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Stabilization Policy in an Open-Economy Equilibrium Model*+ 

by 

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Whether the interests of a nation are best served by maintaining fixed exchange rates or by allowing exchange rates to vary continues to be a much debated issue among economists. Most of the earlier literature on this topic focused attention on the two extremes of completely fixed exchange rates versus the perfectly flexible or free floating system, examining the circumstances (degree of factor mobility, structure and origin of disturbances, etc.) under which one or the other would be preferred. More recently, several authors have adopted a different perspective by which the fixed and free float regimes are considered endpoints on a continuous spectrum of exchange rate flexibility (e.g. Boyer (1978), Frenkel and Aizenman (1982), Roper and Turnovsky (1980)). Undoubtedly, this view was stimulated by the move toward a system of managed floating among the major world currencies since 1973, where governments intervene in foreign exchange markets to varying extents, but not enough to peg the exchange rate. The apparent payoff from this change in perspective is that it allows addressing the question: Under a given set of circumstances, what is the optimal level of exchange rate intervention, or the optimally managed float? Clearly, the ability to answer this question should lead to improved policy recommendations since, instead of having to choose between two, generally suboptimal exchange rate regimes, we have here the possibility of tailoring an exchange rate regime to a particular set of national circumstances.
The objective in this paper is to address the issue of an optimally managed float in the context of a rational expectations macromodel of an open economy. As Buiter (1979) already recognized in a similar endeavor, the search for an optimal foreign exchange market intervention strategy raises the same issues which had already been subjected to heated debates in earlier Keynesian-monetarist confrontations; these being the issue of "rules versus discretion" and the controversy over "targets, instruments and indicators" of policy. With respect to the first of these, Sargent and Wallace (1976) state that "there is no longer any serious debate about whether monetary policy should be conducted according to rules or discretion," but that "the central practical issue... is the appropriate form of the monetary policy rule". By casting the problem of optimal exchange rate management in the language of rational expectations equilibrium models, it becomes transparent that this issue also consists in essence of finding the appropriate feedback rule. Also, this treatment makes more transparent the distinction between a "managed float" and an "adjustable peg" system; something which earlier disequilibrium treatments, such as Frenkel and Aizenman's (1982) discussion of "the optimal degree of fixity of exchange rates", left unclear. Here it becomes simply a question of which target variable is to be controlled by an optimal rule.

To obtain an optimal intervention strategy, the analysis will focus on the behaviour of real variables under different policy
rules. This confronts us with the well-known policy-ineffectiveness results which are common to a large class of equilibrium macro-models that display money neutrality; see McCallum (1980). The central message from these results is that for government demand management policies to be systematically effective in influencing real behavior, public policymakers must either be in possession of superior information or be in a better position to respond and adjust to new information than private agents, say because there are economies of scale or public goods involved. This paper does not strive to contribute to this policy-ineffectiveness debate. Instead, it grants the premise that the government policymakers are in an advantageous position vis-à-vis private decisionmakers with regard to the acquisition of information and the speed of response. This assumption is slightly relaxed in a second version of the model where, even though public and private decisionmakers have the same access to information, contractual rigidities provide a hindrance to the speed with which private, but not public, agents can respond to current information.

In the next section, a basic macromodel for a small open economy is presented. Thereafter, the analysis is structured in three parts: First is presented the hypothetical case, to be used as a benchmark, where policymakers have perfect contemporaneous information on all economic shocks so that an intervention rule can be constructed on the basis of these shocks. Secondly, the analysis is repeated for the case where policymakers only receive current
information on a restricted set of variables. In this case the optimal intervention rule amounts to an indexation rule by which variations in either the exchange rate or the money supply are tied to variations in the other observed variables. In the last part, several foreign exchange market intervention schemes which have been variously proposed in the past will be reinterpreted in the terms of the present model, and then compared to the optimal rule derived in the preceding section. A short summary concludes the paper.

Two papers, already mentioned, which are closely related to the present paper in both the issues addressed and the approach used are those by Buiter (1979) and Roper and Turnovsky (1980). Buiter uses a more general objective function by which an optimal intervention rule is defined than is done here. However, in his framework, government policymakers respond only to past realizations of market data, whereas here the market period is so defined that policymakers are assumed able to respond to current data. Favoring this construction is the fact that data from international financial markets, where the intervention takes place, is available after very short intervals. Roper and Turnovsky (1980) also emphasize this point. The present analysis can in turn be differentiated from theirs in that it is conducted in a less restrictive macroeconomic framework allowing for flexible quantities and prices. This has the advantage of not only allowing for greater flexibility in modeling the process of exchange rate determination, but also of expanding the range of issues which can be addressed.
I. An Open-Economy Macromodel

The model of macroeconomic activity used to analyze the effects of alternative policy measures is taken from Weber (1981). It represents a fairly straightforward extension to an open economy of a set of models typically used in the macro-rational expectations literature and popularized, for example, by Barro (1976) and Sargent and Wallace (1975). The model consists of the following linear system of equations:

1. \[ y_t = a_0 + a_1 (P_t - E(P_t/\pi_{t-1})) + a_2 y_{t-1} + u_t; \quad 0 < a_2 < 1 \] (aggregate supply)

2. \[ y_t = b_0 - b_1 (i_t - E(P_{t+1} - P_t/\pi_{t-1})) + b_2 (e_t + P^*_t - P_t) + v_t \] (aggregate demand)

3. \[ M_t = c_0 P_t + c_1 y_t - c_2 i_t + c_3 (e_t + P^*_t) + \eta_t; \quad c_0 + c_3 = 1 \] (LM schedule)

4. \[ i_t = i^*_t + E(e_{t+1} - e_t/\pi_{t-1}) \] (interest parity condition)

