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Local public goods, adaptive migration decisions and agglomerative bias

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**Local Public Goods, Adaptive Migration Decisions
and Agglomerative Bias**

by

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1 Introduction

In this paper we examine how uncertainty might lead to an agglomerative bias in the domain of competition between jurisdictions for mobile citizens. Since Tiebout's (1956) seminal work asserting that competition between jurisdictions would lead to a first best allocation even in the presence of public goods much of the research has addressed normative questions. The models employed differ with respect to what extent and in which way the public good is congestible, making it a *local* public good, with respect to incorporating a land market, with respect to the possible entry of new communities, the mobility of the consumers and the ability of the communities' authorities to control the residence in their communities. From a first best perspective only the consequences of population (congestion) entering consumers' utility functions and the population (congestion) entering the production function are of analytical interest. All the other features taken as given in other models of local public finance should be at the discretion of the social planner (Berliant 1998).

While the focus of this paper pertains to normative questions as well we adopt a model that contains the characteristics that are considered to be indispensable for an *empirically relevant* model. Similar to the uncertainty that follows inherently from the frictions of spatial markets we study uncertainty of consumers or potential migrants with respects to the characteristics of the communities they are migrating to. This uncertainty is the result of the fact that the consumers have limited knowledge of the bundles of public services and tax policies which will prevail in the future places of residence.

The empirically important features that the underlying model of local public goods economies should contain are the following: First, since the supply of local public goods is decided upon by local governments the model should attempt to explain government behavior. It should therefore set forth an explicit and realistic political process by which local governments arrive at fiscal policy decisions. We use majority voting as the jurisdiction's decision mechanism in the actual economy. Second, since many jurisdictions use property taxes in their local tax bases the local public goods model should contain land endowments with market prices, as well as local property taxation. Third, Tiebout's insight that consumers shop between communities by moving should be incorporated in the model

by allowing free mobility of individuals or households. That is, consumers express their preferences in two kind of voting processes: voting with their feet as emphasized by the Tiebout models and voting by ballot.

These requirements have first been put forward by Rose-Ackerman (1979) and subsequently adopted by Epple et al. (1984,1993), Dunz (1985,1989), Nechyba (1994), and Konishi (1996). Rose-Ackerman proposed a model with a finite number of mobile agents, majority rule voting and property taxes on land. Land is rented from absentee landlords and assumed to be perfectly homogeneous and divisible. Consumers choose their residential locations by taking the tax-public good - bundles as given. The local public good supply is financed by tax revenue collected in each jurisdiction. Majority voting by the residents determines the level of public good provision.

Rose-Ackerman's model extended previous work on a majority voting equilibrium with an arbitrary finite number of jurisdictions in a two good economy, without a land market of Westhoff (1977, 1979): Westhoff assumed that there is a continuum of consumers who can be sorted according to their marginal rates of substitution between a public and a composite private good. The local public good in Westhoff's model is financed by a wealth tax. That Westhoff's simpler model contained a strong

agglomerative tendency was noticed but as a central weakness of the model: Since there is no land market and no congestion at all, it is difficult to explain why consumers are spread over different communities. They could enjoy a much higher level of public services at the same cost by living together. Without any form of congestion it is always advantageous to have just one jurisdiction, basically because there is no *local* public good. For the study in this chapter we accept the postulates of Rose-Ackerman for our model building.

The results of Rose-Ackerman's analysis, on the provision of local public goods with voting and a land market, were primarily negative: Equilibria were found not to exist generally. The equilibria which were identified by restricting the space of admissible preferences were shown to be unstable. As Konishi (1996) has pointed out, these negative results are due to a non-convexity in the extended budget sets under land taxes when land is assumed to be perfectly divisible. To identify stable equilibria of public good provision in a system of jurisdictions other models have been developed. These models either depend on restrictive assumptions as well or deviate from Rose-Ackerman's set of postulates for an empirically meaningful model.

Epple et al. (1984, 1993) proposed a model with a finite number of jurisdictions and a continuum of consumers with identical preferences but endowed with different amounts of the public good. This assumption, first employed by Ellickson (1973) and later by Westhoff (1977), ensures that the slopes of the indifference curves change continuously with income. That is, the agents can be ordered by their marginal rates of substitution between the private and the public good. There are no land endowments, and hence incomes are unaffected by decisions on the supply of public goods. Consumers can purchase any quantity of a homogeneous housing good given by an exogenous supply function. The housing good is taxed proportionately at a rate determined by the majority voting rule. Therefore, the agents' budgets are influenced by public decisions only to the extent that the housing good becomes relatively more expensive with higher tax rates. To circumvent the non-convexity problem of the budget set which caused the equilibrium existence problems in the Rose-Ackerman model, they assume concavity of indifference curves of indirect utility functions. This possible way out of the non-existence problem had already been discussed by Rose-Ackerman, pointing out that it is a restrictive assumption which is not even satisfied by Cobb-Douglas utility functions. The equilibria in the model of Epple et al. share the "stratification

property" of Westhoff's equilibrium results: Equilibria on the community level are characterized by individuals with incomes and corresponding marginal rates of substitution between the public and the private good in a single interval. The level of public good provision rises with community income. The stratification phenomenon is a consequence of the exogeneity of individual incomes which allows for the ordering assumption according to the marginal rates of substitution between the public and the private good¹

Yet another solution to the equilibrium existence problems which had been encountered in the analysis of Rose-Ackerman is provided by Dunz (1986, 1989). He modifies Rose-Ackerman' model by assuming indivisibility of the non-transferable good (land holdings or houses) and a wealth tax which is employed to finance the local public services. Since housing supply is fixed, the model is transformed into an assignment

¹ Epple and Platt (1992) have shown that the same results can be obtained in a model with all agents differing additionally in a taste parameter. The ordering assumption is made for both the incomes differing with the taste parameter held fixed and the taste parameter differing with the income held fixed for all individuals. The equilibria then show two dimensional stratification.

model. Since there are no absentee landlords, consumers' wealth levels vary with house prices. Instead of assuming the sorting condition for consumers' preferences, he assumes that each agent's most preferred tax rate is independent of the total wealth level in the jurisdiction. Like the sorting conditions this independence assumption is a strong assumption. If, for example, the production function of the public good is linear, only the Cobb-Douglas utility function allows for the independence. Nechyba (1994) has extended the framework of Dunz by adding a national government to the level of the local governments providing a national public good financed by a wealth tax in addition to the local public goods financed by a land tax. For the question of whether the tax competition between local jurisdictions favors agglomeration this approach is inadequate as the assumption of indivisible land implies that the populations in the individual jurisdictions are fixed. Dunz (1989) has shown that there is in general no stratification equilibrium if land ownership is included in the model. The (restrictive) additional conditions under which such a stratification equilibrium might occur are identified by Nechyba (1994).

