

Lächler, Ulrich

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## On optimal factor proportions in a competitive firm under factor and output price uncertainty

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# Kieler Arbeitspapiere

## Kiel Working Papers

Working Paper No. 184

On Optimal Factor Proportions in a Competitive Firm under Factor and Output Price Uncertainty \*

by  
Ulrich Lächler

Institut für Weltwirtschaft an der Universität Kiel

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Kiel Institute of World Economics  
Department IV  
Düsternbrooker Weg 120, 2300 Kiel 1

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## 1. Introduction\*

Most observers of economic events have noticed a considerable increase in the general volatility of prices over the last decade. An important byproduct often attributed to this increased price variability is greater uncertainty perceived by individual decisionmakers in the process of formulating intertemporal plans. This paper seeks to clarify and provide some extensions to previous theoretical work on the question of how economic agents adjust to increased price uncertainty in the context of the competitive firm. In particular, the question asked is whether the optimal choice of inputs in a competitive firm is affected by the advent of increased factor and output price uncertainty. The answer given in earlier studies such as those by Baron (1970), Batra and Ullah (1974), Leland (1972) and Sandmo (1971) is quite straightforward: If competitive-firm managers are risk-neutral profit maximizers, the optimal input mix remains unaffected by increased uncertainty, while under risk-averse managers, firms either reduce their scale of operations or adjust their input mix towards relatively greater use of less risky inputs. A similar result is advanced in Holthausen (1976), who treats this issue in the context of demand uncertainty and finds that under risk neutrality only the behavior of price-setting imperfect competitors is affected. The analysis by Stewart (1978), written explicitly for the case of factor-price uncertainty, adds further support to these conclusions.

Before discussing other subsequent treatments of this issue, it might be useful to point out what led these authors to their conclusion. In all cases mentioned so far, a rather crucial limitation is placed upon the intertemporal choice flexibility available to the firm, namely, it is assumed that all input choices are made under the same amount of uncertainty. This may be illustrated by briefly examining the typical analytic framework employed in these studies: A competitive firm maximizes expected profits subject to a production function relating output to two factor inputs, say capital and labor. The fixed input, capital, takes time to install and therefore must be chosen ex ante, before the market price of output or of the other input is known. Ex post, after the random price variables take on actual values, the firm is left free to choose an optimal level of the variable input, labor. However, additional constraints are then generally imposed which, in a two-factor setting, do not allow for any ex post freedom of choice. Either the firm is constrained to produce at a predetermined output level or to meet all demand at the market price. In both cases, the decision on how much of the variable factor to use was effectively made prior to the realization of the random elements. It is easy to see, then, that if output and both inputs are chosen ex ante, the ex post profit is a linear function in both factor and output prices. Its expected value is therefore invariant to changes in the probability distributions of these prices, provided their means are held constant. Within this framework, the only channel

left by which changes in uncertainty could have non-neutral effects is through the possibly risk-averse behavior of a firm's managers. However, the introduction of risk-averse behavior into the theory of the firm raises a number of well-known problems, and thus has met with only limited acceptance among economists. The main problem being that risk-averse behavior on the part of firm managers cannot be justified on the same grounds as risk-averse behavior among individual households. With well-functioning markets, the most efficient allocation of resources would result with firms maximizing expected profits while asset-owners, which includes the managers, hold appropriately diversified portfolios depending on their risk preferences. It would therefore be in the interests of stockholders to choose risk-neutral behaving managers.

Subsequent work on this question has recognized that in general firms do have more room to maneuver in the face of increased uncertainty than was implied by the preceding example. This is done by relaxing the assumption that all effective decisions are made under the same uncertainty, but that some can be postponed until after the uncertainty is resolved. It is intuitively apparent that with the ability to postpone decisions a firm can increase its ability to respond to realized variations in prices. That this could benefit the firm was already recognized by Hart (1942), who concluded that when production decisions can be staggered so that some input choices can be deferred