All variables are expressed in logarithms and have the following definitions: \( y_t \) is the current level of real output or income, \( P_t \) is the domestic goods price level, \( e_t \) is the exchange rate (units of domestic currency per unit of foreign currency), \( M_t \) is the domestic nominal money supply, \( i_t \) is one plus the domestic nominal interest rate and \( i^*_t \) is the corresponding world interest rate. \( P^*_t \) is
the foreign currency price of foreign goods and will be assumed to obey the following stochastic process:

\begin{equation}
\mathbf{p}_t^* = \mathbf{p}_{t-1}^* + k + \epsilon_t,
\end{equation}

where \( k \) is the (average) foreign inflation rate. The terms \( u_t, v_t, \eta_t \) and \( \epsilon_t \) are mutually independent white noise disturbances. The expression \( E(x_t/\Omega_{t-1}) \) is used to denote the mathematical expectation of the value of \( x \) in period \( t \) conditioned on the information set, \( \Omega_{t-1} \), which contains all variables dated \((t-1)\) and earlier. Later on the model will also be altered to include expectations based on current information. The letters \( a_i, b_i, c_i \), symbolizing the parameters of the model, are all taken to be non-negative.

Since this model has already been described in some detail by Weber (1981), this need not be repeated here. Instead, only some of the more prominent features which make the model attractive for the purposes of this study will be commented on. First, embodied in the supply equation (1) is the natural rate hypothesis, stating that only the unanticipated component of changes in the price level (or inflation) will have an effect on output. The lagged output term in equation (1) is included to account for the empirical observation that output movements over time are serially correlated. This could be theoretically justified as a proxy for capacity effects or wealth effects. Equations (2) and (3) are standard IS and LM equations. In equation (4) is reflected the joint assumption that domestic and foreign bonds are perfect substitutes, capital is perfectly mobile, and that the domestic country is too small to
affect the world nominal interest rate. Domestic and foreign goods, on the other hand, are not assumed to be perfect substitutes as evidenced by the fact that no restrictions have been placed on the behavior of the terms of trade, $e_t + P_t^* - P_t$. All markets are assumed to clear in every period. Finally, if for simplicity all constants in the model are set to zero, including all foreign variables, then it is easy to derive that in the absence of shocks in steady-state equilibrium $y = i = 0$ and $e = P = M$. This serves to illustrate that the system is homogeneous of degree 1 in all nominal variables.

Throughout most of the main text it is assumed that private agents receive macroeconomic information with a one period time lag, or else that their reaction time to new information takes that long. This is made explicit in equations (1)-(4) by the fact that all expectations are conditioned on set $\Omega_{t-1}$. In contrast, government policymakers are assumed to have immediate access to some current information and able to react instantaneously (in terms of adjusting their policy variables) within one period to new information. Weber (1981) motivates the sluggish reaction time on the part of private agents, in the specific case of equation (1), by pointing to the existence of contracts which remain in force longer than one period, so that current employment decisions will partly depend on past expectations based on old information. This argument leans on previous work by Fischer (1977) and Gray (1976). The same argument is more difficult to defend in the case of the remaining equations of the model. It appears particularly unrealistic for some
financial variables and prices, such as exchange rates, which are quoted on a daily basis. In order to meet this criticism, the analysis will also be repeated for the case where the expectations in equations (2)-(4) are based on the same updated information set, \( \Omega_t \), available to government policymakers. However, because this does not alter the qualitative results derived from the lagged-information model, the results from the updated-information model are relegated to the footnotes.

The solution technique applied in this analysis is the method of undetermined coefficients which can now be described. First, the model can be simplified by substituting for \( i_t \) and \( P_t^* \) using (4) and (5) into (1) and (2). This yields

\[
(6) \quad (a_1 + b_2)P_t + (b_1 - a_1)E(P_t/\Omega_{t-1}) - b_1E(P_{t+1}/\Omega_{t-1}) = b_2e_t - b_1E(e_{t+1} - e_t/\Omega_{t-1}) - a_2y_{t-1} + v_t - u_t - b_1i^* + b_2P_{t-1} + b_2\epsilon_t
\]

Similarly, equation (3) can be rewritten as

\[
(7) \quad M_t = (c_0 + c_1a_1)P_t - c_1a_1E(P_t/\Omega_{t-1}) + c_3e_t - c_2E(e_{t+1} - e_t/\Omega_{t-1}) + c_1a_1y_{t-1} - c_2i^* + c_1u_t + c_3P_{t-1} + c_3\epsilon_t + n_t
\]

This leaves us with two equations in the three variables \( P_t \), \( e_t \) and \( M_t \). The remaining equation needed to determine the system is given by the as yet unspecified government policy rule. The specification of that rule is the subject of subsequent sections. Mean-
while, provided we restrict ourselves to linear feedback rules, it is possible to express the general solution of the endogenous variables as linear functions of the exogenous and predetermined variables in the model. Unless the government policy rule responds to additional variables not included in (6) and (7), this solution can be written as

\[ P_t = \Pi_0 + \Pi_1 Y_{t-1} + \Pi_2 u_t + \Pi_3 v_t + \Pi_4 \eta_t + \Pi_5 P^*_{t-1} + \Pi_6 \varepsilon_t, \]

\[ e_t = \lambda_0 + \lambda_1 Y_{t-1} + \lambda_2 u_t + \lambda_3 v_t + \lambda_4 \eta_t + \lambda_5 P^*_{t-1} + \lambda_6 \varepsilon_t, \]

\[ M_t = \psi_0 + \psi_1 Y_{t-1} + \psi_2 u_t + \psi_3 v_t + \psi_4 \eta_t + \psi_5 P^*_{t-1} + \psi_6 \varepsilon_t, \]

where the \( \Pi \)'s, \( \lambda \)'s and \( \psi \)'s are coefficients to be determined. Under the assumption that private agents perceive new information with a lag, their expectations can be expressed as

\[ E(P_{t+1}/\Omega_{t-1}) = \Pi_0 + \Pi_1 E(Y_t/\Omega_{t-1}) + \Pi_5 E(P^*/\Omega_{t-1}) = (\Pi_0 + \Pi_5 k) + \Pi_1 a_2 Y_{t-1} + \Pi_5 P_{t-1}. \]

\[ E(e_{t+1}/\Omega_{t-1}) = (\lambda_0 + \lambda_5 k) + \lambda_1 a_2 Y_{t-1} + \lambda_5 P_{t-1}. \]