The most general framework for proving the existence of an equilibrium of the provision of public goods in a system of multiple

jurisdictions was provided by Konishi (1996). He developed a model with perfectly divisible land and without ordering assumptions building on a model of Greenberg and Shitovitz (1988). Greenberg and Shitovitz had developed a model with many local public goods which are provided in many jurisdictions and the provision level of which is decided upon by majority voting. The existence proof of this model however relied upon the immobility of consumers. Without mobility of consumers Greenberg and Shitovitz could assume the convexity of preferences and consumption sets. In the case of consumer mobility this is unreasonable. The consumers' location choice problem leads to non-convexity of consumption sets across locations even though consumption sets in each location might be convex. To be able to apply Kakutani's fixed point theorem Konishi introduces a dummy consumer for each type to each jurisdiction. The locations of these dummy consumers are fixed. The dummy consumers' demand bundles are determined by the postulate to recover the convexity of consumption sets. To avoid integer problems (discussed by Bewley (1981)) he assumes a continuum of individuals entailing the conceptual problem that with unrestricted mobility there might be empty jurisdictions in the sense that only a zero measure of consumers is living there in equilibrium. This is a conceptual problem as it is unclear how policy packages are determined in

empty jurisdictions. The problem is solved by assuming that there are no empty jurisdictions.

In this paper we address a different conceptual problem that is common to all of the work reviewed: In most of the reviewed models it is assumed that the number of jurisdictions is very large to justify a "utility taking equilibrium". That is, whatever voting equilibrium occurs and which migration streams these might induce the utility levels prevailing in the other jurisdictions will be unaffected. Adopting this decision in order to model perfect competition between the local economies the conceptual problem of the Konishi model, i.e. the possibility of empty jurisdictions is most prevalent. With truly geographic frictions, e.g. distance dependent migration costs, the competition between jurisdictions is restricted to a limited number of jurisdictions in the same way as the competition between firms on spatial markets is restricted to a limited number of geographical neighbors. In such a case the assumption that decision makers act as if all the other jurisdiction remain unaffected by what happens in their own jurisdiction appears to be particularly difficult to maintain. If the competition is among the few a fully rational behavior by the migrants and voters requires that the voting, migration and market equilibrium is anticipated by the individuals. (cf. Scotchmer 1994 and

Starrett 1993). The immense data collection and data processing requirements associated with this situation have been discussed for local public good models without a political process. It has been argued that it is too restrictive to assume that local governments can anticipate the equilibrium distribution of types across communities as this would require that the government decision makers knew the preferences of the individuals. After all, the decentralized provision of public goods was meant to solve the problem of the revelation of the individuals' preferences for public goods. In a model with voting and free mobility of the individuals between jurisdictions each consumer would have to anticipate the equilibrium if it were to be attained instantaneously.

This problem may be reflected in the models of Rose-Ackerman and Epple et al. in that the otherwise fully rational agents are in some respects assumed to be rather naive: Calculating the effects of specific fiscal policies on the internal equilibrium the households in Rose-Ackerman's model believe that the net land rents are unaffected by the extent of property taxation and possible induced migration streams. Similarly, in the model of Epple et al. (1993) the individuals believe that the net-of-tax housing price is exogenously given. In an adjustment process towards the equilibria that have been identified to exist, to maintain such naive beliefs

experiencing changing land or house market equilibria would be a very restrictive assumption.

Here we try to take explicit account of the fact that the information collection and processing capacities of the individuals are limited. Migration decisions of households are based on a non-sophisticated way of estimating the distribution of future fiscal policy regimes across communities. The individuals use limited information to arrive at such estimates. What is more, agents are allowed to make mistakes or to have different motives of choosing their residential location than picking the optimal fiscal policy. The equilibrium that will be studied here differs from the equilibrium concept employed in the literature reviewed above. There an equilibrium is defined as the situation in which all markets clear and no individual has an incentive to change his residential location after a round of voting has taken place. Due to the possibility of the agents making "mistakes" in the equilibrium concept we study here even such a situation would be constantly in danger of being upset. What will be studied here is the stochastically stable equilibrium. The stochastically stable equilibrium will describe that configuration of types of households distributed over the localities that will be observed in the long run most of the time (Young 1998). Perhaps surprisingly, assuming an adaptive behavior on the part of

the agents and studying the stochastically stable equilibrium Tiebout's claims with respect to the outcome of the competition of jurisdictions seem more justified than in the traditional framework.

2 The model

2.1 Outline: voting and adaptive residential choice

The individuals select their residential location out of a fixed number of jurisdictions on the basis of selecting the most preferred amount of public services, financed by a property tax. The fiscal package implemented had been determined before at the beginning of the periods considered by majority voting. Only those individuals are eligible to participate in the voting process who have residence in that community. The voting equilibrium is in turn determined by past location decisions of the agents and the consequent distributions of consumer types in the individual communities. This interplay of migration and voting decisions constitutes an n -person game which is played once each period. A migration phase is preceded by voting on the fiscal policy package by the residents of a local community. The n players are drawn at random from a large finite population of N individuals.

In principle players choose optimal strategies based on their beliefs about the economic environment. This environment is held to be stationary. The beliefs are formed by looking at how individuals in the same situation have acted in the past. Since the means of collecting and processing information are limited, each player knows only a fraction of the relevant events in the past. In other words, current actions are based on samples of play from recent time periods. In some periods the individuals do not optimize in the sense of choosing the optimal reaction to the information obtained.. This may be due to the agents' making mistakes and/or experimenting.

Current actions become part of the relevant sample space in the next period when another random set of n players is drawn from the fixed total population. Each of these agents takes again a random sample of previous plays and reacts optimally to the information obtained, or with some probability makes mistakes or ventures to use different strategies. In our case the sampling does not refer directly to the migration decisions taken by individual players but to their consequences which are revealed in a possibly new set of voting equilibria in the individual communities. In any case, there is no continuous, systematic learning process over time, during which the players update the information on the voting equilibria in all

communities after each round of voting and implementation of a new voting equilibrium. Rather the information consists of fragmentary pieces of the entire history of relevant events in the past.

