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**Working Paper — Digitized Version**

## Social security and redistribution within generations in an overlapping generations model

Kiel Working Paper, No. 957

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Suggested Citation: Lorz, Jens Oliver (1999) : Social security and redistribution within generations in an overlapping generations model, Kiel Working Paper, No. 957, Institut für Weltwirtschaft (IfW), Kiel

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# Kiel Working Papers

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**Social Security and  
Redistribution Within Generations  
in an Overlapping Generations Model**

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ISSN 0342 - 0787

The Kiel Institute of World Economics  
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D-24105 Kiel

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# **Social Security and Redistribution Within Generations in an Overlapping Generations Model**

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**October 1999**

**Keywords:** Overlapping generations, redistribution, political economy, asymmetric information.

**JEL-Classification:** E 62, D 78, D 82.

**Abstract:** An overlapping generations model is set up in this paper to analyze social security policy in a representative democracy with asymmetric information. The model considers not only redistribution between generations but also redistribution within generations according to individual labor incomes. Labor supply and savings are endogenous. The government is able to observe labor incomes, but not labor supply, savings or capital incomes. Two main results are derived in this setting: First, consumption levels are perfectly equalized within both generations. Second, a redistribution bias exists in favor of the old generation: the old generation receives a higher level of consumption than the young generation although both generations have the same weight in the objective function of the government.

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## 1. Introduction<sup>1</sup>

In this paper, social security policy is analyzed in an overlapping generations growth model of a representative democracy. In each period, the respective government is assumed to maximize aggregate welfare of the young generation of working individuals and of the old generation of retirees. The government does not consider the utility of individuals born in later periods. To redistribute between the young and the old generation, the government raises social security contributions from the young and pays benefits to the old. In addition to this redistribution between generations, the paper also considers redistribution within generations between individuals who receive a high wage during their working life and individuals with a lower wage. To redistribute within generations, the government can differentiate social security contributions and benefits according to individual labor incomes. For example, contributions to social security may increase with the individual labor income, or income dependent taxes and transfers may supplement the social security system.

As in the theory of optimum income taxation, the government is assumed to be able to observe labor incomes, but not individual labor supply or wages.<sup>2</sup> In addition, it is assumed that savings and capital incomes are not observable and consequently cannot be taxed or subsidized. Each government then has to determine optimal social security contributions and benefits as a function of labor incomes – given that the young individuals can adjust their labor supply and their savings and given that subsequent governments determine their optimal social security policy.

The following results are derived in this setting: First, consumption levels are perfectly equalized within both generations. Individuals receiving a high wage end up with the same consumption levels in both periods of their lives as individuals receiving a lower wage. This result follows from the sequential structure of decision-

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<sup>1</sup> This paper was written during a visit of the National Bureau of Economic Research. Its hospitality and a grant by the Deutsche Forschungsgemeinschaft (DFG) are gratefully acknowledged.

<sup>2</sup> See Lang (1996), Hori (1997) or Lorz (1999a) for the alternative setting where governments can observe and control savings.

making with respect to social security policy and from the individual savings decision. In each period, the respective government can redistribute within the old generation without any consequences for labor supply or savings. The government therefore completely equalizes consumption levels within the old generation. Individual savings then also lead to equal consumption levels within the young generation.

The policy of perfect redistribution within generations has consequences for the supply of labor: Because all differences in labor income are completely redistributed, high-wage individuals have no incentive to earn a higher labor income than individuals receiving a lower wage. As a result, all individuals receive the same labor income in the equilibrium. This outcome is Pareto-inefficient, and the paper shows the potential for a Pareto-improvement.<sup>3</sup>

As a second result, the paper shows that a redistribution bias exists in favor of the old generation: in each period, the respective old generation receives a higher level of consumption than the young generation although both generations are assumed to have the same weight in the objective function of the governments. This redistribution bias leads to a capital-labor ratio in the steady state below the modified golden rule level. The redistribution bias can be explained with the fact that the governments do not consider the utility of unborn generations in their objective function. They set social security contributions on a high level to reduce aggregate savings, because a part of the return of aggregate savings accrues to individuals born in later periods.

The paper also derives the influence of demographic changes – i.e. changes in population growth and in life expectancy – and of changes in the political weight of the old generation on the redistribution equilibrium. It is shown that the contribution rate to social security increases, if (i) the rate of population growth declines, if (ii) life

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<sup>3</sup> The perfect redistribution result and the Pareto-inefficient supply of labor are also derived in Lorz (1999b) using a 2-period model of intergenerational redistribution. This paper generalizes the results of the 2-period model where the capital stock in the first period is exogenously given to a growth model where capital is accumulated over time to a long-run steady state equilibrium.

expectancy increases or, if (iii) the weight of the old generation in the objective function of the government increases.

From the literature on social security in a representative democracy, most closely related to this paper is the work of Meijdam and Verbon (1996).<sup>4</sup> However, there are two important differences between this paper and Meijdam, Verbon (1996). First, Meijdam and Verbon do not consider heterogeneity within generations, and they assume a given labor supply. Consequently, they do not derive the results of this paper with respect to intra-generational redistribution and its effects on labor supply. Second, they assume a different sequence of moves between the government and the private sector than this paper. In this paper, the government sets the scheme of social security transfers before individuals determine their labor supply and savings. The government then takes into account the effects of its policy on individual factor supply and on the social security policy of subsequent governments. In contrast to this setting, Meijdam, Verbon (1996) assume that the government and the private sector move simultaneously. The government then takes individual savings and future social security transfers as given when deciding about its social security policy.<sup>5</sup>

The paper is structured as follows: Section 2 analyzes intergenerational redistribution in a basic version of the model without intra-generational heterogeneity and with an exogenous labor supply. Section 3 then introduces wage differences within the young generation and an endogenous labor supply into the model. In section 4, comparative statics show the influence of demographic changes and of the political weight on the redistribution equilibrium.

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<sup>4</sup> See Breyer (1994) or Verbon (1993) for a survey on the political economy of social security. See also Grossman, Helpman (1998) who employ the same objective function of the government as this paper. Their economic model, however, differs fundamentally from the one used in this paper.

<sup>5</sup> Van de Ven (1996) assumes the same sequential structure of moves as this paper. However, he analyzes social security policy in a partial equilibrium model with a given interest rate and a given wage rate. This paper in contrast derives the general equilibrium solution for social security policy in a model where factor prices are determined by the supply of labor and the accumulated capital stock.