until at least part of the uncertainty has been resolved, the firm's expected value of profits could be raised above the value that would result if all decisions had to be made before the realization of the random elements. However, the acquisition of this response flexibility will also in general involve some costs. Fuss and Mc Fadden (1978) have drawn attention to the existence of a tradeoff between the degree of production technique flexibility chosen by the firm and the static, or on-average, efficiency of a production process. See also Stigler (1939), for a ground-breaking contribution in this respect. What these results imply is that changes in uncertainty associated with increased price variability can have an effect on the optimal structure of the firm for reasons independent of the managers' attitudes toward risk. Hartman (1976) also points this out in his analysis of factor demands by competitive firms, using the typical model described earlier, but relaxing the assumption that output be held fixed exogenously. In order to obtain a determinate solution in this case, however, it became necessary to assume decreasing returns to scale in production.<sup>1</sup> This raises a different problem. What has to be explained under this assumption is why firms would not continue to split themselves up into smaller equal-sized units as this would bring about greater output at no increase in costs. In Hartman (1972) this problem is avoided by introducing the nowadays more accepted notion of costs-of-adjustment in investment to make the size of the firm determinate. But since this analysis takes place in the context of a dynamic

model of the firm, the results obtained there are not readily comparable to the others. (This model is discussed in greater detail below). Finally, Perrakis (1980) reexamines the problem posed in Stewart (1978) but arrives at different conclusions, namely that the optimal capital intensity choice will not remain invariant to changes in uncertainty even under risk-neutrality. In terms of the typical framework earlier, this result is achieved by introducing more than two factors of production into the analysis, several of which are variable in the short-run. Unfortunately, on account of a minor error, Perrakis erroneously concludes that, "Under risk neutrality,... [ $\bar{}$ uncertainty in the prices of some inputs $\bar{}$ ] ... results in most cases of practical interest, in a heavier relative use of the riskless input as compared to the certainty case with input prices set equal to their expectations." (p. 1086). In fact, the opposite occurs, as will be shown in the next section.

In section 2, the analysis first presented in Perrakis (1980) is reconsidered and corrected. This model is then extended in section 3, by relaxing the assumption that the output produced by the firm be a predetermined constant. This analysis builds on the model presented in Hartman (1972). The results obtained here are similar to those derived in Hartman (1976), to which a more detailed comparison follows. Overall, the conclusions obtained in both sections come out guardedly in favor of the proposition that the advent of increased price uncertainty



induces the risk-neutral competitive firm to reduce the capital intensity of its input mix. It still remains true in both cases that the separate introduction of risk-averse behavior would tend to operate in the opposite direction, as earlier authors had found, so that the net effect under this assumption would be indeterminate. However, in the subsequent analysis, the effects of risk-aversion will be ignored, both in order to isolate the effects of uncertainty which operate through a pure profit motive, and for dissatisfactions on theoretical grounds, attached to the risk-aversion assumption, which were touched upon earlier. The final section summarizes the conclusions and discusses some of the broader implications.

## 2. The Model of a Risk-Neutral Competitive Firm Producing a Fixed Output

The production function of a neoclassical firm is characterized by constant returns to scale in capital  $K$  and  $n$  variable factors,  $\{x_1, \dots, x_n\}$ . Output is sold in a competitive market at price  $p$ . The firm is also faced with a competitive market for factor inputs, whose prices are denoted  $w_K$  (for one unit of capital), and  $w_1, \dots, w_n$ , for the variable factors. The firm regards  $p$  and  $\{w_1, \dots, w_n\}$  as independently distributed random variables with known means and variances.

The firm must make the following decision: Because it takes

time to install capital, the level of  $K$  must be chosen before the values of the random variables are realized, while  $x_1, \dots, x_n$  are then chosen after their realization. Conceptually this can be restated in reverse order. For any given stock,  $\bar{K}$ , the firm chooses a variable factor mix which maximizes ex post profits, depending upon the realized factor prices. This results in a set of conditional demand functions,  $\{x_1^*, \dots, x_n^*\}$ . The firm then chooses an optimal  $K$  which maximizes the expected value of the conditional, ex ante profit function. Now, it turns out that under constant returns to scale the ex ante profit function is linear in  $K$ , so that the optimal level of  $K$  is indeterminate. To overcome this problem it will be assumed that the level of output to be produced is exogenously predetermined. This problem is reconsidered in section 3.

