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Debt Versus Equity in Development Finance*

by

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Debt Versus Equity in Development Finance

Ulrich Lächler

I. Introduction

This essay aims to provide a theoretical foundation for two widely debated issues in development economics. The first concerns a current controversy associated with the external indebtedness of developing countries. The second relates to a longer standing debate over the effects of foreign capital inflows on domestic savings and growth.

In reference to the developing countries' external debt problems, it is frequently argued that the source of these problems lies not so much in the high volume of funds that were transferred (i.e., that creditors lent too much or debtors borrowed too much), but rather lies in the form in which the transfers took place. After all, if the potential real return to capital in the more labor-intensive developing economies is higher than in the capital-intensive developed economies, as generally conceded, then according to standard efficiency criteria even further transfers should take place. However, to avert the recurrence of payment crises, it is suggested that a greater portion of transfers be channeled in the form of equity participation instead of debt. Greater equity involvement could be achieved, for example, by encouraging more foreign portfolio investment or direct investment, but also through the issue of so-called commodity-linked bonds. What this proposal recommends, in other words, is a change in the external financial structure of developing countries. To evaluate this recommendation would presuppose an understanding of what constitutes the appropriate debt-equity ratio of an economy receiving external funds. However, the impression received from surveying the development literature on this subject is that such an understanding is largely lacking.

The determinants of the debt-equity ratio of a firm or industry have, on the other hand, been a long and much discussed subject in financial economics. A central position is occupied in this litera-

ture by the well-known Modigliani-Miller Theorem, whose basic message is that the amount of leverage of a firm has no effect upon its real value. However, few economists became convinced that debt-equity ratios are entirely determined by chance, and the consequence has been an outpour of writings examining the conditions under which the M-M irrelevance result would not hold. Early efforts concentrated on the existence of bankruptcy costs and on differential tax treatment of debt and equity returns. Later work has focused on the agent-principal relationships arising from the separation of ownership and control that characterizes the modern corporation. The central point recognized by this new approach is that a change in financial structure changes how the returns to capital become distributed, which in turn alters the incentive structures by which the actions of managers and owners are motivated. According to one strand of thought originating with Stiglitz (1974) and Jensen and Meckling (1976), the benefit of raising external capital through equity instead of debt is that the owner-entrepreneur can shed some of the risk associated with the firm's operations, but the drawback is that it also provides the managing agent with greater disincentives in his productive effort. The optimal debt-equity ratios would then be attained in a given environment at the point where both forces balance. The objective of this essay is to apply the insights gained from this literature in the broader context of development finance.

The previously mentioned argument is favor of more equity participation in transfers to developing countries is obviously motivated by the risk-sharing benefits of equity versus debt. The transfer recipients would gain through reduced domestic income variability, as their economies become less susceptible to sudden dislocations occasioned by unavoidable defaults. What this argument ignores, in view of the agent-principal analogy, is that the transfer recipients might alter their consumption and investment behavior in response to the changed transfer arrangement, with possibly adverse consequences to growth.

The impact of foreign transfers on domestic savings and growth is, of course, not a recent subject of controversy in development economics, but rather, has been the focus of heated discussions since at least the early 1960's. Despite this long tradition, the degree of consensus achieved is modest at best. Furthermore, the issue that equity and debt financed transfers might have a different impact, if addressed at all, has been confined to descriptive discussions. Most early development models simply took for granted that a dollar of foreign inflows would translate into a dollar of investment without affecting domestic savings. When it was realized that such an assumption violated all basic theoretical principles, efforts were made to explicitly model and test the relation between foreign transfers and domestic savings, but usually with mixed empirical results. A possible explanation that may account for these unsatisfactory results is that previous work has generally failed to draw a clear distinction between alternative forms of capital transfers, rigorously grounded in economic theory. Another aim of this paper, therefore, is to offer a consistent analytical framework for making such distinctions.

The following section begins by modelling the aggregate behavior of a stylized developing economy as the decisions of one intertemporal utility-maximizing risk-averse agent. This is basically the same paradigm applied in modern consumption theory. Alternatively, foreign capital owners are stylized as a risk-neutral principal operating in a competitive market for investible funds. The objective then is to analyze the responses of the agent to different types of transfers, and to derive the agent's optimal financial structure as a function of various underlying parameters. It is assumed throughout that the agent retains control over the disposal of funds within the economy (that is, aggregate consumption and investment) independent of whether foreign capital is introduced via debt finance, equity finance or foreign aid. Another assumption, initially, is that the transfer recipient does not willfully default on his future obligations. This subject is treated separately at the end, where the model modifications needed to capture the notion of sovereign risk, and some implications, are discussed. In place of a summary, the paper concludes with a discussion of related work.

II. The Basic Model of Agent-Principal Relationships

Consider an economy initially endowed with a limited amount of resources that can be either consumed or invested. The amount of output produced in the future, Q_{t+1} , is taken to be a stochastic function of the amount invested in the present, I_t , as shown by the following expression:

$$1) \quad \tilde{Q}_{t+1} = \tilde{x}_{t+1} F(I_t).$$

Tildes denote random variables. The production function, $F(I)$, is assumed to have the following properties: $F(0) = 0$, $\lim_{I \rightarrow \infty} F(I) = \infty$, $F'(I) > 0$, $\lim_{I \rightarrow 0} F'(I) = \infty$ and $F''(I) \leq 0$. The random variable, \tilde{x}_{t+1} , attains values in the non-negative interval $(0, \bar{x})$ subject to a known stationary probability distribution described by the density function, $g(x)$, has a mean equal to 1 and a finite variance. The distribution of \tilde{x}_{t+1} is independent of I_t , and its realized value is revealed only after the investment has been made.

The investment decision is determined from the solution to an intertemporal maximization problem yielding an optimal anticipated consumption stream where all savings are automatically invested. The setting considered here is a two-period Fisherian consumption model, where the aggregate of individual consumption decisions within the economy is modelled as the decision of one agent who seeks to maximize the following time-additive, concave, von Neumann-Morgenstern utility function:

$$2) \quad U = U_1(C_t) + \beta E_t U_2(\tilde{C}_{t+1}).$$

E_t denotes the expected value conditioned on all information available in period t , β is a time-discount factor, and the instantaneous utility functions in each period are characterized by these properties: $U'_1 > 0$, $U''_1 \leq 0$ and $U(0) = 0$. In an autarchic context, the agent's objective can then be written as:

$$3) \quad \text{Max}_I U_1(\bar{y}-I) + \beta \int_0^{\bar{x}} U_2(xF(I))g(x)dx,$$

subject to the inequality constraints, $0 \leq I \leq \bar{y}$, where \bar{y} represents the initial endowment. Time-subscripts have been deleted in this formulation of the agent's objective for notational convenience, without giving rise, I hope, to problems of interpretation.

Consider next the situation where the agent described above receives a given capital transfer from abroad in the initial period. The donor of this transfer will be referred to as the principal from now on. The capital transfer can take place either in the form of a loan or in exchange for equity participation. In the first case, the agent owes the foreign principal a predetermined sum to be paid back in the second period, while in the second case, the foreign principal receives a predetermined share of the agent's second period output. A third case that will also be examined later for comparative purposes is where the transfer is made in the form of a gift (say, foreign aid). The reason for distinguishing between these separate transfer arrangements is that each has a different impact on the incentive structure facing the agent that receives the transfer, and thus is likely to influence his investment, or savings, decision independent of the amount transferred. Before proceeding to analyze the agent's response under the various arrangements, however, some assumptions regarding the institutional environment and the principal's behavior have to be made explicit.

The foreign principal is assumed in this model to consist of one rational, risk-neutral wealth-holder. It is a matter of indifference to him whether the capital transfer takes place in return for debt or equity participation as long as the expected return on either claim remains the same. In the event of a pure equity participation arrangement, let $(1-\gamma)$ denote the share of the agent's output in the second period to which the principal is entitled in return for an initial capital transfer, T ; such that $0 \leq \gamma \leq 1$. The expected value of this share to the principal is then $E((1-\gamma)xF(I)) = (1-\gamma)F(I^e)$,

where I^e denotes the level of investment that the agent is expected to undertake ex post, upon receipt of the transfer. The amount that the principal is currently prepared to pay for this share in future output is then assumed to be $T = (1-\gamma)F(I^e)/(1+R)$, where R is the principal's rate of time preference. For simplicity, and without damage to all subsequent arguments, R is henceforth assumed to be 0. In the second period, once the random variables have been determined, the principal would receive $(1-\gamma)xF(I)$ and the agent would be left with the remaining amount, $\gamma xF(I)$.

