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Price Search, Bargaining and Agglomeration

by Andreas Kopp
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1. Introduction

The main idea of this contribution is that the geographic concentration of firms selling a homogeneous product can be explained by consumers' imperfect information. Imperfectly informed consumers tend to turn to clusters of sellers because the clusters provide a superior setting for search. This advantage may outweigh higher transportation costs due to neglecting dispersed suppliers being closer to the consumers' locations. On the firms' side it may overcompensate the centrifugal forces which are central to the traditional models of spatial competition.

The general model of price determination is based on the bargaining and search model of Wolinsky (1987). That is, the price determination is the result of bilateral bargaining between producers and consumers who both have the opportunity to search for alternatives during the process of price negotiations. Delays of finding a bargaining solution imply bargaining costs on the firms' and the consumers' side. Moving to other locations in search for bargaining opportunities leads additionally to transportation costs for the consumers.

That is, in contrast to other models of spatial competition with perfect price information, referring to Hotelling's path breaking article, it is not assumed that the sellers set prices and the consumers act as price takers. These assumptions allow to side-step any bargaining problem by presuming that sellers can commit themselves to prices while consumers cannot. If, as is introduced by the usual assumption of "mill pricing", the consumers have to bear the transportation costs, firms do not act strictly rational: They could gain by demanding higher prices than originally announced after the consumers have incurred the sunk transportation costs. If commitment of the firms is impossible a bargaining approach to price determination is more appropriate, an example being the model of Bester (1989). Bester has shown that adopting a non-cooperative bargaining approach solves the notorious problem of the existence of a price equilibrium in models of spatial competition.

An analogous argument has been made with respect to search theory in general and price search in particular. Diamond (1971) noticed that if search is costly, consumers' valuation is greater than the net costs of production, firms just quote prices too buyers and
buyers just decide whether to accept this offer all firms would charge prices such that they appropriate all of the surplus of consumers' valuations over the production costs. Let \( p \) be the lowest price charged by any store, and let \( k \) be the cost of searching again. Then the firm charging \( p \) can increase its profits by increasing its price slightly; if it raises the price by less than \( k \), no customer who has turned to the firm will refuse to buy. Any one who did would have to pay the search cost \( k \) and will save less than \( k \) by searching again. That is, the slight increase of the price would not reduce the number of sales and hence increase profits. As this argument holds for all firms there is no equilibrium in which any firm charges a price less than the consumer's valuation. If, on the other hand, only consumers were able to make commitments they would get the whole surplus.

From this follows that also a complete analysis of the process of how spatial agglomerations can be due to incomplete price formation has to rely on a combination of search and bargaining theory. Wolinsky developed a model with uncertainty on product quality in a market of horizontally differentiated products. The price determination was modelled by adopting Salop's (1979) model of monopolistic competition in a horizontally differentiated industry. Apart from the fact that the model can be criticised because of the assumption that firms can commit to prices, Economides (1989) has pointed out that the model lacks a full-fledged stage of choice of varieties, but allowed only for a symmetric configuration of varieties. He showed that if the choice of varieties is completely taken account of there is no existence of any price subgame, although an equilibrium exists in the price subgame defined by a symmetric equilibrium configuration of varieties produced.

The approach chosen here thus provides a general framework which can treat a broad class of spatial oligopolistic competition models in a unifying manner (cf. Fujita 1990) without having to resort to a number of ad hoc assumptions which have been employed the ensure the existence of the price equilibrium of the Hotelling model. What is more, it does not exclude the "principle of minimum differentiation" to hold.\(^1\)

In this paper, the analysis concentrates on consumers' imperfect information in goods' prices in a market for a homogeneous good. This market serves a geographical area. Firms can be located throughout it, and the consumers residing in the area bear the transportation costs as they search for the lowest price. It will be shown that, under cer-

\(^1\) For an appreciation of the more general importance of the type of model used here cf. Diamond 1994, ch. 2.
tain conditions concerning the size of the market region and the number of consumers, there exists a price-location equilibrium with all firms agglomerating at one point. In the next section the general model of spatial competition in a search cum bargaining framework will be developed. This will be done by first presenting the time structure of the bargaining and search process, then isolating the search process in detail before looking at the equilibrium of the bargaining and search process. In the third subsection the geographical dimension is added leading to the main result of the paper.