## 2. Redistribution Between Generations

In this section, intergenerational redistribution is analyzed in a standard overlapping generations model with two generations, a young generation and an old generation. In period  $t$ , the young generation supplies labor,  $L_t$ , and it receives a wage rate of  $w_t$  and a labor income of  $w_t L_t$ . Labor supply is assumed to be exogenously given in this section, and it is constant over time. The young generation consumes a part of its labor income and saves the remaining part  $s_t$  for retirement. Savings of period  $t$  determine the capital stock in the subsequent period  $t+1$ :  $K_{t+1} = s_t$ . The old generation receives a capital income  $r_t K_t$  for its savings of the previous period, with  $r_t$  as the return to capital. Labor and capital are used to produce an aggregate consumption good. For simplicity, a Cobb-Douglas production function,  $Y_t = K_t^\alpha L_t^{1-\alpha}$ , is assumed. All markets are competitive, and the price of the consumption good is set equal to one. The labor income of the young is given by  $w_t L_t = [1 - \alpha] K_t^\alpha L_t^{1-\alpha}$ , and the capital income of the old is  $r_t K_t = \alpha K_t^\alpha L_t^{1-\alpha}$ . Utility is given by  $\tilde{u}_{y_t} = \ln c_{y_t} + \beta \ln c_{o_{t+1}}$  for the young individual and by  $\tilde{u}_{o_t} = \ln c_{o_t}$  for the old, with  $c_{y_t}$  and  $c_{o_t}$  as the respective consumption levels. The economy lasts for  $T$  periods from  $t = 0$  to  $t = T$ , with  $T$  approaching infinity. Each generation is assumed to consist of one representative member.

To redistribute between both generations, the government in  $t$  can raise social security contributions,  $\tau_t$ , from the young generation and pay benefits,  $b_t$ , to the old.<sup>6</sup> Without loss of generality, the government is assumed to balance its budget in each period, so that  $b_t = \tau_t$ .<sup>7</sup>

As mentioned in the introduction, savings and individual capital incomes are assumed to be non-observable for the government. This explains why the government cannot tax or subsidize savings or differentiate social security benefits according to the

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<sup>6</sup> Contributions and benefits may be positive or negative.

<sup>7</sup> A deficit or surplus of the government as well as individual contributions to a mandatory funded pension system would have no effects on the equilibrium allocation in this paper.



individual capital income of the old. In this setting, the representative young individual chooses a savings level that equalizes the marginal rate of intertemporal substitution with the marginal productivity of capital, i.e.  $c_{o,t+1}/\beta c_{y,t} = \alpha K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha}$ . The consumption levels of both generations are given by  $c_{y,t} = [1 - \alpha]K_t^\alpha L_t^{1-\alpha} - K_{t+1} - b_t$  and  $c_{o,t} = \alpha K_t^\alpha L_t^{1-\alpha} + b_t$ .

In each period  $t$ , the respective government maximizes the aggregate utility of both generations. Its objective function is given by  $W_t = \tilde{u}_{y,t} + \tilde{u}_{o,t}$ . The government decides about the level of  $b_t$ ; the representative member of the young generation decides about  $s_t$ . With respect to the sequence of moves, the government is assumed to set  $b_t$  before the young individual decides about the level of  $s_t$ .<sup>8</sup>

A strategic relationship exists between the governments in the subsequent periods: Social security contributions in  $t$  have an influence on savings in  $t$  and thereby on the capital stock and the aggregate income in  $t+1$ . The transfers chosen by the government in  $t+1$  in turn influence the lifetime utility of the young in  $t$  and thereby also the payoff of the government in  $t$ . Intergenerational redistribution is a sequential game between the subsequent governments. Figure 1 illustrates this game in extensive form. The equilibrium of the sequential game is derived by backward induction, beginning with the terminal period  $T$  (see appendix a). The following two equations characterize this equilibrium:

$$(1) \quad c_{o,t} = [1 + \alpha\beta]c_{y,t},$$

$$(2) \quad k_{t+1} = \frac{\alpha\beta}{1 + \alpha\beta} k_t^\alpha, \quad t = 0 \dots T-2$$

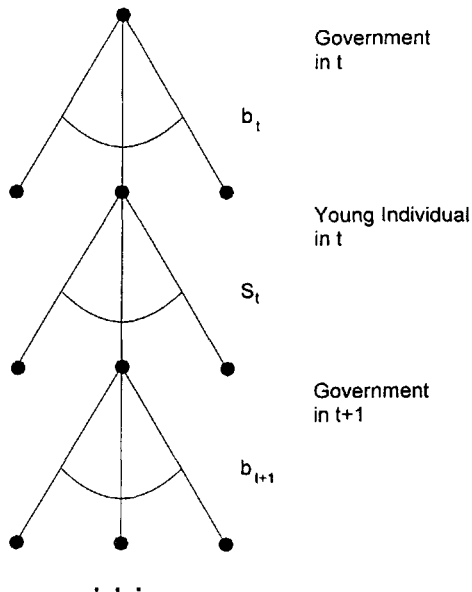
According to (1), the consumption level of the old generation exceeds the consumption level of the young generation. Intergenerational redistribution results in

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<sup>8</sup> This sequence of moves appears to be the most plausible to analyze policy design issues. However, it should be noted that in an overlapping generations model with only two generations the government is then assumed to be able to set the contribution level for the whole period  $t$ , that is the whole working life of the young.

an unequal distribution of consumption levels although both generations have the same weight in the objective function of each government.

Figure 1 – Social Security Game



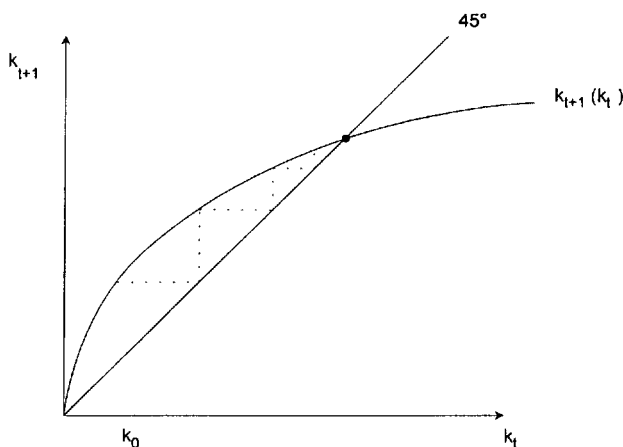
This result can be explained with the fact that the governments do not consider the welfare of unborn generations in their objective function. In  $t+1$  aggregate income is divided up into consumption of the old generation, consumption of the young generation and savings. The marginal return of aggregate savings in  $t$  therefore not only accrues to the old generation in  $t+1$  but also to the young generation in  $t+1$  and to individuals in subsequent periods. The government in  $t$ , however, only cares for the generations already living in  $t$ . It has an incentive to raise high social security contributions in order to reduce aggregate savings. As a result, social security contributions and benefits are above the level that equalizes consumption of both generations.

The term  $k_t$  in (2) denotes the capital-labor ratio in  $t$ :  $k_t \equiv K_t/L_t$ . It approaches gradually the steady-state level of  $k^*$ :

$$(3) \quad k^* = \left[ \frac{\alpha\beta}{1 + \alpha\beta} \right]^{\frac{1}{1-\alpha}}$$

Figure 2 illustrates the adjustment path to the steady state, starting from  $k_0 < k^*$ .

Figure 2 – Adjustment Path to the Steady State



The social security contribution rate as a percentage of labor income,  $\tilde{\tau}_t = \tau_t/w_t L_t$ , is given by

$$(4) \quad \tilde{\tau}_t = \frac{1 - \alpha[2 + \alpha\beta]}{[1 - \alpha][2 + \alpha\beta]}, \quad t = 0 \dots T - 1.$$

The contribution rate is constant over time. It is strictly positive for  $\alpha[2 + \alpha\beta] < 1$ .

