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## A multicountry, multisector general equilibrium model system with endogenous trade

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# Kieler Arbeitspapiere

# Kiel Working Papers

Working Paper No. 174

A Multicountry, Multisector General Equilibrium Model System with Endogenous Trade\*

by

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## 1. Introduction\*

Over the last decade there has been increasing interest shown by economists and policy makers in country-specific, economy-wide general equilibrium models. Salient features of these models are their construction around input-output systems of accounts, thus permitting a high degree of sectoral detail, and their firm basis in microeconomic theory. Such models distinguish two trading blocks, (a) the domestic economy of the specific country and, (b) a residual block encompassing the rest of the world. Trade flows, at the commodity level, between the two blocks are endogenous.

The range of policy applications of this type of model is enormous.<sup>1</sup> It can be used to analyse the macroeconomic and sectoral implications for the country concerned of economic policy shocks which;

- (i) originate in the rest of the world but to which the specific country must adjust, and
- (ii) are internal to the specific country. Examples under (i)

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<sup>1</sup> Recent work carried out within the Sonderforschungsbereich 86 (Hamburg-Kiel) "Teilprojekt 3" provides a good example of the policy applications of specific country models. Models for Chile, Colombia, India, Ivory Coast, Kenya, Mexico, South Korea and Turkey have been used to analyse the implications for such countries of a range of shocks in the world economy and the effectiveness of alternative stabilisation measures (wage and expenditure policies, exchange rate changes, changes in trade taxes and subsidies etc.) in accommodating the shocks. A selection of papers describing this work is Gerken and Vincent (1981), Dick, Gupta, Vincent and Voigt (1981), Dick, Gerken and Vincent (1982a), (1982b). Dick, Gupta Mayer and Vincent (1982a), (1982b). Dick, Gerken, Mayer and Vincent (1982) and Vincent (1982) and (1983).

include changes in world prices for commodity exports and imports, and quantitative restrictions placed on the specific country's trade with the rest of the world. Examples under (ii) include changes in commodity tariffs and export subsidies, changes in wages, changes in the exchange rate, and in the level and composition of government expenditure. Some of the changes represent internal policy responses to an exogenous shock under (i). Alternatively, they may come about solely from domestic factors.

Given however their restriction to a two block system, country specific models have nothing to say about the effects of such shocks on trade flows with, and economic activity in, other countries possessing some degree of economic integration with the specific country. To answer such questions requires that the basic, one country-rest of the world framework be extended to a  $c$  country (where  $c > 1$ )- rest of the world (world -  $c$  country) framework.

A number of multicountry general equilibrium models have been reported in the literature in recent years. Perhaps the best known of these is the IMF's multilateral exchange rate model (MERM) which distinguishes 20 countries or country groups (mainly industrialized), each producing 6 goods. Its main purpose is to provide projections of the medium term effects of exchange rate changes and divergent inflationary tendencies

among industrial countries on their trade balances.<sup>1</sup> Another industrial country oriented model is the four block (USA-Japan-EEC-rest of the world) system of Walley (1980), with each block distinguishing 33 commodities. This system was constructed primarily to study trade distortions between the blocks. In the developing country area Gunning et al (1982) have recently constructed a 12 block system consisting of 11 LDC regions and the rest of the world with each block distinguishing six sectors. The largest, in terms of country and commodity detail, of the multicountry general equilibrium models reported to date is that of Deadorff and Stern (1982). This model has a world perspective incorporating 18 industrialized and 16 developing economies each distinguishing 29 industries. However, it contains only a rudimentary treatment of the linkages between producing and consuming sectors within each of the 34 economies.

As the number of countries to be included in the multicountry system increases, data and parameter limitations, particularly those pertaining to commodity flows between the various demand categories in each country, and substitution prospects between commodities from alternative sources in different end uses, become binding. Thus in moving from the single to the multicountry framework a trade-off must be made between the number of countries

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<sup>1</sup> See Artus and McGuirk (1981) for a description of the revised version of MERM.



and the amount of input-output (IO) detail incorporated into each country model. The problem of lack of information about price responsiveness at a disaggregated level can be to some extent "overcome" by the imposition of appropriate theoretical restrictions on the parameter space.

Our aim in this paper is to describe the theory of a multi-country multisector general equilibrium model system whose specification preserves as far as is possible; (i) the considerable amount of IO detail, (ii) the numerous prospects for relative price induced substitution and (iii) the operational flexibility, now found in the most advanced of the single country models. To do this, we take as our reference point the Australian ORANI model (Dixon et al (1982)) which is the most comprehensive (in terms of its treatment of within country IO linkages and substitution prospects) and flexible (in terms of its ease of application to a range of policy shocks) of the single country models developed to date. Starting with a somewhat simplified version of the ORANI system<sup>1</sup> (which describes intersectoral linkages within one economy and commodity trade flows - exports from it to the rest of the world and imports from the rest of the world to the demand categories in which they are used -) we extend it to a c+1 block system describing intersectoral linkages between

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<sup>1</sup> The simplifications essentially involve the omission of a treatment of (i) margins and taxes on domestic flows and (ii), technical change. Both of these issues are comprehensively covered in the ORANI system.

the  $c$  countries into intermediate input and final demand categories of each country and trade flows from each of the countries to the  $c+1$  block (which depicts an aggregation of all remaining countries). Such a framework would be suitable for studying effects of internal (originating in any of the  $c$  countries) and external (to the group of  $c$  countries) shocks on macroeconomic and sectoral variables in each country and on trade flows between countries. Given its considerable attention to regionalised (country to country) flows at the IO level the system is especially applicable to a group of  $c$  countries between which the degree of economic integration is high. The most obvious example is the 10 member customs union of the EEC. Other examples include the five member countries of ASEAN, the developed countries (USA, Japan, Australia) of the so-called Pacific rim and the trans-Tasman economic integration between Australia and New Zealand.

The plan of the paper is as follows. In Section 2 we discuss the data base required for such a model system taking as an example the case where  $c$  consists of two countries. In Section 3 we work through the theoretical structure of this two country - three block system. In Section 4 we discuss the solution procedure and illustrate a typical model closure. Section 5 provides some computational examples using an artificial data base and parameter settings. Concluding remarks are contained in Section 6. The generalised model system, where the number of countries is specified as  $c$ , is presented in the Appendix.

## 2. Data Base Underlying the Model System

Figure 1 sets out schematically the IO data base required for our multicountry system in the case where the number of countries,  $c$ , = 2. The data refer to flows in a particular base year. In Figure 1 each country has  $h$  domestic industries producing  $g$  commodities. These commodities compete for domestic market share with  $g$  commodities from the other country and from the rest of the world. Thus Figure 1 contain  $2h$  industries and  $(2+1)g$  commodities. (The generalised system of  $c$  countries contains  $ch$  industries and  $(c+1)g$  commodities). Each industry also employs industry specific capital and land, labour ( $r$  occupational types), and a residual input category termed other costs. Each of the  $(2+1)g$  commodities can be used for the activities current production, capital creation, household demands and a residual other demand category in each of the two countries. In addition each country can export its  $g$  domestically produced commodities to the rest of the world.

Matrices  $\tilde{A}_{11}$  and  $\tilde{B}_{11}$  show the flows of the  $g$  commodities produced in country 1 into the  $h$  industries of country 1 for current production and capital creation respectively. Vectors  $\tilde{C}_{11}$ ,  $\tilde{D}_{11}$  and  $\tilde{E}_{13}$  show these commodity flows to households and other demands in country 1 and to exports to the rest of the world respectively. Matrices  $\tilde{A}_{21}$ ,  $\tilde{B}_{21}$  and vectors  $\tilde{C}_{21}$  and  $\tilde{D}_{21}$  contain

the flows of the  $g$  commodities imported from country 2 into the above mentioned demand categories of country 1. Similarly matrices  $\tilde{A}_{31}$ ,  $\tilde{B}_{31}$  and vectors  $\tilde{C}_{31}$  and  $\tilde{D}_{31}$  contain the corresponding flows for the  $g$  commodities imported from the rest of the world into country 1. All the above flows are valued in country 1's currency.

Matrices  $\tilde{A}_{22}$ ,  $\tilde{B}_{22}$  and vectors  $\tilde{C}_{22}$ ,  $\tilde{D}_{22}$  and  $\tilde{E}_{23}$  show the flows of the  $g$  commodities produced in country 2 for use in the demand categories of country 2 and for exports to the rest of the world. Similarly,  $\tilde{A}_{12}$ ,  $\tilde{B}_{12}$ ,  $\tilde{C}_{12}$  and  $\tilde{D}_{12}$  contain the flows to the various demand categories in country 2 of commodities imported from country 1.  $\tilde{A}_{32}$ ,  $\tilde{B}_{32}$ ,  $\tilde{C}_{32}$  and  $\tilde{D}_{32}$  contain the flows to these demand categories from commodities imported from the rest of the world. All of these flows are valued in country 2's currency.

Matrices  $\tilde{G}_1$  and  $\tilde{G}_2$  contain payments to occupational labour by industry in countries 1 and 2 respectively.  $\tilde{H}_1$  and  $\tilde{H}_2$  contain rentals on industry capital in countries 1 and 2,  $\tilde{I}_1$  and  $\tilde{I}_2$  industry land rentals and  $\tilde{J}_1$  and  $\tilde{J}_2$  inputs of other costs in countries 1 and 2. Matrices  $\tilde{K}_1$  and  $\tilde{K}_2$  show the commodity composition of industry outputs in countries 1 and 2. Vectors  $\tilde{F}_{12}$  and  $\tilde{F}_{13}$  contain respectively the negative of the duty paid on imports by country 1 from country 2 and the rest of the world respectively. Vectors  $\tilde{F}_{21}$  and  $\tilde{F}_{23}$  contain respectively the negative of the duty paid on imports by country 2 from country 1 and the rest of

Figure 1 Schematic Input-Output Data Base: Two Country-rest of the World System

Country 1					Country 2						
Current production					Current production					Duty (-)	
Capital creation					Capital creation					Between country margins	
Household consumption					Household consumption						
Other domestic					Other domestic						
Exports to rest of world					Exports to rest of world						
n					n					1	
Domestic commodities from country 1	$\tilde{A}_{11}$	$\tilde{B}_{11}$	$\tilde{C}_{11}$	$\tilde{D}_{11}$	$\tilde{E}_{13}$	$\tilde{A}_{12}$	$\tilde{B}_{12}$	$\tilde{C}_{12}$	$\tilde{D}_{12}$	$\tilde{F}_{21}$	$\tilde{L}_{21}$
Domestic commodities from country 2	$\tilde{A}_{21}$	$\tilde{B}_{21}$	$\tilde{C}_{21}$	$\tilde{D}_{21}$	$\tilde{E}_{23}$	$\tilde{A}_{22}$	$\tilde{B}_{22}$	$\tilde{C}_{22}$	$\tilde{D}_{22}$	$\tilde{F}_{12}$	$\tilde{L}_{12}$
Rest of world imports by country 1	$\tilde{A}_{31}$	$\tilde{B}_{31}$	$\tilde{C}_{31}$	$\tilde{D}_{31}$						$\tilde{F}_{13}$	Row sums = total rest of world imports by country 1 (cif)
Rest of world imports by country 2						$\tilde{A}_{32}$	$\tilde{B}_{32}$	$\tilde{C}_{32}$	$\tilde{D}_{32}$	$\tilde{F}_{23}$	Row sums = total rest of world imports by country 2 (cif)
Labour	$\tilde{G}_1$					$\tilde{G}_2$					
Capital	$\tilde{H}_1$					$\tilde{H}_2$					
Land	$\tilde{I}_1$					$\tilde{I}_2$					
Other costs	$\tilde{J}_1$					$\tilde{J}_2$					
Column sums = output of domestic industries		Column sums = industry investment expenditure	Column sums = household expenditure	Column sums = other domestic expenditure	Column sums = exports to rest of world demands	Column sums = output of domestic industries	Column sums = industry investment expenditure	Column sums = household expenditure	Column sums = other domestic expenditure	Column sums = exports to rest of world demands	

$\tilde{K}_1$	Row sums = domestic output by commodity
Column sums = output by industry	

$\tilde{K}_2$	Row sums = domestic output by commodity
Column sums = output by industry	

Rest of World Supplies

Supplying source	commodity g	
Country 1	$\tilde{p}$	Row sums = total commodity supplies to rest of world by source
Country 2		
Rest of world		
		Column sums = total supplies to rest of world by commodity

Duty (-)			
-12	1		5
-1	1		89
-1	1		92
-12	1		5

Supply- ing source	Commodity	
Country 1	93	1
Country 2	1	100
Rest of world	836	1000
	930	1101

the world. Vector  $\tilde{L}_{1,2}$  contains the margins flows on the import of goods by country 1 from country 2 (valued in country 2's currency) while vector  $\tilde{L}_{2,1}$  contains the margins flows on the import of goods by country 2 from country 1 (valued in country 1's currency). Finally, matrix  $\tilde{P}$  contains the composition, by supplying sources, of commodities consumed in the rest of the world.

In Figure 2 we present a hypothetical data base in the case where for each country  $g = h = r = 2$ . For simplicity in constructing Figure 2 we have assumed that one unit of currency in country 1 = one unit of currency in country 2 = one unit of currency in the rest of the world, for a particular year, the base year of the model.

By reference to the numbers in Figure 2 we can see that Figure 1 imposes various balancing requirements. For example, if we add down the columns of  $\tilde{A}_{11}, \tilde{A}_{21}, \tilde{A}_{31}, \tilde{G}_1, \tilde{H}_1, \tilde{I}_1$  and  $\tilde{J}_1$  we obtain the industry outputs in country 1. This is also obtainable from the column sums of  $\tilde{K}_1$ . Adding across the rows of  $\tilde{A}_{11}, \tilde{B}_{11}, \tilde{C}_{11}, \tilde{D}_{11}, \tilde{E}_{13}, \tilde{A}_{12}, \tilde{B}_{12}, \tilde{C}_{12}$  and  $\tilde{D}_{12}$  gives the commodity outputs of country 1. This is also obtainable from the row sums of  $\tilde{K}_1$ . Similarly, adding down the columns of  $\tilde{A}_{12}, \tilde{A}_{22}, \tilde{A}_{32}, \tilde{G}_2, \tilde{H}_2, \tilde{I}_2, \tilde{J}_2$  yields the industry outputs in country 2 which are also obtainable from the column sums of  $\tilde{K}_2$ . Adding across the rows of  $\tilde{A}_{21}, \tilde{B}_{21}, \tilde{C}_{21}, \tilde{D}_{21}, \tilde{A}_{22}, \tilde{B}_{22}, \tilde{C}_{22}, \tilde{D}_{22}$  and  $\tilde{E}_{23}$  gives the commodity outputs of country 2, which are also obtainable from the row sums of  $\tilde{K}_2$ .

Note that because of the accounting procedures adopted in the construction of Figure 2, in particular the valuation of between country flows in the prices paid by users in the country in which the goods are consumed, the production and demand sides, while balanced for the system as a whole, are slightly unbalanced with respect to each country. Thus GDP calculated from the sales side (household consumption expenditure + industry investment expenditure + other domestic demands + exports (fob) - imports (cif) gives;  $204 + 79 + 50 + 120 - 120 = 333$  (country 1) and  $130 + 68 + 49 + 125 - 125 = 247$  (country 2). Calculating GDP from the income side (payments to labour + capital + land + other costs + duty) gives;  $137 + 100 + 42 + 4 + 65 = 348$  (country 1) and  $94 + 45 + 57 + 4 + 62 = 262$  (country 2). That is, GDP measured from the income exceeds GDP measured from the sales side for each country by the duty plus margins on trade between the two countries. Taking the two countries together however, consumption expenditure + industry investment expenditure + other domestic demands + exports (to the rest of the world) - imports (from the rest of the world), i.e.,  $334 + 147 + 99 + 195 - 191$  equals payments to labour + capital + land + other costs + duty (on imports from the rest of the world) i.e.,  $231 + 145 + 99 + 8 + 101$ .

## 2.1 Trade Flows

Table 1 provides a summary of the trade flows of commodities 1 (C 1) and 2 (C 2) depicted in Figure 2. Both countries are in



Table 1: Trade Flows in Figure 2

Trade flow category	Value	Share of total exports/imports
<u>Country 1</u>		
<u>Exports (fob)</u>		
C 1 } to the rest of the world	93	0.775
C 2 }	1	0.008
C 1 } to country 2	22	0.183
C 2 }	4	0.034
	<u>120</u>	<u>1.000</u>
<u>Imports (cif)</u>		
C 1 } from the rest of the world	5	0.042
C 2 }	89	0.742
C 1 } from country 2	5	0.042
C 2 }	21	0.174
	<u>120</u>	<u>1.000</u>
<u>Country 2</u>		
<u>Exports (fob)</u>		
C 1 } to the rest of the world	1	0.008
C 2 }	100	0.800
C 1 } to country 1	4	0.032
C 2 }	20	0.160
	<u>125</u>	<u>1.000</u>
<u>Imports (cif)</u>		
C 1 } from the rest of the world	92	0.736
C 2 }	5	0.040
C 1 } from country 1	23	0.184
C 2 }	5	0.040
	<u>125</u>	<u>1.000</u>

overall balance on their trade accounts (exports fob-imports cif). From country 1's viewpoint it is in balance with country 2. From country 2's viewpoint it has a deficit with country 1 of 4 units. The absolute difference between the two trade positions is equal to twice the transport costs between country 1 and 2.<sup>1</sup> About 22 per cent of country 1's trade is with country 2, the remainder being with the rest of the world. For country 2, exports with country 1 represent around 19 per cent of its total exports while imports from country 1 represent about 22 per cent of its total imports.

Country 1 is a strong net exporter of good 1 to country 2 and the rest of the world and a strong net importer of good 2 from country 2 and the rest of the world. On the other hand, country 2 is a strong net exporter of good 2 to the rest of the world and country 1 and a strong net importer of good 1 from the rest of the world and from country 1.

Country 1 supplies 10 per cent of the rest of the world's consumption of good 1. Similarly, country 2 supplies 9 per cent of the rest of the world's consumption of good 2.

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<sup>1</sup> Note that for country  $n$  ( $n \in c$ ), the cif imports of good  $i$  from country  $m$  ( $m \in c$ ), equal the fob export value in country  $m$  plus the transport margins between  $m$  and  $n$ .

## 2.2 Production and Sales Structure in Each Country

Country 1's base period output of industry 1 is composed predominantly of commodity 1 while that for industry 2 is predominantly of commodity 2. Industry 1 is relatively capital intensive and industry 2 relatively labour intensive. Primary factor returns are distributed to labour (49 per cent), returns to capital (36 per cent) and returns to land (15 per cent). On the sales side GDP is comprised of household consumption expenditure (61 per cent), private investment expenditure (24 per cent), other (mainly government) expenditure (15 per cent) and trade (52 per cent).

Country 2's base period output of industry 1 is predominantly commodity 1 while that of industry 2 is predominantly commodity 2. Industry 1 is relatively labour intensive and industry 2 relatively land intensive. Aggregate primary factor returns are distributed to labour (50 per cent), capital (22 per cent) and land (28 per cent). GDP is composed of household consumption expenditure (53 per cent), private investment expenditure (27 per cent), other (mainly government) expenditure (20 per cent) and trade (51 per cent).

Before leaving this section we note that Figures 1 and 2 do not include any accounting of the trade margins required to facilitate the flow of goods and services within a particular

country. An implication of this omission, which greatly simplifies the ensuing equation system, is that domestic prices of a commodity are the same for all activities in that country. The figures do allow for the cost of transporting goods between countries (but not between each country and the rest of the world). However, no provision is made for the production of these transport services in either country.

### 3. The Theoretical Structure of the Model System

In what follows we specify the equations necessary to explain all the flows distinguished in Figure 1. The variable notation is unavoidably messy. We use lower case letters to indicate the percentage change in the corresponding upper case variables. Also used is an extensive system of superscripts and subscripts to indicate source of origin, country of end use and type of end use. For example  $X_{(is)j}^{(n)(k)}$  is used to denote the demand by using industry  $j$  for commodity  $i$  from source  $s$  for purpose  $k$  in country  $n$ . Possible values for  $k$  are 1 (current production), 2 (capital creation), 3 (household consumption) 4 (exports to the rest of the world) and 5 (other demands). There are  $c+1$  possible values for  $s$ . In our two country example ( $c = 2$ ),  $n = 1$  denotes that the commodity is produced in country 1,  $n = 2$  denotes that the commodity is produced in country 2 and  $n = 3$  denotes that the commodity is produced in the rest of the world. In the case of variables denoting primary factor prices or quantities a superscript  $P$  replaces the  $k$  superscript while sub-

scripts denote the type of primary factor. Thus for example  $(n)P$

$X_{vj}$  denotes the demand for primary factor  $v$  ( $v = 1$  aggregate labour,  $v = 2$  land,  $v = 3$  fixed capital) in industry  $j$  in country  $n$ .

### 3.1 Production Technology for Current Goods

Following Dixon et al (1982) we describe the production technology available to each of the  $h$  industries in a particular country in two parts;

- (i) the relationship between the industry's inputs and its activity level and
- (ii) the relationship between its activity level and commodity outputs.

On the input side we assume that industry production functions exhibit constant returns to scale (CRTS) and are of a three level form. At the first level we have the Leontief assumption. That is, there is no substitution between the IO commodity groups or between them and an aggregate of the primary factors (aggregate labour, capital, land) and the residual input category termed other costs. At the second level we have CRESH functions (see Hanoch (1971)) describing substitution possibilities between different sources of goods of the same commodity category where the  $c+1$  sources consist of the  $c$  countries and the rest of the world. At this level we also have CRESH functions

describing substitution possibilities between the three groups of primary factors, aggregate labour, fixed capital, agricultural land. These factors are assumed here to be country specific. They could however be modelled as being mobile between countries. This would be desirable in the case of labour if the model system was applied to the EEC for example, where labour mobility between member countries is largely unrestricted. At the third level we have CRESH functions describing substitution prospects between the  $r$  labour occupations comprising the aggregate labour category.<sup>1</sup>

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<sup>1</sup> This three level specification of production technology is that used in the ORANI system. While in the Australian context it represented a reasonable tradeoff between the desire to provide a comprehensive treatment of input substitution on the one hand and the availability of estimates of the relevant substitution parameters on the other, it might well prove restrictive in other countries. Production technologies which allowed for a greater range of substitution prospects could easily be included in instances where the available microeconomic evidence suggested they were warranted. In particular, the Leontief restriction between intermediate input categories and between intermediate inputs and primary factors could easily be relaxed in specific industries for which contradictory evidence of substitution prospects is available. In specifying an operationally "optimal" production theory it is a knowledge of substitution prospects at the industry level, rather than theory, which is the limiting factor.

