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Determinants of the Expected Real Long-term Interest Rates in the G7-countries

by

Jörg W. Krämer
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June 1996

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Abstract

The paper investigates which factors determine the expected real long-term interest rates of the G7-countries as a whole within a single equation error correction model. Inflationary expectations are generated using the low frequency component of inflation provided by the Hodrick-Prescott filter. A comparison of the calculated expected inflation rates with those resulting from index-linked and conventional UK bonds suggests this approach to be appropriate. Expected real long-term interest rates turn out to be influenced positively by real short-term interest rates, capacity utilization and structural public borrowing.

JEL Classification: C22, E43
I. Introduction

Expected real long-term interest rates largely influence saving and investment decisions. Accordingly, monetary policy makers have to know whether and to what extent a central bank is able to influence real long-term rates. Moreover, the government should take into account the effects of public borrowing on real bond yields. Finally, for all policy makers it is interesting to know whether interest rate determinants exist which cannot be controlled directly.

This paper aims at explaining the expected real long-term interest rate at the G7-level. Aggregating interest rates of individual countries implies the existence of a unified capital market for the G7-countries. Given the financial deregulation mainly at the beginning of the eighties and the synchronised development of interest rates in most time periods this assumption seems justifiable.¹

To determine expected real long-term interest rates, a simple loanable funds equilibrium approach is used (section ii). Inflationary expectations are generated using the low frequency component of inflation provided by the Hodrick-Prescott filter. The appropriateness of this approach is judged by comparing the calculated expected inflation to inflationary expectations resulting from index-linked and conventional government bonds for the UK (section iii). Data issues involving the use of structural rather than actual public deficits are discussed in section iv. With respect to the degree of integration of the variables selected (section v) econometric methodology is chosen (section vi). Empirical results include a simulation of the period starting in 1993Q4 and ending in 1995Q3, when bond yields changed markedly (section vii). Conclusions will be drawn in the last section of the paper.

¹ Studies following Feldstein and Horioka (1980) suggest that domestic investment is mainly financed by domestic saving and thus capital mobility is low (Obstfeld 1994: 44–65). However, these findings can be criticized for several reasons (see e.g. Lapp 1996).
II. The model

Several theories exist explaining expected real long-term interest rates. For empirical purposes the loanable funds equilibrium approach turns out to be well suited. Following this approach, expected real long-term interest rates, $i_l^r$, are determined by supply and demand of bonds:\footnote{See e.g. Hoelscher (1986), Cebula et al. (1988), Correia-Nunes and Stemitsiotis (1995).}

\[
B^S(i_l^r, is^r, cap, gb) = B^D(i_l^r, is^r, gb).
\]

The supply of bonds, $B^S$, depends negatively on the expected real cost of borrowing, i.e. the expected real long-term interest rate. Rising short-term real interest rates, $is^r$, lead to a higher supply of bonds because long-term borrowing becomes cheaper in relation to short-run funds. Furthermore, the supply of bonds is positively related to capacity utilization, $cap$, because according to the accelerator principle fixed capital formation is largely determined by the business cycle. Finally, government borrowing, $gb$, is regarded as an exogenous variable which positively influences expected long-term interest rates.

The demand for bonds, $B^D$, depends positively on the expected real long-term interest rate. Rising short-term yields induce investors to increase their holdings of short-term funds and to lower their demand for bonds. According to the Ricardian equivalence theorem (Barro 1974) public long-term borrowing may influence positively the bond-demand. Realizing that higher public deficits must be repaid in the future, rational economic agents increase their savings now in order to intertemporally optimize consumption.

The solution of the structural model [1] for $i_l^r$ leads to the following reduced form:

\[
i_l^r = f(is^r, cap, gb).
\]

\footnote{Economic agents are implicitly assumed to be interested in real rather than nominal yields.}
Real long-term interest rates depend positively on the real short-term interest rate and on capacity utilization. The impact of government borrowing is theoretically ambiguous. If a rise in the public deficit is totally offset by higher savings, interest rates are not affected (marked by 0). On the other hand, if the increase in savings falls short of the rise in deficit, interest rates go up.

