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The role of nonrivalries in the new growth theory

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The Role of Nonrivalries in the New Growth Theory.

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✓

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1. Introduction

This paper discusses the role of the assumption of nonrival production factors¹ in models of the New Growth Theory.² It is shown that this assumption is neither necessary nor sufficient to generate endogenous growth. It is not necessary, because the assumption of non-diminishing marginal returns to accumulatable³ production factors is sufficient to generate endogenous growth. It is not sufficient, because without the explicit assumption of non-diminishing marginal returns to accumulatable production factors, a growth model cannot exhibit the property of endogenous growth.

This paper is organized as follows. Section 2 surveys the basic versions of a neoclassical and an endogenous growth model. It is shown that the assumption of non-diminishing marginal returns to scale is sufficient to generate endogenous growth. Section 3 illustrates that the assumption of partial externalities of production and nonrival production factors in the Grossman/Helpman (1991) model of vertical product differentiation is not sufficient to generate endogenous growth. Section 4 shows that non-diminishing marginal returns to accumulatable production factors can not be derived from the assumption of nonrival production factors. Section 5 draws the conclusions.

¹By analogy to nonrival consumption goods, Romer (1990) defines a nonrival production factor as one that can be "simultaneously" and "repeatedly" used in production "by arbitrarily many different firms and people".

²Since the late eighties a new class of growth models has been developed that explain long run growth of per capita GDP *endogenously*, contrary to the neoclassical growth models that rely on an *exogenous* rate of technological growth. The discussion has been initiated by Romer (1986), who showed that endogenous growth can be generated by postulating non-diminishing returns to production factors that can be accumulated. Since then a large number of different versions of endogenous growth models has evolved. For a survey of literature see Stolpe (1992). Throughout the literature all models that exhibit the property of endogenous growth are referred to as "New Growth Theory".

³Throughout this paper I use the term "accumulatable production factors" instead of "production factors that can be accumulated". I hold the former term to be self-explanatory.

2. Comparison of the basic versions of a neoclassical and an endogenous growth model.⁴

This section points out the basic reasons for permanent growth in the presence of non-diminishing marginal returns to accumulatable production factors:

- First, the instantaneous resource constraint opens and permanent growth becomes *physically possible* (see figure1).
- Second, *net* marginal productivity of capital⁵ can permanently be higher than the discount rate and permanent net investment, and hence permanent output growth becomes *utility maximizing* (see equation (2.4)).

To show this, a simple growth model is used that allows to compare the *basic* mechanics of the neoclassical growth theory and the "new growth theory" (NGT). The model is taken from Rebelo (1990). He postulates a rudimentary one-factor-production function of the following type:

$$(2.1) \quad Y_t(K_t) = A K_t$$

where Y_t is gross domestic product, A is an exogenous productivity parameter and K_t is a measure for all accumulatable production factors, such as physical capital, human capital and technological knowledge.⁶ The Rebelo production function (2.1) postulates constant returns to K . In the following a more general version of (2.1) is used:

⁴The set-up of the Rebelo (1990) model is taken from Sala-i-Martin (1990).

⁵Let *net* marginal productivity of capital be marginal productivity of capital minus capital depreciation rate.

⁶This specification of a production function neglects non-accumulatable production factors, because Rebelo (1990) used it to analyse the long run growth process. Rebelo postulated that non-accumulatable production factors such as raw labour can be neglected in the long run. However, this assumption is of no importance in the following, because the purpose of this section is to compare the basic mechanics of the neoclassical and the endogenous growth models. It can be shown that these basic mechanics do not change once non-accumulatable production factors are included.

$$(2.2) \quad Y(K) = A K^\beta.$$

For $0 < \beta < 1$ returns in accumulatable production factors are decreasing and the model is neoclassical; for $\beta \geq 1$ returns are non-diminishing, i.e. the stage for endogenous growth is set and the model belongs to the NGT. Gross domestic product per period can be decomposed in the following way:

$$(2.3) \quad A K^\beta = (dK/dt) + C + \delta K,$$

where dK/dt is gross investment, C is aggregate consumption and δK are capital depreciation allowances. Equation (2.3) can be interpreted as the instantaneous resource constraint of the economy. For $dK/dt = 0$ equation (2.3) implies:

$$(2.4) \quad C = A K^\beta - \delta K$$

From this follows that - depending on β - the $dK/dt = 0$ locus takes the following form in the (C,K) - plane:⁷

⁷Above the $dK/dt=0$ locus $dK/dt < 0$ holds, below the $dK/dt=0$ locus $dK/dt > 0$ holds.

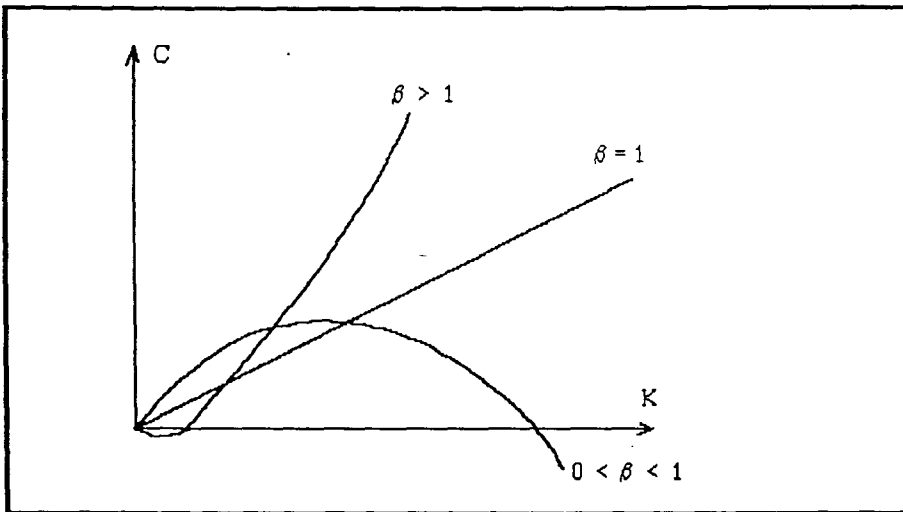


Figure 1: Instantaneous resource constraints depending on β .

In the neoclassical case $0 < \beta < 1$ depreciation allowances set a physical border to capital accumulation, as can be seen by taking the first derivation of (2.4) with respect to K . Hence the instantaneous resource constraint becomes closed, i.e. there exists always a K' such that every $K > K'$ has a return that is lower than the depreciation allowances of capital ($AK^\beta < \delta K$). In case of the NGT ($\beta \geq 1$) the instantaneous resource constraint is opened, i.e. there exists always a K' such that for every $K > K'$ the return is always higher than the depreciation allowances of capital ($AK^\beta > \delta K$) and permanent growth of consumption and capital becomes *physically possible*.⁸

So far the analysis has neglected the consumer's choice between present consumption and capital accumulation. To model this decision, the following intertemporal utility function of a representative household is introduced:⁹

⁸In case of constant returns ($\beta = 1$), the $dK=0$ locus will have a positive slope, if and only if $A - \delta > 0$, i.e. if the marginal productivity of capital is larger than the rate of depreciation.

⁹Equation (2.6) postulates a simple instantaneous CES utility function. At least two points of criticism can be made against the intertemporal utility function (2.5). First, (2.5) postulates intertemporal separability of preferences. For a discussion of the restrictiveness of this assumption

$$(2.5) \quad U(0) = \int_0^{\infty} e^{(n-\rho)t} u(c_t) dt,$$

$$(2.6) \quad u(c_t) = (c_t^{1-\sigma} - 1) / (1-\sigma)$$

where c_t is per capita consumption at date t ; ρ is the discount rate; L_t is the number of members of representative household at time t ; and σ is a parameter of taste that determines the intertemporal elasticity of substitution and ensures the concavity of the instantaneous utility function ($0 < \sigma < 1$). $\rho > 0$ can be interpreted as the exogenously given probability of decreasing ability to enjoy future consumption because of disease or death of the household dynasty.¹⁰

The intertemporal utility function (2.5) together with a per capita version of the instantaneous resource constraint (2.3) yield a dynamic optimization program. For $0 < \beta \leq 1$, the optimization program is concave, and an optimum exists. For $\beta > 1$, the optimization program is non-concave, and an optimum does not exist without further assumptions. Appendix A1 derives the solution for the neoclassical case ($0 < \beta < 1$) and the NGT case with constant returns ($\beta = 1$). From appendix A1 follows that an intertemporal utility optimum implies the following condition to be fulfilled:

$$(2.7) \quad (dk/dt)/k = \sigma^{-1} ([y'(k) - \delta] - \rho)$$

This equation explains the basic difference between the neoclassical theory and the NGT:

see Deaton/Muellbauer (1980). Second, (2.5) postulates that all members of the household dynasty agree to maximize the *sum* of individual utility functions. This assumption is sufficient but not necessary to yield a Pareto optimal allocation of goods between household members.

¹⁰Ramsey (1928) holds a positive, non-zero discount rate to be "ethically indefensible". Romer (1989) holds a positive, non-zero discount rate to be justified in positive models only.

- In case of the *neoclassical growth theory* (i.e. $0 < \beta < 1$); $y''(k)$ equals $A\beta(\beta-1)k^{(\beta-2)}L^{(\beta-1)} < 0$ (see equation A5). Therefore, as long as per capita capital is increasing ($dk/dt/k > 0$), *net* marginal productivity of capital¹¹ is decreasing. When it reaches the level where it equalizes the discount rate (ρ), the marginal return of further accumulation of capital ($y'(k) - \delta$) is lower than the discount rate (ρ). Consequently, further capital accumulation would lead to an intertemporal loss of utility. A zero growth rate of the per capita capital stock implies a zero growth rate of per capita output. Hence, growth of per capita output peters out in the long run under neoclassical assumptions.

- In case of the *new growth theory* (NGT) with constant returns to scale to capital ($\beta = 1$), $y''(k)$ equals zero. Therefore, as long as per capita capital is increasing ($dk/dt/k > 0$), *net* marginal productivity of capital is constant and equals $A - \delta$ (see A(6)). If $A - \delta > \rho$, the accumulation of per capita capital will never stop, because the marginal net return of further capital accumulation will always be higher than the discount rate. Consequently, capital accumulation always implies an intertemporal gain of utility. A positive non-zero growth rate of the per capita capital stock leads to a positive non-zero growth rate of per capita output. Hence, growth of per capita output does not peter out in the long run under the assumptions of the NGT.

Both cases can be graphically described in the (c,k) - plane:

¹¹Let *net* marginal productivity of capital be marginal productivity of capital minus capital depreciation: $y'(k) - \delta$. $y(k)$ is the per capita version of $Y(K)$, see (A5) and (A6).

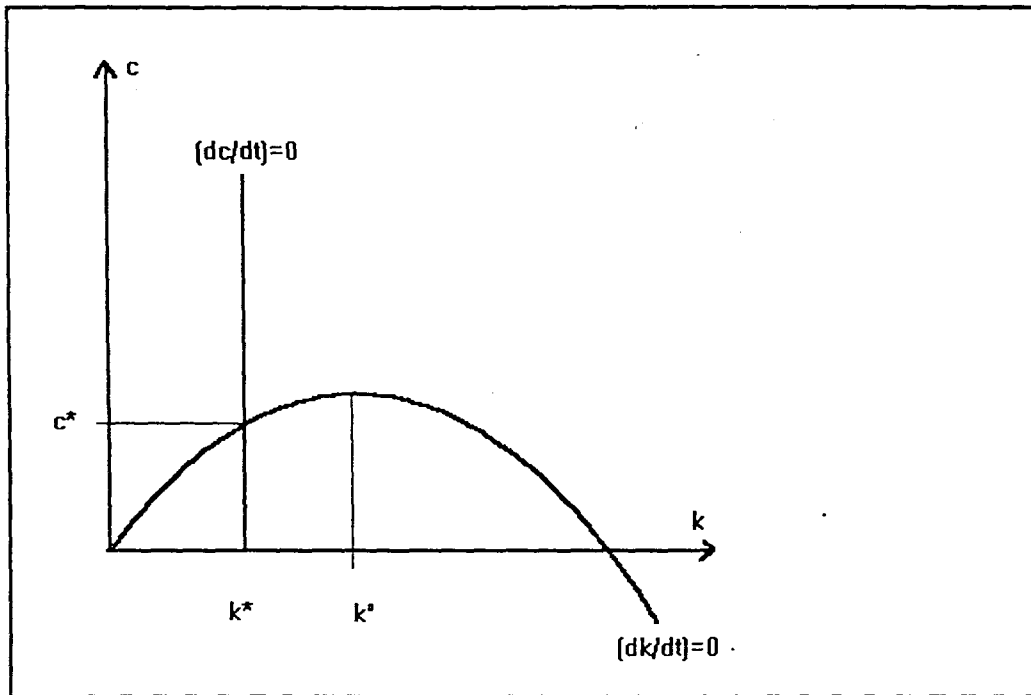


Figure 2: Neoclassical steady state.

