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by Christian M. Scholz
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A NOTE ON THE DOUBLE DIVIDEND HYPOTHESIS*

by

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Abstract

This paper tries to clear the confusion in the literature about the potential of environmental tax reforms to yield a double dividend. In opposition to a number of recent papers it is found that the possibility for a double dividend depends largely on the substitutability characteristics of taxed commodities and not on the uncompensated elasticities. It is found that a double dividend is possible, if the following conditions are met. First, the initial tax system has to be inefficient from a non-environmental point of view. Second, it is possible to raise the tax on the externality creating commodity and in exchange to reduce the tax on a commodity that is a gross substitute for the externality creating commodity. Third, under the existing distortionary tax system the commodity whose tax is reduced is relatively difficult to substitute through other taxed commodities and hence, easier to substitute through the untaxed numeraire.

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1. Introduction

In the discussion of environmental taxation there has been a considerable confusion about the effects of environmental tax reforms on non-environmental welfare. Earlier contributions, e.g. Pearce (1991), claim that raising environmental taxes gives the possibility to the government to reduce the overall distortion of the existing tax system through a reduction of distorting taxes and thus, raising environmental and non-environmental welfare. This result gains importance when designing environmental tax reforms. In the public and also academic debate on environmental tax reforms it is sometimes postulated that an environmental tax reform should have two aims: first, it should create environmental benefits and second, it should reduce preexisting tax distortions. The second postulate results from the argument that the magnitude of environmental benefits is largely unknown due to missing markets for environmental quality (See Goulder (1995)). A tax reform that satisfies both postulates is said to yield a double dividend. If the tax reform exacerbates preexisting tax distortions it is not guaranteed that the net welfare effect of the tax reform is still positive. Therefore, in order to guarantee positive welfare effects, an environmental tax reform must fulfill the two mentioned postulates. However, recently a number of papers, Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994), Goulder (1995), find that the double dividend hypothesis is subject to serious doubt. These authors find that as long as the uncompensated

elasticity of household labor supply is positive, in a model with a dirty consumption good causing a negative externality, a tax reform that aims at raising the tax on the dirty consumption good and reducing the labor income tax „will typically exacerbate, rather than alleviate, preexisting tax distortions“, (Bovenberg, de Mooij (1994), p. 1085.).

However, in a way this result contradicts standard results in modern public finance. Since Atkinson, Stern (1974) it is known that the uncompensated elasticity of labor supply determines the marginal social cost of public funds (MSCF) of a tax rate. When calculating the welfare effects of tax reforms one compares the MSCF of different tax rates. Incorporated in the MSCF are the income effects of tax rates. But since all marginal tax changes create the same income effects all that matters in comparing the welfare effects of different tax rates are the substitution effects which are described by compensated rather than uncompensated elasticities.

In this paper it is shown that the results of Bovenberg, de Mooij (1994) and Bovenberg, van der Ploeg (1994) depend on compensated elasticities and not on uncompensated elasticities. As a consequence this paper is able to show that the results of these authors depend largely on assumptions about preferences and the

choice of the numeraire.¹ Further it is shown that an alternative environmental tax reform in the Bovenberg de Mooij (1994) model is able to yield a double dividend in which non-environmental welfare and employment increase.

2. The framework

Tax reforms have the feature of changing a vector composed of price vector q^0 and income vector I^0 , from $(q^0; I^0)$ to $(q^1; I^1)$. Using the indirect utility function $W(q; I)$ it is easy to express the resulting welfare change as $W(q^0; I^0) - W(q^1; I^1)$. But since utility functions are ordinal and not cardinal, there is not a unique number that represents this welfare change. For this reason Hicks (1943) introduced the concepts of compensating variation and equivalent variation. As shown in Mayshar (1990) both measures are equal when only marginal variations are considered. Consider the indirect utility function

$$(2.1) \quad W(q; I - T; z) = \max_x \{u(x; z) : I - T \geq qx\}$$

where $u(x; z)$ denoted a concave and continuously differentiable utility function. x denotes the $(n+1) \times 1$ vector of private commodities, with x_0 as the numeraire. $q = p + t$ denotes consumer price vector, p the producer price vector and t the

¹ Orosel, Schöb (1995) find that the assumptions on preferences and the choice of the numeraire also affect the relationship between the Pigou tax and the second best tax rate. However, these authors do not show the implications of their results for the double dividend hypothesis.