This pattern of behavior defines a Markov chain whose states are the histories of voting equilibria in the finite number of communities truncated to a finite number of periods.¹ The belief formation and best reply behavior with occasional mistakes is called adaptive play. As will be detailed below, adaptive play need not converge to a Nash equilibrium for general n -person games. It does, however, converge for games in which sequences of best reply choices lead from any initial choice of strategies to a strict, pure strategy Nash equilibrium. This class contains coordination games and common interest games which are of interest for our model. For these games which are called weakly acyclic games, adaptive play leads with probability one to a pure strategy Nash equilibrium, provided that information collection is sufficiently incomplete, and the players never

¹ The prototype of such an adaptive dynamics is fictitious play. In fictitious play agents react optimally with respect to the entire history of previous actions taken by other players. Originally, this was not meant to represent a learning process but considered as an algorithm to compute Nash equilibria (Brown 1951, Robinson 1951).

make mistakes. If the equilibrium is not unique it cannot be said in advance to which of the equilibria the process will converge as this depends on the random sampling process and on the initial state.

If the possibility that players make mistakes or experiment occasionally is taken into account more can be said. Then the process does not converge to an equilibrium and does not stay in an absorbing state as long as the environment remains constant. Rather, adaptive play with a positive probability of making mistakes has a stationary distribution around a particular subset of pure strategy Nash equilibria. Moreover, the process usually puts almost all the probability weight on a particular equilibrium. This equilibrium is called the stochastically stable equilibrium which will be observed almost surely when the disturbance created by the mistakes is close to zero.¹

2.2 Voting and location decisions

There is a fixed number of K communities. The number is exogenously given, K being a positive integer. An individual can be a resident of only

¹ The analytical techniques of studying perturbed Markov processes has been developed by Freidlin and Wentzell (1984). They have been first applied to general evolutionary processes by Foster and Young (1990).

one of these communities. In our model we consider three types of goods, a local public good g , a local housing good h as a non-tradable good and a composite private good c . The private good is the numéraire good. The consumption of the public good is restricted to the residents of the community where it is supplied. The level of provision of the public good is decided by majority voting. Only the residents of the community are eligible to vote. The public good is financed by an ad valorem tax on the housing good. In presenting the core of the model we assume in accordance with the Tiebout model and the Rose-Ackerman postulates that the costs of changing the residential relocation are negligible. The role of positive migration costs will be considered in the next section when the geographical structure is added.

Our equilibrium definition must comprise an intercommunity component and an intracommunity component: The intercommunity equilibrium is characterized by the fact that each community has a population of positive size, and the fact that no individual wants to move to another community, i.e. that no individual strictly prefers the bundle available in another community to the one available where he or she currently resides. The equilibrium within the community is defined by the following: Each household is in equilibrium, i.e. no household could

increase its utility by changing the bundle of housing services and the composite private good. The community is in political equilibrium, i.e. the preferences of the median voter are implemented. The budget of the community is balanced, and the aggregate supply of housing services equals its aggregate demand.

Defining different types of consumers with respect to their preferences for the public good we assume following Epple et al. (1993) that all individuals have the same preferences but different incomes and therefore differing willingness to pay for the public good in units of the composite private good. More specifically, all individuals are assumed to have a utility function U which is a continuous increasing function of (g, c, a) , with g denoting the local public good, c the private composite consumption good and a the housing good. Moreover, U is assumed to be separable in g and (c, a) , strictly quasi-concave and twice continuously differentiable in (g, c, a) over all $(g, c, a) \gg 0$. If $(g, c, a) \gg 0$, then

$$U(g, c, h) > U(0, \bar{c}, \bar{h})$$

$$U(g, c, h) > U(\bar{g}, 0, \bar{h}), \text{ for all } (\bar{g}, \bar{c}, \bar{h}) \geq 0.$$

$$U(g, c, h) > U(\bar{g}, \bar{c}, 0)$$

The individuals differ in their incomes y . In the total population of N agents we observe n different values of incomes. That is the N members of the total population are partitioned into n non-empty income classes C_1, C_2, \dots, C_n .

Each community finances the local public good with a proportional tax on the non-tradable good, a housing tax. Let p be the gross of tax price of housing. The individuals then maximize their utility $U(g,c,a)$ with respect to the private good c and the housing good a subject to the budget constraint $y \geq pa + c$. Substituting the optimal values of c and a as functions of the gross price of housing, household income and the level of local public services g we obtain the indirect utility function

$$V(g, p, y) \equiv \max_{(c,a)} U(g, c, a) \text{ s.t. } y \geq ah + c. \quad (1)$$

From this indirect utility function we derive the marginal rate of substitution M

$$M(g, c, h) \equiv \frac{\partial V(g, p, y)}{\partial g} / -\frac{\partial V(g, p, y)}{\partial p}$$

M indicates by how much the gross price of the housing good might increase, what agents are willing to pay (accompanied by an adjustment of the consumption of h relative to c) to finance a marginal increase of the

public services without affecting the indirect utility of the individual. If all agents have identical utility functions the value of the individual income determines the value of M . That is, the classes of agents C_i ($i = 1, 2, \dots, n$) are characterized by identical income levels for all members. Given the above assumptions the willingness to pay is positive for finite values of the gross price of the housing good. To have a one-to-one correspondence between the income levels and the willingness to pay it is further assumed that the willingness to pay is continuously increasing with income

$$\frac{\partial M(g, p, y)}{\partial y} > 0, \text{ for all } (x, p) \gg 0.$$

This assumption implies that the indifference curves of individuals with differing incomes cross only once, with the indifference curve of a poorer individual crossing that of a wealthier individual from above.¹

Each locality $i \in \{1, 2, \dots, K\}$ is endowed with a supply function $A_s^i(p_h^i)$ where p_h^i denotes the net-of-tax price of a unit of housing services at i . The population of community i be N^i . Let the tax rate in the community be t^i , and let p^i be the gross-of-tax price of housing in

