), \( \sum_{i=1}^{3} k_i = 1 \), and \( k_i > 0, \forall i \). One can think of \( k_i \) as the amount available to firm \( i \) of a production factor whose total supply in the industry is given. The more a firm has available of this factor, the lower are the costs of production. Since the general case is difficult
to analyse, we focus on a special case of asymmetrically sized firms where firm 1 is of a different size than the other two firms, which both are of the same size. In particular, we make the assumption that $k_1 = k \in (0, 1)$, whereas $k_2 = k_3 = \frac{1-k}{2}$.

A merger creates a new unit that has lower costs than the merging firms. In particular, if a set $M \subseteq S$ of firms merge, then the merged entity has a cost function

$$C_M(x_M) = \frac{x_M}{k_M},$$

where $x_M$ is the production quantity of the merged entity, and $k_M = \sum_{i \in M} k_i$. Denote by $2^S$ the power set of $S$, i.e., the set of all possible subsets of $S$. A firm’s profit is given by:

$$\pi_i = p(X) x_i - C_i(x_i) = \left( a - X - \frac{1}{k_i} \right) x_i, \ i \in 2^S.$$

Because of linearity in demand, the consumer surplus is simply:

$$CS = \frac{1}{2} X^2.$$

Moreover, as long as all firms’ production quantities are positive, which is assumed in the following, the equilibrium profit of firm $i$ equals:

$$\pi_i = x_i^2, \ i \in 2^S.$$

There are four principally different outcomes of the merger game: SQ - Status Quo, with no merger and the configuration \{1,2,3\}; PO - Partial Out, with a merger between the two firms other than firm 1 and the configuration \{1,23\}; PI - Partial In, with a merger between firm 1 and one other firm and a configuration such as \{12,3\}; and CM - Complete Monopoly, with a merger between all three firms and the configuration \{123\}.

We assume that both the AA and the firms are forward-looking. This means that, when making a decision, each player compares the eventual outcomes that follow each choice. In Section 5 below, we discuss an alternative assumption, letting firms be forward-looking while the AA is myopic.

We assume that the AA applies the total-welfare standard when assessing merger proposals, i.e., they maximize $TW := CS + \Pi$, where $\Pi = \sum_i \pi_i$. In Section 3 below, we discuss how equilibrium outcomes are affected by variations in the relative weights put by the AA on consumer surplus and total profit.

The model has two exogenous parameters: $a$, which measures the market size; and $k$, which measures firm asymmetry. We restrict interest to those combinations $(a,k)$ for which all firms present produce positive quantities in all

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See for example Motta (2004, sec. 8.4.1.2). Conditions ensuring positive quantities are discussed in the Appendix.

By the convention we adopt here, \{1,23\} denotes a two-firm industry consisting of firm 1 and the entity stemming from the merger between firms 2 and 3. With this notation, $2^S = \{1,2,3,12,13,23,123\}$.
the four outcomes outlined above. We denote by Z the set of all such parameter combinations.

A merger is never proposed if it will subsequently be turned down by the AA. In assessing whether a merger is profitable, a firm compares the profit it gains from this merger with the alternative, which is not to merge. In most such comparisons, how the extra profit that is obtained from merging is split between the merging parties is of no relevance for the assessment. However, firms are far-sighted and therefore compare profits obtained from the outcomes that eventually prevail after the various alternatives. This calls for a comparison of profits from merging that, in principle, takes into account the way profits are split. We assume that the two firms involved in a merger split evenly the extra profit gained from merging. For example, when firm 1 merges with firm 2 and we move from \( SQ \) to \( PI \), firm 1’s share of the merged entity’s profit is:

\[
\frac{1}{2} \left( \pi^{PI}_{12} - \pi^{SQ}_{1} - \pi^{SQ}_{2} + \pi^{SQ}_{1} \right) + \frac{1}{2} \left( \pi^{PI}_{12} + \pi^{SQ}_{1} - \pi^{SQ}_{2} \right),
\]

where superscripts denote outcomes of the merger game.

The merger game - details of which are provided in Figure 1 - consists of 8 decision nodes. The game starts out with firm 1 deciding whether or not to merge; this is node 1 in Figure 1. Since the other two firms are of equal size, we randomly assign firm 2 the role of firm 1’s merging mate. A merger is proposed if the joint profit of the merging firms is higher following a merger proposal than following a decision not to merge. Note that the crucial issue is not simply whether the profit of the merged firm 12 in situation \( PI \) is higher than the sum of their profits in situation \( SQ \). Rather, firm 1 takes into consideration the actions along the equilibrium path following each of its alternatives.

\footnote{In a game-theoretic sense, the number of decision nodes is greater than 8. As will become clear in the text, some of our 8 nodes can be reached by different routes through the graph in Figure 1.}
If firm 1 proposes a merger, then the AA makes a decision whether to approve the merger or not; this is node 2 in Figure 1. If AA says No, then we are at node 5, in the same situation as if firm 1 had decided not to merge; see below. If AA says Yes, then firm 3, the remaining small firm, is given the choice to propose.

Firm 3 chooses between no merger and a merger with 12; this is node 3 in Figure 1. If firm 3 decides not to merge, then the process stops, and we end in a \( PI \) situation, with the firms in \( \{12,3\} \) playing a Cournot game. If firm 3 decides to merge with 12, then AA makes a decision whether to approve or not; this is node 4 in Figure 1. If AA says No, then the merger game again ends in a \( PI \) situation. If AA says Yes, then we arrive at \( CM \) with 123 a monopolist in the industry.

If, at node 1, firm 1 decides not to merge (or if, at node 2, AA says No), then the two small firms decide whether or not to merge. Formally, we let firm 2 decide whether or not to merge with firm 3; this is node 5 in Figure 1. If firm 2 decides not to merge with firm 3, then both alternatives of pairwise mergers have been tried - firm 1 with one other firm, and the two other firms together - with negative responses, so that the merger game ends with the \( SQ \) outcome.

If firm 2 proposes a merger with firm 3, however, we move on to the AA deciding whether or not to approve this merger; this is node 6 in Figure 1. If the AA does not approve, then again the merger game ends with \( SQ \). If the AA approves, on the other hand, then a new situation has arisen, and it is natural to give firm 1 a new chance to consider a merger. Now, the only possible merger partner is the already merged combination of the two other firms. So firm 1 chooses whether or not to merge with 23; this is node 7 in Figure 1. If firm 1 proposes no merger, then the merger game ends in the \( PO \) outcome, with the firms in \( \{1,23\} \) playing a Cournot game. If firm 1 proposes to merge with 23, however, the AA has to approve the merger or not; this is the final decision node 8 in Figure 1. If the AA says No to the merger, then the merger game ends in \( PO \), while if it says Yes, then we end in \( CM \).

As can be seen from Figure 1, there are two different ways to arrive at \( CM \), and we want to keep the two apart in the continuation. Therefore, we denote by \( CM_1 \) complete monopoly following from firm 1 merging with firm 2 and then firm 3 merging with 12 (nodes 1-2-3-4 in Figure 1), while we denote by \( CM_2 \) complete monopoly following from firm 1 first not merging, then firm 2 merging with firm 3 and finally firm 1 merging with firm 23 (nodes 1-5-6-7-8 in Figure 1). This completes the description of the merger game.

