Lorz, Jens Oliver

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with International Capital Mobility

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Wage Competition 
with International Capital Mobility 

Oliver Lorz*
The Kiel Institute of World Economics

Abstract: This paper analyzes wage competition between national trade 
unions caused by the international mobility of capital. Perfect capital mobility 
leads to a Bertrand result for the outcome of wage competition: A pure 
strategy equilibrium implies full employment in all countries. It is shown that 
such an equilibrium exists for a sufficiently large number of countries. As 
extensions of the basic model, decreasing returns to scale and capital 
adjustment costs are introduced.

Keywords: Capital mobility, trade unions 

JEL-classification: F2, J5

* Address: Oliver Lorz, Institut für Weltwirtschaft, Düsternbrooker Weg 120, 24105 Kiel, Germany, Tel.: ++49-431-8814-483; Fax: ++49-431-8814-500, E-Mail: O.Lorz@ifw.uni-kiel.de,
1. Introduction

This paper analyzes wage competition between national trade unions caused by international capital mobility. Capital mobility gives investors the possibility to transfer their capital to the country offering the most favorable investment conditions. With their influence on the wage level, national trade unions determine an important aspect of these investment conditions. Other things equal, a country with comparatively low wage costs will therefore attract more mobile capital than other countries. As labor demand and therefore employment increases with the invested capital stock, trade unions concerned about the aggregate employment level will take this relationship into account in their wage demands.

A widely disseminated model of trade union behavior is that of a monopoly union fixing the wage level and firms subsequently adjusting employment. This paper replaces the monopoly union model of a closed country by that of an oligopoly of national trade unions—tied together through the international mobility of capital. It analyzes wage competition in a simple symmetric one-good-two-factor model of the open economy. Both the structure and results

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2 For a description of the monopoly model of the trade union, see e.g. Oswald (1985) or Holmlund (1989).

3 The implications of trade in consumption goods for the behavior of labor unions are treated for example in Davidson (1988), Dowrick (1989) or Kemp et al. (1991).

4 Oswald (1979) also investigates wage competition caused by the possibility of firms to substitute labor offered by different trade unions. In contrast to this paper, he derives a continuous labor demand function from a partial equilibrium model assuming an aggre-
of wage competition in this framework resemble that of the Bertrand model of price competition with capacity constraints: First, an equilibrium in pure strategies implies full employment in all countries. Second, such an equilibrium may not exist for certain parameter values.\textsuperscript{5}

The properties of the factor market equilibrium with international capital mobility are derived in section 2. Section 3 is devoted to the equilibrium of wage competition. In section 4, decreasing returns to scale and capital adjustment costs are introduced as extensions of the basic model. In both cases, the Bertrand character of the model disappears and an interior equilibrium with unemployment becomes possible. It is shown, however, that wage competition still leads to a decline of the equilibrium wage rate, compared to a closed economy.

2. Wages and Employment in a Closed and Open Economy

Assume that the world is divided symmetrically into \( N \) countries, each country endowed with \( K \) units of capital and \( L \) units of labor.\textsuperscript{6} Capital is perfectly mobile internationally, whereas labor is immobile. A representative price-taking firm employs capital \( K^i \) and labor \( L^i \) to produce in each country \( i (i=1...N) \) a tradable composite consumption good. A linear homogeneous production function with a positive and declining marginal product of both

gate production function with a declining marginal productivity of labor and no capital. As the following analysis will show, the basic model of the capital market equilibrium employed here does not lead to a labor demand function with such a property.

\textsuperscript{5} The capacity constraint extension of the Bertrand model of price competition goes back to Edgeworth (1925). See also Beckman (1965) and Leivithan, Shubik (1972) for examples of a game theoretical treatment of this model.

\textsuperscript{6} Assuming a fixed capital endowment, the model abstracts from an intertemporal savings decision. This simplification, however, is not essential for the main results of this paper as the following analysis will show.
factors describes technology. All countries have the same production function.