For a further interpretation, the redistribution equilibrium may be compared with the utilitarian optimum where a social planner maximizes the discounted utility of all

individuals in all periods:  $W_{IT} = \sum_{t=0}^T \beta^t [\ln c_{y_t} + \ln c_{o_t}]$ . In the utilitarian optimum, both

generations get the same levels of consumption in each period, i.e.  $c_{y_t} = c_{o_t}$ . The steady state capital-labor ratio is given by the modified golden rule:  $k_{UT}^* = [\alpha\beta]^{1-\alpha}$  (see Ithori 1996). Comparing  $k^*$  with  $k_{UT}^*$  shows that  $k^* < k_{UT}^*$ . The capital-labor ratio in the redistribution steady state is below the capital-labor ratio that satisfies the modified golden rule. This result can be explained with the property  $c_{o_t} > c_{y_t}$  of the redistribution equilibrium in combination with the first order condition for individual savings  $c_{o_{t+1}}/\beta c_{y_t} = \alpha K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha}$ . According to the first order condition, the capital-labor ratio is at the golden rule level, if the representative individual consumes at the same level during retirement as during the working period. Because of  $c_{o_{t+1}} > c_{y_t}$ , the steady state capital-labor ratio is below the modified golden rule level.

### 3. Redistribution Within and Between Generations

This section introduces heterogeneity within generations into the basic model of the previous section. Each generation is now assumed to consist of two representative individuals –  $i = 1, 2$  – differing with respect to their labor productivity during the working life. Individual 1 has a higher productivity than individual 2 and consequently receives a higher wage rate. Labor supply of both young individuals is variable. Each individual is endowed with  $\bar{l}$  units of labor and supplies  $l_t^i$  units on the labor market.

Differences in productivity are represented by a productivity term that transforms physical units of supplied labor into efficiency units. This productivity term is set equal to 1 for individual 1, and it is equal to  $a$  for individual 2, with  $a < 1$ . For an interior solution of the model, it is necessary to restrict the range of  $a$  to  $a > 1/3$ . Aggregate labor supply,  $L_t$ , is given by  $L_t = l_t^1 + al_t^2$ . Individual 1 of the young generation receives a wage rate of  $w_t^1 = [1 - \alpha]K_t^\alpha L_t^{1-\alpha}$  and individual 2 gets a wage rate of  $w_t^2 = a[1 - \alpha]K_t^\alpha L_t^{1-\alpha}$ .

To incorporate an endogenous supply of labor, the individual utility function is extended to include the disutility of working. For simplicity, a special utility function is chosen in this paper that is linear in labor supply:  $\bar{u}_{y_t}^i = \ln c_{y_t}^i + \beta \ln c_{o_{t+1}}^i - v l_t^i$ . This

utility function allows an explicit and straightforward solution, in spite of the comparatively general set-up of the model.

The government in  $t$  maximizes  $W_t = \bar{u}_{y_t}^1 + \bar{u}_{y_t}^2 + \bar{u}_{o_t}^1 + \bar{u}_{o_t}^2$ . As in the previous section, the government pays social security benefits to the old and raises contributions from the young. The government can differentiate individual contributions and benefits according to labor incomes in order to redistribute within both generations. The government can observe labor incomes,  $w_t l_t^i$ , but not the wage rate,  $w_t$ , or individual labor supply,  $l_t^i$ . This setting reflects the information pattern that is underlying the theory of optimum labor income taxation.<sup>9</sup> To derive the equilibrium, the maximization problem of the government in  $t$  is redefined as finding the  $l_t^i$ ,  $\tau_t^i$  and  $b_t^i$  that maximize  $W_t$ , subject to the budget constraint  $\tau_t^1 + \tau_t^2 = b_t^1 + b_t^2$ , the self-selection constraint  $\ln c_{y_t}^1 + \beta \ln c_{o_{t+1}}^1 - v l_t^1 = \ln c_{y_t}^2 + \beta \ln c_{o_{t+1}}^2 - a v l_t^2$ , and subject to the solution of the model in subsequent periods.

The following equations describe individual consumption levels in the redistribution equilibrium (appendix b):

$$(5) \quad c_{o_t}^1 = c_{o_t}^2, \quad t = 0 \dots T$$

$$(6) \quad c_{y_t}^1 = c_{y_t}^2, \quad t = 0 \dots T - 1$$

$$(7) \quad c_{o_t} = [1 + \alpha\beta] c_{y_t}, \quad t = 0 \dots T - 2.$$

According to (5) and (6) consumption levels are perfectly redistributed within both generations. The high-productivity individual gets the same level of consumption as the low-productivity individual in both periods of life. With respect to intergenerational redistribution, (7) shows that the consumption level of the old generation exceeds the consumption level of the young generation as in the basic version of the model of the previous section.

The perfect redistribution result of (5) and (6) can be explained with the sequential nature of decision-making in determining redistribution policy and savings. The

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<sup>9</sup> See Mirlees (1971) or Stiglitz (1985, 1987).

government in  $t+1$  can redistribute lump-sum within the old generation. This leads to perfectly redistributed consumption levels within the old generation, i.e.  $c_{o,t+1}^1 = c_{o,t+1}^2$ . In period  $t$ , the young individuals save up to the point where the marginal rate of intertemporal substitution is equal to the marginal productivity of capital, i.e.  $c_{o,t+1}^1 / \beta c_{y,t}^1 = \alpha K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha}$ . Because of  $c_{o,t+1}^1 = c_{o,t+1}^2$  individual savings imply  $c_{y,t}^1 = c_{y,t}^2$ . The savings decision transfers the result of perfectly equalized consumption levels within the old generation to the young generation of the preceding period. The government in  $t$  then cannot influence the distribution of consumption within the young generation. For example, if the government in  $t$  decides to increase the contribution level of the high-wage individual and to lower the contribution level of the low wage individual, then both individuals adjust their savings and the government in the following period adjusts social security benefits to completely offset the effects of this policy change.

If consumption levels are equalized within both generations, then the self-selection constraint implies that  $l_t^1 = a l_t^2$ . The individual with the high wage rate supplies less labor than the individual with the low wage rate to receive the same labor income. This outcome is Pareto-inefficient.<sup>10</sup>

The following policy experiment shows the potential for a Pareto-improvement of the redistribution equilibrium: Assume that in period  $t$  ( $t < T$ ) individual labor supply  $l_t^1$  is increased by  $\Delta l_t^1$  ( $\Delta l_t^1 \rightarrow 0$ ) and  $l_t^2$  is reduced by  $\Delta l_t^2 = -\Delta l_t^1 / a$  such that aggregate labor supply remains constant, i.e.  $\Delta L_t = 0$ . In addition, individual contributions  $\tau_t^i$  and benefits  $b_{t+1}^i$  are adjusted to keep the self-selection constraint in period  $t$  satisfied as an equality and to keep individual savings constant. Totally differentiating the self-selection constraint leads to

$$(8) \quad \frac{dc_{y,t}^1}{c_{y,t}^1} + \beta \frac{dc_{o,t+1}^1}{c_{o,t+1}^1} - v dl_t^1 = \frac{dc_{y,t}^2}{c_{y,t}^2} + \beta \frac{dc_{o,t+1}^2}{c_{o,t+1}^2} - v dl_t^2.$$

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<sup>10</sup> See Stiglitz (1985, 1987) for the Pareto-optimal redistribution policy in an overlapping generations setting.