The solution technique now involves substituting equations (9)-(12) for the appropriate terms in equations (6)-(7) and the policy rule equation. For an equilibrium solution, it is necessary that each equation hold identically in each of the predetermined and exogenous variables. This should yield enough restrictions to solve for all of the coefficients in (8)-(10).
For purposes of illustration, let us focus momentarily on the exogenous shock, $u_t$. Because equations (6) and (7) must hold for all values of $u_t$, the coefficients involving this term obey the following restrictions: (i) $(a_1 + b_2)\Pi_2 - b_2\lambda_2 + 1 = 0$, and (ii) $\psi_2 - (c_0 + c_1a_1)\Pi_2 - c_3\lambda_2 - c_1 = 0$. With one additional equation provided by the policy rule, it would be possible to solve for $(\Pi_2', \lambda_2', \psi_2)$ in terms of the structural parameters of the model.

Finally, it is necessary to describe the manner by which alternative intervention rules are assessed. Despite some reservations, only the simplest criterion, namely unconditional output-variance minimization, will be applied here. From equation (1) and using equation (8), the prospective solution for $P_t$, output can be written

$$(13) \quad y_t = a_1(P_t - E(P_t/\Omega_{t-1})) + a_2y_{t-1} + u_t$$

$$= a_1(\Pi_2u_t + \Pi_3v_t + \Pi_4\eta_t + \Pi_6\epsilon_t) + a_2y_{t-1} + u_t,$$

and its unconditional variance is therefore

$$(14) \quad \sigma_y^2 = (1 - a_2)^{-1} \left( (1 + a_1^2)\sigma_u^2 + (a_1\Pi_2)^2 \sigma_v^2 + (a_1\Pi_3)^2 \sigma_\eta^2 + (a_1\Pi_4)^2 \sigma_\epsilon^2 \right)$$

where cross-correlations between different shocks are ignored. This is to be minimized by choice of an appropriate policy rule.

Alternative suggestions for a policy objective have included the minimizing of variations in real consumption expenditures, as
dictated by microeconomic considerations. That proposal becomes
impractical in this model since consumption has not been explicitly
defined. Others have suggested the maximization of an intertemporal
aggregate welfare function; Buiter (1980). Insofar as the argument
in the utility function is restricted to output, it would turn out
that this criterion would also on general reduce to some sort of
output-variability minimization. The inclusion of other variables
in the utility function, on the other hand, has to be motivated more
carefully. It is not clear, for example, whether nominal variables,
such as the price level or inflation rates, should enter the utility
function as arguments in their own right or simply as proxies to
reflect the misallocations in the real sector that may occur on
account of excessive variability of these terms. Lastly, an argument
can be made that the optimal economic behavior is that which would
occur if all agents made their decisions under perfect contemporane-
ous information - the perfect markets solution. Therefore, the
objective of government policy should be to minimize the deviations
of actual output from this benchmark behavior. This argument ignores
the possible existence of public goods and other externalities. For
present purposes, however, it is enough to note that this
criterion would also lead to the same recommendations in this model
as the output-variance minimizing criterion, with the one exception
of adjustments to the real supply shock, $u_t^3$.3
II. Optimal Stabilization Policy with Complete Current Information

Consider the hypothetical case where the central authorities can observe and react to all current realizations of the random shocks. This would in principle allow the formulation of a policy feedback rule directly as a function of these shocks. The problem which has not yet been addressed concerns the target variable that is to be controlled by the feedback rule. The two alternative candidates which are most often proposed in the literature are the exchange rate and the money supply. Even though any other currently observed variable, such as $P_t$ in this model, could potentially serve as a candidate, the focus here will remain on those two alternatives. The issue of whether it is operationally easier to target prices or quantities will also be side-stepped here by assuming that both could be equally well controlled by the authorities. It can then be shown that it is irrelevant whether the exchange rate or the money supply are targeted optimally.

Begin with an exchange rate rule. In this case equation (9) might be proposed as a possible feedback rule, where the $\lambda$'s are parameters to be set by the government. This would mean that only equations (6) and (7) are needed to solve for the remaining undetermined coefficients, the $\pi$'s and $\psi$'s, as functions of the structural parameters and the policy parameters. Applying the method outlined in the previous section, the following values are obtained for the coefficients of the price and money supply equations:
where $A = (b_2 + a_1(a_1 + c_3))/(a_1 + b_2)$ and $B = (c_0 + c_1 a_1)/(a_1 + b_2)$.

The optimal exchange rate rule can now be derived by substituting the expressions for the $\pi$'s in (15) into the objective function (14), and minimizing it with respect to the control parameters, the $\lambda$'s. Notice that an absolute minimum is achieved when $\pi_2 = -a_1^{-1}$ and $\pi_3 = \pi_4 = \pi_6 = 0$. In this case all output variance is eliminated.