$(n \times 1)$ tax vector. x_i , p_i , q_i and t_i denote the i th element of the vectors x , p , q and t , respectively. T denotes lump-sum taxes, and z denotes environmental quality, i. e. $\frac{\partial u(x; z)}{\partial z} = u_z > 0$, which is a public good. In order to simplify the analysis we follow a large part of the literature and assume weak separability in the utility function between environmental quality z and the vector of private commodities x , hence $\frac{\partial x}{\partial z} = 0$. It is also assumed that environmental quality depends on the consumption of the commodity x_d , which is an element of the vector x , in the following way:

$$(2.2) \quad z = e(x_d(q, I - T)), \text{ with } e' < 0$$

Therefore, equation (2.2) describes a consumption externality. Thus, the government can change the provision of z only via induced changes of dirty consumption x_d . This means that the government has to change taxation in order to change environmental quality z . The revenue constraint of the government is

$$(2.3) \quad R = T + \sum_{i=1}^n t_i x_i(q, I - T).$$

where R denotes government revenue and t_i commodity tax i . Note that x_0 is untaxed. Consider a change in the tax rate t_k . Assuming a linear transformation curve, i.e. constant producer price vector p , differentiating (2.1), while leaving the

utility level constant, one can express the marginal costs of raising t_k in terms of income I in the following way (this expression is derived in the appendix)

$$(2.4) \quad \frac{dI}{dt_k} = x_k(q, I - T) - \frac{u_z(x, z)}{\lambda} e' \frac{\partial x_d(q, I - T)}{\partial t_k}$$

where use of Roy's identity $x_k = -W_{q_k}/W_I$ has been made and $\lambda = W_I$ is the marginal household utility of income. (2.4) gives the marginal compensating variation in lump-sum income I that is necessary to fully compensate the household for raising t_k . Differentiating (2.3) one gets

$$R_k = \frac{dR}{dt_k} = x_k(q, I - T) + \sum_{i=1}^n t_i \frac{\partial x_i(q, I - T)}{\partial t_k}$$

We get the marginal social cost of public funds of t_k as defined in Schöb (1995) by dividing (2.4) through the marginal government revenue R_k

$$(2.5) \quad \frac{dI/dt_k}{R_k} = MSCF_k = MCF_k - MEI_k = \frac{x_k}{R_k} - \frac{1}{R_k} \frac{u_z}{\lambda} e' \frac{\partial x_d}{\partial t_k}$$

where MCF_k denotes the private marginal cost of public funds of t_k as defined commonly in modern public finance, e. g. Mayshar (1990), and MEI_k denotes the marginal environmental impact of t_k . $MSCF_k$ measures the welfare costs of a tax increase per unit of additional government revenue raised by the marginal increase in t_k . Expressing the welfare costs per unit of additional government

revenue, allows the comparison of welfare costs of different tax rates. If $MSCF_k$ is negative, raising t_k yields a welfare gain, since lump sum income could be taken away from the household without changing its pretax utility level.

The measure MCF_k gives the welfare costs that result from a change in consumption of commodity k . MCF_k denotes therefore the marginal private welfare costs of t_k .

The measure MEI_k gives the marginal welfare costs that are associated with a change in environmental quality. MEI_k denotes therefore environmental benefits

note that this benefit is negative if $\frac{\partial x_d}{\partial t_k} > 0$, i.e. commodity k and d are gross substitutes. MEI_k has to be subtracted from MCF_k since a positive

environmental benefit reduces the marginal welfare costs of raising t_k . This definition of $MSCF$ allows us to separate the environmental benefits and the private benefits of a tax reform. Closely related to $MSCF$ is the marginal social excess burden $MSEB$. The $MSEB$ wants to compensate the household only over the additional revenue that the government raises by a tax change. Differentiating (2.1) while leaving the utility level constant gives

$$(2.6) \quad MSEB_k = x_k - R_k - \frac{u_z}{\lambda} e' \frac{\partial x_d}{\partial t_k}$$

Note that $MSEB_k = (MSCF_k - 1)R_k$. With this framework one can analyze the welfare costs of environmental tax reforms.

3. Environmental tax reform and the double dividend

In this section environmental tax reforms are considered in which the government raises an environmental tax t_d on the externality creating dirty consumption good x_d and in exchange adjusts another tax t_c , such that the tax revenue R remains constant. Throughout the analysis it is assumed that the marginal revenue of a tax is always positive, which is the normal case. The marginal social excess burden that results when in exchange for raising an environmental tax t_d a distortionary tax t_c is reduced, is derived from

$$(3.7) \quad W_{q_d} dt_d + W_{q_c} dt_c + W_I (R_d dt_d + R_c dt_c + dI) + W_z dz = 0$$

Note that revenue neutrality means $R_d dt_d + R_c dt_c = 0$. Equation (3.7) can be rearranged to give

$$(3.8) \quad \frac{dI}{dt_d} = x_d - x_c \frac{R_d}{R_c} - \frac{u_z}{\lambda} e' \left(\frac{\partial x_d}{\partial t_d} - \frac{\partial x_d}{\partial t_c} \frac{R_d}{R_c} \right)$$