In the event of a pure debt financing arrangement, let B denote the amount (in terms of output) that the agent promises to pay the principal in the second period in exchange for the initial transfer, T . In evaluating this claim, the principal recognizes that the agent can only fulfill his obligation if the realized future output, $xF(I)$, equals or exceeds the amount of the debt, B ; that is, if $x \geq B/F(I)$. In that case, also, the agent would receive the difference, $xF(I)-B$, in period 2. Should \bar{x} attain a value less than $B/F(I)$, however, the principal would receive only $xF(I)$ in period 2, while the agent receives nothing. Assuming again that the principal's rate of time preference is 0, the present expected value of this claim can be expressed as:

$$4) \quad T = B \int_b^{\bar{x}} g(x) dx + F(I^e) \int_0^b xg(x) dx$$

$$= F(I^e) \left[1 - \int_b^{\bar{x}} (x-b)g(x) dx \right],$$

such that $b \equiv B/F(I^e)$.

Two points may be observed from expression (4): For a given anticipated investment level, I^e , the maximal value of T is $F(I^e)$, which occurs when $b = \bar{x}$, or $B = \bar{x}F(I^e)$. Another point is that even though the principal's time preference rate has been assumed to be constant, the observed interest rate charged on the loan made does not stay constant, but rather varies positively with the level of b . The observed interest rate in this setting is $(B-T)/T = r$, which can be derived as

$$5) \quad r = \frac{\int_0^b (b-x)g(x)dx}{\int_0^{\bar{x}} (x-b)g(x)dx} \geq 0.$$

Upon differentiating this expression with respect to b, we have:

$$\frac{\delta r}{\delta b} = \frac{G(b) \left(1 - \int_0^b xg(x)dx\right) + \int_0^b xg(x)dx (1-G(b))}{\left(1 - \int_0^{\bar{x}} (x-b)g(x)dx\right)^2} \geq 0,$$

such that $G(b) \equiv \int_0^b g(x)dx.$

Consider, finally, a combination of debt and equity claims. Using the standard convention that debt repayment takes precedence over disbursements to shareholders, the expected value of a $(1-\gamma)$ share of net output becomes $(1-\gamma)(F(I^e) - V^D)$, where V^D is the expected value of outstanding bond claims as derived in expression (4). The total expected value of foreign debt plus equity participation then becomes:

$$\begin{aligned} 6) \quad T &= (1-\gamma)(F(I^e) - V^D) + V^D \\ &= (1-\gamma)F(I^e) + \gamma V^D \\ &= F(I^e) \left(1 - \gamma \int_0^{\bar{x}} (x-b)g(x)dx\right), \end{aligned}$$

where γ and b are defined to lie in the intervals $(0,1)$ and $(0,\bar{x})$ respectively. In different words, T is taken to represent the amount that a foreign principal is prepared to transfer in exchange for a combined claim defined by the values of γ and B, given that the agent is expected to undertake the level of investment, I^e .

A few comments for clarification may be necessary before addressing the question of how the anticipated and actual levels of investment become determined. Ignored so far in the discussion is the possibility that the agent receiving a transfer may subsequently decide to renege on his obligations, either by expropriating all or part of the foreign principal's share of output in the case of equity finance, or in the case of debt finance, by willfully defaulting despite an ability to pay. In a purely domestic financial context such an option is assumed to be eliminated by an adequate legal enforcement

system. However, when the participants in a financial transaction are sovereign entities or fall under different legal jurisdictions, then this possibility is not so easily dismissed. Numerous writers have dealt with the risk of debt repudiation by introducing default penalties or costs to the debtor as a deterrent; examples are Allen (1983), Eaton and Gersovitz (1981), Sachs (1982), and Sachs and Cohen (1982). Generally the problem has been posed as one where a country defaults when the expected gain from default exceeds the expected penalties, such that the net expected gain rises with the amount of credit extended. That places an upper bound on the amount of capital that rational principals would transfer. From this viewpoint, the primary results of this investigation may be interpreted as applying to the situation where the amount transferred lies below that upper limit. Furthermore, in the model developed here, the possibility that defaults might be triggered by non-economic motives is ignored. For example, Hanson (1974) argues that the risk of expropriation or default rises with the ratio of foreign to nationally owned capital (without distinguishing between foreign debt and equity finance), while Feder and Regev (1975) assume that default on loans is basically an economic decision, but that expropriation of foreign (equity) capital is largely a political decision. In section VII of this essay, where sovereign risk is dealt with explicitly, both forms of default are assumed to be entirely motivated by economic considerations.

III. The Determination of Equilibrium

Consider a situation where the amount of the capital transfer from the principal to the agent is fixed exogenously. What remains to be determined, then, are the terms of the transfer, γ and B , and the levels of actual and anticipated investment, I and I^e . It is assumed that the capital transfer takes place after the terms, γ and B , have been negotiated, but before the agent makes his investment decision, so that I and I^e could conceivably be different. When both the agent and principal are rational, however, the anticipated and actual investment levels will be the same. There exist two solutions, termed cooperative and non-cooperative, that satisfy this equilibrium condition, and they are distinguished by whether or not the agent can credibly precommit himself to undertake certain actions. Both solutions are illustrated next, first for the case of pure equity finance and secondly for the case of pure debt finance.

Given a fixed transfer amount, \bar{T} , the agent's expected utility may be expressed as

$$7) U(\gamma, B, I) = U_1[\bar{Y} + \bar{T} - I] + \beta \int_b^{\bar{x}} U_2[\gamma F(I)(x-b)]g(x)dx + \beta \int_0^b U_2(0)g(x)dx.$$

Letting $B=0$, we can represent this utility function by a set of indifference curves (C_i) in (γ, I) space, as shown in figure 1. The slope of each curve is given by $-(\delta U / \delta I) / (\delta U / \delta \gamma)$, where

$$\delta U / \delta \gamma = \beta F(I) \int_0^{\bar{x}} U_2'(\cdot) x g(x) dx \geq 0$$

and

$$\delta U / \delta I = -U_1'(\cdot) + \gamma \beta F'(I) \int_0^{\bar{x}} U_2'(\cdot) x g(x) dx \geq 0 \text{ as } I \leq I^*$$

Moving upward across the family of indifference curves yields higher utility.

Also portrayed in figure 1, by the curve denoted TT' , is the combination of points which yield a constant expected return to the principal, and is derived from expression (6) under the assump-

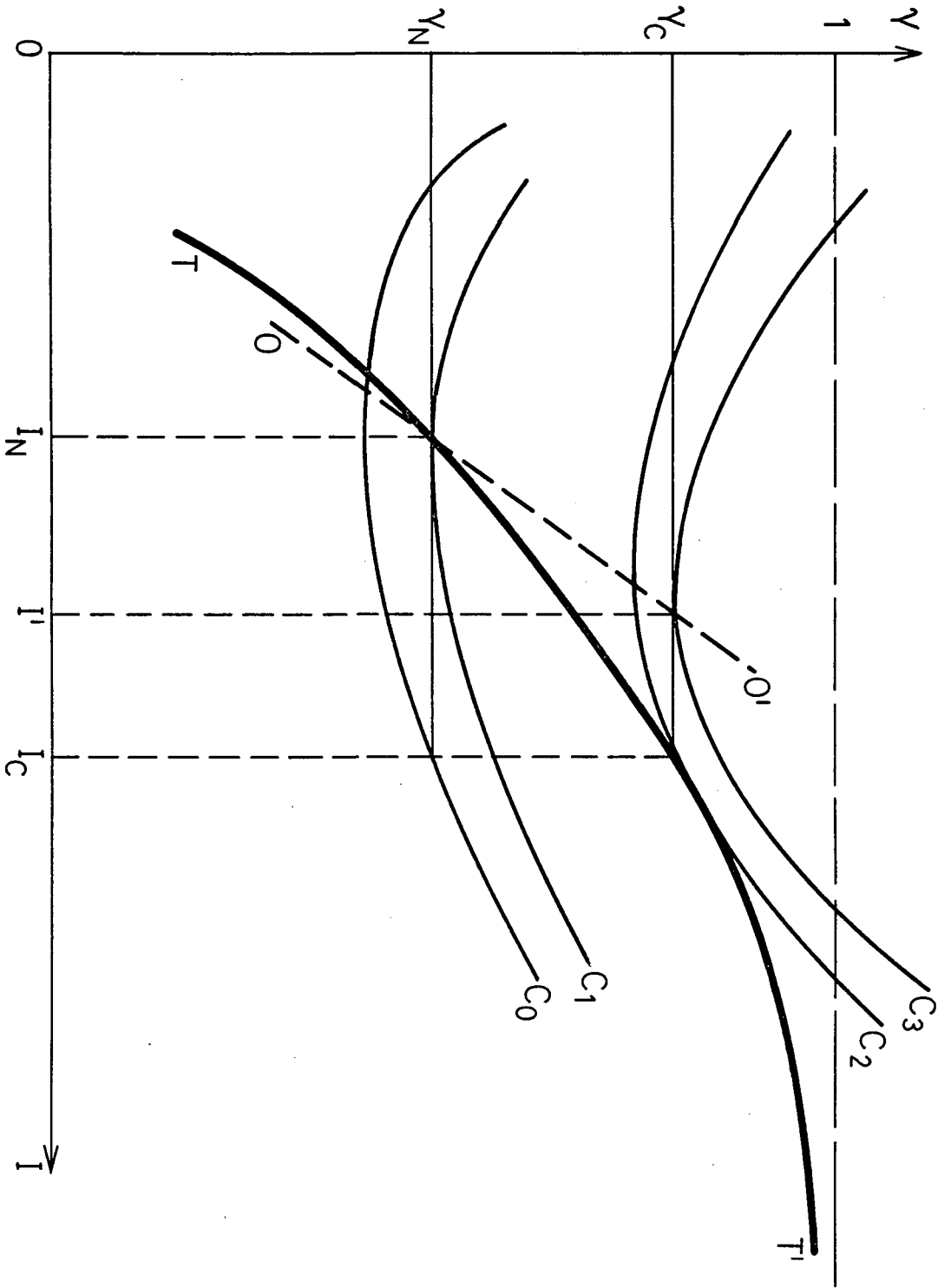


Figure 1

tion that $B = 0$. The slope of that curve is $(1-\gamma)F'(I)/F(I) \geq 0$. Points above and to the left of TT' yield an expected return less than \bar{T} , while points below and to the right imply a higher expected value.