III. Calculating inflationary expectations

The equilibrium on the bond market determines ex-ante real interest rates, i.e. the difference between nominal interest rates $i$ and inflationary expectations $\pi^e$:

$$i' = i - \pi^e$$

Economists have developed various methods to calculate these unobservable expectations. As far as short time periods like half a year are concerned, ARIMA-models turn out to describe expected inflation quite well (Barro and Sala-i-Martin 1990: 17/8). Severe problems arise when calculating long-term real interest rates. Following Tease et al. (1991: 119), Correia-Nunes and Stemitsiotis (1995: 433) and Orr et al. (1995: 5), expected inflation is modeled by the low frequency component of inflation generated by the Hodrick-Prescott (HP) filter. The inflationary expectations are calculated solving the following minimization problem (Hodrick, Prescott 1981: 5):

$$\text{Min} \sum_{t=1}^{T} (\pi^e_t - \pi^e_{t-1})^2 + \lambda \sum_{t=1}^{T} \left[ (\pi^e_t - \pi^e_{t-1}) - (\pi^e_{t-1} - \pi^e_{t-2}) \right]^2$$

The parameter $\lambda$ penalizes variability in the expected inflation rate, the larger the value of $\lambda$, the smoother the low frequency component which is a two-sided average of the observed inflation. According to Nunes and Stemitsiotis (1995: 434/5) backward-oriented as well as forward-oriented information is relevant when economic agents form rational expectations in a world of sticky prices and slow adjustment.

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4 For quarterly figures Hodrick and Prescott (1981: 7) suggest $\lambda = 1600$. 
In order to assess whether the HP filter is suited for the problem in question, the low-frequency component of UK-inflation will be compared to inflationary expectations calculated from the yield on index-linked and conventional government bonds.\(^5\) Index-linked bonds link the nominal coupon payments and the redemption payment to the broad index of retail prices. Therefore, from an investor's point of view the real value of both income and capital is certain\(^6\) and the yield on index-linked bonds corresponds to a real long-term interest rate. According to the Fisher equation, average inflationary expectations over the next ten years are calculated as the difference between the (nominal) yield on conventional bonds with a time to maturity of ten years and the (real) yield on index-linked bonds with the same time to maturity.\(^7\)

Until 1992, the low frequency component of the year-over-year change of the retail price index and the average inflationary expectations calculated by yield data followed more or less a similar pattern (figure 1). Expected price level changes declined until 1986, then gained momentum before starting to decrease again in 1990. After 1992, however, the two time series exhibited different trends. The expected inflation calculated from nominal and real yields stagnated whereas the low frequency component of inflation was directed downwards. The difference observed in this time period may result from the departure of the Sterling from the ERM in September 1992. Since then the Bank of England has not been forced to defend exchange rate targets by tight monetary policy inducing market participants to form their inflationary expectations more cautiously compared to the period before the ERM exit. Furthermore, one has to take into account a technical argument: The difference between the low frequency component and the actual values of a time series tends to be too small in the last few observations of the estimation period. All in all, the comparison shows that HP-

\(^5\) Unfortunally, the UK is the only G7-country where index-linked bonds have been traded during a time period providing enough observations to run econometric analysis.

\(^6\) In reality, there is no complete certainty because coupon payments are linked to inflation with a lag of eight months (Deacon, Derry 1994: 235/6).

\(^7\) The data are taken from yield curve estimations provided by the Bank of England (Breedon 1995: 164).
filtered inflation rates – although not a perfect measure of price change expectations – can be used to approximate expected inflation.

Figure 1 — Expected inflation calculated by a Hodrick-Prescott (HP) filter and by yield data of the Bank of England (BoE).

![Figure 1](image)

\[ i_t = \sum_{j=1}^{7} \left( \frac{\text{GDP}_{j}^{90} \cdot \text{PPP}_{j}^{90}}{\sum_{j=1}^{7} \text{GDP}_{j}^{90} \cdot \text{PPP}_{j}^{90}} \right) \]

8 Unless otherwise stated, the figures being mentioned now and in the following are taken from OECD (1996). Starting in 1991Q1 GDP data refer to unified Germany. Prior to that date western German figures are used (Statistisches Bundesamt, various issues).

9 Country specific real GDP data not referring to the base year 1990 were approximately adjusted using GDP deflators.
Expected real long-term interest rates equal nominal long-term interest rates minus the HP low frequency component of the year-over-year inflation rate of the G7-countries. Following Orr et al. (1995: 37) short-term inflationary expectations are approximated by actual inflation rate, because in the short-run actual and expected inflation differ only by small amounts. Thus, expected real short-term interest rates of the G7-countries are determined as the difference between nominal rates and actual inflation.

Capacity utilization is calculated on the basis of the real GDP of the G7-countries:

\[ \text{cap}_i = \left( \log \sum_{j=1}^{7} \frac{GDP_{j}^{90} \cdot PPP_{j}^{90}}{Pot} - \log Pot \right) \times 100, \]

where potential output, \( Pot \), is the HP low frequency component of the real GDP of the G7-countries.