2. Monopolistic price competition with imperfect information

2.1 Structure of the search and bargaining process

The process of information gathering on goods' prices is confined here to the simple case where information is exclusively gained by visiting firms sequentially, combining the costs of collecting goods and of gathering information. For simplicity, search is modelled in the classical way: individuals get, by bearing search costs $k$, the opportunity to take a random sample from a distribution $F()$ of economic opportunities with non-negative values over the support $[0, m^+]$. The search cost is assumed to depend on the search intensity chosen, i.e. there is a search cost function $k(s)$, which is twice differentiable, $k' > 0$, $k'' > 0$ and $k$ being bounded away from zero for all $s > 0$. In what follows the opportunity consists of striking a bargain on the difference of the valuations of the firm and the buyer. This surplus is revealed after the parties have met. The bargaining process takes place over time; the time periods are denoted by $t=0,1,2,3...$. At the beginning of each period $t$ for those parties that enter a bargaining stage a chance move determines who has the right (with probability 1/2 and independently of previous selections) to propose a partition of the surplus $m$.\(^2\) The other bargaining partner responds immediately by accepting the offer or rejecting it. If the offer is accepted, it is implemented and the game ends. Upon rejection the parties proceed into the search stage of period $t$. in this stage the parties (i=1,2) choose first their search intensities $s_i$ and incur the costs $k(s_i)$ $\Delta$, $\Delta$ denoting the length of the discrete time period. The search activities enable both bargaining partners to assess their "outside opportunities". Consequently, what the parties have to expect in case of disagreement is not fixed and given, like in axiomatic bargaining theory, but depends on the behaviour and in particular the choice of search intensities of all

\(^2\) This assumption serves to avoid the arbitrary assignment of the first mover advantage to either the firms or the consumers.
agents. Upon encountering an outside opportunity a party finds out its value and decides immediately whether or not to adopt it. As a party can participate only in one bargaining process at a time, the adoption of an outside opportunity ends the original bargaining game: The person withdraws from the bargaining and does not search for further opportunities. If both parties do not adopt their outside opportunities, they will proceed into the bargaining stage of period t+1. The bargaining partners are assumed to be utility maximisers, discounting future utilities. The distribution function of bargaining opportunities F(), the preferences as captured by the discount factors δi (i=1,2) and the search cost functions kj(s) are common knowledge. The parties, however, do not observe the search intensities of the rival and the timing and values of the outside options she or he has encountered.

To summarise the order of events of the bargaining and search process: At each time period there are four consecutive instances at which a party might have to make a decision. She or he has to decide: (1) What proposal to make (if the bargaining was resumed and the party was selected to propose); (2) whether or not to accept the partner's proposal (if the other party was selected to propose); (3) which search intensity to employ (if no agreement was reached); (4) whether or not to adopt an outside opportunity (if such has been met). Strategies in this game are sequences of decision rules which describe the party's behaviour at each time period and each of the decision points (1) to (4), conditional on that party's history up to that point. An equilibrium is obtained if the strategy of each bargaining partner is the best reply to the other agent's strategy after any possible history of the game.

2.2 Equilibrium analysis

For the equilibrium analysis it is assumed that there are N1 firms and N2 buyers. N1 and N2 are assumed to be large such that strategic interactions between agents on one side of the market can be neglected. Specifically, it is assumed that opportunities arrive as a Poisson process with a certain arrival rate s. A person searching with an intensity s for a period of length Δ in curs search costs of k(s) Δ. The Poisson assumption means that in a short time interval Δ the probability that precisely one opportunity arrives is exactly s + σ(Δ). For simplicity Δ is chosen to be so small that the σ(Δ) possibility that more than one opportunity arrives in Δ can be neglected.
Let $V^*$ denote the value of a search if the player follows an optimal strategy. The timing of the search activity is such that costs are incurred immediately, and benefits from search are received in the next period. The searcher discounts future benefits using the discount factor $\delta$. $V^*$ must then satisfy

\[ V^* = sA \cdot \delta \int_{-\infty}^{\infty} \max\{m, V^*\} dF(y) - k(s)A. \] (1)

That is, searching for outside opportunities the searcher accepts bargaining situations where the surplus $m$ implies a value greater or equal to $V^*$. Otherwise she or he continues to search. As long as this search problem is stationary, i.e. the searcher's situation in terms of the distribution function $F(\cdot)$ and the search cost function $k(s)$ is constant, this reservation value property is maintained.

Switching to a continuous time formulation and replacing the discount factor $\delta$ by the function $e^{-rA}$ with $r$ denoting the agent's instantaneous rate of time preference, we obtain the following general expression for the value of searching:

\[ V(s, x) = e^{-rA} \left\{ \left[ 1 - sA + sA \cdot F(x) \right] V(s, x) + s \Delta x \int_{x}^{\infty} ydF(y) \right\} - k(s)A \] (2)

$x$ denotes the cut-off value for the surplus at which the agent is indifferent between what has been obtained and continuing to search. In each period the probability that the next draw will show a surplus higher than $x$ and hence accepted is equal to $sA(1-F(x))$. With probability $[1-sA+sA \cdot F(x)]$ the searcher will encounter a bargaining surplus lower than $x$, reject the opportunity and be back where she or he started with an expected value of $V(s, x)$. The search cost $k(s)$ is incurred immediately independently of the realisation of the random variable.