Because of the fixed wage rate, the firm may be rationed in its labor input by the labor endowment of the representative country. The firm takes the endowment constraint into account as a restraint of its maximization problem. The paper only considers wages leading to a strictly positive employment level in all countries. In addition, it is assumed that the marginal product of capital gets sufficiently high as the capital input approaches zero and therefore also the capital input in all countries is strictly positive. The following first order conditions then describe the factor market equilibrium ($w^i$ denotes the wage rate in country $i$, $r$ the world interest rate, the price of the consumption good is set to one, $i=1...N$, $\lambda^i \geq 0$, and $L^i \leq \bar{L}$):

\begin{align*}
F_1(K^i, L^i) - r &= 0, \quad [1] \\
F_2(K^i, L^i) - w^i - \lambda^i &= 0, \quad [2] \\
\lambda^i (L - L^i) &= 0, \quad [3] \\
\sum_{i=1}^{N} K^i &= N\bar{K}. \quad [4]
\end{align*}

For $N=1$, equations [1] - [4] describe the factor market equilibrium in a closed country. In such a closed country, there is full employment as long as the wage rate does not exceed the level $w^c = F_2(\bar{K}, \bar{L})$. For wages below $w^c$, the term $\lambda^1$ is strictly positive. It measures the shadow price of the full employment constraint or the rent per unit of labor that the representative

\begin{itemize}
\item For more than one firm and an excess labor demand, the allocation of labor across the single firms has to be modeled explicitly. A parallel rationing rule, where all firms get the same share of total labor endowment, corresponds to the case of a representative firm considered here.
\end{itemize}
firms earns. Wage increases reduce this rent but have no effects on employment. As the wage rate reaches the limit \( w^c \), the rent takes a value of zero and further wage increases reduce employment. The following equation measures the employment effect of a marginal wage increase for \( w^1 \geq w^c \):

\[
\frac{dL^1}{dw^1} = \frac{1}{F_{22}(K^1, L^1)} < 0.
\]  

[5]

In an open country wages not exceeding \( w^c \) also secure full employment. In addition, because of international capital mobility, there is full employment as long as the wage rate in the representative country is lower than the wage rate in any other country (see appendix a). This property of the labor demand function follows from the assumption of the same constant returns to scale technology in all countries. As equation [1] shows, the capital market equilibrium leads to an equal marginal product of capital and an equal capital intensity in all countries. Because of constant returns to scale, the marginal product of labor is then also equalized internationally. With different wage rates, an interior solution with unemployment in all countries is therefore not possible. Instead, in the low wage countries, labor is fully employed. The country with the highest wage rate only gets a residual employment level that results from the fact that employment in the low wage countries is limited to \( \bar{L} \).

Suppose, country 1 is the country with the internationally highest wage rate and that \( w^1 > w^c \). A marginal increase of the wage rate in country 1 then decreases employment according to the following equation (see appendix b):

\[
\frac{dL^1}{dw^1} = \frac{[N - 1]\bar{L} + L^1}{F_{22}(K^1, L^1) \bar{L}}.
\]  

[6]

As \( w^1 \) approaches \( w^c \), the employment level in country 1 approaches full employment. Thus for \( w^1 < w^c \) and \( w^1 = w^c \), the impact of a marginal wage increase is given by the following expression:
According to equation \([7]\), the number of countries influences the employment effects of a marginal wage increase. The larger the number of countries, the more pronounced are these employment effects.

For equal wage rates in all countries the firm is indifferent about the level of production in the single locations as long as the average employment level equals the comparable employment level in a closed economy. For simplicity, it is assumed that in this case the employment level is the same in all countries.

3. The Equilibrium Wage Rate

Given the impact of the wages on employment, the properties of the wage competition equilibrium can be derived in this section.