Figure 2 shows the steady state in the neoclassical scenario. The resource constraint (2.4) is "closed" and the intersection point (c^*, k^*) represents the zero growth steady state of the neoclassical model.¹² At this point, the *net* marginal return to capital investment has decreased to a level where it equals the discount rate. Therefore the process of per capita capital accumulation peters out. As shown in appendix A.2, k^* will be less or equal to k° (the maximum per capita consumption rate that is physically within reach), if and only if $\rho \geq n$.¹³ This ensures that future consumption in steady state cannot be increased without increasing the present capital stock, i.e. without decreasing present consumption. Therefore the steady state is dynamically efficient.¹⁴

¹²As can be seen from (A23) k^* will be the higher the lower the discount rate ρ (i.e. the higher the preference to save) and the lower the rate of depreciation, if and only if $\beta < 1$.

¹³This condition - the so called bounded utility condition - is a necessary (but not sufficient) condition to ensure the existence of a solution of the dynamic optimization problem.

¹⁴Dynamic efficiency is equivalent to intertemporal Pareto efficiency: A state will be called dynamically efficient, if it is not possible to increase future consumption (resp. utility) without

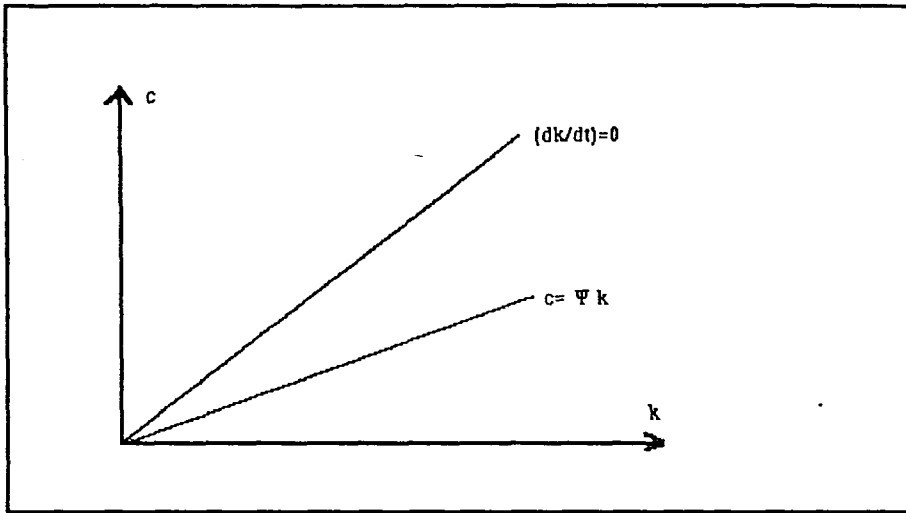


Figure 3: New Growth Theory steady state with constant returns.

Figure 3 shows the steady state growth path in case of the NGT with constant returns ($\beta=1$). The resource constraint (2.4) will be opened and c and k will grow permanently at equal rates along the growth path $c = \Psi k$, if $A - \delta > \rho$ (see A(25), A(26), A(27)). Consequently, in this case a *permanent incentive* for capital accumulation exists.¹⁵ The growth rate depends negatively on the discount rate ρ and on the capital depreciation rate δ , but positively on the capital productivity parameter A . Following (A27) a high ρ yields a high value of c / k ; following (A25) a high ρ slows down the speed of growth. Every point on the growth path is dynamically efficient, because it is not possible to increase present consumption without slowing down the speed of growth, i.e. future consumption.

The optimal time paths of the neoclassical theory and the above version of the NGT evolve only, if an appropriate *institutional framework* is given. A benevolent central planner, who has *all* information about preferences and production possibilities *and* has the power to enforce the optimal allocation of

decreasing present consumption (rsp. utility). It follows that all reachable points on the right hand side of k^0 are dynamically inefficient.

¹⁵As shown in appendix A.3, this condition ensures also that the growth path never cuts the instantaneous resource constraint.

goods, is such an institutional framework. Yet it is a very unrealistic one. A more realistic institutional framework is a competitive market economy based on the assumption of utility maximizing individuals. The existence of a competitive market equilibrium is well established given the above made assumptions concerning preferences and technologies.¹⁶ Consequently, a competitive market economy is an institutional framework that can yield the optimal time paths of the neoclassical theory and the above version of the NGT with constant returns.

The existence of an optimal growth path is not proved for this simple model in case of the *NGT with increasing returns* ($\beta > 1$). Romer (1986) presented a theorem¹⁷ stating a set of conditions that are sufficient for the existence of an optimal growth path in case of increasing returns to accumulatable production factors. Roughly speaking, these conditions require the discount rate ρ to be "high enough" and a concave "transformation function" that *has to be* used to transform output y into physical capital k . Given these conditions the optimal growth path implies c and k to grow without bound.

Yet these conditions do not ensure the existence of a competitive market equilibrium. Romer (1986) presented a second theorem¹⁸ stating *additional* conditions to ensure the existence of a market solution. Roughly speaking, these conditions require *individual* returns to accumulatable production factors to be decreasing, but *social* returns¹⁹ to accumulatable production factors to be increasing. This implies that external returns of accumulatable production factors have to be postulated.²⁰ Consequently, individuals face concave maximization

¹⁶Romer (1989), pp 118-119, gives a simple example of a prove of a market equilibrium existence for concave utility and production functions in a dynamic setting. The basic idea of this proof is based on the equivalence of an optimal time path and a saddle point of the choice variables c and k and the Lagrangian multipliers λ . A saddle point will result, if c° and k° maximize the Lagrangian, and λ° minimizes the Lagrangian. The Lagrangian multipliers can be interpreted as the price vector of the economy. To prove the existence of a competitive market equilibrium that supports the optimal growth path, it has to be shown that price taking individuals chose $c=c^\circ$ and $k=k^\circ$ for a given price vector λ° , such that $(c^\circ, k^\circ, \lambda^\circ)$ is a saddle point.

¹⁷Romer (1986), p. 1021, Theorem 1.

¹⁸Romer (1986), p. 1024, Theorem 2.

¹⁹Social returns are *individual* returns plus *external* returns.

²⁰ Some authors believe that strong external returns of accumulatable production factors generate - somehow - non-diminishing returns and are consequently the ultimate reason for endogenous growth (See e.g. Sachs/Larrain (1993), p 571: "When the externalities of investment

problems with well established solutions.²¹ Yet, the evolving *competitive market equilibrium with externalities* is not Pareto efficient, because individuals do not consider the positive externalities of accumulation.

are sufficiently great, however, diminishing returns to capital do not set in." However, as will be seen in Section 3, it is not the strength of external returns of accumulatable production factors that generate non-diminishing returns. The fact is that external returns have to be imposed to allow for an competitive equilibrium solution of the model in the presence of increasing returns to scale. Once diminishing returns to accumulatable production factors are assumed, no endogenous growth will evolve - no matter how strong external returns of accumulatable returns are.

²¹Karl Marx (1867) gave an *economic* explanation why market economies in the presence of *increasing returns* to accumulatable production factors *on the individual level* have no competitive equilibrium solution: Increasing returns to capital imply capital accumulation to increase the productivity of "individual" capital stocks. Consequently owners of large capital stocks (capitalists) can pay higher interest rates than owners of lower capital stocks and acquire *all* the capital, offered by financial markets. Owners of small capital stocks are "defeated" in the competition for capital funds. This implies a permanent monopolization of capital stocks. The number of capitalists shrinks, and the monopoly power of capitalists increases. Capitalists will use their monopoly power to pull wages of non-capitalists down. Output demand decreases, because income elasticity of demand is decreasing as income increases. Output supply increases, because capital productivity increases, as capital stocks increase. Consequently, a growing excess supply results. Strategic interacting between capitalists and non-capitalists determines the further dynamics of the model.

Marx (1867) build this analysis on the empirical observation of rapid technological progress in the beginning of industrialization that led him to the conclusion that ever increasing incentives for capital investments, i.e. increasing returns to capital, exist (Marx (1867), pp 585-591). He believed in the existence of increasing returns to capital on the individual level as an eternal law. He based his so called "General Law of Capital Accumulation on this belief (Marx (1867), chapter 23).

3. What causes endogenous growth in the Grossman/Helpman model of expanding product variety.²²

This section illustrates that the assumption of nonrival production factors in the GH (1991) model of expanding product variety is neither necessary nor sufficient to generate endogenous growth. The model exhibits endogenous growth only, if non-diminishing marginal returns to an accumulatable production factor are explicitly postulated - at least asymptotically.

The total set-up of this model is delivered in appendix B.1. In the following only the basic elements are described. Households maximize an intertemporal utility function with $u(D(t)) = \ln(D(t))$. The number of households stays constant. The production function of the household good D is given by

$$(3.1) \quad D(t) = \left(\int_0^n x(j,t)^\alpha \right)^{1/\alpha}, \quad 0 < \alpha < 1,$$

where n is the number of different products $x(j,t)$ that are the inputs of the household production function. Given the budget constraint of the representative household this yields an intertemporal optimization program with the typical first order conditions (see appendix B.1). The production of $x(j,t)$ obeys for all brands j the following constant returns technology:

²²GH (1991) analysed the influence of the interactions between innovation and trade on the development of open economies. They use two basic models of endogenous growth for this purpose. The first is based on the idea that an increasing variety of intermediate products enables individuals to produce a greater amount of the "household product" D, which is equivalent to the Gross Domestic Product (GDP) of this economy. This model is called the model of expanding product variety. The other model is based on the idea that a higher quality of intermediate products enables consumers to produce a greater amount of the "household good" D, which is equivalent to GDP. This model is called the model of rising product quality. It is not analysed here, because it is not build on the assumption of nonrival production factors. As is shown in appendix B.3. the model of rising product quality contains the explicit assumption that product quality (which plays in this model the role of an accumulatable production factor) exhibits non-diminishing returns to scale.

$$(3.2) \quad x(j,t) = (1/b) L, \quad \text{with } j \in [0,n],$$

where L is the amount of labour and b a productivity parameter. The production function of new brands per unit of time obeys a similar technology:

$$(3.3) \quad dn/dt = (1/a) L,$$

where a is a productivity parameter. Hence, the only resource of this economy is labour. The only accumulatable production factor is the number of products n . The assumed technologies do not imply constant returns to an accumulatable production factor, because labor is not accumulatable and - as is shown in appendix B.2. - the household production function (3.1) exhibits in steady state decreasing returns to n .