On the output side we allow industries in each country to produce a combination of the  $g$  commodities where the aggregation of commodities to the industry level is described by CRETH functions (see Dixon et al (1982)). These allow us to capture the idea of imperfect transformation between commodities that constitute an industry's output according to changes in relative prices of these commodities and the ease of transformation between commodities in producing the industry's output.

### 3.2 Country Specific Demands for Inputs for Current Production

For a particular country, demand functions for the various types of inputs into current production are derived under the assumption that producers minimise their costs of producing a given output level subject to the constraints imposed by the nested production functions outlined above. That is, the typical producer in industry  $j$  in country  $n$  must choose the input level

$^{(n)}_{(1)} X_{ij}$	$i = 1, \dots, g$	"effective" intermediate inputs
$^{(n)}_P X_j$		"effective" primary inputs
$^{(n)}_{(1)} X_{(is)j}$	$i = 1, \dots, g$ $s = 1, \dots, c+1$	intermediate inputs from source $s$
$^{(n)}_P X_{v,j}$	$v = 1, 2, 3$	aggregate labour ( $v = 1$ ), fixed capital ( $v = 2$ ), land ( $v = 3$ )
$^{(n)}_P X_{1,q,j}$	$q = 1, \dots, r$	input of labour of occupation type $q$

$(n)o$   
 $X_j$

input of other costs

to minimise

$$\sum_{i=1}^g \sum_{s=1}^{c+1} P_{is}^{(n)} X_{(is)j}^{(1)} + \sum_{v=2}^3 P_{vj}^{(n)P} X_{vj}^{(n)P} + \sum_{q=1}^r P_{1,q}^{(n)P} X_{1,q,j}^{(n)} +$$

$(n)o$   
 $P_j$   $(n)o$   
 $X_j$

(1)

subject to;

$$\text{Leontief} \quad \left\{ \frac{X_{ij}^{(n)(1)}}{A_{ij}^{(n)(1)}}, \frac{X_j^{(n)P}}{A_j^{(n)P}}, \frac{X_j^{(n)o}}{A_j^{(n)o}} \right\} = \frac{(n)z_j}{z_j} \quad (2)$$

$$i = 1, \dots, g$$

$$\text{and} \quad X_{ij}^{(n)(1)} = \text{CRESH} \quad X_{(is)j}^{(n)(1)} \quad (3)$$

$$s = 1, \dots, c+1$$

$$X_j^{(n)P} = \text{CRESH} \quad X_{vj}^{(n)P} \quad (4)$$

$$v = 1, 2, 3$$

$$X_{1j}^{(n)P} = \text{CRESH} \quad X_{1,q,j}^{(n)P} \quad (5)$$

$$q = 1, \dots, r$$



where  $Z_j^{(n)}$  denotes the activity level of industry  $j$  in country  $n$  and the  $P$ 's denote the respective prices of the  $X$ 's. (From the point of view of the producer the  $Z$  and  $P$ 's are treated as exogenous).  $P_{is}^{(n)}$  is the price of good  $i$  from source  $s$  to industry  $j$  for current production in country  $n$ . In the absence of taxes and margins on commodity flows in each country this price will be the same to all end users hence the omission of the  $k$  superscript. Similarly  $P_{1,q}^{(n)}$  is the price to industry  $j$  of a unit of labour of skill type  $q$  in country  $n$ . It is sufficient here to treat occupational labour as being homogenous across industries hence the omission of a  $j$  subscript on this price variable. The  $P_{vj}^{(n)}$  ( $v = 2, 3$ ), are for country  $n$  the rental costs to industry  $j$  of capital ( $v = 2$ ) and agricultural land ( $v = 3$ ). By retaining the  $j$  subscripts we can, if required, model these factors as being industry specific.  $P_j^{(n)o}$  is the price of a unit of other costs in industry  $j$  in country  $n$ . Finally the  $A$ 's are a set of Leontief IO coefficients.  $A_{ij}^{(n)(1)}$  for example represents the minimum amount of "effective" input of good  $i$  to support a unit of current production in industry  $j$  in country  $n$ . We treat the  $A$ 's as coefficients. That is, production technology in all countries is assumed constant.<sup>1</sup>

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<sup>1</sup> The  $A$ 's could be treated as variables should it be desired to model the effects of changes in technology. See Dixon et al (1982).

The solution to the above cost minimising problem yields, in the case of our two country example, input demands of the form;

$$x_{(is)j}^{(n)(1)} = z_j^{(n)} - \sigma_{(is)j}^{(n)(1)} \left( p_{is}^{(n)} - \sum_{s=1}^3 S_{(is)j}^{(n)(1)*} p_{is}^{(n)} \right) \quad (6)$$

$$\begin{aligned} i &= 1, \dots, g \\ s &= 1, \dots, 3 \\ j &= 1, \dots, h \\ n &= 1, 2 \end{aligned}$$

$$x_{vj}^{(n)P} = z_j^{(n)} - \sigma_{vj}^{(n)} \left( p_{vj}^{(n)P} - \sum_{v=2}^3 S_{vj}^{(n)*} p_{vj}^{(n)P} - S_{1j}^{(n)*} p_1^{(n)P} \right) \quad (7)$$

$$\begin{aligned} v &= 2, 3 \\ j &= 1, \dots, h \\ n &= 1, 2 \end{aligned}$$

$$x_{1j}^{(n)P} = z_j^{(n)} - \sigma_{1j}^{(n)} \left( p_1^{(n)P} - \sum_{v=2}^3 S_{vj}^{(n)*} p_{vj}^{(n)P} - S_{1j}^{(n)*} p_1^{(n)P} \right) \quad (8)$$

$$\begin{aligned} j &= 1, \dots, h \\ n &= 1, 2 \end{aligned}$$

$$x_{1,q,j}^{(n)P} = x_{1j}^{(n)P} - \sigma_{1,q,j}^{(n)} \left( p_{1,q}^{(n)P} - \sum_{q=1}^r S_{1,q,j}^{(n)*} p_{1,q}^{(n)P} \right) \quad (9)$$

$$\begin{aligned} n &= 1, 2 \\ q &= 1, \dots, r \\ j &= 1, \dots, h \end{aligned}$$

$$x_j^{(n)O} = z_j^{(n)} \quad \begin{aligned} n &= 1, 2 \\ j &= 1, \dots, h \end{aligned} \quad (10)$$

$$\text{where } p_1^{(n)P} = \sum_{q=1}^r p_{1,q}^{(n)P} S_{1,q}^{(n)} \quad n = 1, 2. \quad (11)$$

In equation (6)  $\sigma_{(is)j}^{(n)(1)}$  is the CRESH substitution parameter for commodity  $i$  from source  $s$  used as a current input in industry

$j$  in country  $n$ . The  $S_{(is)j}^{(n)*}$  denotes the CRESH modified share<sup>1</sup> of good  $i$  from source  $s$  in the total costs of input  $i$  into industry  $j$  for current production in country  $n$ . If there are no changes in the relative prices of good  $i$  from the three sources of supply then a 1 per cent increase in  $z_j^{(n)}$  leads to a 1 per cent increase in  $X_{(is)j}^{(n)(1)}$  for each  $s$ . This reflects the CRTS assumption. If however the price of good  $i$  from say source 1 increases relative to a weighted average of the prices of good  $i$  from the three sources then producers in industry  $j$  in country  $n$  will substitute away from source 1 in favour of other sources. Hence  $X_{(i1)j}^{(n)(1)}$  will increase less rapidly than  $z_j^{(n)}$ . The strength of this substitution effect will depend on the value of the parameter  $\sigma_{(i1)j}^{(n)(1)2}$ .

Equations (7), (8) and (9) have a similar form to (6). That is, the left hand side variable is explained by a scale factor and a substitution factor. In (7) and (8)  $\sigma_{vj}^{(n)}$  is the substitution parameter for primary factor  $v$  in industry  $j$  in country  $n$  and  $S_{vj}^{*(n)}$  is the CRESH modified primary factor cost share. In (9)  $\sigma_{1,q,j}^{(n)}$  is the CRESH parameter for labour of type  $q$  in industry  $j$  and country  $n$  and  $S_{1,q,j}^{*(n)}$  is the CRESH modified share of labour of type  $q$  in the total labour costs of industry  $j$  in country  $n$ . Equations (7) and (8) imply that increases in the cost to industry  $j$  in country  $n$  of any primary factor relative to a weighted average of the prices of the three factors leads to substitution away from that factor.

<sup>1</sup> The relationship between a CRESH modified share ( $S^*$ ) and a conventional share ( $S$ ) is given by;

$$S^* = S \sigma_s / \sum_{s=1}^3 \sigma_s S \quad \text{where } \sigma_s \text{ is the CRESH substitution parameter for the good from source } s.$$

<sup>2</sup> The restrictions on the CRESH function (see Dixon et al (1982) Section 11) ensure that  $\sigma_{(is)j}^{(n)(1)}$ ,  $\sigma_{vj}^{(n)}$  and  $\sigma_{1,q,j}^{(n)}$  will be positive.

Similarly (9) indicates that if there is no change in the relative prices of the different types of labour then the occupational composition of industry j's workforce in country n will remain unchanged. However if the price of one type of labour increases relative to a weighted average of all the occupational wage rates payable by that industry then j's use of this type of labour will increase more slowly than j's use of labour in general.

Equation (10) depicts a Leontief relationship between other costs and output in all country industries while (11) expresses the price of labour in general in country n as a share weighted average of the prices of each of the labour occupations in country n, where the weights  $S_{1,q}$  represent the cost share of labour of type q in country n's total labour costs.

### 3.3 Commodity Supplies

Commodity supply equations for country n are derived assuming that at any given activity level  $z_j^{(n)}$  producers in industry j choose the commodity output combination to maximise their revenue. That is, we assume that for each industry j

$$X_{(in)j}^{(n)} \quad i = 1, \dots, g$$

are chosen to maximise

$$\sum_{i=1}^g P_{in}^{(n)} X_{(in)j}^{(n)}$$

subject to

$$\text{CRETH} \quad X_{(in)j}^{(n)} = Z_j^{(n)} \quad i = 1, \dots, g \quad (12)$$

where the P's and Z are treated as exogenous to producers. The solution to the above revenue maximising problem yields, for our two country example, supply equations of the form;

$$x_{(in)j}^{(n)} = z_j^{(n)} + \sigma_{(in)j}^{(n)T} \left( p_{in}^{(n)} - \frac{g}{\sum_{i=1}^g C_{(in)j}^{(n)*}} p_{in}^{(n)} \right) \quad (13)$$

$$\begin{aligned} i &= 1, \dots, g \\ j &= 1, \dots, h \\ n &= 1, 2. \end{aligned}$$

Equation (13) relates industry j's supplies of commodities in country n to the industry's overall activity level and to the relative prices of the g commodities produced by that industry. If there are no relative commodity price changes then a one per cent increase in industry j's activity level generates a 1 per cent increase in the supplies of the g commodities it produces. If however the price of domestic commodity i increases relative to a weighted average of the prices of all the commodities produced by j then j transforms the commodity composition of its output in favour of commodity i. The strength of this transformation effect is governed by the transformation parameter  $\sigma_{(in)j}^{(n)T}$ .<sup>1</sup> The  $C_{(in)j}^{(n)*}$  are the CRETH modified revenue shares of commodity i in the total commodity revenue of industry j for country n.<sup>2</sup>

While (13) allows each domestic industry to produce each of the g domestic commodities, in implementing the model system it is likely that multiproduct production would be confined to a few industries. Thus  $C_{(in)j}^{(n)}$  for most i and j will be zero in each n.

<sup>1</sup> The restrictions placed on CRETH (see Dixon et al (1982) Section 11) ensure that  $\sigma_{(in)j}^{(n)T}$  will be positive.

<sup>2</sup>  $C_{(in)j}^{(n)*} = \frac{C_{(in)j}^{(n)} \sigma_{(in)j}^{(n)T}}{\sum_{i=1}^g C_{(in)j}^{(n)} \sigma_{(in)j}^{(n)T}}$  where

$C_{(in)j}^{(n)}$  is the revenue share of domestic commodity i in industry j's output in country n.

### 3.4 Demands for Inputs for the Production of Fixed Capital

We assume that a unit of capital for use in industry  $j$  in country  $n$  can be created according to the production function

$$Y_j^{(n)} = \text{Leontief} \left\{ \begin{array}{c} X_{ij}^{(n)(2)} \\ A_{ij}^{(n)(2)} \end{array} \right\} \quad (14)$$

$i = 1, \dots, g$

where

$$X_{(ij)}^{(n)(2)} = \text{CRESH} \quad X_{(is)j}^{(n)} \quad (15)$$

$s = 1, \dots, c+1.$

$Y_j^{(n)}$  is the number of units of capital created for industry  $j$  in country  $n$ ,  $X_{ij}^{(n)(2)}$  is the effective input of good  $i$  into industry  $j$  for capital creation in country  $n$ , the  $A$ 's are a set of Leontief IO coefficients, and  $X_{(is)j}^{(n)}$  is the input of good  $i$  from source  $s$  for the production of capital for industry  $j$  in country  $n$ . We assume that producers of capital for industry  $j$  treat input prices as beyond their control and for any given level of capital creation  $Y_j^{(n)}$  they choose  $X_{(is)j}^{(n)}$  to minimise;

$$\sum_{s=1}^{c+1} \sum_{i=1}^g P_{is}^{(n)} X_{(is)j}^{(n)(2)} \quad \text{subject to (14) and (15).}$$

The solution yields, in the case of our two country system, a set of demand functions for capital creation of the form

$$X_{(is)j}^{(n)(2)} = Y_j^{(n)} - \sigma_{(is)j}^{(n)} \left( P_{is}^{(n)} - \sum_{s=1}^3 S_{(is)j}^{(n)(2)*} P_{is}^{(n)} \right) \quad (16)$$

$$\begin{aligned} i &= 1, \dots, g \\ s &= 1, \dots, 3 \\ j &= 1, \dots, h \\ n &= 1, 2 \end{aligned}$$

where  $S_{(is)j}^{(n)(2)*}$  is the CRESH modified share of good  $i$  from source  $s$  in the total cost of good  $i$  used for creation of capital in industry  $j$  in country  $n$ , and  $\sigma_{(is)j}^{(n)(2)}$  is the CRESH substitution parameter between the various sources of good  $i$  as inputs to capital creation of type  $j$  in country  $n$ . The interpretation of (16) is similar to that of (6).

### 3.5 Household Demands

For each country we specify one type of household whose demands are explained by the conventional utility maximising framework. Letting  $Q^{(n)}$  be the number of households in country  $n$  we assume that the consumption bundle of effective inputs  $(X_i^{(n)(3)} / Q^{(n)})$  for the typical household is chosen to maximise household utility  $U(X_i^{(n)(3)} / Q^{(n)})$

subject to

$$X_i^{(n)(3)} = \text{CRESH} \sum_{s=1, \dots, c+1} X_{is}^{(n)(3)} \quad \text{and} \quad (17)$$

$$\sum_{s=1}^{c+1} \sum_{i=1}^g P_{is}^{(n)} X_{is}^{(n)(3)} = C^{(n)} \quad (18)$$

where  $X_{is}^{(n)(3)}$  is the demand for good  $i$  from source  $s$  to consumers in country  $n$  and  $C^{(n)}$  is country  $n$ 's aggregate consumer budget. The solution to the above utility maximising problem yields, in the case of our two country system, consumer demand functions of the form

$$X_{is}^{(n)(3)} = X_i^{(n)(3)} - \sigma_{is}^{(n)(3)} \left( P_{is}^{(n)} - \sum_{s=1}^3 S_{is}^{(n)(3)*} P_{is}^{(n)} \right) \quad (19)$$

$$\begin{aligned} i &= 1, \dots, g \\ s &= 1, \dots, 3 \\ n &= 1, 2, \end{aligned}$$

$$x_i^{(n)(3)} - q = \epsilon_i^{(n)} (c - q) + \sum_{k=1}^g \eta_{ik}^{(n)} p_k^{(n)(3)} \quad (20)$$

$i = 1, \dots, g$   
 $n = 1, 2$

where

$$p_k^{(n)(3)} = \sum_{s=1}^3 s_{ks}^{(n)(3)} p_{ks}^{(n)} \quad \begin{matrix} k = 1, \dots, g \\ n = 1, 2 \end{matrix} \quad (21)$$

$\sigma_{is}^{(n)(3)}$  is the CRESH substitution parameter between alternative sources of good  $i$  in consumption in country  $n$ ,  $s_{is}^{(n)(3)}$  and  $s_{is}^{(n)(3)*}$  are respectively the ordinary and CRESH modified shares of total consumer spending on good  $i$  in country  $n$  which is devoted to good  $i$  from source  $s$ ,  $p_k^{(n)(3)}$  is the percentage change in the price of composite good  $k$  in consumption in country  $n$  and the  $\epsilon_i^{(n)}$  and  $\eta_{ik}^{(n)}$  are expenditure and own and cross price elasticities of consumption in country  $n$  respectively.

### 3.6 Rest of the World Demands for Exports from the $c$ Countries

We assume that the rest of the world purchasers of the products of the  $c$  countries are able, in the pursuit of cost minimisation, to substitute between the  $c+1$  alternative sources of supply of a particular product category. The constrained cost minimisation problem facing rest of the world purchasers of good  $i$  is of the form,

$$\begin{aligned} & \text{choose} && x_{is}^{(s)(4)} && s = 1, \dots, c+1 \\ & \text{to minimise} && p_{is}^{(4)} x_{is}^{(s)(4)} \\ & \text{subject to} && x_i^{(4)} = \text{CRESH } x_{is}^{(s)(4)} && s = 1, \dots, c+1 \end{aligned} \quad (22)$$



where  $X_{is}^{(s)(4)}$  denotes rest of the world demand for good  $i$  from source  $s$  ( $s = 1, \dots, c$  representing the  $c$  supplying countries and  $s = c+1$  representing the rest of the world),  $p_{is}^{(4)}$  is the price in the rest of the world of good  $i$  from source  $s$  and  $X_i^{(4)}$  denotes the demand for good  $i$  in the rest of the world undifferentiated by source.

The solution of the above cost minimisation problem yields rest of the world demand functions of the form;

$$X_{is}^{(s)(4)} = X_i^{(4)} - \sigma_{is}^{(4)} \left( p_{is}^{(s)(4)} - \sum_{s=1}^{c+1} S_{is}^{(s)(4)*} p_{is}^{(s)(4)} \right) \quad (23)$$

$$i = 1, \dots, g$$

$$s = 1, \dots, c+1$$

where  $\sigma_{is}^{(4)}$  is the CRESH substitution parameter in the rest of the world for each of the  $c+1$  sources of good  $i$ ,  $S_{is}^{(s)(4)*}$  is the CRESH modified share of the total purchases of good  $i$  in the rest of the world represented by purchases of good  $i$  from source  $s$  and the lower case variables represent percentage changes in the respective upper case variables. The interpretation of (23) is as follows. If there are no changes in the relative prices between alternative sources of supply of good  $i$  in the rest of the world then demands for good  $i$  from source  $s$  in the rest of the world will increase proportionately with increases in the demand for good  $i$  in general in the rest of the world. If however the price of good  $i$  in the rest of the world from source  $s$  increases relative to a weighted average of the prices of good  $i$  in the rest of the world from the  $c+1$  sources then the rest of the world

demand for good  $i$  from source  $s$  will increase more slowly than the rest of the world demand for good  $i$  in general.<sup>1</sup>

To consider the operational significance of (23) we refer back to our two country - rest of the world example. From (23) we have that;

$$\begin{aligned} (1)(4) \quad x_{i1} = & \boxed{x_i^{(4)} + \sigma_{i1}^{(4)} S_{i3}^{(4)} P_{i3}^{(3)(4)*} (3)(4)} - \sigma_{i1}^{(4)} (1 - S_{i1}^{(1)(4)*}) P_{i1}^{(1)(4)} \\ & + \sigma_{i1}^{(4)} S_{i2}^{(2)(4)*} (2)(4) P_{i2}^{(2)(4)} \quad i = 1, \dots, g \quad (23A) \end{aligned}$$

$$\begin{aligned} (2)(4) \quad x_{i2} = & \boxed{x_i^{(4)} + \sigma_{i2}^{(4)} S_{i3}^{(4)} P_{i3}^{(3)(4)*} (3)(4)} - \sigma_{i2}^{(4)} (1 - S_{i2}^{(2)(4)*} (2)(4)) P_{i2}^{(2)(4)} \\ & + \sigma_{i2}^{(4)} S_{i1}^{(1)(4)*} (1)(4) P_{i1}^{(1)(4)} \quad i = 1, \dots, g \quad (23B) \end{aligned}$$

$$\begin{aligned} (3)(4) \quad x_{i3} = & \boxed{x_i^{(4)} - \sigma_{i3}^{(4)} (1 - S_{i3}^{(3)(4)*} (3)(4)) P_{i3}^{(3)(4)}} + \sigma_{i3}^{(4)} S_{i1}^{(1)(4)*} (1)(4) P_{i1}^{(1)(4)} \\ & + \sigma_{i3}^{(4)} S_{i2}^{(2)(4)*} (2)(4) P_{i2}^{(2)(4)} \quad i = 1, \dots, g. \quad (23C) \end{aligned}$$

Equations (23A) and (23B) describe rest of the world demands for good  $i$  exported from countries 1 and 2 respectively. Equation (23C) is of lesser interest, referring to rest of the world demands for good  $i$  produced in the rest of the world. In each equation, the terms on the RHS enclosed in the rectangle are exogenous to the system. They may be thought of as representing a demand shift factor for good  $i$  in the rest of the world. The remaining two terms in each equation describe the own and cross

<sup>1</sup> The restrictions on the CRESH function (see Dixon et al (1982) Section 11), ensure that  $\sigma_{is}^{(4)}$  will be positive.

price responsiveness in the rest of the world of good  $i$  exported from country 1 and 2. That is, they illustrate the path by which, if for example the price of cars from country 1 in the rest of the world increases relative to the price of cars from country 2 in the rest of the world, exports of cars from country 1 to the rest of the world will decrease relative to exports of cars from country 2 to the rest of the world.