Suppose now that the agent is able to precommit himself to undertake a given ex post level of investment, while negotiating with the principal over the terms of the transfer. Then the optimal solution would be given by the point, (γ_C, I_C) , where the agent receives \bar{T} in exchange for the share $(1-\gamma_C)$ of future output, and proceeds to invest an amount, I_C . This cooperative solution is derived by maximizing (7) with respect to γ and I subject to the constraint equation (6).

Alternatively, consider what would happen if the agent does not precommit himself beforehand to invest, I_C , but nonetheless were to receive the transfer on the favorable terms given by γ_C . Then we observe from figure 1 that the best attainable position for the agent is to invest only I' . That is the solution obtained from the unconstrained maximization of (7) with respect to I , given that $\gamma = \gamma_C$. At point (γ_C, I') , however, the expected value of the principal's equity share falls below \bar{T} . A rational principal would anticipate this potential capital loss and offer to exchange \bar{T} only in return for a higher share of future output, namely $(1-\gamma_N)$. The non-cooperative solution (γ_N, I_N) is characterized by the property that the ex post incentive of the agent would be to invest precisely the amount I_N , given $\gamma = \gamma_N$; which happens to be consistent with an anticipated return of \bar{T} to the principal.

Both solutions have their counterpart in the case of pure debt finance: The expected utility function (7) can also be represented by a family of indifference curves (C_j) in (B, I) space, as shown in figure 2. In this case, downward movements across indifference curves imply increasing utility. The slope of each curve is given

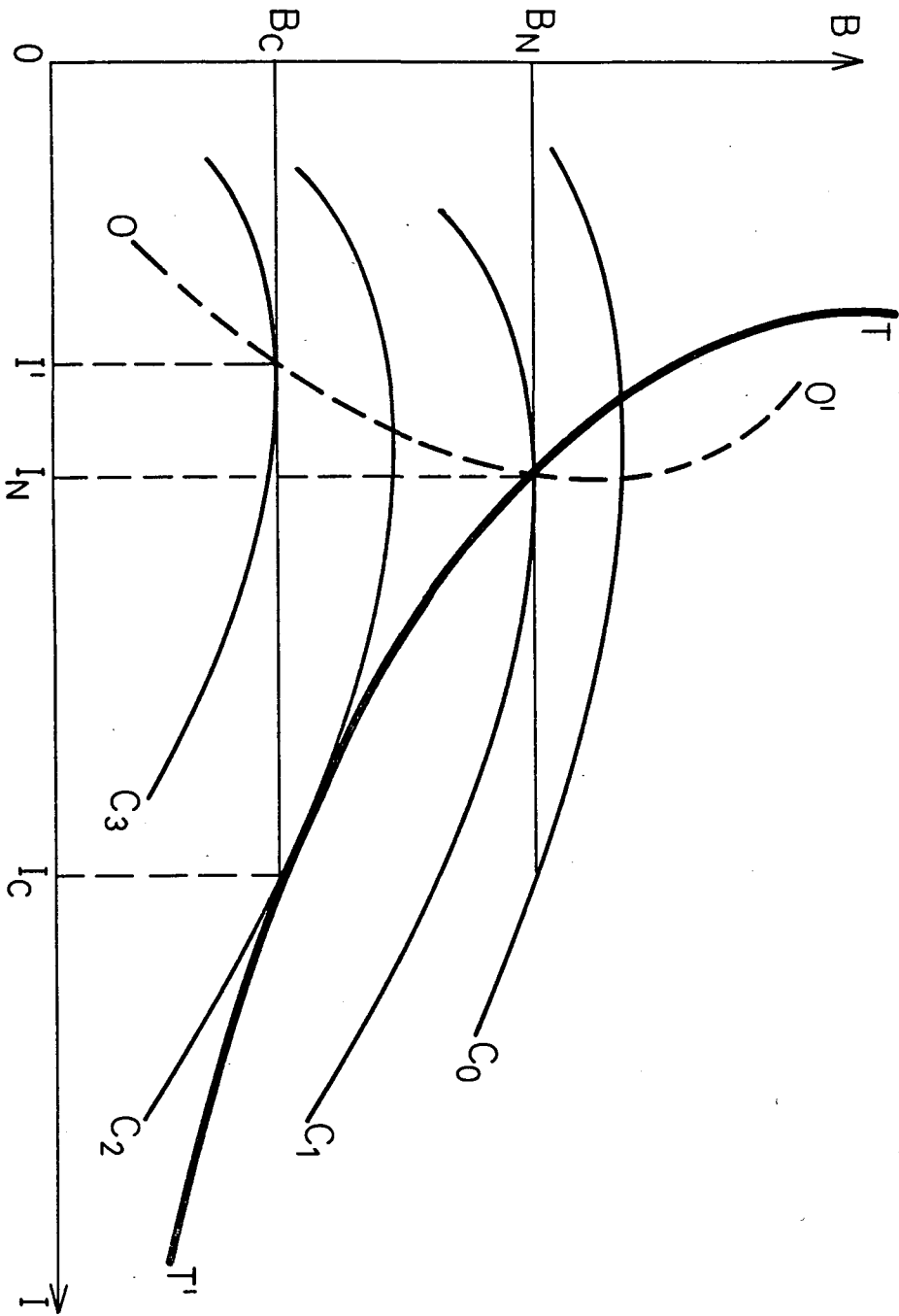


Figure 2

by $-(\delta U/\delta I)/(\delta U/\delta B)$, such that

$$\delta U/\delta B = -\beta \gamma \int_b^{\bar{x}} U_2'(\cdot) g(x) dx \leq 0$$

and

$$\delta U/\delta I = -U_1'(\cdot) + \beta F'(I) \gamma \int_b^{\bar{x}} U'(\cdot) x g(x) dx \stackrel{\geq}{\leq} 0 \text{ as } I \stackrel{\leq}{\geq} I^*.$$

Similarly, the curve denoted TT' in figure 2 portrays the combination of points such that $T = \bar{T}$, from expression (6). The slope of that curve is

$$dB/\delta I \Big|_{T=\bar{T}} = -F'(I) [1 - \gamma \int_b^{\bar{x}} x g(x) dx] / \int_b^{\bar{x}} g(x) dx \leq 0.$$

Points above and to the right of TT' imply that $T > T'$, while to the left and below $T < T'$. Applying the same logic as before, it can be readily observed that the point (B_C, I_C) represents the cooperative solution, while the non-cooperative solution is given by (B_N, I_N) . An interesting point to observe from figure 2 is that the non-cooperative solution always involves a higher probability of default than the cooperative solution.

Whether the market equilibrates at the cooperative solution or at the non-cooperative solution cannot be determined without some additional assumptions. It will depend, among other things, on the time horizon with which the agent operates and on the ability of the principal to monitor the agent's behavior. Jensen and Meckling (1976) refer to the difference in utility obtained by the agent in the cooperative solution case relative to the non-cooperative case as the agency costs of external finance, and these would be borne entirely by the agent. To reduce these costs, in a national environment agents can engage in bonding activities; that is, legally bind themselves to follow a preannounced course of action in the future. In an international context, such contracts are not only more difficult to enforce, but also involve higher monitoring costs, such that these might outweigh the difference in expected utilities. Attempts in this direction occur, nonetheless, as when a nation

voluntarily subjects itself to an IMF conditionality agreement, or when banks extend loans, not in one lump-sum, but spread out over time, with each installment made conditional on prior actions by the agent.

A non-cooperative solution may not always exist. That happens if the locus of points OO' in figures 1 and 2 lies everywhere to the left of the principal's transfer constraint, TT' . It can be shown, however, that for low enough values of $\bar{y} + \bar{T}$, both curves will generally intersect. This raises the possibility that in a non-cooperative environment the agent would be rationed by the foreign credit supplier. In the cooperative case, on the other hand, a non-rationing solution always exists.

