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Workforce or Workfare?

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Workforce or Workfare?

Abstract

This article explores the use of workfare as part of an optimal tax mix when labor supply responses are along the extensive margin. Particular attention is paid to the interaction between workfare and an earned income tax credit, two policies that are designed to provide additional incentives for individuals to enter the labor force. This article shows that, despite their common goal, these policies are often at odds with each other.


Keywords: extensive margin, optimal income taxation, workfare.
1 Introduction

The Great Recession has placed additional strain on public programs that provide transfers to individuals. On the one hand, increased levels of unemployment have added to the need for welfare and related benefits. On the other, many governments face considerable budget deficits, making funds increasingly scarce. It is little wonder, therefore, that the proper design of benefit programs is high on the policy agenda. (OECD, 2009)

In Britain, for example, legislation was tabled in February, 2011 with the intent of completely overhauling that country’s benefits programs. (Government of Great Britain, 2011)

Against this backdrop, it seems appropriate to re-visit the role of workfare — the practice of requiring those who receive public benefits to spend time on some mandated activity — in the policy mix. In this article, we offer some results on when and how workfare can fit into the policy mix and argue that the case for workfare is weakened if earned income tax credits are part of the optimal policy mix. We do so in a model of labor supply choice along the extensive margin in the tradition of Diamond (1980) and more recent elaborations by Saez (2002) and Choné and Laroque (2005).

Private individuals in our model differ along two dimensions, labor market skill and preference for leisure. The labor market is perfectly competitive so that market wages are the respective skill levels. Given these wages and the tax-transfer program adopted by the government, individuals choose whether to work or not work. All workers are assumed to provide a fixed quantity of labor supply. The tax-transfer program might include the requirement that anyone who does not work must spend time in workfare activities. Given heterogeneity in preferences, some portion of workers of each skill type (those with the least taste for leisure) choose to work. The others remain out of the labor force and receive public benefits. The government chooses an income tax schedule for those in work, the level of the public benefit and the intensity of workfare to maximize a utilitarian social welfare function subject to a government budget constraint.

We derive conditions under which the addition of a completely unproductive workfare
requirement to an optimal tax-transfer scheme is welfare-improving. Of course, these conditions are expressed in terms of the costs and benefits of workfare. One cost of workfare is the direct utility losses of those who must do the required work. But workfare has behavioral effects, too. By increasing the *ceteris paribus* cost of remaining out of the labor force, workfare induces workforce participation. To a first order, this is a matter of indifference to marginal participants. However, increasing the size of the workforce affects the public budget. If marginal participants pay a net tax, workfare eases the public budget constraint and generates gains that can be weighed against the direct utility loss. However, if marginal participants receive a net wage subsidy, as would be in the case if the optimal tax-transfer system features an earned income tax credit, then the behavioral effect of workfare actually worsens the public budget. Thus, the presence of an earned income tax credit weakens the case for workfare when labor supply choices are along the extensive margin.

Most of the existing literature on workfare as part of an optimal tax-transfer mix follows Besley and Coate (1992, 1995) by modeling labor supply with variable hours of work. In this framework, workfare helps to dissuade highly productive workers from claiming benefits because it is the highly productive that have the highest opportunity cost of time. As Brett (1998) notes, this channel is not available when claimants are out-of-work, because the opportunity cost of voluntarily unemployed labor is the marginal rate of substitution between labor and consumption, which does not depend on labor market productivity.

Cuff (2000) introduces preference heterogeneity in a framework with intensive labor supply responses. In some variants of her model, it is desirable to give public benefits to individuals with a low preference for leisure. For the obvious reason, workfare helps to screen out would-be claimants with a higher preference for leisure. Cuff also notes that the same desire to screen out the “lazy” provide the government with a reason to provide marginal wage subsidies.\(^1\) Thus, along the intensive margin, workfare and wage subsidies

\(^1\)Due to the countervailing force of incentive effects operating along the skill dimension, the optimal
push in the same direction and it is reasonable to expect the two policies to coexist. Our results show that this affinity between wage subsidies and workfare disappears when labor is supplied along the extensive margin. Thus, when assessing the compatibility of workfare with other elements of welfare policy, it is important to distinguish between average wage subsidies and marginal wage subsidies.

The remainder of the article is organized as follows. In the next section, we introduce and analyze our basic model, which is a straightforward extension of the Diamond (1980) model of optimal taxation. We offer some extensions and qualifications to our analysis in Section 3. Concluding remarks are found in Section 4. The proofs of our results are contained in an appendix.

2 The Basic Model

All individuals in the economy are capable of working in the market. They can choose to work, but not the number of hours they work. Those who enter the market produce \( n \) units of a composite consumption good \( c \). Individuals differ in their market productivity along some interval \([\bar{n}, \tilde{n}]\). The labor market is perfectly competitive, so that \( n \) also measures the before-tax income of the individual.