The appropriate policy setting to bring this about would be (from (15)) $\lambda_2 = a_1^{-1}$, $\lambda_3 = -b_2^{-1}$, $\lambda_4 = 0$ and $\lambda_6 = -1$. In words, the exchange rate should be devalued in response to a positive supply shock, revalued in response to a positive aggregate demand shock and to a positive shock to the foreign price level, but should not respond to random changes in money demand.}

Suppose that instead of an exchange rate rule, the central authorities contemplate a money supply rule. This would mean that the $\psi$'s in equation (10) would be considered policy parameters, while the $\lambda$'s and $\pi$'s become endogenously determined. It is obvious, however, that the variance of output will be minimized as before. 

\[ \Pi_2 = -a_1^{-1} \text{ and } \Pi_3 = \Pi_4 = \Pi_6 = 0. \]

Introducing this restriction into the linear system of equations (15) would therefore yield the same optimal solution for the relevant \( \lambda \)'s and \( \psi \)'s as with an exchange rate rule. An interesting aspect of this result is that if the monetary authorities were optimally targeting the exchange rate, the resulting behavior of the money supply should be the same as if they had been optimally targeting the money supply in the first place. It bears repeating, however, that this applies only to the extent that prices and quantities are equally controllable.

One observation which will be useful for later reference concerns the elements which have been included in the policy feedback rule. In principle, any number of lagged variables could have been added to the policy feedback rules, which would imply that all the proposed solutions, equations (8)-(10), would contain terms involving these lagged variables. However, as the preceding solution has shown, and as can be observed directly from equation (1), the behavior of real output remains unaffected by policy responses to lagged variables, even though this would clearly have an impact on the behavior of nominal variables. To the extent that only real variables are of concern to us, more elaborate dynamic feedback rules involving past variables can therefore be ignored.

III. Optimal Stabilization Policy with Restricted Current Information

In practice, "shocks" are rarely observed directly, only their effects, namely price changes or quantity changes. In fact, it is not at all clear whether shocks can even be meaningfully defined.
independently of their effects. At any rate, the policymakers are now assumed to receive current information on a limited set of variables. In the case of our model, let this set consist of $P_t$, $e_t$, $M_t$, $P^*_t$. The task facing the policymakers is then to find an optimal policy rule by which either $e_t$ or $M_t$ are indexed to the remaining observable variables. Since the values of the current untargeted variables depend on current realizations of the underlying shocks, this is simply an indirect means of responding to the shocks. In the fortuitous case where there are at least as many linearly independent observable variables as there are shocks in the system, it would be possible to extrapolate the exact values of all the current shocks and this would place us in the situation dealt with in the preceding section. This does not apply, however, for this particular model.

Let us then consider the following exchange rate rule:

\[
(16) \quad e_t = g_0 + g_1 M_t + g_2 P_t + g_3 P^*_t
\]

where the $g_i$'s are parameters to be chosen by the policymakers. In principle, the exchange rate rule could be expanded to include also lagged values of the variables. However, as was argued before, this would not have an impact on the behavior of $y_t$. Equation (16) along with (6) and (7) can now be used to solve for the undetermined coefficients in (8)-(10) as functions of the structural parameters and the $g_i$'s. Since from expression (14) the variance of $y_t$ only depends on the values of the $\Pi$'s, attention is focused on these coefficients. Their solution values turn out to be:
where \( A = a_1 + b_2(1 - g_2) - g_1(b_2 + a_1(c_3 + c_1b_2)). \)

By inserting these values of the \( \Pi \)'s into the expression for the variance of output, we could then minimize (14) with respect to the control parameters \( g_1, g_2 \) and \( g_3 \). From (17), we can observe directly, however, that there does not exist a policy setting which will eliminate all variation in \( y_t \), as in the preceding full information example. The only unambiguous recommendation which can be made is to set \( g_3 = -1 \). This will equate \( \Pi_6 \) to 0. In words, the exchange rate should be appreciated in the same proportion as the foreign price level. This will insulate the terms of trade from foreign price shocks and thus eliminate this source of variability from the domestic economy. With regard to the other two policy parameters, \( g_1 \) and \( g_2 \), all that can be said is that their optimal (i.e., output-variance minimizing) values will depend on the relative sizes of the disturbances and on the sizes of the structural parameters.

As an illustration, if the demand for money provides the primary source of variability, then by setting \( g_1 = 0 \), this leads to \( \Pi_4 = 0 \),
so that this shock is no longer transmitted to the price level and hence to output. With this policy setting, variations in money demand are entirely absorbed by variations in the money supply. Alternatively, if aggregate demand were to become the primary source of variability, this could be eliminated by setting $g_1 = c_3^{-1}$. However, there do not exist values for $g_1$ and $g_2$ that will simultaneously set $\Pi_3 = \Pi_4 = 0$ and $\Pi_2 = -a_1^{-1}$.

Now consider the case of an optimal money supply rule. Since the set of currently observable variables remains the same, the exchange rate rule (16) can be rewritten as

\[
M_t = -(g_0/g_1) + (1/g_1)e_t - (g_2/g_1)P_t - (g_3/g_1)P_t^*,
\]

and thereby converted into a money supply rule. It could be easily shown then that the optimal values of the monetary rule parameters are the same as would be obtained by substituting into (18) the optimal values for the exchange rate rule parameters. This result mirrors the conclusion of the preceding section, namely that, apart from the controllability issue mentioned earlier, the optimal monetary rule and the optimal exchange rate rule both lead to the same macroeconomic behavior.