### 3.7 Other Demands

This is a residual category consisting mainly of government demands. These may be satisfied by any of the  $c+1$  sources of supply. At this stage, no formal theory is presented. We simply write that

$$\begin{aligned} x_{is}^{(n)(5)} &= c_R^{(n)} h_{is}^{(n)(5)} + f_{is}^{(n)(5)} & i &= 1, \dots, g \\ & & s &= 1, \dots, 3 \\ & & n &= 1, 2 \end{aligned} \quad (24)$$

where  $x_{is}^{(n)(5)}$  denotes other demands in country  $n$  for good  $i$  from source  $s$ ,  $c_R^{(n)}$  is the percentage change in aggregate real consumption expenditure in country  $n$ ,  $h_{is}^{(n)(5)}$  is a parameter and  $f_{is}^{(n)(5)}$  is a shift variable. If for example the  $h_{is}^{(n)(5)}$  were set to one and the  $f_{is}^{(n)(5)}$  to zero then the vector of other demands in country  $n$  would remain a constant share of country  $n$ 's aggregate real consumption. We define  $c_R^{(n)}$  as

$$c_R^{(n)} = c^{(n)} - \epsilon^{(n)(3)} \quad n = 1, 2 \quad (25)$$

where  $c^{(n)}$  is the percentage change in aggregate consumption expen-

diture in money terms in country  $n$  and  $\epsilon^{(n)(3)}$  is an appropriately constructed index of consumer goods prices in country  $n$ .  $\epsilon^{(n)(3)}$

is in turn defined by

$$\epsilon^{(n)(3)} = \sum_{i=1}^g \sum_{s=1}^3 W_{is}^{(n)(3)} P_{is}^{(n)} \quad n = 1, 2 \quad (26)$$

where  $W_{is}^{(n)(3)}$  represents the share of aggregate consumer spending in country  $n$  devoted to good  $i$  from source  $s$ .

### 3.8 The Price System

Because of the absence of margins and taxes on domestic commodity flows there is only one set of prices for goods in each country. However, prices for goods of the same commodity category differ between the  $c$  countries according to transport margins incurred in moving goods from one country to another.

We assume for each country that there are no pure profits in<sup>1</sup> (i) the production of current goods, (ii) the production of capital goods, (iii) importing from other countries in the group of  $c$  and the rest of the world, (iv) exporting to the rest of the world and (v) exchange rate movements between countries.

Hence we can write, in the case of our two country system that,

Domestic Production

$$\begin{aligned} \sum_{i=1}^g P_{in}^{(n)} X_{(in)}^{(n)} &= \sum_{i=1}^g \sum_{s=1}^3 X_{(is)}^{(n)(1)} P_{is}^{(n)} + \sum_{q=1}^r P_{1,q}^{(n)} X_{1,q,j}^{(n)} \\ &+ \sum_{v=2}^3 P_{vj}^{(n)} X_{vj}^{(n)} + P_j^{(n)o} X_j^{(n)o} \quad \begin{matrix} j = 1, \dots, h \\ n = 1, 2. \end{matrix} \end{aligned} \quad (27)$$

<sup>1</sup> That is, profits must accrue to factors of production. This follows from the assumptions of CRTS and competitive behaviour.

The left hand side of (27) is the value of the output of industry  $j$  in country  $n$  and the right hand side is the total payment for inputs (intermediate input costs, labour costs, capital plus land costs, other costs). In percentage change form (27) becomes

$$\begin{aligned} \sum_{i=1}^g P_{in}^{(n)} C_{(in)j}^{(n)} &= \sum_{i=1}^g \sum_{s=1}^3 P_{is}^{(n)} H_{(is)j}^{(n)(1)} + \sum_{q=1}^r P_{1,q}^{(n)} H_{1,q,j}^{(n)} \\ &+ \sum_{v=2}^3 P_{vj}^{(n)} H_{vj}^{(n)} + P_j^{(n)o} H_j^{(n)o} \quad \begin{matrix} j = 1, \dots, h \\ n = 1, 2 \end{matrix} \end{aligned} \quad (28)$$

where the  $C$ 's are revenue shares and the  $H$ 's are cost shares. Thus  $C_{(in)j}^{(n)}$  is the revenue share of commodity  $i$  in the output of industry  $j$  in country  $n$  while  $H_{(is)j}^{(n)(1)}$  for example is the share of industry  $j$ 's current production costs in country  $n$  accounted for by the cost of its inputs of good  $i$  from source  $s$ .

#### Capital creation

$$\Pi_j^{(n)} Y_j^{(n)} = \sum_{i=1}^g \sum_{s=1}^3 P_{is}^{(n)} X_{(is)j}^{(n)(2)} \quad \begin{matrix} j = 1, \dots, h \\ n = 1, 2 \end{matrix} \quad (29)$$

In (29)  $\Pi_j^{(n)}$  is the cost of producing a unit of capital for industry  $j$  in country  $n$ . Equation (29) imposes the condition that the value of new capital in industry  $j$  in country  $n$  equals the cost of its production. In percentage change form (29) becomes

$$\pi_j^{(n)} = \sum_{i=1}^g \sum_{s=1}^3 P_{is}^{(n)} H_{(is)j}^{(n)(2)} \quad \begin{matrix} j = 1, \dots, h \\ n = 1, 2 \end{matrix} \quad (30)$$

where  $H_{(is)j}^{(n)(2)}$  is for country  $n$  the share of good  $i$  from source  $s$  in industry  $j$ 's total purchases of good  $i$  for inputs to capital creation.

## Importing

In the multicountry system each country can import goods from  $c$  sources, comprising the other  $c-1$  countries included in  $c$ , and the rest of the world. Hence we write that;

$$\begin{aligned} P_{is}^{(n)} &= P_{is}^{(n)*} \phi_{n,s} T_{i(n,s)} & n &= 1, \dots, c \\ & & s &= 1, \dots, c+1 \\ & & s &\neq n \\ & & i &= 1, \dots, g. \end{aligned} \quad (31)$$

Equation (31) equates for country  $n$  the domestic selling price of good  $i$  (LHS) to the cost of importing (RHS). In (31)  $P_{is}^{(n)}$  is the domestic price in country  $n$  of commodity  $i$  from other country or rest of the world source  $s$ ,  $P_{is}^{(n)*}$  is the cif import price for country  $n$  of good  $i$  from source  $s$  valued in source  $s$ 's currency,  $\phi_{n,s}$  is the exchange rate (currency  $n$ /currency  $s$ ) between country  $n$  and other country or rest of the world source  $s$  and  $T_{i(n,s)}$  is 1 plus the ad valorem tariff levied on commodity  $i$  from source  $s$  by country  $n$ . We interpret  $T_{i(n,s)}$  to include the tariff equivalent of other forms of protection in addition to tariffs. Thus it represents the extent to which the domestic price of  $i$  in country  $n$  imported from source  $s$  is raised by protection granted the corresponding domestically produced commodity in country  $n$ . The inclusion of the  $s$  subscript on  $T_{i(n,s)}$  allows the model to capture any discriminating protection policy practised by the countries towards alternative foreign supply sources. Thus for example country  $m$  might allow imports of good category  $i$  from country  $n$  to come in duty free while levying a heavy tariff on good  $i$  category supplied from the rest of the world. In a genuine customs union the  $T_{i(n,m)}$  ( $n, m = 1, \dots, c; n \neq m$ ) would be unity. The  $T_{i(n,w)}$  ( $w$  indicates rest of the world sources) would be adjusted to a common value for each  $n$ .

Expressing (31) in percentage change form gives, for our two country system,

$$\begin{aligned} {}^{(n)}P_{is} &= {}^{(n)*}P_{is} + \phi_{n,s} + t_{i(n,s)} & n &= 1, 2 \\ & & s &= 1, \dots, 3 \\ & & s &\neq n \\ & & i &= 1, \dots, g. \end{aligned} \quad (32)$$

Note that the  $n$  superscript on  ${}^{(n)*}P_{i3}$  in (32) makes provision for the foreign (rest of the world) cif import prices of goods to differ between importing countries. As is discussed in Section 3, the  ${}^{(n)*}P_{i3}$  are treated as exogenous. For countries within  $c$  which are geographic neighbours it might be appropriate to impose the restriction that the  ${}^{(n)*}P_{i3}$  are the same for all  $n$ .

The next step is to link the cif import prices facing an importing country with the domestic price in the country in which the good is produced and the costs incurred in transporting the good from its country of production to the importing country. We write that;

$$\begin{aligned} {}^{(n)*}P_{im} &= {}^{(m)}P_{im} + A_{i(m,n)} P_{i(m,n)}^T & i &= 1, \dots, g \\ & & n &= 1, \dots, c \\ & & m &= 1, \dots, c \\ & & m &\neq n. \end{aligned} \quad (33)$$

Equation (33) says that the cif import price to country  $n$  of commodity  $i$  imported from country  $m$  ( ${}^{(n)*}P_{im}$ ) equals the domestic price of commodity  $i$  in country  $m$  expressed in country  $m$ 's currency ( ${}^{(m)}P_{im}$ ) plus the quantity of margins required to transport  $i$  from country  $m$  to country  $n$  ( $A_{i(m,n)}$ ) times the unit price of margins (in country  $m$ 's currency) on commodity  $i$  transported from country  $m$  to country  $n$  ( $P_{i(m,n)}^T$ ). Note that (33) applies only to the movement of goods between the  $c$  countries.

Expressing (33) in percentage changes and treating  $A_{i(m,n)}$  as a constant gives, for our two country system;

$${}^{(n)*}P_{im} = \xi_{1i(m,n)} {}^{(m)}P_{im} + \xi_{2i(m,n)} P_{i(m,n)}^T \quad \begin{matrix} i = 1, \dots, g \\ n = 1, 2 \\ m = 1, 2 \\ m \neq n \end{matrix} \quad (34)$$

where  $\xi_{1(i(m,n))}$  and  $\xi_{2(i(m,n))}$  are respectively the shares in the cif import price of commodity  $i$  from country  $m$  to country  $n$  accounted for by the domestic price of  $i$  in  $m$  and the margins costs in transporting  $i$  from country  $m$  to country  $n$ .

There are a number of possible ways we could treat the  $P_{i(m,n)}^T$ . They could for example be indexed to movements in domestic costs in either the country of origin or the country of destination. For the time being we simply treat them as exogenous.

### Exporting

Our next set of zero pure profits conditions equates the revenue from exporting to the rest of the world by each of the  $c$  countries to the relevant costs. We write that

$${}^{(n)}P_{in} {}^{(n)}S_i \phi_{n,w} = {}^{(n)}P_{in} \quad \begin{matrix} i = 1, \dots, g \\ n = 1, \dots, c \end{matrix} \quad (35)$$

where  $S_i^{(n)}$  is one plus the ad valorem rate of export subsidy given to commodity  $i$  by country  $n$  and  $\phi_{n,w}$  is the exchange rate between country  $n$  and the rest of the world.<sup>1</sup> On the LHS of (35) we have the value (in country  $n$ 's currency) of exporting a unit of commodity  $i$  to the rest of the world and on the right hand

<sup>1</sup> Note that no explicit provision is made for transport costs between country  $n$  and the rest of the world. If for example the domestic and hence fob export price of cars from country  $n$  increases by 10 per cent then the selling price of country  $n$ 's cars in the rest of the world increases by 10 per cent.



side we have the cost of doing so, that is, the domestic price in country  $n$  of a unit of commodity  $i$ . Expressing (35) in percentage change terms for our two country system gives;

$$\begin{matrix} (n) & (4) & (n) & & (n) \\ P_{in} + s_i + \phi_{n,3} & = & P_{in} & & i = 1, \dots, g \\ & & & & n = 1, 2. \end{matrix} \quad (36)$$

### Exchange Rate Arbitrage

Our final set of pure profits conditions imposes arbitrage between exchange rates. The necessary conditions are;

$$\phi_{w,n} \phi_{n,w} = 1.0 \quad n = 1, \dots, c \quad (37)$$

$$\begin{matrix} \phi_{n,m} \phi_{m,w} = \phi_{n,w} & n = 1, \dots, c \\ & m = 1, \dots, c \\ & n \neq m \\ & n < m \end{matrix} \quad (38)$$

$$\begin{matrix} \phi_{n,m} \phi_{m,n} = 1.0 & n = 1, \dots, c \\ & m = 1, \dots, c \\ & n \neq m \\ & n < m \end{matrix} \quad (39)$$

where  $\phi_{w,n}$  is the exchange rate between foreign (rest of the world) source  $w$  and country  $n$ , i.e., (currency rest of world/ currency  $n$ ).

Thus in the generalised system of  $c$  countries there are

$c + 2 \left[ \frac{1}{2} (c^2 - c) \right]$  such conditions. In our two country system there are six possible combinations of exchange rates

$(\phi_{1,2}; \phi_{2,1}; \phi_{1,3}; \phi_{3,1}; \phi_{2,3}; \phi_{3,2})$ . The required equations to impose arbitrage, expressed in percentage change form are;

$$\phi_{3,1} + \phi_{1,3} = 0.0 \quad (37A)$$

$$\phi_{3,2} + \phi_{2,3} = 0.0 \quad (37B)$$

$$\phi_{1,2} + \phi_{2,3} = \phi_{1,3} \quad (38A)$$

$$\phi_{1,2} + \phi_{2,1} = 0.0 \quad (39A)$$

### 3.9 The Allocation of Investment Across Industries

The theory set out in Section 19 of Dixon et al (1982) is followed. Five steps are involved.

(i) The current rate of return on capital in industry  $j$  and country  $n$  ( $R_j^{(n)}$ ) is defined as

$$R_j^{(n)} = \frac{P_{2j}^{(n)}}{\Pi_j^{(n)}} - D_j^{(n)} \quad \begin{matrix} j = 1, \dots, h \\ n = 1, \dots, c \end{matrix} \quad (40)$$

where  $D_j^{(n)}$  is the rate of depreciation in industry  $j$  in country  $n$  (assumed constant) and  $P_{2j}^{(n)}$  and  $\Pi_j^{(n)}$  were previously defined as for country  $n$  the rental on capital in industry  $j$  and the cost of producing a unit of capital in industry  $j$  respectively.

(ii) Capital is assumed to take one period to install. (iii)

Investors in country  $n$  are assumed to be cautious in assessing the effects of expanding the capital stock in industry  $j$ . They behave as if they expect that industry  $j$ 's rate of return schedule in one period's time will have the form

$$R_{j(1)}^{(n)} = R_j^{(n)} \left( \frac{K_j^{(n)}(1)}{K_j^{(n)}} \right)^{-\beta_j} \quad \begin{matrix} j = 1, \dots, h \\ n = 1, \dots, c \end{matrix} \quad (40A)$$

where  $\beta_j^{(n)}$  is a positive parameter,  $K_j^{(n)}$  is the current level of capital stock in industry  $j$  in country  $n$ , and  $K_{j(1)}^{(n)}$  is the level at the end of one period. (iv) It is assumed that total investment in country  $n$ ,  $I^{(n)}$ , is allocated across industries so as to equate expected rates of return. This implies for each country that there exists some rate of return  $\Lambda^{(n)}$  such that

$$\left( \frac{{}^{(n)}K_{j(1)}}{{}^{(n)}K_j} \right) - \beta_j {}^{(n)}R_j = \frac{{}^{(n)}I}{\Lambda} \quad \begin{matrix} n = 1, \dots, c \\ j = 1, \dots, h. \end{matrix} \quad (41)$$

(v) Equations are defined for  $K_{j(1)}^{(n)}$  and  $I^{(n)}$ . We write that

$$K_{j(1)}^{(n)} = K_j^{(n)} (1 - D_j) + Y_j^{(n)} \quad \begin{matrix} j = 1, \dots, h \\ n = 1, \dots, c, \end{matrix} \quad (42)$$

$$I^{(n)} = \sum_{j=1}^h \pi_j Y_j^{(n)} \quad n = 1, \dots, c. \quad (43)$$

Equation (42) assumes that for each country the effects of past investment decisions are fully incorporated in the current capital stock, with the only variables influencing capital stock at the end of one period being current capital stock and current investment.

Expressing (40), (41), (42) and (43) in percentage change form gives, for our two country system,

$$r_j^{(n)} = Q_j^{(n)} (p_{2j}^{(n)} - \pi_j^{(n)}) \quad \begin{matrix} j = 1, \dots, h \\ n = 1, 2, \end{matrix} \quad (44)$$

$$-\beta_j^{(n)} (k_{j(1)}^{(n)} - k_j^{(n)}) + r_j^{(n)} = \lambda \quad \begin{matrix} j = 1, \dots, h \\ n = 1, 2, \end{matrix} \quad (45)$$

$$k_{j(1)}^{(n)} = k_j^{(n)} (1 - G_j^{(n)}) + y_j^{(n)} G_j^{(n)} \quad \begin{matrix} j = 1, \dots, h \\ n = 1, 2, \end{matrix} \quad (46)$$

$$\sum_{j=1}^h (\pi_j^{(n)} + y_j^{(n)}) T_j^{(n)} = i^{(n)} \quad n = 1, 2, \quad (47)$$

where  $Q_j^{(n)} = (R_j^{(n)} + D_j^{(n)}) / R_j^{(n)}$  i.e., the ratio of the gross rate of return in industry  $j$  for country  $n$  to the net rate of return,  $G_j^{(n)} = Y_j^{(n)} / K_{j1}^{(n)}$  i.e., the ratio of gross investment in industry  $j$  in country  $n$  to its future capital stock, and  $T_j^{(n)}$  is the share of aggregate fixed investment in country

n accounted for by investment in industry j in country n, i.e.,

$$T_j^{(n)} = Y_j^{(n)} \pi_j^{(n)} / \sum_{j=1}^h Y_j^{(n)} \pi_j^{(n)}.$$

### 3.10 Market Clearing Equations

We next specify equations which ensure that demand equals supply for domestically produced commodities and labour, capital and land in each country. We write for country n that

$$X_{in}^{(n)} = \sum_{j=1}^h \sum_{m=1}^c X_{(in)j}^{(m)(1)} + \sum_{j=1}^h \sum_{m=1}^c X_{(in)j}^{(m)(2)} + \sum_{m=1}^c X_{in}^{(m)(3)} + X_{in}^{(n)(4)} + \sum_{m=1}^c X_{in}^{(m)(5)} \quad \begin{matrix} i = 1, \dots, g \\ n = 1, \dots, c \end{matrix} \quad (48)$$

where

$$X_{in}^{(n)} = \sum_{j=1}^h X_{(in)j}^{(n)} \quad \begin{matrix} n = 1, \dots, c \\ i = 1, \dots, g. \end{matrix} \quad (49)$$

$$L_q^{(n)} = \sum_{j=1}^h X_{1,q,j}^{(n)P} \quad \begin{matrix} n = 1, \dots, c \\ q = 1, \dots, r. \end{matrix} \quad (50)$$

$$K_j^{(n)} = X_{2j}^{(n)P} \quad \begin{matrix} n = 1, \dots, c \\ j = 1, \dots, h. \end{matrix} \quad (51)$$

$$N_j^{(n)} = X_{3j}^{(n)P} \quad \begin{matrix} n = 1, \dots, c \\ j = 1, \dots, h. \end{matrix} \quad (52)$$

Equation (48) equates, for each country, supply and demand for domestically produced goods. Total supply of good i in country n is the sum over the outputs of i by each of the h industries in n. Total demand is composed of intermediate input demands of goods for use in (1) current production, (2) capital creation, (3) household consumption and (5) other demands in each of the c countries together with export demands by country n to the rest

of the world. Equation (50) equates occupational labour supply in each country to the country demand for it. Note that (50) merely states that labour demands must be satisfied. It does not necessarily impose full employment assumptions. Equations (51) and (52) equate supply and demand for capital and land respectively for industries in each country.

Expressing (48) to (52) in percentage change terms gives, for our two country system,

$$\begin{aligned} x_{in}^{(n)} = & \sum_{j=1}^h \sum_{m=1}^2 x_{(in)j}^{(m)(1)} B_{(in)j}^{(m)(1)} + \sum_{j=1}^h \sum_{m=1}^2 x_{(in)j}^{(m)(2)} B_{(in)j}^{(m)(2)} + \\ & \sum_{m=1}^2 x_{in}^{(m)(3)} B_{in}^{(m)(3)} + x_{in}^{(n)(4)} B_{in}^{(n)(4)} + \sum_{m=1}^2 x_{in}^{(m)(5)} B_{in}^{(m)(5)} \end{aligned} \quad \begin{aligned} i &= 1, \dots, g \\ n &= 1, 2, \end{aligned} \quad (53)$$

$$x_{in}^{(n)} = \sum_{j=1}^h x_{(in)j}^{(n)} D_{(in)j}^{(n)} \quad \begin{aligned} i &= 1, \dots, g \\ n &= 1, 2, \end{aligned} \quad (54)$$

$$l_q^{(n)} = \sum_{j=1}^h x_{1,q,j}^{(n)} B_{1,q,j}^{(n)} \quad \begin{aligned} n &= 1, 2 \\ q &= 1, \dots, r, \end{aligned} \quad (55)$$

$$k_j^{(n)} = x_{2j}^{(n)P} \quad \begin{aligned} n &= 1, 2 \\ j &= 1, \dots, h, \end{aligned} \quad (56)$$

$$n_j^{(n)} = x_{3j}^{(n)P} \quad \begin{aligned} n &= 1, 2 \\ j &= 1, \dots, h. \end{aligned} \quad (57)$$

The B's in (53) are coefficients which reflect the shares of the sales of goods produced in country n absorbed by the various types of demands on the right hand side. For example  $B_{(in)j}^{(m)(1)}$  refers to the share of the total sales of good i produced in country n absorbed by sales to industry j in country m for current production. In (54) the D's are production shares.  $D_{(in)j}^{(n)}$  is the

share of country n's output of good i produced by industry j in country n. In (55)  $B_{1,q,j}^{(n)}$  is for country n the share of the total employment of labour of type q which is accounted for by industry j.

### 3.11 Aggregate Imports, Exports and the Balance of Trade

We use  $X_{iF}^{(n)}$  (where F denotes foreign sources of supply) to represent aggregate import demands for good i by country n. These demands can be met by any of the c sources of foreign supply, i.e., the remaining c-1 countries plus the rest of the world.

Hence,

$$X_{iF}^{(n)} = \sum_{\substack{s=1 \\ s \neq n}}^{c+1} X_{is}^{(n)} \quad \begin{matrix} n = 1, \dots, c \\ i = 1, \dots, g. \end{matrix} \quad (58)$$

In percentage change terms (58) becomes, for our two country system,

$$x_{iF}^{(n)} = \sum_{\substack{s=1 \\ s \neq n}}^3 x_{is}^{(n)} F_{is}^{(n)} \quad \begin{matrix} i = 1, \dots, g \\ n = 1, 2 \end{matrix} \quad (59)$$

where  $F_{is}^{(n)}$  is the share of country n's total imports of good i represented by imports of good i from foreign (to country n) source s. Next we write that;

$$X_{is}^{(n)} = \sum_{j=1}^h X_{(is)j}^{(n)(1)} + \sum_{j=1}^h X_{(is)j}^{(n)(2)} + X_{is}^{(n)(3)} + X_{is}^{(n)(5)} \quad (60)$$

$$\begin{matrix} i = 1, \dots, g \\ n = 1, \dots, c \\ s = 1, \dots, c+1 \\ s \neq n. \end{matrix}$$

Equation (60) says that country n's total imports of good i from foreign source s equals the sum of the separate demands in country n of commodity i from foreign source s for use as an input into

current production, capital creation, household consumption and other demands. Note that the model excludes imports for the purpose of reexports without any further processing. Expressing (60) in percentage change form gives, for our two country system,

$$\begin{aligned}
 x_{is}^{(n)} &= \sum_{j=1}^h x_{(is)j}^{(n)(1)} F_{(is)j}^{(n)(1)} + \sum_{j=1}^h x_{(is)j}^{(n)(2)} F_{(is)j}^{(n)(2)} + x_{is}^{(n)(3)} F_{is}^{(n)(3)} \\
 &+ x_{is}^{(n)(5)} F_{is}^{(n)(5)} \quad \begin{aligned} i &= 1, \dots, g \\ n &= 1, 2 \\ s &= 1, 2, 3 \\ s &\neq n. \end{aligned} \quad (61)
 \end{aligned}$$

In (61) the  $F$ 's denote shares of total import flows. For example  $F_{(is)j}^{(n)(1)}$  is the share of the total imports of good  $i$  by country  $n$  from foreign source  $s$  which is absorbed by industry  $j$  in country  $n$  for current production.