The assumption of infinitely granted patents for each brand or the assumption of a subgame perfect limit pricing scenario yields monopolistic pricing of the producers of $x(j,t)$. Hence brand owners earn non-zero profits π despite the constant returns technology. The stock value of their firms is given by the present value of their instantaneous profits. Bubbles are excluded by the assumption of rational expectations and infinitely living household dynasties. The first derivative of the stock value equation with respect to time yields the dynamics of the stock value (B15). The dynamics of brand innovation are derived by combining the resource constraint (B20) with the equilibrium condition for finite innovation activities per unit time (B19). Figure 5 scatters the system of dynamic equations (B 24),(B 25),(B 26):

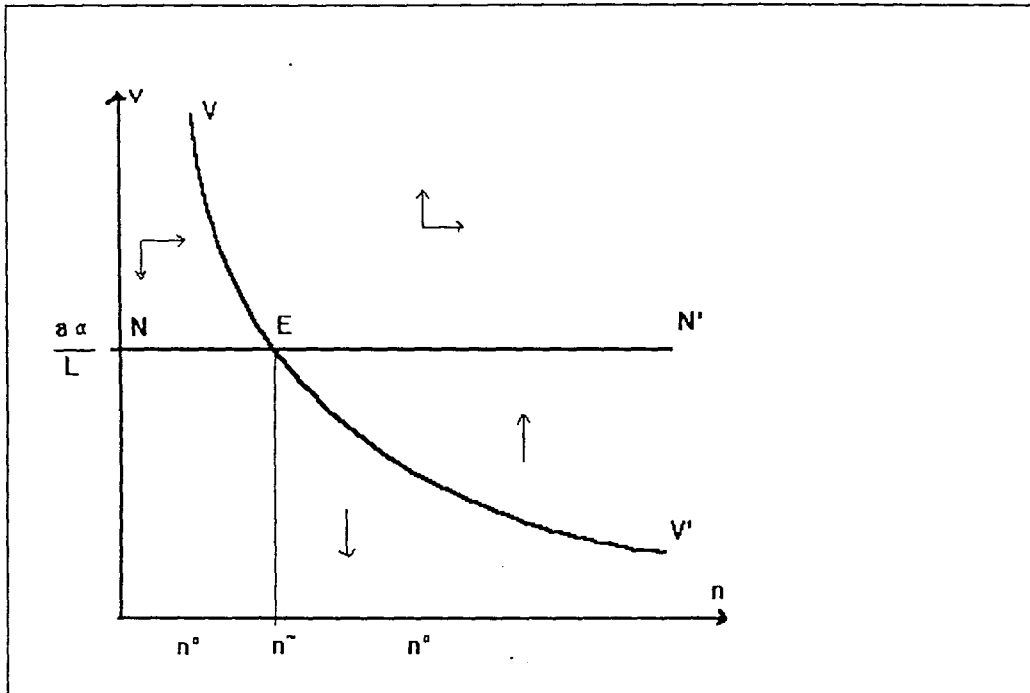


Figure 5: Steady state in the model of expanding product variety *with diminishing marginal returns* to the accumulatable production factor and *without externalities* (Source: GH (1991), Fig.3.1).

The hyperbola VV' describes the $dv/dt = 0$ locus. Given these combinations of stock value v and "number" of brands; the stock value remains constant. The curve is negatively sloped, as a higher "number" of brands n implies lower (nominal) instantaneous profits ((B13): $\pi = (1-\alpha) / n$) and hence a lower (nominal) stock value of a firm.²³ In the area below and on the horizontal NN' the "number" of brands is constant ($dn/dt=0$), because the present value of a new brand ($=v$) is too low to make further research in the development of new brands profitable. From figure 5 follows that for any initial endowment with brands n_0 a unique steady state with zero growth in v and n exists.²⁴ As the growth rate of D

²³As there exists a range of n where a higher value of n implies a higher amount of the household good D , a growing n may cause an deflation of the price of D (money amount is held constant by (B 8) such that a lower nominal profit can go along with a higher real profit. Yet, this is of no importance for the dynamic analysis.

²⁴See appendix B.1.2.3.

is given by $dD/dt = ((dn/dt)/n) (1-\alpha)/\alpha$, zero growth of n implies zero growth of the household good D .

This result is equivalent to the result of the typical neoclassical growth model without (exogenous) technological progress. To generate endogenous growth in per capita GDP, GH reformulate the model. They argue that the above

"specification of the R&D process may miss an important feature of knowledge as an economic commodity"(..., because) "we treated knowledge capital as a private good (...). Investment in R&D gave rise to a design for a new product, the returns to which were fully appropriated by the inventor. As we have seen, this specification implies the cessation of growth in the long run. But arguably, it neglects an important characteristic of many types of knowledge. (...) the originators of many new ideas often cannot appropriate all of the potential benefits from their creations. Some uses of the information may not be recognized by the original inventors, or their pursuit may require more expertise than the innovators have or can readily acquire in the market. In other cases property rights may be difficult to define and enforce, so the inventors will be unable to exclude others from making free use of their innovative ideas. (Therefore) we modify our formulation of knowledge creation to allow for the existence of such non-appropriable benefits from industrial research."²⁵

They do this by writing instead of (3.3):

$$(3.3') \quad dn/dt = (1/a) f(n) L \quad \text{with } f(n) = n^\beta \quad \text{and } \beta = 1.$$

Hence, they assume that technological knowledge, which had been accumulated in the research for new brands and is therefore indicated by the "number" of brands (n), facilitates the production of new brands. Consequently technological knowledge is assumed to have two characteristics: *First*, once it is produced, non-payers cannot be excluded from its usage, i.e. it exhibits external returns. *Second*, as everybody can use technological knowledge at the same time without depriving others of using it, i.e. it is non-rival. Given (3.3') for $\beta = 1$ figure 5 is replaced by figure 6:

²⁵GH (1991), page 57.

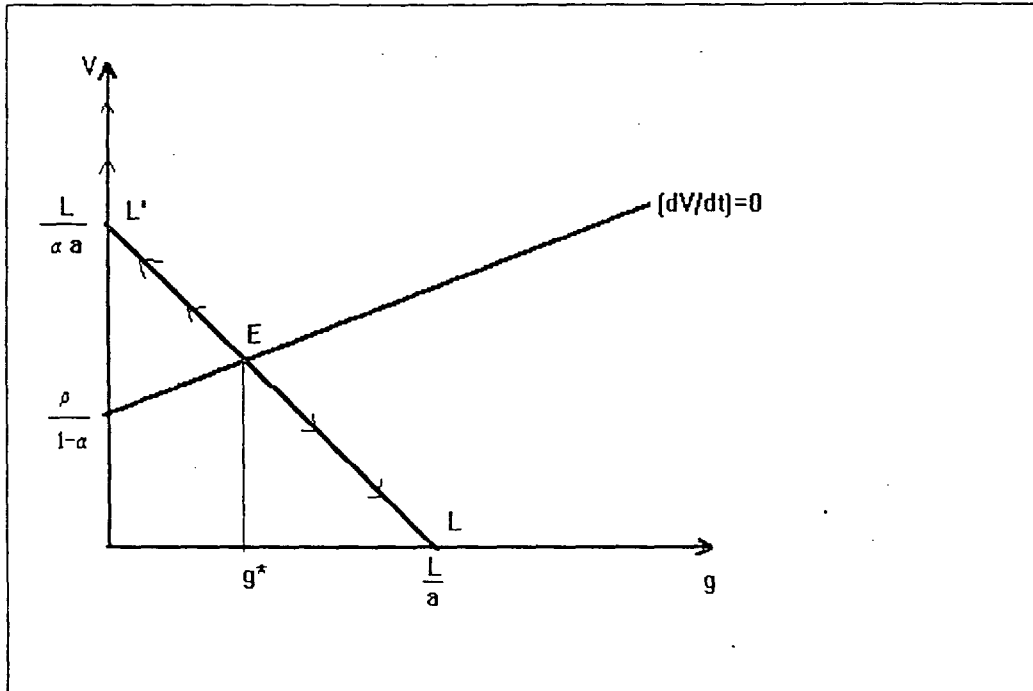


Figure 6: : Steady state in the model of expanding product variety *with externalities* and *non-diminishing marginal returns* to an accumulatable production factor (Source: GH (1991), Fig. 3.2).

Figure 6 is explained in appendix B. Given the new set of assumptions, it follows that the economy immediately jumps to point E, as any other trajectory is not in keeping with rational expectations of the stock value.²⁶ In point (E) the stock value stays constant and the number of brands is permanently growing at a constant rate g^* . Consequently the amount of household good (D) produced per unit of time grows at a constant rate too. The model exhibits endogenous growth. From this result GH conclude:

"Sustained innovation is possible in this case because the cost of product development falls with the accumulation of knowledge capital, even as the return to the marginal innovation declines. The nonappropriable benefits from R&D

²⁶ V is defined as $1/vn$, g is $(dn/dt/n)$. See appendix B.1.4 for an explicit derivation of the dynamics of the system.

keep the state of knowledge moving forward, and so the private incentives for further research are maintained."²⁷

Yet if β is chosen to take a value $0 < \beta < 1$ no steady state with non zero growth will emerge any longer. Projected on the (v,n) -plane the dynamic system in this case is described by figure 7, as is shown in appendix B 1.4.

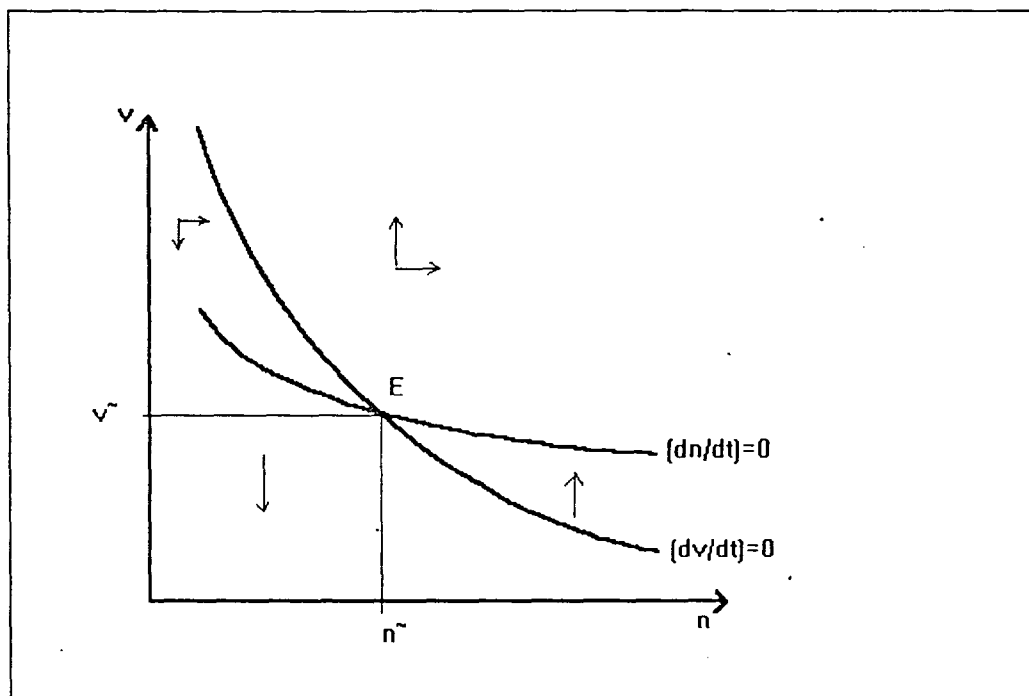


Figure 7: Steady state in the model of expanding product variety *with externalities* and *diminishing marginal returns* to an accumulatable production factor (Source: GH (1991), Fig.A 3.1).