IV. The Equilibrium Debt-Equity Ratio

In the preceding section, two equilibrium outcomes determining investment and the terms of transfer were shown to be possible under either debt finance or equity finance. This section seeks to determine the equilibrium combination of both forms of finance given a cooperative environment, and then in a non-cooperative environment. As before, the total amount transferred, \bar{T} , continues to be held constant.

Given that the principal is a rational, risk-neutral agent, as assumed earlier, he is indifferent as to the proportion of debt to equity claims he receives in return for the capital transfer, provided that the combined value of those claims remains the same. The equilibrium combination of both claims is therefore entirely left up to the agent to determine. Given that he too behaves rationally, this equilibrium combination will then turn out to be the agent's optimal debt-equity ratio.

The cooperative case

Recall that in obtaining the cooperative solution the agent maximizes his expected utility function (7) taking the principal's transfer constraint (6) explicitly into account. Given that now both γ and B are variable, the constraint equation (6) can be rearranged as

$$6) \quad \gamma = \frac{(F(I) - \bar{T})}{\bar{T}} / \int_b^{\bar{x}} (x-b) g(x) dx.$$

Substituting this expression for γ into the utility function (7) yields:

$$7) \quad U(B, \gamma(B, I), I) = U_1(\bar{Y} + \bar{T} - I) + \beta \int_b^{\bar{x}} U_2 \left(\frac{(F(I) - \bar{T})(x-b)}{\int_b^{\bar{x}} (x-b) g(x) dx} \right) g(x) dx.$$

The agent's preferred position is obtained by maximizing (7)' with respect to I and B, and substituting those values into (6)' to obtain the optimal γ . Differentiating (7)' with respect to B, however, yields

$$\begin{aligned}
 8) \quad \delta U / \delta B &= \frac{\beta (F(I) - \bar{T}) / F(I)}{(\int_b^{\bar{x}} (x-b) g(x) dx)^2} \int_b^{\bar{x}} U_2'(\cdot) [(x-b) \int_b^{\bar{x}} g(x) dx \\
 &\quad - \int_b^x (x-b) g(x) dx] g(x) dx \\
 &= A \int_b^{\bar{x}} U_2'(\cdot) [x \int_b^{\bar{x}} g(x) dx - \int_b^{\bar{x}} x g(x) dx] g(x) dx
 \end{aligned}$$

The term, A, in expression (8) is always positive, except in the extreme case where $\bar{T} = F(I)$. Recall from the discussion in section II that $F(I)$ is the maximal amount that a rational principal would transfer. On the other hand, the term following A on the right of expression (8) is shown in the appendix to be always negative for all feasible values of I and B (such that $0 < b < \bar{x}$), if $U_2(\cdot)$ is a strictly concave function of its argument. This means that, as long as the agent is risk-averse, he will always prefer equity finance over debt finance. The equilibrium debt-equity ratio in the cooperative solution would then be 0. In the case where $U_2(\cdot)$ is a linear function, $U_2'(\cdot)$ is a constant, and the integral in expression (6) is easily seen to be always equal to zero. This result is simply restating the Modigliani-Miller Theorem, which says that the source of finance is irrelevant in an environment with rational, risk-neutral participants. In that case, the debt-equity ratio would be indeterminate.

The non-cooperative case

Recall that in this case the agent does not take into account the principal's transfer constraint when making his investment decisions. Rather, he takes the terms of transfer, γ and B, as given, and simply maximizes $U(\bar{\gamma}, \bar{B}, I)$ with respect to I; that is $\delta U / \delta I = 0$. With this in mind, let us take the total derivative of $U(\gamma, B, I)$, as given by expression (7), with respect to all of its arguments:

$$\begin{aligned}
 9) \quad dU &= U_{\gamma} d\gamma + U_B dB + U_I dI \\
 &= U_{\gamma} d\gamma + U_B dB \\
 &= (\beta F(I) \int_b^{\bar{x}} U_2'(\cdot) (x-b) g(x) dx) d\gamma - (\gamma \beta \int_b^{\bar{x}} U_2'(\cdot) g(x) dx) dB
 \end{aligned}$$

We also know that in equilibrium the agent's optimal combination of γ and B must satisfy the transfer constraint. Therefore, taking the total derivative of (6), such that $dT = 0$, yields:

$$\begin{aligned}
 10) \quad d\gamma &= \left[\int_b^{\bar{x}} (x-b) g(x) dx \right]^{-1} \left[(F'(I)/F(I) (1 - \gamma \int_b^{\bar{x}} x g(x) dx) dI \right. \\
 &\quad \left. + ((\gamma/F(I)) \int_b^{\bar{x}} g(x) dx) dB \right]
 \end{aligned}$$

Substituting expression (10) for $d\gamma$ into expression (9), dividing everything by dB , and rearranging terms, yields:

$$\begin{aligned}
 11) \quad dU/dB &= A \gamma \beta \left[\int_b^{\bar{x}} U_2'(\cdot) \left[x \int_b^{\bar{x}} g(x) dx - \int_b^{\bar{x}} x g(x) dx \right] g(x) dx \right] \\
 &\quad + A \beta \int_b^{\bar{x}} U_2'(\cdot) (x-b) g(x) dx (F'(I) (1 - \gamma \int_b^{\bar{x}} x g(x) dx)) \frac{dI}{dB},
 \end{aligned}$$

$$\text{where } A \equiv \left[\int_b^{\bar{x}} (x-b) g(x) dx \right]^{-1}.$$

In expression (11), dU/dB refers to the change in utility resulting from an increase in debt obligations accompanied by a commensurate change in γ so as to keep the total value of claims held by the principal constant. Similarly, dI/dB represents the impact on the agent's utility maximizing investment behavior of a switch from equity finance to debt finance without altering the sum transferred. Note that the coefficient of dI/dB is always non-negative, independent of the concavity of $U_2(\cdot)$.

The first term on the right hand side of (11) is similar in structure to the result obtained in equation (8), and therefore is always negative provided that the agent displays some degree of risk aversion. Equity finance is a means of sharing risks in contrast to debt finance, and therefore a rise in the debt-equity ratio reduces the agent's utility to the extent that risk matters to him.

Now consider the second term on the RHS of (11). A standard proposition in much of the related finance literature (e.g. Jensen and Meckling (1976), Stiglitz (1972)) is that equity finance involves greater disincentive effects in a non-cooperative environment than debt finance. That is because, except for bankrupt states, the agent receives the full benefit of profit increases under debt finance, while under equity finance part of those profits would accrue to the outside principals. Applying that proposition to the present context, an increase in debt finance relative to equity would increase the incentives of the agent to provide more in the future, as the share of future output increases going to the agent is greater. In other words, dI/dB would be positive. As is shown later, this same proposition also emerges from the present model, assuming that a non-cooperative solution exists.

In summary, what expression (11) shows is that in a non-cooperative environment a risk-averse agent faces a trade-off between risk and future return in the sense of output growth. The optimal debt-equity ratio would then be derived by solving for the combination of (γ, B) such that $dU/dB = 0$ (provided an interior solution exists). Contrary to the cooperative case, where all-equity finance is optimal for risk-averse agents, we would expect here to obtain a solution with positive amounts of both types of claims. Only when the agent is risk-neutral, the case generally assumed in the theory of the firm, would all-debt financing be optimal. The parallel drawn here for the case of a country is that by shifting the source of finance toward more equity participation and less debt, greater income stability could be achieved, but at the expense of lower expected future growth.

V. The Impact on Investment of Alternative Financial Policies

The last section has shown that the agent's optimal debt-equity ratio depends in part upon his investment response in the non-cooperative case, while in the cooperative case his choice is entirely determined by his attitude toward risk. This section examines the agent's investment behavior under alternative financing constraints.

The non-cooperative case

As stated previously, the agent's non-cooperative investment response is found by maximizing the expected utility function (7) with respect to I , while regarding the terms of the transfer, γ and B , as fixed parameters. This yields the necessary condition,

$$12) \quad \delta U / \delta I = -U'_1(.) + \gamma \beta F'(I) \int_b^{\bar{x}} U'_2(.) x g(x) dx \stackrel{\geq}{=} 0.$$

Provided that an interior solution exists, so that the equality sign obtains, we then also have the sufficiency condition: $\delta^2 U / \delta I^2 < 0$. An endpoint solution may apply, however, in the case of a risk-neutral agent (the expected value maximizing firm), when the endpoint constraint, $0 \leq I \leq \bar{y} + \bar{T}$, is binding.