The aggregate value of imports by country  $n$   $(M)^{(n)}$ , expressed in rest of the world currency units, is given by;

$$\begin{aligned}
 M^{(n)} &= \sum_{i=1}^g \sum_{\substack{m=1 \\ m \neq n}}^c X_{im}^{(n)} P_{im}^{(n)*} / \phi_{m,w} + \sum_{i=1}^g X_{iw}^{(n)} P_{iw}^{(n)*} \quad (68) \\
 n &= 1, \dots, c.
 \end{aligned}$$

In (68),  $P_{im}^{(n)*}$  denotes the cif import price for country  $n$  of good  $i$  from other country source  $m$  expressed in  $m$ 's currency. Similarly,  $P_{iw}^{(n)*}$  represents the cif import price to country  $n$  of good  $i$  from the rest of the world and  $X_{iw}^{(n)}$  is the corresponding import quantity from the rest of the world. The first term in (68) therefore represents the cost to country  $n$  of imports from the other countries within the group of  $c$  expressed in rest of the world currency. The second term represents the cost to country

n of imports from the rest of the world, also expressed in rest of the world currency. Rewriting (68) in percentage change form gives, for our two country example,

$$\begin{aligned} \begin{matrix} (n) \\ m \end{matrix} &= \sum_{\substack{m=1 \\ m \neq n}}^2 \left[ \sum_{i=1}^g (x_{im}^{(n)} + p_{im}^{(n)*} - \phi_{m,3}^{(n)}) M_{im}^{(n)} \right] M_m^{(n)} \\ &+ \left[ \sum_{i=1}^g (x_{i3}^{(n)} + p_{i3}^{(n)*}) M_{i3}^{(n)} \right] M_3^{(n)} \quad n = 1, 2. \end{aligned} \quad (69)$$

In (69),  $M_m^{(n)}$  is the share in the total rest of the world currency costs of imports to country n represented by imports from country m,  $M_{im}^{(n)}$  is the share in the total import costs by country n from country m represented by good i from country m,  $M_3^{(n)}$  is the share in the total rest of the world currency costs of imports to country n represented by imports from the rest of the world and  $M_{i3}^{(n)}$  is the share in the total import costs of n from the rest of the world represented by commodity i from the rest of the world. Note that

$$M_3^{(n)} + \sum_{\substack{m=1 \\ m \neq n}}^2 M_m^{(n)} = 1.0 \quad V_n,$$

$$\sum_{i=1}^g M_{im}^{(n)} = 1.0 \quad m = 1, 2 \quad V_n,$$

$$\sum_{i=1}^g M_{i3}^{(n)} = 1.0 \quad V_n.$$

Next we define aggregate foreign (rest of the world) currency export receipts earned by country n,  $E$ , as

$$E^{(n)} = \sum_{\substack{m=1 \\ m \neq n}}^c \sum_{i=1}^g X_{in}^{(m)} P_{in}^{(n)} / \phi_{n,w} + \sum_{i=1}^g P_{in}^{(n)(4)} X_{in}^{(n)(4)}. \quad (70)$$



The first term on the RHS of (70) represents the foreign exchange earnings (in the rest of the world currency) of country n's exports to other countries within the group of c. The second term represents the foreign exchange earnings from country n's exports to the rest of the world. Expressing (70) in percentage changes gives, for our two country system,

$$\begin{aligned} e^{(n)} &= \sum_{m=1, m \neq n}^2 \left[ \sum_{i=1}^g (x_{in}^{(m)} + p_{in}^{(n)} - \phi_{n,3}) E_{in}^{(m)} \right] E_n^{(m)} + \left[ \sum_{i=1}^g (p_{in}^{(n)} + x_{in}^{(4)}) \right] E_n^{(4)} \\ \left[ \begin{matrix} (3) \\ E_{in} \end{matrix} \right] \left[ \begin{matrix} (3) \\ E_n \end{matrix} \right] & \quad n = 1, 2 \end{aligned} \quad (71)$$

where  $E_n^{(m)}$  is the share of the foreign currency exports of country n accounted for by exports to country m,  $E_{in}^{(m)}$  is the share of the foreign currency exports of country n to country m accounted for by commodity i to country m,  $E_n^{(3)}$  is the share of the foreign currency export earnings of country n represented by exports to the rest of the world, and  $E_{in}^{(3)}$  is the share of the foreign currency exports of country n to the rest of the world accounted for by exports of good i to the rest of the world. Note that

$$\begin{aligned} \sum_{m=1, m \neq n}^2 E_n^{(m)} + E_n^{(3)} &= 1.0 & V_n, \\ \sum_{i=1, i \neq n}^g E_{in}^{(m)} &= 1.0 & m = 1, 2 & V_n, \\ \sum_{i=1}^g E_{in}^{(3)} &= 1.0 & V_n. \end{aligned}$$

Next we define the balance of trade in country n,  $B^{(n)}$ , as

$$B^{(n)} = E^{(n)} - M^{(n)} \quad n = 1, \dots, c. \quad (72)$$

From (72) we can write, for our two country system that,

$$100 \Delta B^{(n)} = E^{(n)} e^{(n)} - M^{(n)} m^{(n)} \quad n = 1, 2 \quad (73)$$

where  $\Delta B^{(n)}$  is the change (not the percentage change) in B. Because  $B^{(n)}$  can change sign we avoid the percentage change form of (72).

Thus  $\Delta B^{(n)}$  require units of measurement. These will depend on the units for  $E^{(n)}$  and  $M^{(n)}$ .

We can also write expressions for the balance of trade between pairs of countries within the group of c. For example,

$$B_{n,m} = E_{n,m} - M_{m,n} \quad \begin{matrix} n \neq m \\ m > n \end{matrix} \quad (73A)$$

where  $B_{n,m}$  is the balance of trade between country n and country m in country n's currency,  $E_{n,m}$  is the fob value of exports earned by country n in exporting to country m and  $M_{m,n}$  is for country n, the cif value of its imports from country m. There will be  $\frac{1}{2} (c^2 - c)$  such conditions. Expressing (73A) in first difference form gives, for our two country system,

$$100 \Delta B_{1,2} = E_{1,2} e_{1,2} - M_{2,1} m_{2,1} \quad (73B)$$

where  $\Delta B_{1,2}$  is for country 1, the change in its balance of trade with country 2,  $e_{1,2}$  and  $m_{2,1}$  are respectively for country 1 the percentage change in the fob value of its exports to country 2 and the percentage change in the cif value of its imports from country 2, and  $E_{1,2}$  and  $M_{2,1}$  are the base period values of exports to 2 (fob) and imports from 2 (cif).

The variables  $e_{1,2}$  and  $m_{2,1}$  are defined by

$$e_{1,2} = \sum_{i=1}^g E_{i(1,2)} \left( x_{i1}^{(2)} + p_{i1}^{(1)} \right) \quad (73C)$$

$$m_{2,1} = \sum_{i=1}^g M_{i(2,1)} \left( x_{i2}^{(1)} + p_{i2}^{(1)*} + \phi_{1,2} \right) \quad (73D)$$

where  $E_{i(1,2)}$  and  $M_{i(2,1)}$  are coefficients representing respectively, the share of the total value of country 1's exports to country 2 represented by exports of good  $i$  from country 1 to country 2, and the share of the total value of country 1's imports from country 2 represented by country 1's imports of good  $i$  from country 2.

### 3.12 Macro and Other Miscellaneous Equations

At this stage we provide only a very simple treatment of the behaviour of the macroeconomic aggregates. First we define the percentage change in real aggregate investment in country  $n$

$$\begin{aligned} &^{(n)}(i_R) \text{ as} \\ &^{(n)}i_R = i^{(n)} - \epsilon^{(n)} \quad n = 1, 2 \end{aligned} \quad (74)$$

where  $i^{(n)}$  is the percentage change in aggregate nominal investment in country  $n$  and

$$\epsilon^{(n)} = \sum_{j=1}^n T_j^{(n)} \pi_j^{(n)} \quad n = 1, 2. \quad (75)$$

$\epsilon^{(n)}$  is an investment goods price index made up of a weighted average of the percentage changes in capital goods prices where the weights  $T_j^{(n)}$  reflect the shares of total investment spending in country  $n$  accounted for by investment spending in each

industry of country n. Next we add the equation

$$i_R^{(n)} = c_R^{(n)} + f_R^{(n)} \quad n = 1, 2 \quad (76)$$

where  $f_R^{(n)}$  is an exogenous shift variable whose role is to fix the relationship between movements in real aggregate consumption and investment in country n.

We also need to add equations to define aggregate employment and the aggregate capital stock in each country. We write

$$l^{(n)} = \sum_{q=1}^r l_q^{(n)} \psi_{1,q}^{(n)} \quad n = 1, 2, \quad (77)$$

$$K^{(n)} = \sum_{j=1}^h k_j^{(n)} \psi_{2j}^{(n)} \quad n = 1, 2 \quad (78)$$

where  $l^{(n)}$  and  $K^{(n)}$  are respectively the percentage changes in aggregate employment and aggregate current capital stock in country n,  $\psi_{1,q}^{(n)}$  is the share of employment of occupation q in total employment in country n and  $\psi_{2j}^{(n)}$  is the share of capital of type j in the total value of fixed capital in the base year economy of country n.

Next we define several indexing equations to increase the flexibility of operation of the model. These are

$$p_j^{(n)o} = h_j^{(n)o} \epsilon^{(n)(3)} + f_j^{(n)o} \quad n = 1, 2 \quad (79)$$

$$j = 1, \dots, h,$$

$$p_{1,q}^{(n)p} = h_{1,q}^{(n)} \epsilon^{(n)(3)} + f_{1,q}^{(n)} + f_1^{(n)} \quad n = 1, 2 \quad (80)$$

$$q = 1, \dots, r.$$

In (79) and (80) the  $h_j^{(n)o}$  and  $h_{1,q}^{(n)}$  are indexing parameters and the  $f_j^{(n)o}$ ,  $f_{1,q}^{(n)}$  and  $f_1^{(n)}$  are shift variables.<sup>1</sup> Equation (79) allows us to manipulate the price of other costs in each industry and country. Equation (80) allows us to exogenously set real and nominal occupational wage rates and the economy wide wage in each country.

Finally we define the percentage change in gross domestic product (gdp) in each country as

$$\text{gdp}^{(n)} = S_c^{(n)} c_R^{(n)} + S_i^{(n)} i_R^{(n)} + S_g^{(n)} x_G^{(n)} + S_e^{(n)} e^{(n)} + S_m^{(n)} m^{(n)} \quad (81)$$

$n = 1, 2$

i.e., as a share weighted sum of the percentage changes in aggregate real household consumption ( $c_R^{(n)}$ ), aggregate real investment ( $i_R^{(n)}$ ), aggregate other demands ( $x_G^{(n)}$ ), aggregate foreign currency exports ( $e^{(n)}$ ) and aggregate foreign currency imports ( $m^{(n)}$ ). The  $x_G^{(n)}$  refers to a weighted average of other demands in country  $n$  from domestic and imported sources. That is

$$x_G^{(n)} = \sum_{s=1}^3 \sum_{i=1}^g x_{is}^{(n)(5)} S_{is}^{(n)(5)} \quad n = 1, 2 \quad (82)$$

where  $S_{is}^{(n)(5)}$  is for country  $n$  the share of other demands for good  $i$  from source  $s$  in total other demands. The coefficients  $S_c^{(n)}$ ,  $S_i^{(n)}$ ,  $S_g^{(n)}$ ,  $S_e^{(n)}$  and  $S_m^{(n)}$  are the shares of the GDP in country  $n$  represented by the respective final demand categories. They are constructed so that  $S_c^{(n)} + S_i^{(n)} + S_g^{(n)} + S_e^{(n)} + S_m^{(n)}$  equal unity for each  $n$ .

<sup>1</sup> There are many potential uses of these shift variables in policy analysis. For example, a decrease in  $f_j^{(n)}$  ( $j$  = agricultural industry), thereby shifting the agricultural industry supply curve to the left, could be undertaken to simulate the effects on the multicountry system of a drought in country  $n$ .

#### 4. The Complete Model System

Tables 2 and 3 list the equations of the two country, rest of the world system in percentage change form. Table 4 defines the system's parameters. The equations are linear and could be arranged as

$$Az = 0 \quad (83)$$

where A is a u by v matrix of coefficients and z is a v x 1 vector of variables expressed in percentage changes. From Tables 1 and 2 we see that

$$v = 14gh + 30h + 55g + 2hr + 6r + 45.$$

$$u = 14gh + 24h + 37g + 2hr + 4r + 35.$$

Thus, before solving the model,  $v - u = 6h + 18g + 2r + 10$  variables must be declared exogenous.

##### 4.1 Alternative Model Closures

It is a feature of models which can be represented by (83) that the v-u variables can be chosen in many different ways, thus allowing the model to be applied to a wide range of economic policy issues without changing its structure or solution procedure. Table 4 gives one possible model closure designed to reflect a short run macroeconomic environment. In discussing the list of exogenous variables in Table 5 we also draw attention to some alternative selections of exogenous variables.

Table 2: Equations of the Two Country, Rest of the World System

Identifier	Equation	Range	Number	Description
(6)	$x_{(is)j}^{(1)} = z_j^{(n)} - \sigma_{(is)j}^{(n)} \left( p_{is}^{(1)} - \sum_{s=1}^3 S_{(is)j}^{(1)*} p_{is}^{(n)} \right)$	$i = 1, \dots, g$ $s = 1, \dots, 3$ $j = 1, \dots, h$ $n = 1, 2$	6gh	Country demands for intermediate inputs by source for current production
(7)	$x_{vj}^{(n)P} = z_j^{(n)} - \sigma_{vj}^{(n)} \left( p_{vj}^{(n)P} - \sum_{v=2}^3 S_{vj}^{(n)*} p_{vj}^{(n)P} - S_{1j}^{(n)P} p_1 \right)$	$v = 2, 3$ $j = 1, \dots, h$ $n = 1, 2$	4h	Country demands for capital and land
(8)	$x_{1j}^{(n)P} = z_j^{(n)} - \sigma_{1j}^{(n)} \left( p_1^{(n)P} - \sum_{v=2}^3 S_{vj}^{(n)*} p_{vj}^{(n)P} - S_{1j}^{(n)P} p_1 \right)$	$j = 1, \dots, h$ $n = 1, 2$	2h	Country demands for labour in general
(9)	$x_{1,q,j}^{(n)P} = x_{1j}^{(n)P} - \sigma_{1,q,j}^{(n)} \left( p_{1,q}^{(n)P} - \sum_{q=1}^r S_{1,q,j}^{(n)*} p_{1,q}^{(n)P} \right)$	$q = 1, \dots, r$ $j = 1, \dots, h$ $n = 1, 2$	2hr	Country demands for labour by occupation and industry
(10)	$x_j^{(n)O} = z_j^{(n)}$	$j = 1, \dots, h$ $n = 1, 2$	2h	Country demands for other costs
(11)	$p_1^{(n)P} = \sum_{q=1}^r p_{1,q}^{(n)P} S_{1,q}^{(n)}$	$n = 1, 2$	2	Country price of labour in general
(13)	$x_{(in)j}^{(n)} = z_j^{(n)} + \sigma_{(in)j}^{(n)} \left( p_{in}^{(n)T} - \sum_{i=1}^g C_{(in)j}^{(n)*} p_{in}^{(n)} \right)$	$i = 1, \dots, g$ $j = 1, \dots, h$ $n = 1, 2$	2gh	Country supplies of commodities by industry
(16)	$x_{(is)j}^{(2)} = y_j^{(n)} - \sigma_{(is)j}^{(n)} \left( p_{is}^{(2)} - \sum_{s=1}^3 S_{(is)j}^{(2)*} p_{is}^{(n)} \right)$	$i = 1, \dots, g$ $j = 1, \dots, h$ $s = 1, \dots, 3$ $n = 1, 2$	6gh	Country demands for intermediate inputs by source for capital creation
(19)	$x_{is}^{(n)(3)} = x_i^{(n)(3)} - \sigma_{is}^{(n)} \left( p_{is}^{(n)(3)} - \sum_{s=1}^3 S_{is}^{(n)(3)*} p_{is}^{(n)} \right)$	$i = 1, \dots, g$ $s = 1, \dots, 3$ $n = 1, 2$	6g	Country household demands by commodities classified by source
(20)	$x_i^{(n)(3)} - q = \epsilon_i \left( c - q \right) + \sum_{k=1}^g \eta_{ik} p_k^{(n)(3)}$	$i = 1, \dots, g$ $n = 1, 2$	2g	Country household demands for commodities undifferentiated by source
(21)	$p_k^{(n)(3)} = \sum_{s=1}^3 S_{ks}^{(n)(3)} p_{ks}^{(n)}$	$k = 1, \dots, g$ $n = 1, 2$	2g	Country general price of each commodity to households
(23)	$x_{is}^{(s)(4)} = x_i^{(s)(4)} - \sigma_{is}^{(s)} \left( p_{is}^{(s)(4)} - \sum_{s=1}^3 S_{is}^{(s)(4)*} p_{is}^{(s)} \right)$	$i = 1, \dots, g$ $s = 1, \dots, 3$	3g	Commodity demands in the rest of the world by supplying source
(24)	$x_{is}^{(n)(5)} = c_R^{(n)} h_{is}^{(n)(5)} + f_{is}^{(n)(5)}$	$i = 1, \dots, g$ $s = 1, \dots, 3$ $n = 1, 2$	6g	Country other demands for commodities by source
(25)	$c_R^{(n)} = c - \epsilon$	$n = 1, 2$	2	Defines country real aggregate consumption
(26)	$\epsilon^{(n)(3)} = \sum_{i=1}^g \sum_{s=1}^3 W_{is}^{(n)(3)} p_{is}^{(n)}$	$n = 1, 2$	2	Defines country index of consumer prices
(28)	$\sum_{i=1}^g p_{in}^{(n)} C_{(in)j}^{(n)} = \sum_{i=1}^g \sum_{s=1}^3 p_{is}^{(n)} H_{(is)j}^{(n)(1)} + \sum_{q=1}^r p_{1,q}^{(n)P} H_{1,q,j}^{(n)} + \sum_{v=2}^3 p_{vj}^{(n)P} H_{vj}^{(n)P} + p_j^{(n)O} H_j^{(n)O}$	$j = 1, \dots, h$ $n = 1, 2$	2h	Zero pure profits in production in each country
(30)	$\pi_j^{(n)} = \sum_{i=1}^g \sum_{s=1}^3 p_{is}^{(n)} H_{(is)j}^{(n)(2)}$	$j = 1, \dots, h$ $n = 1, 2$	2h	Zero pure profits in capital creation in each country

Table 2: Equations of the Two Country, Rest of the World System (cont.)

Identifier	Equation	Range	Number	Description
(32)	$P_{is}^{(n)} = P_{is}^{(n)*} + \phi_{n,s} + t_{i(n,s)}$	$i = 1, \dots, g$ $s = 1, \dots, 3$ $s \neq n$ $n = 1, 2$	4g	Zero pure profits in imports by country n from source s.
(34)	$P_{im}^{(n)*} = \xi_{1i(m,n)} P_{im}^{(m)} + \xi_{2i(m,n)} P_{i(m,n)}^T$	$i = 1, \dots, g$ $m = 1, 2$ $n = 1, 2$ $m \neq n$	2g	Zero pure profits in the transport of goods between countries
(36)	$P_{in}^{(n)} + S_i^{(4)} + \phi_{n,3}^{(n)} = P_{in}^{(n)}$	$i = 1, \dots, g$ $n = 1, 2$	2g	Zero pure profits in exporting to the rest of the world by each country
(37)	$\phi_{3,n} + \phi_{n,3} = 0.0$	$n = 1, 2$	2	Exchange rate arbitrage
(38)	$\phi_{n,m} + \phi_{m,3} = \phi_{n,3}$	$n = 1, 2$ $m = 1, 2$ $n \neq m$ $n < m$	1	
(39)	$\phi_{n,m} + \phi_{m,n} = 0.0$	$n = 1, 2$ $m = 1, 2$ $n \neq m$ $n < m$	1	
(44)	$r_j^{(n)} = Q_j^{(n)} (P_{2j}^{(n)} - \pi_j^{(n)})$	$j = 1, \dots, h$ $n = 1, 2$	2h	Rates of return on capital in each country industry
(45)	$-\beta_j^{(n)} (k_{j(1)}^{(n)} - k_j^{(n)}) + r_j^{(n)} = \lambda^{(n)}$	$j = 1, \dots, h$ $n = 1, 2$	2h	Equality of rates of return across industries in each country
(46)	$k_{j(1)}^{(n)} = k_j^{(n)} (1 - G_j^{(n)}) + y_j^{(n)} G_j^{(n)}$	$j = 1, \dots, h$ $n = 1, 2$	2h	Country capital accumulation by industry
(47)	$\sum_{j=1}^h (\pi_j^{(n)} + y_j^{(n)}) T_j^{(n)} = i^{(n)}$	$n = 1, 2$	2	Country investment budget
(53)	$x_{in}^{(n)} = \sum_{j=1}^h \sum_{m=1}^2 x_{(in)j}^{(m)} B_{(in)j}^{(m)} + \sum_{j=1}^h \sum_{m=1}^2 x_{(in)j}^{(m)} B_{(in)j}^{(m)} + \sum_{m=1}^2 x_{in}^{(m)} B_{in}^{(m)} + \sum_{m=1}^2 x_{in}^{(m)} B_{in}^{(m)}$	$i = 1, \dots, g$ $n = 1, 2$	2g	Demand equals supply for domestic commodities of each country
(54)	$x_{in}^{(n)} = \sum_{j=1}^h x_{(in)j}^{(n)} D_{(in)j}^{(n)}$	$i = 1, \dots, g$ $n = 1, 2$	2g	Output of commodities by country
(55)	$l_q^{(n)} = \sum_{j=1}^h x_{1,q,j}^{(n)} B_{1,q,j}^{(n)}$	$q = 1, \dots, r$ $n = 1, 2$	2r	Supply equals demand for occupational labour by country
(56)	$k_j^{(n)} = x_{2j}^{(n)}$	$j = 1, \dots, h$ $n = 1, 2$	2h	Supply equals demand for capital by country
(57)	$n_j^{(n)} = x_{3j}^{(n)}$	$j = 1, \dots, h$ $n = 1, 2$	2h	Supply equals demand for land by country
(59)	$x_{if}^{(n)} = \sum_{s=1}^3 x_{is}^{(n)} F_{is}^{(n)}$ $s \neq n$	$i = 1, \dots, g$ $n = 1, 2$	2g	Import volume by country n from other countries and the rest of the world



Table 2: Equations of the Two Country, Rest of the World System (cont.)