The budget constraint of period  $t$  implies  $dc_{y_t}^2 = -dc_{y_t}^1$ . The budget constraint of period  $t+1$  gives  $dc_{o_{t+1}}^2 = -dc_{o_{t+1}}^1$ . Inserting these equations in (8) leads to

$$(9) \quad \frac{dc_{y_t}^1}{c_{y_t}^1} + \beta \frac{dc_{o_{t+1}}^1}{c_{o_{t+1}}^1} - vdl_t^1 = 0.$$

The effects of this policy experiment on individual utilities are given by

$$(10) \quad d\bar{u}_{y_t}^1 = \frac{dc_{y_t}^1}{c_{y_t}^1} + \beta \frac{dc_{o_{t+1}}^1}{c_{o_{t+1}}^1} - vdl_t^1.$$

The utility of individual 1 is not influenced by the policy experiment ( $d\bar{u}_{y_t}^1 = 0$ ), whereas the utility of individual 2 increases:

$$(11) \quad \Delta \bar{u}_{y_t}^2 = v \frac{1-a}{a} \Delta l_t^1 > 0.$$

The utility of individual 2 in period  $t$  can be increased without lowering the utility of any other individual. The equilibrium is therefore not Pareto-efficient.

The equilibrium capital-labor ratio is given by the same equation (2) as in the basic version of the model without intra-generational redistribution. The economy therefore approaches the same steady state capital-labor ratio as in the basic model. This capital-labor ratio is again lower than the capital-labor ratio defined by the modified golden rule. Average social security contributions as a percentage of labor income are again given by (3).

Individual labor supply is determined by the following condition (see appendix b):

$$(12) \quad v_{c_{y_t}} = \frac{2a[1+\alpha\beta]}{[1+a][1+2\alpha\beta]} [1-\alpha]k_t^\alpha, \quad t = 0 \dots T-1.$$

The left-hand-side of (12) denotes the marginal rate of substitution between consumption and leisure. The term  $[1-\alpha]k_t^\alpha$  on the right-hand-side of (12) denotes the wage rate  $w_t^1$  of individual 1. As (12) shows, the wage rate  $w_t^1$  exceeds the marginal rate of substitution; labor supply of the high-wage individual is taxed at the margin. This outcome is in contrast to a standard result of the theory of optimum

income taxation according to which the marginal tax rate for the highest income group should be zero.<sup>11</sup>

#### 4. Comparative Statics

This section shows the effect of a change in exogenous parameters on the redistribution equilibrium. For this purpose, the model of the previous section is extended in three directions: First, instead of assuming that both generations have the same weight in the objective function of the government, the weight of the old generation is allowed to differ from one. Second, population growth is included, and third, an exogenous probability of dying at the beginning of the retirement period is introduced into the model to analyze the effects of changes in life-expectancy.

Instead of assuming only one representative individual per group, the size of each young group in  $t$  is now given by  $N_t$ . The term  $N_t$  grows with the constant rate of  $n$ . Each individual faces a given probability  $1 - \pi$  of dying at the beginning of the retirement period. The expected utility of a young individual is then given by  $\tilde{u}_{y_t}^t = \ln c_{y_t}^t + \pi\beta \ln c_{o_{t+1}}^t - v l_t^t$ . It is assumed that the young individuals can invest in a mutual fund that pays capital income only to the surviving old. The rate of return to capital is then given by  $r_t = [1 - \alpha]k_t^{\alpha-1}/\pi$  and consumption of the old is  $c_{o_t}^t = \alpha s_{t-1}^t k_t^{\alpha-1}/\pi + b_t^t$ .

Let  $\omega$  denote the weight of the old generation in the objective function of the government. The objective function of the government is given by  $W_t = N_{t-1} \left[ [1+n] \tilde{u}_{y_t}^t + [1+n] \tilde{u}_{o_t}^t + \omega \pi \tilde{u}_{o_t}^t + \omega \pi \tilde{u}_{o_t}^t \right]$ . The budget constraint is  $[1+n] [\tau_t^1 + \tau_t^2] = \pi [b_t^1 + b_t^2]$ . The following equations characterize the equilibrium of the extended model (see appendix c):

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<sup>11</sup> Labor supply of individual 2 may be taxed or subsidized at the margin depending on the values of the parameters  $a$ ,  $\alpha$  and  $\beta$ .



$$(13) \quad k^* = \left( \frac{\alpha\beta}{\omega + \alpha\beta[1+n]} \right)^{\frac{1}{1-\alpha}},$$

$$(14) \quad c_{o_t} = \{\omega + \alpha\beta[1+n]\}c_{y_t}, \quad t = 0 \dots T-2$$

$$(15) \quad \bar{\tau}_t = \frac{\omega\pi[1-\alpha] - \alpha[1+\alpha\beta\pi][1+n]}{\{\omega\pi + [1+n][1+\alpha\beta\pi]\}[1-\alpha]}, \quad t = 0 \dots T-1.$$

The term  $\bar{\tau}_t$  denotes the average contribution rate to social security. From (13), (14) and (15) the influence of the exogenous parameters  $\omega$ ,  $n$  and  $\pi$  can be derived. With respect to the political weight,  $\omega$ , equation (15) implies that  $\frac{d\bar{\tau}}{d\omega} > 0$ . An increase in the political weight of the old generation causes an increase in the social security contribution rate. As a consequence, the consumption level of the old generation relative to the consumption level of the young generation increases, and the steady state capital-labor ratio declines:  $\frac{dc_o/c_y}{d\omega} > 0$  and  $\frac{dk^*}{d\omega} < 0$ .

An increase in the rate of population growth,  $n$ , causes the steady state capital-labor ratio and the contribution rate to decline. The relative consumption level of the old increases:<sup>12</sup>  $\frac{dk}{dn} < 0$ ,  $\frac{dc_o/c_y}{dn} > 0$  and  $\frac{d\tau}{dn} < 0$ .

With respect to life expectancy, (13) and (14) show that  $\frac{dk}{d\pi} = 0$  and  $\frac{dc_y/c_o}{d\pi} = 0$ .

According to (15), the contribution rate increases with an increase in the probability of survival:  $\frac{d\bar{\tau}}{d\pi} > 0$ .

For a given level of social security transfers, the consumption level of the old generation declines with an increase in  $\pi$ , because the aggregate capital income has to be shared between a greater number of retired. As a consequence, the relative consumption level of the old compared to the young decreases. The government can

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<sup>12</sup> See also Verbon et al. (1998) who derive a similar result with respect to the influence of population growth on the contribution rate.

offset this distribution effect of a change in life expectancy by paying higher social security benefits to the old and accordingly by raising higher contributions from the young. Because the relative consumption level of the old then remains unchanged, a change in life expectancy has no effects on the capital-labor ratio in the steady state.<sup>13</sup>

## 5. Concluding Remarks

As this paper has shown, the social security and redistribution policy in a representative democracy may differ from the policy in the utilitarian second-best optimum. On the one hand, a redistribution bias in favor of the old generation exists that leads to a capital-labor ratio below the modified golden rule level. On the other hand, labor supply in the redistribution equilibrium is not Pareto-efficient, because consumption levels are perfectly equalized within both generations. The perfect redistribution result of this paper may appear to be somewhat extreme, if it is compared to reality. Additional factors that are not considered in this paper seem to moderate redistribution within the old generation and prevent perfectly equalized consumption levels. However, the main driving force behind the Pareto-inefficiency result is expected to hold also in a generalized framework: the government that redistributes within the old generation does not need to consider labor supply effects of previous periods.