Our aim is, for each combination \( (a,k) \in Z \) of market size and firm asymmetry, to find the corresponding equilibrium outcome. We do this through backward induction by first solving the product-market game in each of the four situations. Thereafter, we proceed by looking at each node \( n \in N \) to determine, for each \( (a,k) \in Z \), what the eventual outcome of the merger game is. The details are in the Appendix.

Note that all the graphs below depict the set \( Z \) of combinations \( (a,k) \) for which there are positive quantities by all firms present in the product market in all outcomes as the collection of all shaded areas.
Before we move on to discussing the equilibrium outcome of our game, we take a look at the socially optimum market structure, *i.e.*, what a social planner would implement if he could decide the market structure without playing the merger game.\(^5\) The social planner’s choice is presented in Figure 2 where, for each \((a, k) \in \mathbb{Z}\), the socially optimal outcome is given. We see that all four possible market structures are represented. When the market is large (high \(a\)) and the firms not very asymmetric (\(k\) not very small or very large), the social planner prefers the status quo (\(SQ\)). If firm 1 is relatively small, then the social planner prefers the Partial In (\(PI\)) outcome, with firm 1 merging with one of the other firms. If, on the other hand, firm 1 is relatively big, the social planner prefers the Partial Out (\(PO\)) outcome, with firm 1 sitting on the outside while the other two firms merge. When the market size \(a\) is small, the social planner prefers Complete Monopoly (\(CM\)).

![Figure 2. Socially optimum market structures.](image)

Notice that the picture in Figure 2 coincides exactly with the equilibrium outcomes at node 2, depicted in Figure A3 in the Appendix. This means that the equilibrium outcomes, presented below, deviate from social optimum solely because firm 1 has the option not to merge at node 1, *i.e.*, to move the merger game to node 5 rather than to node 2.

### 3 Equilibrium outcomes

The details of the equilibrium analysis is relegated to the Appendix. From this analysis, the picture in Figure 3 emanates. Note how we have split the \(CM\) area in two, \(CM_1\) and \(CM_2\), corresponding to complete monopoly being reached by firm 1 at node 1 choosing to merge or not, respectively. The outcomes \(SQ\), \(PO\), and \(CM_2\) are reached by firm 1 not merging at the start of the game, while the outcomes \(PI\) and \(CM_1\) are reached by firm 1 starting with proposing a merger.\(^5\)

\(^5\)The procedure to determine the socially optimum market structures is: for each \((a, k) \in \mathbb{Z}\), identify the market structure that maximizes \(TW\).
Most of the cases where firm 1 chooses not to merge, it would not make a difference whether it did or not, since a merger proposal would have been rejected by the AA at node 2 anyway. The interest centres, therefore, on those cases where firm 1’s profit is strictly better from not merging than from merging at node 1. Accordingly, in Figure 4, we have drawn the solution again, highlighting the areas in which firm 1 chooses not to merge for such strategic reasons.

Figure 4 highlights the three reasons, discussed in the Introduction, that a firm may have for not merging at the first opportunity:

- **The bargaining-power motive**
  There are cases where the outcome will be complete monopoly whatever the choice of firm 1 at node 1 is; in such cases, firm 1 can decide whether to get complete monopoly by merging immediately with firm 2 and then with 3, or letting the two other firms merging first, joining them later. The latter option is convenient in case firm 1 is relatively small (k low) and the market is relatively big (a large, at least conditioned on the AA allowing complete monopoly). In these cases, firm 1 - being small - has limited bargaining power in the status quo and would get a low share of
the equilibrium (monopoly) profit if it chooses the first option; on the contrary, by letting the other two firms merge, firm 1 acquires a stronger bargaining position as it obtains an outside option of free-riding on the others’ merger. That is why, by letting the other two firms merge first, firm 1 can achieve a higher share of the monopoly profit. This is the bargaining-power motive for waiting to merge; see the horizontally hatched area in Figure 4.

- **The pill-sweetening motive**
  The no-merger strategy might be adopted by firm 1 also in order to get the AA to accept a complete monopoly outcome in situations where complete monopoly would not be allowed in case of an immediate merger between firms 1 and 2. Such cases occur when firm 1 is smaller than the other two firms: while the AA would not allow firm 3, a big firm, to join firm 12, it accepts firm 1, a small firm, joining firm 23. This is the pill-sweetening motive for waiting to merge; see the vertically hatched area in Figure 4.

- **The external-effect motive**
  There are cases in which firm 1 benefits from not merging where the complete monopoly would not be allowed at any rate, but where the AA prefers one merger taking place to none; we are talking about cases with a market of medium size. When firm 1 gets more out of not merging than of merging in these one-merger situations, it is because it is better for it to be the outsider to a merger in the PO outcome than being on the inside of a merger in the PI outcome. This happens when firm 1 is slightly smaller than the others: it is more profitable for it to let the other two firms merge enjoying the free-riding effect, than merging with one of them and suffering from the contraction in output that follows. This is the external-effect motive for not merging; see the cross hatched area in Figure 4.

As discussed in the Introduction, there are four features of our model that are crucial in producing our results. These are: the presence of efficiency gains; asymmetry among firms; the presence of an AA; and a sufficient alignment of interest among firms and the AA. The importance of these four features of our model is discussed in the following.

Note, first, that the presence of efficiency gains is instrumental in creating an interest in the AA in having mergers to take place. As is evident from Figure 4 above, there is nothing (strictly) to gain from not merging, if no merger will be allowed anyway, with SQ being the outcome.

Secondly, Figure 4 also illustrates clearly the importance of firm asymmetry. Symmetry, i.e., all three firms being of identical size, shows up in the Figure along the vertical \( k = \frac{1}{3} \) line. At \( k = \frac{1}{3} \), there is a range of values of \( \alpha \), \( [9 + \frac{2}{3}\sqrt{105}, \frac{117}{17} + \frac{18}{37}\sqrt{37}] \approx [15.1, 32.4] \), for which the choice for firm 1 is between the outcomes PI and PO. The AA is indifferent between the two outcomes for the borderline case of symmetry. But in line with previous work, such as Fridolfsson and Stennek (2005a), we find that firm 1 strictly prefers PO
to \( PI \) because of the benefit accruing from the external effect of the other two firms merging in outcome \( PO \). What does not come out of an analysis of the symmetry case is that firm 1, by not merging, can enjoy the external effect also in many cases where the AA would prefer that the firm be involved in a merger rather than it being on the outside of one, notably when \( k < \frac{1}{3} \). Our analysis also shows that a value of \( k \) slightly above \( \frac{1}{3} \) takes away the firm’s incentives not to merge, since now also the AA prefers \( PO \) to \( PI \).

Note that, while other authors, like Fridolfsson and Stennek (2005a), simply assume that exactly one merger takes place, the range of parameter values for which exactly one merger occurs here is endogenously determined from an explicit consideration of the AA’s objectives.