In every country, a national trade union completely controls labor supply and sets the wage rate for the whole country. With this assumption the model abstracts from intra-national competition between sectoral or regional trade unions as well as from competition between unionized and non-covered labor. This assumption also assigns the whole power in determining the wage rate to the trade unions whereas employer organizations are assumed to have no influence at all in the wage negotiations.\(^8\) The paper thus models the role of trade unions in determining the wage rate in a quite simplistic way and probably overstates the true monopoly power of trade unions. This representation is chosen nevertheless, as it characterizes the essence of the monopoly union view most clearly.

\[^8\] Allowing for a positive influence of the employer organizations in a "right-to-manage" model of wage negotiations would not affect the main results derived in the following analysis.
All national trade unions decide about their respective wage demand simultaneously. Given the expected wages in other countries, each union maximizes a differentiable objective function $G' = G\left( w', L' \right)$, which is strictly increasing in both arguments. It is assumed that for all given foreign wage demands, a union prefers at least one possible wage-employment combination with a strictly positive employment to any wage rate that leads to zero employment. Thus, the existence of an equilibrium with zero employment in one country can be excluded directly. The following lemma and its proof show that with perfect capital mobility all trade unions set the wage rate at the level $w^c$ in equilibrium.9

Lemma: Suppose $N > 1$. An equilibrium in pure strategies can exist only for $w^i = w^c$, $i = 1...N$.

Proof: First, for any country $i$, a wage rate below $w^c$ can not be an optimal strategy: The wage rate $w^c$ leads to full employment, irrespectively of the wage rate in other countries. Since the objective function of the unions is strictly increasing in the wage rate, the wage $w^c$ dominates all lower wages. Second, there exists no symmetric equilibrium $(\bar{w}, ..., \bar{w})$ with $w > w^c$ and therefore $L^i < L$, $i = 1...N$. Setting $w^h = \bar{w} - \epsilon$, $\epsilon > 0$, a union $h$ would expect a payoff of $G\left( \bar{w}-\epsilon, L^i, L^h \right)$. The union's objective function is continuous, so that $\lim_{\epsilon \to 0} G\left( \bar{w}-\epsilon, L^i, L^h \right) = G\left( \bar{w}, L^i, L^h \right) > G\left( \bar{w}, L^i \right)$. Thus there exists a strategy $w^h = \bar{w}-\epsilon$, $\epsilon > 0$, leading to a higher payoff for trade union $h$ than the strategy $w^h = \bar{w}$. Third, no asymmetric equilibrium in pure strategies $\left( \tilde{w}^1, \tilde{w}^2, ..., \tilde{w}^N \right)$ exists. In an asymmetric equilibrium, there are two unions $k$ and $l$ with $w^k < \tilde{w}^l$ and $L^k = L$. Since the set of wages satisfying $\tilde{w}^k < \tilde{w}^l$

9 As an equilibrium this paper considers only the concept of a Nash-equilibrium in pure strategies. Lorz (1997) also investigates the properties of mixed strategy equilibria in the wage competition game.
is an open set, for every \( \tilde{w}^k \) there exists a strategy \( \tilde{w}^k + \varepsilon, \varepsilon > 0 \), also leading to \( t^k = \overline{t} \) and a higher payoff for union \( k \).

The model of wage competition with international capital mobility thus exhibits a Bertrand paradox similar to the model of price competition in a product market: Even for a small number of competitors, a pure strategy equilibrium can exist only at the "competitive" full employment wage level \( w^c \). However, because of the full employment constraint of the countries, the existence of such an equilibrium is not ensured: With the employment level in the low wage countries limited to their total labor endowment, there may remain a strictly positive residual employment level in the high wage country. At \( w^i = w^c, i=1...N \), a trade union may thus find it profitable to increase the desired wage rate and to content itself with the resulting residual employment level.

The following proposition specifies the condition for the existence of the pure strategy equilibrium at \( w^i = w^c, i=1...N \). To prove this proposition, the payoff function of any trade union \( i \) is assumed to be strictly concave in \( w^i \) for \( w^i > w^i \) and \( w^i > w^c \).

**Proposition:** Define the following function:

\[
\theta(N, R, L) = G_1(w^c, L) + G_2(w^c, L) \frac{N}{F_{22}(R, L)}.
\]

The model of wage competition with international capital mobility has an equilibrium in pure strategies at \( w^i = w^c, i=1...N \), if \( \theta() \leq 0 \). For \( \theta()>0 \), no equilibrium in pure strategies exists.