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<sup>13</sup> However, this does not mean that a change in life expectancy has no real effects on the economy. Instead, aggregate labor supply increases with the increase in  $\pi$  (see appendix c).

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## Appendix a: Equilibrium with Intergenerational Redistribution

In this appendix the equilibrium of the basic model of section 2 is derived. The government in the final period  $T$  maximizes  $W_T = \ln c_{yT} + \ln c_{oT}$ , with  $c_{yT} = [1 - \alpha]K_T^\alpha L_T^{1-\alpha} - b_T$  and  $c_{oT} = \alpha K_T^\alpha L_T^{1-\alpha} + b_T$ . The solution of this maximization problem is given by

$$(a.1) \quad c_{yT} = c_{oT} = \frac{1}{2} K_T^\alpha L_T^{1-\alpha}.$$

In period  $T-1$  the representative individual of the young generation decides about its savings. In the savings equilibrium, the first order condition  $\frac{c_{oT}}{\beta c_{yT-1}} = \alpha K_T^{\alpha-1} L_T^{1-\alpha}$  has to be satisfied. Inserting from (a.1) into this first order condition yields

$$(a.2) \quad c_{yT-1} = \frac{K_T}{2\alpha\beta}.$$

Inserting (a.2) into  $c_{yT-1} = [1 - \alpha]K_{T-1}^\alpha L_{T-1}^{1-\alpha} - K_T - b_{T-1}$  and differentiating leads to

$$(a.3) \quad \frac{dc_{yT-1}}{db_{T-1}} = -\frac{1}{1 + 2\alpha\beta},$$

$$(a.4) \quad \frac{dc_{oT}}{db_{T-1}} = -\frac{\alpha^2\beta}{1 + 2\alpha\beta} K_T^{\alpha-1} L_T^{1-\alpha}.$$

The government in  $T-1$  sets  $b_{T-1}$  to maximize  $W_{T-1} = \ln c_{yT-1} + \ln c_{oT-1} + \beta \ln c_{oT}$ , with  $c_{oT-1} = \alpha K_{T-1}^\alpha L_{T-1}^{1-\alpha}$  and given the influence of  $b_{T-1}$  on  $c_{yT-1}$  and  $c_{oT}$  as described by (a.3) and (a.4). From the first order condition for  $b_{T-1}$  the following equation can be derived:

$$(a.5) \quad c_{oT-1} = \frac{1 + 2\alpha\beta}{2\alpha\beta[1 + \alpha\beta]} K_T.$$

Inserting (a.2) and (a.5) into the equation  $c_{yT-1} + c_{oT-1} + K_T = K_{T-1}^\alpha L_{T-1}^{1-\alpha}$  gives

$$(a.6) \quad K_T = \frac{2\alpha\beta[1 + \alpha\beta]}{[1 + 2\alpha\beta][2 + \alpha\beta]} K_{T-1}^\alpha L_{T-1}^{1-\alpha},$$

$$(a.7) \quad c_{oT-1} = \frac{K_{T-1}^\alpha L_{T-1}^{1-\alpha}}{2 + \alpha\beta}.$$

In period  $T-2$ , the consumption level of the young individual is given by

$$(a.8) \quad c_{yT-2} = \frac{K_{T-1}}{\alpha\beta[2 + \alpha\beta]}.$$

This implies

$$(a.9) \quad \frac{dc_{yT-2}}{db_{T-2}} = -\frac{1}{[1 + \alpha\beta]^2},$$

$$(a.10) \quad \frac{dc_{oT-1}}{db_{T-2}} = -\frac{\alpha^2\beta}{[1 + \alpha\beta]^2} K_{T-1}^{\alpha-1} L_{T-1}^{1-\alpha}.$$

The government in  $T-2$  chooses  $b_{T-2}$  to maximize  $W_{T-2}$ . This leads to

$$(a.11) \quad c_{oT-2} = \frac{1 + \alpha\beta}{\alpha\beta[2 + \alpha\beta]} K_{T-1}.$$

Inserting (a.8) and (a.11) into  $c_{yT-2} + c_{oT-2} + K_{T-1} = K_{T-2}^\alpha L_{T-2}^{1-\alpha}$  then yields

$$(a.12) \quad K_{T-1} = \frac{\alpha\beta}{1 + \alpha\beta} K_{T-2}^\alpha L_{T-2}^{1-\alpha},$$

$$(a.13) \quad c_{oT-2} = \frac{K_{T-2}^\alpha L_{T-2}^{1-\alpha}}{2 + \alpha\beta}.$$

The solution for the period  $T-3$  and all previous periods is the same as for  $T-2$ .

The following equations then describe the equilibrium for  $t = 0 \dots T-2$ :

$$(a.14) \quad c_{o_t} = [1 + \alpha\beta]c_{y_t},$$

$$(a.15) \quad K_{t+1} = \frac{\alpha\beta}{1 + \alpha\beta} K_t^\alpha L_t^{1-\alpha}.$$

With  $k_t \equiv K_t/L_t$ , equation (a.15) may be rewritten as

$$(a.18) \quad k_{t+1} = \frac{\alpha\beta}{1 + \alpha\beta} k_t^\alpha.$$

## Appendix b: Equilibrium with Intra- and Intergenerational Redistribution

In this appendix, the equilibrium is derived for the model with intra- and intergenerational redistribution of section 3. As in appendix a, the equilibrium is derived by backward induction beginning with the terminal period  $T$ . The government in  $T$  maximizes

$$W_T = \sum_{i=1}^2 \left[ \ln c'_{y_T} + \ln c'_{o_T} - \nu_T^i \right], \quad \text{with} \quad c'_{y_T} = l_T^1 [1 - \alpha] K_T^\alpha L_T^{-\alpha} - \tau_T^1,$$

$c'_{y_T} = a l_T^2 [1 - \alpha] K_T^\alpha L_T^{-\alpha} - \tau_T^2$  and  $c'_{o_T} = s_{T-1}^1 \alpha K_T^{\alpha-1} L_T^{1-\alpha} + b_T^1$ . The capital stock  $K_T$  is given by  $K_T = s_{T-1}^1 + s_{T-1}^2$ . The government in  $T$  has to obey the budget constraint  $b_T^1 + b_T^2 = \tau_T^1 + \tau_T^2$  and the self-selection constraint  $\ln c'_{y_T} - \nu_T^1 = \ln c'_{y_T} - \nu_T^2$ . The following equations can be derived from the first order conditions of this constrained maximization problem:

$$(b.1) \quad c'_{o_T} = c_{o_T}^2, \quad c'_{y_T} = [1 + \sigma_T] c_{o_T}, \quad c_{y_T}^2 = [1 - \sigma_T] c_{o_T}.$$

The term  $\sigma_T$  denotes the shadow price of the self-selection constraint ( $\sigma_T > 0$ ).