Further simplifying by approximating $e^{-rA}$ by the linear function $1-rA$, and choosing $A$ so small that we can discard all $A^2$, we obtain as the present value of the net benefit of searching:

\[ V(s, x) = \frac{s \int_{-\infty}^{\infty} ydF(y) - k(s)}{r + s(1-F(x))}. \] (3)
Let $x^*$ be the optimal cut-off value, i.e. the value of $x$ which maximises $V(s, x)$. Then it is easy to verify (by differentiating equation (3) and setting the derivative equal to zero) that $V(s, x^*) = x^*$. We can then derive the implicit solution for $x^*$:

$$x^*(1 + r) = x^* + s \int_{x^*}^{\infty} (y - x^*) dF(y) - k(s), \quad \text{or} \quad (4)$$

$$r V(s, x^*) = s \int_{x^*}^{\infty} (y - x^*) dF(y) - k(s). \quad \text{(5)}$$

The optimal reservation value for the bargaining surplus establishes the equivalence of keeping $x^*$ now or searching again incurring the search costs and enjoying the expected improvement compared to what has been obtained up to now.

From equation (4) we can easily choose the optimal search intensity $s^*$. The $s^*$ maximizing $V(s, x^*)$ is implicitly given by

$$k'(s) = \int_{x^*}^{\infty} (y - x^*) dF(y). \quad \text{(6)}$$

Turning now to the search and bargaining process, we have the agents on both sides of the market choosing their search intensities $s_i$ ($i=1,2$), $s_i$ indicating the probability that, within a unit interval of time, the searcher initiates a contact with an agent of the opposite type. Search activity costs $k_i(s_i)$ per unit of time, $k_i$ having the properties defined above. $s_i^0$ be the probability per unit of time that the type $i$ agent is contacted as a result of the search activity on the opposite side of the market. Given the numbers of firms and consumers we have

$$s_i^0 = \frac{N_j}{N_i} s_j, \quad i \neq j = 1, 2. \quad \text{(7)}$$

Having assumed that firms as well as buyers are many, the probability that a type $i$ agent finds a partner is $(s_i + s_i^0)$ per unit of time. It is assumed that there is common rate of time preference $r$. Additionally, let the (exogenous) rate of entry of new searchers equal the (endogenous) rate of exit as agreements are made.
The equilibrium of the game is characterised by an agreement \( \{w_1(m), w_2(m)\} \) that divides the surplus of a certain match \( m \). \( w_i(m) \) (\( i=1,2 \)) denotes the actual payoff of the player \( i \). Following the discussion in the introduction, it is assumed that none of the players can make commitments. Instead it is required that they behave in a subgame-perfect way. The perfect equilibrium is derived by combining the techniques of the optimal search decision which is elaborated above with the backward-induction method of finding the perfect equilibrium of a non-cooperative bargaining model (Binmore et al. 1992). Every player chooses the reservation value \( m_i \) and the search intensity according to the above model. This implies that the searcher accepts the first opportunity whose surplus exceeds \( m_i \). Let \( V(s_i, m_i; s_i^0) \) denote the discounted present value of such a search policy. Then \( V_i \) is, as a modified version of equation (5):

\[
V_i(s_i, m_i; s_i^0) = \left( s_i + s_i^0 \right) \int_{m_i}^{m^*} \left[ w_i(y) - V_i(s_i, m_i; s_i^0) \right] dF(y) - k_i(s_i) \tag{8}
\]

Now, a player being at the stage of searching may find herself or himself in two different situations: Either the searcher is without a partner or is in a search stage of the bargaining cum search process as described under 2.1. For those who are unmatched we may define:

\[
V_i^* = V_i(s_i^*, m_i^*, s_i^0) = \max_{s_i, m_i} V_i\left(s_i, m_i; s_i^0\right). \tag{9}
\]

The choices of the optimal reservation value and the search intensity characterise optimal search by an agent who is unmatched, given \( s_i^0, w_i(m) \) and \( F() \). It is evident that a bargaining process will be initiated only if \( m > V_i^* + V_i^* \).