The intersection point E of the two hyperbolas represents the steady state of the economy. Appendix B.1.4 describes the trajectories of the system for different initial endowments of n . For every initial endowment of n the economy will eventually reach the steady state E, where v and n stay constant. As n stays

²⁷See HG (1991), pp. 61-62.

constant the amount of household good D produced per unit time stays constant too. Hence the economy does not grow despite the fact that technological knowledge is nonrival and partially non-exclusive. Consequently, the assumption of nonrival and non-exclusive technological knowledge is not sufficient to yield endogenous growth. What is needed to generate endogenous growth in this model is the explicit assumption that there are non-diminishing marginal returns to n . In their appendix A3.1 GH show that $f(n)$ (see equation (B16')) needs not exactly exhibit constant returns to n . It is sufficient that the elasticity of $f(n)$ with respect to n approaches unity as n approaches infinity. As can be seen from the definition of elasticity, this implies that $f(n)$ approaches constant returns in the limit:

$$(3.1) \quad \varepsilon(f(n), n) = 1$$

$$\Leftrightarrow \quad n f' = f(n)$$

$$\Rightarrow \quad f' = (f')' = (f(n)/n)'$$

$$\Leftrightarrow \quad f'' = [n f' - f(n)] / n^2$$

$$\Leftrightarrow \quad f'' = 0 \quad | \text{ as } n f' = f(n)$$

Consequently GH must assume at least asymptotical constant returns to n , in order to yield endogenous growth in their model of expanding product variety. The assumption of nonrival production factors that exhibit external returns is not sufficient to generate endogenous growth.

Furthermore the assumption of nonrival production factors is - even in this model - not necessary to generate endogenous growth. As shown in appendix B.1.5., if equation (3.3) is reformulated in a way that implies that knowledge is perfectly excludable and rival, the model can still exhibit endogenous growth. The only difference is that under this assumption the $L'L$ line intersects the ordinate at point $IL/\alpha a$, (where I is the number of producers engaged in the production of new brands) that is higher than $L/\alpha a$ (the intersection point in case of nonrival and partially nonexcludable knowledge) and the abscissa at point L/aI that is lower than L/A (the intersection point in case of nonrival and partially nonexcludable

knowledge); to see this compare figure 6 with figure B5. Consequently the growth rate of technological knowledge under the assumption of rival and excludable knowledge may be *lower or higher* than under the assumption of nonrival and partially nonexcludable knowledge.

4.Can non-diminishing marginal returns of accumulatable production factors be derived from the assumption of nonrivalries?

In the preceding section it was illustrated that the assumption of nonrival production factors is neither necessary nor sufficient to generate endogenous growth in the GH (1991) model of expanding product variety. The GH (1991) model of expanding product variety exhibits endogenous growth only, if non-diminishing marginal returns to accumulatable production factors are *explicitly* postulated.

However - as Romer (1990) suggests - there may be another way to construct a link between the assumption of nonrivalries and endogenous growth. It may be possible to derive the assumption of non-diminishing marginal returns of accumulatable production factors from the assumption of the existence of nonrival production factors. If this were possible, nonrival production factors could serve as an microeconomic explanation for the assumption of non-diminishing returns.

Yet in the following paragraph it is shown that, although the assumption of nonrival production factors implies a nonconvex technology, it does not necessarily imply non-diminishing marginal returns to accumulatable production factors.

Romer (1990) showed that the existence of nonrival production factors implies a nonconvex technology by an argument of the following kind: Let $F(R,N)$ be a production function with rival inputs R and nonrival inputs N . As nonrival inputs can be used repeatedly without being consumed, a doubling of rival inputs alone will be sufficient to double output:

$$(4.1) \quad F(\lambda R, N) = \lambda F(R, N)$$

Romer draws from this relation the conclusion that doubling rival *and* nonrival inputs must more than double output:

$$(4.2) \quad F(\lambda R, \lambda N) > \lambda F(R, N)$$

Therefore, the production function must exhibit increasing returns to *all* production factors in the presence of nonrivalries and the technology is nonconvex. Yet, it is obvious that increasing returns to *all* production factors do not imply increasing or non-diminishing marginal returns to accumulatable production factors. This can be shown by the following example: Let the rival input R of production function (4.2) consist of accumulatable input K and non-accumulatable input L , let N be an accumulatable nonrival input and let $F(K, L, N)$ be a production function of the following type:

$$(4.3) \quad F(K, L, N) = K^\alpha L^\beta N^\gamma$$

Romer's argument states that $\alpha + \beta + \gamma > 1$. But this does not ensure that accumulatable inputs K and N exhibit the property of non-diminishing marginal returns, i.e. it does not ensure that $\alpha + \gamma \geq 1$. A fortiori, it does not ensure that N exhibits the property of non-diminishing marginal returns (i.e. $\gamma \geq 1$).

In the following the same argument is stated in a more intuitive way: Think of accumulatable technological inputs like chemical formulas, blueprints or computer programs. Obviously, these inputs can be used nonrival in the sense Romer (1990) defined it (see footnote 2, above). However a doubling of all accumulatable inputs (these technological inputs and/or physical capital and/or human capital inputs) does *not necessarily* imply that output is doubled. Hence, the fact that nonrival inputs are used in production does *not necessarily* imply that accumulatable inputs exhibit non-diminishing marginal returns. Of course, it cannot be excluded that a doubling of these inputs leads to a doubling of output. This depends on the technological state of the real world production function and is open to empirical investigation.

5. Conclusions.

This paper has discussed the role of the assumption of nonrival production factors in models of the New Growth Theory. It was shown that the assumption of nonrival inputs is *neither necessary nor sufficient* to generate endogenous growth. In the simple Rebelo-model of section 2, the assumption of constant returns in accumulatable production factors is sufficient to generate endogenous growth. Hence the assumption of nonrivalries is *not necessary* to generate endogenous growth. In the more complex Helpman/Grossman model of expanding product variety the assumption of nonrivalries is *not sufficient* to generate endogenous growth.

The discussion of Romer's (1990) paper has shown that there is no direct link between the property of nonrivalry of accumulatable production factors and the property of non-diminishing marginal returns of accumulatable production factors. This implies that the assumption of non-diminishing marginal returns to accumulatable production factors cannot be based on the assumption of nonrival production factors.

Of course, this is by no means a refutation of the assumption of non-diminishing marginal returns to accumulatable production factors. As this assumption is an empirical statement it can of course not be refuted by a priori reasoning. Whether the hypothesis of non-diminishing marginal returns to accumulatable production factors proves to be empirically successful, can only be answered by empirical research.

Appendix A: Comparison of the basic versions of a neoclassical and an endogenous growth model.

A1: Solution of the dynamic optimization program.

The per capita version of resource constraint (2.3) has to be derived to write the dynamic maximization program in per capita terms. The following definitions are useful:

- (A1) $\dot{x} := dx/dt$
 (A2) $k := K/L$
 (A3) $c := C/L$
 (A4) $n := \dot{L}/L$
 (A5) $y(K) := Y(K)/L = AK^\beta L^{-1} = Ak^\beta L^{\beta-1}$
 (A6) $y'(k) := \beta Ak^{(\beta-1)} L^{(\beta-1)}$

Time subscripts are suppressed where possible. n is the fertility rate of the representative household. n is assumed to be constant ($\dot{n} = 0$). Consequently $L_t = L_0 e^{nt} = e^{nt}$, if L_0 is normalized $L_0 = 1$. (A2) implies:

$$(A7) \quad \dot{k} = \frac{1}{L^2} (\dot{K}L - \dot{L}K) \quad | (2.3)$$

$$\Leftrightarrow \dot{k} = AK^\beta/L - c - \delta k - nk$$

$$\Leftrightarrow \dot{k} = Ak^\beta L^{(\beta-1)} - c - (\delta + n)k$$

The dynamic optimization program is:

$$(A8) \quad \max! U(0) = \max! \int_0^\infty e^{-\rho t} \frac{c_t^{(1-\sigma)} - 1}{1-\sigma} L_t \, dt \quad \text{s.t. (A7)}$$

The corresponding Hamiltonian is¹:

$$(A9) \quad H(c, k, \lambda) = e^{(n-\rho)t} \frac{c^{(1-\sigma)} - 1}{1-\sigma} + \lambda \left(Ak^\beta L^{(\beta-1)} - c - (\delta+n)k \right)$$

The first order conditions for an intertemporal utility maximum are:

$$(A10) \quad H_c \stackrel{!}{=} 0 \quad \Leftrightarrow \quad e^{(n-\rho)t} c^{-\sigma} - \lambda \stackrel{!}{=} 0$$

$$(A11) \quad H_k \stackrel{!}{=} -\dot{\lambda} \quad \Leftrightarrow \quad \dot{\lambda} \stackrel{!}{=} -\lambda \left(\beta AL^{(\beta-1)} k^{(\beta-1)} - (\delta+n) \right)$$

(A12) Transversality condition:

$$\lim_{t \rightarrow \infty} (k_t \lambda_t) \stackrel{!}{=} 0$$

Taking logarithms and derivatives of (A10) with respect to time yields:

$$(A13) \quad (n-\rho) - \sigma \frac{\dot{c}}{c} = \frac{\dot{\lambda}}{\lambda}$$

Inserting (A11) yields:

$$(A14) \quad (n-\rho) - \sigma \frac{\dot{c}}{c} = -\beta AL^{(\beta-1)} k^{(\beta-1)} + (\delta+n)$$

$$\Leftrightarrow \quad \gamma_c := \frac{\dot{c}}{c} = \sigma^{-1} \left(\beta AL^{(\beta-1)} k^{(\beta-1)} - \delta - \rho \right)$$

$$\Leftrightarrow \quad \gamma_c = \sigma^{-1} (y'(k) - \delta - \rho)$$

Dividing (A7) by k yields

$$(A15) \quad \gamma_k := \frac{\dot{k}}{k} = Ak^{(\beta-1)} L^{(\beta-1)} - \frac{c}{k} - (\delta+n)$$

Inserting (A14) yields:

$$(A16) \quad \frac{c}{k} = (\gamma_c \sigma + \delta + \rho) \beta^{-1} - \gamma_k - \delta - n =: z$$

A steady state implies γ_c and γ_k to be constant. Therefore, z is constant in a steady state. Taking logarithms and derivatives with respect to time yields:

¹ For $U(0)$ to be bounded, $\rho > n$ is needed. See Sala-i-Martin (1990a), p. 19.

$$(A17) \quad \frac{\dot{c}}{c} = \frac{\dot{k}}{k} \Leftrightarrow \gamma_c = \gamma_k$$

Consequently, a steady state implies equal growth rates for capital and consumption. Equation (A14) can therefore be written as follows:

$$(A18) \quad \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \sigma^{-1}(y'(k) - \delta - \rho)$$

Taking logarithms of (A14) yields:

$$(A19) \quad (\beta - 1) \ln k = \ln \left[(\gamma_c \sigma + \delta + \rho) / (\beta A L^{\beta-1}) \right]$$

In steady state the right hand side of (A19) is constant. Taking derivatives with respect to time yields:

$$(A20) \quad (\beta - 1) \frac{\dot{k}}{k} = 0$$

As this equation has to be fulfilled in the intertemporal optimum, the neoclassical model ($\beta - 1 \neq 0$) implies a zero growth steady state ($\dot{k}/k = 0$). The NGT model with constant returns ($\beta - 1 = 0$) is compatible with permanent growth ($\dot{k}/k > 0$).

A2: The neoclassical steady state ($\beta < 1$)

The ($\dot{k} = 0$)-locus follows from the resource constraint (A4):

$$(A21) \quad c = Ak^\beta L^{(\beta-1)} - k(\delta + n)$$

The physical maximum per capita consumption level follows from the first derivation of the resource constraint with respect to k :

$$(A22) \quad \frac{\delta c}{\delta k} = 0 \quad \Leftrightarrow \quad k^0 = \left(\frac{\delta + n}{\beta AL^{(\beta-1)}} \right)^{1/(\beta-1)}$$

From optimality condition (A14) follows the steady state k^* :

$$0 = \beta AL^{(\beta-1)} k^{(\beta-1)} - \delta - \rho \quad | \quad \gamma_c = 0$$

$$(A23) \quad k^* = \left(\frac{\delta + \rho}{\beta AL^{(\beta-1)}} \right)^{1/(\beta-1)}$$

The steady state k^* is dynamically efficient if

$$(A24) \quad k^* \leq k^0$$

$$\Leftrightarrow \quad \left(\frac{\delta + \rho}{\beta AL^{(\beta-1)}} \right)^{1/(\beta-1)} \leq \left(\frac{\delta + n}{\beta AL^{(\beta-1)}} \right)^{1/(\beta-1)}$$

$$\Leftrightarrow \quad \rho \geq n$$

Consequently, the bounded utility condition implies a dynamically efficient k^* .