Inserting (b.1) into the constraint  $c_{o_T}^1 + c_{o_T}^2 + c_{y_T}^1 + c_{y_T}^2 = K_T^\alpha L_T^{1-\alpha}$  produces

$$(b.2) \quad c_{o_T} = \frac{K_T^\alpha L_T^{1-\alpha}}{4}.$$

The first order conditions for individual savings in T-1 are given by

$$\frac{c_{o_T}^1}{\beta c'_{y_{T-1}}} = \alpha K_T^{\alpha-1} L_T^{1-\alpha}. \quad \text{Because of } c_{o_T}^1 = c_{o_T}^2 \text{ these first order conditions imply}$$

$$c_{y_{T-1}}^1 = c_{y_{T-1}}^2. \quad \text{Inserting from (b.2) yields}$$

$$(b.3) \quad c_{y_{T-1}} = \frac{K_T}{4\alpha\beta}.$$

$$\text{Inserting (b.3) into } c_{y_{T-1}}^1 = l_{T-1}^1 [1 - \alpha] K_{T-1}^\alpha L_{T-1}^{-\alpha} - \tau_{T-1}^1 - s_{T-1}^1 \quad \text{and}$$

$$c_{y_{T-1}}^2 = a l_{T-1}^2 [1 - \alpha] K_{T-1}^\alpha L_{T-1}^{-\alpha} - \tau_{T-1}^2 - s_{T-1}^2 \text{ leads to}$$

$$(b.4) \quad l_{T-1}^1 [1 - \alpha] K_{T-1}^\alpha L_{T-1}^{-\alpha} - \tau_{T-1}^1 - s_{T-1}^1 = \frac{s_{T-1}^1 + s_{T-1}^2}{4\alpha\beta},$$

$$(b.5) \quad a l_{T-1}^2 [1 - \alpha] K_{T-1}^\alpha L_{T-1}^{1-\alpha} - \tau_{T-1}^2 - s_{T-1}^2 = \frac{s_{T-1}^1 + s_{T-1}^2}{4\alpha\beta}.$$

Totally differentiating (b.4) and (b.5) gives

$$(b.6) \quad \frac{dK_T}{d\tau_{T-1}^1} = \frac{dK_T}{d\tau_{T-1}^2} = -\frac{2\alpha\beta}{1+2\alpha\beta},$$

$$(b.7) \quad \frac{dc_{oT}}{d\tau_{T-1}^1} = \frac{dc_{oT}}{d\tau_{T-1}^2} = -\frac{\alpha^2\beta}{2[1+2\alpha\beta]} K_T^{\alpha-1} L_T^{1-\alpha},$$

$$(b.8) \quad \frac{dc_{yT-1}}{d\tau_{T-1}^1} = \frac{dc_{yT-1}}{d\tau_{T-1}^2} = -\frac{1}{2[1+2\alpha\beta]}.$$

The government in  $T-1$  maximizes  $W_{T-1} = \sum_{i=1}^2 [\ln c_{oT-1}^i + \ln c_{yT-1}^i + \beta \ln c_{oT}^i - v l_i^i]$

subject to the budget constraint and the self-selection constraint. Because of  $c_{oT}^1 = c_{oT}^2$  and  $c_{yT-1}^1 = c_{yT-1}^2$  the self selection constraint is given by  $l_{T-1}^1 = a l_{T-1}^2$ .

The first order conditions for  $b'_{T-1}$  imply  $c_{oT-1}^1 = c_{oT-1}^2$ . The first order conditions for  $\tau_{T-1}^i$  lead to the following equation:<sup>14</sup>

$$(b.9) \quad c_{oT-1} = \frac{1+2\alpha\beta}{4\alpha\beta[1+\alpha\beta]} K_T.$$

Inserting (b.3) and (b.9) into  $2c_{yT-1} + 2c_{oT-1} + K_T = K_{T-1}^\alpha L_{T-1}^{1-\alpha}$  gives

$$(b.10) \quad K_T = \frac{2\alpha\beta[1+\alpha\beta]}{[1+2\alpha\beta][2+\alpha\beta]} K_{T-1}^\alpha L_{T-1}^{1-\alpha},$$

$$(b.11) \quad c_{oT-1} = \frac{K_{T-1}^\alpha L_{T-1}^{1-\alpha}}{2[2+\alpha\beta]}.$$

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<sup>14</sup> Because  $\tau_{T-1}^1$  and  $\tau_{T-1}^2$  have the same influence on consumption levels, one of the first order conditions for  $\tau_{T-1}^i$  is redundant.

The consumption level of the young in period T-2 is again determined by

$$\frac{c_{oT-1}}{\beta c_{yT-2}} = \alpha K_{T-1}^{\alpha-1} L_{T-1}^{1-\alpha}. \text{ Inserting for } c_{oT-1} \text{ from (b.11) leads to}$$

$$(b.12) \quad c_{yT-2} = \frac{K_{T-1}}{2\alpha\beta[2 + \alpha\beta]}.$$

The following equations can be derived from (b.12):

$$(b.13) \quad \frac{dK_{T-1}}{d\tau_{T-2}} = -\frac{\alpha\beta[2 + \alpha\beta]}{[1 + \alpha\beta]^2},$$

$$(b.14) \quad \frac{dc_{oT-1}}{d\tau_{T-2}} = -\frac{\alpha^2 \beta K_{T-1}^{\alpha-1} L_{T-1}^{1-\alpha}}{2[1 + \alpha\beta]^2},$$

$$(b.15) \quad \frac{dc_{yT-2}}{d\tau_{T-2}} = -\frac{1}{2[1 + \alpha\beta]^2}.$$

The government in  $T-2$  maximizes  $W_{T-2}$ . The first order conditions of this maximization problem lead to

$$(b.16) \quad c_{oT-2} = \frac{1 + \alpha\beta}{2\alpha\beta[2 + \alpha\beta]} K_{T-1}.$$

Inserting (b.12) and (b.16) into  $2c_{yT-2} + 2c_{oT-2} + K_{T-1} = K_{T-2}^\alpha L_{T-2}^{1-\alpha}$  yields

$$(b.17) \quad K_{T-1} = \frac{\alpha\beta}{1 + \alpha\beta} K_{T-2}^\alpha L_{T-2}^{1-\alpha},$$

$$(b.18) \quad c_{oT-2} = \frac{K_{T-2}^\alpha L_{T-2}^{1-\alpha}}{2[2 + \alpha\beta]}.$$

This gives in general ( $t = 0 \dots T-2$ ):

$$(b.19) \quad c_{o_t} = [1 + \alpha\beta]c_{y_t},$$

$$(b.20) \quad K_{t+1} = \frac{\alpha\beta}{1 + \alpha\beta} K_t^\alpha L_t^{1-\alpha},$$

$$(b.21) \quad k_{t+1} = \frac{\alpha\beta}{1 + \alpha\beta} k_t^\alpha.$$



The average contribution rate as a percentage of labor income,  $\bar{\tau}_t = [\tau_t^1 + \tau_t^2] / [w_t^1 l_t^1 + w_t^2 l_t^2]$ , is given by

$$(b.22) \quad \bar{\tau}_t = \frac{1 - \alpha[2 + \alpha\beta]}{\{2 + \alpha\beta\}[1 - \alpha]}$$