This expression can be rewritten as

$$(3.9) \quad \frac{dI}{dt_d} = R_d [(MEI_c - MEI_d) + (MCF_d - MCF_c)]$$

Equation (3.9) says that the described environmental tax reform will yield a welfare gain if the expression on the right hand side is negative, or in other words

$$MSCF_d < MSCF_c$$

In the literature it is usually stated that there is uncertainty about the environmental benefits. An environmental tax reform yields environmental benefits as long as

$$MEI_c < MEI_d$$

which is equivalent to

$$(3.10) \quad \frac{1}{R_c} \frac{\partial x_d}{\partial t_c} > \frac{1}{R_d} \frac{\partial x_d}{\partial t_d}$$

From (3.10) it can be seen that environmental benefits are always be positive, if

$\frac{\partial x_d}{\partial t_c} > 0$. As pointed out by Schöb (1995), a sufficient condition for achieving

environmental benefits is that an environmental tax reform reduces only taxes on commodities that are gross substitutes for the dirty commodity. This means that as long as the commodities c and d are gross substitutes an environmental tax reform will yield an environmental benefit.

Under the assumption that an environmental tax reform reduces taxes on commodities that are gross substitutes for the dirty consumption good, the environmental benefits are guaranteed to be positive. There is uncertainty about

the magnitude of the environmental benefit, such that it is not known if the environmental benefits outweigh possible decreases in non-environmental welfare. The uncertainty stems from the term

$$MEI_k = \frac{1}{R_k} \frac{u_z}{\lambda} e' \frac{\partial x_d}{\partial t_k}$$

Since there is no market for environmental quality, $\frac{u_z}{\lambda}$ cannot be observed. Also e' is not known in a lot of cases due to informational lacks. All that is known with certainty about $\frac{u_z}{\lambda}$ and e' are the signs.

Under the assumption that environmental tax reforms reduce taxes on commodities that are gross substitutes for the dirty consumption good a necessary and sufficient condition for a double dividend is

$$(3.11) \quad MCF_d < MCF_c$$

That is the marginal cost of public funds should be higher for the tax rate that is about to be reduced then the specified environmental tax reform will yield a double dividend. Substituting the demand functions into (3.11) gives after some manipulations

$$(3.12) \quad x_d x_c \left(\sum_{i=1}^n \frac{t_i}{q_i} \varepsilon_{ci} - \sum_{i=1}^n \frac{t_i}{q_i} \varepsilon_{di} \right)$$

where use of the symmetry of the Slutsky matrix has been made. ϵ_{ji} denotes the compensated demand elasticity of commodity j with respect to commodity price i . If expression (3.12) is negative and commodity c is a gross substitute for the dirty commodity d then an environmental tax reform that raises the environmental tax t_d and reduces the tax t_c will yield a double dividend. For a value added tax with $t_i = p_i \tau_i$ and $q_i = (1 + \tau_i) p_i$ expression (3.12) gives the following condition for a double dividend under the assumption that commodity c is a gross substitute for commodity d

$$(3.13) \quad \sum_{i=1}^n \frac{\tau_i}{1 + \tau_i} (\epsilon_{ci} - \epsilon_{di}) < 0$$

Expression (3.13) shows that the double dividend depends on initial tax rates and on compensated demand elasticities. In order to yield a double dividend the government has to reduce taxes on commodities c that are relatively more difficult to substitute through other taxed commodities. Commodity d should be easier to substitute through other taxed commodities relative to c . Since

$\sum_{i=1}^n \epsilon_{ci} = -\epsilon_{c0}$ one can interpret (3.13) that the commodity whose tax is reduced

should in average, where the weights are $\frac{\tau_i}{1 + \tau_i}$, be a better Hicksian substitute

for the numeraire relative to the dirty commodity.

4. An important special case: The Bovenberg, de Mooij model

An important special case of the result obtained above is described in the contributions of Bovenberg, de Mooij (1994b) and Bovenberg, van der Ploeg (1994).² To these two contributions we refer to as the Bovenberg, de Mooij model. In these papers the model consists in a linear production technology that employs only labor L and produces a clean and a dirty consumption good denoted with C and D , respectively. β denotes labor productivity. With C as the numeraire, whose price is $q_C = 1$ and hence $q_D = 1 + t_D$. Utility is weakly separable in environmental quality z and the private goods leisure $V = 1 - L$, dirty and clean consumption, D and C . Consumption D and C is weakly separable from leisure V . The utility function is therefore $u = u(z; M(V; Q(C; D)))$, where $M(\cdot)$ and $Q(\cdot)$ are the corresponding subutility functions which are assumed to be homothetic. The model is presented in the following table:

Production	$Y = \beta L$
Equilibrium	$w = \beta$
Household utility	$u = u(z; M(V; Q(C; D)))$
Household budget	$C + (1 + t_D)D = (1 - t_L)wL$

² In this section a simplified version of these models is presented that nevertheless captures the main features of the mentioned models.

constraint	
Government budget	$G = t_D D + t_L wL$
constraint	
Equilibrium	$Y = C + D + G$
Environment	$z = e(D)$

Table 1

In order to derive a welfare measure for a taxreform we totally differentiate the utility function

$$du = 0 = u_V dV + u_C dC + u_D dD + u_z dz$$

From the necessary conditions of a household optimum it is known that the marginal utility of a commodity equals the price times the Lagrange multiplier λ . Substituting the necessary conditions characterizing household behavior into the total differential of the utility function we get

$$du = 0 = -\lambda(1 - t_L)w dL + \lambda dC + \lambda(1 + t_D)dD + u_z dz$$

Differentiating the household budget constraint delivers

$$dC + (1 + t_D)dD - (1 - t_L)w dL = dI - dt_D D - dt_L wL$$

where dI describes the additional lump-sum household income that is needed to restore the household to the indifference curve that his utility was on before the

tax change. From the last two equations and total differentiation of the government budget constraint with $dG = 0$ one can derive

$$(4.14) \quad \tilde{I} = -\frac{u_z dz}{(1-t_L)wL} \frac{1}{\lambda} - \frac{(1+t_D)D}{(1-t_L)wL} \frac{t_D}{(1+t_D)} \tilde{D} - \frac{t_L}{(1-t_L)} \tilde{L}$$

where $\tilde{I} = dI/(1-t_L)wL$. (4.14) gives additional lump-sum household income that is needed to restore the household to the indifference curve that his utility was on before the tax change. The first term measures the environmental benefit of an environmental tax reform. The last two terms measure the non-environmental benefit. If the first term and the sum of the last two terms is negative, there is a double dividend since environmental and private welfare are increasing. In the next section the welfare effects of an environmental tax reform are analyzed.

4.1. Environmental tax reform in the Bovenberg, de Mooij model

In this section the government changes increases the environmental tax t_D and adjusts the tax on labor income t_L such that government revenue remains constant. In order to analyze the welfare effects of such a tax reform the model in table 1 is log linearized. In the appendix the following log-linearized equations are derived:

Production	$\tilde{Y} = \tilde{L}$
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Equilibrium	$\tilde{w} = 0$
Labor supply	$\tilde{L} = -\theta(\tilde{t}_L + \phi_D \tilde{t}_D)$
Demand for D	$\tilde{D} = \tilde{C} - \sigma_Q \tilde{t}_D$
Government budget constraint	$\tilde{t}_D \phi_D + \tilde{t}_L = -\frac{t_D}{1-t_L} a_D \tilde{D} - \frac{t_L}{1-t_L} \tilde{L}$
Equilibrium	$\tilde{Y} = a_C \tilde{C} + a_D \tilde{D}$
Environment	$\tilde{z} = \gamma \tilde{D}$

Table 2

A tilde denotes a relative change, i. e. $\tilde{x} = \frac{dx}{x}$. For the tax rates we have

$\tilde{t}_L = \frac{t_L}{1-t_L}$ and $\tilde{t}_D = \frac{t_D}{1+t_D}$. The variable γ denotes the elasticity of

environmental quality with respect to dirty consumption. θ denotes the

uncompensated elasticity of labor supply, $a_C = C/wL$, $a_D = D/wL$, σ_Q denotes

the elasticity of substitution between C and D , and $\phi_D = (1+t_D)D/(1-t_L)wL$.

From table 2 the following system of equations can be derived:

$$(4.15) \quad \begin{pmatrix} -\frac{t_L}{1-t_L} & -\frac{t_D a_D}{1-t_L} & -1 \\ -1 & 0 & -\theta \\ 1 & -(a_C + a_D) & 0 \end{pmatrix} \begin{pmatrix} \tilde{L} \\ \tilde{D} \\ \tilde{t}_L \end{pmatrix} = \begin{pmatrix} \phi_D \\ \phi_D \theta \\ a_C \sigma_Q \end{pmatrix} \tilde{t}_D$$

one can derive

$$(4.16) \quad \tilde{L} = \frac{1}{|Det|} a_C \sigma_Q \frac{t_D a_D}{1 - t_L} \theta \tilde{t}_D$$

$$(4.17) \quad \tilde{D} = -\frac{1}{|Det|} a_C \sigma_Q \left(\frac{(\theta + 1)t_L - 1}{1 - t_L} \right) \tilde{t}_D$$