In empirical work, it is sometimes difficult to find a significant positive relationship between government long-term borrowing and interest rates. These findings may result from using actual instead of structural borrowing. Actual borrowing is highly correlated with capacity utilization and other cyclical variables which probably makes it difficult to identify the "true" parameters (multicollinearity). To eliminate potential impacts of the business cycle, public borrowing is corrected for deviations between actual and average capacity utilization. Each country’s ratio of structural borrowing to potential output is taken from Lapp et al. (1996), who refer to OECD data. The annual figures are converted to a quarterly frequency by linear interpolation. Each country’s structural borrowing ratios are aggregated using GDP shares.

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10 Individual long-term interest rates usually have a time to maturity of 10 years.
11 Inflation is calculated as the year-over-year instead of the quarter-over-quarter change of the G7 price index in order to eliminate irregular components.
12 All short-term interest rates are three-month money market rates.
V. The statistical properties of the time series

The econometric methods to be applied crucially depend on the degree of integration of the variables in question. For all time series, unit root tests as proposed by Dickey and Fuller (1981) will be used in different specifications: first without intercept, second only an intercept is included, and third – for levels – an intercept and a trend are added. The test equation is augmented by endogeneous variables to ensure white-noise residuals (Downes 1987: 231).

The upper half of table 1 shows the results for the levels of real short-term and long-term interest rates, the capacity utilization and the structural borrowing ratio.

Table 1 — Unit Root Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF</th>
<th>ADF + constant</th>
<th>ADF + constant + trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>il'</td>
<td>-0.21 (1)</td>
<td>-3.48** (1)</td>
<td>-3.09 (4)</td>
</tr>
<tr>
<td>is'</td>
<td>-0.53 (0)</td>
<td>-2.35 (2)</td>
<td>-2.66 (1)</td>
</tr>
<tr>
<td>cap</td>
<td>-3.35*** (1)</td>
<td>-3.35** (1)</td>
<td>-3.39* (1)</td>
</tr>
<tr>
<td>gb</td>
<td>-0.15 (1)</td>
<td>-2.36 (3)</td>
<td>-1.45 (0)</td>
</tr>
<tr>
<td>Δil'</td>
<td>-5.60*** (0)</td>
<td>-5.60*** (0)</td>
<td>.</td>
</tr>
<tr>
<td>Δis'</td>
<td>-6.94*** (1)</td>
<td>-6.91*** (1)</td>
<td>.</td>
</tr>
<tr>
<td>Δcap</td>
<td>-5.84*** (0)</td>
<td>-5.81*** (0)</td>
<td>.</td>
</tr>
<tr>
<td>Δgb</td>
<td>-1.20** (7)</td>
<td>-2.82* (2)</td>
<td>.</td>
</tr>
</tbody>
</table>

*Augmented Dickey-Fuller (ADF) test, figures in brackets indicate the number of lagged endogenous variables in the test equation, estimation period: 1980Q1–1995Q3.

With the exception of capacity utilization, all variables appear to be non-stationary. In contrast, capacity utilization seems to be a stationary variable. The lower half of the table presents unit root test statistics involving the first difference of the variables. The evidence is unambiguous: the first difference of each variable is stationary, implying the levels – with the exception of capacity utilization – to be integrated of order one.
VI. Econometric methods

Relationships between integrated variables are usually analyzed within the framework of error correction models. As proposed by Stock (1987: 1040), the coefficients of the following single equation error correction model are estimated by OLS:

\[ \Delta i_{t} = \beta_{1} + \beta_{2} \Delta il_{t-1} + \beta_{3} \Delta is_{t-1} + \beta_{4} \Delta cap_{t-1} + \beta_{5} \Delta gb_{t-1} + \sum_{i=1}^{s} \beta_{6i} \Delta il_{t-i} \]

The first differences (\( \Delta \)) of expected real long-term interest rates are explained by both the levels and the first differences of the variables discussed above. Rearranging the equation yields

\[ \Delta il' = \beta_{2} \left[ \frac{\beta_{1}}{\beta_{2}} + \frac{\beta_{3}}{\beta_{2}} \right] + \sum_{i=0}^{s} \beta_{6i} \Delta il_{t-i} + \ldots \]

whereby the expression in brackets implicitly defines the long-run relationship

\[ il' = \gamma_{1} + \gamma_{2} \Delta is' + \gamma_{3} \Delta cap_{t} + \gamma_{4} \Delta gb_{t-1} + \epsilon_{t} \]

\[ \gamma_{1} = -\frac{\beta_{1}}{\beta_{2}}, \gamma_{2} = -\frac{\beta_{3}}{\beta_{2}}, \gamma_{3} = -\frac{\beta_{4}}{\beta_{2}}, \gamma_{4} = -\frac{\beta_{5}}{\beta_{2}}. \]