The solution derived so far holds for any given aggregate labor supply. The equilibrium level of labor supply is derived in the following. In period T, the first order conditions for the government with respect to  $l_T^1$  are given by

$$(b.23) \quad v c_{y_T}^1 = [1 - \alpha] K_T^\alpha L_T^{-\alpha},$$

$$(b.24) \quad v c_{y_T}^1 [1 - a \sigma_T] = [1 + \sigma_T] a [1 - \alpha] K_T^\alpha L_T^{-\alpha}.$$

Dividing (b.24) by (b.23) leads to  $\sigma_T = \frac{1 - a}{2a}$ . Inserting (b.1) and (b.2) into (b.23) then gives

$$(b.25) \quad L_T = \frac{8a[1 - \alpha]}{v[1 + a]}.$$

In period T-1 the government obeys the following first order conditions:

$$(b.26) \quad v[1 + \sigma_{T-1}] = \frac{2\alpha[1 - \alpha] K_{T-1}^\alpha L_{T-1}^{-\alpha}}{c_{o_{T-1}}} + \frac{2}{c_{y_{T-1}}} \frac{dc_{y_{T-1}}}{dl_{T-1}^1} + \frac{2\beta}{c_{o_T}} \frac{dc_{o_T}}{dl_{T-1}^1},$$

$$(b.27) \quad v[1 - a\sigma_{T-1}] = \frac{2a\alpha[1 - \alpha] K_{T-1}^\alpha L_{T-1}^{-\alpha}}{c_{o_{T-1}}} + \frac{2}{c_{y_{T-1}}} \frac{dc_{y_{T-1}}}{dl_{T-1}^2} + \frac{2\beta}{c_{o_T}} \frac{dc_{o_T}}{dl_{T-1}^2}.$$

The terms  $c_{o_{T-1}}$ ,  $c_{y_{T-1}}$  and  $c_{o_T}$  are given by (b.9), (b.3) and (b.2) respectively. The terms  $dc_{y_{T-1}}/dl_{T-1}^1$  and  $dc_{o_T}/dl_{T-1}^1$  can be derived from (b.4) and (b.5):

$$(b.28) \quad \frac{dK_T}{dl_{T-1}^1} = \frac{1}{a} \frac{dK_T}{dl_{T-1}^2} = \frac{2\alpha\beta[1 - \alpha]^2 K_{T-1}^\alpha L_{T-1}^{-\alpha}}{1 + 2\alpha\beta},$$

$$(b.29) \quad \frac{dc_{y_{T-1}}}{dl_{T-1}^1} = \frac{1}{a} \frac{dc_{y_{T-1}}}{dl_{T-1}^2} = \frac{[1 - \alpha]^2 K_{T-1}^\alpha L_{T-1}^{-\alpha}}{2[1 + 2\alpha\beta]},$$

$$(b.30) \quad \frac{dc_{oT}}{dl_{T-1}^1} = \frac{1}{a} \frac{dc_{oT}}{dl_{T-1}^2} = \frac{\alpha^2 \beta K_T^{\alpha-1} L_T^{1-\alpha} [1-\alpha]^2 K_{T-1}^\alpha L_{T-1}^{-\alpha}}{2[1+2\alpha\beta]}.$$

Inserting gives

$$(b.31) \quad v[1+\sigma_{T-1}] = \frac{4\alpha\beta[1+\alpha\beta][1-\alpha]K_{T-1}^\alpha L_{T-1}^{-\alpha}}{[1+2\alpha\beta]K_T},$$

$$(b.32) \quad v[1-a\sigma_{T-1}] = a \frac{4\alpha\beta[1+\alpha\beta][1-\alpha]K_{T-1}^\alpha L_{T-1}^{-\alpha}}{[1+2\alpha\beta]K_T}.$$

This gives  $\sigma_{T-1} = \frac{1-a}{2a}$  and

$$(b.33) \quad K_T = \frac{8a\alpha\beta[1+\alpha\beta][1-\alpha]K_{T-1}^\alpha L_{T-1}^{-\alpha}}{[1+2\alpha\beta][1+a]v}.$$

Inserting from (b.10) then implies

$$(b.34) \quad L_{T-1} = \frac{4a[2+\alpha\beta][1-\alpha]}{v[1+a]}.$$

The same procedure for  $T-2$  and previous periods leads to  $\sigma_t = \frac{1+a}{2a}$  and

$$(b.35) \quad L_t = \frac{4a[2+\alpha\beta][1-\alpha]}{v[1+a]}, \quad \text{for all } t = 0 \dots T-1.$$

### Appendix c: Comparative Statics

In this appendix the redistribution model is extended to derive the comparative static results of section 4. The objective function of the government in period T is given by

$$W_T = N_{T-1} \sum_{i=1}^2 \left\{ [1+n] \ln c'_{yT} - [1+n] v l'_T + \omega \pi \ln c'_{oT} \right\} \quad \text{with} \quad c'_{yT} = l_T^1 [1-\alpha] K_T^\alpha L_T^{-\alpha} - \tau_T^1,$$

$$c'_{yT} = a l_T^2 [1-\alpha] K_T^\alpha L_T^{-\alpha} - \tau_T^2 \quad \text{and} \quad c'_{oT} = \frac{\alpha K_T^{\alpha-1} L_T^{1-\alpha}}{\pi} s'_{T-1} + b'_T. \quad \text{The aggregate capital stock}$$

$K_T$  is given by  $K_T = N_{T-1} [s_{T-1}^1 + s_{T-1}^2]$ . The budget constraint is

$$N_{T-1} \left\{ [1+n] [\tau_T^1 + \tau_T^2] - \pi [b_T^1 + b_T^2] \right\} = 0, \quad \text{and the self-selection constraint is}$$

$N_{T-1}[1+n]\{\ln c_{yT}^1 - \nu_T^1 - \ln c_{yT}^2 + \nu_T^2\} = 0$ . The following equations then determine the equilibrium consumption levels in T:

$$(c.1) \quad c_{oT}^1 = c_{oT}^2, \quad c_{yT}^1 = \frac{1 + \sigma_T}{\omega} c_{oT}^2, \quad c_{yT}^2 = \frac{1 - \sigma_T}{\omega} c_{oT}^2.$$

Inserting (c.1) into  $N_{T-1}\{[1+n][c_{yT}^1 + c_{yT}^2] + \pi[c_{oT}^1 + c_{oT}^2]\} = K_T^\alpha L_T^{1-\alpha}$  leads to

$$(c.2) \quad N_{T-1}c_{oT}^2 = \frac{\omega K_T^\alpha L_T^{1-\alpha}}{2[1+n+\omega\pi]}.$$

The first order conditions for individual savings then imply

$$(c.3) \quad N_{T-1}c_{yT-1} = \frac{\omega K_T}{2\alpha\beta[1+n+\omega\pi]}.$$