The determinant $|Det|$ is negative due to stability considerations.³ Hence, for a positive elasticity of labor supply, employment decreases as a consequence of the environmental tax reform, specified above. In order to determine the sign of (4.17) a closer look at the determinant is needed. The determinant is:

$$(4.18) \quad |Det| = \frac{t_D a_D}{1 - t_L} \theta + (a_C + a_D) \left(\frac{t_L(\theta + 1) - 1}{1 - t_L} \right) < 0$$

From (4.18) it is clear that the demand for dirty consumption will decrease as a consequence of an increase in t_D . A look at equation (4.14) that measures the welfare change and $\tilde{z} = \gamma \tilde{D}$ tells us that the environmental tax reform will lead to an increase in environmental welfare, but a decrease in non-environmental welfare. Hence, there is no double dividend in the Bovenberg, de Mooij model.

³ A sufficient condition for local stability is that the matrix in (4.15) is negative definite. The trace is clearly negative. Hence, a necessary and sufficient condition for the negative definiteness of the matrix is that the determinant is also negative. Instead of analyzing the stability properties of the model, one can also argue that decreasing government expenditure and keeping the dirt tax constant should enable the government to decrease the labor tax. This also requires a negative determinant. In this model the stability postulate is equivalent to postulating that a decrease in government spending with keeping t_d constant should result in a smaller t_L .

This result is valid as long as the initial tax t_D is positive and the uncompensated elasticity of labor supply is positive.

The result of the Bovenberg, de Mooij model suggests that uncompensated elasticities play an important role in the evaluation of an environmental tax reform. In order to show that this reasoning is wrong we derive the compensated elasticities of the Bovenberg, de Mooij model. The compensated demand elasticities can be expressed as a function of the elasticities of substitution σ_{ij} between commodity i and j . In the appendix we derive the following compensated demand elasticities:

ε_{ij}	p_C	$p_D = t_D$	$p_L = -t_L$
L	$-\sigma_M V(1 - \phi_D)$	$-\sigma_M \phi_D V$	$\sigma_M V$
D	$(1 - \phi_D)(\sigma_Q - \sigma_M V)$	$\phi_D(\sigma_Q - \sigma_M V) - \sigma_Q$	$\sigma_M V$
C	$-\phi_D(\sigma_Q - \sigma_M V) - \sigma_M V$	$\phi_D(\sigma_Q - \sigma_M V)$	$\sigma_M V$

Table 3

where σ_M denotes the elasticity of substitution between labor and private consumption goods. Equation (3.12) becomes

$$DL \left[\frac{\tau_L}{1 - \tau_L} (-\varepsilon_{LL} + \varepsilon_{DL}) + \frac{\tau_D}{1 + \tau_D} (\varepsilon_{LD} - \varepsilon_{DD}) \right]$$

$$(4.19) \quad DL \frac{\tau_D}{1 + \tau_D} \sigma_Q (1 - \phi_D)$$

Equation (4.19) indicates the reason for the failure for the double dividend hypothesis. The price of the dirty consumption good and the price of leisure increases through the tax changes. In order to yield a double dividend, labor has to be the better Hicksian substitute for the clean good, compared to the dirty good. With the specified utility function from above, the dirty consumption good is the better Hicksian substitute for the clean consumption good. Hence, in the Bovenberg, de Mooij model a double dividend cannot be achieved, if in exchange for raising the environmental tax, the labor tax is reduced.

The reason for the failure of the double dividend hypothesis in this model is that the government is unable to reduce the labor tax sufficiently, if it has to maintain an unchanged revenue. This is because raising the environmental tax will cause substitution away from the dirty consumption good to the untaxed clean consumption good. Thus the decrease in demand for dirty consumption will not only improve environmental quality, but also erode the tax base of the government. This effect, that Bovenberg, de Mooij (1994) call the tax base erosion effect, is responsible for the inability of the government to reduce the labor tax sufficiently.

4.2. An alternative environmental tax reform in the Bovenberg, de Mooij model

The reason for the failure of the double dividend hypothesis in the Bovenberg, de Mooij model, is the assumption that dirty and clean consumption are closer Hicksian substitutes than labor and clean consumption. To strengthen this point, let us consider a simple modification of the environmental tax reform in the Bovenberg, de Mooij model. Instead of reducing the tax on labor income the tax on the clean consumption good is reduced, while labor becomes the untaxed commodity. This simple modification will give some insights into the mechanisms at work.