The government can observe a worker’s before-tax income and implement a tax schedule \( t(n) \). A worker’s consumption is equal to after-tax income and is given by \( c(n) = n - t(n) \).\(^2\) Negative taxes are interpreted as subsidies. In addition to the tax schedule, the government provides a welfare benefit of \( b \) units of the consumption good. Receipt of this benefit is conditional on the individual engaging in some required activity, denoted \( r \).

Individuals’ utilities are increasing in consumption and decreasing in an index of the onerousness of labor. For market work, this onerousness is measured by a variable tax system might feature either marginal wage taxes or marginal wage subsidies.

\(^2\)This set of informational assumptions is consistent with Diamond (1980). Choné and Laroque (2011) allow workers to generate income less than their productivity \( n \) should it be in their interest to do so in a model of extensive labor supply choice. They characterize optimal tax schedules without workfare.
\( m \in [\underline{m}(n), \bar{m}(n)] \), where this interval may vary by skill type. Taking the tax-transfer into account, a worker experiences utility level \( u(c(n), m) \), where the function \( u \) is increasing in its first argument, decreasing in its second argument, continuously differentiable and strictly concave. Someone outside the labor market experiences utility \( u(b, r) \), where the workfare variable \( r \) measures some combination of the duration, intensity, and unpleasantness of the required activity. In this basic version of the model, we assume that all individuals find workfare equally onerous; in other words, \( r \) is the same for everyone. We assume that \( r \) is measured in such a way that \( r = 0 \) when there are no work requirements. Under these conditions, individuals differ along two dimensions and can be characterized by an ordered pair \((m, n)\). The population is described by a continuous distribution function \( F(m, n) \) with a density \( f(m, n) \) with support \([\underline{m}(n), \bar{m}(n)] \times [\underline{n}, \bar{n}]\).

Given the policies adopted by the government, individuals decide whether to work or not. An individual works if
\[
u(c(n), m) \geq u(b, r).
\]
(1)
For each \( n \), those individuals with the lowest values of \( m \) choose to work, while those with a relatively higher preference for leisure remain outside the labor force. Indeed, for each \( n \), there exists a critical value \( m^*(c(n), b, r) \) such that the workers with skill type \( n \) are exactly those that have \( m < m^*(c(n), b, r) \). This critical value is determined by the equation
\[
u(c(n), m^*(c(n), b, r)) = u(b, r).
\]
(2)
It follows immediately from the properties of the utility function that \( m^*(c(n), b, r) \) — and, consequently the size of the workforce — increases when \( c(n) \) or \( r \) increases or when \( b \) decreases. The greater is after-tax income from employment, the more desirable is work. The more unpleasant the workfare activity, the less desirable is being out-of-work. In either case, the relative return to working increases, thereby encouraging labor market participation. The greater is the public welfare benefit, the more desirable is remaining out of the labor market, so participation falls. We assume that \( m^*(c(n), b, r) \in (\underline{m}, \bar{m}) \) for all skill types \( n \), so that small changes in program parameters always have some effect.
on participation. Alternatively, our analysis carries through with minor modifications under the proviso that it pertains to those skill types whose participation is affected by the policies modeled in this article.

Following Diamond (1980), the government maximizes a utilitarian social welfare function. Given the participation decisions, social welfare can be written

$$W = \int\int n \left[ \int_{m(n)}^{m^*(c(n),b,r)} u(c(n), m) f(m, n) dm + \int_{m^*(c(n),b,r)}^{\bar{m}(n)} u(b,r) f(m, n) dm \right] dn. \quad (3)$$

To focus on the issue of optimal redistribution, we assume that the sole motives for taxation are to finance the public welfare benefit and go cover an exogenous revenue requirement $R$. Additional, so as to not conflate other effects of workfare with its role in influencing labor market participation, we assume that workfare is completely unproductive. Under these conditions, the public constraint of the government can be written

$$\int\int n \left[ \int_{m(n)}^{m^*(c(n),b,r)} [n - c(n)] f(m, n) dm - \int_{m^*(c(n),b,r)}^{\bar{m}(n)} b f(m, n) dm \right] dn = R. \quad (4)$$

The government’s decision problem is to choose $c(n)$, $b$ and $r$ to maximize the social welfare criterion (3), subject to the resource constraint (4).

The goal of our analysis is to examine the conditions under which workfare is a desirable addition to the policy mix. Thus, we proceed in two steps. First, we consider the optimal tax-transfer mix (the choice of $c(n)$ and $b$) for an arbitrary intensity of workfare. Second, we ask if, starting from no workfare — that is, $r = 0$ — a small increase in the intensity of workfare enhances social welfare. The first step is formally identical to the existing literature on optimal taxation with work choice along the extensive margin.