To determine the investment response to perturbations in the parameters $(\bar{y}, \bar{T}, \gamma, B)$, take the total differential of equation (12). For mathematical simplicity, consider first the initial situation of an agent who has yet to receive any foreign transfers. That is, evaluate the total differential at the point $(T=0, \gamma=1, B=0)$. This yields:

$$13) \quad d(\delta U / \delta I) = (\delta^2 U / \delta I^2) dI - U''_1(.) (d\bar{y} + d\bar{T}) \\ + (\beta F'(I) \int_0^{\bar{x}} U'_2(.) x g(x) dx + \beta F'(I) \int_0^{\bar{x}} U''_2(.) x^2 F(I) g(x) dx) d\gamma \\ - (\beta (F'(I) / F(I)) \int_0^{\bar{x}} U'_2(.) x g(x) dx) dB = 0$$

To further simplify the exposition, confine the utility function to the set of functions displaying a constant degree of relative risk aversion, using the Arrow-Pratt measure. In other words, $U(Z) \equiv (Z)^{1-\alpha}/(1-\alpha)$, so that $-ZU''(Z)/U'(Z) = \alpha$, and where $0 < \alpha < 1$, is the degree of risk aversion. Equation (13) may then be rewritten as,

$$\begin{aligned}
 13)' \quad d(\delta U/\delta I) &= (\delta^2 U/\delta I^2) dI - U_2'(\cdot) (d\bar{y} + d\bar{T}) \\
 &+ (\beta F'(I) (1-\alpha) \int_0^{\bar{x}} U_2'(\cdot) xg(x) dx) d\gamma + (\alpha\beta (F'(I)/F(I))^2) \int_0^{\bar{x}} U_2'(\cdot) g(x) dx) dB \\
 &= (\delta^2 U/\delta I^2) dI - U_2'(\cdot) (d\bar{y} + d\bar{T}) \\
 &+ (1-\alpha) U_1'(\cdot) d\gamma + (\alpha\beta F'(I)/F(I))^2 \int_0^{\bar{x}} U_2'(\cdot) g(x) dx dB = 0,
 \end{aligned}$$

where the last substitution is made using (12).

Consider as a benchmark the situation where the agent receives a gift from abroad, say as foreign aid. This would be tantamount to an increase in the agent's initial endowment ($d\bar{y} > 0$). The investment response can then be solved from (13)', by letting $d\bar{T} = d\gamma = dB = 0$, to yield:

$$14) \quad 0 \leq (dI/d\bar{y}) = U_1'(\cdot)/(\delta^2 U/\delta I^2) \leq 1.$$

The second inequality easily follows from the fact that at the initial position, $\delta^2 U/\delta I^2 = U_1'(\cdot) + U_1'(\cdot)[F''(I)/F'(I) - \alpha F'(I)/F(I)] \leq U_1'(\cdot) \leq 0$. Equation (14) simply states that the agent would allocate part of his higher endowment toward raising current consumption and part toward raising future consumption. A transfer constraint is not binding here because the agent has no obligations to pay back. Alternatively, under debt or equity finance, the agent's response would be constrained by the transfer condition (6), the total differential of which yields:

$$15) \quad dT = F'(I) (1-\gamma \int_b^{\bar{x}} xg(x) dx) dI + (\gamma \int_b^{\bar{x}} g(x) dx) dB - (F(I) \int_b^{\bar{x}} (x-b)g(x) dx) d\gamma$$

Also evaluating this expression at the initial position, where $(\gamma, B) = (1, 0)$, yields

$$15)' \quad dT = dB - F(I)d\gamma.$$

The effects of an equity financed transfer may be obtained by letting $dB = 0$, and substituting for $d\gamma$ from (15)' to (13)', which yields:

$$16) \quad \frac{dI}{dT} \Big|_{\gamma} = \frac{U_1'(\cdot)}{(\delta^2 U / \delta I^2)} + \frac{(1-\alpha)\beta U_1'(\cdot)/F(I)}{(\delta^2 U / \delta I^2)}.$$

The first term on the right is positive and identical to the expression derived in eq. (14), while the second expression is generally negative. Therefore, provided that the agent is not "infinitely" risk averse, meaning $\alpha = 1$, the investment response to an equity-financed transfer is less than the response to a gift, and possibly negative.

In contrast, the effects of a debt-financed transfer are obtained by letting $d\gamma = 0$, and substituting for dB from (15)' into (13)' to yield:

$$17) \quad \frac{dI}{dT} \Big|_B = \frac{U_1'(\cdot)}{(\delta^2 U / \delta I^2)} - \frac{\alpha\beta (F'(I)/F(I)^2) \int_0^{\bar{x}} U_2'(\cdot)g(x)dx}{(\delta^2 U / \delta I^2)} > 0$$

Both terms on the right hand side of (17) are generally positive, while the first term is the same as derived in eq. (14), so that the total investment response to a debt-financed transfer is greater than the response to an equal-sized gift.

The results given by equations (14), (16) and (17) show that in a non-cooperative environment, a debt-financed transfer in general leads to a higher level of investment than an equity-financed transfer of equal size. Recalling the discussion in the previous section, this means that when debt-financing is substituted for equity-financing, while keeping the total transfer unchanged, the structure of incentives facing the agent change in favor of raising the amount of investment. These results, however, are not insensitive to the initial conditions that were assumed, namely that the agent starts out in an

autarchic position. Nevertheless, the same result continues to apply as long as there exists a non-cooperative solution.

To prove this last proposition, let the total differential of equation (12) and equation (15) be represented as:

$$12)'' \quad d(\delta U/\delta I) = a_1 dT + a_2 dI + a_3 dB + a_4 d\gamma = 0$$

$$15)'' \quad dT = b_1 dI + b_2 dB + b_3 d\gamma$$

The impact on investment of a switch from equity to debt finance, such that $dT = 0$ is:

$$18) \quad \left. \frac{dI}{dB} \right|_{dT=0} = \frac{(a_3 - a_4 b_2/b_3)}{-(a_2 - a_4 b_1/b_3)}$$

For all permissible distributions and utility functions, we know that (a_1, a_4, b_1, b_2) are positive and $b_3 < 0$. From the second order condition, $\delta^2 U/\delta I^2 < 0$, we also have that $a_2 < 0$. Only $a_3 \equiv \delta^2 U/\delta I \delta B$ remains indeterminate.

Now recall from figure 1 that if a non-cooperative solution exists in the case of equity finance, the slope of locus OO' must be at least as steep as the slope of the budget constraint, TT' , when both intersect. That is, from (12)'' and (15)'', $-a_2/a_4 \geq -b_1/b_3$, or $a_2 \leq a_4 b_1/b_3$. This, along with the already known parameter signs, serves to determine the denominator in expression (18) as non-negative. Similarly, from figure 2, the necessary condition for existence of a non-cooperative equilibrium under debt finance is that $-a_3/a_2 \geq -b_2/b_1$, or $a_3 \geq a_2 b_2/b_1$. Combining this result with the existence condition under equity finance, yields $a_3 \geq a_4 b_2/b_3$, which determines the numerator in expression (18), also as non-negative. Provided a non-cooperative solution exists, therefore, a switch from equity to debt finance always encourages more investment.

The cooperative case

Although, in a cooperative environment, the risk-averse agent always chooses equity finance over debt finance, it may nonetheless be of interest to examine his investment behavior in response to a transfer. To obtain the cooperative solution, the agent maximizes the following Lagrangian expression with respect to (I, γ) or (I, B) :

$$19) \quad L = U_1(\bar{y} + \bar{T} - I) + \beta \int_b^{\bar{x}} U_2(\gamma F(I)(x-b))g(x)dx + \lambda(\bar{T} - F(I)(1 - \gamma \int_b^{\bar{x}} (x-b)g(x)dx))$$

where λ is the Lagrangian multiplier associated with the principal's transfer constraint. The first-order conditions for a maximum are:

$$\delta L / \delta I = \delta U / \delta I - \lambda F'(I) (1 - \gamma \int_b^{\bar{x}} xg(x)dx) = 0$$

$$\delta L / \delta B = \delta U / \delta B - \lambda \gamma \int_b^{\bar{x}} g(x)dx = 0, \text{ or,}$$

$$\delta L / \delta \gamma = \delta U / \delta \gamma + \lambda F(I) \int_b^{\bar{x}} (x-b)g(x)dx = 0$$

$$\delta L / \delta \lambda = \bar{T} - F(I) (1 - \gamma \int_b^{\bar{x}} (x-b)g(x)dx) = 0$$

The comparative statics effects of an increase in \bar{T} or \bar{y} are found by taking the total derivatives of the necessary conditions, and rearranging the resulting equations to obtain the following two systems: Under equity-financing we have,

$$A_\gamma (dI, dB, d\lambda)^T = (U_1'(\cdot), 0, -1)^T d\bar{T} + (U_1'(\cdot), 0, 0)^T d\bar{y},$$

and under debt-financing,

$$A_B (dI, dB, d\lambda)^T = (U_1'(\cdot), 0, -1)^T d\bar{T} + (U_1'(\cdot), 0, 0)^T d\bar{y}.$$

The superscript, T, denotes a transposed vector, and (A_γ, A_B) denote the (3x3) bordered Hessians associated with the Lagrangian maximization problem under each financing form. Both matrices are symmetric,

and negative-definite as required by the second-order condition for a maximum (that is, the determinant in this case is positive), and have the following sign patterns (as the interested reader may verify):

$$\text{sgn } A_{\gamma} = \begin{pmatrix} - & - & - \\ - & - & + \\ - & + & 0 \end{pmatrix} ; \quad \text{sgn } A_{\beta} = \begin{pmatrix} - & + & - \\ + & - & - \\ - & - & 0 \end{pmatrix}$$

Applying Cramer's rule in the two equations above, it is then easily observed that $(dI/d\bar{T})_{\gamma} \geq (dI/d\bar{y})_{\gamma} \geq 0$ and $(dI/d\bar{T})_{\beta} \geq (dI/d\bar{y})_{\beta} \geq 0$. In other words, the cooperative agent would always invest more in response to a transfer entailing future obligations than in response to a gift.