¹ More generally the single crossing property has been used to ensure the existence of voting equilibria. Cf. Roberts (1977)

community i . The level of public services provided in community i is g^i . Let r_j^i denote the number of individuals of class j ($j = 1, 2, \dots, n$) living in community i . $\{r_1^i, r_2^i, \dots, r_n^i\}$ then describes the discrete distribution of types in community i . The tax rate in that community be t^i . The cost of providing the public services g^i is given by the cost function $c(g^i, N^i)$. With this notation we can make the equilibrium definitions more precise: Given a distribution of types in community i , p_h^i, t^i is an internal equilibrium if and only if

a. The relationship between the gross-of-tax price p^i and the net-of-tax price for housing is given by:

$$p^i = p_h^i (1 + t^i) \quad (2)$$

b. The sum of the individual demands for housing within each community is equal to the aggregate supply of the housing good.:

$$\sum_{j=1}^n a(p^i, y_j) r_j^i = A_s^i(p_h^i) \text{ where for any } y_j, a(p^i, y_j) \text{ solves the problem}$$

$$\max_a U(g^i, a, y_j - p^i h) \quad (3)$$

c. The community budget is balanced:

$$t^i p_h^i \sum_{j=1}^n a(p^i, y_j) = c(g^i, N^i) \quad (4)$$

d. The voting equilibrium is characterized by the following:

Over all $(g,t) \neq (g^i,t^i)$ on the community budget constraint it must hold that

$$t p_h^i \sum_{j=1}^n a(p^i, y_j) = c(g, N^i), \text{ and}$$

$$U(g^i, a(p^i, y_j), y_j - p^i a(p^i, y_j)) \geq U(g^i, a(p^i, y_j), y_j - p_h^i (1+t) a(p^i, y_j)) \quad (5)$$

for strictly more than half of the voters.

To define the intercommunity equilibrium we denote a set of individuals forming a community at location i as R^i , and the internal equilibrium of that community as (g^i, t^i) leaving t^i and p_h^i as implicitly defined by (7.1) to (7.4). We then have the following for the formal definition of the intercommunity equilibrium

$R^1, R^2, \dots, R^K; (g^1, p^1), \dots, (g^K, p^K)$ is an equilibrium with K distinct communities iff

$$\bigcup_{i=1}^K R^i = \{N\}, \text{ and } |R^i| > 0 \text{ for all } i = 1, 2, \dots, K; \quad (6)$$

(g^i, p^i) is an internal equilibrium in community R^i , and

every individual residing in R^i , $i \in \{1, 2, \dots, K\}$ weakly prefers (g^i, p^i) to

(g^j, p^j) , $j \neq i$, $j = 1, 2, \dots, K$.

Let $t = 1, 2, \dots$ denote successive time periods. A stage game G is played once in each period. In each period t one individual is drawn randomly from each of the classes of players C^i ($i = 1, 2, \dots, n$) and assigned to choose his or her preferred place of residence with respect to the fiscal policy implemented. We refer to the member of the class C^i who has the right to choose as player i , although the identity would be the same only by chance. All players choose a strategy $s^i(t)$ in period t ($i = 1, 2, \dots, n$), i.e. they move to some other jurisdiction or they prefer to remain in the community of their current residence. The strategy tuple of the random selection of players $s(t) = (s^1(t), s^2(t), \dots, s^n(t))$ changes the distribution of types in each of the communities and consequently the distribution of median voter preferences in each jurisdiction. Let the willingness to pay of the median voter in community R^i be denoted as M^{mi} . The median voter preferences are recorded as the K -tuples $M^m(t) = (M^{m1}(t), M^{m2}(t), \dots, M^{mk}(t))$. Each of these K -tuples will be referred to as the record of time t . The history up to time t consists of the sequences $h(t) = (M^m(1), M^m(2), \dots, M^m(t))$. These histories are anonymous, i.e. the individuals are unable to keep track of due to whose movement the median voter profile changed. That is, the stochastic process evolves on two levels. The first one is the level of actions taken by the n individuals in taking residence in one of the K

communities. These actions change directly the distribution of types over communities. The change of the distribution of consumer types, in turn, changes the voting equilibrium in the communities.. This means that underlying the sequence of K -tuples of indices of median voter preferences is a changing distribution of intra-community distributions of consumer types. The latter distribution is directly affected by the actions of the n players $s^j \in S^j$ selected from the individual action spaces. The Markov process on the state space consisting of all distributions of intra-community distributions of consumer types is mapped into a Markov process of voting equilibria, driving the development of the histories of distributions of median voter preferences.

The individuals now decide how to choose their strategies as follows: The individuals form expectations on the future fiscal policies implemented in the communities by examining the histories of policies that have been pursued there. The most distant record they might investigate is the record of the period $m - t + 1$.¹ Older records are considered irrelevant

¹ For the sake of completeness we assume that that the first m lays are randomly selected. Thus we can think of the sampling process as beginning in period $t = m + 1$ from some arbitrary initial sequence of m records $h(m)$.

or are disregarded due to limited capacities of holding information. Each player inspects these recent records by drawing randomly a sample of size k ($1 \leq k \leq m$) without replacement. The restriction of the sample might reflect a limited capacity to process information or reflect a passive attitude towards information gathering with the effect of getting to know about a limited number of historical records. The specific values of the restrictions of the length of the history to m records and the sample size to k randomly drawn records is not important. What matters is the level of completeness of the individuals measured by the ratio k/m . The feasible samples of size k must not necessarily have an equal probability weight. It suffices to assume that all feasible samples of size k have a positive probability of being drawn by each potential migrant i . We then obtain a finite Markov chain on the state space H consisting of all m -sets of K -vectors of median voter indices drawn from $\prod_{i \in K} M^{m_i}$ beginning with some arbitrary initial state $h(m)$.

The histories h are the states of the Markov chain. A successor of a state $h \in H$ is any state $h' \in H$ obtained by deleting the left-most element of h and adjoining a new right-most element. The finite Markov process moves from the current state h to a successor state h' in each period according to

the following transition rule: For each $s^i \in S^i$, $p(s^i | h)$ be the probability that agent i chooses s^i . $p(s^i | h)$ is positive if and only if there exists a sample of size k to which s^i is player i 's best response. It is also assumed that $p(s^i | h)$ is independent of t . All this defines a time-homogeneous Markov process: If s^i is the right-most element of the history h , the probability of moving from h to h' is

$$P_{hh'}^0 = \prod_{i=1}^n p(s^i | h).$$

h' is then the new K -vector of indices of median voter preferences induced by the n -vector of actions s^i . The Markov process P^0 is called adaptive play with memory m and sample size k (Young 1993a).