A3: The New Growth Theory steady state with constant returns ($\beta = 1$)

$$(A25) \quad \gamma_c = (A - \delta - \rho) \sigma^{-1} \quad | (A14)$$

$$\gamma_k = (A - \delta - \rho) \sigma^{-1} \quad | (A17)^2$$

$$(A26) \quad \gamma_k = A - \delta - n - \frac{c}{k} \quad | (A15)$$

$$(A27) \quad c = \Psi k, \quad | (A25)$$

where $\Psi = (\sigma^{-1} (\rho - (1 - \sigma)(A - \delta) - \sigma n))$. (A27) is the growth path given constant returns. An intertemporal utility maximum implies that for a given initial capital endowment (k_0) initial consumption (c_0) is chosen such that (A27) is fulfilled. The resource constraint is in case of constant returns:

$$(A7) \quad c = (A - \delta - n)k$$

A positive growth rate implies that the growth path (A27) never cuts the resource constraint:

$$(A14) \quad A - \delta - n > (\sigma^{-1} (\rho - (1 - \sigma)(A - \delta) - \sigma n))$$

$$\Leftrightarrow \quad A - \delta > \rho$$

²(A17) was derived under the assumption that γ_K is constant. This is evident for the neoclassical model, because the "closed" resource constraint implies γ_K to be zero and hence constant in steady state. It is not evident for the NGT model, because the resource constraint is "opened". Hence, the conditions for an intertemporal utility maximum, A10, A11, A12, may imply a varying growth rate of the capital stock g_K . Yet Sala-i-Martin (1990), pp. 5-6, showed that the transversality condition implies γ_K to be constant for the case of $\beta = 1$ too.

Appendix B

B.1. The Grossman/Helpman model of expanding product variety.

B.1.1. The consumption sector.

The representative household is maximizing an intertemporal utility function of the following type:

$$(B1) \quad u_t = \int_0^{\infty} e^{-\rho(\tau-t)} \log D(\tau) d\tau$$

The production function for household good $D(\tau)$ is given by:

$$(B2) \quad D(\tau) = D(x(j, \tau), n) = \left[\int_0^n x(j, \tau)^\alpha dj \right]^{1/\alpha}, \quad 0 < \alpha < 1,$$

where $x(j, \tau)$ is the amount of intermediate goods $x(j)$ combined at time τ with other intermediate goods to the household good $D(\tau)$. The range of different intermediate goods is given by the interval $j \in]0, n]$.

The instantaneous demand of $x(j, \tau)$ is independent of time and can be found by maximizing:¹

$$(B3) \quad \mathcal{L} = \left(\int_0^n (x_j)^\alpha dj \right)^{1/\alpha} + \lambda \left(E - \int_0^n p_j x_j dj \right)$$

$$(1) \quad \frac{\delta \mathcal{L}}{\delta x_j} = x_j^{(\alpha-1)} \left(\int_0^n (x_j)^\alpha dj \right)^{\left(\frac{1}{\alpha}-1\right)} - \lambda \quad | p_j = 0$$

$$\Leftrightarrow p_j^\varepsilon x_j = D \lambda^{-\varepsilon} = \text{constant} \quad | \quad \varepsilon = 1/(1-\alpha)$$

$$(2) \quad \frac{\delta \mathcal{L}}{\delta \lambda} = E - \int_0^n p_j x_j dj$$

¹ Time subscripts are suppressed where possible.

$$\Leftrightarrow \frac{\delta \mathcal{L}}{\delta \lambda} = 0$$

$$(B4) \quad x_j = E p_j^{-\varepsilon} \int_0^n p_j^{(\varepsilon-j)} dj \quad | \quad p_j^\varepsilon x_j = \text{constant}$$

(B4) is needed in B.1.2.1 to determine a monopoly price. As GH, (1991), chapter 3, pp. 46-47 show, an ideal price index (p_D) can be constructed such that aggregate expenditure is:

$$(B5) \quad E(\tau) = D(\tau) p_D$$

Combining (B5) and (B1) yields:

$$(B6) \quad u(\tau) = \log D(\tau) = \int_t^\infty e^{-\rho(\tau-t)} \log E(\tau) dt - \int_t^\infty e^{-\rho(\tau-t)} \log p_D(\tau) dt$$

Hence the household maximizes intertemporal utility in two steps: First, it chooses its instantaneous composition of goods such that $p_D(\tau)$ is minimized. Second, it chooses for the given intertemporal budget constraint a time path of spending such that the first integral is maximized. This program yields the following condition for optimal spending.²

$$(B7) \quad \frac{\dot{E}}{E} = r - \rho,$$

where r is the interest rate and ρ the discount rate. If aggregate spending is chosen as numéraire ($E(\tau) = 1 \forall t$) then (B7) simplifies to:

$$(B8) \quad r = \rho$$

This normalization of spending implies a permanent deflation of prices as the output of the economy grows. Households spread their expenditures over time

² See GH (1991), chapter 2, pp. 27-28.

according to (B8) via an asset market, where riskless bonds and stocks are offered by firms and by other households (see B14).

B.1.2. The production sector

B.1.2.1. Production of intermediate goods

It is assumed that once a new product has been invented in the research labor, it can be produced with the same production function for all product lines:

$$(B9) \quad x_j(\ell) = \frac{1}{b} \ell^c, \quad \forall j \in]0, n],$$

where ℓ represents units of labour. GH (1991) assume $c = 1$, i.e. constant returns. This is not a critical assumption, since non-diminishing returns in production factors that cannot be accumulated are neither necessary nor sufficient to yield endogenous growth. The assumption is just a simplification as it ensures equality of marginal and average variable production costs. (B9) yields the following cost function:

$$(B10) \quad C(x_j) = w b x_j \quad | \quad b := 1 \\ = w x_j$$

GH assume that an inventor of a new brand becomes monopolist of this brand. They offer two sets of further assumptions to justify this assumption: First, infinitely lived patents are legally granted. Second, Bertrand competition in case of marked entrance and positive entrance costs deter marked entrance. The latter is the typical subgame perfect limit pricing scenario, based on the very restrictive assumption that even a small number of producers plays uncooperatively. Given these conditions each producer can practice monopolistic pricing by maximizing:

$$(B11) \quad \pi_j = p_j x_j - w x_j, \quad x_j \equiv (B4)$$

This yields a monopoly price of:

$$(B12) \quad p_j = w/\alpha \quad \forall j \in]0, n]$$

Using (B11), (B12), $E = 1$ and the fact that demands are identical for all $j \in]0, n]$ $\left(\Rightarrow \sum_r^n x_j = nx_j \right)$ the profit rate per producer is:

$$(B13) \quad \pi = (1 - \alpha)/n$$

π is the sum of dividends paid to shareholders by a producer, who has the blueprint for a certain product j . The value of the firm in a perfect foresight equilibrium is then:

$$(B14) \quad v(t) = \int_t^{\infty} e^{-\rho(\tau-t)} \pi(\tau) d\tau$$

This implies that assets are valued according to their fundamentals. A sufficient condition for the absence of bubbles are immortal households that maximize "lifetime" utility. Hence, (B1) implies the absence of bubbles and justifies (B14). Taking the first derivative with respect to time and inserting (B8) yields:

$$(B15) \quad \dot{v} = rv - \pi$$

(B15) determines the dynamics of asset prices.

B.1.2.2. Production of blueprints

The available set of brands can only incrementally be increased per unit of time:

$$(B16) \quad \dot{n} = \frac{1}{a} \ell dt$$

Again GH assume a constant return to scale technology. Yet, (B16) does not set the stage for endogenous growth because the only production factor is "non-accumulatable" labour.

From (B16) follows the cost function:

$$(B17) \quad c(\dot{n}) = aw\dot{n},$$

where w is the wage of a unit labour (ℓ). From (B14) follows the sales function:

$$(B18) \quad s(\dot{n}) = v\dot{n}$$

Hence given this constant returns framework $v > wa$ implies that all resources (ℓ) are used for R&D activities; $v < wa$ implies that no research is undertaken. Hence equilibrium implies

$$(B19) \quad v \leq wa \quad \text{with equality whenever } \dot{n} > 0.$$

To close the model, the labour market equilibrium has to be determined. From (B16) follows that the labour demand of the R&D-sector equals $\ell(\dot{n}) = a\dot{n}$. The labour demand of the manufacturing sector follows from (B9), $E=1$ and $nx_j = E/p_j$: $\ell(nx_j) = 1/p$, where $p_j = p \forall j$. Hence market equilibrium is given by:

$$(B20) \quad a\dot{n} + \frac{1}{p} = L$$

(B20) can be interpreted as resource constraint, because labour is exogenously given in this model. Since $\dot{n} \geq 0$ in equilibrium it follows

$$(B21) \quad p \geq 1/L \quad \text{with equality whenever } \dot{n} = 0$$

B.1.3. The dynamics of the model

So far, the model contains two time dependent variables \dot{n} and \dot{v} . The dynamics of these variables are sufficient to determine the steady state of the economy:

First, (B19) implies $wa = v$ whenever $\dot{n} > 0$. Inserting (B12) and respecting (B21) yields

$$(B22) \quad \dot{n} > 0 \Leftrightarrow v > \alpha a/L$$

Resource constraint (B20) implies:

$$(B23) \quad \dot{n} > 0 \Rightarrow \dot{n} = \frac{L}{a} - \frac{1}{ap} \quad | \quad v = wa \text{ (B19)}, p = w/\alpha \text{ (B12)}$$

$$\Leftrightarrow \dot{n} = \frac{L}{a} - \frac{\alpha}{v}$$

(B22) and (B23) imply:

$$(B24) \quad \dot{n} = \frac{L}{a} - \frac{\alpha}{v} \Leftrightarrow v > \alpha a/L$$

Second, (B19), (B12) and (B21) imply:

$$(B25) \quad \dot{n} = 0 \quad \Leftrightarrow v \leq \alpha a/L$$

Hence (B24) and (B25) determine the dynamics of \dot{n} dependent on the structural parameters of the model and the (exogenous) resource endowment (L). Inserting (B8) and (B13) in (B15) yields the dynamics of the firm value:

$$(B26) \quad \dot{v} = \rho v - \frac{1-\alpha}{n}$$

Hence the $\dot{v} = 0$ locus is a downward sloping hyperbola, while the $\dot{n} = 0$ locus is the area below the horizontal $v = \alpha a/L$ in the (v, n) -plane (see figure B1). Two different cases have to be distinguished, because the initial endowment with brands (n_0) may be lower or higher than \bar{n} :

First, in case of $n_0 < \bar{n}$, the intersection point of the horizontal and the hyperbola (E) represents the unique steady state of the model. In this case three types of trajectories have to be regarded: *First*, if the financial markets "choose" a stock value per firm corresponding with point A , the dynamics of the system will drive the stock value permanently down, such that it becomes eventually negative. Yet, this trajectory is not consistent with rational expectations, because a constant n_0 implies constant positive non-zero profits of firms, consequently the firm value cannot approach zero (see B14). *Second*, if the financial markets "chose" a stock value per firm corresponding with point C , the dynamics of the system will drive the stock value and the number of brands permanently up. Yet, this trajectory is

not consistent with rational expectations, because a growing number of brands implies ceasing profits, consequently the stock market value of a firm *and* the number of brands cannot grow together (see (B13) and (B14)). *Third*, it follows that the only trajectory consistent with rational expectations starts at point *B*. This trajectory must lead to the steady state *E*.

Second, in case of $n_0 > \bar{n}$, the same rational expectation argument apply as in case of $n_0 < \bar{n}$. Yet, the result implies that the financial markets must choose the stock value such that the economy jumps immediately in a steady state. The steady state is now given by the intersection of the $(\dot{v} = 0)$ -hyperbola and the vertical standing on n_0 .