Identifier	Equation	Range	Number	Description
(61)	$x_{is}^{(n)} = \sum_{j=1}^h x_{(is)j}^{(n)(1)} F_{(is)j}^{(n)(1)} + \sum_{j=1}^h x_{(is)j}^{(n)(2)} F_{(is)j}^{(n)(2)}$ $+ x_{is}^{(n)(3)} F_{is}^{(n)(3)} + x_{is}^{(n)(5)} F_{is}^{(n)(5)}$	$i = 1, \dots, g$ $n = 1, 2$ $s = 1, 2, 3$ $s \neq n$	4g	Import volume for country n from source s
(69)	$m_m^{(n)} = \sum_{\substack{i=1 \\ m \neq n}}^g \left[ \sum_{i=1}^g (x_{im}^{(n)} + p_{im}^{(n)} - \phi_{m,3}^{(n)}) M_{im}^{(n)} \right] M_m^{(n)}$ $+ \left[ \sum_{i=1}^g (x_{i3}^{(n)} + p_{i3}^{(n)}) M_{i3}^{(n)} \right] M_3^{(n)}$	$n = 1, 2$	2	Foreign currency cost (in rest of the world currency) of imports by country n
(71)	$e_n^{(m)} = \sum_{\substack{i=1 \\ m \neq n}}^g \left[ \sum_{i=1}^g (x_{in}^{(m)} + p_{in}^{(m)} - \phi_{n,3}^{(m)}) E_{in}^{(m)} \right] E_n^{(m)}$ $+ \left[ \sum_{i=1}^g (p_{in}^{(m)} + x_{in}^{(m)}) E_{in}^{(3)} \right] E_n^{(3)}$	$n = 1, 2$	2	Aggregate foreign currency export earnings of country n
(73)	$100\Delta B^{(n)} = E_n^{(n)} - M_m^{(n)}$	$n = 1, 2$	2	Balance of trade for country n
(73B)	$100\Delta B_{n,m} = E_{n,m} e_{n,m} - M_{m,n} m_{m,n}$	$m, n = 1, 2$ $n \neq m$ $n < m$	1	Bilateral trade balance of country n with respect to country m
(73C)	$e_{n,m} = \sum_{i=1}^g E_{i(n,m)}^{(m)} (x_{in}^{(m)} + p_{in}^{(n)})$	$n, m = 1, 2$ $n \neq m$ $n < m$	1	Value of exports of country n to country m
(73D)	$m_{m,n} = \sum_{i=1}^g M_{i(m,n)}^{(n)} (x_{im}^{(n)} + p_{im}^{(n)} + \phi_{n,m}^{(n)})$	$n, m = 1, 2$ $n \neq m$ $n < m$	1	Value of imports by country n from country m
(74)	$i_R^{(n)} = i^{(n)} - \epsilon^{(n)(2)}$	$n = 1, 2$	2	Defines real aggregate investment in each country
(75)	$\epsilon^{(n)(2)} = \sum_{j=1}^h T_j^{(n)} \pi_j^{(n)}$	$n = 1, 2$	2	Defines investment goods price index in each country
(76)	$i_R^{(n)} = c_R^{(n)} + f_R^{(n)}$	$n = 1, 2$	2	Sets relationship between real aggregate consumption and investment in each country
(77)	$l^{(n)} = \sum_{q=1}^r l_q^{(n)} \psi_{1q}^{(n)}$	$n = 1, 2$	2	Defines aggregate employment in each country
(78)	$\kappa^{(n)} = \sum_{j=1}^h k_j^{(n)} \psi_{2j}^{(n)}$	$n = 1, 2$	2	Defines aggregate capital stock in each country
(79)	$p_j^{(n)0} = h_j^{(n)0} \epsilon^{(n)(3)} + f_j^{(n)0}$	$n = 1, 2$ $j = 1, \dots, h$	2h	Allows for exogenous setting of the price of other costs in each country
(80)	$p_{1,q}^{(n)} = h_{1,q}^{(n)} \epsilon^{(n)(3)} + f_{1,q}^{(n)} + f_1^{(n)}$	$n = 1, 2$ $q = 1, \dots, r$	2r	Allows for exogenous setting of wages for each country
(81)	$gdp^{(n)} = S_c^{(n)} c_R^{(n)} + S_i^{(n)} i_R^{(n)} + S_g^{(n)} x_G^{(n)} + S_e^{(n)} + S_m^{(n)} m$	$n = 1, 2$	2	Defines real gdp in each country
(82)	$x_G^{(n)} = \sum_{s=1}^3 \sum_{i=1}^g x_{is}^{(n)(5)} S_{is}^{(n)(5)}$	$n = 1, 2$	2	Defines real other demands in each country

Total equations = 14gh + 24h + 37g + 2hr + 4r + 35

Table 3 Variables of the Two Country, Rest of the World System

Variable	Subscript Range	Number	Description
(n) (1) $x_{(is)j}$	$n = 1, 2$ $i = 1, \dots, g$ $s = 1, \dots, 3$ $j = 1, \dots, h$	6gh	Demand for good i from source s for current production in industry j and country n.
(n) $z_j$	$n = 1, 2$ $j = 1, \dots, h$	2h	Activity level in industry j and country n.
(n) $p_{is}$	$n = 1, 2$ $i = 1, \dots, g$ $s = 1, \dots, 3$	6g	Price of good i from source s in country n (denoted in country n's currency) for all end uses in country n.
(n)P $x_{vj}$	$n = 1, 2$ $v = 1, 2, 3$ $j = 1, \dots, h$	6h	Demand for primary factor v in industry j and country n.
(n)P $p_{vj}$	$n = 1, 2$ $v = 2, 3$ $j = 1, \dots, h$	4h	Price of primary factor v (v = 2 capital), (v = 3 land) in industry j and country n.
(n)P $p_1$	$n = 1, 2$	2	Price of aggregate labour in country n.
(n)P $x_{1,q,j}$	$n = 1, 2$ $q = 1, \dots, r$ $j = 1, \dots, h$	2hr	Demand for labour of occupation q in industry j and country n.
(n)P $p_{1,q}$	$n = 1, 2$ $q = 1, \dots, r$	2r	Price of labour of occupation q in country n.
(n)O $x_j$	$n = 1, 2$ $j = 1, \dots, h$	2h	Demand for other costs in industry j in country n.
(n) $x_{(in)j}$	$n = 1, 2$ $i = 1, \dots, g$ $j = 1, \dots, h$	2gh	Output of commodity i in industry j and country n.
(n) (2) $x_{(is)j}$	$n = 1, 2$ $i = 1, \dots, g$ $s = 1, \dots, 3$ $j = 1, \dots, h$	6gh	Demand for good i from source s for capital creation in industry j and country n.
(n) $y_j$	$n = 1, 2$ $j = 1, \dots, h$	2h	Investment in industry j in country n.
(n) (3) $x_{is}$	$n = 1, 2$ $i = 1, \dots, g$ $s = 1, \dots, 3$	6g	Demand for good i from source s for household consumption in country n.
(n) (3) $x_i$	$n = 1, 2$ $i = 1, \dots, g$	2g	Demand for good i undifferentiated by source for household consumption in country n.
(n) $q$	$n = 1, 2$	2	Number of households in country n.
(n) (3) $p_k$	$n = 1, 2$ $k = 1, \dots, g$	2g	Price of good k undifferentiated by source to consumers in country n.
(s) (4) $x_{is}$	$s = 1, \dots, 3$ $i = 1, \dots, g$	3g	Commodity demands by source in the rest of the world.
(4) $x_i$	$i = 1, \dots, g$	g	Commodity demands, undifferentiated by source, in the rest of the world.
(s) (4) $p_{is}$	$s = 1, \dots, 3$ $i = 1, \dots, g$	3g	Price of good i from source s in the rest of the world.
(n) (5) $x_{is}$	$n = 1, 2$ $i = 1, \dots, g$ $s = 1, \dots, 3$	6g	Other demands for good i from source s for use in country n.

Table 3 Variables of the Two Country, Rest of the World System (cont.)

Variable	Subscript Range	Number	Description
(n) $c_R$	$n = 1, 2$	2	Aggregate real household consumption in country n.
(n) (5) $f_{is}$	$n = 1, 2$ $i = 1, \dots, g$ $s = 1, \dots, 3$	6g	Country shift factors for other demands by source.
(n) (3) $\epsilon$	$n = 1, 2$	2	Index of consumer prices in country n.
(n) $c$	$n = 1, 2$	2	Aggregate nominal household expenditure in country n.
(n) O $P_j$	$n = 1, 2$ $j = 1, \dots, h$	2h	Country prices of other costs.
(n) $\pi_j$	$n = 1, 2$ $j = 1, \dots, h$	2h	Cost of capital creation in industry j in country n.
(n)* $P_{is}$	$n = 1, 2$ $i = 1, \dots, g$ $s = 1, \dots, 3$ $s \neq n$	4g	Cif import price to country n of commodity i from foreign source s, expressed in source s's currency.
$\phi_{n,m}$	$n, m = 1, 2$ $n \neq m$	2	Exchange rate between country n and country m (currency country n/ currency country m).
$\phi_{n,3}$	$n = 1, 2$	2	Exchange rate between country n and rest of the world (currency in n/currency in rest of the world).
$\phi_{3,n}$	$n = 1, 2$	2	Exchange rate between rest of the world and country n.
$t_{i(n,s)}$	$i = 1, \dots, g$ $n = 1, 2$ $s = 1, \dots, 3$ $s \neq n$	4g	One plus the ad valorem rate of protection on country n's imports of good i from source s.
$P_{i(m,n)}^T$	$m, n = 1, 2$ $i = 1, \dots, g$ $m \neq n$	2g	Price, in country m's currency, of a unit of margins required to move commodity i from country m to country n.
(n) $S_i$	$i = 1, \dots, g$ $n = 1, 2$	2g	One plus the ad valorem export (to the rest of the world) subsidy on good i in country n.
(n) $r_j$	$n = 1, 2$ $j = 1, \dots, h$	2h	Rate of return to capital in industry j and country n.
(n) $k_j(1)$	$n = 1, 2$ $j = 1, \dots, h$	2h	Future period capital stock in industry j in country n.
(n) $k_j$	$n = 1, 2$ $j = 1, \dots, h$	2h	Current capital stock in industry j and country n.
(n) $\lambda$	$n = 1, 2$	2	Economy-wide rate of return to capital for country n.
(n) $i$	$n = 1, 2$	2	Aggregate nominal investment in country n.
(n) $x_{in}$	$n = 1, 2$ $i = 1, \dots, g$	2g	Total output of good i in country n.
(n) $l_q$	$q = 1, \dots, r$ $n = 1, 2$	2r	Employment of labour of occupation q in country n.
(n) $n_j$	$n = 1, 2$ $j = 1, \dots, h$	2h	Employment of land in industry j in country n.
(n) $x_{iF}$	$n = 1, 2$ $i = 1, \dots, g$	2g	Country n's commodity import volume (from other countries and the rest of the world).

Table 3 Variables of the Two Country, Rest of the World System (cont.)

Variable	Subscript Range	Number	Description
(n) $x_{is}$	$n = 1, 2$ $s = 1, \dots, 3$ $s \neq n$ $i = 1, \dots, g$	$4g$	Country n's imports of good i from source s (other countries or rest of the world).
(n) $m$	$n = 1, 2$	2	Aggregate foreign currency (rest of the world currency units) value of imports by country n.
(n) $e$	$n = 1, 2$	2	Aggregate foreign currency (rest of the world currency units) value of exports by country n.
(n) $\Delta B$	$n = 1, 2$	2	Balance of trade in country n.
$\Delta B_{n,m}$	$m, n = 1, 2$ $m \neq n$ $n < m$	1	Bilateral trade balance of country n with respect to country m.
$e_{n,m}$	$m, n = 1, 2$ $m \neq n$ $n < m$	1	Value of exports of country n to country m (fob country n).
$m_{m,n}$	$m, n = 1, 2$ $m \neq n$ $n < m$	1	Value of imports by country n from country m (cif country n).
(n) $i_R$	$n = 1, 2$	2	Real aggregate investment in country n.
(n) (2) $\epsilon$	$n = 1, 2$	2	Investment goods price index in country n.
(n) $f_R$	$n = 1, 2$	2	Fixes relationship between aggregate real investment and consumption in country n.
(n) $l$	$n = 1, 2$	2	Aggregate employment in country n.
(n) $\kappa$	$n = 1, 2$	2	Aggregate capital stock in country n.
(n) O $f_j$	$n = 1, 2$ $j = 1, \dots, h$	$2h$	Shift factor for the price of other costs in industry j in country n.
(n) $f_{1,q}$	$n = 1, 2$ $q = 1, \dots, r$	$2r$	Shift factor for the price of labour of occupation q in country n.
(n) $f_1$	$n = 1, 2$	2	Shift factor for the price of labour in general in country n.
(n) $gdp$	$n = 1, 2$	2	Real GDP in country n.
(n) $x_G$	$n = 1, 2$	2	Real other demands in country n.

Total variables =  $14gh + 30h + 55g + 2hr + 6r + 45$

Table 4 Parameters of the Two Country, Rest of the World System

Equation	Parameter	Description	Source and Example <sup>a</sup>
(6)	$(n)(1)$ $\sigma_{(is)j}$	Substitution parameter for alternative sources of good i for use as input into current production in industry j in country n.	Econometric.
	$(n)(1)^*$ $S_{(is)j}$	CRESH share of good i from source s in industry j's purchases of i for inputs to current production in country n.	IO and econometric. (1)(1) $S_{(is)j}$ is the ijth element of $A_{11}$ divided by sum of ijth elements of $A_{11} + A_{21} + A_{31}$ $S_{(is)j} = S_{(is)j} \sigma_{(is)j} / \sum_{s=1}^3 \sigma_{(is)j}$ (n)(1) (n)(1) $\sigma_{(is)j} S_{(is)j}$ .
(7)	$(n)$ $\sigma_{vj}$	Substitution parameter for primary factor v in industry j in country n.	Econometric.
to			
(9)	$(n)^*$ $S_{vj}$	CRESH cost share of primary factor v (v=1, labour, v=2 capital, v=3 land) in total primary factor cost in industry j in country n.	IO and econometric. (1) $S_{2j}$ is jth element of $\bar{H}_1$ divided by jth column total of $\bar{G}_1 + \bar{H}_1 + \bar{I}_1$ $(n)^* (n) \bar{G}_1 + \bar{H}_1 + \bar{I}_1 (n)$ $S_{vj} = S_{vj} \sigma_{vj} / \sum_{v=1}^3 \sigma_{vj} S_{vj}$ .
	$(n)$ $\sigma_{1,q,j}$	Substitution parameter of labour of type q in industry j in country n.	Econometric.
	$(n)^*$ $S_{1,q,j}$	CRESH cost share of labour of occupation q in total labour costs of industry j in country n.	IO and econometric. (1) $S_{1,q,j}$ is the qjth element of $\bar{G}_1$ divided by jth column total of $\bar{G}_1$ . $(n)^* (n) \bar{G}_1 (n)$ $S_{1,q,j} = S_{1,q,j} \sigma_{1,q,j} / \sum_{q=1}^r \sigma_{1,q,j} S_{1,q,j}$ .
(10)		No parameters.	
(11)	$(n)$ $S_{1,q}$	Cost share of labour of type q in country n's total labour cost.	(1) IO. $S_{1,q}$ is the qth row sum of $\bar{G}_1$ divided by the sum of all elements in $\bar{G}_1$ .
(13)	$(n)T$ $\sigma_{(in)j}$	Transformation parameter for commodity i produced in the multiproduct bundle of industry j in country n.	Econometric.
	$(n)^*$ $C_{(in)j}$	CRETH revenue share of commodity i in the total revenue of industry j in country n.	IO and econometric. (1) $C_{(i1)j}$ is the ijth element of $\bar{K}_1$ divided by the jth column sum of $\bar{K}_1$ . $(n)^* (n) (n)T$ $C_{(i1)j} = C_{(i1)j} \sigma_{(in)j} / \sum_{i=1}^g \sigma_{(in)j} C_{(i1)j}$ (n)T (n) $\sigma_{(in)j} C_{(i1)j}$ .
(16)	$(n)(2)$ $\sigma_{(is)j}$	Substitution parameter for alternative sources of good i for use as an input into capital production in industry j in country n.	Econometric.

Table 4 Parameters of the Two Country, Rest of the World System (cont.)

Equation	Parameter	Description	Source and Example <sup>a</sup>
	$(n)(2)^* S_{(is)j}$	CRESH share of good i from source s in industry j's total purchases of i for inputs to capital creation in country n.	IO and econometric. $(1)(2)$ $S_{(i1)j}$ is the ijth element of $\tilde{B}_{11}$ divided by the sum of the ijth elements of $B_{11} + B_{21} + B_{31}$ . $(n)(2)^* (n)(2) (n)(2)$ $S_{(is)j} = S_{(is)j} \sigma_{(is)j} / \sum_{s=1}^3 \sigma_{(is)j} S_{(is)j}$ .
(19)	$(n)(3) \sigma_{is}$	Substitution parameter for good i from source s when used in household consumption in country n.	Econometric.
	$(n)(3)^* S_{is}$	CRESH share of the value of good i from source s in the total purchases of good i by households in country n.	IO and econometric. $(1)(3)$ $S_{i1}$ is the ith element of $\tilde{C}_{11}$ divided by the sum of the ith elements of $C_{11} + C_{21} + C_{31}$ . $(n)(3)^* (n)(3)(n)(3) 3$ $S_{is} = S_{is} \sigma_{is} / \sum_{s=1}^3 \sigma_{is} S_{is}$ . $(n)(3)(n)(3)$ $\sigma_{is} S_{(is)}$ .
(20)	$(n) \epsilon_i$	Household expenditure elasticity of good i in country n.	Econometric.
	$(n) \eta_{ik}$	Household cross price elasticities of demand for good i in general with respect to changes in the general price of good k in country n.	Econometric.
(21)	$(n)(3) S_{ks}$	Defined in (19).	IO.
(23)	$(4) \sigma_{is}$	CRESH substitution parameter in the rest of the world for good i from source s.	Econometric.
	$(s)(4)^* S_{is}$	CRESH share of the total purchases of good i in the rest of the world represented by purchases of good i from source s.	IO and econometric. $(1)(4)$ $S_{i1}$ is first element of the ith column of P divided by total of all elements in ith column of P. $(1)(4)^* (1)(4) (4) 3 (4)(1)(4)$ $S_{i1} = S_{i1} \sigma_{i1} / \sum_{s=1}^3 \sigma_{i1} S_{i1}$ .
(24)	$(n)(5) h_{is}$	Indexing parameter to set relationship between aggregate real consumption and other demands for good i from source s in country n.	Specified by user.
(25)		No parameters.	
(26)	$(n)(3) W_{is}$	Weight of good i from source s in the consumer price index in country n.	$(1)(3)$ IO. $W_{i1}$ is the ith element of $C_{11}$ divided by sum of all elements in $C_{11} + C_{21} + C_{31}$ .
(28)	$(n) C_{(in)j}$	Defined in (13).	
	$(n)(1) H_{(is)j}$	Cost share of good i from source s in the total current production costs of industry j in country n.	$(1)(1)$ IO. $H_{(i1)j}$ is ijth element of $\tilde{A}_{11}$ divided by the total costs of industry j in country 1. These are computed as the jth column sum of $A_{11} + A_{21} + A_{31} + G_1 + H_1 + I_1 + J_1$ .

Table 4 Parameters of the Two Country, Rest of the World System (cont.)

Equation	Parameter	Description	Source and Example <sup>a</sup>
(29)	(n) $H_{1,q,j}$	Cost share of labour of occupation q in the total costs of industry j in country n.	(n) IO. $H_{1,q,j}$ is the qjth element of $G_j$ divided by the total costs of industry j in country 1.
	(n)P $H_{vj}$ $v=2,3$	Cost share of primary factor v in the total costs of industry j in country n.	(1)P IO. $H_{2j}$ is the jth element of $H_1$ divided by the total costs of industry j in country 1.
	(n)o $H_j$	Cost share of other costs in the total costs of industry j.	(1)o IO. $H_j$ is jth element of $J_1$ divided by total costs of industry j in country 1.
(30)	(n)(2) $H_{(is)j}$	Share of good i from source s in the total costs of capital creation in industry j in country n.	(1)(2) IO. $H_{(i1)j}$ is ijth element of $\bar{B}_{11}$ divided by sum of the jth column elements of $\bar{B}_{11} + \bar{B}_{21} + \bar{B}_{31}$ .
(32)		No parameters.	
(34)	$\xi_{1i(m,n)}$	Share in the cif import price to country n of commodity i from country m accounted for by the domestic price of i in m.	IO. $\xi_{1i(2,1)}$ is the ith row sum of $\bar{A}_{21} + \bar{B}_{21} + \bar{C}_{21} + \bar{D}_{21} + \bar{E}_{12}$ divided by the ith row sum of $\bar{A}_{21} + \bar{B}_{21} + \bar{C}_{21} + \bar{D}_{21} + \bar{E}_{12} + \bar{L}_{1,2}$ .
	$\xi_{2i(m,n)}$	Share in the cif import price to country n of commodity i from country m accounted for by the costs of transporting i from country m to country n.	$\xi_{2i(m,n)} = 1 - \xi_{1i(m,n)}$ .
(36)-(39)		No parameters.	
(44)	(n) $Q_j$	Ratio of gross (before depreciation) to net (after depreciation) rate of return in industry j in country n.	Econometric.
(45)	(n) $\beta_j$	Elasticity of the expected rate of return schedule in industry j with respect to increases in the planned capital stock in industry j in country n.	Econometric.
(46)	(n) $G_j$	Ratio of gross investment to following period capital stock in industry j in country n.	Econometric.
(47)	(n) $T_j$	For country n share of total investment accounted for by industry j.	IO. For country 1 first sum the column elements of $\bar{B}_{11} + \bar{B}_{21} + \bar{B}_{31}$ . $T_j$ is the jth element in the array of the column sums divided by the sum of the elements in the array.
(53)	(m)(1) $B_{(in)j}$	Share of total sales of good i produced in country n absorbed by sales to industry j in country m for current production.	(1)(1) IO. $B_{(i1)j}$ is the ijth element of $\bar{A}_{11}$ divided by the total sales of country 1's good i i.e., the sum over the ith row of $\bar{A}_{11} + \bar{B}_{11} + \bar{C}_{11} + \bar{D}_{11} + \bar{A}_{12} + \bar{B}_{12} + \bar{C}_{12} + \bar{D}_{12} + \bar{E}_{13}$ .