The next question we have to address is whether such a process converges to an absorbing state. An absorbing state would be reached if the above pattern of decisionmaking would move the process to a m -history where in all successive periods the members of all classes of individuals take identical decisions, no matter what sample they happened to draw from sequence of histories. In our context the repetition of identical actions means that agents of a particular type choose the same residential location. That is, there are no movements between communities.

This implies that only such histories can be absorbing states which consist of a strict pure Nash equilibrium being played m times in succession. Suppose that the m -set of n -vectors of actions $(s(1), s(2), \dots, s(m))$ is an absorbing state. For each agent i let s^i be the best response to some subset of k records drawn from this m -set, and let $s = (s^1, s^2, \dots, s^n)$. By assumption there is a positive probability of moving to a successor $(s(2), s(3), \dots, s(m), s)$ in one period. Since the m -set is absorbing $s(1) = s(2)$. Continuing in this way we conclude that $s(1) = s(2) = \dots = s(m) = s$. Therefore, if the m -set of vectors of actions is to be an absorbing state it has to be an m -tuple of identical s . By construction all of the elements of s are best replies. Furthermore, the best replies are unique, otherwise the process could move to a different state. Thus, s is a strict, pure-strategy Nash equilibrium. Conversely, any strict pure-strategy Nash equilibrium of the stage game which is repeated m times is an absorbing state.

The existence of a strict, pure-strategy equilibrium is, on the other hand, no sufficient condition for the convergence of the process as was shown by Shapley (1964). As has been mentioned before, a sufficient condition for the convergence of adaptive play when there is no experimentation is that the game Γ is weakly acyclic. To define a weakly acyclic game we have to

introduce a best reply graph of Γ . Each vertex of a best reply graph is an n -tuple of strategies $s \in \prod_{i=1}^n S^i$. For every two vertices s and s' there is a directed edge $s \rightarrow s'$ if and only if $s \neq s'$ and there exists exactly one agent i such that $(s^i)'$ is the best reply to the actions of all other agents and these actions are in turn best replies to $(s^i)'$. A game is now acyclic if its best reply graph contains no directed cycles. It is weakly acyclic if, from any initial vertex s there exists a directed path to some vertex s^* from which there is no exiting edge. Such a vertex s^* is called a sink. In other words, a game is weakly acyclic if and only if from every strategy tuple there exists a finite sequence of best replies by one agent at a time which ends in a sink. For this class of games adaptive play without mistakes converges with probability one, provided that sampling is sufficiently incomplete (Young 1993a, Theorem 1, see Appendix 1).

That is, the limited degree of being informed prevents that the agents never forget about past miscoordination and enables them to move towards an absorbing state. The incomplete sampling introduces stochastic variation into the players' responses. Because of this stochastic variation it is possible that players initially coordinate by chance and by doing so

often enough the process might lock in to an absorbing state which is a pure strategy equilibrium.

For our concerns, however, not all absorbing states are desirable. If, for example, each community had the same distribution of consumer types with the consequence of identical voting equilibria in all communities no individual would have an incentive to move. The identity of the voting equilibria would continue to be a convention although the sorting of individuals of different types into different communities would be Pareto-superior. (Bewley 1981)

Such a lock-in into a Pareto - inferior allocation is avoided if the agents do make mistakes or experiment with choices of residence which are not best replies to samples of the distribution of voting equilibria. In this case the stochastic process does not converge to an absorbing state, because there are no longer absorbing states. Even if the process has settled on an m -times repetition of the strategy tuple, mistakes or experiments could cause the process to leave the rest point. Assuming, however, that all mistakes can occur with positive probability and that these probabilities are time-independent, the process does have a unique stationary distribution. When the mistake probability is small, the distribution is centered around a particular absorbing state of the unperturbed process.

These are the stochastically stable states which will be observed in the long run when the perturbation of the process due to the mistake probabilities is small but non-vanishing.

To define adaptive play with perturbations we fix the sample size k and the number of records in a history m . We assume that with some small probability $\varepsilon\lambda_i > 0$ that a player of type i experiments by choosing a strategy from his or her strategy set S^i instead of choosing a best-reply with respect to the information gained by sampling. The factor ε determines the probability with which players in general experiment. The event that individual i experiments is assumed to be independent of the event that agent j experiments for every $i \neq j$. The ratio λ_i/λ_j is the relative probability with which a player i experiments as compared to a player of type j . For every player of type i let $q_i(s | h)$ to be the conditional probability that he or she chooses $s \in S^i$ given that she or he experiments and that the process is in h , where $\sum_{s \in S^i} q_i(s | h) = 1$ for every i and h . $q_i(s | h)$ is assumed to be time independent and positive for all $s \in S^i$. the latter assumption is meant to ensure in a direct way that every state is reachable from every other state in a finite number of periods by agents who experiment.

The fact that we do not have any further information on the distribution $q = (q_1(\cdot), q_2(\cdot), \dots, q_n(\cdot))$ or the relative probabilities of experimentation $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$ of the different types does not matter for the long run behavior of the stochastic process: If the overall probability of experimentation is small and the individuals experiment independently of one another, the selected stochastically stable equilibria are independent of q and λ .

Which consequences does the positive probability of mistakes have for the stochastic process? Let the stochastic process be in state h at time t . Let J be a subset of j players, $1 \leq j \leq n$. the probability that exactly the members of the subset J experiment and the others do not is

$$\varepsilon \left(\prod_{j \in J} \lambda_j \right) \prod_{j \notin J} (1 - \varepsilon \lambda_j).$$

Conditional on this event the transition probability

of moving from h to h' is

$$Q_{hh'}^j = \prod_{j \in J} q_j(s_j|h) \prod_{j \notin J} p_j(s_j|h)$$

if h' is a successor of h , s being the right-

most element of h' , and $Q_{hh'}^j = 0$ if h' is not a successor of h .