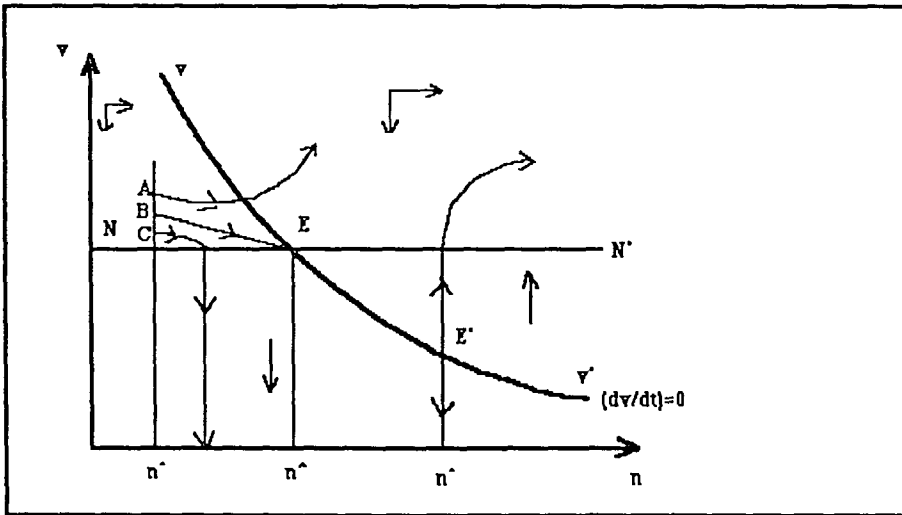


Figure B1: Steady state in the model of expanding product variety *without non-diminishing returns* to an "accumulatable" production factor and *without externalities* (GH(1991), Fig. 3.1).

B.1.4. Reformulation of the model of expanding product variety

GH (1991), pp 57, chapter 3.2, suggest that the zero growth steady in the above model arises, because knowledge capital is treated as a private good. To change this, they modify (B16):

$$(B16') \quad \dot{n} = \frac{1}{a} K_n \ell dt \quad \text{with} \quad K_n := n^\beta, \quad \beta = 1$$

Hence privately produced knowledge (\dot{n}) adds to a common available stock of knowledge (K_n), such that the costs (measured in working hours (ℓ)) of the production of new blueprints are reduced. GH mention two typical characteristics of technological knowledge to justify this assumption. First, technological knowledge is nonrival. Second, technological knowledge is - at least partially - nonexcludable. Yet, these characteristics do not necessarily imply that n joints production function (B16') with constant returns ($\beta = 1$). In the following it is shown that the modified version of the GH-model of expanding product variety implies a non-zero-growth steady state only for $\beta = 1$. For $\beta < 1$, there is always zero growth in steady state. Given (B16') the original equations are changed in the following way:

$$(B17') \quad c(\dot{n}) = \frac{aw}{n^\beta} \dot{n}$$

$$(B18') \quad s(\dot{n}) = v \dot{n}$$

$$(B19') \quad \frac{aw}{n^\beta} \geq v \quad \text{with equality whenever } \dot{n} > 0$$

$$(B20') \quad \frac{a\dot{n}}{n^\beta} + \frac{1}{p} = L$$

\Rightarrow

$$(B21') \quad p \geq 1/L \quad \text{with equality whenever } \dot{n} = 0$$

Now, the dynamic equations of the model have to be modified:

From (B19') follows $\frac{wa}{n^\beta} = v$, whenever $\dot{n} > 0$. Inserting (B12) and respecting (B21') yields

$$(B22') \quad \dot{n} > 0 \Leftrightarrow v > \alpha a / Ln^\beta$$

From resource constraint (B20') follows:

$$(B23') \quad \dot{n} > 0 \Leftrightarrow \frac{\dot{n}}{n^\beta} = \frac{L}{a} - \frac{1}{ap} \quad | v = aw/n^\beta \quad (B19'), \quad p = w/\alpha \quad (B12),$$

$$\Leftrightarrow \frac{\dot{n}}{n} = \frac{L}{an^{(1-\beta)}} - \frac{\alpha}{vn} \quad \left| \frac{1}{n^{(1-\beta)}} \right.$$

From (B22') and (B23') follows:

$$(B24') \quad \frac{\dot{n}}{n} = \frac{L}{an^{(1-\beta)}} - \frac{\alpha}{vn} \Leftrightarrow v > \alpha a / Ln^\beta$$

From (B19'), (B12') and (B21') follows:

$$(B25') \quad \frac{\dot{n}}{n} = 0 \quad \Leftrightarrow v \leq \alpha a / Ln^\beta$$

Hence (B24') and (B25') determine the dynamics of \dot{n}/n dependent on the structural parameters of the model and the (exogenous) resource endowment (L).

Inserting (B8) and (B13) in (B15) yields the dynamics of the firm value:

$$(B26') \quad \frac{\dot{v}}{v} = \rho - \frac{1-\alpha}{vn}$$

(B24'), (B25'), (B26') determine the dynamics of the modified model.

GH set $\beta = 1$ and $V := 1/vn$. Hence it follows:

$$(B32) \quad \frac{\dot{n}}{n} = \begin{cases} \frac{L}{a} - \alpha V & \text{for } V < \frac{L}{\alpha a} \\ 0 & \text{for } V \geq \frac{L}{\alpha a} \end{cases}$$

$$(B33) \quad \frac{\dot{V}}{V} = (1-\alpha)V - \frac{\dot{n}}{n} - p \quad \left| \quad \dot{V} = \frac{-\dot{n}}{vn^2} - \frac{\dot{v}}{nv^2} \right.$$

Projected on the $(V, \dot{n}/n)$ plane, the zero loci of this differential equations take the following form:

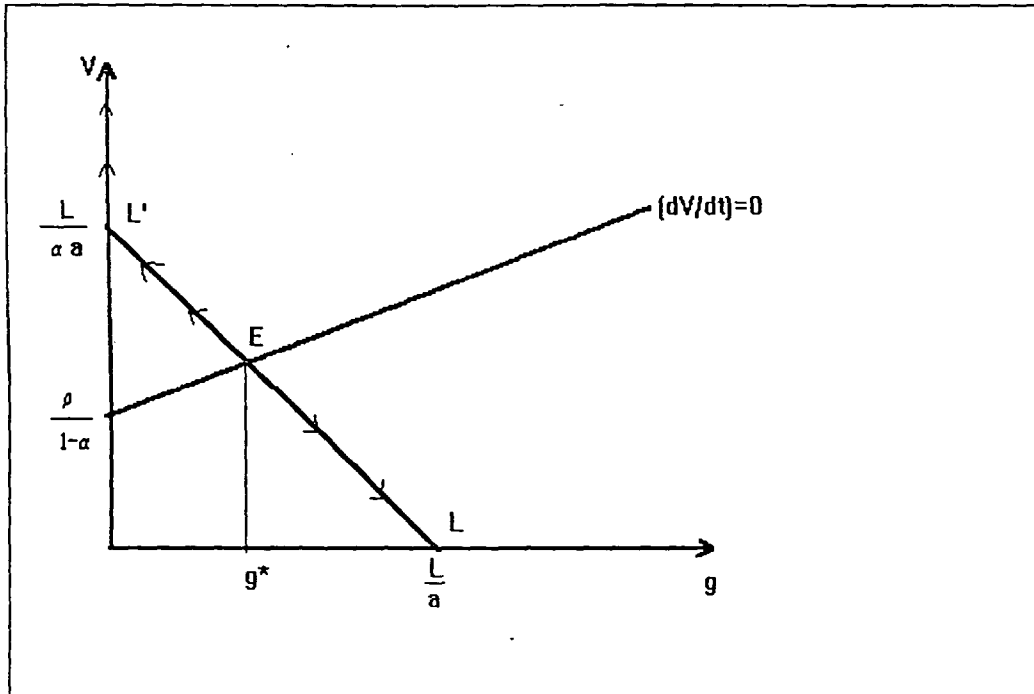


Figure B2: Steady state in the model of expanding product variety *with externalities* and *constant retruns to scale* to an "accumulatable" production factor (GH (1991), Fig. 3.2).

E is the steady state of the economy, where the value of firms ($\dot{V} = 0$) and the rate if innovation stags constant (g^*)³. As growth rate of the household product (D) equals $\dot{D}/D = \frac{\dot{n}}{n} \left(\frac{1}{\alpha} - 1 \right)$, this implies constant growth.⁴

The economy must immediately jump to the steady state $E = (\bar{V}, g^*)$, because any other trajectory is not in keeping with rational expectations: *First*, if V greater than \bar{V} , V will grow without bounds along line EL' , while n stays constant. As

³ Let $g := \frac{\dot{n}}{n}$.

⁴ As all $p_j = p = w/\alpha$ (see B12) $x_j = (\sum x_j)/n = x/n$ (see B4). Hence (B2) can be written $D = n^{1-\alpha} x_j = x n^{\alpha}$. Taking logarithm and derivative with respect to time yields: $\dot{D}/D = \frac{\dot{n}}{n} \left(\frac{1}{\alpha} - 1 \right)$, as $x = nx_j$ is constant in equilibrium.

$V = 1/nv$, this implies an ever diminishing stock value v . But, a constant value of n implies constant profits per firm and hence a constant stock value (see (B14)). Hence rational expectations exclude such a trajectory. *Second*, if V is smaller than \bar{V} , V will finally approach zero. This implies v to approach infinity. Yet, in this case the profit of a single firm, and consequently its stock value, is permanently driven down, because n growth with its maximum rate L/a . Hence $V < \bar{V}$ is not in keeping with rational expectations too.

Yet if $\beta < 1$ is chosen, the $(\dot{v} = 0)$ -locus is (see (B26')):

$$(B34) \quad v = \frac{1-\alpha}{\rho} n^{-1}$$

and from (B25') the $(\dot{n} = 0)$ -locus is

$$(B35) \quad v \leq \frac{\alpha a}{L} n^{-\beta}$$

Projected on the (v, n) -plane these two equations imply two hyperbolas (see figure B3). Since $\beta < 1$ there exists always an intersection point (E) at

$$(B36) \quad \bar{n} = \left(\frac{(1-\alpha)L}{\alpha a \beta} \right)^{\frac{1}{1-\beta}}$$

$$(B37) \quad \bar{v} = \frac{1-\alpha}{\rho} \left(\frac{(1-\alpha)L}{\alpha a \rho} \right)^{\frac{\beta-1}{1-\beta}}$$

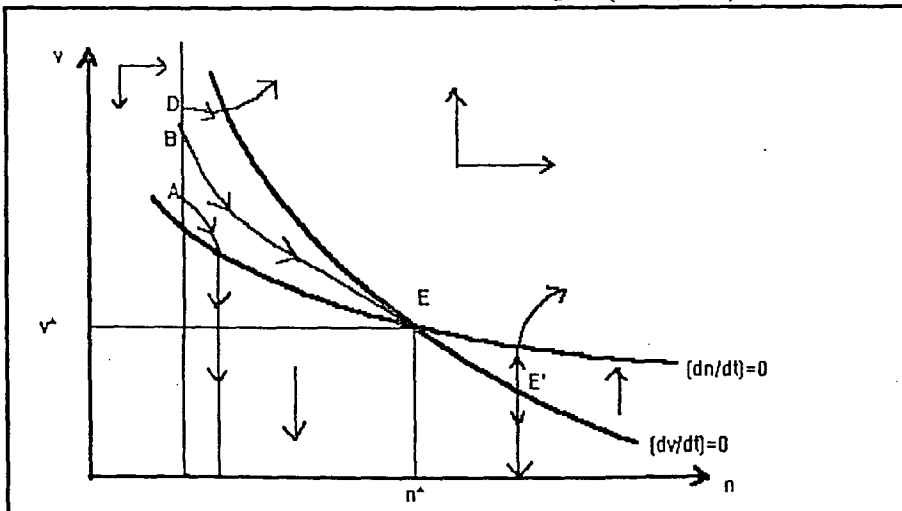


Figure B3: Steady state in the model of expanding product variety with externalities and diminishing returns to an "accumulatable" production factor (GH (1991), Fig 3.1).