VI. The Optimal Transfer

So far in the discussion, the amount transferred by the principal to the agent was exogenously given. This section examines what would be the optimal amount transferred from the viewpoint of the agent, given that the possibility of voluntary default is absent. As in previous sections, this question is addressed separately for the cooperative and non-cooperative cases. The only difference is that here the principal's rate of time preference (read, world market interest rate, R) is explicitly introduced.

The cooperative case

The cooperative agent's preferred position can be derived by substituting the transfer constraint,

$$20) \quad T = F(I) \left(1 - \gamma \int_b^{\bar{x}} (x-b)g(x)dx \right) / (1+R)$$

directly into the utility function, (7), for the variable T , and maximizing with respect to either (I, γ) or (I, B) . In the case of pure equity finance, we obtain as necessary conditions:

$$\begin{aligned} -U'_1(.) \left(1 - (1-\gamma)F'(I)/(1+R) \right) + \gamma \beta F'(I) \int_0^{\bar{x}} U'_2(.) xg(x)dx &= 0 \\ -U'_1(.)F'(I)/(1+R) + \beta F'(I) \int_0^{\bar{x}} U'_2(.) xg(x)dx &= 0 \end{aligned}$$

Combining both conditions, the following relation can be easily derived, which must apply at the optimum:

$$21) \quad F'(I^*) = 1+R.$$

In other words, the agent would sell equity shares until the level of investment associated with the optimal transfer is such that the expected marginal return from investment equals the world interest rate; the standard, traditional result. (To find the actual transfer amount, simply substitute I^* and the associated γ^* into (20).)

In the case of pure debt finance, the maximization is done with respect to (I,B), yielding as necessary conditions:

$$-U'_1(.) (1-F'(I) (1 - \int_b^{\bar{x}} xg(x)dx)/(1+R)) + \beta F'(I) \int_b^{\bar{x}} U'_2(.) xg(x)dx = 0$$

$$U'_1(.) \left(\int_b^{\bar{x}} g(x)dx \right) / (1+R) - \beta \int_b^{\bar{x}} U'_2(.) g(x)dx = 0$$

Both conditions can be combined to arrive at the following relation, analogous to equation (21):

$$22) \quad \beta \int_{b^*}^{\bar{x}} U'_2(.) F'(I^*) [x \int_{b^*}^{\bar{x}} g(x)dx - \int_{b^*}^{\bar{x}} xg(x)dx] g(x)dx \\ + [F'(I^*) - (1+R)] \beta \int_{b^*}^{\bar{x}} U'_2(.) g(x)dx = 0$$

The first term in equation (22) is always negative when the agent is risk-averse, as mentioned previously and proved in the appendix. This means that the second term must then be positive for the equality to hold, or that $F'(I^*) > (1+R)$. In other words, a risk-averse agent stops borrowing and investing before the expected marginal return from investment equals the world interest rate. Given that $F''(I) < 0$, the optimal amount of investment by the cooperative risk-averse agent is smaller under debt finance than under equity finance. Only in the case of a risk-neutral agent (whence the first term in expression (22) is 0) would the optimal investment levels under either debt or equity finance be the same.

The non-cooperative case

To begin, take the total differential of the agent's utility function, (6), divide by dT, and set the resulting expression equal to zero:

$$23) \quad dU/dT = U'_1(.) + U_I (dI/dT) + U_Y (dY/dT) + U_B (dB/dT) = 0$$

The non-cooperative agent's behavior is characterized by the assumption that $U_I = \delta U / \delta I = 0$ (given an interior solution) both when the transfer

is financed through equity or through debt.

In the case of pure equity finance, expression (23) reduces to:

$$dU/dT = U'_1(.) + (U'_1(.)F(I)/\gamma F'(I))(d\gamma/dT) = 0$$

Taking the total derivative of the transfer constraint (7) and rearranging terms, yields

$$d\gamma/dT = (1-\gamma)(F'(I)/F(I)) \left. \frac{dI}{dT} \right|_{\gamma} - (1+R)/F,$$

where $(dI/dT)|_{\gamma}$ is the non-cooperative investment response to marginal increments in the transfer, as derived in the preceding section. Substituting this expression into the previous equation, for $d\gamma/dT$, yields the following necessary condition for an optimum:

$$24) \frac{U'_1(.)}{\gamma F'(I^*)} [F'(I^*) - (1+R) - (1-\gamma^*)F'(I^*)(1 - \left. \frac{dI}{dT} \right|_{\gamma})] = 0$$

Given that $(dI/dT)|_{\gamma} < 1$, then it is easily observed from equation (24) that the optimal investment level is such that $F'(I^*) > (1+R)$. In other words, the maximal level of investment undertaken in a non-cooperative environment is smaller than in a cooperative environment. (The premise, that $(dI/dT)|_{\gamma} < 1$, obtains, not only from the autarchic position, as already shown in section V, but also could be shown to hold when T is evaluated at T^* .)

Under pure debt-financing, a similar situation arises. Equation (23) reduces to

$$dU/dT = U'_1(.) - \beta \int_b^{\bar{x}} U'_2(.)g(x)dx(dB/dT) = 0,$$

and the constraint (7), upon differentiation can be rearranged as

$$dB/dT = (1+R)/\left(\int_b^{\bar{x}} g(x)dx\right) - [F'(I)(1 - \int_b^{\bar{x}} xg(x)dx)/\left(\int_b^{\bar{x}} g(x)dx\right)] \frac{dI}{dT}\Big|_B.$$

Inserting this expression into the preceding equation yields the following necessary condition (analogous to eq. (24)),

$$25) U'_1(.) \int_b^{\bar{x}} g(x)dx - \beta \int_b^{\bar{x}} U'_2(.)g(x)dx((1+R) - F'(I)(1 - \int_b^{\bar{x}} xg(x)dx) \frac{dI}{dT}\Big|_B) = 0$$

Using equation (12) to solve for $U'_1(.)$ and substitute in (25), yields after some manipulation:

$$\begin{aligned} 25)' \quad & \beta \int_b^{\bar{x}} U'_2(.)F'(I^*) \left[x \int_b^{\bar{x}} g(x)dx - \int_b^{\bar{x}} xg(x)dx \right] g(x)dx \\ & + [F'(I^*) - (1+R)] \beta \int_b^{\bar{x}} U'_2(.)g(x)dx \\ & + \left(\frac{dI}{dT}\Big|_B - 1 \right) \beta F'(I^*) \left(1 - \int_b^{\bar{x}} xg(x)dx \right) \int_b^{\bar{x}} U'_2(.)g(x)dx = 0 \end{aligned}$$

Notice that if $(dI/dT)|_B = 1$, equation (25)' reduces to the same optimality condition as in the cooperative case, equation (22). Therefore, given that $(dI/dT)|_B$ lies below 1, it is also easily observed that the non-cooperative optimal investment level must lie below the cooperative level.

VII. Introducing Sovereign Risk

The possibility that a country might willfully default or expropriate foreign-owned assets has been ignored in the analysis so far. In a national context, principals are protected from such actions of an agent by the judicial system. In the international context considered here, however, the ability to enforce legal rights across sovereign nations is considerably more limited. The purpose of this section is to examine various types of voluntary default within the previously constructed framework.

The term, voluntary default, is taken here to refer to any situation where the agent refuses to pay the principal a share of output that rightfully belongs to him despite the agent's physical ability to pay. What constitutes an ability to pay, however, is often subject to conflicting interpretation. In the preceding analysis, an agent defaulted on his debts if realized future output fell below the debt commitment, B . In that case, it was assumed that the foreign creditor would receive whatever output was realized, while the agent remained with nothing. This type of scenario corresponds to the usual bankruptcy proceedings within a domestic context. In the case of a country, on the other hand, it is arguable that there exists some minimum level of consumption to which a population is entitled before the foreign principals are repaid. Alternatively, this could be interpreted as a maximal limit on the government's ability to tax its residents or enforce repayment without risking an overthrow. This would then raise the possibility that the foreign principal would receive nothing in return for his transfer even though the output realized is positive.