If no agent experiments, then the transition probability of moving from h to h' in one period is given by the unperturbed process $P_{hh'}^0$. The event

"no one experiments" occurs with the probability $\prod_{i=1}^n (1 - \varepsilon \lambda_i)$. The

perturbed Markov process then has the transition function

$$P_{hh'}^\varepsilon = \left(\prod_{i=1}^n (1 - \varepsilon \lambda_i) \right) P_{hh'}^0 + \sum_{J \subseteq N, J \neq \emptyset} \varepsilon^{|J|} \left(\prod_{j \in J} \lambda_j \right) \left(\prod_{j \notin J} (1 - \varepsilon \lambda_j) \right) Q_{hh'}^j \quad (7)$$

The process P^ε is called adaptive play with memory m , sample size k , experimentation probabilities $\varepsilon \lambda_i$ and experimentation distributions q_i . P^0 will be referred to as the unperturbed process.

The asymptotic behavior of process (6), when the overall probability of mistakes or experiments is close to zero, characterizes the stochastically stable equilibrium. Let h and h' be two distinct states. If P^ε is in state h at time t , there is a positive probability that all players will experiment for m periods in succession. Thus there is a positive probability that the process arrives at state h' at time $t + m$, so P^ε is irreducible¹. The process is also

¹ A state s is accessible from state s , if there is a positive probability of moving from z to z' in a finite number of periods. If both z and z' are accessible from each other they are said to communicate. States which communicate form an equivalent class of states, a communication class. A communication class is called a recurrent class if no state outside the class is accessible from any state inside it. A process is

aperiodic because the process can move from h to h in exactly m periods and also in exactly $m + 1$ periods.¹ As any other irreducible aperiodic Markov process P^e has a unique stationary distribution μ^e satisfying the equation $\mu^e P^e = \mu^e$. For a process that is irreducible and aperiodic, not only is it true that the time-average behavior of the process converges to the unique stationary distribution μ ; its position at each point in time t is also approximated by μ when t is sufficiently large. In other words, the probability that the process is in a particular state and the relative frequency with which a state has been observed after t periods converge to the same probability, independently of the initial state. Consequently, the perturbed Markov process is ergodic, i.e. independent of the starting point of the process.

called irreducible if it has exactly one recurrent class, which consists of the whole state space. Equivalently, a process is irreducible if and only if there is a positive probability of moving from any state to any other state in a finite number of periods.

- ¹ Let P be any finite Markov process on the set S , and for each state s , let N_s be the set of all integers $n \geq 1$ such that there is a positive probability of moving from s to s in exactly n periods. The process is called aperiodic if for every s , the greatest common divisor of N_s is unity.

A state $h \in H$ is stochastically stable relative to the process P^ϵ if

$$\lim_{\epsilon \rightarrow 0} \mu^\epsilon(h) > 0. \quad (8)$$

States which are not stochastically stable will be observed infrequently in the long run compared to states which are, provided that the mistake probability ϵ is small.

To discuss how the stochastically stable states are computed, we first make precise what is a mistake. Let h' be a successor of h and let s be the right-most element of h' . A mistake in the transition $h \rightarrow h'$ is a component s^i of s that is not an optimal response of by agent i to any sample of size k from h . For any two states h and h' the resistance $r(h, h')$ is the total number of mistakes involved in the transition $h \rightarrow h'$ if h' is a successor of h ; otherwise $r(h, h') = \infty$.

Now we consider all states of the entire state space H as vertices of a directed graph. For every pair of states h, h' we insert a directed edge $h \rightarrow h'$ if $r(h, h')$ is finite, and let $r(h, h')$ be its weight or resistance. The edges of zero resistance correspond to the transitions which occur under P^0 with positive probability. Let H_1, H_2, \dots, H_J be recurrent communication classes of P^0 . These disjoint classes have the following characteristics: From every state there is a path of zero resistance to at least one of the classes. Within

the recurrent classes there is a path of zero resistance from every state to every other state. Every edge exiting from a recurrent class has positive resistance.

Given any two recurrent classes H_i and H_j consider all directed paths that begin in H_i and end in H_j in the perturbed process P^ϵ . There is at least one such path, because the perturbed process is irreducible. Among all such paths we have to identify the one with the least total resistance. We denote this resistance by r_{ij} . By definition $r_{ij} \geq 0$. Computing the r_{ij} amounts to solving a shortest path problem in a directed graph. r_{ij} is independent of which vertex of H_i is the starting point and of which vertex is the end point in H_j because every two states within the same class are accessible from each other by paths of zero resistance.

We then define a directed graph \mathcal{G} as follows: There is one vertex i for each recurrent communication class H_i , and for every distinct $1 \leq i, j \leq J$ the directed edge (i,j) has resistance r_{ij} . An i -tree in \mathcal{G} is now defined as a spanning tree such that from every vertex $j \neq i$ there is a unique path directed from j to i .

For every vertex i let \mathcal{T}_i denote the set of all i -trees on \mathcal{G} . The resistance of an i -tree $T \in \mathcal{T}_i$ is the sum of the resistances of its edges:

$$r(T) = \sum_{(i,j) \in T} r_{ij} \quad (9)$$

The *stochastic potential* γ_i of a recurrent class H_i is now the least resistance among all i -trees:

$$\gamma_i = \min_{T \in \mathcal{T}_i} r(T) \quad (10)$$

Computing γ_i for a given set of weights r_{ij} is a standard problem in combinatorial optimization, known as the abhorescence problem. There exist algorithms for solving it in the order of $|J|^2$ steps (Tarjan 1977). At this point we can see that the potential function is independent of the values of the relative probabilities of making mistakes λ_j and of the probabilities q with which certain actions are taken when experimenting. We can summarize these results by stating

Proposition 1: Let Γ be an n -person game on a finite strategy space. The stochastically stable states of adaptive play with mistakes are the states contained in the recurrent communication classes of the unperturbed process, i. e. the equilibria of with rational agents, which have minimum stochastic potential. The states of minimum stochastic potential are independent of the relative probabilities with which the agents resort to actions which are not best responses and of the probabilities with which

certain actions are taken when the agents are experimenting as long as these probabilities have full support.

That is, if a game Γ is weakly acyclic and the relative sample size $k/m \leq 1/(L_\Gamma+2)$ the stochastically stable states of adaptive play is the convention or are the conventions with minimum stochastic potential.