A negative value of parameter \( \beta_{2} \) ensures the stability of the error correction model. If, for instance, expected real long-term interest rates are below their equilibrium level, a negative \( \beta_{2} \) will cause expected real long-term interest rates to increase in the following periods. Integrated variables which can be described by a stable error correction model are called cointegrated, the long-run relationship [10] is referred to as a cointegration relationship (Engle, Granger 1987). As stressed by Hansen and Juselius (1995: 1), a cointegration relationship may also include stationary variables. Therefore, capacity utilization enters the long-run...
relationship of the error correction model. However, due to the non-stationarity of
the other variables, the t-statistic of $\beta_2$ is not normally distributed. Instead, the
critical values proposed by Banerjee et al. (1992) are used.

In order to calculate the standard errors of the elements of the cointegration
vector, equation [8] is rearranged (Bewley 1979: 358):

$$\begin{align*}
\text{[11]} \quad il_t' &= \frac{\beta_1}{\beta_2} + \frac{\beta_2 + 1}{\beta_2} \Delta il_t' - \frac{\beta_3}{\beta_2} \Delta is_{t-1}' - \frac{\beta_4}{\beta_2} \Delta cap_{t-1} - \frac{\beta_5}{\beta_2} \Delta gb_{t-1} \\
&= -\sum_{i=0}^{5} \frac{\beta_{8i}}{\beta_2} \Delta il_{t-i}' - \sum_{i=0}^{5} \frac{\beta_{7i}}{\beta_2} \Delta is_{t-i}' - \sum_{i=0}^{5} \frac{\beta_{6i}}{\beta_2} \Delta cap_{t-i}' - \sum_{i=0}^{5} \frac{\beta_{9i}}{\beta_2} \Delta gb_{t-i}' - \frac{\mu_t}{\beta_2}
\end{align*}$$

Because $\Delta il_t$ is correlated with $il_t$, OLS leads to inconsistent estimators. The
problem can be solved using $il_{t-1}$ as an instrument for $\Delta il_t$ (Wickens, Breusch 1988: 197).

Having estimated the coefficients of [8] and [11], various tests are used to
ensure residuals are white noise processes and normally distributed and the para-
parameters are invariant with respect to time. To find the most parsimonious speci-
fication the final prediction error, FPE, will be applied as an information criterion.

VII. Empirical results

The t-statistic of the lagged endogeneous level-variable indicates the variables $il_t'$, $is_t'$, $cap$ and $gb$ to form a stable long-run relationship at a marginal significance
level lower than 1 percent (table 2). Moreover, the signs of the estimated long-run coefficients seem to be significantly different from zero and can be interpreted
economically. The estimated coefficient of real short-run interest rates is positive
and lower than unity. This is consistent with the expectations theory of the term
structure stating that long-term interest rates as an average of actual and expected
short-term interest rates are less volatile than short-term interest rates. Assuming
that the central bank can set real short-term interest rates, the finding suggests
Table 2 — Estimated error correction model of expected real long-term interest rates

\[ \Delta i_{t} = -0.40 \left( i_{t-1} - 0.72 \, is_{t-1} - 0.45 \, cap_{t-1} - 0.58 \, gb_{t-1} \right) \]

\[ \begin{align*}
&+ 0.40 \, \Delta i_{t-1} + 0.26 \, \Delta is_{t-3} - 0.17 \, \Delta is_{t-3} \\
&+ 0.29 \, \Delta is_{t} - 0.25 \, \Delta is_{t-1} + 0.43 \, \Delta cap_{t} \\
&+ 0.20 \, \Delta cap_{t-1} - 0.10 \, \Delta cap_{t-2} - 0.14 \, \Delta cap_{t-3} \\
&+ 0.97 \, \Delta gb_{t-1} - 1.24 \, \Delta gb_{t-3} + 1.04 \, \Delta gb_{t-4} + \Delta u_{t}
\end{align*} \]

Estimation period = 1980Q1–1995Q3; \( R^2 = 0.75 \); \( SE = 0.27 \); \( AC(1) = 0.00 \); \( AC(4) = 3.97 \); \( ARCH(1) = 2.50 \); \( BJ = 0.73 \); \( CHOW = 0.89 \); \( CSMAX = 0.71 \); \( CSQMAX = 0.19 \); t-values in brackets.