The welfare measure for the tax reform is now:

$$(4.20) \quad \tilde{I} = -t_C a_C \tilde{C} - t_D a_D \tilde{D} - \frac{u_z}{\lambda w L} dz$$

The log-linearized version of the model is now:

Production	$\tilde{Y} = \tilde{L}$
Equilibrium	$\tilde{w} = 0$
Labor supply	$\tilde{L} = -\theta((1 - \phi_D)\tilde{t}_C + \phi_D\tilde{t}_D)$
Demand for D	$\tilde{D} = \tilde{C} - \sigma(\tilde{t}_D - \tilde{t}_C)$
Government budget	$\tilde{t}_C(1 - \phi_D) + \tilde{t}_D\phi_D = -\frac{t_C C}{wL}\tilde{C} - \frac{t_D D}{wL}$

constraint	
Equilibrium	$\tilde{Y} = a_C \tilde{C} + a_D \tilde{D}$
Environment	$\tilde{z} = \gamma \tilde{D}$

Table 4

From table 4 the following system of equations can be derived:

$$(4.21) \quad \begin{pmatrix} -a_C & -a_D & -\theta(1-t_C)a_C \\ 1 & -1 & \sigma \\ -t_C a_C & -t_D a_D & -(1-t_C)a_C \end{pmatrix} \begin{pmatrix} \tilde{C} \\ \tilde{D} \\ \tilde{t}_C \end{pmatrix} = \begin{pmatrix} -\theta(1-t_D)a_D \tilde{t}_D \\ \sigma \tilde{t}_D \\ (1-t_D)a_D \tilde{t}_D \end{pmatrix}$$

The determinant, which again has to be negative, is:

$$(4.22) \quad |Det| = (1-t_C)a_C[(\theta t_D - 1)a_D + (\theta t_C - 1)a_C] - \sigma a_C a_D [t_D - t_C] < 0$$

One can derive:

$$(4.23) \quad \frac{\tilde{C}}{\tilde{t}_D} = \frac{-1}{|Det|} \sigma a_D (1 - t_D \theta)$$

$$(4.24) \quad \frac{\tilde{D}}{\tilde{t}_D} = \frac{1}{|Det|} \sigma a_C (1 - t_C \theta)$$

Substituting equations (4.23) and (4.24) into the non-environmental welfare terms of (4.20) gives

$$\frac{\tilde{I}}{\tilde{t}_D} = \frac{a_C \sigma a_D}{|Det|} [t_C - t_D] - \frac{u_z z}{\lambda \omega L} \frac{\tilde{z}}{\tilde{t}_D}$$

The condition for the double dividend is

$$(4.25) \quad t_C > t_D$$

One can show that (4.25) is also the condition for increasing employment.

This result emphasizes two aspects: First, the welfare costs of an environmental tax reform can be influenced substantially by the initial tax rates, as Bovenberg, de Mooij (1994) have noticed. Second, compared to the original Bovenberg, de Mooij analysis the modified analysis shows that the welfare costs of an environmental tax reform can be influenced by the choice of tax rates to be reduced.

If the tax system is initially efficient from a non-environmental point of view, i. e. $t_C = t_D$, the non-environmental welfare costs of the environmental tax reform are zero. This result shows that non-environmental welfare costs depend largely on how far the initial tax system is away from non-environmental efficiency. From standard results of optimal taxation it is known, however, that non-environmental efficiency depends on the compensated demand elasticities. The compensated demand elasticities are shown in table 3. Inserting the relevant compensated demand elasticities into (3.12) gives

$$(4.26) \quad DC(t_D - t_C) \sigma_M V$$

Expression (4.26) has to be negative in order to yield a double dividend.

Expression (4.25) confirms the result expressed in (4.26).

5. Conclusions

The above analysis underlines that an environmental tax reform yields a double dividend if the tax on a commodity is reduced that is a better substitute for the numeraire than the dirty good and the numeraire. In this case the tax base erosion effect can be limited such that the government is able to sufficiently reduce the tax on the other good, so a double dividend is possible. In this case there will be still substitution away from the dirty good. But at the same time there will be substitution from the dirty good to the good whose tax is reduced. This limits and possibly erases the tax base erosion effect. Definitely, one cannot say that „environmental taxes *typically* exacerbate, rather than alleviate, preexisting tax distortions“ when revenues are used to cut preexisting distortionary taxes. It all depends on which tax rates are cut. The above analysis shows that there will be a double dividend if the tax system approaches to the tax system that is efficient from a non-environmental point of view and the commodities with reduced taxes are gross substitutes to the dirty goods.

In general there will be a double dividend if the following conditions are met. First, the initial tax system has to be inefficient also from a non-environmental

point of view.⁴ Second, it is possible to raise the tax on the externality creating commodity and in exchange to reduce the tax on a commodity that is a gross substitute for the externality creating commodity. Third, under the existing distortionary tax system the commodity whose tax is reduced is relatively difficult to substitute through other taxed commodities and hence easier to substitute through the untaxed numeraire. If the last two conditions are met, the tax base erosion effect that results from raising the tax on the externality creating commodity will be eliminated such that the government is able to sufficiently reduce another distorting tax.