An interesting possibility which has been raised by Roper and Turnovsky (1980) and which also appears here is that it is conceivable that the optimal intervention policy is one of "leaning with
the wind" rather than against. "Leaning against the wind" is generally understood as a policy of contracting the money supply (selling foreign reserves) in response to a depreciating exchange rate to offset further depreciation, and vice-versa. This would imply a negative value for the term \((1/g_1)\) in equation (18). But now consider again the previous example where only aggregate demand shocks matter. It was found for that case that the optimal setting for \(g_1\) would be \(c_3^{-1}\). The optimal monetary rule would then turn out to be \(M_t = c_3 a_t\), where \(0 < c_3 < 1\). It could similarly be shown that a positive response is also optimal in the case where only aggregate supply shocks matter. This result has certain potentially disturbing implications. From the coefficient solutions in (17) observe that for certain values of \(g_1\) and \(g_2\) the term A vanishes, which means that the price level would exhibit explosive behavior. This problem can arise when either \(g_1\) or \(g_2\) assume positive values. Given that in practice it is unlikely that policymakers can set intervention parameters with the precision implied in theory, attempts to implement the optimal stabilization policy may instead lead to a magnification of economic fluctuations. This danger may be illustrated by considering once more the variance of output when only aggregate demand disturbances matter. This becomes

\[ \sigma_Y^2 = \frac{a_1^2 \sigma_N^2}{(1 - a_2^2)} \pi_3^2. \]

Assuming for simplicity that \(g_2 = 0\), \(\pi_3\) can be represented as a function of \(g_1\) as illustrated in Figure 1. A fixed exchange rate rule occurs when \(g_1 = 0\). Notice then that as \(g_1\) is increased toward its optimal setting, \(c_3^{-1}\), the absolute value
\[ \Pi_3 = \frac{(1 - c_3 g_1)}{(a_1 + b_2) - g_1 (b_2 + a_1 (c_3 + c_4 b_2))} \]
of Π_3, and therefore c^2_y, will first increase before declining again. Moving closer to the optimal setting is therefore no guarantee for better performance.

IV. A Comparison of Several Intervention Proposals

This section examines a small sample of intervention rules which have been proposed in the literature in the context of the equilibrium model developed before. Following the distinction made earlier, the discussion focuses first on some exchange rate rules before considering a monetary rule.

Although the Bretton Woods system did allow parity changes in the event of "fundamental disequilibria", the definition of such a state was never spelled out explicitly. Williamson (1965) was one of the first to suggest a systematic rule to govern exchange rate changes which became known as the crawling peg. Since then there have appeared numerous additional proposals, many of which are conveniently described in Williamson (1981). A useful distinction here is between what McKinnon (1981) terms "active" and "passive" exchange rate rules. In the language of optimal control this would be referred to as open loop versus closed loop solutions or, more casually, rules without and rules with feedback.
Active Rules - Under an active rule, the government proposes to effect certain exchange rate changes in the foreseeable future which are not contingent on future economic developments, e.g. such as depreciating the exchange rate by 2 percent every month. We could express this rule as: \( e_t = e_{t-1} + .02 \). But more elaborate formulas could also be devised such as
\[
e_t = k + \sum_{i=0}^{n} g_i e_{t-i}.
\]
The question to focus on here is then to what extent the behavior of real output is a function of the government parameters, \( g_i \). Recall from the previous analysis that the only channel by which policymakers can systematically influence real output is by reacting to new information not yet available to private agents or not available at the time that contracts had been formed. The active feedback rule described here, however, does not involve any information which is not known to private agents, and hence the policy-ineffectiveness result, that changes in the \( g_i \)'s will leave real output unchanged, applies. The same can be said also for active monetary rules of the form:
\[
M_t = k + \sum_{i=0}^{n} g_i M_{t-1}.
\]

On the other hand, the model may not be sufficiently rich to accommodate the primary argument for which active rules had been proposed. McKinnon (1981) regards active exchange rate rules as an indirect way of targeting the inflation rate. It is suggested that a preannounced pattern of devaluations would help coalesce price expectations around a particular rate of inflation and thus lead to greater market efficiency. One way to introduce this argu-
ment into the earlier model might be to assume that private agents' expectations contain also an unsystematic term, such as \( E(e_{t+1}/\hat{\alpha}_t) + \xi_t \), where \( \xi_t \) is an independent source of variability whose variance depends on the costs of forming accurate expectations. If the government announces (credibly) what \( e_{t+1} \) will be, rather than letting it be market-determined, this reduces private prediction costs and thus \( \sigma^2_\xi \). If this is the only argument, however, there does not appear any advantage to choosing one predetermined path of devaluation, including a fixed rate, over another, unless the behavior of nominal variables constitute independent policy objectives.\(^9\)

**Passive Rules** - Williamson (1981) suggests that in most countries, where some form of "crawling peg" was implemented, the practical motivation was essentially to neutralize inflation differentials between such countries and their trading partners or competitors. This could be modelled by considering a rule which keeps the Terms of Trade constant,\(^10\)

\[
(19) \quad e_t = c + P_t - P^*_t
\]

This can be seen as a special case of the exchange rate equation (16), where \( g_2 = -g_3 = 1 \) and \( g_1 = 0 \). Feeding these values into the expressions for the price coefficients in (17) yields the second row of values in Table 1. It is obvious that this rule could never be superior to the optimal rule developed in Section III. However,
it may be of interest to observe that this rule completely insulates real output from supply shocks and from foreign price level shocks. In that respect a PPP rule is an improvement over a constant exchange rate rule or a constant money rule. The other side of the coin, however, is that the economy would show a highly unstable response to all other shocks. This was also recognized by McKinnon (1981) and can be seen in Table 1 by the explosive values of \( \pi _1 \). The reason is easy to explain: Notice that the aggregate demand equation (2) can be written as

\[
y_t = b_0 + b_1 E(P^*_{t+1} - \frac{P^*}{\Omega_{t-1}}) + b_2 (e_t + P^* - P_t) + v_t
\]

\[
= b_0 + b_1 k + b_2 (e_t + P^* - P_t) + v_t
\]

after substituting (4) and (5). Consider now the effect of a once-for-all positive aggregate demand shock. Since the monetary authorities are keeping the terms of trade fixed, aggregate demand increases by the full amount of the shock. This requires an equal increase in aggregate supply (brought about by an unexpected price increase) to clear the goods market. In the next period, aggregate supply would be higher than before, on account of the persistence term, and in the absence of any further shocks to the system. Normally, this should lead to a fall in the terms of trade (as prices fall) in order to redirect demand toward domestic goods. However, the PPP rule by the monetary authorities prevents the terms of trade from serving their market clearing role. Instead every incipient fall in prices is countered by a revaluation with
the consequence that further price falls are required.