The ex post objective of the firm is then to

$$(1) \text{ Maximize } p\bar{Q} - w_1x_1 - \dots - w_nx_n - w_KK,$$

$$\{x_1, \dots, x_n\}$$

subject to  $\bar{Q} = F[\bar{K}, x_1, \dots, x_n]$ ;  $\bar{Q}, \bar{K}$  both fixed.

Performing the operation leads to a set of conditional factor demand functions,  $\{x_i^* (\bar{Q}, \bar{K}; w_1, \dots, w_n)\}$ , which do not involve  $p$ . The ex ante conditional profit function may then be written as

$$(2) \Pi^* = p\bar{Q} - w_1x_1^* - \dots - w_nx_n^* - w_KK$$

$$= p\bar{Q} - C(\bar{Q}, \bar{K}; w_1, \dots, w_n) - w_KK,$$

where  $C(\cdot)$  is the firm's variable cost function corresponding to the production function  $F[\cdot]$ .

The ex ante objective of the firm is then to maximize the expected value of  $\Pi^*$  given by (2). Performing the differentiation yields

$$(3) -E [C_K(\bar{Q}, K; w_1, \dots, w_n)] - w_K = 0,$$

where  $E$  is the expectations operator taken over all random variables. The second-order condition for a maximum is<sup>2</sup>

$$(4) -E C_{KK}(\cdot) < 0, \text{ or } E C_{KK} > 0.$$

We are now in a position to evaluate the firm's optimal response, in choosing  $K$ , to increases in uncertainty. The notion of increased uncertainty employed here (and also in most of the connected literature) is that the random variable in question undergoes a mean-preserving spread.<sup>3</sup> Rothschild and Stiglitz (1971) established the result that the expected value of a concave function decreases as its argument undergoes a mean-preserving spread, and increases for the case of convex functions. Therefore, if  $C_K$  is a convex function of  $w_j$ , increased uncertainty will raise the value of  $E [C_K]$ , which would warrant a decrease in  $K^*$  (since  $E [C_{KK}] > 0$ ) in order to reestablish the maximizing condition (3). Conversely, if  $C_K$  is a concave function of  $w_j$ , the optimal response to increased variable factor price uncertainty is to increase  $K$ .

From duality theory we know that the variable cost function  $C(\bar{K}, \bar{Q}; w_1, \dots, w_n)$  is concave in factor prices. However, this property does not necessarily carry over to the partial  $C_K(\cdot)$ . What can be said at this stage is simply that, contrary to the original neutrality results, the optimal capital intensity of a risk-neutral competitive firm will change in response to increased factor price uncertainty. Now, if we place some restrictions on the production function it is possible to give a more definite answer. Consider then the set of functions of the form

$$(5) \quad F[\bar{K}, x_1, \dots, x_n] = \hat{F}[\bar{K}, g(x_1, \dots, x_n)],$$

where the function  $g(x_1, \dots, x_n)$  is homogenous of any degree other than 0 in all its arguments. This restriction is satisfied by many commonly used functions such as,

$Q = [\alpha_K \bar{K}^\rho + \beta [\alpha_1 x_1^\mu + \dots + \alpha_n x_n^\mu]^{\rho/\mu}]^{\nu/\rho}$ , as well as by the standard CES function assuming equal substitution elasticities between all factors. It can then be shown [see Perrakis (1980)] that the variable cost function associated with such a production function has the following separable form:<sup>4</sup>

$$(6) \quad C(\bar{Q}, \bar{K}; w_1, \dots, w_n) = A(\bar{Q}, \bar{K}) \cdot B(w_1, \dots, w_n).$$

Since  $C(\cdot)$  is concave in  $w_j$ , it must be the case that  $B(\cdot)$  is concave in  $w_j$ ; and since  $C_K < 0$ , also  $A_K < 0$ . We therefore have that  $C_K(\cdot) \equiv A_K(\cdot) B(w_1, \dots, w_n)$ , which is a convex function of  $w_j$  (since a concave function multiplied by a negative constant

is convex). Therefore, from the preceding discussion, it is possible to conclude that with production functions of the type given by (5), a risk neutral firm will always decrease its demand for capital in response to increased uncertainty in factor prices. In other words, since  $Q$  is being held fixed, the firm will opt for a less capital-intensive input mix.