To model this situation, let \bar{C} denote the minimum domestic consumption that needs to be attained before any return transfers to the principal can take place. Then, a country would not default if the realized value of \bar{x} is such that $\gamma F(I)(x-b) \geq \bar{C}$, or $x > b + \bar{C}/\gamma$; where $\bar{c} \equiv \bar{C}/F(I)$. For values of x below this level,

the foreign principal would receive only $x F(I) - \bar{C}$, if $\bar{c} \leq x \leq b + \bar{c}/\gamma$; and would receive nothing if $x \leq \bar{c}$. The agent's utility function under these circumstances would be written as:

$$26) \quad U(\gamma, B, I, \bar{C}) = U_1(y+T-I) + \beta \int_{b+\bar{c}/\gamma}^{\bar{x}} U_2(\gamma F(I)(x-b)) g(x) dx \\ + \beta \int_{\bar{c}}^{b+\bar{c}/\gamma} U_2(\bar{C}) g(x) dx + \beta \int_0^{\bar{c}} U_2(x F(I)) g(x) dx.$$

The principal's transfer constraint would in turn become:

$$27) \quad T = F(I) \left[(1-\gamma) \int_{b+\bar{c}/\gamma}^{\bar{x}} x g(x) dx + \gamma b \int_{b+\bar{c}/\gamma}^{\bar{x}} g(x) dx + \int_{\bar{c}}^{b+\bar{c}/\gamma} (x-\bar{c}) g(x) dx \right]$$

Observe that equations (26) and (27) reduce the equations (7) and (6) as $\bar{C} \rightarrow 0$. Furthermore, it could be shown that if $\bar{C} > 0$ (but not too large), the same qualitative results derived earlier would continue to apply.

The type of scenario described above is one where defaults or expropriations would occur only in times of economic distress, on account of, say, subsistence needs. Given that \bar{C} is a known exogenous parameter, it would in principle be possible to take this term into account during the transfer negotiations as part of the terms of transfer, along with γ and B . As a rule, however, \bar{C} is not known. It is simply difficult, if not impossible, to define what constitutes a minimum "subsistence" level of aggregate consumption. Given that \bar{C} can therefore not be monitored, a similar situation arises as discussed earlier with respect to the agent's ex post investment decisions. That is, the agent may wish to default on all his obligations not because of need, but because the utility gain from default or expropriation would exceed the loss.

In the context of the two-period model developed previously, if the principal could not retaliate in the event of a default, then it is clear that the agent would always be better off, ex post, by

keeping the entire realized output for himself. A rational principal would foresee this behavior and therefore not make any transfers (ignoring foreign aid granted for humanitarian reasons). The only way a transfer would then take place is if the principal can impose some form of penalty; examples being the appropriation of the agent's assets abroad, a boycott on trade or a withholding of future credit.

Consider two types of penalties that the principal could impose on the agent in the event that the latter defaults or expropriates the former's assets: An absolute penalty, P , and a proportional penalty, $sxF(I)$, where $0 \leq s \leq 1$. A voluntary default is then assumed to occur whenever the agent's utility in a default situation is higher than were he not to default. In the two period model considered previously, this would mean that a default occurs whenever the realized output in period 2 is such that the agent could attain a higher consumption level by defaulting than by not defaulting. This will be illustrated separately for the case of equity and debt financed transfers.

In the case of a pure equity financed transfer, the agent's period 2 consumption level, in the event that he expropriates the principal's share, would be $xF(I) - sxF(I) - P$. If he does not expropriate, his consumption level is $\gamma xF(I)$. For expropriation to occur, therefore, it has to be the case that

$$xF(I) - sxF(I) - P > \gamma xF(I),$$

or,

$$28) \quad (1-\gamma-s)x > P/F(I) \equiv p.$$

If $(\gamma+s) \geq 1$, then from (28), the agent would never expropriate, since x only attains non-negative values. Expropriations could only occur, then, if $(\gamma+s) < 1$, and when the realized value of x is $x > p/(1-\gamma-s)$. To introduce this possibility into the earlier model, assume for simplicity that in the event of expropriation the principal receives the proportional component of the penalty while the absolute component, P , represents a loss to the agent without gain

to anyone else. The agent's utility function and the transfer constraint can then be written as

$$29) \quad U(\gamma, I, s, P) = U_1(\bar{y}+T-I) + \beta \int_0^{\bar{x}} U_2(\gamma x F(I)) g(x) dx \\ + \beta \int_{p/(1-\gamma-s)}^{\bar{x}} U_2(F(I) ((1-s)x-p)) g(x) dx$$

$$30) \quad T = (1-\gamma)F(I) \int_0^{p/(1-\gamma-s)} xg(x) dx + sF(I) \int_{p/(1-\gamma-s)}^{\bar{x}} xg(x) dx.$$

The model examined earlier can be interpreted as a special case of equations (29) and (30) where $s = (1-\gamma)$. However, as long as γ is large enough ($\gamma \geq (1-s)$) the same results derived in previous sections would apply. Furthermore, an interesting implication of this model, in contrast to a debt financed transfer examined next, is that voluntary expropriations would only occur when the economy is prospering.

In the pure debt case, the necessary condition for default, corresponding to expression (28), is:

$$xF(I) - sxF(I) - P > xF(I) - B, \text{ or}$$

$$31) \quad x \leq \frac{B - P}{sF(I)} \equiv (b-p)/s.$$

It is easy to observe from (31) that the agent would never default when the absolute penalty, P , exceeds the debt commitment, B . Only if $B > P$ is voluntary default possible. For illustrative purposes, let $P = 0$. Then the utility function and transfer constraint pertaining to this case become:

$$32) \quad U(B, I, s) = U_1(\bar{y}+T-I) + \beta \int_0^{b/s} U_2(F(I)x(1-s)) g(x) dx \\ + \beta \int_{b/s}^{\bar{x}} U_2(F(I)(x-b)) g(x) dx$$

$$33) \quad T = sF(I) \left[1 - \int_{b/s}^{\bar{x}} ((b/s)-x) g(x) dx \right]$$

Notice that expressions (32) and (33) have the same structure as the earlier expressions (7) and (6), so that the same qualitative results derived previously would apply here as well, except that the range of admissible values for b is reduced to $0 \leq b \leq s\bar{x}$. In contrast to the equity financed transfer case, observe also that a default would only occur here when the realized future output falls below a certain level, and not in periods of economic prosperity.

In the two examples of voluntary default that were just considered, it was taken for granted that the agent's decision to default or not to default would be taken after the decision to invest had been made, and after the random variable takes on an actual value. That is, the agent chooses an optimal investment level taking into account the possibility that he might rationally default later, depending on the realized random variable. Another modelling alternative would have been to assume that the agent makes his decision to default or not to default after he receives the transfer, but before he has decided how much to invest. The expected outcome of this sequence of decisions, however, would always be inferior from the agent's viewpoint to that attained under the former sequence, and is therefore not explicitly considered. This result is due to the fact that by deciding whether or not to default after the random variable is determined, the agent has more information at his disposal at the time of decision than in the case where the decision is made beforehand. Given that the ability to postpone the default decision has no costs attached, the advantage of waiting is clear. (See, e.g., Hart (1942) and Lächler (1984) on this issue).

In summary, if defaults and expropriations are mainly motivated by narrow economic considerations, a systematic pattern of default behavior should emerge according to the preceding discussion: Debt repudiation would be more likely to occur during economic recessions, while equity expropriations would be more likely during economic booms.

The preceding analysis has also shown that penalties and the notion of sovereign risk can be introduced into the basic model in various ways. Unfortunately, without better knowledge of what constitutes the "true" model of sovereign risk, it is difficult to derive a set of general conclusions that would apply in every case. However, what has been shown is that for several specific penalty structures the basic analytic structure employed earlier remains largely unchanged. The primary consequence to the basic model is to reduce the range of permissible values of γ and B . In other words, provided that the values of (T, γ, B) do not exceed certain bounds, the same results with regard to the choice between debt and equity finance apply under the assumption of sovereign default risk as were derived without that assumption. Where the impact of this assumption is likely to be more significant is with regard to the optimal transfer amount. That subject, however, is best treated separately in another investigation.

VIII. Discussion of Related Work

The conceptual foundations of the preceding model are primarily derived from Jensen and Meckling (1976) and Stiglitz (1974). Jensen and Meckling depart from the traditional profit-maximizing paradigm by considering an owner-entrepreneur intent on attracting external capital, whose utility is a positive function of the value of the firm and of non-pecuniary benefits (perquisites). The more perquisites the manager takes, the lower is the value of the firm. In the present model, the counterparts to non-pecuniary benefits and firm value are current consumption and future output.* Another contrast is that this model considers a risk-averse, expected utility maximizer operating in a stochastic, intertemporal context, whereas the agent considered in Jensen and Meckling is implicitly risk-neutral, so that risk-sharing plays no role in their model.