A searcher who looks for outside opportunities being in an ongoing bargaining process has to take a simpler decision: His reservation value is fixed at the surplus level she or he has already found. This surplus level must exceed the above \( m_i^* \), otherwise the player would have rejected the opportunity. The players already matched have only to decide on the search intensity \( s_i^{**} \) which is implicitly given by the following equation:

\[
\int_{m}^{m^*} \left[ w_i(x) - w_i(m) \right] dF(x) = k_i(s_i). \tag{10}
\]
Since \( m > m^* \), this equation implies a lower search intensity \( s_i^{**} < s_i^* \). The expected return of an optimal search in case of an agent being unmatched is then denoted by \( V_i^{**} = V_i(s_i^{**}, m; s_i^0) \).

The results of the analyses of the searches in an unmatched or matched position are then used to determine the definite bargaining agreement. Using the general results on sequential noncooperative bargaining games (Binmore et al. 1992), we have as the division of the surplus at a perfect equilibrium:

\[
 w_i(m) = \left[ \frac{m + d_i(m) - d_j(m)}{2} \right], \quad i \neq j = 1, 2, \tag{11} 
\]

where \( d_i(m) (i=1,2) \) is a weighted average of the returns from the two kinds of search:

\[
d_i(m) = \alpha_i V_i^{**} + (1 - \alpha_i) V_i^*. \tag{12}
\]

The weights \( \alpha_i \) reflect the search intensities, the discount rate, and the probability of finding a better match:

\[
\alpha_i = \frac{r + (s_i + s_i^0)(1 - F(m))}{r + (s_i + s_i^0 + s_j + s_j^0)(1 - F(m))}, \quad i \neq j = 1, 2 \tag{13}
\]

To interpret equation (11) we may now recall the fact that the limiting perfect equilibrium in a sequential bargaining game is identical to the Nash cooperative bargaining solution, provided that the disagreement points for the Nash solution are appropriately chosen. Equation (11) defines the Nash cooperative bargaining solution for dividing the sum \( m \) with the disagreement point \((d_1, d_2)\). Equation (12) defines the disagreement points to be a weighted average of \( V_i^* \) and \( V_i^{**} \) as developed above. The weights in turn depend on the instantaneous rate of time preference of the players, the search intensities chosen and the distribution function of the bargaining surplus. Additionally, the search costs determine the opportunity costs of a disagreement. The player with lower costs of finding a new partner is in an advantageous position. Hence, the bargaining result depends on both the costs and expected benefits of search as well as the chances of bargaining in a given match.
3. The geographical consequences of search in a spatial market

Turning to the geographical dimension we recall that the market under consideration is assumed to serve an inhabited area throughout which firms can locate. Consumers are characterised by their willingness to pay for the homogeneous good and their residential location. If the location of a firm differs from the residential location of the consumer, consumption will involve transportation costs. The transportation costs are assumed to be a linear function of the distance between the locations of the firm and the consumer, \( t \) denoting the unit transportation cost and \( D \) the distance. A single point of the market area can accommodate more than one firm, in fact an arbitrary number of firms. Consumers are assumed to know the number of firms at each location and use this information in planning their search.

To put forward the argument that under certain conditions price uncertainty and the search behaviour of consumers will lead to single agglomeration in the market area under consideration, it suffices to look at a very simple geographical configuration as depicted in Figure 1.

**Figure 1: Geographical configuration of \( n+1 \) firms**

There are \( n + 1 \) firms, \( n \) of them located in one geographic point denoted \( h_1 \). The remaining firm is located at \( h_2 \) at a travelling distance of \( d \) from \( h_1 \). The decision problem of the consumers is discussed in terms of the problem of one consumer who resides in \( \alpha \), at travelling distances \( D_1 \) and \( D_2 \) from \( h_1 \) and \( h_2 \), respectively. The consumer seeks a search
plan that will maximise his net expected benefit, given his expectations on the distribution of bargaining surpluses.

The search plan consists of a decision on where to start, i.e. at the agglomeration or at the isolated firm and on when to stop searching. Starting the search process at $h_2$, it will be continued only if the expected gain from continuing search covers the search costs plus the transportation costs $t_d$. This implies that the reservation value of the accepted bargaining surplus at $h_2$, $m'_i$ ($i=1,2$) will be lower than what has been developed above for a point market. Consequently, the probability of stopping the search process $[1-F(m'_i)]$ will be higher.

If the search process starts at $h_1$, the next (n-1) search acts do not entail transportation costs. That is, for the search process within the cluster $h_1$ we will have the reservation value $m$ as determined above in the general discussion of the search and bargaining process. For the n-th search there will again be a lower $m'_i$.