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\(^{10}\) This assumption is satisfied for a strictly quasi-concave objective function of the trade union and a non-increasing marginal impact of the wage rate on the employment level.
Proof: The wage rate $w^c$ dominates all wages below $w^c$, so that only $w^i = w^c + \varepsilon, \varepsilon > 0$ may lead to a higher payoff for the representative trade union than $w^i = w^c$. Equation [7] denotes the employment effect of a marginal increase of the wage rate. Thus, the following equation holds:

$$\lim_{\varepsilon \to 0^+} \frac{G(w^c + \varepsilon, L^i(w^c + \varepsilon)) - G(w^c, L)}{\varepsilon} = 0(N, \bar{K}, \bar{L})$$

For $0(\cdot) > 0$, the expected payoff of the representative union increases with a marginally rising wage rate. The condition $0(\cdot) \leq 0$ thus is necessary for the existence of a pure strategy equilibrium $w^i = w^c, i=1...N$. Because the payoff function of the representative union is assumed to be strictly concave in $w^i$ for $w^i > w^{i-1}$ and $w^i > w^c$, the condition $0(\cdot) \leq 0$ is also sufficient for the existence of such an equilibrium.\[\]

Although the number of countries $N$ does not influence the wage level in equilibrium, it may have an impact on the outcome of wage competition: With a rising number of countries, the function $0(\cdot)$ declines. An equilibrium in pure strategies exists for a broader range of parameter values as the number of countries increases. For $N$ sufficiently high, an equilibrium in pure strategies exists for all $\bar{K}$ and $\bar{L}$. The outcome of oligopolistic wage competition thus converges via its existence condition to the competitive outcome, where a union takes the wage level as given.

4. Extensions

The Bertrand paradox and the possible nonexistence of an equilibrium in pure strategies may appear as quite extreme results of wage competition with international capital mobility. The question therefore arises, whether these results prove to be robust against realistic modifications and extensions of the model or whether these extensions lead to less strict outcomes of international wage competition. This section introduces decreasing returns to scale and capital adjustment costs as such extensions of the basic model. In
both cases the Bertrand paradox disappears and a symmetric equilibrium with unemployment in all countries becomes possible.

Decreasing returns to scale with respect to the variable factors capital and labor may result from negative production externalities or from immobile factors fixed in supply in the separate countries. To incorporate decreasing returns to scale into the model, the production function \( F(K,L) \) is still assumed to be homogenous, but of degree less than one. The marginal product of capital and labor then depends not only on the capital intensity but also on the scale of production. An interior solution for the equations [1] - [4] describing the factor market equilibrium may then exist for differing wage rates in the separate countries.

To illustrate this, suppose all unions impose the same wage rate above the full employment level. The union in country 1 now increases its demanded wage level marginally. With constant returns to scale, this implies a jump to full employment in all low wage countries \( 2...N \), whereas country 1 only gets the residual employment. With decreasing returns to scale, however, a marginal wage increase or decrease causes only marginal employment effects. These employment effects are given by the following equations (see appendix c):^{12}

\[ \text{Equations} \]

^{11} The factors land – including environmental resources – or labor of a different skill level than unionized labor are examples for such immobile, fixed factors. Production factors supplied by the public sector may also lead to decreasing returns to scale – as long as they can not be employed without any negative congestion effects with respect to the scale of production.

^{12} With constant returns to scale the term in the denominator of equations [9] and [10] would be zero, whereas with decreasing returns this term is strictly positive.
With these employment effects wage competition loses its Bertrand character and an interior, symmetric equilibrium with unemployment in all countries may exist. The first order condition of this equilibrium is given by the following equation – with country 1 as representative country and $w^1 = F_2 \left( \bar{K}, l^1 \right)$:

$$
\frac{dL^1}{dw^1} = \frac{1}{F_{22}} \cdot \frac{F_{11}F_{22} - \left[ F_{12} \right]^2}{F_{11}F_{22} - \left[ F_{12} \right]^2} < 0, \quad [9]
$$

$$
\frac{dL^{-1}}{dw^1} = \frac{1}{NF_{22}} \cdot \frac{\left[ F_{12} \right]^2}{F_{11}F_{22} - \left[ F_{12} \right]^2} > 0. \quad [10]
$$