### 3. A Steady-State Model Involving Costs-of-Adjustment

In contrast to the previous assumption made, that output be exogenously predetermined, it is often more realistic to assume that the actual level of output to be produced is an ex post decision variable. The problem with relaxing this assumption in the preceding framework was that the size of the firm would become indeterminate. Hartman (1972) provides an elegant solution to this problem by introducing a cost-of-adjustment term applicable to investment in a dynamic model of the firm; a solution which does not entail the same logical difficulties associated with the postulate of decreasing returns to scale, directly applied to the production function in the previous model. From this model, Hartman derives the proposition that a mean-preserving spread in output prices and wage rates has the effect of stimulating a firm's investment expenditures. This result has no counterpart in the static model employed previously, the results of which are only comparable to the steady-state solutions of the dynamic model. For that reason, the corollary proposition is proven here, that mean-preserving spreads in prices

will always lead to an increase in the desired steady-state capital stock of the risk-neutral firm. But more importantly for our purposes, it will also be shown that this need not imply that the firm would choose a more capital-intensive output mix. Rather, under a sufficient flexible production technology, as measured by the elasticity of substitution, it can be shown that the average steady-state capital intensity of the firm is reduced.

The following exposition is considerably simplified if we revert to the assumption of only two factors of production: Capital and a factor  $X$  which is variable in the short-run. The variable  $X$  can be interpreted as an aggregate measure of various short-run factors of production,  $g(x_1, \dots, x_n)$ , the price of which is given by the scalar  $w$ . The firm's profits in period  $t$  may then be written as:

$$(7) \quad \Pi_t = p_t F[\bar{K}_t, X_t] - w_t X_t - D(I_t),$$

where  $I_t$  is gross investment and  $D(I_t)$  is a cost-of-adjustment term with  $D' > 0$  and  $D'' > 0$ . For further simplicity, it is assumed that the investment cost function is non-stochastic. This is motivated by the result derived in Hartman (1972, p. 263) that investment is invariant to mean-preserving spreads in random terms which shift the investment cost function. The objective of the firm is to maximize the discounted expected value of (7)

summed over all  $t$ , subject to the constraint

$$(8) K_{t+1} = (1 - \gamma) K_t + I_t,$$

where  $\gamma$  is the (constant) rate of capital depreciation. By this formulation of the problem, the capital stock for period  $t$  must be chosen in period  $t-1$ , before  $p_t$  and  $w_t$  are known, while  $X_t$  and thus output,  $F[\bar{K}_t, X_t]$ , are chosen after these prices are known.

Given that this firm approaches an equilibrium size, we have in the steady-state that  $K_{t+1} = K_t = \bar{K}$ , and therefore that  $I_t = \bar{I} = \gamma\bar{K}$ . This allows us to delete the time subscript, so that the steady-state profit function in every subsequent period is  $\Pi = pF[\bar{K}, X] - wX - C(\gamma\bar{K})$ . By virtue of constant returns to scale in production, this may be rewritten as

$$(9) \Pi = \bar{K}p [f(x) - (w/p)x] - D(\gamma\bar{K}),$$

where  $x = X/\bar{K}$ . The necessary ex post profit-maximizing condition is

$$(10) f'(x) = w/p,$$

which does not involve  $\bar{K}$  directly. From (10) the conditional demand for the relative factor  $x$  is derived as  $x^* = x^*(w/p)$ . The conditional profit function for every period in the steady-state may then be written as

$$(11) \Pi^* = \bar{K}h(p, w) - D(\gamma\bar{K}).$$

Since we know from duality theory that profit functions are convex in factor and output prices, it must be the case that  $h(p,w)$  is a convex function of its arguments.