A variant of the PPP rule is the suggestion by the German Council of Economic Experts in the mid 60's that the rate of depreciation be equated to the difference between a target national inflation rate and the actual foreign inflation rate. This rule can be written as

\[ e_t - e_{t-1} = T - (p^*_t - p^*_{t-1}) \]

or

\[ (20) \quad e_t = e_{t-1} + T - k - \varepsilon_t, \]

where \( T \) is a (constant) target inflation rate, \( k \) is the systematic component of the foreign inflation rate and \( \varepsilon_t \) is the unsystematic component. This rule would be another special case of the optimal rule in Section III, equation (16) where \( g_3 = -1 \) and \( g_1 = g_2 = 0 \). The values of the \( \Pi \)'s under this rule are given in the third row of Table 1. For the greater part, this rule yields the same operating characteristics in this model as a constant exchange rate system, except that it insulates the real sector from unsystematic foreign price shocks.

It may be interesting to observe further that the primary concern of the German Council of Experts at the time this rule was proposed was with maintaining price stability in the face of inflationary pressures originating abroad rather than with reducing output fluctuations. By that criterion, however, note from the values of the reduced form coefficients in table 1 (rows 3 and 4) that a free float would have been preferrable to the crawling peg. That is
because the crawling peg rule (20) simply postpones the impact of foreign price changes on domestic prices (from the fact that $\Pi_b = 0$, but $\Pi_7 = 1$), whereas the free float insulates the domestic price level from foreign price changes altogether (since $\Pi_6 = \Pi_7 = 0$).

Another suggested formula for parity changes, attributed to James Meade and Thomas Willetby Williamson (1981), includes tying the rate of exchange depreciation to the rate at which foreign reserves are being lost or gained. If for simplicity it is assumed that the high-powered money multiplier remains constant, this proposal amounts to an indexation of exchange rate changes to money supply changes

\[ e_t - e_{t-1} = \alpha(M_t - M_{t-1}) \quad \text{or} \]

(21) \[ e_t = \alpha M_t \]

In terms of the optimal rule in Section III this reduces to the proposal of setting $g_1 = \alpha$, to be chosen optimally, and $g_2 = g_3 = 0$. As we found earlier, however, this rule could be improved on at least by setting $g_3 = -1$ and thereby eliminating the destabilizing effects of foreign price disturbances; that is, under the assumption that the information flow on $P_t^*$ is rapid enough to be observable on a current basis.

Frenkel and Aizenman (1982) define an intervention index in the context of a disequilibrium monetary model as the ratio of actual exchange rate changes over the equilibrium exchange rate
change in the absence of exchange rate targeting by the monetary authorities. They propose setting this index at an optimal level. However, it is not clear whether this should be considered an exchange rate rule or a monetary rule, or whether it is simply an ex post measure of the relative variability of the exchange rate and the money supply, written as

\[ y = \frac{e_t - e_{t-1}}{M_t - M_{t-1}}. \]

This rule could be interpreted either as an exchange rate rule

\[ e_t = e_{t-1} + y(M_t - M_{t-1}). \]

or as a monetary rule

\[ M_t = M_{t-1} + \left(1/y\right)(e_t - e_{t-1}). \]

As was argued in Section III, it should not matter as far as the optimal choice of \( y \) is concerned which way the feedback rule is written, provided money and exchange rates could be equally well targeted. That being the case then, this proposal also reduces to the special case of the optimal rule in Section III where \( g_1 = y \) and \( g_2 = g_3 = 0. \)

In 1976, the OPTICA Report to the European Commission proposed a monetary intervention rule requiring the monetary authorities to buy foreign exchange whenever domestic inflation exceeds foreign inflation lest the exchange rate depreciate by more than the difference in inflation rates. The original proposal did not consider a symmetric response in the event of appreciations. However, De Grauwe, Steinherr and Basevi (1980) consider an expanded version of this rule which directs the monetary authorities to buy foreign exchange (and thus expand the domestic money supply) whenever the terms of trade worsen and to sell foreign exchange if they improve. This rule may be expressed as
(22) \[ M_t = M_{t-1} - g(c + P_t - e_t - P^*_t), \]

where \( g \) is a policy parameter. Note that if \( g \) is set to zero, (22) reduces to a constant money rule, while letting \( g \) approach infinity reduces (22) to the constant PPP.rule examined earlier. Rewriting (22) as

(22)' \[ e_t = c + P_t - P^*_t + \left(1 \over g\right) M_t - \left(1 \over g\right)(M_{t-1}). \]

we observe that this rule can be viewed as a special case of the optimal rule in Section III, where \( g_1 = (1/g) \) is chosen optimally and \( g_3 = -1 \), also the optimal setting, but where \( g_2 \) is arbitrarily restricted to be equal to 1. Furthermore, the reduced form solutions for the endogenous variables (8)-(10) will also involve the term \( M_{t-1} \), but that should have no bearing on the behavior of real output.