In another range of values of \( a \), \( [7 + \sqrt{21}, 9 + \frac{2}{5}\sqrt{105}] \approx (11.6, 15.1) \), there will be complete monopoly anyway for \( k = \frac{1}{3} \), but firm 1 benefits from getting in late and so chooses not to merge at the first opportunity. This is the bargaining-power motive for not merging. By introducing firm asymmetry, we are able to show that this motive is present both when the firm in decision is relatively large \((k > \frac{1}{3})\) and when it is relatively small \((k < \frac{1}{3})\).

Observe that the case of symmetry is not at all able to accommodate the pill-sweetening motive for not merging, since the AA is not affected by the sequence in which mergers occur when firms are of identical sizes. Accordingly, the pill-sweetening motive only shows up for \( k < \frac{1}{3} \), as indicated by Figure 4.

Thirdly, the importance of having the AA present can be illustrated by repeating our analysis, with the difference that all decisions by the AA are taken away. The result of this exercise is presented in Figure 5 below showing that, in case there is no AA who can veto mergers, the equilibrium outcome of the simplified game is trivial: complete monopolization of the market for all parameters.

![Figure 5. Equilibrium outcomes without an antitrust authority.](image)

Still, firm 1 has the option to choose the equilibrium path through which such a market structure is formed. Thus, the bargaining-power motive not to merge immediately plays a role. If firm 1 were to join with one of the firms in the very beginning, then it would have a weak outside option in the ensuing bargaining to get the last firm to join (except if it itself is very large to start with), while
after the merger between the other firms, firm 1 becomes an outsider and enjoys free-rider profits, strengthening its bargaining position when joining the merger at a later stage. The outsider’s position is stronger the bigger is the market, as it has more opportunities to expand output. Thus, unless firm 1 is very big \((k\) high) or the market is very small \((a\) low), firm 1 will decide to wait for the other two firms’ merger. The exact formula for the curve splitting the two \(CM_1\) and \(CM_2\) regions in Figure 5 is given in (10) in the Appendix.

Finally, as discussed in the introduction, the weight put on firms’ profits in the AA’s objective function need to be sufficiently high in order to ensure the AA is interested in allowing mergers at the same time as firm 1 is interested in waiting in order to obtain more, either from other firms’ merging or from itself merging later on. A simple way of exploring the consequences of various assumptions on the AA’s objective is to write the latter’s objective function as

\[
U = \alpha CS + (1 - \alpha) \Pi,
\]

where \(\alpha \in [0, 1]\). Here \(\alpha\) measures the weight put on consumer surplus relative to industry profit by the AA. In the analysis above, we put \(\alpha = \frac{1}{2}\), which implies the total welfare standard. With \(\alpha = 1\), the AA would be applying the consumer welfare standard, while a value of \(\alpha\) decreasing from 1 to \(\frac{1}{2}\) would imply a steady movement from the consumer welfare standard towards the total welfare standard. As discussed in the introduction, both US and EU merger policies have in effect an \(\alpha\) at or close to 1, while Canadian and Norwegian merger policy has an \(\alpha\) close to \(\frac{1}{2}\). Allowing values of \(\alpha\) less than \(\frac{1}{2}\) is mostly for expositional reasons.

When analyzing our model for different variations of the AA’s objective, we find naturally that a stronger weight on consumer surplus makes the AA less interested in allowing mergers. When \(\alpha\) is high, there is still some scope for the AA allowing mergers. However, this tends to happen only in cases where there are no incentives for firm 1 to pass up the opportunity to merge at stage 1. In Figure 6 we illustrate this for the case where market size is fixed at \(a = 20\). Thus, the Figure depicts various outcomes in \((\alpha, k)\) space. Note that the restriction that all firms be active in all possible outcomes of the merger game implies that we only consider cases where \(k \in [0.12, 0.81]\).

Figure 6. Equilibrium outcomes for different specifications of the welfare standard.
Figure 6 shows the presence of the three motives introduced above for a firm not to make a merger proposal at the first opportunity. As pointed out above, alignment of interests between the AA and the firms helps in obtaining our results: if, in the case of \( a = 20 \) depicted in Figure 6, the AA were to apply the consumer welfare standard, with \( \alpha = 1 \), no merger or, when firm 1 is big, only a merger not involving firm 1 would be allowed, deleting \textit{de facto} any opportunity to strategically not propose a merger.

4 The four-firm case

Our results essentially carry through to the case where there are four firms initially, instead of three. In this section, we explain the modifications necessary in our model to accommodate four firms, as well as the outcome of the modified game. The set of firms at the outset is now \( S := \{1, 2, 3, 4\} \). Firm 1 is again of a different size then the other three, who are again identical. Thus, \( k_1 = k \in (0, 1) \), and \( k_2 = k_3 = k_4 = \frac{1-k}{3} \). The symmetric case is at \( k = \frac{1}{4} \).

The merger game in the four-firm case is depicted in Figure 7 and consists of 22 decision nodes.

![Figure 7. The four-firm merger game.](image)

In the four-firm case, there are seven principally different outcomes of the merger game: \textit{SQ} - Status Quo, with no merger and the configuration \( \{1,2,3,4\} \); \textit{PO} - Partial Out merger, with a merger between two small firms and a configuration such as \( \{1,23,4\} \); \textit{FI} - Partial In merger, with a merger between firm 1 and one small firm and a configuration such as \( \{12,3,4\} \); \textit{FO} - Full Out merger, with a merger between all three small firms and the configuration \( \{1,234\} \); \textit{FI} - Full In merger, with a merger between firm 1 and two of the small firms and a configuration such as \( \{123,4\} \); \textit{AD} - Asymmetric Duopoly, with two pairwise mergers, one involving firm 1, and a configuration such as \( \{12,34\} \); and \textit{CM} -
Complete Monopoly, with a merger between all four firms and the configuration \{1234\}. The new outcomes are \(FO\), \(FI\), and \(AD\), all three featuring two mergers in sequence, which now are one merger short of complete monopoly.

Note that we now have five different paths to arrive at the Complete Monopoly outcome; they are marked \(CM_1\) through \(CM_5\) in the Figure. As it turns out, though, only \(CM_2\) and \(CM_5\) occur in equilibrium. Note also that firm 1 now has a total of three chances to merge (nodes 1, 12, and 21 in Figure 7), compared to two chances in the three-firm case: It may want to merge after two of the others have merged (node 12); and if not, we need to check whether it wants to merge after all three other firms have merged (node 21). For a complete description and a detailed analysis of the four-firm merger game, see our supplementary note, Fumagalli and Nilssen (2008). The equilibrium outcomes derived from this analysis are depicted in Figure 8.

Figure 8. Equilibrium outcomes in the four-firm case.

Figure 8 shows that, as in the three-firm case, there is a great variety in possible outcomes, although the \(FI\) outcome does not occur in equilibrium for any combination \((a, k)\). It also shows that firms’ decisions to merge again are heavily influenced by what will get through at the AA. When \(k\) is large, so that the industry at the outset consists of one big firm and three small ones, firm 1 realizes that its best shot is sitting outside the merger process and letting the small firms merge, either all three (\(FO\)) or at least two of them (\(PO\)). When \(k\) is small, so that the industry consists of one small firm and three large ones, then there is a potential for at least one of the large firms to get involved in a merger. If \(a\) is not very large, then it is actually possible for all the large firms to get involved in a merger, as in the \(AD\) outcome, with two of the larger firms merging with each other and the third one merging with the small firm. When \(a\) is very small, then there is not room for more than one firm in the industry, also by the AA’s standard, and so we end up with CM. In the opposite end, when \(a\) is very large, there is no scope for a merger seen from the AA’s point of view, and SQ, the situation we start out with, is also the final outcome.