The ex ante objective of the firm in the steady-state is to maximize the expected value of  $\Pi^*$  with respect to  $\bar{K}$ . Differentiating (11), we obtain the necessary condition for a maximum,

$$(12) \quad \delta E(\Pi^*)/\delta \bar{K} = E h(p,w) - \gamma D'(\gamma \bar{K}) = 0,$$

and the second-order condition

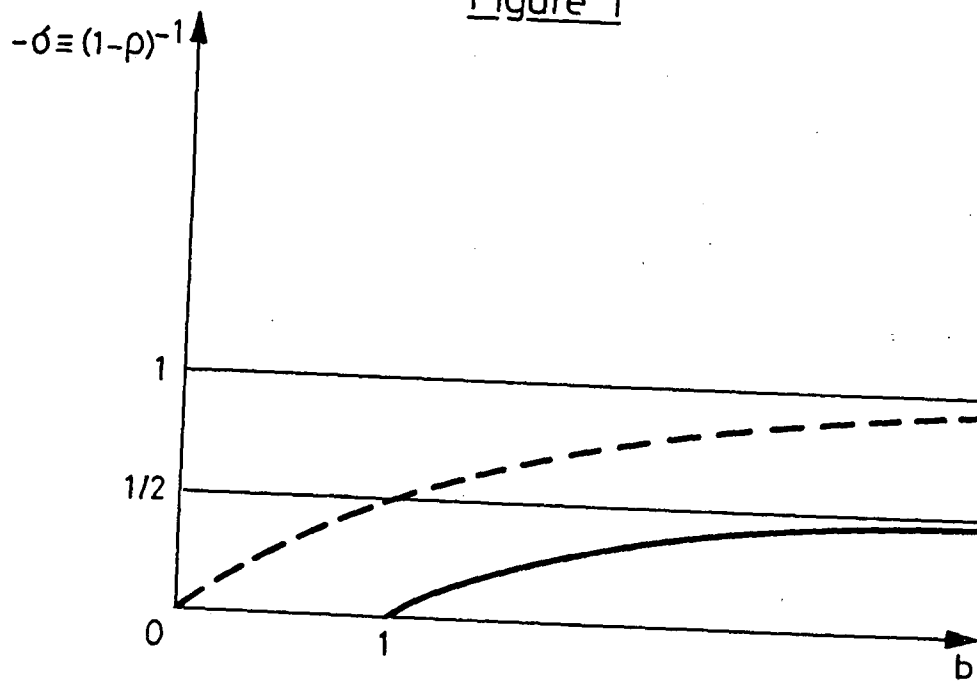
$$(13) \quad \delta^2 E(\Pi^*)/(\delta \bar{K})^2 = -\gamma^2 D''(\gamma \bar{K}) < 0.$$

The first proposition asserted in the beginning is now easily established. Since  $h(p,w)$  is a convex function of both arguments, a mean-preserving spread of the distribution of these variables raises  $E h(p,w)$ , which by expression (13) warrants an increase in  $\bar{K}$  in order to reestablish the equality in (12). In other words, increased uncertainty in factor and output prices increases the desired steady-state capital stock of the firm.

Consider now the effect of increased uncertainty on the desired steady-state factor proportions of the firm. From the necessary optimality condition (10), notice that the optimal level of  $x$  is only a function of  $w/p$  and independent of  $K$ . This means that when  $w$  and  $p$  are evaluated at their mean values, the



Figure 1



desired level of  $x$  remains the same as before the increase in uncertainty. However, the relevant question to ask is, what happens to the average level of  $x$  chosen by the firm. Assuming that the subjective probability distribution of  $w$  and  $p$  used by the firm's decisionmakers is the same as the true distribution, the average level of  $x$  is  $E(x)$ . As before, then, the behavior of  $E(x)$  following increased uncertainty hinges on whether  $x^*$  is a convex or concave function of  $w$  and  $p$ . Without further restrictions on the production function this is indeterminate, but in general it will be the case that  $x^*$  is not a linear function, so that increased uncertainty will affect the average level of  $x$  chosen.