One point to emerge from this brief review is that, in contrast to the unrestricted optimal rule outlined in section 3, each of these policy proposals involves extra restrictions placed on the government policy parameters \( (g_1, g_2, g_3) \). It therefore follows, as a logical necessity, that none of the suggested rules could within this framework lead to a superior economic performance than the unrestricted optimal rule. At best it might be the case that the configuration of disturbance term variances happens to be such that the optimal policy setting happens to coincide with the proposed policy parameter restrictions. This result would have obtained also
Table 1: The Price Equation Coefficients under Alternative Intervention Rules:

\[ P_t = \Pi_0 + \Pi_1 Y_{t-1} + \Pi_2 u_t + \Pi_3 v_t + \Pi_4 q_t + \Pi_5 P^*_{t-1} + \Pi_6 \epsilon_t + \Pi_7 \epsilon_{t-1} \]

<table>
<thead>
<tr>
<th>Rule</th>
<th>( \Pi_1 )</th>
<th>( \Pi_2 )</th>
<th>( \Pi_3 )</th>
<th>( \Pi_4 )</th>
<th>( \Pi_5 )</th>
<th>( \Pi_6 )</th>
<th>( \Pi_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( e_t = \bar{e} )</td>
<td>(-\frac{a_2}{b_2 + b_1(1-a_2)})</td>
<td>(-\frac{1}{a_1 + b_2})</td>
<td>(\frac{1}{(a_1 + b_2)})</td>
<td>0</td>
<td>*</td>
<td>1</td>
<td>(\frac{b_2}{a_1 + b_2})</td>
</tr>
<tr>
<td>2. ( e_t = c + P_t - P^* )</td>
<td>(-\frac{1}{0})</td>
<td>(-\frac{1}{a_1})</td>
<td>*</td>
<td>0</td>
<td>*</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3. ( e_t = e_{t-1} + T - k - \epsilon_t )</td>
<td>(-\frac{a_2}{b_2 + b_1(1-a_2)})</td>
<td>(-\frac{1}{a_1 + b_2})</td>
<td>(\frac{1}{(a_1 + b_2)})</td>
<td>0</td>
<td>*</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4. ( M_t = \bar{M} )</td>
<td>(-\frac{a_2(c_1 + (c_2(1-a_2) + c_3)}{b_2 + b_1(1-a_2)})</td>
<td>(-\frac{1}{a_1 + b_2/(c_3 + b_2c_1)})</td>
<td>(\frac{1}{(a_1 + b_2) + b_2(c_0 + c_1a_1)/c_3})</td>
<td>(-\frac{1}{1 + a_1c_1 + a_1c_3/b_2})</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The stars signify that, of the four alternatives, the indicated intervention rule minimizes the influence of the relevant disturbance on the behavior of real output.
in a more general setting where more than output-variance enters into the objective function. However, this criticism must be qualified by recognizing the possibility that not all of these rules had been proposed with the same assumptions in mind concerning the availability of current information. For example, it is arguable that Meade, Willett, and Frenkel and Aizenman, whose rule proposals are captured by equation (21), did not consider price information to be currently available. In that case it would be infeasible to index exchange rates or reserves to current prices, and (21) turns out to be the same as the optimal rule. This same extenuating argument is, on the other hand, not applicable to the remaining proposals, such as the PPP rule, since they are implicitly premised on the availability to policymakers of current price information.

While the preceding discussion has emphasized the limitations of previous policy proposals from the viewpoint of an unrestricted optimal rule, this should not necessarily be construed as an enthusiastic endorsement of the optimal rule. Instead, it might be advisable to conclude on a less sanguine note by again pointing out a possible danger in attempting to implement optimal policies. In order that policy rules be successful in stabilizing economic fluctuations it is necessary not only that government policymakers have better information than private agents, as was assumed in this paper, but also that this information be sufficiently accurate. In the hypothetical case treated in section 2, where policymakers had perfect current information, the possibility of destabilizing poli-
cies did not arise. But in the analysis of section 3 involving limited current information, it was shown that merely setting the policy parameters in the neighborhood of their optimal values might actually magnify rather than stabilize economic fluctuations.

It is not for lack of trying that reports on international financial developments are often couched in such nebulous phrases as "leaning against the wind" or "dirty floating". Another example is provided by the former German Bundesbank president's statement that the problem of defining "disorderly conditions" in the exchange market "is similar to that of the definition of a pretty girl: difficult to define, but one recognizes her when one meets her". (Emminger (1982), p. 15). Though laudable for their honesty, statements such as these do not inspire confidence in the precision with which central banks operate or perhaps can operate. Upon recognizing these limitations, it may well be that as a practical matter the best course of action remains one of either a constant exchange rate rule or a constant money rule, both of which involve relatively undemanding operating procedures.

V. Summary

This paper examines the question of optimal foreign exchange market intervention in the context of a common macro-model of an open economy with rational expectations. It is assumed that government policymakers receive better current information, or have a
shorter reaction time to new information, than private agents. An optimal intervention rule is derived, using the criterion of output-variance minimization, for the case where policymakers have perfect contemporaneous information and for the case where only limited current information is available. This optimal rule was then shown to perform at least as well, but generally better, than a sample of other rules which had been variously proposed in the literature. Less encouraging for proponents of activist intervention, however, is the finding that the successful implementation of the optimal rule may under some circumstances require more accurate information that is generally available to central banks. When this requirement is not met, attempts to implement the optimal rule may instead destabilize the economy.
If, as explained in the text, private agents also had current information available when making all decisions except for the supply decision, which remains a function of contracts formed last period, then equations (6) and (7) would respectively become:

\[(6)\quad (a_1 + b_2 + b_1)P_t - a_1E(P_t/\Omega_{t-1}) + b_1E(P_{t+1}/\Omega_t) = (b_2 + b_1)e_t - b_1E(e_{t+1}/\Omega_t) - a_2Y_t + v_t - u_t - b_1i^* + b_2P^* + b_2\varepsilon_t\]

and

\[(7)\quad M_t = (c_0 + c_1a_1)P_t - c_1a_1E(P_t/\Omega_{t-1}) + (c_2 + c_3)e_t - c_2E(e_t/\Omega_t) + c_1Y_{t-1} + c_1u_t - c_2i^* + c_3P^*_{t-1} + c_3\varepsilon_t + \eta_t\]