3 Stochastic stability and the size bias in the voting and mobility model

The noise, caused by the individuals' mistakes or experimentation, the consequent perturbed stochastic processes and the equilibrium concept of stochastic stability imply that even in a stationary environment the equilibrium will be upset from time to time. In our context this means that the Nash equilibrium of the competitive process between jurisdictions might be upset, leading to a process of changing voting equilibria and new rounds of residential relocations. As long as the residential relocation is costless, the agents do not care about how often they have to change the community of residence. If we assume instead that the agents have to bear moving costs to change their residence they will care about how often they have to change their location. In other words they will care about how long

the periods of inertia will last before a phase of turbulence is caused by the pressure of the constant noise in the Markov process.

To show an agglomerative bias of the adaptive model of voting within the communities and moving between communities we have to demonstrate that the periods of stability are longer in relatively large jurisdictions. The individuals have to have additional information on the size of the communities. Taking account of positive mobility costs the agents would then have an incentive to favor large jurisdictions in taking their decisions where to locate, the decisions being guided by the local fiscal policies.

To show that relatively large communities have longer periods of inertia we turn to a technique of computing the stationary distribution of adaptive play, using a technique developed by Freidlin and Wentzell (1984).¹ More specifically, we want to determine the stationary distribution when the noise parameter ϵ is not converging to zero but small and positive. If ϵ were known precisely it is in principle possible to compute

¹ The computation of stochastically stable states as set out in the previous subsection is a special case of applying that technique.

the stationary distribution $\mu^\varepsilon(h)$. One way would be to solve the system of equations

$$\mu^\varepsilon P^\varepsilon = \mu^\varepsilon, \text{ with } \mu^\varepsilon \geq 0 \text{ and } \sum_{z \in Z} \mu^\varepsilon(z) = 1 \quad (11)$$

In our model this system has as many equations as there are conceivable distributions of median voter preferences indexed by their willingness to pay for the public good. The technique of Freidlin and Wentzell offers a route to solve for the stationary distribution at lower computational costs. Like the computation of the stochastic potential function the determination of μ^ε is based on the notion of rooted trees. It does, however, not proceed from the construction of trees with the recurrent classes as vertices but on trees which comprise all states of the process as vertices. Recall that adaptive play P^ε is an irreducible Markov process defined on a finite state space H . We consider a directed graph with the vertex set H . The edges of this graph form a h -tree, for one particular state $h \in H$, if it consists of $|H| - 1$ edges and from each vertex $h' \neq h$ there is a unique directed path from h' to h . The directed edges can be represented by ordered pairs of vertices (h, h') and we can represent a h -tree T as a subset of all ordered

pairs of neighboring vertices. Let \mathcal{T}_h be the family of all h-trees. We can then define the likelihood of a h-tree $T \in \mathcal{T}_h$ as

$$P(T) = \prod_{(h,h') \in \mathcal{T}_h} P_{hh'} \quad (12)$$

The probability $\mu^{\epsilon}(h)$ of each state h can now be determined with the help of the following lemma (cf. the simplified proof in Young 1998, Appendix, pp. 151-153).

Lemma (Freidlin and Wentzell 1984): Let P be an irreducible Markov process in a finite state space Z . Its stationary distribution μ has the property that the probability $\mu(z)$ of each state z is proportional to the sum of the likelihoods of its z -trees, that is

$$\mu(h) = v(h) / \sum_{w \in H} v(w), \text{ where } v(h) = \sum_{T \in \mathcal{T}_h} P(T).$$

We define a state z as the relative distribution of types of an individual community j . The state space of all distributions for an individual community is denoted by Z . A z -tree is then given by

$$P(Z) = \prod_{(z,z') \in Z} P_{z,z'}$$

For each z -tree we have

$$\frac{1}{R_j} \ln P(Z) = \frac{1}{R_j} \sum_{(z, z') \in Z} \ln P_{zz'}.$$

For every $z \in [0, 1]$ we can then define

$$V(z) = \int_0^1 P_u du$$

Taking logarithms we have for individual z -trees:

$$\lim_{R_j \rightarrow \infty} \frac{1}{m} \ln v^N(z) = V(z),$$

and the convergence is uniform on the vector w with elements $[0, 1]$. Let z^* be the global maximum. For every small $\delta > 0$ let $F_\delta = \{z: |z - z^*| \geq \delta\}$ and $N_{\delta/2} = \{z: |z - z^*| \leq \delta/2\}$. Then $\sup\{V(z): z \in F_\delta\} < \inf\{V(z): z \in N_{\delta/2}\}$, and

$$\lim_{R_j \rightarrow \infty} \frac{\int_{F_\delta} e^{mV(z)} dz}{\int_{N_{\delta/2}} e^{mV(z)} dz} = 0$$

That is for large populations of individual communities the probability that the distribution of types is in a very small neighborhood of the Nash equilibrium is equal to one. From this we can conclude that for not too

small mobility cost the changes of the voting equilibria will be small enough not to induce relocations when the stochastically stable equilibria are upset.

4 Conclusions

We have shown that the conjectures of the literature following the Tiebout model can be supported by a model of adaptive behavior of the economic agents. If the number of the jurisdictions is not very large, the assumption of adaptive behaviour overcomes the conceptual problems of assuming common knowledge of the game.

Adaptive play with a positive error probability avoids the results of standard equilibrium models that the inter-community equilibrium may be pareto-inferior. We show that the stochastically stable equilibria will be selected the more sharply, and that therefore the periods of inertia are the longer the larger the population of individual communities. If the agents have to incur positive mobility costs, other things equal they will therefore prefer relatively large communities in terms of the absolute size of the population.

Appendix 1

Let Γ be a weakly acyclic n -person game. For each strategy tuple s let $L(s)$ denote the shortest directed path in the best reply graph from the strategy-tuple s to a strict Nash equilibrium. L_T be the longest of all such shortest paths:

$$L_T = \max_s L(s)$$

Theorem (Young 1993a): Γ be a weakly acyclic n -person game. If the sample size k is smaller or equal to the number of records of a history m divided by the length of the longest of all shortest directed paths leading from an arbitrary strategy tuple to a Nash equilibrium plus 2 then adaptive play converges almost surely to a convention, i. e. histories which contain only identical elements.

Proof: We arbitrarily fix a sample size k and a length of a history m with $k \leq m/(L_T+2)$. To prove the theorem it is shown that there exists a positive integer M and a positive probability p such that from any state h the probability is at least p that adaptive play will converge to a convention within M periods. M and p are time- and state-independent. Hence the probability of not reaching an absorbing state after at least rM periods is at most $(1-p)^r$.