Let us then confine ourselves to a CES production function,  $F(K, X) = (\alpha_K K^\rho + \alpha_X X^\rho)^{1/\rho}$ , where  $-\infty < \rho < 1$ , and  $\sigma \equiv 1/(\rho-1)$  is the elasticity of substitution. Output per unit of capital is then

$$(14) \quad f(x) = (\alpha_K + \alpha_X x^\rho)^{1/\rho}.$$

Using this functional form, the desired level of  $x$  can be solved from (10) to be

$$(15) \quad x^* = \left( \frac{\alpha_K w}{\alpha_X p} \right)^{\frac{\rho}{1-\rho}} - \frac{\alpha_X}{\alpha_K} \Big)^{\frac{-1}{\rho}}.$$

By taking the second derivatives of this expression with respect to  $w$  and  $p$  it is possible to determine whether the function is concave or convex. Performing this operation yields

$$(16) \quad \delta^2 x^*/(\delta w)^2 = a \bar{L}^{-2} - \rho + (1 + \rho)b_- \bar{L}$$

$$(17) \quad \delta^2 x^*/(\delta p)^2 = c \bar{L}^{-1} + \rho b_- \bar{L}.$$

where  $a$ ,  $b$  and  $c$  are positive terms. Of primary interest is the term  $b$  which ends up to be  $(\alpha_X/\alpha_X) \bar{x}^\rho$ , or the relative factor share of income accruing to  $X$  over the share accruing to  $K$ . In figure 1, the unbroken curve gives the combinations of  $\sigma$  and  $b$  for which expression (16) is zero. For combinations of  $\sigma$  and  $b$  that lie above that curve,  $x^*$  is a convex function of  $w$ . Similarly, the dashed curve in figure 1 gives the combination of points for which expression (17) is zero. Above that curve,  $x^*$  is a convex function of  $p$ .

From figure 1 it is possible to conclude that the advent of increased uncertainty in output and factor prices will decrease the average steady-state capital intensity of the firm given that the elasticity of substitution is sufficiently large. For the case of factor price uncertainty this would always be true for an elasticity of 1/2 or more, while for output price uncertainty the corresponding critical value is 1. For illustrative purposes, let us take the traditional example where the variable  $X$  stands for labor. Although there is considerable controversy as to the true elasticity of substitution between capital and labor, most empirical estimates at the aggregate level have resulted in values of approximately 1. Empirical values for

b, the relative income term of labor to capital in this case, range between 2 and 4 (approximately 3 in the United States). Given these parameter values, increased price uncertainty leads to a less capital intensive input mix.

As mentioned earlier, Hartman (1976) analyzed the absolute demand for capital by a firm subjected to increasing output price uncertainty in a model with decreasing returns to scale in production. In his specific example, production is given by  $Q = [\alpha_K K^\rho + \alpha_L L^\rho]^{\mu/\rho}$ , where  $0 < \mu < 1$ , which exhibits a constant degree of homogeneity,  $\mu$ . He then proceeds to derive a set of critical values for  $\mu$  and  $\sigma$  such that, for any given  $\mu$  less than 1, if the elasticity of substitution,  $\sigma$ , lies above the associated critical value, increased uncertainty leads to a lower demand for capital. Since the analysis in this paper focuses on optimal factor proportions rather than absolute values, a straightforward comparison is not possible. However, there is a parallel in that this analysis also relies on decreasing returns to scale in profits, but as a consequence of increasing investment costs. The other similarity is that the desired capital intensity of a firm is more likely to fall with increased uncertainty the higher is the elasticity of substitution.

#### 4. Concluding Remarks

The general conclusion derived in this paper is that the

choice of factor proportions by a risk-neutral competitive firm will be affected by increased output and factor price uncertainty provided that not all choices must be taken under the same amount of uncertainty. More specifically, when the production function is restricted to a certain class most commonly used in practical applications, increased factor price uncertainty always reduces the capital intensity of the output-constrained firm. For firms subject to cost-of-adjustment in investment, but with no constraints on output, the average desired steady-state capital intensity of the firm is also reduced in response to increased uncertainty, provided the substitution elasticities in production are not too inelastic. One way to interpret these results is that they arise as the consequence of Oi's Price Instability Theorem and of something akin to Samuelson's Le Chatellier Principle, applied to the firm. Oi's Theorem states that so long as profits are a convex function of prices, increased variability in their realized values will raise the average profit level above what it would be had these prices remained constant at their mean values. This presupposes that the firm be able to respond to changing prices and not have to make all input decision, as in the case of capital, before the actual price changes are known. Samuelson's Le Chatellier Principle, as originally applied to household demand theory, says that the fewer rationing constraints are placed on the household, the more elastic will be the response to price changes; see Samuelson (1965, p. 168). The analogy to draw here is that, for those

practical cases examined, with a less capital-intensive input mix a firm is in a better position to respond more elastically to ex post price changes by varying the amounts of variable production factors used, and thereby take better advantage of the profit opportunities afforded by increased price variability.