In Figure 9 we highlight the combinations \((a, k)\) for which the firm has a strategic motive for not merging at once. As with three firms, there is the
bargaining-power motive for not merging: If the AA is going to accept a merger to complete monopoly anyway, firm 1 may want to postpone joining in the sequence of mergers; this happens in Region 1 of Figure 9, where firm 1 holds up two opportunities to merge, only to join as the last firm at node 21 of Figure 7.

Figure 9. Motives for not merging in the four-firm case

The pill-sweetening motive not to merge now shows up in two different shapes. One of them is completely parallel to what we saw in the three-firm case: In order to obtain complete monopoly, firm 1 chooses to pass up on an opportunity to merge, not only once but twice; this is Region 2 of Figure 9. But in the four-firm case, another reason for sweetening the pill occurs: In order to obtain a sequence of two mergers, turning the industry into an asymmetric duopoly \( (AD) \), firm 1 abstains from a merger at the outset, thereby avoiding the \( PI \) outcome and instead going for a merger at the next opportunity by joining forces with the other remaining outside firm at node 12 in Figure 7; this is Region 3 in Figure 9.

The external-effect motive not to merge is present also in the four-firm case in much the same shape as in the three-firm case: By saying no twice to a merger, firm 1 can realize the \( FO \) outcome in cases where the AA would otherwise like the \( AD \) outcome to happen; this is Region 4 in Figure 9.

Finally, we observe a new motive for not merging that did not occur in the simple three-firm case: In Region 5 of Figure 9, there are cases where the AA would allow firm 1 to merge, but where the merger is not profitable for the firms involved simply because of the output contraction involved; we might call this the contraction motive for not merging. Although this is a rather prosaic reason for not merging, we note that it does occur in our model solely because of the presence of an AA; as in the three-firm case, without an AA, there would be merger to monopoly for all parameter combinations.
It seems safe to conclude, if only by a visual inspection of Figures 4 and 9, that the prevalence of a firm passing up on opportunities to merge is only increasing as the number of firms in the industry increases.

5 Discussion

In this Section, we discuss some alternatives to our modeling approach and argue that these alternatives, for various reasons, do not demand the same interest as the model we have chosen above.

Legal restrictions on the antitrust authority’s decisions. In the model we presented above we assumed that the AA is forward-looking when making its decisions. One might object to this that a merger proposal should be judged per se, a view that might put limits on the AA’s ability to take into account all repercussions of its decision. However, we regard our assumption to be not too far from reality since, when an AA considers a merger proposal, it always takes into account the possible development of the industry after that merger, such as increased/decreased chances for entry, for collusion - and for further mergers.

Nevertheless, with the view of checking for the robustness of our results with respect to this criticism, we consider here a variation of our model where the AA’s ability to make its decision based on the future development of the industry is restricted. The extreme way of modelling such restrictions is to assume that the AA is myopic, i.e., that it takes its decision without considering that other mergers might follow. For example, at node 2, this myopic AA is comparing total welfare in $PI$ with that in the status quo, ignoring the fact that, after the acceptance/rejection of the proposed merger between firm 1 and 2, further mergers might be proposed and accepted.

Solving this modified version of our model, we find that our qualitative results are confirmed: there still are strategic reasons for not merging, and all three motives survive in the new model. Figure 10 depicts the equilibrium outcomes and the occurrence of strategic non merging of this new game.

![Figure 10. Equilibrium outcomes and strategic regions with a restricted antitrust authority.](image-url)
Comparing this Figure with Figure 4 above, we find that a restricted AA implies a higher prevalence of firm 1 proposing a merger at stage 1 (leading eventually to CM in most cases because of the AA’s preferences at node 4, instead of PO as in the standard model), but also a higher prevalence of firm 1’s non-merging being caused by the external effect motive. This is due to the fact that the AA at node 2 now compares PI to SQ only, so that it is going to accept the merger also in cases where firm 1 is big, i.e., where \( k > \frac{1}{3} \). This opens up both for firm 1 to merge when it is very big (CM or PI), and for firm 1 not to merge even if the merger would be accepted, because it prefers being an outsider (PO). Not surprisingly, weakening the AA this way leads essentially to equilibrium outcomes that are further away from the social optimum than in the standard model, a little caveat being in order since the prevalence of the pill-sweetening motive for not merging is slightly reduced. Details are provided in Section A.4 in the Appendix.

**Altering the move sequence.** Suppose we let one of the other two firms, the equal-sized firms 2 and 3, make the first decision whether or not to merge at the first stage of the game. Say firm 2 is the first-mover. Since it has two potential merger partners of different sizes, firm 2 has three alternatives to choose from: \{Merge with firm 1; Merge with firm 3; No merger\}. If firm 2 now chooses not to merge, this ends the game. The two possibilities of a first merger have been considered already at the first stage: a merger between two equal-sized firms (firms 2 and 3) and a merger between two different-sized firms (firm 1 and one of the others). This move sequence therefore does not make us able to discuss a firm’s incentives not to merge for strategic reasons: when the first mover chooses not to merge, it is simply because a merger at stage 1 is unprofitable.

**Alternative ways to model firm asymmetry.** An alternative model of firm asymmetry is the one used by Barros (1998) and Catalão-Lopes (2007). Instead of invoking Perry and Porter (1985), as we do here, they simply posit a Cournot oligopoly with asymmetric costs. In line with their approach, we could have assumed three firms, one of which having a constant marginal cost \( c > 0 \) and the other two with an identical and constant marginal cost equal to \( c + \delta \), where \( \delta \in [-c, c] \). In a merger between two firms with different costs, the merged entity makes use of the more efficient technology and therefore gets a constant marginal cost equal to \( \min \{c, c + \delta\} \). One problem with this approach, in relation to the issues we take up to discussion here, is the inability to distinguish between the two cases PI and PO. In both cases, the industry consists of two firms with different costs, one with marginal cost \( c \) and the other with marginal cost \( c + \delta \).

One could, of course, try to take the analysis a step further by letting all three firms have different marginal costs. Along the lines of Barros (1998), one could for example think of letting the three firms have constant marginal costs equal to \( c_1 = c, c_2 = c + \delta, \) and \( c_3 = c - \delta \), where now \( \delta \in (0, c) \). This would, however, complicate the analysis a lot, and there would not be any obvious choice of a move sequence.