Table 4 Parameters of the Two Country, Rest of the World System (cont.)

Equation	Parameter	Description	Source and Example <sup>a</sup>
	(m) (2) $B_{(in)j}$	Share of total sales of good i produced in country n absorbed by sales to industry j in country m for capital creation.	(1) (2) IO. $B_{(i1)j}$ is the ijth element of $\tilde{B}_{11}$ divided by the total sales of country 1's good i.
	(m) (3) $B_{in}$	Share of total sales of good i produced in country n absorbed by sales to households in country m.	(1) (3) IO. $B_{i1}$ is ith element of $\tilde{C}_{11}$ divided by the total sales of country 1's good i.
	(1) (4) $B_{in}$	Share of total sales of good i produced in country n absorbed by sales to exports (rest of the world).	(1) (4) IO. $B_{i1}$ is ith element of $\tilde{E}_{13}$ divided by the total sales of country 1's good i.
	(m) (5) $B_{in}$	Share of total sales of good i produced in country n absorbed by sales to other demands in country n.	(1) (5) IO. $B_{i1}$ is ith element of $\tilde{D}_{11}$ divided by the total sales of country 1's good i.
(54)	(n) $D_{(in)j}$	Share of the total output of domestic commodity i in country n which is produced in industry j in country n.	(1) IO. $D_{(i1)j}$ is ijth element of $\tilde{K}_1$ divided by the sum of the elements in the ith row of $\tilde{K}_1$ .
(55)	(n) $B_{1,q,j}$	Share of country n's total employment in occupation q which is accounted for by industry j.	(1) IO. $B_{1,q,j}$ is qjth element of $\tilde{G}_1$ divided by qth row sum of $\tilde{G}_1$ .
(57)		No parameters.	
(59)	(n) $F_{is}$	Share of country n's total cif imports of good i represented by imports of i from source s.	(1) IO. $F_{i2}$ is ith row sum of $\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12}$ divided by total of all elements in $\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12} + \tilde{B}_{31} + \tilde{C}_{31} + \tilde{D}_{31} + \tilde{F}_{13}$ .
(61)	(n) (1) $F_{(is)j}$	Share of the total imports of good i by country n from source s which is absorbed by industry j in country n for current production.	(1) (1) IO. $F_{(i2)j}$ is ijth element of $\tilde{A}_{21}$ divided by total imports of good i to country 1 from country 2, i.e., ith row sum of $\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21}$ .
	(n) (2) $F_{(is)j}$	Share of the total imports of good i by country n from source s which is absorbed by industry j in country n for capital creation.	(1) (2) IO. $F_{(i2)j}$ is ijth element of $\tilde{B}_{21}$ divided by total imports of good i to country 1 from country 2.
	(n) (3) $F_{is}$	Share of the total imports of good i by country n from source s which is absorbed by households in country n.	(1) (3) IO. $F_{i2}$ is ith element of $\tilde{C}_{21}$ divided by total imports of i to country 1 from country 2.
	(n) (5) $F_{is}$	Share of the total imports of good i by country n from source s which is absorbed by other demands in country n.	(1) (5) IO. $F_{i2}$ is ith element of $\tilde{D}_{21}$ divided by total imports of i to country 1 from country 2.
(69)	(n) $M_{im}$	Share of total foreign currency cost of imports to country n from country m represented by imports of good i from country m.	IO. First compute row sums of $\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{21}$ . (1) $M_{i2}$ is ith element of vector sum divided by all elements in vector sum.



Table 4 Parameters of the Two Country, Rest of the World System (cont.)

Equation	Parameter	Description	Source and Example <sup>a</sup>
(71)	(n) $M_m$	Share of total foreign currency cost of imports to country n represented by imports from country m.	(1) IO. $M_2$ is $(\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12}) / (\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12} + \tilde{A}_{31} + \tilde{B}_{31} + \tilde{C}_{31} + \tilde{D}_{31} + \tilde{F}_{13})$ .
	(n) $M_{i3}$	Share of total foreign currency cost of imports to country n from rest of the world represented by imports of good i from rest of the world.	IO. First compute row sums of $\tilde{A}_{31} + \tilde{B}_{31} + \tilde{C}_{31} + \tilde{D}_{31} + \tilde{F}_{13}$ . (n) $M_{i3}$ is ith element of vector sum divided by all elements in vector sum.
	(n) $M_3$	Share of total foreign currency cost of imports to country n represented by imports from the rest of the world.	(1) IO. $M_3$ is $(\tilde{A}_{31} + \tilde{B}_{31} + \tilde{C}_{31} + \tilde{D}_{31} + \tilde{F}_{13}) / (\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12} + \tilde{A}_{31} + \tilde{B}_{31} + \tilde{C}_{31} + \tilde{D}_{31} + \tilde{F}_{13})$ .
	(m) $E_{in}$	Share of the foreign currency exports of country n to country m accounted for by commodity i to country m.	IO. First sum $\tilde{A}_{12} + \tilde{B}_{12} + \tilde{C}_{12} + \tilde{D}_{12} + \tilde{F}_{21} - \tilde{L}_{21}$ . (2) $E_{i1}$ is ith element of vector sum divided by all elements in vector sum.
	(m) $E_n$	Share of the total foreign currency exports of country n represented by exports to country m.	(2) IO. $E_1$ is $(\tilde{A}_{12} + \tilde{B}_{12} + \tilde{C}_{12} + \tilde{D}_{12} + \tilde{F}_{21} - \tilde{L}_{21}) / (\tilde{A}_{12} + \tilde{B}_{12} + \tilde{C}_{12} + \tilde{D}_{12} + \tilde{F}_{21} - \tilde{L}_{21} + \tilde{E}_{13})$ .
	(3) $E_{in}$	Share of the foreign currency exports of country n to the rest of the world accounted for by commodity i to the rest of the world.	(3) IO. $E_{i1}$ is ith element of $\tilde{E}_{12}$ divided by total of all elements of $\tilde{E}_{13}$ .
(73)	(3) $E_n$	Share of the total foreign currency exports of country n represented by exports to the rest of the world.	(3) IO. $E_1$ is $\tilde{E}_{13} / (\tilde{A}_{12} + \tilde{B}_{12} + \tilde{C}_{12} + \tilde{D}_{12} + \tilde{F}_{21} - \tilde{L}_{21} + \tilde{E}_{13})$ .
	(n) $E$	Total foreign currency export earnings of country n.	(1) IO. $E$ is $\tilde{A}_{12} + \tilde{B}_{12} + \tilde{C}_{12} + \tilde{D}_{12} + \tilde{F}_{21} - \tilde{L}_{21} + \tilde{E}_{13}$ .
	(n) $M$	Total foreign currency import costs of country n.	(1) IO. $M$ is $\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12} + \tilde{A}_{31} + \tilde{B}_{31} + \tilde{C}_{31} + \tilde{D}_{31} + \tilde{F}_{13}$ .
(73B)	$E_{n,m}$	Fob value of exports earned by country n in exporting to country m.	IO. $E_{1,2}$ is total of all elements in $\tilde{A}_{12} + \tilde{B}_{12} + \tilde{C}_{12} + \tilde{D}_{12} + \tilde{F}_{21} - \tilde{L}_{21}$ .
	$M_{m,n}$	For country n, the cif value of its imports from country m.	IO. $M_{2,1}$ is total of all elements in $\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12}$ .

Table 4 Parameters of the Two Country, Rest of the World System (cont.)

Equation	Parameter	Description	Source and Example <sup>a</sup>
(73C)	$E_{i(n,m)}$	Share of the total value of country n's exports to country m represented by exports of good i from country n to country m.	IO. $E_{i(1,2)}$ is ith row sum of $\tilde{A}_{12} + \tilde{B}_{12} + \tilde{C}_{12} + \tilde{D}_{12} + \tilde{F}_{21} - \tilde{L}_{21}$ divided by total of all elements in $\tilde{A}_{12} + \tilde{B}_{12} + \tilde{C}_{12} + \tilde{D}_{12} + \tilde{F}_{21} - \tilde{L}_{21}$ .
(73D)	$M_{i(m,n)}$	Share of the total value of country n's imports from country m represented by country n's imports of good i from country m.	IO. $M_{i(2,1)}$ is ith row sum of $\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12}$ divided by total of all elements in $\tilde{A}_{21} + \tilde{B}_{21} + \tilde{C}_{21} + \tilde{D}_{21} + \tilde{F}_{12}$ .
(75)	$T_j^{(n)}$	Defined in (47).	
(76)		No parameters.	
(77)	$\psi_{1q}^{(n)}$	Share of country n's aggregate employment accounted for by employment of occupation q.	Econometric.
(78)	$\psi_{2j}^{(n)}$	Share of country n's aggregate capital stock accounted for by the capital stock in industry j.	Econometric.
(79)	$h_j^{(n)o}$	Other costs indexing parameter.	Set by model user.
(80)	$h_{1,q}^{(n)}$	Occupational wage indexing parameter.	Set by model user.
(81)	$S_c^{(n)}$ $S_i^{(n)}$ $S_g^{(n)}$ $S_e^{(n)}$ $S_m^{(n)}$	Respectively the shares of gross domestic product accounted for by aggregate consumption demand, investment demand, other demand, export demand and import demand in country n.	Calculated by model user. Sum of all shares is unity.
(82)	$S_{is}^{(n)(5)}$	Share of total other demands in country n accounted for by other demands of good i from source s.	(1)(5) IO. $S_{i1}$ is the ith element of $D_{11}$ divided by the total of all elements in $D_{11} + D_{21} + D_{31}$ .

<sup>a</sup> Parameter source is either econometric or from the base period input-output (IO) tables. The IO based parameters are defined in terms of the elements in Figure 1.

The first variables in Table 5 are the current capital stocks. Their exogenous treatment specifies a short run environment. (Implicit is the assumption that changes in industry capital stocks associated with the exogenous shock under study can be ignored). For long run simulations they could be replaced on the exogenous list by the industry rates of return variables,  $r_j^{(n)}$ . The underlying assumption in such a simulation would be that, over the longer term, industry capital stocks in each country adjust to levels at which they earn their exogenously specified rates of return.

The second group of variables are the  $n_j^{(n)}$ , the country employment levels of agricultural land. With the  $n_j^{(n)}$ 's exogenous, the model determines changes in the rental prices of agricultural land,  $P_{3j}^{(n)}$ , following the exogenous shock.

Next we have the wage shift variables  $f_{1,q}^{(n)}$  and  $f_1^{(n)}$ . Their inclusion on the exogenous list indicates that wages, rather than employment levels of labour are determined exogenously (demand determined labour markets). One alternative, perhaps more applicable for the long run, would be to endogenise employment levels of labour by endogenising wages. In addition, it might be considered applicable to model the labour market as being demand determined in country 1 (exogenous wages) and supply determined in country 2 (exogenous employment level).

Table 5 One Possible Selection of Exogenous Variables

Variable	Subscript Range	Number	Description
$(n)$ $k_j$	$n = 1, 2$ $j = 1, \dots, h$	$2h$	Current capital stock in industry $j$ in country $n$ .
$(n)$ $n_j$	$n = 1, 2$ $j = 1, \dots, h$	$2h$	Employment of land in industry $j$ in country $n$ .
$(n)$ $f_{1,q}$	$n = 1, 2$ $q = 1, \dots, r$	$2r$	Shift factor for the price of labour of occupation $q$ in country $n$ .
$(n)$ $f_1$	$n = 1, 2$	$2$	Shift factor for the price of labour in general in country $n$ .
$(4)$ $x_i$	$i = 1, \dots, g$	$g$	Commodity demands, undifferentiated by source, in the rest of the world.
$(3) (4)$ $p_{i3}$	$i = 1, \dots, g$	$g$	Local price of good $i$ in the rest of the world.
$(n)^*$ $p_{i3}$	$n = 1, 2$ $i = 1, \dots, g$	$2g$	Cif import price to country $n$ of commodity $i$ from the rest of the world.
$t_{i(n,s)}$	$n = 1, 2$ $s = 1, \dots, 3$ $s \neq n$ $i = 1, \dots, g$	$4g$	One plus the ad valorem rate of protection on country $n$ 's imports of good $i$ from source $s$ .
$p_{i(m,n)}^T$	$m, n = 1, 2$ $m \neq n$ $i = 1, \dots, g$	$2g$	Price, in country $m$ 's currency, of a unit of margins required to move commodity $i$ from country $m$ to country $n$ .
$(n)$ $s_i$	$n = 1, 2$ $i = 1, \dots, g$	$2g$	One plus the ad valorem export subsidy on good $i$ in country $n$ .
$(n)$ $c_R$	$n = 1, 2$	$2$	Aggregate real household consumption in country $n$ .
$(n)$ $i_R$	$n = 1, 2$	$2$	Real aggregate investment expenditure in country $n$ .
$(n) (5)$ $f_{is}$	$n = 1, 2$ $i = 1, \dots, g$ $s = 1, \dots, 3$	$6g$	Country shift factors for other demands by source.
$(n)$ $f_j$	$n = 1, 2$ $j = 1, \dots, h$	$2h$	Shift factor for the price of other costs in industry $j$ in country $n$ .
$(n)$ $q$	$n = 1, 2$	$2$	Number of households in country $n$ .
$\phi_{n,3}$	$n = 1, 2$	$2$	Exchange rate between country $n$ and the rest of the world.

Total variables =  $6h + 18g + 2r + 10$

The next groups of variables are the commodity demands undifferentiated by source, and the local commodity prices in the rest of the world. The system contains no equations to explain these variables. They would always be set exogenously.

These are followed by the  $p_{i3}^{(n)*}$ , the cif import prices for commodities imported from the rest of the world. By placing these on the exogenous list we are adopting the small country assumption for imports from the rest of the world, i.e. prices, in rest of the world currency, are independent of each of the  $n$  countries' import demands from the rest of the world. We are also allowing for the computation of answers to questions of the form: what will be the effects of projected changes in rest of the world supply prices of imports for any of the  $n$  countries?

The next group of variables are the one plus ad valorem rates of protection on country  $n$ 's imports from other countries within  $c$  and from the rest of the world. With these variables exogenous the model can be used to tackle questions of the form, what would be the macroeconomic and sectoral effects on each country of changes in protection levied by a country or countries against imports from other countries or the rest of the world. Suppose for example that country 1 was Australia and country 2 was New Zealand. Exogenous reductions in  $t_{i(1,2)}$  and  $t_{i(2,1)}$  could be undertaken to investigate the adjustment pressures on sectors in each of the two countries likely to arise from the implementation of the proposed "free trade" agreement between countries.

Next on the list are the  $p_{i(m,n)}^T$ , representing the prices of the transport margins required to move goods between countries within  $c$ . These will be always exogenous as the model now stands. Their endogenisation would require the addition of appropriate explanatory equations.

The next variables are the ad valorem export subsidies on country  $n$ 's exports to the rest of the world. Their inclusion on the exogenous list indicates that the corresponding commodity exports are determined endogenously. We see from equation (36) that with the export subsidies exogenous, movements in selling prices in the rest of the world,  $p_{in}^{(n)(4)}$ , are set by movements in domestic prices in the respective country,  $p_{in}^{(n)}$ , after taking into account exchange rate changes between country  $n$  and the rest of the world. Of course, how much is sold at these prices depends, in addition, on local prices for competing products in the rest of the world, the ease of substitution between alternative supplying sources and the size of the market in the rest of the world (see equation (23)).

We consider the next three sets of variables,  $c_R^{(n)}$ ,  $i_R^{(n)}$  and  $f_{is}^{(n)}$  together. By treating these exogenously we are making the assumption that real domestic absorption in each country is determined independently of the exogenous shock under consideration. The underlying assumption is that policy makers in each country have available macro instruments not explained in this system,

by which they can influence the components of absorption  
 $(c_R^{(n)}, i_R^{(n)} \text{ and } x_G^{(n)})$ . Alternatively, we could set  $\Delta B^{(n)}$  and  $f_R^{(n)}$  exo-  
 genously in place of  $c_R^{(n)}$  and  $i_R^{(n)}$ . In this case the model would  
 indicate the change in the level of real absorption in each  
 country which would need to accompany, say, a shift in the rest  
 of the world demand curve for commodity  $i$ , to maintain given  
 levels for the balance of trade for both countries. The allo-  
 cation of the change in absorption between investment and con-  
 sumption in each country would however, be imposed exogenously  
 via the  $f_R^{(n)}$  shift variables.

Next on the list are the variables,  $f_j^{(n)o}$ , which allow for  
 the exogenous manipulation of the price of other costs in each  
 country industry, and  $q^{(n)}$ , the number of households in each  
 country. These groups of variables are always exogenous in the  
 model system.

Table 5 concludes with two exchange rate variables, the  
 exchange rate between each country and the rest of the world.  
 The model requires that any  $c$  ( $c$  = number of countries) of the  
 $c^2 + c$  combinations of exchange rates be set exogenously. (The  
 system contains  $2 \left( \frac{1}{2}(c^2 - c) \right) + c$  i.e.,  $c^2$  exchange rate arbi-  
 trage conditions covering  $c^2 + c$  possible combinations of ex-  
 change rates). The exchange rate variables can be thought of

as the numeraire of the system.<sup>1</sup> If these exchange rates were to be endogenised a set of country price level variables (for example the consumer price indexes in each country) would need to be exogenised. Thus the model can endogenise the real exchange rate between each country and the rest of the world but not the nominal exchange rate.

The system is well suited to investigating the effects of exchange rate changes in one country on that and other countries within the group of  $c$ . Suppose for example that  $c$  included the member countries of the EEC. The effects on member countries of changing the form of the currency snake linking the currencies of participating countries could be investigated by appropriately manipulating the exchange rate relativities. Alternatively, the model could be used to answer questions relating to the degree of currency realignment necessary to achieve specified targets following a policy change or price shock in any one of the  $c$  countries or in the rest of the world.

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<sup>1</sup> In fact, this property was exploited to check the numerical construction of the model's equations. It was reassuring to note that (after implementing the model with the data and parameters in Section 5) with the closure as set out in Table 5, and with money wages fully indexed to domestic prices, a 10 per cent increase in  $\phi_{1,3}$  and  $\phi_{2,3}$  resulted in solution values of zero for all real endogenous variables and solution values of 10 for all domestic prices and monetary quantities.



## 4.2 Solution of the Model System<sup>1</sup>

The model can be written as

$$F(Z) = 0 \quad (84)$$

where  $F$  is a vector of  $v$  twice differentiable functions of the  $u$  vector  $Z$  of model variables. A model solution can be written as a system of  $v$  equations of the form

$$Y = G(X) \quad (85)$$

where  $Y$  is the  $v \times 1$  subvector of  $Z$  consisting of endogenous variables,  $X$  is the  $u-v \times 1$  subvector of exogenous variables, and  $G$  is a vector of  $v$  differential functions with the property that

$$F(X, Y) = 0 \quad (86).$$

The essence of a model solution involves computing

$$\Delta Y = G(X_F) - G(X_I)$$

where  $X_I$  and  $X_F$  are the initial and final values for the exogenous variables and  $\Delta Y$  is the change in the endogenous variables caused by the change,  $\Delta X$  in  $X$  from  $X_I$  to  $X_F$ . Given the large size of the non-linear system represented by (84), direct derivation of an explicit form for  $G$  is impractical. A computational formula for  $G_X(X)$ , the  $v \times (u-v)$  matrix of first order derivatives of  $G$ , can however be obtained.

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<sup>1</sup> This section draws extensively on Dixon et al (1982), chapter 5.

From (85) we can write that

$$dY = G_X(X) dX \quad (87)$$

which in percentage change form becomes

$$\hat{Y} = \hat{Y}^{-1} + G_X(X) \times \hat{X} \quad (88)$$

In (88)  $\hat{Y}$  and  $\hat{X}$  are the diagonal matrices formed from the vectors  $Y$  and  $X$ .

For a sufficiently small change in the exogenous variables, it follows from (86) that,

$$\frac{\partial F}{\partial X}(X_I, Y_I) dx + \frac{\partial F}{\partial Y}(X_I, Y_I) dy = 0 \quad (89)$$

which in percentage change form can be written

$$A_1 y + A_2 x = 0 \quad (90)$$

where  $A_1 = \frac{\partial F}{\partial Y}(X_I, Y_I) Y_I$  and  $A_2 = \frac{\partial F}{\partial X}(X_I, Y_I) X_I$  and the nomenclature  $(X_I, Y_I)$  indicates that  $A_1$  and  $A_2$  are evaluated at the initial values for  $X$  and  $Y$ . Note that  $A_1$  is the  $v \times v$  matrix formed by the  $v$  columns of  $A$  (see equation (83)) corresponding to the endogenous variables and  $A_2$  is the  $v \times (u-v)$  matrix formed by the  $u-v$  columns of  $A$  corresponding to the exogenous variables. From (88) and (90) we see that

$$G_X(X) = -\hat{Y}A_1^{-1} (X,Y) A_2(X,Y) \hat{X}^{-1} \quad (91)$$

$$\text{and } G_X(X_I) = -\hat{Y}A_1^{-1} (X_I,Y_I) A_2(X_I,Y_I) \hat{X}^{-1} \quad (92)$$

In the results presented in section 5 we have obtained solutions via (88) with  $G_X(X)$  evaluated according to (92). However, because the elements of  $A_1$  and  $A_2$  are assumed constant, being evaluated only at initial values for  $X$  and  $Y$ , solutions via (92) are approximate, being strictly valid only for small changes in the exogenous variables. As  $\Delta X$  increases it becomes increasingly difficult to justify the assumption that the components of  $G_X(X)$  are constant.<sup>1</sup> Dixon et al (1982, Sections 31.5 and 47.1) have investigated the linearisation errors introduced by (92) by comparing the results with those, arbitrarily close to the true solution, obtained by an n-step Euler procedure. The latter procedure involves dividing the exogenous change into

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<sup>1</sup> Recall from Table 4 that the elements of  $A_1$  and  $A_2$  are functions of the various substitution parameters and cost and sales shares computed from the base period IO data of Figures 1 and 2. Where for example the prices and quantities of commodities which are identified as model variables are changing as a result of the exogenous change then clearly the shares of these commodities in the total costs of using industries or in the total sales of producing industries are likely to change.

n small components and computing a series of solutions for these small changes, updating the A matrix (i.e., reevaluating its components on the basis of the newly computed cost and sales shares) at each step. The comparison has suggested that these errors can be safely ignored, even for fairly large exogenous changes. While the routine application of the n step update procedure would be prohibitively expensive, solutions subject to negligible linearisation error can be obtained from the application of only a two-step Euler procedure supplemented by a simple extrapolation.<sup>1</sup>

#### 5. Some Illustrative Applications of the Model

We illustrate the key mechanisms of the model by way of two simulations. Both make use of the model closure set out in Table 5, which depicts a short run adjustment environment. Recall that the salient features of this environment are, for each country, fixed exchange rates with the rest of the world

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<sup>1</sup> The relevant algebra for the updating procedure together with the extrapolation rule is given in Dixon et al (1982, pp. 204-207).

fixed industry specific capital stocks, exogenous (real) wages with employment being demand determined and exogenous real domestic absorption with the endogenous adjustment of each country's balance of trade.