With current information available, these expectations would be expressed as:

\[(11)\quad E(P_{t+1}/\Omega_t) = \Pi_0 + \Pi_1E(Y_t/\Omega_t) + \Pi_5E(P^*/\Omega_t) = \Pi_0 + \Pi_1a_1(\Pi_2u_t + \Pi_3v_t + \Pi_4\eta_t + \Pi_6\varepsilon_t) + a_2\Pi_1Y_{t-1} + \Pi_5(P^*_{t-1} + \varepsilon_t)\]

\[(12)\quad E(e_{t+1}/\Omega_t) = \lambda_0 + a_1\lambda_1(\Pi_2u_t + \Pi_3v_t + \Pi_4\eta_t + \Pi_6\varepsilon_t) + a_2\lambda_1Y_{t-1} + \lambda_5(P^*_{t-1} + \varepsilon_t)\]

In the absence of contractual rigidities and with perfect contemporaneous information, the behavior of output would be given by \(Y_t = a_2Y_{t-1} + u_t\). The squared deviations of actual output from this level would be

\[\sigma^2_y = a_1^2(\Pi_2\sigma^2_u + \Pi_3\sigma^2_v + \Pi_4\sigma^2_n + \Pi_6\sigma^2_\varepsilon).\]
The corresponding values of the $\pi$'s for the current information case would be:

\[
\begin{align*}
\pi_2 &= \frac{(b_2 + b_1) \lambda_2 - 1}{A} \\
\pi_3 &= \frac{(b_2 + b_1) \lambda_3 + 1}{A} \\
\pi_4 &= \frac{(b_2 + b_1) \lambda_4}{A} \\
\pi_6 &= \frac{(b_2 + b_1)(\lambda_6 + 1)}{A}
\end{align*}
\]

where

\[
A = a_1 + b_2 + b_1(1 + a_1 a_2/(b_2 + b_1(1 - a_2)))
\]

The values of $\pi_0$, $\pi_4$, and $\pi_5$ are the same as for the lagged information case. Note that the optimal policy setting in this case involves the same qualitative responses as with the lagged information case treated in the text.

To consider exchange rates as variables that are observed on a current basis would probably not meet with many objections, nor would the inclusion of $M_t$ as long as this is taken to refer to high-powered money or foreign reserves. On the other hand, to also include $P_t$ and $P^*_t$ in this set is bound to raise considerably more eyebrows, particularly when these are taken to be aggregate price indices. Whether $P_t$ and $P^*_t$ belong with the currently observed variables is partly a matter of defining the length of a market period in the model. Most important for the logical consistency of this analysis, however, is that data on price indices be available over shorter intervals than the data on aggregate quantities such as $y_t$.

The reason for not also considering $i_t$ in this set is that by equation (5) the interest rate does not reflect any additional information in this model that is not already contained in $e_t$, and therefore would not be useful in constructing a policy response function.

The optimal exchange rate rule derived in the preceding section could be written as

\[
e^*_t = \lambda^*(u_{t'}, v_{t'}, \eta_{t'}, \epsilon_{t'})
\]

where $\lambda^*$ is a row vector of optimal settings $(\lambda^*_2, \lambda^*_3, \lambda^*_4, \lambda^*_6)$. Suppose there exist four linearly independent variables, denoted by the column vector $x_{t'}$, which are functions of the four random variables. Then in reduced form we can write

\[
\tilde{x}_{t'} = A(u_{t'}, v_{t'}, \eta_{t'}, \epsilon_{t'})
\]

where $A$ is a $4 \times 4$ nonsingular matrix. The optimal feedback response function can finally be written $e_{t} = \lambda^* A^{-1} \tilde{x}_{t'}$. 
The solution values for the \( n \)'s corresponding to the same exchange rate rule but using the current information model, equations (6)' and (7)' in footnote 1, are:

\[
\begin{align*}
\hat{n}_2 &= (g_1((b_2 + b_1)c_1 + c_2 + c_3) - 1)A^{-1} \\
\hat{n}_3 &= (1 - g_1(c_2 + c_3))A^{-1} \\
\hat{n}_4 &= (b_2 + b_1)g_1A^{-1} \\
\hat{n}_6 &= ((b_2 + b_1)(1 + g_3) + g_1(b_1c_3 - b_2c_2))A^{-1}
\end{align*}
\]

where \( A = (1 - g_1(c_2 + c_3)(a_1 + b_2 + b_1(1 + a_1a_2/(b_2 + b_1(1 - a_2)))) \\
- (b_2 + b_1)(g_2 + g_1(c_0 + a_1(c_1 - c_2a_1)))), \)

Only negligible differences distinguish the values of \( \hat{n}_0, \hat{n}_1 \) and \( \hat{n}_5 \) from those obtained in the lagged information model in (17). But since the variance of \( y \) is not a function of these terms, their values will not be reproduced here.

This policy recommendation will not turn out quite that simple in the current information case. However, from the values in footnote (7), the optimal setting for \( g_3 \) can be seen to be \(-1-g_1(b_1c_3 - b_2c_2)/(b_2 + b_1)\).

There may also be other criteria, such as the operational ease with which different rules can be implemented, by which certain active rules would be preferred to others. But since the present framework of analysis is not equipped to handle these, they are not discussed further.

See also Genberg (1981).

This is discussed in both Williamson (1981, Ch. I) and in Giersch (1973).
References