**Paying for the right to merge.** It can be argued that our results on firm 1’s motivation to pass up on its merger opportunity stem from the firm not getting enough out of its potential popularity as a merging partner with the current
structure of the merger game. If each of the two identical firms 2 and 3 would prefer merging immediately with firm 1 to waiting until the two firms’ chance comes to merge with each other later on, one would think that firm 1 should be able to play its two suitors up against each other. One way to accommodate this would be to replace firm 1’s merger with firm 2 at node 1 with an auction between firms 2 and 3 for the right to merge with firm 1. We have experimented with a simpler set-up, in which the Nash bargaining between firms 1 and 2 in the merger process at node 1 is substituted with firm 1 making a take-it-or-leave-it offer to firm 2 while all other merger processes at later nodes are kept as before. Although this change gives firm 1 a larger gain from merging immediately, it turns out that our results stand essentially unaltered. The only change is that the incidence of the bargaining-power motive for not merging is slightly reduced.

6 Conclusion

In this paper, we have introduced a little model of sequential mergers in order to study firms’ incentives to pass up an opportunity to merge. We found three motives for not merging at the first opportunity. One motive, already discussed in the literature, is what we call the external-effect motive: when the antitrust authority will allow only a single merger, it might be better to be on the outside of a merger than on the inside. Therefore, a firm might wait and see if other firms want to merge instead. The two other motives we point out are not recorded in the literature so far, to our knowledge. The bargaining-power motive occurs when the antitrust authority is willing to allow complete monopoly and leads a firm to waiting to merge because it obtains an advantage from arriving late at a grand merger compared to initiating it. The pill-sweetening motive occurs in intermediate cases when the number of mergers the antitrust authority is willing to accept depends on the order of merger proposals. By holding back its own merger and letting other combinations form first, a firm may get more mergers through acceptance of the antitrust authority.

Among the crucial assumptions of our model, it is worth pointing out two: size asymmetry among firms in the industry; and some alignment of interest between firms and the antitrust authorities. Both of them have policy implications. While it has been pointed out earlier how a merger leading to a more symmetric industry also leads to a more collusion-prone industry, we see here that more symmetry may also reduce firms’ incentives to strategically hold back merger proposals. The importance of the antitrust authority’s objectives has implications for the current discussions on the best objective to impose on a government’s competition agency. In line with other contributions in the literature, we find that an antitrust authority maximizing total welfare leads to strategic behaviour among firms - in this case strategically holding back on merger proposals - that are counter to the interests of society and that do not show up when the antitrust authority is strongly consumer biased.
A Appendix

A.1 Some notation

In sections A.2 and A.3 of this Appendix, we provide the complete solution of the model. In order to do this, we have to introduce some notation that might seem a bit elaborate for the present three-firm case, but it has been chosen in order to facilitate extensions to cases with more than three firms.

We denote the set of possible outcomes by \( \Xi := \{SQ, PO, PI, CM\} \). In order to ease notation, we will sometimes need to express an outcome by a single letter: \( Q = SQ, O = PO, I = PI, C = CM \); and \( \Xi = \{Q, O, I, C\} \). Furthermore, we denote the set of decision nodes in the merger game by \( N := \{1, \ldots, 8\} \); see Figure 1 in the text.

The model has two exogenous parameters: \( a \), which measures the market size; and \( k \), which measures firm asymmetry. As noted in section 2, we restrict attention to those combinations \((a, k)\) for which all existing firms produce positive quantities in all the four outcomes outlined above. We do this by, for every outcome \( \xi \in \Xi \) and every \( k \), restricting \( a \) such that \( a \geq a^\xi (k) \), where, for each \( \xi \in \Xi \), \( a^\xi (k) \) is described in the next Section. These outcome-wise restrictions can be summarized in the restriction

\[
a \geq a (k) := \max \left\{ a^{SQ} (k), \ldots, a^{CM} (k) \right\}.
\]

In the following, our attention is thus limited to parameter combinations \((a, k) \in Z := \{(a, k) \mid a \geq a (k)\} \).

We assume that the AA applies the total-welfare standard when assessing merger proposals, i.e., it maximizes

\[
TW := CS + \sum_i \pi_i = \frac{1}{2} \left( \sum_i x_i \right)^2 + \left( \sum_i x_i^2 \right) = X^2 \left( \frac{1}{2} + H \right),
\]

where \( H := \sum_i s_i^2 \) is the Herfindahl index, with \( s_i := \frac{2x_i}{X} \) the market share of firm \( i \). The index for each outcome is computed over all firms active in that outcome. In other words, by the total-welfare standard, there are two effects of a merger: it increases concentration and efficiency, and therefore firms’ profits, which is good, and it changes total output, which is good when the change is positive, but very often is bad because the merger entails a lower total output. In the text, we discuss a relaxation of this assumption, allowing the welfare standard to be a weighted average of consumers’ and producers’ surplus.

Our aim is, for each combination \((a, k) \in Z \) of market size and firm asymmetry, to find the corresponding equilibrium outcome. We do this through backward induction by first solving the product-market game in each of the four situations. Thereafter, we proceed by looking at each node \( n \in N \) to determine, for each \((a, k) \in Z \), what the eventual outcome of the merger game is; i.e., we are looking for an outcome partition \( \Omega^n \) of \( Z \) at each node, where \( \Omega^n := \{Z^n_1, Z^n_2, \ldots\} \), and \( Z^n_\xi \) consists of all \((a, k) \in Z \) such that the outcome of
the merger subgame starting at node \( n \in N \) is \( \xi \in \Xi \). The equilibrium outcome of the whole merger game then corresponds to \( \Omega^1 \), the outcome partition at node 1.

At each decision node \( n \in N \), the entity who has to make a decision at that node compares the possible outcomes that can follow each of its decisions. Let \( \Gamma^n \subseteq \Xi \) denote the set of outcomes that can occur after node \( n \). For example, at node 8, \( \Gamma^8 = \{ \text{PO}, \text{CM} \} \). Denote by \( V^n_{\xi} \subseteq Z \) the relevant region of the parameter space at node \( n \in N \) for the comparison between outcomes \( \xi, \iota \in \Gamma^n \); that is, \( V^n_{\xi} \) is the set of parameter combinations such that taking one of the feasible actions at node \( n \) would lead to outcome \( \xi \) and taking another one would lead to outcome \( \iota \). Define \( \xi \in \Gamma^n \) as the set of parameter combinations for which decision maker \( m \) prefers outcome \( \xi \) to outcome \( \iota \), where \( \xi, \iota \in \Xi \).

Let \( \Gamma^e \subseteq \Xi \) denote the set of end notes of the merger game. Outcome partitions at these end nodes are degenerate: if the merger game ends in outcome \( \xi \in \Xi \) at end node \( e \in \tilde{N} \), then the outcome partition of that end node of the merger game is \( \{ \xi \} \). The relevant region at a decision node can thus be constructed recursively through the outcome partitions of the node’s immediate successors:

\[
V^n_{\xi} = \left( \bigcup_{\iota \in \Gamma^n \cap \Phi, \xi \not= \xi} \left( V^n_{\iota} \cap \xi \in \Gamma^M(n) \right) \right) \cap \left( \bigcup_{\iota \in \Gamma^n \cap \Phi, \xi \not= \xi} \left( V^n_{\iota} \cap \xi \in \Gamma^H(n) \right) \right),
\]

where \( \Phi_{\xi} := \{ n \in N | Z^n_{\xi} \neq \emptyset \} \) is the set of decision nodes from which outcome \( \xi \) is a possible outcome. At every decision node in the present three-firm model, however, \( \Gamma^n \) consists of two nodes, so that the expression simplifies to:

\[
V^n_{\xi} = Z^n_{\xi} \cap Z^n_{\iota},
\]

where \( l, h \in \Gamma^n \) and \( l \neq h \), such that \( l \in \Phi_{\xi} \) and \( h \in \Phi_{\iota} \).