References

- Atkinson, A. B., and N. H. Stern (1974), "Pigou, Taxation and Public Goods", Review of Economic Studies 41, 119-128.
- Bovenberg, A. L., and R. A. de Mooij (1994), "Environmental Levies and Distortionary Taxation", American Economic Review (Papers and Proceedings) 94 (4), 1085-1089.
- Bovenberg, A. L., and R. A. de Mooij (1995), "Environmental Taxation and the Double-Dividend: The Role of Factor Substitution and Capital Mobility", Discussionpaper prepared for the SFB- Workshop 1995: Environmental Policy in Open Economies, June 5-7, 1995, in Konstanz, p. 29.
- Bovenberg, A. L., and F. van der Ploeg (1994), "Environmental Policy, Public Finance and the Labour Market in a Second-Best World", Journal of Public Economics 55, 349-390.
- Goulder, L. H. (1995), "Environmental Taxation and the Double Dividend: A Reader's Guide", International Tax and Public Finance 2 (2), 157-184.
- Hicks, J. R. (1943), "The Four Consumers' Surpluses", Review of Economic Studies 11, 31-41.
- Mayshar, J. (1990), "On Measures of Excess Burden and their Application", Journal of Public Economics 43 (3), 263-289.

⁴ A similar result is found in Bovenberg, de Mooij (1995).

Orosel, G. O., and R. Schöb (1995), "Internalizing Externalities in Second-Best Tax Systems", CES Working Paper of the University of Munich, No. 101

Pearce, D. (1991), "The Role of Carbon Taxes in Adjusting to Global Warming", Economic Journal **101**, 938-948.

Schöb, R. (1995), Ökologische Steuersysteme. Umweltökonomie und optimale Besteuerung, Campus Verlag, Frankfurt a. M.

Appendix

Derivation of (2.4) (2.5)

Differentiating

$$\bar{u} = W(q; I - T; z)$$

yields

$$0 = W_{qk} dt_k + W_I dI + W_z dz$$

Rearranging gives

$$\frac{dI}{dt_k} = -\frac{W_{qk}}{W_I} - \frac{W_z}{W_I} \frac{dz}{dt_k}$$

Since $-\frac{W_{qk}}{W_I} = x_k$, $\frac{W_z}{W_I} = \frac{u_z}{\lambda}$ and $\frac{dz}{dt_k} = e' \frac{\partial x_d}{\partial t_k}$, substitution yields (2.4). Dividing

(2.4) through $\frac{dR}{dt_k} = R_k$, delivers $MSCF_k$ in equation (2.5). For readers familiar

with optimal taxation, the intuition of the measure $MSCF$ can be seen from the optimal taxation problem:

$$\max_t W(q; I - T; z) \text{ subject to } R = T + \sum_{i=1}^n t_i x_i$$

The first order condition for t_k can be written as

$$W_{q_k} + W_z e' \frac{\partial x_d}{\partial t_k} = -\mu R_k$$

where μ is the Lagrange multiplier associated with the government's revenue constraint. Dividing through $W_l = \lambda$ and rearranging gives

$$\frac{x_k}{R_k} - \frac{1}{R_k} \frac{u_z}{\lambda} e' \frac{\partial x_d}{\partial t_k} = \frac{\mu}{\lambda} = MSCF_k = MCF_k - MEI_k$$

In the optimum the $MSCF$ is equal for all taxes. When analyzing tax reforms, where the reform starts from an arbitrary equilibrium, the $MSCF$ are different, in general.

Derivation of table 2

From $Y = C + D + G$ one obtains with $dG = 0$, $\tilde{Y} = \frac{C}{Y} \tilde{C} + \frac{D}{Y} \tilde{D}$, with $a_C = \frac{C}{Y}$ and $a_D = \frac{D}{Y}$. The loglinearized government budget constraint, loglinearized production function, and the log linearized equilibrium condition for w are obtained in the same way.