Abbreviations: \( SE \) = Standard error of residuals; \( AC(i) \) = LM test for autocorrelation of order \( i \) (Breusch 1978, Godfrey 1978); \( ARCH(1) \) = Test for autoregressive conditional heteroscedasticity of order 1 (Engle 1982); \( LM \) test for normality of residuals (Bera, Jarque 1981); \( CHOW \) = F-Test for stability of coefficients, sample splitted into equal halves (Chow 1960); \( CSMAX \) = Cusum test statistic for stability of coefficients (Brown et al. 1975); \( CSQMAX \) = Cusum of squares test statistic for stability of coefficients (Brown et al. 1975, Edgerton, Wells 1994).

that monetary policy makers are able to influence the expected real long-term interest rates. However, one should take into account at least two reservations. If the central banks changed their behaviour in order to exploit intensively the empirical relationship, the estimated coefficient would probably alter. Moreover, monetary policy is likely to influence other variables which themselves are related to expected real long-term interest rates. One of these variables is capacity utilization. On the one hand it is widely accepted that capacity utilization is influenced negatively by real short-term interest rates. On the other hand the estimated error correction model shows that real long-term interest rates depend positively on capacity utilization. Thus, a decrease of expected real long-term
interest rates resulting from lower real short-term interest rates is at least partly offset by higher capacity utilization. Finally, the estimation results suggest that the level of expected real long-term interest rates positively depends on government borrowing. Obviously, a rise in government borrowing is not totally offset by higher savings of private economic agents. This is in line with recent findings (Correia-Nunes and Stetsitsiotis 1995: 142, Ford and Laxton 1995: 9). Thus, politicians have to take into account that expansionary fiscal policy increases the level of expected real long-term interest rates.

Specification tests suggest not to reject the hypothesis of serially uncorrelated, homoscedastic and normally distributed random errors. Both the tests based on recursive regressions (CSMAX and CSQMAX) and the Chow split test do not support the rejection of the hypothesis of stable regression coefficients. In order to assess the forecast ability of the model, ex-post dynamic simulations are carried out. In contrast to the so-called static simulation the dynamic simulation includes values of lagged endogeneous variables which are generated by the model itself. If the levels of actual and simulated real long-term interest rates are compared for the estimation period, the root mean square error, RMSE, amounts to 0.44 percentage points (table 3). The value of Theil's inequality coefficient

<table>
<thead>
<tr>
<th>RMSE&lt;sup&gt;b&lt;/sup&gt;</th>
<th>0.44</th>
<th>UM&lt;sup&gt;c&lt;/sup&gt;</th>
<th>0.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAP&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.34</td>
<td>US&lt;sup&gt;f&lt;/sup&gt;</td>
<td>0.02</td>
</tr>
<tr>
<td>TU&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.57</td>
<td>UCG</td>
<td>0.98</td>
</tr>
</tbody>
</table>

<sup>a</sup>Dynamic simulation, 1980Q1-1995Q3. — <sup>b</sup>Root mean squared error. — <sup>c</sup>Mean absolute error. — <sup>d</sup>Theil's (1966: 28) inequality coefficient. — <sup>e</sup>Bias proportion. — <sup>f</sup>Variance proportion. — <sup>g</sup>Covariance proportion.

indicates that the RMSE of a „naive“ no-change forecast of the level of interest rates is about twice as high as the RMSE of the model's forecast. The inequality

<sup>13</sup> In the literature, there are different versions of the inequality coefficient. The formula used in this paper stems from Theil (1966: 28).
proportions suggest that 98 percent of the errors concerning the forecast of interest rate changes is caused by random influences. Nevertheless, the model fails to explain a considerable part of the 1994 sharp rise in expected real interest rates (figure 2). However, in the second quarter of 1995 the error is totally corrected largely by a decrease of actual interest rates. This indicates that a part of the real yield increase during 1994 may not be backed by the fundamentals of the model chosen. Perhaps, market participants have overvalued the expected tightening of monetary policy as well as the strength and duration of the cyclical upswing.

Figure 2 — Actual versus simulated real long-term interest rates

VIII. Implications

A crucial issue in estimating real long-term interest rates is the modelling of expected inflation. Comparing UK inflationary expectations calculated by the Hodrick-Prescott (HP) filter to expected inflation resulting from index-linked and conventional government UK bonds I suggest that the HP filter can approximate expected inflation.

Expected real long-term interest rates in the G7-countries turn out to be explained by real short-term interest rates, capacity utilization and structural govern-
ment borrowing within a single equation error correction model. One result is that monetary policy is able to influence real long-term interest rates by systematically changing the real short-term rate. However, capacity utilization which itself may be influenced by monetary policy turns out to be positively related to real long-term interest rates. The estimation results also show that higher government borrowing leads to an increase of real long-term interest rates.
References