In the first simulation the model is used to quantify the short run adjustment implications for the two economies of the elimination of tariffs on trade between them. Tariffs levied by each country against imports from the rest of the world remain constant however. From Figure 2 we see that country 1 levies tariffs of 20 and 57 per cent ad valorem respectively against good 1 and good 2 imported from country 2. Country 2 levies tariffs of 52 and 20 per cent respectively against goods 1 and 2 from country 1. To eliminate these tariffs requires that the model variables  $t_i(n,s)$  ( $i = 1,2; n,s = 1,2$ ) representing the percentage changes, in one plus the ad valorem tariffs, be decreased by 16.67 and 36.31 per cent respectively for country 1's imports of good 1 and 2 from country 2, and 34.29 and 16.67 per cent respectively for country 2's imports of good 1 and 2 from country 1.<sup>1,2</sup>

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<sup>1</sup>  $T = 1 + \tau$  where  $T$  is the power of the tariff and  $\tau$  the ad valorem rate. Hence  $\frac{dT}{T} = \frac{\tau}{1+\tau} \frac{d\tau}{\tau}$ . That is, an  $x$  per cent change in the ad valorem tariff rate requires that the power of the tariff be changed by  $x \frac{\tau}{1+\tau}$  per cent

<sup>2</sup> The data base specifies protection as being comprised entirely of tariffs. The tariff variables could however be interpreted more broadly to imply the tariff equivalent of restrictive quotas should these be used as a form of protection between the countries.

The second simulation is in two parts. Part A involves a uniform 10 per cent increase in the components of real aggregate domestic absorption (household consumption, investment and government spending) in country 1. Underlying this shock is the idea that country 1 undertakes a Keynesian-style stimulation of its domestic economy in an effort to increase growth and employment. This stimulus however evokes no active policy response from country 1's close trading partner (country 2) which does no more than hold its real domestic absorption constant in the face of the stimulation in country 1.<sup>1</sup> We use the model to determine the consequences for both countries of the unilateral stimulation of country 1's economy. However, as can be seen from the results in Table 6, the aggregate demand stimulation in country 1 increases the rate of consumer price inflation in country 2. Part B of the simulation involves country 2 in addition revaluing its exchange rate against country 1 by an amount just sufficient to hold its rate of inflation at the level prevailing before the aggregate demand expansion undertaken by country 1.<sup>2</sup>

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<sup>1</sup> With a good deal of imagination we might presume that country 1 is Keynesian France and country 2 is neoclassical Germany.

<sup>2</sup> In terms of the exogenous variable selection in Table 5 part B involves the replacement of  $\phi_{2,3}$  with  $\phi_{2,3}^{(2)(3)}$  (assigned a value of zero). The bilateral exchange rate,  $\phi_{2,1}$ , is endogenised via (38).

### 5.1 Parameter Settings

As noted earlier, the elements of the A matrix in equation (83) are functions of the various substitution parameters, indexing parameters, and cost and sales shares listed in Table 4. The cost and sales shares are computed from the hypothetical data in Figure 2. All indexing parameters are set to unity.

In implementing the model system with actual country data, values for the substitution parameters would ideally be based on econometric estimates. Here we simply assign values to these parameters according to the following procedure. On the input side we set all substitution parameters  $(\sigma_{(is)j}^{(n)(1)}, \sigma_{vj}^{(n)}, \sigma_{1,q,j}^{(n)}, \sigma_{(is)j}^{(n)(2)})$  to unity. This is tantamount to assuming that the flexible CRESH nests of the industry production functions for current production and capital creation as set out in (3), (4), (5) and (15) collapse to the restrictive Cobb-Douglas form. On the commodity output side we set all components of  $\sigma_{(in)j}^{(n)T}$  to 1.0. This is equivalent to replacing the flexible CRETH specification in (12) by a transformation frontier described by a quarter ellipse (see Dixon et al (1982, p. 32)). In our treatment of household consumption we assume, by setting all components of  $\sigma_{is}^{(n)(3)}$  to 1.0, that the CRESH aggregation function given by (17) is replaced by the Cobb-Douglas form. Further, by setting all household expenditure elasticities,  $\epsilon_i^{(n)}$ , to 1.0, all own price elasticities,

$\eta_{ii}^{(n)}$ , to -1.0 and all cross price elasticities,  $\eta_{ij}^{(n)}$   $j \neq i$ , to 0.0 we assume that the underlying household utility functions in each country are of the Cobb-Douglas form. Substitution parameters between alternative sources of supply in the rest of the world,  $\sigma_{is}^{(4)}$ , are set to unity for all  $i$  and  $s$  thus reducing the CRESH specification in (22) to the Cobb-Douglas form. An implication of the above simplifications is that all the modified share coefficients (denoted with an asterisk) in Tables 2 and 4 collapse to the corresponding conventional shares.

Of the remaining parameters denoted in Table 4 as econometric, the investment parameters  $Q_j^{(n)}$ ,  $\beta_j^{(n)}$  and  $G_j^{(n)}$  were assigned values of 2.0, 20.0 and 0.10 respectively for all  $i, j$ , and  $n$ . Proxy values for the  $\psi_{1j}^{(n)}$  and  $\psi_{2j}^{(n)}$  were calculated as the share of country  $n$ 's aggregate wage bill accounted for by wages in occupation  $q$  and as the share of country  $n$ 's aggregate rents on capital accounted for by capital rents in industry  $j$  respectively.

## 5.2 Results: Elimination of Between Country Tariffs

Projections for selected variables are given in columns 1 and 2 of Table 6. The key to understanding these results is the effect of the tariff abolition on the domestic price level in each country. The domestic consumer price index declines



by 3.1 per cent in country 1 and by 4.3 per cent in country 2.<sup>1</sup> With fixed exchange rates with the rest of the world this implies a corresponding decrease in domestic relative to world prices, providing a substantial boost to the competitiveness, against the rest of the world, of the traded goods sectors in both countries. The tariff elimination also causes large decreases (20-37 per cent) in the prices at which goods are traded between countries, which, with the assumed Armington substitution elasticities of unity), causes substantial increases in commodity trade flows between countries at the expense of rest of the world suppliers. The net outcome of the increased trade between countries and the increased competitiveness against the rest of the world is, for both countries, a faster growth rate of foreign currency export earnings over import expenditures. With fixed domestic absorption and real wages this results in increased GDP and employment demand.

The trade flow results indicate a massive boost to bilateral trade, with bilateral import and export volumes increasing by 15 to 29 per cent. Exports to the rest of the world expand while imports from the rest of the world contract, reflecting the enhanced international competitiveness in both countries.

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<sup>1</sup> The main reason for the larger consumer price index fall in country 2 than in country 1 lies in country 2's strong consumption expenditure bias (64 per cent of base period expenditure) towards commodities imported from country 1 (the local prices for which are falling sharply because of the abolition of import tariffs). In comparison, only 2 per cent of country 1's household expenditure is on products from country 2.

Moving to the output results it is interesting to note that all industries and commodities in both countries expand output even though some are strongly import competing.<sup>1</sup> While a particular import competing industry loses market share to imports from the other country, it is able to win back market share from imports from the rest of the world.

The commodity price projections indicate large declines in the local prices of goods imported from the other country, i.e., the goods directly affected by the abolition of tariffs. The local prices of domestically produced commodities fall in both countries reflecting the reduction in the overall price level in these countries. Note that the reductions in cif import prices are a little less than the reductions in the corresponding domestic prices in the country of origin reflecting the influence of between country margins, the prices of which are assumed to remain constant.

### 5.3 Results: Domestic Demand Stimulation in Country 1

Projections are contained in columns 3 and 4 of Table 6. Again it is appropriate to focus first on the domestic price index projections. With fixed supplies of capital and land in country 1 implying an upward sloping supply curve for the

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<sup>1</sup> For strongly export oriented industries such as industry 1 in country 1 and industry 2 in country 2 the output expansion is due primarily to the increased export sales of their respective commodities.

Table 6 Selected Short Run Projections for Two Simulations <sup>a</sup>

Variable	1. Tariff elimination between the two countries		2. Aggregate Demand Stimulation in Country 1			
			A. 10 per cent absorption increase in country 1		B. 10 per cent absorption increase in country 1 plus 1.84 per cent revaluation of $\phi_{2,1}$	
	Country 1	Country 2	Country 1	Country 2	Country 1	Country 2
<u>Macro</u>						
Real GDP	1.42	1.62	5.72	0.21	5.71	0.20
Aggregate real domestic absorption	0.00(EX) <sup>c</sup>	0.00(EX) <sup>c</sup>	10.00(EX) <sup>c</sup>	0.00(EX) <sup>c</sup>	0.00(EX) <sup>c</sup>	0.00(EX) <sup>c</sup>
Aggregate exports <sup>b</sup>	4.82	4.90	1.28	2.25	1.29	2.24
Aggregate imports <sup>b</sup>	2.10	1.72	9.51	1.84	9.52	1.84
Overall balance of trade <sup>b</sup>	3.27	3.98	-9.87	0.51	-9.89	0.50
Exports of country 1 to country 2 <sup>b</sup>	22.65		3.29		3.29	
Imports by country 1 from country 2 <sup>b</sup>	25.42		10.64		6.99	
Country 1's trade balance with country 2 <sup>a</sup>	-0.21		-1.70		-0.82	
Aggregate employment	4.76	7.63	6.09	0.92	6.09	0.90
Consumer price index	-3.09	-4.25	7.40	1.84	7.42	0.00(EX) <sup>c</sup>
Exchange rate $\phi_{2,1}$		0.00		0.00		-1.84
Exchange rate $\phi_{1,3}$	0.00(EX) <sup>c</sup>		0.00(EX)		0.00(EX) <sup>c</sup>	
Exchange rate $\phi_{2,3}$		0.00(EX)		0.00(EX) <sup>c</sup>		-1.84
<u>Trade Volumes</u>						
Exports of good 1 to rest of world	0.94	5.72	-6.62	-1.41	-6.63	-1.49
Exports of good 2 to rest of world	3.60	0.42	-9.52	-1.98	-9.54	-1.98
Imports of good 1 from rest of world	-2.31	-4.43	12.82	1.52	12.86	1.52
Imports of good 2 from rest of world	-4.33	-1.35	8.97	2.25	8.98	2.45
Imports of good 1 from country 2	15.63		11.33		11.12	
Exports of good 1 to country 1		15.63		11.33		11.12
Imports of good 2 from country 2	28.79		8.04		7.98	
Exports of good 2 to country 1		28.79		8.04		7.98
Exports of good 1 to country 2	25.73		-4.41		-4.43	
Imports of good 1 from country 1		25.73		-4.41		-4.43
Exports of good 2 to country 2	15.08		-4.51		-4.53	
Imports of good 2 from country 1		15.08		-4.51		-4.53
<u>Outputs</u>						
Industry 1	2.53	4.56	1.24	0.77	1.24	0.76
Industry 2	2.05	3.00	5.40	0.21	5.39	0.21
Commodity 1	2.86	2.14	1.82	0.72	1.81	0.71
Commodity 2	1.03	4.25	6.14	0.36	6.13	0.35
<u>Prices</u>						
Local price of domestic good 1	-1.04	-5.82	7.35	2.14	7.37	0.30
Local price of domestic good 2	-3.64	-0.46	9.74	2.20	9.76	0.36
Local price of good 1 imported from other country	-21.52	-35.29	1.79	7.04	2.09	5.22
Local price of good 2 imported from other country	-36.75	-19.70	2.10	8.12	2.18	6.29
Cif import price of good 1 from other country	-4.85	-1.00	1.79	7.04	0.25	7.06
Cif import price of good 2 from other country	-0.44	-3.03	2.10	8.11	0.34	8.13

<sup>a</sup> All projections are in percentage changes except for the trade balance variables which are in first differences at the base period valuation units. - <sup>b</sup> Expressed in rest of the world currency units. - <sup>c</sup> Denotes exogenous setting to zero.

economy as a whole the increased domestic expenditure increases the price level in country 1 relative to that in the rest of the world and in country 2. This causes a deterioration in the cost-price competitiveness of the traded goods sector in country 1 against country 2 and against the rest of the world leading to a sharp decline in exports, increase in imports and movement to balance of trade deficit. This leakage on the balance of trade ensures that the increased GDP that results (5.72 per cent) is far below the exogenous increase in absorption.

At the given degree of imperfect substitution between commodities from alternative sources, part of the price inflation originating in country 1 is transmitted to country 2. (Country 2's index of consumer prices increases by 1.8 per cent). Hence, with a fixed exchange rate, country 2 also suffers a deterioration in its competitiveness relative to the rest of the world. This goes some way towards offsetting, in terms of its effect on output and employment, country 2's vastly increased competitiveness relative to country 1. The net effect for country 2 is an increase in GDP of only 0.2 per cent and an increase in aggregate employment of 0.9 per cent.

The commodity trade results emphasise the relative shifts in competitiveness implied by the changes in price level be-

tween the three trading blocks. Thus country 1's exports to the rest of the world are hard hit with its exports to country 2 suffering to a smaller extent because of the price inflation in country 2. Import penetration into country 1 from the rest of the world is particularly high. Bilateral trade is now strongly in favour of country 2.

The output results follow from the base period commodity sales patterns. Thus for country 1 the strongly export oriented industry 1 performs poorly relative to the more domestic oriented industry 2. Although the main output of industry 2 (commodity 2) loses domestic market share to imports of good 2 from country 2 and from the rest of the world this is outweighed by the expansion in the domestic market it supplies. (92 per cent of the base period sales of commodity 2 are to domestic absorption in country 1 which is expanding by 10 per cent). Industry 2 in country 2, whose output is essentially the export (to the rest of the world) oriented commodity 2 performs poorly relatively to industry 1 whose output is more balanced between the export oriented commodity 1 and the import competing commodity 2.

In summary then, given the assumed data base and parameter settings, a unilateral demand stimulus by keynsian country 1 is of limited effectiveness in boosting that country's level of economic activity, with higher inflation costing export sales and ensuring that a substantial part of the domestic demand

stimulus is met by imports. The stimulus is also harmful in terms of inflation, for its close trading partner, orthodox country 2. Given its preference for price stability, orthodox country 2 must invoke compensatory policies (for example a revaluation of its exchange rate with country 1 or a contraction in its real absorption) to "sterilize" the effect on its domestic price level. A second worry for country 2 is that the large improvement in its trade balance with country 1 might lead to retaliation by country 1, in the form of say trade barriers to imports from country 2, to the detriment of both countries.

Columns 5 and 6 of Table 6 contain the results of the demand stimulation in country 1 with an offsetting revaluation of country 2's exchange rate against country 1 and hence the rest of the world such that the percentage change in country 2's consumer price index remains constant. The required exchange rate revaluation turns out to be 1.84 per cent. The projections for most variables in columns 5 and 6 are very close to those for columns 3 and 4. There are however a number of prominent exceptions. As a result of country 1's devaluation relative to country 2 imports by country 1 from country 2 and the bilateral trade deficit of country 1 with respect to country 2 are both sharply reduced. Local prices of goods in country 1 imported from country 2 are now higher while cif import prices are lower.

## 6. Concluding Remarks

This paper has outlined a multicountry multisector general equilibrium model capable of endogenising, on the basis of standard neoclassical postulates of price responsiveness and substitution, commodity trade flows between demand categories in each country and between each country and the rest of the world. Despite its unavoidable notational complexity the system is simple and orthodox representing a straight forward extension of the single country ORANI framework developed for the Australian economy. As such it is tightly constrained by microeconomic theory.

Empirical progress to date has involved only the testing of a very restricted version of the system using hypothetical model coefficients. Nevertheless, preliminary results are encouraging. Subject to the restricted version continuing to perform plausibly on a range of policy shocks with different model closures, its development will proceed with a view to implementing the model system initially for a subset of EEC countries and later for the entire EEC.

High on the agenda for improving the theoretical specification of the model system is a macro-monetary closure for each country together with appropriate cross-country linkages to capture the workings of the European Monetary System. Experience to date suggests that, working within the linear framework,

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such theoretical improvements will be relatively straight forward. By far the most difficult task in implementing the model system will involve the construction of a suitably integrated data base for member countries and the estimation of key behavioural parameters.

The potential range of policy applications of such a system is enormous. A number of policy issues to which the system could be addressed have been mentioned in the paper.



### References

- Artus, J.R. and McGuirk, A.K., "A Revised Version of the Multilateral Exchange Rate Model", International Monetary Fund Staff Papers, Washington, D.C., 1981, pp. 275-309.
- Deardorff, A.V. and Stern, R.M., "A Disaggregated Model of World Production and Trade. An Estimate of the Impact of the Tokyo Round", Journal of Policy Modeling, Vol. 3, No. 2, 1981, pp. 127-152.
- Dick, H., Gupta, S., Vincent, D. and Voigt, H., "Comparing the Effects of the Second OPEC Oil Price Shock on Income and Resource Allocation in Four Oil-Poor Developing Economies: Ivory Coast, Kenya, South Korea, Turkey", Kiel Institute of World Economics, Working Paper No. 123, August 1981, pp. 48 (revised version forthcoming in Energy Economics, October 1983).
- Dick, H., Gerken, E. and Vincent, D.P., "Anpassungsstrategien der Entwicklungsländer unter veränderten Rahmenbedingungen", paper presented to twenty-second annual conference of the Gesellschaft für Wirtschafts- und Sozialwissenschaften des Landbaues e.V. (German agricultural economics society), Stuttgart, October 1981. Published in Landwirtschaft unter veränderten Rahmenbedingungen, pp. 385-403, Landwirtschaftsverlag GmbH, Münster-Hiltrup 1982(a).
- Dick, H., Gerken, E. and Vincent, D.P., "The Benefits of the CAP for Developing Countries: a Case Study of the Ivory Coast", European Review of Agricultural Economics 9 (2), November 1982(b).
- Dick, H., Gupta, S., Mayer, T. and Vincent, D., "Indexation of UNCTAD Core Commodity Prices by Buffer Stocks or Export Quotas? A Comparison of the Benefits for Two Developing Economies", Journal of Development Economics, 11, 1982, pp. 379- 401, (a).
- Dick, H., Gupta, S., Mayer, T. and Vincent, D., "The Short-run Impact of Fluctuating Commodity Prices on Three Developing Economies: Colombia, Ivory Coast and Kenya", Kiel Institute of World Economics, Working Paper No. 155, August 1982, pp. 27, forthcoming in World Development, (b).

- Dick, H., Gerken, E., Mayer, T. and Vincent, D., "Stabilization Policies in Developing Countries: a Case Study of Chile", Kiel Institute of World Economics, Working Paper No. 144, June 1982. Forthcoming in Journal of Development Economics.
- Dixon, P.B., Parmenter, B.R., Sutton, J. and Vincent, D.P., ORANI: A Multisectoral Model of the Australian Economy, North-Holland Publishing Company, 1982.
- Gerken, E. and Vincent, D.P., "Die Wirkung von Rohstoffpreiserhöhungen auf rohstoffarme Entwicklungsländer: Das Beispiel Südkorea", Die Weltwirtschaft, 1981, Heft 1, pp. 108-121.
- Gunning, J.W., Carrin, G., Waelbroeck, J. and Associates, "Growth and Trade of Developing Countries: a General Equilibrium Analysis", Discussion Paper 8210, Centre d'Economie Mathématique et d'Econometrie, Université Libre de Bruxelles, 1982.
- Hanoch, G., "CRESH Production Functions", Econometrica, 39, September 1971, pp. 695-712.
- Vincent, D.P., "Quantifying the Effects of Higher World Oil Prices on Resource Allocation and Living Standards in an Energy Poor Open Economy: the Case of Korea", Developing Economies, Vol. XX, No. 3, September 1982, pp. 279-300, (a).
- Vincent, D.P., "Exchange Rates, Monetary Policy and Wages: a Case Study of Chile", Kiel Institute of World Economics, Working Paper No. 164, January 1983.
- Walley, J., "General Equilibrium Analysis of US-EEC Japanese Trade and Trade Distorting Policies: A Model and Some Initial Findings", Economie Appliquée (33) (1980), pp. 191-230.

# Appendix      The Generalised, Multicountry, Rest of the World System

Table A1 contains the equations of the c country, rest of the world system. The system's variables are defined in Table A2. The parameters can be identified from the description in Table 4. The equation and variable count is as follows;

Variables:  $(2c^2 + 3c)gh + 15ch + (8c^2 + 10c + 3)g + chr + 3cr + \frac{35}{2}c + \frac{5}{2}c^2$

Equations:  $(2c^2 + 3c)gh + 12ch + (5c^2 + 8c + 1)g + chr + 2cr + 15c + \frac{5}{2}(c^2 - c)$

Hence to close the model requires setting values for

$3ch + (3c^2 + 2c + 2)g + cr + \frac{10}{2}c$  variables endogenously.