The focus of this analysis was on factor proportions rather than on absolute amounts. This appears to be a more appropriate reference point when the general equilibrium implications of this partial equilibrium analysis are considered. If the absolute levels of capital and labor, say, are exogenously given to an economy as a whole, then increased factor and output price uncertainty would lead to a fall in the rental-wage ratio, again under the proviso that the substitution elasticities are not too low. However, if we allow for possible factor unemployment, we should observe a decrease in capital intensity, and thus also a decline in average labor productivity. Perhaps it is not such a coincidence, then, that the increased price variability observed in the 1970's should have appeared at about the same time as the dramatic slowdown in labor productivity growth and in the growth of capital-labor ratios observed in most industrialized economies; see, e.g., Baily (1981).

### Footnotes

- <sup>1</sup> See also Dietrich and Heckerman (1980).
- <sup>2</sup> Perrakis' (1980) erroneous conclusions can be traced back, I think, to an oversight of this condition.
- <sup>3</sup> Let us represent the random variable  $w$  as  $w = a\varepsilon + b$ , where  $\varepsilon > 0$  is a random variable and  $a$  and  $b$  are non-stochastic parameters. A mean-preserving spread in  $w$  then involves both an increase in  $a$  (which amplifies the variability of  $\varepsilon$ ), and an appropriate decrease in  $b$  so that  $E[w]$  remains the same.
- <sup>4</sup> This can also be proven as follows: Since  $g(X_1, \dots, X_n)$  is homogenous of degree  $\rho$ , the production function in (5) may be written as  $\bar{Q} = F(\bar{K}, X_j^\rho f(X_1/X_j, \dots, X_n/X_j))$ . Minimizing variable costs with respect to  $\{X_1, \dots, X_n\}$  subject to this output constraint yields  $n$  equations representing the first-order conditions. Dividing every equation by the first equation yields the following  $(n-1)$  equations:

$$w_2/w_1 = f_2(X_1/X_j, \dots, X_n/X_j) / f_1(X_1/X_j, \dots, X_n/X_j)$$

$$w_j/w_1 = \frac{\bar{Q}^\rho f(X_1/X_j, \dots, X_n/X_j) - \sum_{i=1}^n (X_i/X_j) f_i(\cdot)}{f_1(X_1/X_j, \dots, X_n/X_j)}$$

$$w_n/w_1 = f_n(X_1/X_j, \dots, X_n/X_j) / f_1(X_1/X_j, \dots, X_n/X_j)$$

From these equations we can solve for the  $(n-1)$  variables,  $\{X_1/X_j, \dots, X_n/X_j\}$ , as functions of the  $(n-1)$  relative prices  $\{w_2/w_1, \dots, w_n/w_1\}$ . We can then invert the production function to yield

$$X_j^\rho f(X_1/X_j, \dots, X_n/X_j) = G(\bar{Q}, \bar{K}), \text{ or}$$

$$\begin{aligned} X_j &= G(\bar{Q}, \bar{K})^{1/\rho} \left[ f(X_1/X_j, \dots, X_n/X_j) \right]^{-1/\rho} \\ &= G(\bar{Q}, \bar{K})^{1/\rho} \left[ \hat{f}(w_1, \dots, w_n) \right]^{1/\rho}, \end{aligned}$$

which is the conditional demand for factor  $X_j$ . This process can be repeated for all  $j$ . But in all cases, the term involving  $Q$  and  $K$  will be the same, so that the cost function may be written as:  $C = [G(\bar{Q}, \bar{K})]^{1/\rho} [G(w_1, w_2, \dots, w_n)]$ .

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