### A.2 Product-market competition

The outcome of the quantity competition depends on which situation we are in. Below, we go through the four different situations that may occur in order to characterize the equilibrium in each of them.

**Status Quo (SQ): \{1,2,3\}** In this situation, one firm of size \( k \) and two firms each of size \( \frac{k}{2} \) compete. The first-order condition of firm 1 is:

\[
a - X - x_1 - \frac{1}{k} = 0,
\]

while the first-order condition of firm \( s \in \{2,3\} \) is:

\[
a - X - x_s - \frac{2}{k} = 0.
\]

Imposing symmetry on the identical firms 2 and 3, we can write these conditions
as: \(2x_1 + 2x_s = a - \frac{1}{k}\), and \(x_1 + 3x_s = a - \frac{2}{1-k}\). Solving this system, we have:

\[
x_1^{SQ} = \frac{1}{4} \left( a - \frac{3-7k}{k(1-k)} \right),
\]

\[
x_2^{SQ} = \frac{1}{4} \left( a - \frac{5k-1}{k(1-k)} \right),
\]

so that \(a^{SQ}(k) := \max \left\{ \frac{3-7k}{k(1-k)}, \frac{5k-1}{k(1-k)} \right\} \), and total quantity is:

\[
X^{SQ} = \frac{1}{4} \left( 3a - \frac{3k+1}{k(1-k)} \right).
\]

**Partial OUT (PO): \{1,23\}** We have two firms: firm 1 of size \(k\) and firm 23 of size \(1-k\). The first-order conditions of the firms are: \(a - X - x_1 - \frac{1}{k} = 0\); and \(a - X - x_{23} - \frac{1}{1-k} = 0\). Rewriting, we have: \(2x_1 + x_{23} = a - \frac{1}{k}; x_1 + 2x_{23} = a - \frac{1}{1-k}\). Solving this system, we have:

\[
x_1^{PO} = \frac{1}{3} \left( a - \frac{2-3k}{k(1-k)} \right),
\]

\[
x_{23}^{PO} = \frac{1}{3} \left( a - \frac{3k-1}{k(1-k)} \right).
\]

Thus, \(a^{PO}(k) := \max \left\{ \frac{2-3k}{k(1-k)}, \frac{3k-1}{k(1-k)} \right\} \). Total quantity is:

\[
X^{PO} = \frac{1}{3} \left( 2a - \frac{1}{k(1-k)} \right).
\]

**Partial IN (PI): \{12,3\}** We have one big firm, 12, of size \(k + \frac{1-k}{2} = \frac{1+k}{2}\) and one small firm, firm 3, of size \(\frac{1-k}{2}\). The first-order condition of firm 12 is: \(a - X - x_{12} - \frac{2}{1+k} = 0\), while the first-order condition of firm 3 is: \(a - X - x_3 - \frac{2}{1-k} = 0\). We rewrite to obtain: \(2x_{12} + x_3 = a - \frac{2}{1+k}; x_{12} + 2x_3 = a - \frac{2}{1-k}\). Solving the system, we have:

\[
x_{12}^{PI} = \frac{1}{3} \left( a - \frac{2(1-3k)}{1-k^2} \right),
\]

\[
x_3^{PI} = \frac{1}{3} \left( a - \frac{2(1+3k)}{1-k^2} \right),
\]

so that non-negative quantities require \(a \geq a^{PI}(k) := \max \left\{ \frac{2(1-3k)}{1-k^2}, \frac{2(1+3k)}{1-k^2} \right\} \). Total quantity is:

\[
X^{PI} = \frac{2}{3} \left( a - \frac{2}{1-k^2} \right).
\]
Complete Monopoly (CM): \{123\} In complete monopoly, there is a single firm, 123, whose first-order condition is: 
\[a - 2x_{123} - 1 = 0.\]
In other words:
\[X^{CM} = x^{CM}_{123} = \frac{a - 1}{2},\]
so that \(g^{CM}(k) := 1.\)

Based on the above, we can now be specific about the function \(g(k),\) which restricts the set \(Z\) of combinations \((a, k)\) of interest and is given by the following piecewise relationship:
\[
g(k) := \begin{cases} 
  g^{SQ}(k) = \frac{3-7k}{k(1-k)}, & \text{if } k \in (0, \frac{1}{2}) \\
  g^{PO}(k) = \frac{2-3k}{k(1-k)}, & \text{if } k \in \left[\frac{1}{2}, \frac{3}{4}\right] \\
  g^{PI}(k) = \frac{2(1+3k)}{k(1-k)}, & \text{if } k \in \left[\frac{3}{4}, 1\right].
\end{cases}
\]

A.3 The merger game

In order to solve the game, we proceed by backward induction. Consider, therefore, node 8, where AA decides on whether to approve a merger between 1 and 23. If AA says no to the merger, then the merger game stops in the PO situation, whereas a yes leads to CM; in other words, \(\Gamma^8 = \{PO, CM\}.\) The two immediate successors to node 8 are both end nodes, implying that \(V^8 = Z.\) AA compares TW in the two outcomes and approves the merger if and only if
\[
(a, k) \in CY^A := \{(a, k) \in Z \mid a \leq a_{CO}^A(k)\},
\]
where
\[
a_{CO}^A(k) := \frac{27k^2 - 27k + 16 + 6\sqrt{24k^4 - 48k^3 + 28k^2 - 4k + 1}}{5k(1-k)}.
\]

Intuitively, the merger is approved if the market is so small that there is no room for two firms in the market. Thus, the outcome partition at node 8 is \(\Omega^8 = \{Z_{CM}^8, Z_{PO}^8\},\) where \(Z_{CM}^8 = V_{CO}^A \cap CY^A_0 = CY^A_0,\) and \(Z_{PO}^8 = Z \setminus Z_{CM}^8.\)

At node 7, firm 1 decides whether or not to propose a merger with 23. Possible outcomes are \(\Gamma^7 = \Gamma^8 = \{PO, CM\}.\) Firm 1 prefers to merge with 23 if
\[
\pi_1^{CM} \geq \pi_1^{PO}.
\]
Since
\[
\pi_1^{CM} = \frac{1}{2} \left(\pi_{123}^{CM} + \pi_1^{PO} - \pi_2^{PO}\right),
\]
the condition in (3) amounts to
\[
\pi_{123}^{CM} \geq \pi_1^{PO} + \pi_2^{PO},
\]
in other words: firm 1 wants to merge with firm 23 if the profit of the merged unit is larger than what the two firms can get separately. The condition holds
for all \((a, k) \in Z\), so \(cY^1_F = Z\). Thus, \(\Omega^7 = \Omega^8\), and a merger is proposed at node 7 if and only if (1) holds.