The payoff function of the representative union is assumed to be strictly concave in $w^1$, so that the second order condition for an interior equilibrium is satisfied. In addition, a symmetric increase of the wage rate in all countries is assumed to have a negative impact on the marginal payoff of the representative union. The equilibrium wage rate then declines continuously as the number of countries rises: For a given wage rate, the left hand side of equation [11] declines with a rising number of countries. The equilibrium wage rate then has to adjust accordingly to ensure that equation [11] is satisfied. Because the marginal payoff is assumed to decline with a symmetrically increasing wage rate in all countries, this wage adjustment has to be negative. It should be noted, however, that with decreasing returns to scale even for a large number of countries wage competition does not necessarily lead to full employment. The marginal employment effect of a wage increase converges
to a finite value as \( N \) approaches infinity, so that an interior solution satisfying equation [11] still remains possible.\(^{13}\)

With adjustment costs as the second extension of the basic model, the assumption of perfect international capital mobility is dropped. Instead, the adjustment of the invested capital stock causes strictly convex costs for the representative firm. To incorporate adjustment costs in the static framework of the model, it is assumed that initially the invested worldwide capital stock is distributed equally across all countries. Adjustment costs are then represented by the twice differentiable function \( \psi(K' - \bar{K}) \), with \( \psi(0) = 0 \), \( \psi'(0) = 0 \), \( \psi''(\cdot) > 0 \). The following equation [12] replaces equation [1] of the factor market equilibrium in this case:

\[
F_1(K^1, L^1) - \psi'(K^1 - \bar{K}) - r = 0. \tag{12}
\]

An increasing wage rate in the representative country then causes the following adjustment of employment (see appendix c):

\[
\frac{dL^{1}}{dw^1} = \frac{1}{F^{22}_1} \left( 1 - \frac{F_{11}[N - 1]}{\psi''(\cdot)N} \right) < 0. \tag{13}
\]

As a marginally increasing or decreasing wage rate causes only marginal employment effects, an interior solution for the equilibrium wage rate becomes possible in this case. The first order condition then satisfies equation [11] with equation [13] as the respective marginal wage effect on employment. As with decreasing returns to scale, an increasing number of

---

\(^{13}\) Anderson, Devereux (1988) have investigated union wage setting with a variable capital stock in a partial equilibrium model. They assume a fixed interest rate and decreasing returns to scale. Their model can be interpreted as a representation of wage competition in a small open economy. For a large number of countries, the marginal employment effect of equation [9] and the equilibrium wage rate satisfying equation [11] converge to the respective results in Anderson, Devereux (1988).
countries increases the wage effect on employment and therefore leads to an increasing wage level in the interior, symmetric equilibrium.\textsuperscript{14} However, even for a large number of countries an interior equilibrium with prevailing unemployment in all countries cannot be excluded.\textsuperscript{15}

5. Concluding Remarks

This paper has introduced a stylized model of wage competition between national trade unions. In this model, international capital mobility increases the employment effects of unilateral wage variations. This forces the unions to lower their wage demands – at least as long as wages exceed the full employment level in a closed economy. Whether wage competition is as intense as in the Bertrand model or whether some unemployment still may remain in an open economy equilibrium depends crucially on the conditions under which wage competition is taking place. However, in all considered variants of the model, national trade unions lose at least a part of their monopoly power and employment in all countries increases with the introduction of capital mobility. Capital mobility thus causes an international institutional competition between trade unions that is able to compensate insufficient intra-national competition on the labor market.

\textsuperscript{14} Again, the objective function of the representative union is assumed to be strictly concave and a symmetric increase of the wage rate in all countries is assumed to have a negative impact on the marginal payoff of the representative union.

\textsuperscript{15} Van der Ploeg (1987) has also investigated trade union behavior in a dynamic framework with a variable capital stock and strictly convex adjustment costs. As in Anderson, Devereux (1987), the interest rate is assumed to be exogenous. A marginal increase of the wage rate leads to marginally declining steady-state employment level in his model. This result is comparable to the effect described by equation [13] for the number of countries approaching infinity.
References


Appendix

(a) Full Employment in an Open Country
Assume that there exists a country $h$, with $h \neq 1$ and $w^h > w^1$. Equation [1] implies that the marginal product of capital equates in both countries $h$ and 1.\textsuperscript{16} Then also the marginal product of labor then equalizes in both countries, and according to equation [2] the following equation has to be satisfied in equilibrium:

$$w^1 + \lambda^1 = w^h + \lambda^h. \quad [\Lambda.1]$$

As $w^h > w^1$, $\lambda^1$ has to be larger than $\lambda^h$ and therefore also larger than zero. Equation [3] then can only be satisfied for $L^1 = \bar{L}$.