For $\beta = 1$ (constant returns) no cutting point exists. Hence (B34) and (B35) are two parallel hyperbolas with a locus of constant growth of n inbetween (see figure (B4)). Figure (B4) is the equivalent of figure (B2) projected on the (v, n) plane.

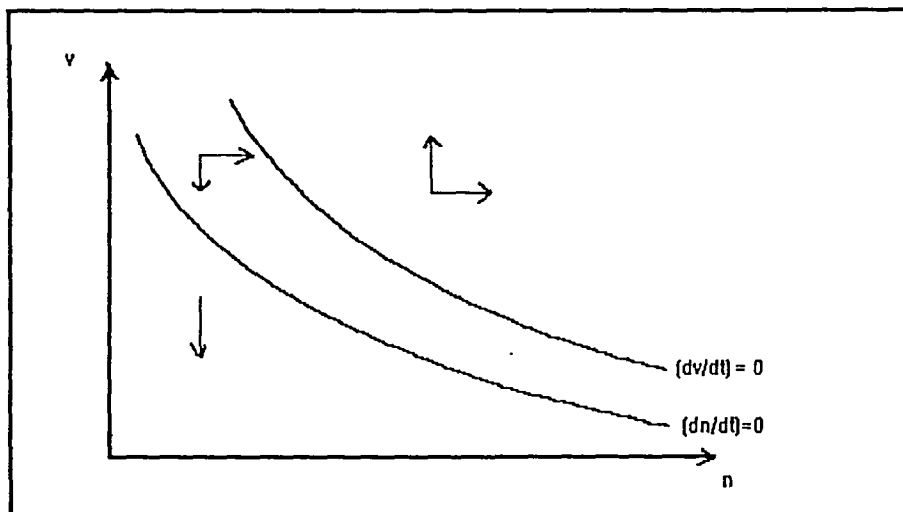


Figure B4: Steady state in the model of expanding product variety *with externalities* and *constant returns* to an "accumulatable" production factor. (GH (1991), Fig. A.3.2).

As can be seen from figure 3, the intersection point E is a saddle point steady state with a zero growth rate. By applying the same rational expectations arguments as before, it follows that, given an initial endowment of technological knowledge $n^* > \bar{n}$, the economy immediately jumps to E , given an initial endowment $n^* < \bar{n}$, there exists an trajectory, which is in keeping with rational expectations, which will eventually reach the steady state E .

B.1.5. Reformulation of the model of expanding product variety under the assumption of rival and excludable knowledge.

In this section it is assumed that knowledge is rival and excludable in the production of new brands, i.e. every firm engaged in the production of new

brands can only use its individual stock of knowledge n_i as an input in its production function. This modification is undertaken only to show that even under this assumption in non-zero steady state growth rate is possible, if knowledge (n_i) exhibits non-diminishing returns in the production function of new goods. Under this assumption equation (B16') has to be modified in the following way:

$$(B16'') \dot{n}_i = \frac{1}{a} \mathcal{K}_{n_i} \ell_i dt \quad \text{with} \quad \mathcal{K}_{n_i} := n_i^\beta, \beta = 1$$

\Leftrightarrow

$$\dot{n}_i = \frac{1}{a} \mathcal{K}_{n_i} \ell_i dt \quad \text{with} \quad \dot{n}_i I = \dot{n}$$

Hence privately produced knowledge \dot{n}_i only adds to the privately available stock of knowledge \mathcal{K}_{n_i} . It is assumed that an arbitrary number of I firms are engaged in the production of new brands. To ensure that every firm has the same incentive to produce new brands, it is assumed that the initial endowment with brands (i.e. knowledge) per firm is equal: $\dot{n}_i = \bar{n} \quad \forall i = 1, 2, 3, \dots, I$. Given (B16'') the original equations are changed in the following way:

$$(B17'') c(\dot{n}_i) = \frac{aw}{n_i} \dot{n}_i$$

$$(B18'') s(\dot{n}_i) = v \dot{n}_i$$

$$(B19'') \frac{aw}{n_i} \geq v \quad \text{with equality whenever} \quad \dot{n}_i > 0$$

$$(B20'') \frac{a\dot{n}_i}{n_i} + \frac{1}{p} = \mathcal{L}$$

\Rightarrow

$$(B21'') p \geq 1/L \quad \text{with equality whenever} \quad \dot{n}_i = 0$$

Now, the dynamic equations of the model have to be modified:

From (B19'') follows $\frac{wa}{n_i} = v$, whenever $\dot{n}_i > 0$. Inserting (B12) and respecting

(B21'') yields

$$(B22'') \quad \dot{n} > 0 \Leftrightarrow v > \alpha a / \mathcal{L} n_i$$

From resource constraint (B20'') follows:

$$(B23'') \quad \dot{n} > 0 \Leftrightarrow \frac{\dot{n}}{n_i} = \frac{\mathcal{L}}{a} - \frac{1}{ap} \quad | \quad v = azw/n_i \quad (B19'), \quad p = w/\alpha \quad (B12),$$

$$\Leftrightarrow \frac{\dot{n}}{n_i} = \frac{\mathcal{L}}{a} - \frac{\alpha}{v n_i}$$

From (B22'') and (B23'') follows:

$$(B24'') \quad \frac{\dot{n}}{n_i} = \frac{\mathcal{L}}{a} - \frac{\alpha}{v n_i} \Leftrightarrow v > \alpha a / \mathcal{L} n_i$$

From (B19'), (B12') and (B21') follows:

$$(B25'') \quad \frac{\dot{n}}{n_i} = 0 \quad \Leftrightarrow v \leq \alpha a / \mathcal{L} n_i$$

Inserting (B8) and (B13) in (B15) yields the dynamics of the firm value:

$$(B26'') \quad \frac{\dot{v}}{v} = \rho - \frac{1-\alpha}{vn}$$

(B24''), (B25''), (B26'') determine the dynamics of the modified model.

GH set $\beta = 1$ and $V := 1/vn$. Hence it follows:

$$(B32'') \quad \frac{\dot{n}}{n} = \begin{cases} \frac{\mathcal{L}}{Ia} - \frac{\alpha}{I^2} \mathcal{V} & \text{for } \mathcal{V} < \frac{I\mathcal{L}}{\alpha a} \\ 0 & \text{for } \mathcal{V} \geq \frac{I\mathcal{L}}{\alpha a} \end{cases}$$

$$(B33'') \quad \frac{\dot{V}}{V} = (1-\alpha)V - \frac{\dot{n}}{n} - p \quad \Bigg| \quad \dot{V} = \frac{-\dot{n}}{vn^2} - \frac{\dot{v}}{nv^2}$$

Projected on the $(V, \dot{n}/n)$ plane, the zero loci of this differential equations take the following form:

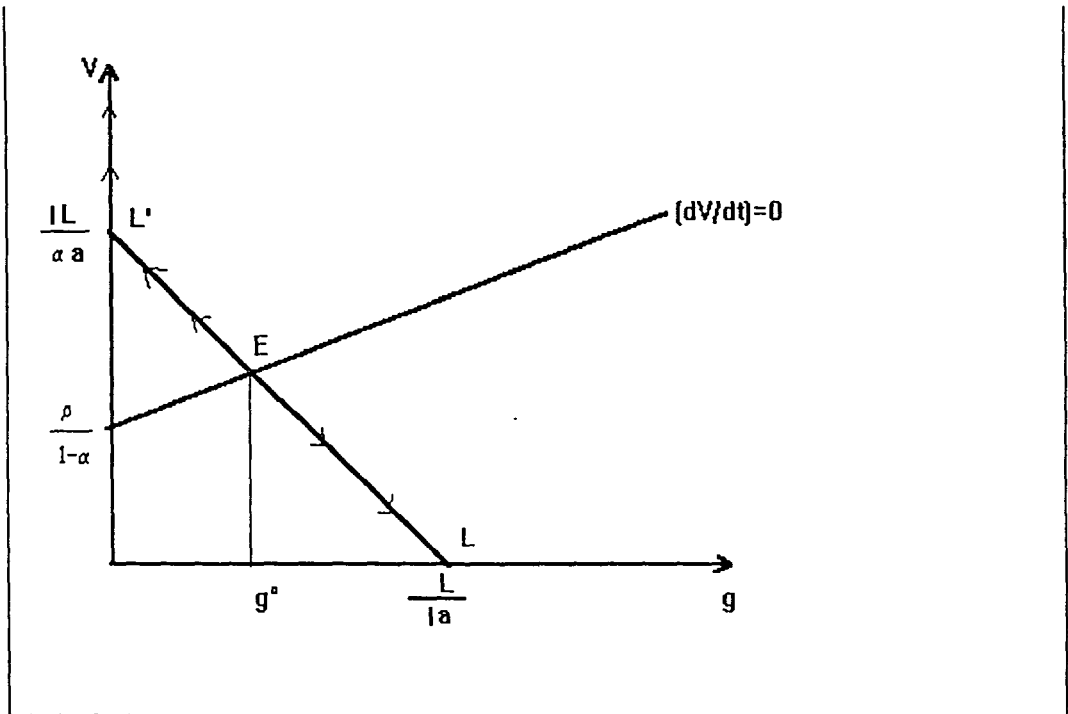


Figure B5: Steady state in the model of expanding product variety *without externalities* but *with constant retruns* to an "accumulatable" production factor.

Figure B5 has the same interpretation as figure B2. It is straightforward to show that the growth rate of technical knowledge under the assumption of rival and excludable knowledge (g^o) can be higher or lower as the growth rate under the assumption of nonrival and non-excludable knowledge (g^* , see figure B'2). The relation between both growth rates is given by

$$g^o = g^* \left(\frac{I}{I^2(1-\alpha) + \alpha} \right) + \left(\frac{\alpha\rho}{I^2(1-\alpha) + \alpha} \right).$$

This implies $g^o = g^*$ for $I=1$. It is interesting to note that there may be indeed parameter constellations for which the growth rate of technical knowledge is

higher under the assumption of rival and excludable knowledge than under the assumption of nonrival and non-excludable knowledge.

B.2. Revealing returns to accumulatable production factors in the Grossman/Helpman model of expanding product variety.