Let $h = (s(t-m+1), \dots, s(t))$ be the state in period $t \geq m$. In period $t+1$ there is a positive probability that each of the n agents samples the last k plays in h , denoted by η . There is also a positive probability that from periods $t+1$ to $t+k$ inclusive, every agent draws the sample η every time. Finally there is a positive probability that, if an agent has a choice of several bet replies to η , he will choose the same one k times in succession. Thus there is a positive probability of a run (s, s, \dots, s) from periods $t+1$ to $t+k$ inclusive.¹

Suppose that s happens to be a strict Nash equilibrium. There is a positive probability that from periods $t+k+1$ through $t+m$, each agent will sample only the last k plays, in which case the unique best response of each agent i is s . In that case they play s for $m-k$ more periods. At that point the process has reached an absorbing state, i.e. the players continue to play s forever.

In case s is not a strict Nash equilibrium, there exists a directed path s, s', \dots, s^r in the best reply graph such that s^r is a strict Nash equilibrium. The first edge on this graph is $s \rightarrow s'$. Let i be an index such that $s'_i = s_i$. That

¹ This argument requires that the agents' memory is at least $2k-1$, since otherwise they could not choose the sample η in period $t+k$.

is, if player i plays a best response to strategies the other players play in s , these other players have no reason to revise their strategies. Consider the event in which agent i samples from the run of s established in periods $t + 1$ to $t + k$, and responds by playing s'_i , while every agent $j \neq i$ draws the sample $\eta = (s(t-k+1), \dots, s(t))$. By assumption, a best response of every agent j to this sample is s_j . These events occur together with positive probability, and there is a positive probability that they occur in every period from $t + k + 1$ to $t + 2k$, assuming that $m \geq 3k - 1$. The result is a run of $s' = (s'_i, s_{-i})$ for k periods in succession.

Continuing in this way, we see that there is a positive probability of obtaining a run of s , followed by a run of s' etc., followed eventually by a run of s'' . Each run is of length k , and the run of s'' occurs from period $t + kr + 1$ to $t + kr + k$. To reach this point may require that some agent look back $kr + 2k - 1$ periods, namely, from period $t + kr + k$ to period $t - k + 1$. This is possible because of the assumption that $k \leq m/(L_T+2)$.

On this basis the process can converge to the absorbing state (s^f, s^f, \dots, s^f) by period $t + kr + m$ if each agent samples the previous k plays from periods $t + kr + k + 1$ to $t + kr + m$ inclusive.

Since $r \leq L_\Gamma$, it is established that, given an initial state h , there is a probability $p_h > 0$ of converging to an absorbing state within $M = k L_\Gamma + m$ periods. Letting $p = \min_{h \in H} p_h > 0$, it follows that from any initial state the process converges with probability at least p to an absorbing state within at most M periods.

References

- Berliant, M. (1998). Local public goods and clubs: A unified theory of first best. mimeo. Department of Economics/Washington University, St. Louis, MO.
- Bewley, T. (1981). A critique of Tiebout's theory of local public expenditures. *Econometrica* 49: 713-740.
- Dunz, K. (1985). Existence of equilibrium with local public goods and houses. Discussion paper Department of Economics 201. SUNY, Albany.
- (1986). The existence of majority rule equilibria in local public good economies. Discussion paper Department of Economics. SUNY, Albany.
- (1989). Some comments on majority rule equilibria in local public good economies. *Journal of Economic Theory* 47: 228-234.

- Ellickson, B. (1973). A generalization of the pure theory of public goods. *American Economic Review* 63: 417-432.
- Epple, D., and G. Platt (1992). Equilibrium among jurisdictions when households differ by preferences and income. Working Paper. Carnegie Mellon University.
- Epple, D., R. Filimon, and T. Romer (1984). Equilibrium among local jurisdictions: Toward an integrated treatment of voting and residential choice. *Journal of Public Economics* 24: 281-308.
- (1993). Existence of voting and housing equilibrium in a system of communities with property taxes. *Regional Science and Urban Economics* 23: 585-610.
- Foster, D.P., and P.H. Young (1990). Stochastic evolutionary game dynamics. *Theoretical Population Biology* 38: 219-232.
- Freidlin, M., and A. Wentzell (1984). *Random Perturbations of Dynamical Systems*. Berlin: Springer.
- Greenberg, J., and B. Shitovitz (1988). consistent voting rules for competitive local public good economies. *Journal of Economic Theory* 46: 223-236.
- Konishi, H. (1996). Voting with ballots and feet: Existence of equilibrium in a local public goods economy. *Journal of Economic Theory* 68: 480-509.

- Nechyba, T.J. (1994). *Fiscal federalism and local public finance: a general equilibrium approach with voting*. Rochester, New York.
- Roberts, K.W.S. (1977). Voting over income tax schedules. *Journal of Public Economics* 8: 329-340.
- Rose-Ackerman, S. (1979). Market models with local government. *Journal of Urban Economics* 6: 319-337.
- Scotchmer, S. (1994). Public goods and the invisible hand. In J. Quigley, and E. Smolensky (ed.), *Modern Public Finance*. Cambridge, Mass.
- Shapley, L.S. (1964). Some topics of two-person games. In M. Dresher, L.S. Shapley, and A.W. Tucker (ed.), *Advances in Game Theory. Annals of Mathematical Studies*, Vol. 52. Princeton.
- Starrett, D.A. (1993). On the equilibrium size and composition of communities. Technical report series No. 53. Stanford Institute for Theoretical Economics, Stanford University, Stanford, CA.
- Tarjan, R.E. (1977). Finding optimal branchings. *Networks* 7: 25-35.
- Tiebout, C. (1956). A pure theory of local public expenditures. *Journal of Political Economy* 64: 416-424.
- Westhoff, F. (1977). Existence of equilibria in economies with a local public good. *Journal of Economic Theory*, 14: 84-112.
- (1979). Policy inferences from community choice models: A critique. *Journal of Urban Economics*, 6: 535-549.

Young, H.P. (1993a). The evolution of conventions. *Econometrica* 61: 57-84.

— (1998). *Individual Strategy and Social Structure. An Evolutionary Theory of Institutions*. Princeton, N. J.: Princeton University Press.