Restating the exogenous variable list in Table 5 in terms of the variable dimensions of the multicountry system gives;

Variable	Subscript Range	Number
(n) $k_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	ch
(n) $n_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	ch
(n) $f_{1,q}$	$n = 1, \dots, c$ $q = 1, \dots, r$	cr
(n) $f_1$	$n = 1, \dots, c$	c
(4) $x_i$	$i = 1, \dots, g$	g
(w) (4) $p_{iw}$	$i = 1, \dots, g$	g
(n)* $p_{iw}$	$n = 1, \dots, c$ $i = 1, \dots, g$	cg

Variable	Subscript Range	Number
$t_{i(n,s)}$	$n = 1, \dots, c$ $s = 1, \dots, c+1$ $s \neq n$ $i = 1, \dots, g$	$c^2 g$
$p_{i(m,n)}^T$	$m, n = 1, \dots, c$ $m \neq n$ $i = 1, \dots, g$	$(c^2 - c)g$
$(n) S_i$	$n = 1, \dots, c$ $i = 1, \dots, g$	$cg$
$(n) c_R$	$n = 1, \dots, c$	$c$
$(n) i_R$	$n = 1, \dots, c$	$c$
$(n) (5) f_{is}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $s = 1, \dots, c+1$	$c(c+1)g$
$(n) o f_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	$ch$
$(n) q$	$n = 1, \dots, c$	$c$
$\phi_{n,w}$	$n = 1, \dots, c$	$c$

Total variables =  $3ch + (3c^2 + 2c + 2)g + cr + 5c$

Table A1 Equations of the Multicountry, Rest of the World System

Identifier	Equation	Subscript Range	Number	Description
(6)	$x_{(is)j} = z_j - \sigma_{(is)j} (p_{is} - \sum_{s=1}^{c+1} S_{(is)j} p_{is})$	$i=1, \dots, g$ $s=1, \dots, c+1$ $j=1, \dots, h$ $n=1, \dots, c$	$c(c+1)gh$	Country demands for intermediate inputs by source for current production.
(7)	$x_{vj} = z_j - \sigma_{vj} (p_{vj} - \sum_{v=2}^3 S_{vj} p_{vj} - S_{1j} p_1)$	$v=2, 3$ $j=1, \dots, h$ $n=1, \dots, c$	$2ch$	Country demands for capital and land.
(8)	$x_{1j} = z_j - \sigma_{1j} (p_1 - \sum_{v=2}^3 S_{vj} p_{vj} - S_{1j} p_1)$	$j=1, \dots, h$ $n=1, \dots, c$	$ch$	Country demands for labour in general.
(9)	$x_{1,q,j} = x_{1,j} - \sigma_{1,q,j} (p_{1,q} - \sum_{q=1}^r S_{1,q,j} p_{1,q})$	$n=1, \dots, c$ $q=1, \dots, r$ $j=1, \dots, h$	$chr$	Country demands for labour by occupation and industry.
(10)	$x_j = z_j$	$n=1, \dots, c$	$ch$	Country demands for other costs.
(11)	$p_1 = \sum_{q=1}^r p_{1,q} S_{1,q}$	$n=1, \dots, c$	$c$	Country price of labour in general.
(13)	$x_{(in)j} = z_j + \sigma_{(in)j} (p_{in} - \sum_{i=1}^g C_{(in)j} p_{in})$	$i=1, \dots, g$ $j=1, \dots, h$ $n=1, \dots, c$	$cgh$	Country supplies of commodities by industries.
(16)	$x_{(is)j} = y_j - \sigma_{(is)j} (p_{is} - \sum_{s=1}^{c+1} S_{(is)j} p_{is})$	$i=1, \dots, g$ $s=1, \dots, c+1$ $j=1, \dots, h$ $n=1, \dots, c$	$c(c+1)gh$	Country demands for intermediate inputs by source for capital creation.
(19)	$x_{is} = x_i - \sigma_{is} (p_{is} - \sum_{s=1}^{c+1} S_{is} p_{is})$	$i=1, \dots, g$ $s=1, \dots, c+1$ $n=1, \dots, c$	$c(c+1)g$	Country household demands by commodities classified by source.
(20)	$x_i - q = \varepsilon_i (c - q) + \sum_{k=1}^g \eta_{ik} p_k$	$i=1, \dots, g$ $n=1, \dots, c$	$cg$	Country household demands for commodities undifferentiated by source.
(21)	$p_k = \sum_{s=1}^{c+1} S_{ks} p_{ks}$	$k=1, \dots, g$ $n=1, \dots, c$	$cg$	Country general price of each commodity to households.
(23)	$x_{is} = x_i - \sigma_{is} (p_{is} - \sum_{s=1}^{c+1} S_{is} p_{is})$	$i=1, \dots, g$ $s=1, \dots, c+1$	$(c+1)g$	Commodity demands in the rest of the world by supplying source.
(24)	$x_{is} = c_R h_{is} + f_{is}$	$i=1, \dots, g$ $s=1, \dots, c+1$ $n=1, \dots, c$	$c(c+1)g$	Country other demands for commodities by source.
(25)	$c_R = c - \varepsilon$	$n=1, \dots, c$	$c$	Defines country real aggregate consumption.
(26)	$\varepsilon = \sum_{i=1}^g \sum_{s=1}^{c+1} W_{is} p_{is}$	$n=1, \dots, c$	$c$	Defines country index of consumer prices.
(28)	$\sum_{i=1}^g p_{in} C_{(in)j} = \sum_{i=1}^g \sum_{s=1}^{c+1} p_{is} H_{(is)j} + \sum_{q=1}^r p_{1,q} r(n)P_{1,q}$	$j=1, \dots, h$ $n=1, \dots, c$	$ch$	Zero pure profits in production in each country.
(30)	$\pi_j = \sum_{i=1}^g \sum_{s=1}^{c+1} p_{is} H_{(is)j}$	$j=1, \dots, h$ $n=1, \dots, c$	$ch$	Zero pure profits in capital creation in each country.
(32)	$p_{is} = p_{is} + \phi_{n,s} + t_{i(n,s)}$	$i=1, \dots, g$ $n=1, \dots, c$ $s=1, \dots, c+1$ $s \neq n$	$c^2g$	Zero pure profits in importing by each country.

Table A1 Equations of the Multicountry, Rest of the World System (cont.)

Identifier	Equation	Subscript Range	Number	Description
(34)	$P_{im}^{(n)*} = \zeta_{1i(m,n)} P_{im}^{(m)} + \zeta_{2i(m,n)} P_{i(m,n)}^T$	$i=1, \dots, g$ $n=1, \dots, c$ $m=1, \dots, c$ $m \neq n$	$c(c-1)g$	Zero pure profits in the transport of goods between countries.
(36)	$P_{in}^{(n)(4)} + s_i^{(n)} + \phi_{n,w}^{(n)} = P_{in}^{(n)}$	$i=1, \dots, g$ $n=1, \dots, c$	$cg$	Zero pure profits in exporting by each country.
(37)	$\phi_{w,n} + \phi_{n,w} = 0.0$	$n=1, \dots, c$	$c$	Exchange rate arbitrage
(38)	$\phi_{n,m} + \phi_{m,w} = \phi_{n,w}$	$n=1, \dots, c$ $m=1, \dots, c$ $n \neq m$ $n < m$	$\frac{1}{2}(c^2 - c)$	
(39)	$\phi_{n,m} + \phi_{m,n} = 0.0$	$n=1, \dots, c$ $m=1, \dots, c$ $n \neq m$ $n < m$	$\frac{1}{2}(c^2 - c)$	
(44)	$r_j^{(n)} = Q_j^{(n)} (P_{2j}^{(n)} - \pi_j^{(n)})$	$j=1, \dots, h$ $n=1, \dots, c$	$ch$	Rates of return on capital in each industry and country.
(45)	$-B_j^{(n)} (k_j^{(n)}(1) - k_j^{(n)}) + r_j^{(n)} = \lambda^{(n)}$	$n=1, \dots, c$ $j=1, \dots, h$	$ch$	Equality of rates of return across industries in each country.
(46)	$k_{j(1)}^{(n)} = k_j^{(n)} (1 - G_j^{(n)}) + y_j^{(n)} G_j^{(n)}$	$j=1, \dots, h$ $n=1, \dots, c$	$ch$	Country capital accumulation by industry.
(47)	$\sum_{j=1}^h (\pi_j^{(n)} + y_j^{(n)}) T_j^{(n)} = i^{(n)}$	$n=1, \dots, c$	$c$	Country investment budget.
(53)	$x_{in}^{(n)} = \sum_{j=1}^h \sum_{m=1}^c x_{(in)j}^{(m)(1)} B_{(in)j}^{(m)(1)} + \sum_{j=1}^h \sum_{m=1}^c x_{(in)j}^{(n)(2)} B_{(in)j}^{(n)(2)}$ $B_{(in)j}^{(m)(2)} + \sum_{m=1}^c x_{in}^{(m)(3)} B_{in}^{(m)(3)} + x_{in}^{(n)(4)} B_{in}^{(n)(4)} + \sum_{m=1}^c x_{in}^{(m)(5)} B_{in}^{(m)(5)}$	$i=1, \dots, g$ $n=1, \dots, c$	$cg$	Demand equals supply for domestic commodities for each country.
(54)	$x_{in}^{(n)} = \sum_{j=1}^h x_{(in)j}^{(n)} D_{(in)j}^{(n)}$	$i=1, \dots, g$ $n=1, \dots, c$	$cg$	Output of commodities by country.
(55)	$l_q^{(n)} = \sum_{j=1}^h x_{1,q,j}^{(n)} B_{1,q,j}^{(n)}$	$n=1, \dots, c$ $q=1, \dots, r$	$cr$	Supply equals demand for occupational labour in each country.
(56)	$k_j^{(n)} = x_{2j}^{(n)}$	$n=1, \dots, c$ $j=1, \dots, h$	$ch$	Supply equals demand for capital in each country.
(57)	$n_j^{(n)} = x_{3j}^{(n)}$	$n=1, \dots, c$ $j=1, \dots, h$	$ch$	Supply equals demand for land by country.
(59)	$x_{iF}^{(n)} = \sum_{s=1}^{c+1} x_{is}^{(n)} F_{is}^{(n)}$ $s \neq n$	$i=1, \dots, g$ $n=1, \dots, c$	$cg$	Import volume by country $n$ from other countries and the rest of the world.

Table A1 Equations of the Multicountry, Rest of the World System (cont.)

Identifier	Equation	Subscript Range	Number	Description
(61)	$x_{is} = \sum_{j=1}^{h(n)(1)} x_{(is)j}^{(n)(1)} F_{(is)j}^{(n)(1)} + \sum_{j=1}^{h(n)(2)} x_{(is)j}^{(n)(2)} F_{(is)j}^{(n)(2)}$ $+ x_{is}^{(n)(3)} F_{is}^{(n)(3)} + x_{is}^{(n)(5)} F_{is}^{(n)(5)}$	$i=1, \dots, g$ $n=1, \dots, c$ $s=1, \dots, c+1$ $s \neq n$	$c^2 g$	Import volume by country n from source s.
(69)	$m = \sum_{m=1}^c \left[ \sum_{i=1}^g (x_{im} + p_{im} + \phi_{m,w}) M_{im} \right] M_m^{(n)}$ $+ \left[ \sum_{i=1}^g (x_{iw} + p_{iw}) M_{iw} \right] M_w^{(n)}$	$n=1, \dots, c$	c	Foreign (rest of the world) currency cost of imports for each country.
(71)	$e = \sum_{m=1}^c \left[ \sum_{i=1}^g (x_{in} + p_{in} - \phi_{n,w}) E_{in} \right] E_n^{(m)}$ $+ \sum_{i=1}^g (p_{in} + x_{in}) E_{in}^{(w)} E_n^{(w)}$	$n=1, \dots, c$	c	Aggregate foreign currency export earnings by each country.
(73)	$100\Delta B = E_e^{(n)} - M_m^{(n)}$	$n=1, \dots, c$	c	Balance of trade for each country.
(73B)	$100\Delta B_{n,m} = E_{n,m}^{e_{n,m}} - M_{m,n}^{m_{m,n}}$	$m, n=1, \dots, c$ $n \neq m$ $n < m$	$\frac{1}{2}(c^2 - c)$	Bilateral trade balance of country n with respect to country m.
(73C)	$e_{n,m} = \sum_{i=1}^g E_{i(n,m)}^{(m)} (x_{in} + p_{in})^{(n)}$	$n, m=1, \dots, c$ $n \neq m$ $n < m$	$\frac{1}{2}(c^2 - c)$	Value of exports of country n to country m.
(73D)	$m_{m,n} = \sum_{i=1}^g M_{i(m,n)}^{(n)} (x_{im} + p_{im} + \phi_{n,m})^{(n)*}$	$n, m=1, \dots, c$ $n \neq m$ $n < m$	$\frac{1}{2}(c^2 - c)$	Value of imports by country n from country m.
(74)	$i_R = i - \epsilon$	$n=1, \dots, c$	c	Defines real aggregate investment in each country.
(75)	$\epsilon = \sum_{j=1}^{n(n)} T_j \pi_j^{(n)}$	$n=1, \dots, c$	c	Defines investment goods price index in each country.
(76)	$i_R = c_R + f_R^{(n)}$		c	Sets relationship between real aggregate consumption and investment in each country.
(77)	$l = \sum_{q=1}^r l_q \psi_{1q}^{(n)}$	$n=1, \dots, c$	c	Defines aggregate employment in each country.
(78)	$k = \sum_{j=1}^h k_j \psi_{2j}^{(n)}$	$n=1, \dots, c$	c	Defines aggregate capital stock in each country.
(79)	$p_j = h_j^{(n)o} \epsilon^{(n)(3)} + f_j^{(n)o}$	$n=1, \dots, c$ $j=1, \dots, h$	ch	Allows for the exogenous setting of the price of other costs in each country.
(80)	$p_{1,q} = h_{1,q}^{(n)} \epsilon^{(n)(3)} + f_{1,q}^{(n)} + f_1^{(n)}$	$n=1, \dots, c$ $q=1, \dots, r$	cr	Allows for exogenous setting of wages in each country.

Table A1 Equations of the Multicountry, Rest of the World System (cont.)

Identifier	Equation	Subscript Range	Number	Description
(81)	$\begin{matrix} (n) & (n) & (n) & (n) & (n) & (n) & (n) & (n) \\ \text{gdp} & = & S_c & c_R & + & S_i & i_R & + & S_g & x_G & + & S_e & e \\ & & & & & (n) & (n) & & & & & & \\ & & & & & + & S_m & m \end{matrix}$	$n=1, \dots, c$	$c$	Defines real GDP in each country.
(82)	$\begin{matrix} (n) & c+1 & g & (n) & (5) & (n) & (5) \\ x_G & = & \sum_{s=1} & \sum_{i=1} & x_{is} & S_{is} \end{matrix}$	$n=1, \dots, c$	$c$	Defines real other demands in each country.

$$\text{Total equations} = (2c^2 + 3c)gh + 12ch + (5c^2 + 8c + 1)g + chr + 2cr + \left[ 15c + \frac{5}{2}(c^2 - c) \right].$$



Table A2 Variables of the Multicountry, Rest of the World System

Variable	Subscript Range	Number	Description
(n) (1) $x_{(is)j}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $s = 1, \dots, c+1$ $j = 1, \dots, h$	$c(c+1)gh$	Demand for good i from source s for current production in industry j and country n.
(n) $z_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	ch	Activity level in industry j and country n.
(n) $p_{is}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $s = 1, \dots, c+1$	$c(c+1)g$	Price of good i from source s in country n (denoted in country n's currency) for all end uses in country n.
(n)P $x_{vj}$	$n = 1, \dots, c$ $v = 1, 2, 3$ $j = 1, \dots, h$	3ch	Demand for primary factor v in industry j and country n.
(n)P $p_{vj}$	$n = 1, \dots, c$ $v = 2, 3$ $j = 1, \dots, h$	2ch	Price of primary factor v (v=2: capital), (v=3: land) in industry j and country n.
(n)P $p_1$	$n = 1, \dots, c$	c	Price of aggregate labour in country n.
(n)P $x_{1,q,j}$	$n = 1, \dots, c$ $q = 1, \dots, r$ $j = 1, \dots, h$	chr	Demand for labour of occupation q in industry j and country n.
(n)P $p_{1q}$	$n = 1, \dots, c$ $q = 1, \dots, r$	cr	Price of labour of occupation q in country n.
(n)o $x_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	ch	Demand for other costs in industry j in country n.
(n) $x_{(in)j}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $j = 1, \dots, h$	cgh	Output of commodity i in industry j and country n.
(n) (2) $x_{(is)j}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $s = 1, \dots, c+1$ $j = 1, \dots, h$	$c(c+1)gh$	Demand for good i from source s for capital creation in industry j and country n.
(n) $y_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	ch	Investment in industry j in country n.
(n) (3) $x_{is}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $s = 1, \dots, c+1$	$c(c+1)g$	Demand for good i from source s for household consumption in country n.
(n) (3) $x_i$	$n = 1, \dots, c$ $i = 1, \dots, g$	cg	Demand for good i undifferentiated by source for household consumption in country n.
(n) $q$	$n = 1, \dots, c$	c	Number of households in country n.
(n) (3) $p_k$	$n = 1, \dots, c$ $k = 1, \dots, g$	cg	Price of good k undifferentiated by source to consumers in country n.
(s) (4) $x_{is}$	$s = 1, \dots, c+1$ $i = 1, \dots, g$	$(c+1)g$	Commodity demands by source in the rest of the world.
(4) $x_i$	$i = 1, \dots, g$	g	Commodity demands, undifferentiated by source, in the rest of the world.
(s) (4) $p_{is}$	$s = 1, \dots, c+1$ $i = 1, \dots, g$	$(c+1)g$	Price of good i from source s in the rest of the world.

Table A2 Variables of the Multicountry, Rest of the World System (cont.)

Variable	Subscript Range	Number	Description
(n) (5) $x_{is}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $s = 1, \dots, c+1$	$c(c+1)g$	Other demands for good $i$ from source $s$ for use in country $n$ .
(n) $c_R$	$n = 1, \dots, c$	$c$	Aggregate real household consumption in country $n$ .
(n) (5) $f_{is}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $s = 1, \dots, c+1$	$c(c+1)g$	Country shift factors for other demands by source.
(n) (3) $\epsilon$	$n = 1, \dots, c$	$c$	Index of consumer prices in country $n$ .
(n) $c$	$n = 1, \dots, c$	$c$	Aggregate nominal household expenditure in country $n$ .
(n) o $p_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	$ch$	Country prices of other costs.
(n) $\pi_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	$ch$	Cost of capital creation in industry $j$ in country $n$ .
(n) * $p_{is}$	$n = 1, \dots, c$ $i = 1, \dots, g$ $s = 1, \dots, c+1$ $s \neq n$	$c^2g$	Cif import price to country $n$ of commodity $i$ from foreign source $s$ (expressed in source $s$ 's currency).
$\phi_{n,m}$	$n, m = 1, \dots, c$ $n \neq m$	$c^2 - c$	Exchange rate between country $n$ and country $m$ (currency country $n$ / currency country $m$ ).
$\phi_{n,w}$	$n = 1, \dots, c$	$c$	Exchange rate between country $n$ and rest of the world (currency in $n$ /currency in rest of the world).
$\phi_{w,n}$	$n = 1, \dots, c$	$c$	Exchange rate between rest of the world and country $n$ .
$t_{i(n,s)}$	$n = 1, \dots, c$ $s = 1, \dots, c+1$ $s \neq n$	$c^2g$	One plus the ad valorem rate of protection on commodity $i$ imported by country $n$ from foreign source $s$ .
$p_{i(m,n)}^T$	$i = 1, \dots, g$ $m, n = 1, \dots, c$ $m \neq n$	$c(c-1)g$	Price, in country $m$ 's currency, of a unit of transport margins required to move goods from country $m$ to country $n$ .
(n) $s_i$	$n = 1, \dots, c$ $i = 1, \dots, g$	$cg$	One plus the ad valorem export (to rest of the world) subsidy on good $i$ in country $n$ .
(n) $r_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	$ch$	Rate of return to capital in industry $j$ and country $n$ .
(n) $k_{j(1)}$	$n = 1, \dots, c$ $j = 1, \dots, h$	$ch$	Future period capital stock in industry $j$ in country $n$ .
(n) $k_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	$ch$	Current capital stock in industry $j$ and country $n$ .
(n) $\lambda$	$n = 1, \dots, c$	$c$	Economy-wide rate of return to capital for country $n$ .
(n) $i$	$n = 1, \dots, c$	$c$	Aggregate nominal investment in country $n$ .
(n) $x_{in}$	$i = 1, \dots, g$ $n = 1, \dots, c$	$cg$	Output of good $i$ in country $n$ .
(n) $l_q$	$q = 1, \dots, r$ $n = 1, \dots, c$	$cr$	Employment of labour of occupation $q$ in country $n$ .

Table A2 Variables of the Multicountry, Rest of the World System (cont.)

Variable	Subscript Range	Number	Description
$(n)$ $n_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	ch	Employment of land in industry j in country n.
$(n)$ $x_{iF}$	$i = 1, \dots, g$ $n = 1, \dots, c$	cg	Country n's commodity import volume (from other countries and the rest of the world).
$(n)$ $x_{is}$	$n = 1, \dots, c$ $s = 1, \dots, c+1$ $i = 1, \dots, g$ $n \neq s$	$c^2g$	Country n's imports of good i from foreign source s.
$(n)$ $m$	$n = 1, \dots, c$	c	Aggregate foreign currency (rest of the world currency units) value of imports by country n.
$(n)$ $e$	$n = 1, \dots, c$	c	Aggregate foreign currency (rest of the world currency units) value of exports by country n.
$(n)$ $\Delta B$	$n = 1, \dots, c$	c	Balance of trade in country n.
$\Delta B_{n,m}$	$m, n = 1, \dots, c$ $m \neq n$ $n < m$	$\frac{1}{2}(c^2 - c)$	Bilateral trade balance of country n with respect to country m.
$e_{n,m}$	$m, n = 1, \dots, c$ $n < m$	$\frac{1}{2}(c^2 - c)$	Value of exports of country n to country m (fob country n).
$m_{m,n}$	$m, n = 1, \dots, c$ $m \neq n$ $n < m$	$\frac{1}{2}(c^2 - c)$	Value of imports by country n from country m (cif country n).
$(n)$ $i_R$	$n = 1, \dots, c$	c	Real aggregate investment in country n.
$(n) (2)$ $\epsilon$	$n = 1, \dots, c$	c	Investment goods price index in country n.
$(n)$ $f_R$	$n = 1, \dots, c$	c	Fixes relationship between aggregate real investment and consumption in country n.
$(n)$ $l$	$n = 1, \dots, c$	c	Aggregate employment in country n.
$(n)$ $\kappa$	$n = 1, \dots, c$	c	Aggregate capital stock in country n.
$(n) c$ $f_j$	$n = 1, \dots, c$ $j = 1, \dots, h$	ch	Shift factor for the price of other costs in industry j in country n.
$(n)$ $f_{1,q}$	$n = 1, \dots, c$ $q = 1, \dots, r$	cr	Shift factor for the price of labour of occupation q in country n.
$(n)$ $f_1$	$n = 1, \dots, c$	c	Shift factor for the price of labour in general in country n.
$(n)$ gdp	$n = 1, \dots, c$	c	Real GDP in country n.
$(n)$ $x_G$	$n = 1, \dots, c$	c	Real other demands in country n.

$$\text{Total variables} = (2c^2 + 3c)gh + 15ch + (8c^2 + 10c + 3)g + chr + 3cr + \left[ c^2 + 19c + \frac{3}{2}(c^2 - c) \right]$$