In the following it is shown that the household production (3.2) exhibits diminishing returns to the accumulative production factor n in the steady state of the Grossman/Helpman (1991) model of expanding product variety. From (B4) and (B12) follows that in steady state the same quantity is produced from every product. Consequently, instead of (B2) (resp. 3.2 in Grossman/Helpman (1991)) can be written:

$$(B38) \quad D(\tau) = D(x(j, \tau), n) = D(n) = n^{(1/\alpha)} x = n^{((1-\alpha)/\alpha)} nx.$$

If no research is undertaken, it follows from (B20) that $nx=L$ (L is the exogenous supply of labour). Hence instead of B(38) can be written:

$$B(39) \quad D(\tau) = n^{((1-\alpha)/\alpha)} L.$$

B(39) gives the amount of the household good in steady state in terms of the steady state value of n and the exogenous given labour supply. B(39) exhibits diminishing returns to n in steady state, if its second derivative with respect to n is negative. To determine the second derivative of B(39), it has to be taken into consideration that the production of an additional marginal amount of n implies the use of labour in the research sector. Hence, L , the amount of labour to produce x , is reduced by $a \Delta n$, as follows from (B20). Taking this into consideration, from the definition of the first derivative of a function

$\left(\frac{\delta f}{\delta x} = \lim_{\Delta x \rightarrow 0} ((f(x + \Delta x) - f(x)) / \Delta x) \right)$ follows then the first derivative of B(39):

$$(B40) \quad \frac{\delta D}{\delta n} = \lim_{\Delta n \rightarrow 0} \left(\frac{(n + \Delta n)^{((1/\alpha)-1)} (L - a \Delta n) - n^{((1/\alpha)-1)} L}{\Delta n} \right)$$

$$\begin{aligned}
 &= \lim_{\Delta n \rightarrow 0} \left(-a n^{((1/\alpha)-1)} + L \frac{(n + \Delta n)^{((1/\alpha)-1)} - n^{((1/\alpha)-1)}}{\Delta n} \right) \\
 &= \left(-a n^{((1/\alpha)-1)} + L ((1/\alpha) - 1)((1/\alpha) - 2) n^{((1/\alpha)-3)} \right).
 \end{aligned}$$

In steady state $n = (1-\alpha)L/\alpha pa$ (see Grossman/Helpman (1991), pp.53, equation(3.17)). Substituting this for n yields:

(B42)

$$\frac{\delta^2 D}{n^2} = \left(a \left(\frac{1}{\alpha} - 1 \right) \left(\frac{\alpha pa}{(1-\alpha)L} \right)^{\left(\frac{1}{\alpha} - 2 \right)} \left(\frac{\rho(1-2\alpha)}{1-\alpha} - 1 \right) \right)$$

As Grossman/Helpman (1991) postulate $0 < \alpha < 1$ (see Grossman/Helpman (1991), p. 45, equation (3.2)) this expression is negative, if :

$$(B43) \left(\frac{\rho(1-2\alpha)}{1-\alpha} - 1 \right) < 0$$

For $\rho < 0.5$ this inequality is fulfilled, if $\alpha < \left(\frac{1-\rho}{1-2\rho} \right)$. As $\left(\frac{1-\rho}{1-2\rho} \right) > 1$, for $0 < \rho < 0.5$, the inequality is fulfilled if $\alpha < 1$.

For $\rho > 0.5$ this inequality is fulfilled, if $\alpha > \left(\frac{1-\rho}{1-2\rho} \right)$. As $\left(\frac{1-\rho}{1-2\rho} \right) < 0$, for $1 > \rho > 0.5$, the inequality is fulfilled if $\alpha > 0$.

Consequently, inequality (B43) is fulfilled, if $0 < \alpha < 1$. As Grossman/Helpman (1991) postulate $0 < \alpha < 1$ (see Grossman/Helpman (1991), p. 45, equation (3.2)), (B42), i.e. the second derivative of the household production function (B2) with respect to n , is negative in steady state. Hence n exhibits diminishing returns in steady state.

B.3. Revealing returns to accumulatable production factors in the Grossman/Helpman model of rising product quality.

The GH model of rising product quality differs from the model of expanding product variety mainly by the assumption concerning the production of the household good D. Instead of (3.1) the production of D now obeys the following function:

$$(B38) \quad D(x(m, j, t), q(m, j)) = \exp \left[\int_0^n \ln \left[\sum_m q(m, j) x(m, j, t) \right] dj \right],$$

where $x(m, j, t)$ is the amount of good x of variety j of quality level m that is consumed at time t . $q(m, j)$ is the measure of quality of a unit of the corresponding good. The number of product varieties n is now held constant and normalized to unity ($n=1$). Again, the only resource of the model is labour. The only accumulatable production factor is product quality $q(m, j)$, as n is now hold constant. A discrete number of quality levels m exists for every product variety j . R&D activities are undertaken in this model, to find new product qualities. If R&D has been successful, the quality of a certain product variety will grow with factor $\lambda > 1$. For $q(0, j) = 1$ this implies:

$$(B39) \quad q(m, j) = q(m-1, j)\lambda = \lambda_j^m \quad \text{for } q(0, j) = 1$$

As it is shown in the following, this modelling of the quality rising process (the so called "quality ladder") together with (B.38) and the assumptions modelling the market structure implies increasing returns to m .

The product of the highest quality level (\bar{m}) of each product line (j) has the lowest price for the given market structure (GH (1991), pp 90-91). Consequently, $\sum_{m \neq \bar{m}} q(m, j) x(m, j, t) = 0$. Hence, (B38) can be simplified in the following way:

$$(B38) \quad D(x(\bar{m}, j, t), \bar{m}) = \exp \left[\int_0^1 \ln \left[\lambda_j^{\bar{m}} x(\bar{m}, j, t) \right] dj \right]$$

The second derivation of (B38) with respect to \bar{m} cannot be taken in the usual way, because m is a discrete variable that can only take an integer value. Yet, the second difference of (B39) can be derived as follows:

The first difference is given by:

(B40)

$$D(x(\bar{m}+1, t), \bar{m}+1) - D(x(\bar{m}, t), \bar{m}) = \exp\left[\int_0^1 \ln(\lambda^{\bar{m}+1} x(\bar{m}+1, t)) dj\right] - \exp\left[\int_0^1 \ln(\lambda^{\bar{m}} x(\bar{m}, t)) dj\right]$$

The number of products is given by a unit measure (see GH, (1991), p. 88). This implies that the resource constraint (GH, (1991), p. 94, equation (4.12)) can be written:

$$\begin{aligned} x(\bar{m}+1, t) &= L - at \\ x(\bar{m}, t) &= L, \end{aligned}$$

where $x(\cdot)$ is labour demand of the manufacture sector, at is labour demand of the R&D sector and L is exogenous labour supply. Hence the first difference (B40) is equivalent to:

$$\exp\left[\int_0^1 \ln(\lambda^{\bar{m}+1}(L - at)) dj\right] - \exp\left[\int_0^1 \ln(\lambda^{\bar{m}}L) dj\right]$$

The steady state research intensity t is given by (GH, (1991), p. 96, equation (4.18)):

$$t = (1 - \delta) \frac{L}{a} - \delta\rho, \text{ with } \delta = 1/\lambda, \text{ hence the first difference can be written:}$$

$$\begin{aligned} &\exp\left[\int_0^1 \ln(\lambda^{\bar{m}+1}L - \lambda^{\bar{m}}(\lambda - 1)L + \lambda^{\bar{m}}a\rho) dj\right] - \exp\left[\int_0^1 \ln(\lambda^{\bar{m}}L) dj\right] = \\ &\exp\left[\int_0^1 \ln(\lambda^{\bar{m}}L + \lambda^{\bar{m}}a\rho) dj\right] - \exp\left[\int_0^1 \ln(\lambda^{\bar{m}}L) dj\right] > 0 \end{aligned}$$

Hence *in steady state equilibrium* the first difference of the household production function is positive. It is easy to show that the second difference of (B38) is positive too:

$$\begin{aligned}
 \text{(B41)} \quad & D(x(\bar{m} + 2, t), \bar{m} + 2) - D(x(\bar{m} + 1, t), \bar{m} + 1) \\
 & - [D(x(\bar{m} + 1, t), \bar{m} + 1) - D(x(\bar{m}, t), \bar{m})] = \\
 & \exp \left[\int_0^1 \ln(\lambda^{\bar{m}+2} x(\bar{m} + 2, j, t)) dj \right] - \exp \left[\int_0^1 \ln(\lambda^{\bar{m}+1} x(\bar{m} + 1, j, t)) dj \right] \\
 & - \left[\exp \left[\int_0^1 \ln(\lambda^{\bar{m}+1} x(\bar{m} + 1, j, t)) dj \right] - \exp \left[\int_0^1 \ln(\lambda^{\bar{m}} x(\bar{m}, j, t)) dj \right] \right]
 \end{aligned}$$

From the resource constraint it follows: $x(\bar{m} + 2, j, t) = x(\bar{m} + 1, j, t)$, if research is undertaken, and $x(\bar{m} + 1, j, t) = x(\bar{m}, j, t)$, if no research is undertaken. Hence, the second difference is equivalent to:

$$(\lambda - 1) \left[\exp \left[\int_0^1 \ln(\lambda^{\bar{m}+1} x(\bar{m} + 1, j, t)) dj \right] - \exp \left[\int_0^1 \ln(\lambda^{\bar{m}} x(\bar{m}, j, t)) dj \right] \right]$$

As $\lambda > 1$, the second difference will be positive, if the first difference is positive. Hence, *in steady state equilibrium* the household production function exhibits *increasing returns to the accumulatable intermediary input m* .

It is straightforward to show that this is not true for the accumulatable input (n) of the household production function of the model of expanding product variety (B2). The second derivative of this function with respect to n is negative in steady state equilibrium. Hence the model of expanding product variety does not exhibit endogenous growth without further assumptions.

B.4. Some problems of the GH model of rising product quality

GH assume that in the beginning "every industry has a unique leading firm, which implicitly requires that there has been at least one innovation in each industry prior to time 0" (see GH, (1991), p. 91, footnote 3). Given this initial state in every industry $j \in]0, 1]$ a research success, i.e. a quality jump of factor λ , is reached with probability ι , where ι is determined in equilibrium. Thus research

success in each industry is guided by an independent Poisson process and "the model predicts an evolving distribution of product qualities, with individual products continuously swapping relative positions within the distribution". (GH, (1991), p. 97).

This implies that after "a while", there will be industries which are more than one quality jump back compared with other industries.⁵ Assume that there is an industry j^0 , which is two quality jumps back from industry j^* , which has the highest quality level of all industries (\bar{m}). The assumptions governing market structure and market conduct imply that the producer of the good with the highest quality level in industry j^* (in the following called the leader) sets a price p^* that is a shade lower than the quality adjusted price of his follower in industry j^* , who sets a price equal to his variable average costs w . This implies p^* is incrementally lower than $w\lambda$ because the quality difference is given by λ with ($\lambda > 1$).

Given the household production function (B38), households are indifferent between a product of quality level $(\bar{m} - 1)$ with price w and a product with quality level \bar{m} with price $w\lambda$. Consequently, as $p^* < w\lambda$, households buy only from the product leader, who is able to earn a monopoly profit π , as p^* is higher than his variable average production costs w , while its follower makes a zero profit.

From all this follows that the producer of the leading product of industry j^0 , which is two quality jumps back from the leading product of industry j^* , can at best set a price equal to his variable average costs (w) but will nevertheless make zero profits, because the household production function (B38) evaluates all products $j \in]0,1]$ with the same quality level equally and exhibits constant returns to $x(m,j,t)$, such that nobody has an incentive to buy the leading product of

⁵ GH (1991),p 93, show, that there is no incentive for a product leader of a certain industry to invest in R&D in order to gain a two step quality advantage compared to his followers *within* his industry. Yet, this is not true for the relation between product leaders of different industries. As research success is stochastic, it is always possible that in industry j^* two research successes appear one after the other, while in industry j^0 no research success is made. GH (1991) are clearly aware of this fact, as can be seen from figure 4.1, p. 86, and the second integral on p. 97.

industry j^0 .⁶ Roughly speaking the leading product of industry $j^{0\circ}$ is more expensive than the leading product of industry j^* . Consequently, everybody will only buy the leading product of industry j^* .

From this follows, that no more research is done in industry j^0 , because the costs necessary to reach the monopoly profit π in industry j^0 and the risk of failure is higher than the cost and the risk of failure of reaching the same monopoly profit in industry j^* , for in industry j^0 two quality jumps have to be made, while in industry j^* only one quality jump has to be made, to reach the monopoly position. Hence, all research is concentrated in industry j^* .

From this follows, that every industry that is more than one quality jump back from another industry will stay on this level for ever. This implies a permanent concentration process of industries. After a sufficiently long time interval, only one industry will have survived. Consequently the market result seriously changes and the leader of the industry that has survived has an incentive to deter research by his followers by investing himself in R&D and gaining a unknown research advantage.⁷

In the end no research activities are undertaken and technological progress and endogenous growth fade out.

⁶ The ultimate reason for this are the constant returns to scale to $x(m,j,t)$. They ensure that the productivity of the leading product does not diminish no matter what quantity of the leading product is used.

⁷ The assumption made by GH (1991) in footnote 4, p. 93, does not apply anymore, because even if the followers cannot observe the R&D operations of the leading firm, they know that the leading firm has a strong incentive to conduct R&D in its own industry, as investments in other industries are - given their quality lack - not attractive.

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