The reservation value $m'_i$ depends on the distance $d$ between $h_1$ and $h_2$ as well as the number of firms in the agglomeration. The more firms are located in $h_1$, the more attractive is the search process at $h_1$. The shorter the distance $d$ between the two firms' locations the higher will be $m'_i$. This follows from the fact that with a decrease in $d$ the propensity to continue search at the second location increases.

In what follows it will be shown that, depending on the distance between $h_1$ and $h_2$ and the number of firms in $h_1$, it may be advantageous for the consumers to directly move to the agglomeration even if the consumer lives closer to the single firm location and even if she or he has to pass $h_2$ on the way to the cluster. The argument holds without taking account of the fact that for a given $m$ the bargaining power of the consumers (measured by the $\alpha$, influencing the disagreement payoffs in the general expressions for the results of the search cum bargaining game) is greater in the cluster. It holds a fortiori if the differences in bargaining power are considered as well. The argument rests on a comparison of the expected transportation costs $T_1$ for the search plan starting at the single firm location $h_2$ and those, denoted $T_m$, resulting from starting the search at $h_1$. The expression for the transportation costs $T_1$ is:\footnote{The $\geq$ holds because a return to $h_2$ after having searched in $h_1$ is neglected.}
\[ T_i \geq tD_2 + (1 - F(m'_i)) tD_2 + F(m'_i) tD_1 + F(m'_i) tD, \quad (14) \]

The first term on the right hand side refers to the costs of moving from \( a \) to \( h_2 \), the second to the expected costs of moving back to \( a \) after stopping the search there, the third expression refers to the expected transportation costs of moving on to the cluster, the fourth to expected costs of the return to \( a \) from \( h_1 \).

The expression for the search plan starting at the \( n \)-firm location is:

\[ T_n < tD_1 + \left[ (1 - F(m'_i)) F(m)^{n-1} \right] tD_1 + F(m'_i) F(m)^{n-1} tD_2 + F(m'_i) F(m)^n (td + tD_1) \quad (15) \]

The first term on the right hand side indicates the costs of moving from \( a \) to the location \( h_1 \), the second term the expected costs of moving back to \( a \) after having settled for a buy in \( h_1 \), the third one those of moving on to the second location, the fourth one those of buying in \( h_2 \) and returning to the residential location and the fourth the expected costs of being disappointed in \( h_2 \) and moving back to \( h_1 \) and only then to the residential location \( a \).

Now, the consumer being located at \( a \) would directly move to the cluster if \( T_i \) is greater than \( T_n \). The algebraic expression for the difference between the expected transportation costs is:

\[ T_i - T_n > t(D_2 + d - D_1) \times \left[ (1 - F(m'_i)) F(m)_i + 1 + (1 - F(m'_i)) F(m)^{n-1} + F(m'_i) F(m)^n \right] + F(m'_i) tD_1 - F(m'_i) F(m)^{n-1} tD_2 - F(m'_i) F(m)^n tD_1 \quad (16) \]

The product in the first two rows of (16) is necessarily non-negative. It would just be zero for consumers who reside on the extension of the straight line between \( h_1 \) and \( h_2 \). If the inequality holds for the third row of the right hand side only, it holds a fortiori if we add again the product in the first two rows. If \( n \) gets very large the terms with the negative signs in the third row will get very small such that because of the positive \( F(m'_i) tD_1 \), the whole expression becomes positive. That is for a given \( d \) there is a critical \( n \) that will
lead to a $T_1$ that is greater than $T_n$. That is the consumer, also those passing $h_2$ will go directly to the agglomeration.

The greater the number of firms in the agglomeration, the larger can be the distance between $h_1$ and $h_2$, without removing the result that the consumer goes directly to the cluster. Hence there is a critical distance $d^*(n)$ compatible with the above inequality, looking only at the third row of the inequality (16). There is a number $N$ of firms which ensures the inequality even if $d$ converges to zero. From this follows that for $n>N$ there is a distance $d(n) > 0$ such that for all $d < d(n)$ the consumers go directly to the cluster. If the whole market area lies within the distance $d(n)$ from the cluster, then the number of $n+1$ firms in the agglomeration ensures that no firm can leave the cluster or no additional firm can enter the market and meet positive demand.

4. Conclusion

In this paper it is shown that consumers' imperfect information on goods prices may lead to a single agglomeration within a spatial market. Spatial competition is modelled as a search cum bargaining game with mill pricing and the consumers bearing the transportation costs. The bargaining approach to price determination avoids the problems of the existence of a unique equilibrium of spatial competition. The search process refers to the surplus to be divided by bargaining between the seller and the buyer. Conditions referring to the number of firms in an agglomeration and distances between firms' locations leading to a single cluster equilibrium are derived.
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