Risk neutral behavior among managers is the standard assumption made in the context of the firm, and is often justified by the argument that, with well-functioning capital markets, managers have the opportunity to reduce risk more efficiently by holding their personal wealth in a diversified asset portfolio. In the context of a developing economy examined here, it appears less plausible to assume such a separation between the personal fortunes of domestic

* The agency relationship giving rise to non-cooperative equilibria in the Jensen-Meckling model initially applies only to equity finance. To generate a similar agency problem in the case of debt finance, they assume in addition that the agent has independent control over the riskiness associated with various investment projects. By raising the riskiness of projects in an unanticipated fashion, the owner-entrepreneur can raise the value of his equity at the expense of outside bondholders (see, also Merton (1974) and Galai and Masulis (1976) for an explanation of this process). This situation is not given a counterpart in the model here, first because the logic of the argument yields ambiguous results in the case of risk-averse agents, and secondly because such an argument was not needed to generate an agency problem under debt finance.

residents (internal stockholders) and the country's overall economic fate. One way to motivate risk averse behavior in this case is by assuming that capital markets within the developing economy are sufficiently imperfect to disallow risk reduction through asset diversification abroad. In this regard there is a closer parallel with the model advanced by Stiglitz (1974), who examines the risk sharing and incentive properties of alternative distribution systems in the context of landlords and tenant farmers. The choice to be made there is between renting land at a fixed fee, receiving a fixed wage or sharecropping. Renting land and sharecropping may be regarded, from the agent's viewpoint, as analogous to debt versus equity finance.

Both Jensen-Meckling and Stiglitz point out that the nature of equilibrium reached (that is, the optimal combination of distribution systems) will depend on the supervision, monitoring or bonding costs pertaining to a given environment, but they do not model these costs explicitly. This is done by Grossman and Hart (1980), who motivate the existence of debt as a bonding device. Their model is similar to Jensen and Meckling's in that the manager faces a tradeoff between raising the value of the firm and taking more perquisites. The additional assumption made, however, is that the manager receives utility only in the event that no bankruptcy takes place. This leads to a situation where, by issuing more debt and therefore raising the threat of bankruptcy, ceteris paribus, managers can communicate to prospective shareholders their willingness to perform more profitably (in the interests of principals), which raises the value of the firm and at the same time enables the managers in the end to take more perquisites.

A common feature of the models reviewed so far is that they portray a moral hazard situation, where agents respond to various incentives or disincentives. This is in contrast to the signalling models of Myers and Majluf (1984) and Ross (1977, 1978), which also provide an explanation of debt-equity ratios, but rely less on such a response behavior of agents. Rather the results obtained there are

based on an asymmetric distribution of information regarding certain exogenous characteristics of the firm. For example, agents know more about the prospective profits or the variance of profits of a firm than the current and potential principals. The agent's choice of security, to raise external funds, then conveys to the principals some of that additional information, thereby affecting the value of the firm. In a similar vein, the model by Stiglitz (1972) is based on differences in expectations among principals.

The adverse selection models of borrowing and lending behavior by Jaffee and Russell (1976) and Keeton (1979) are also based on an asymmetric distribution of information concerning certain exogenous characteristics of the borrower. For example, Jaffee and Russell make a distinction between "honest" and "dishonest" (or "lucky" and "unlucky") borrowers, where the attributes, honesty and luck, are exogenously distributed among borrowers. Lenders cannot distinguish beforehand what type of borrower is demanding credit, but know only that the proportion of honest to dishonest borrowers declines as the interest charged rises. That raises the possibility, that the market equilibrium eventually attained involves credit rationing. In moral hazard models, the difference is that the individual agent's behavior in response to various incentives (e.g. interest rates) influences his degree of luck or honesty. This difference is also pointed out explicitly by Stiglitz and Weiss (1981) when they distinguish between the interest rate as a screening device and as an incentive mechanism.

The recent literature on international lending to developing countries (as exemplified by Eaton and Gersovitz, 1981; Folkerts-Landau, 1985; Sachs, 1982; Sachs and Cohen, 1982; and surveyed by McDonald, 1982) also emphasizes credit rationing as an important feature of international credit markets in the presence of potential debt repudiation. This work provides useful insights toward understanding various new developments in the international credit transfer process; helping to explain, e.g., how the channels through which funds are transferred to LDC's have evolved in response to

institutional innovations, and changes in credit practices designed to raise the monitoring capability of creditors. Those authors, however, have mostly been concerned with debt related issues (i.e., bank loans and bond finance) without considering other (equity) forms of transfer, and main attention has centered around the total volume of funds transferred.

Feder and Regev (1975) approach the question of debt versus equity finance using a more traditional, steady-state optimal growth perspective. What distinguishes foreign direct investment and foreign loans in their model is the difference in default risk attached to each type of claim. However, in contrast to the previous papers, which model default as a rational decision by the borrower, Feder and Regev simply assume the risk of expropriation and of default to depend on different parameters (as does Hanson, 1974), giving rise to different supply functions of foreign capital. In Feder and Just (1979), a more interesting situation is portrayed, where the borrowing country can influence its creditworthiness, and thus the amount of credit received, by varying the amount of resources channeled into the exportables sector versus the domestic non-traded sector. This could also prove to be a useful suggestion for extending the model developed in this essay, which only considers one traded-goods sector.

One of the main propositions to emerge from the preceding analysis was that the investment response to debt financed transfers would be higher than the response to equity financed transfers, given that monitoring or supervision costs are equally high in both cases. As already mentioned in the introduction, early development models (e.g., Chenery and Strout, 1966; Leontief, 1965) often ignored the possibility that foreign capital transfers might affect domestic savings. Various authors (e.g., Griffin, 1971; Papanek, 1972; Weisskopf, 1972) have subsequently pointed out that standard theoretical considerations would lead one to expect an additional quantity of resources to be used in part to increase consumption and in part to augment investment; see also Haavelmo (1965). Empirical work (surveyed in Bhagwati, 1978; Fischer, 1981; Dowling and Hiemenz, 1983) has tended to support this conjecture. Both time-series and cross-section regression analyses of domestic savings or

investment have generally yielded a negative impact of foreign transfers, although the results have ranged widely. For example, Papanek (1972) states that the average impact has been to increase investment by only 0.11 to 0.77 for every dollar of inflow. Few attempts have been made, on the other hand, to examine the separate impact of alternative forms of capital transfers. Data limitations may be part of the reason for this neglect. In this regard, mainly foreign aid has been singled out (e.g., by Griffin, 1971; Griffin and Enos, 1970; and Gupta and Islam, 1983), with largely inconclusive results; or else, distinctions between several types of transfers have been drawn along arbitrary lines with little grounding in an explicit theory, so that the results obtained become correspondingly difficult to interpret. The theoretical results obtained in this essay suggest that further empirical research is needed on the subject of savings responses, with explicit attention given to the distinction between equity and debt financed transfers.

Appendix

Proof that $\int_b^{\bar{x}} U_2'(\cdot) [x \int_b^{\bar{x}} g(x) dx - \int_b^{\bar{x}} xg(x) dx] g(x) dx \leq 0$.

Let $E^C(x)$ denote $\int_b^{\bar{x}} xg(x) dx$, and let $\text{Cov}^C(x, y)$ denote $E^C(x - E^C x)(y - E^C y)$. Since $U_2(\cdot)$ is a concave function of x (that is, $U_2''(x) \leq 0$) we know that $\text{Cov}^C(x, U_2'(\cdot)) \leq 0$.

Writing out this expression,

$$\begin{aligned} \text{(A.1)} \quad \text{Cov}^C(x, U_2'(\cdot)) &= E^C(xU_2'(\cdot)) - 2E^C x E^C U_2'(\cdot) + E^C x E^C U_2'(\cdot) \int_b^{\bar{x}} g(x) dx \\ &= E^C(xU_2'(\cdot)) - E^C x E^C U_2'(\cdot) [1 + G(b)] \end{aligned}$$

where $G(b) = \int_b^{\bar{x}} g(x) dx$. Add and subtract the expression $E^C(xU_2'(\cdot))G(b)$ on the RHS of (A.1) to obtain

$$\begin{aligned} \text{(A.2)} \quad \text{Cov}^C(x, U_2'(\cdot)) &= E^C(xU_2'(\cdot))(1 - G(b)) - E^C x E^C U_2'(\cdot) \\ &\quad + G(b) [E^C(xU_2'(\cdot)) - E^C x E^C U_2'(\cdot)] \\ &= E^C(xU_2'(\cdot))(1 - G(b)) - E^C x E^C U_2'(\cdot) \\ &\quad + G(b) [\text{Cov}^C(x, U_2'(\cdot)) + G(b) E^C x E^C U_2'(\cdot)], \end{aligned}$$

where expression (A.1) is used to make the substitution. Taking the second term on the RHS of (A.2) to the left yields:

$$\begin{aligned} \text{(A.3)} \quad [1 - G(b)] \text{Cov}^C(x, U_2'(\cdot)) - (G(b))^2 E^C x E^C U_2'(\cdot) &= E^C(xU_2'(\cdot))(1 - G(b)) \\ &\quad - E^C x E^C U_2'(\cdot) \end{aligned}$$

Notice that the left hand side of (A.3) is always negative for all values of b in the interval $(0, \bar{x})$, and 0 for $b = \bar{x}$, while the right hand side of (A.3) is simply the original expression whose sign value was to be ascertained.

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