At node 6, the AA is to decide whether or not to approve a merger between firms 2 and 3. Possible outcomes are \(\Gamma^6 = \{CM, PO, SQ\}\). In particular, if it says no, then the merger game ends in an \(SQ\) outcome; and if it says yes, then the game ends in \(CM\) if (1) holds, in \(PO\) otherwise. Consider first the comparison between \(CM\) and \(SQ\). The relevant region is \(V^6_{CQ} = Z^7_{CM} = Z^8_{CM}\). The AA prefers \(CM\) to \(SQ\) if and only if \((a, k) \in cY^4_A := \{(a, k) \in Z \mid a \leq a^A_{CQ}(k)\}\), where

\[
a^A_{CQ}(k) := \frac{12k^2 + 3k + 5 + 2\sqrt{45k^2 - 114k^2 + 96k - 11}}{3k(1 - k)}.
\]

Consider next the comparison between \(PO\) and \(SQ\). The relevant region is \(V^6_{OQ} = Z^7_{PO} = Z^8_{PO}\). The AA prefers \(PO\) to \(SQ\) if \((a, k) \in oY^4_A := \{(a, k) \in Z \mid a \leq a^A_{OQ}(k)\}\), where

\[
a^A_{OQ}(k) := \frac{135k - 19 + 12\sqrt{644k^2 - 12k + 1}}{7k(1 - k)}.
\]

Putting this together, we see that \(\Omega^6 = \{Z^6_{CM}, Z^6_{PO}, Z^6_{SQ}\}\); see Figure A1. Here, \(Z^6_{CM} = \{(a, k) \in Z \mid a \leq \min\{a^A_{CQ}(k), a^A_{OQ}(k)\}\}\): when the market, measured by \(a\), is small, then both this merger and the next one (at node 8) is accepted by the AA and the merger game ends in a \(CM\) outcome; \(Z^6_{PO} = \{(a, k) \in Z \mid a^A_{CQ}(k) < a \leq a^A_{OQ}(k)\}\): when firm 1 is big (\(k\) is large), the AA prefers balancing it by accepting the merger between the two small firms 2 and 3 here at node 6, but will not allow a merger to \(CM\) later on at node 8; and finally \(Z^6_{SQ} = \{(a, k) \in Z \mid a > \max\{a^A_{CQ}(k), a^A_{OQ}(k)\}\}\): when the market is large, there is no reason for the AA to allow any merger at all.

At node 5, no merger has taken place so far in the game when firm 2 considers whether or not to merge with firm 3, the other small firm. We have \(\Gamma^5 = \{CM_2, PO, SQ\}\): In parallel to node 6 discussed above, we need to compare \(SQ\) with the outcomes \(CM\) and \(PO\), but this time from the perspective of firm 2.
rather than that of the AA; note that we now need to be explicit on which kind of complete monopoly is obtained. Consider first the comparison between \( CM_2 \) and \( SQ \). The relevant region is \( V_{CM_2} = Z_{CM_2} \). In order to find firm 2’s share of the profit in the completely monopolized industry, \( \pi^{CM_2}_{2} \), we note that firms 2 and 3, if they merge, will eventually end up in the \( CM \) outcome. Thus, for firm 2 at node 5, merger is preferable to no merger if

\[
\pi^{CM_2}_{2} = \frac{1}{2} \left( \pi^{CM}_{23} + \pi^{SQ}_{2} - \pi^{SQ}_{3} \right) \geq \pi^{SQ}_{2} \tag{6}
\]

Since firms 2 and 3 are identical, we have \( \pi^{SQ}_{2} = \pi^{SQ}_{3} \). Using this and inserting from

\[
\pi^{CM}_{23} = \frac{1}{2} \left( \pi^{CM}_{123} + \pi^{PO}_{23} - \pi^{PO}_{1} \right),
\]

we can write (6) as

\[
\frac{1}{4} \left( \pi^{CM}_{123} + \pi^{PO}_{23} - \pi^{PO}_{1} \right) \geq \pi^{SQ}_{2}.
\]

This leads to the finding that firm 2, in the relevant region, always prefers \( CM_2 \) to \( SQ \). In the comparison between \( PO \) and \( SQ \), where \( V_{PO} = Z_{PO} \), we find similarly that also \( PO \) is preferred to \( SQ \) for any \((a, k) \in Z_{PO} \). The conclusion for node 5, therefore, is that a merger is proposed whenever it will be accepted at node 6, i.e., \( \Omega_{5} = \Omega_{6} \).

Next, we move to node 4, where the AA is to decide whether or not to approve a merger between firms 12 and 3. The choice is essentially between outcomes \( PI \) and \( CM \), i.e., \( \Gamma_{4} = \{ PI, CM \} \), and \( V_{C_{B}} = Z \). We find that the AA prefers \( CM \) to \( PI \) if and only if \((a, k) \in Z_{CM} = \{ (a, k) : (a, k) \in Z_{A} \\} \), where

\[
a_{CM}^A (k) := \frac{27k^2 + 37 + 12\sqrt{6k^4 - 8k^2 + 6}}{5(k+1)(1-k)}.
\]

This gives us \( \Omega_{4} = \{ Z_{CM}, Z_{PI} \} \); see Figure A2.

![Figure A2. Outcomes at node 4.](image)

Complete monopoly is fine with the AA if the market is small or if firm 1, and therefore even more so the merged entity 12, is anyway so big that the outside firm 3 does not make up any reasonable balance.
At node 3, firm 3 decides whether or not to join firm 12 and create a complete monopoly. The comparison is also here between $CM$ and $PI$: $\Gamma^3 = \{PI, CM\}$, and $V^3_{CI} = Z$. We find that a merger is always preferable, and so a merger is proposed whenever it will be accepted: $\Omega^3 = \Omega^4$.

At node 2, the AA says yes or no to the merger between firms 1 and 2. If it says no, then the game moves to node 5 in Figure 1. If it says yes, then the game moves to node 3. Thus, all outcomes are possible at this node: $\Gamma^2 = \{CM, PI, PO, SQ\}$. The two outcomes $PO$ and $SQ$ can only occur if the AA says no and moves the game to node 5. Therefore, there is no need to discuss the comparison between the two at node 2. In the comparison between $CM$ and $PO$, we note that $V^2_{CO} = Z^3_{CM} \setminus Z^5_{PO}$. Thus, whenever the comparison between $CM$ and $PO$ is relevant at node 2, the AA prefers $PO$. In the comparison between $PO$ and $PI$, the relevant region is $V^2_{OI} = Z^3_{PI} \setminus Z^5_{PO}$.

Note that, from the AA’s point of view, the two outcomes $PO$ and $PI$ are identical when $k = \frac{1}{3}$, in which case the industry consists of one firm of size $\frac{2}{3}$ (firm 12 in the case of $PI$ and firm 23 in the case of $PO$) and one firm of size $\frac{1}{3}$ (firm 3 in the case of $PI$ and firm 1 in the case of $PO$). With $k$ going slightly below $\frac{1}{3}$, the big firm gets bigger in the case of $PO$ and smaller in the case of $PI$. Thus, the AA prefers $PI$ to $PO$ whenever $k < \frac{1}{3}$: $IA_2 := \{(a, k) \in Z \mid k < \frac{1}{3}\}$.