This leads to

$$(c.4) \quad l_{T-1}^1[1-\alpha]K_{T-1}^\alpha L_{T-1}^{1-\alpha} - \tau_{T-1}^1 - s_{T-1}^1 = \frac{\omega[s_{T-1}^1 + s_{T-1}^2]}{2\alpha\beta[1+n+\omega\pi]},$$

$$(c.5) \quad al_{T-1}^2[1-\alpha]K_{T-1}^\alpha L_{T-1}^{1-\alpha} - \tau_{T-1}^2 - s_{T-1}^2 = \frac{\omega[s_{T-1}^1 + s_{T-1}^2]}{2\alpha\beta[1+n+\omega\pi]}.$$

Totally differentiating (c.4) and (c.5) gives

$$(c.6) \quad \frac{dK_T}{d\tau_{T-1}^1} = -\frac{\alpha\beta[1+n+\omega\pi]N_{T-1}}{\omega + \alpha\beta[1+n+\omega\pi]}.$$

The government in T-1 maximizes  $W_{T-1} = N_{T-2} \sum_{i=1}^2 \{[1+n]\tilde{u}_{yT-1}^i + \pi\omega\tilde{u}_{oT-1}^i\}$ . The first order conditions lead to the following equation:

$$(c.7) \quad N_{T-1}c_{oT-1} = \frac{\omega\{\omega + \alpha\beta[1+n+\omega\pi]\}K_T}{2\alpha\beta[1+n+\omega\pi][1+\alpha\beta\pi]}.$$

Inserting into  $2N_{T-1}c_{yT-1} + 2\pi N_{T-2}c_{oT-1} + K_T = K_{T-1}^\alpha L_{T-1}^{1-\alpha}$  produces

$$(c.8) \quad K_T = \frac{\alpha\beta[1+n+\omega\pi][1+\alpha\beta\pi][1+n]}{\{\omega\pi + [1+n][1+\alpha\beta\pi]\}\{\omega + \alpha\beta[1+n+\omega\pi]\}} K_{T-1}^\alpha L_{T-1}^{1-\alpha},$$

$$(c.9) \quad N_{T-2}c_{oT-1} = \frac{\omega K_{T-1}^\alpha L_{T-1}^{1-\alpha}}{2\{\omega\pi + [1+n][1+\alpha\beta\pi]\}}.$$

This leads to

$$(c.10) \quad N_{T-2}c_{yT-2} = \frac{\omega K_{T-1}}{2\alpha\beta\{\omega\pi + [1+n][1+\alpha\beta\pi]\}}.$$

This implies

$$(c.11) \quad \frac{dK_{T-1}}{d\tau_{T-2}} = -\frac{\alpha\beta\{\omega\pi + [1+n][1+\alpha\beta\pi]\}N_{T-2}}{\{\omega + \alpha\beta[1+n]\}[1+\alpha\beta\pi]}.$$

Maximizing  $W_{T-2}$  gives

$$(c.12) \quad N_{T-2}c_{oT-2} = \frac{\omega\{\omega + \alpha\beta[1+n]\}}{2\alpha\beta\{\omega\pi + [1+n][1+\alpha\beta\pi]\}}K_{T-1}.$$

Inserting into  $2N_{T-2}c_{yT-2} + 2\pi N_{T-3}c_{oT-2} + K_{T-1} = K_{T-2}^\alpha L_{T-2}^{1-\alpha}$  then implies

$$(c.13) \quad K_{T-1} = \frac{\alpha\beta[1+n]}{\omega + \alpha\beta[1+n]}K_{T-2}^\alpha L_{T-2}^{1-\alpha},$$

$$(c.14) \quad k_{t+1} = \frac{\alpha\beta}{\omega + \alpha\beta[1+n]}k_t^\alpha,$$

$$(c.15) \quad k^* = \left(\frac{\alpha\beta}{\omega + \alpha\beta[1+n]}\right)^{\frac{1}{1-\alpha}},$$

$$(c.16) \quad c_{o_t} = \{\omega + \alpha\beta[1+n]\}c_{y_t}, \quad t = 0 \dots T-2,$$

$$(c.17) \quad \bar{v}_t = \frac{\omega\pi - \alpha\{\omega\pi + [1+n][1+\alpha\beta\pi]\}}{\{\omega\pi + [1+n][1+\alpha\beta\pi]\}[1-\alpha]}, \quad t = 0 \dots T-1.$$

The supply of labor in the terminal period  $T$  is determined as in appendix b by the first order conditions

$$(c.18) \quad vc_{y_T}^1 = [1-\alpha]K_T^\alpha L_T^{-\alpha},$$

$$(c.19) \quad vc_{y_T}^1[1-a\sigma_T] = [1+\sigma_T]a[1-\alpha]K_T^\alpha L_T^{-\alpha}.$$

This implies  $\sigma_T = \frac{1-a}{2a}$  and  $\frac{L_T}{N_T} = \frac{4a[1+n+\omega\pi][1-\alpha]}{\nu[1+a][1+n]}$ . In T-1 the first order conditions are

$$(c.20) \quad \nu[1+\sigma_{T-1}] = \frac{\omega\alpha[1-\alpha]K_{T-1}^\alpha L_{T-1}^{-\alpha}}{c_{oT-1}} + \frac{2}{c_{yT-1}} \frac{dc_{yT-1}}{dl_{T-1}^1} + \frac{2\beta\pi}{c_{oT}} \frac{dc_{oT}}{dl_{T-1}^1},$$

$$(c.21) \quad \nu[1-a\sigma_{T-1}] = \frac{a\omega\alpha[1-\alpha]K_{T-1}^\alpha L_{T-1}^{-\alpha}}{c_{oT-1}} + \frac{2}{c_{yT-1}} \frac{dc_{yT-1}}{dl_{T-1}^2} + \frac{2\beta\pi}{c_{oT}} \frac{dc_{oT}}{dl_{T-1}^2}.$$

From (c.4) and (c.5) the following equation can be derived:

$$(c.22) \quad \frac{dK_T}{dl_{T-1}^1} = \frac{1}{a} \frac{dK_T}{dl_{T-1}^2} = \frac{\alpha\beta[1+n+\omega\pi][1-\alpha]^2 N_{T-1} K_{T-1}^\alpha L_{T-1}^{-\alpha}}{\omega + \alpha\beta[1+n+\omega\pi]}.$$

This implies  $\sigma_{T-1} = \frac{1-a}{2a}$  and

$$(c.23) \quad K_T = \frac{4a\alpha\beta[1+\alpha\beta\pi][1+n+\omega\pi][1-\alpha]N_{T-1}K_{T-1}^\alpha L_{T-1}^{-\alpha}}{\{\omega + \alpha\beta[1+n+\omega\pi]\}[1+a]\nu}.$$

Inserting from (c.8) then implies

$$(c.24) \quad \frac{L_{T-1}}{N_{T-1}} = \frac{4a\{\omega\pi + [1+\alpha\beta\pi][1+n]\}[1-\alpha]}{\nu[1+a][1+n]}.$$

The general equation is :

$$(c.25) \quad \frac{L_t}{N_t} = \frac{4a\{\omega\pi + [1+\alpha\beta\pi][1+n]\}[1-\alpha]}{\nu[1+a][1+n]}, \quad \text{for } t = 0 \dots T-1.$$