For $w^1 \geq w^h$ there is also full employment as long as $w^1$ does not exceed $w^c$. To show this, suppose instead that $L^1 < \bar{L}$ and therefore $\lambda^1 = 0$. With $w^1 < w^c$, the following equation has to be satisfied:

$$F_2(K^1, L^1) = F_2(K, \bar{L}) \quad [\Lambda.2]$$

Since $L^1$ is assumed to be smaller than $\bar{L}$, $K^1$ also has to be smaller than $\bar{K}$. Equation [4] then implies that at least for one country $k$, $k \neq 1$, the capital stock $K^k$ has to exceed $\bar{K}$. Equation [1], however, requires an equal capital intensity in all countries. For $K^k > \bar{K}$ this would not be possible, as $L^k$ can not exceed $\bar{L}$.

(b) The Employment Effects of a Marginal Wage Increase
For $w^1 > w^{-1}$ and $w^1 > w^c$, the following equations determine the factor market equilibrium:

$$F_1(K^1, L^1) = F_1(K^{-1}, \bar{L}) \quad [\Lambda.3]$$

\textsuperscript{16} Equilibria with either a capital or labor input of zero are ruled out by assumption.
The total differential of these equations \( dK = 0 \), \( dw^{-1} = 0 \) can be expressed as follows:

\[
F_2\left(K^1, L^1\right) = w^1 \tag{A.4}
\]

\[
K^1 + [N - 1]K^{-1} = NK \tag{A.5}
\]

Using Cramer's rule, the marginal impact of a wage increase on labor demand can now be derived:

\[
\begin{pmatrix}
F_{11}(K^1, L^1) & F_{12}(K^1, L^1) \\
F_{21}(K^1, L^1) & F_{22}(K^1, L^1)
\end{pmatrix}
\begin{pmatrix}
dK^1 \\
dl^1
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}
\]

\[
\begin{pmatrix}
F_{11}(K^{-1}, \bar{L}) \\
F_{21}(K^{-1}, \bar{L})
\end{pmatrix}
\begin{pmatrix}
dK^{-1} \\
dL
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}
\]

Because of the constant returns to scale and internationally equalized capital intensities, the relationship \( F_{11}(K^1, L^1) = \lambda F_{11}\left(K^{-1}, \bar{L}\right) \) holds, so that equation \(6\) can be derived from \(A.7\).

(c) Wages and Employment in the Extended Model

Point of departure is a factor market equilibrium with symmetric wage rates and unemployment. Because of the symmetry assumption, the capital input and the employment level is the same in all countries. Total differentiation of equations \([2] - [5]\) with \( \lambda^1 = \lambda^{-1} = 0 \) and setting \( dw^{-1} = 0 \) gives the marginal impact of a wage increase in country 1 on the factor inputs under the assumption of decreasing returns to scale:

\[
F_{11}dK^1 + F_{12}dL^1 = F_{11}dK^{-1} + F_{12}dL^{-1} \tag{A.8}
\]

\[
F_{21}dK^1 + F_{22}dL^1 = dw^1 \tag{A.9}
\]
Inserting the equations \([A.10]\) and \([A.11]\) in equation \([A.8]\) and then inserting \([A.8]\) in \([A.9]\) gives equation [9] of the text. Equation [10] then follows from \([A.10]\), \([A.11]\) and [9].

With adjustment costs, equation [12] replaces equation [1] in the capital market equilibrium. Total differentiation of [12] gives the following term:

\[
[F_{11} - \Lambda''(\cdot)]dK^1 + F_{12}dl^1 = [F_{11} - \Lambda''(\cdot)]dK^{-1} + F_{12}dl^{-1} \tag{A.12}
\]