In the comparison between $PI$ and $SQ$, the relevant region is $V^2_{IQ} = Z^5_{SQ}$. We have that the AA prefers $PI$ to $SQ$ if and only if $(a, k) \in IA_2 := \{(a, k) \in Z \mid a \leq a^A_{IQ}(k)\}$, where

$$a^A_{IQ}(k) := \frac{135k^2 - 76k + 45 + 24\sqrt{37k^4 - 68k^3 + 38k^2 - 4k + 1}}{7k(k+1)(1-k)}.$$

Finally, in the comparison between $PI$ and $CM$, there is a possibility for the AA to obtain $CM$ in stead of $PI$ when $(a, k) \in V^2_{CI} = Z^3_{PI} \setminus Z^5_{CM}$. However, for any $(a, k) \in V^2_{CI}$, the AA prefers $PI$ to $CM$.

Our findings for node 2 are summarized in Figure A3.

![Figure A3. Outcomes at node 2.](image)

The AA prefers $SQ$ when the market is large (high $a$) and the firms not very asymmetric ($k$ not very small or very large): $Z^2_{SQ} = Z^5_{SQ} \setminus IA_2$. For
intermediate market sizes or for a very small firm 1 \((k \text{ small})\), the AA prefers \(PI\): 
\[
Z_{PI}^{2} = \{(a, k) \in (Z_{PI}^{2} \cap I_{Q}) \mid k < \frac{1}{2}\}.
\]
For intermediate market sizes or for a big firm 1 \((k \text{ large})\), the AA prefers \(PO\): 
\[
Z_{PO}^{2} = \{(a, k) \in Z_{PO}^{2} \mid k \geq \frac{1}{2}\}.
\]
Finally, the AA prefers \(CM\) when the market is small: 
\[
Z_{CM}^{2} = Z_{CM}^{2} \cap Z_{CM}^{5}.
\]
At node 1, firm 1 decides whether or not to merge with firm 2. A merger proposal would move the game to node 2, where the AA decides whether or not to accept, whereas a decision not to merge would move the game to node 5, where firm 2 decides whether or not to merge with firm 3. The first thing to note is that firm 1’s share of the monopolist’s profit in \(CM\) differs between \(CM_{1}\) and \(CM_{2}\). Whereas firm 1’s share of the monopoly profit in \(CM_{2}\) is 
\[
\pi_{CM}^{1} = \frac{1}{2} \left( \pi_{123}^{CM} + \pi_{1}^{PI} - \pi_{3}^{PI} \right) + \frac{1}{2} \pi_{23}^{PO} - \pi_{2}^{PO},
\]
as noted in (4) above, its share in \(CM_{1}\) is 
\[
\pi_{1}^{CM} = \frac{1}{2} \left( \pi_{123}^{CM} + \pi_{1}^{PI} - \pi_{3}^{PI} \right).
\]
At node 1, firm 1’s share in the merged unit’s profit, when the final outcome is complete monopoly, is 
\[
\pi_{12}^{CM} = \frac{1}{2} \left( \pi_{123}^{CM} + \pi_{1}^{PI} - \pi_{3}^{PI} \right).
\]
Now, comparing \(\pi_{1}^{CM_{1}}\) and \(\pi_{1}^{CM_{2}}\), we find that, when \((a, k) \in Z_{CM}^{2} \cap Z_{CM}^{5}\), so that the final outcome is anyway \(CM\), firm 1 prefers not to merge immediately if and only if \(\pi_{1}^{CM_{2}} > \pi_{1}^{CM_{1}}\). After insertions from (4), (8), and (9), this condition can be rewritten as 
\[
\frac{1}{2} \pi_{123}^{CM} + \pi_{1}^{PO} + \pi_{2}^{SQ} + \frac{1}{2} \pi_{1}^{PI} > \pi_{1}^{SQ} + \pi_{23}^{PO} + \frac{1}{2} \pi_{12}^{PI}.
\]
It follows that firm 1 prefers \(CM_{2}\) to \(CM_{1}\) when \((a, k) \in C_{2}Y_{C_{1}} := \{(a, k) \in Z \mid a > a_{C_{2}C_{1}}^{1}(k)\}\), where 
\[
a_{C_{2}C_{1}}^{1}(k) := \frac{-3k^{3} + 18k^{2} + 7k + 2 + 2\sqrt{-27k^{4} + 102k^{3} + 42k^{2} + 10k^{2} - 11k + 4}}{3(k + 1)(1 - k)k}.
\]
Other comparisons at node 1 are more straightforward. We find, in the choice between \(PI\) and \(CM_{2}\), that firm 1 always prefers \(CM_{2}\) in the relevant region. Likewise, it always prefers, in the respective relevant regions, \(PI\) to \(SQ\) and \(PO\) to \(PI\). See Figure 3 in the text for details. Note in particular that the \(CM\) region is split in two by the (10) curve.

### A.4 Restricted AA

In this section we provide details of the alternative model with a myopic AA discussed in section 5.

At node 8, there is no difference between the behavior of a myopic AA and a forward looking one. Therefore, also node 7 is not affected by this new assumption.
At node 6, the AA is now comparing PO with the status quo, without considering that for some parameters, the merger game leads to complete monopoly. This myopic AA will accept the merger proposal between firm 2 and 3 iff $a < a_{OQ}^A(k)$; see (5). Figure A4 presents the outcomes at node 6.

![Figure A4. Restricted AA: outcomes at node 6.](image)

Comparing Figure A4 with Figure A1, one can see that the SQ region now is slightly larger.

At node 5, firm 2 will propose any merger that will be accepted at node 6.

At node 4, as at node 8, there are no changes. Therefore, there are no changes at node 3 as well.

At node 2, the myopic AA makes a comparison only between PI and SQ and accepts the merger proposal between firm 1 and 2 iff $a < a_{IQ}^A(k)$; see (7). The outcomes at node 2 are depicted in Figure A5.

![Figure A5. Restricted AA: outcomes at node 2.](image)

This graph is dramatically different from Figure A3. The myopic AA accepts this merger proposal more often than a forward looking AA would do. In particular, there are now cases where firm 1 will have the merger accepted, and eventually ending up with complete monopoly, even when it is very big.
At node 1, the equilibrium behaviour of firm 1 is almost never affected by the assumption of the AA being myopic. The only difference occurs for a small parameter region where firm 1 no longer can obtain complete monopoly merger because the AA, at node 6, does no longer make any comparison between $CM$ and $SQ$. Instead, firm 1 has to settle with the $PI$ outcome in this case. This means that the parameter region giving rise to a decision not to merge because of the pill-sweetening motive has been slightly reduced.

Changes in the equilibrium outcome, as depicted in Figure 10 in the text, are otherwise not attributable to changes in firm 1’s behaviour at node 1 but rather to changes in the AA’s behaviour at node 2. In particular, we now have a large region of $CM$ for high values of $k$. As seen in Figure 10, there is also a thin slice of a $PI$ region between $CM$ and $PO$.

References