From the necessary conditions characterizing household behavior one can obtain

$$\frac{Q_D}{Q_C} = (1 + t_D)$$

Differentiating this condition and considering the definition of the elasticity of substitution one gets

$$\tilde{D} = \tilde{C} - \sigma_Q \tilde{t}_D$$

Due to the weak separability assumption of the utility function the household decision between consumption of commodities and consumption of leisure can be characterized through the following maximization problem

$$\max_{V; Q} M(1 - L, Q) \text{ subject to } p_Q Q = (1 - t_L) w L$$

where p_Q is the optimal price index defined as $p_Q = \frac{C + (1 + t_D)D}{Q}$. This

maximization problem can be expressed through the following indirect utility function W^M :

$$W^M \left(\frac{(1 - t_L)w}{p_Q} \right) = \max_{V; Q} \left\{ M(1 - L, Q) : Q = \frac{(1 - t_L)w}{p_Q} L \right\}$$

From this follows the labor supply function $L = L \left(\frac{(1 - t_L)w}{p_Q} \right)$. Total

differentiation gives $\tilde{L} = \theta(-\tilde{t}_L - \tilde{p}_Q)$. Since $\tilde{p}_Q = \phi_D \tilde{t}_D$, we have

$\tilde{L} = -\theta(\tilde{t}_L + \phi_D \tilde{t}_D)$. Note that in the price index p_Q the quantities remain

constant, since they refer to the initial equilibrium and that only the tax rates

change. Note also $\phi_D = \frac{(1+t_D)D}{p_Q Q}$.

Derivation of table 3

Weak separability of the utility function implies that the optimal choice of V, C, D can be described by the indirect utility functions

$$W^Q(p_C; (1+t_D); p_Q Q) = \max_{C, D} \{Q(C; D): p_C C + (1+t_D)D = p_Q Q\}$$

$$W^M((1-t_L)\beta; p_Q) \max_{V, Q} \{M(V; Q): (1-t_L)\beta V + p_Q Q = (1-t_L)\beta\}$$

The duals to these problems are the following minimum expenditure functions

$$E^Q(p_C; (1+t_D); \bar{Q}) = \min_{C, D} \{p_C C + (1+t_D)D: \bar{Q} = Q(C; D)\}$$

$$E^M(p_Q; (1-t_L)\beta; \bar{M}) = \min_{V, Q} \{(1-t_L)\beta V + p_Q Q: \bar{M} = M(V; Q)\}$$

The functions $M(\cdot)$ and $Q(\cdot)$ are assumed to be homothetic. This implies that the minimum expenditure functions can be written as

$$A1 \quad p_Q Q = E^Q(p_C; (1+t_D); \bar{Q}) = e^Q(p_C; (1+t_D)) \bar{Q}$$

$$A2 \quad (1-t_L)\beta = E^M(p_Q; (1-t_L)\beta; \bar{M}) = e^M(p_Q; (1-t_L)\beta) \bar{M}$$

The elasticity of substitution is defined in terms of a minimum expenditure function

$$\sigma_{ij} = \frac{E_{ij}E}{E_i E_j}$$

Rearranging gives

$$\sigma_{ij} \frac{E_j p_j}{E} = \varepsilon_{ij}$$

where ε_{ij} is the compensated elasticity of demand from commodity i with respect to commodity j . The compensated demand function for commodity i is described through E_i .

Log linearizing the compensated demand function $C = e_{p_C} Q(p_C; (1 + t_D)) \bar{Q}$ gives

$$\text{A3} \quad \tilde{C} - \tilde{Q} = \sigma_Q \phi_D (\tilde{t}_D - \tilde{p}_C)$$

Note that a minimum expenditure function E is linearly homogenous in prices.

One can derive similar to A3

$$\text{A4} \quad \tilde{D} - \tilde{Q} = \sigma_Q (1 - \phi_D) (\tilde{p}_C - \tilde{t}_D)$$

Similarly, one can derive

$$\text{A5} \quad \tilde{Q} = \sigma_M V (-\tilde{t}_L - \tilde{p}_Q)$$

$$A6 \quad \tilde{V} = \sigma_M(1-V)(\tilde{p}_Q + \tilde{t}_L)$$

Note that $\tilde{M} = 0$.

Substituting A5 and $\tilde{p}_Q = (1 - \phi_D)\tilde{p}_C + \phi_D\tilde{t}_D$ into A4 gives

$$A7 \quad \begin{aligned} \tilde{D} = & (1 - \phi_D)(\sigma_Q - \sigma_M V)\tilde{p}_C \\ & + (\phi_D(\sigma_Q - \sigma_M V) - \sigma_Q)\tilde{t}_D - \sigma_M V\tilde{t}_L \end{aligned}$$

Substituting $\tilde{V} = -\frac{L}{V}\tilde{L}$ and $\tilde{p}_Q = (1 - \phi_D)\tilde{p}_C + \phi_D\tilde{t}_D$ into A6 gives

$$A8 \quad \tilde{L} = -\sigma_M V(1 - \phi_D)\tilde{p}_C - \sigma_M V\phi_D\tilde{t}_D - \sigma_M V\tilde{t}_L$$

The terms connected to the price changes in A7 and A8 give the compensated demand elasticities from table 3.