The results from this literature that we need below are summarized in the Proposition 1 below. In order to simplify the statement of this result, we define the participation tax for individuals of type $n$, denoted $\tau(n)$ to be sum of the tax paid when employed and the public welfare benefit, which is withdrawn upon taking up employment. Formally,

$$\tau(n) = t(n) + b = n - c(n) + b. \quad (5)$$

---

3Our results extend to more general Bergson-Samuelson social welfare functions.
When $\tau(n)$ is negative, employment of individuals of type $n$ is subsidized by the tax system. Such a subsidy is commonly referred to as an earned income tax credit (EITC). When the net tax on employment is positive, it is common to refer to the tax system as a negative income tax (NIT) scheme. Additionally, we denote by $g(n)$ the (average and endogenous) marginal social welfare weight given to workers of skill $n$, expressed in terms of public funds. Formally,  

$$g(n) \equiv \frac{1}{\lambda F(m^*, n)} \int_{m(n)}^{m^*(c(n), b, r)} \frac{\partial u(c(n), m)}{\partial c(n)} f(m, n) \, dm$$  

(6)

where $\lambda$ is the Lagrange multiplier of the budget constraint (4).

**Proposition 1** (Saez, 2002). The optimal participation tax for individuals of skill type $n$ is negative exactly when

$$g(n) > 1.$$  

(7)

The condition governing optimality of an EITC scheme relates the average marginal social welfare weight of workers of skill $n$ (expressed in public funds) to one. This welfare weight represents the dollar equivalent value for the government of distributing an extra dollar uniformly to workers of type $n$. When this value is larger (lower) than one, then these workers should receive a subsidy (should pay taxes) $\tau(n) < 0 (> 0)$. The intensity of required work might influence this condition through the cross-derivatives of the function $u$ (see (6)). If the marginal utility of consumption increases as workfare becomes more intense, there is an increasing tendency for the inequality in (7), thereby moving the optimal tax-transfer scheme in the direction of an EITC. However, given the focus of the second step of our analysis, which is to examine the advisability of adding a small workfare requirement, we will carry out our subsequent analysis under the assumption that $r = 0$. By so doing, we can treat the issue of whether there is an EITC or an NIT as fixed by the optimum without workfare.

We are now in a position to carry out the second step of our analysis. Imagine that the government has solved the optimal-tax transfer problem associated without workfare. In
the notation of this article, it has found the optimal solution with the added constraint that $r = 0$. Using the Envelope Theorem, the impact of a marginal increase in the intensity of workfare on social welfare can be written

$$
\frac{dW}{dr} = \frac{\partial u(b,0)}{\partial r} \int_0^\infty \int_{m^*(c(n),b,0)}^{\bar{m}(n)} f(m,n) dmdn + \lambda \int_0^\infty \tau(n) \frac{\partial m^*(c(n),b,0)}{\partial r} f(m^*,n) dn.
$$

The first term in (8) is negative, and it captures the welfare loss due to the disutility of work felt by those engaged in the required work. The second term captures the effect of workfare on the public budget. An increase in the intensity of required work increases the size of the workforce, which is captured by an increase in $m^*$.\(^4\) The total increase in the workforce of skill type $n$ is given by $\frac{\partial m^*}{\partial r} f(m^*,n)$. Multiplying this change in the workforce by the net tax on working for that type $\tau(n)$ gives the change in tax revenue received from workers of skill type $n$. Integrating over all skill types provides the total effect on the public budget. If skill type $n$ receives a participation subsidy then this revenue effect is negative. In this event workfare induces more people to enter the labor force, which increases the amount of public funds needed to finance the EITC scheme. This discussion is summarized in Proposition 2.

**Proposition 2.** The introduction of a workfare program at low intensity reduces social welfare if marginal participants receive, on average, a participation subsidy.

Proposition 2 is robust to some of the types of modifications to the government objective functions often found in the literature. For example, the income maintenance approach to workfare in Besley and Coate (1992, 1995) and the reduction of poverty in after-tax income approach to nonlinear income taxes found in Kanbur et al. (1994) both put zero social weight on the disutility of labor. Using this class of welfare functions would effectively eliminate the first term from (8), and we could replace the “if” in Proposition 2 with the phrase “if and only if.” The sign of the first term in (8) might

\(^4\)To a first order, these marginal participants experience no change in utility when entering the workforce because they were previously indifferent between working and remaining out of the labor force.
change, and with it the simple statement of Proposition 2, if work itself is seen as a social good, as in Moffit (2006). It is also clear how to modify Proposition 2 to account for productive workfare. In a model with intensive labor supply choice, Brett (1998) shows that workfare for the out-of-work is optimal if and only if the marginal product of the required activity is sufficient to compensate for the marginal disutility of work for program participants. Along the extensive margin and in the presence of an EITC, workfare is optimal if and only if its output is enough to compensate for the disutility of work of program participants and cover the increases in the EITC.

3 Extensions

The model of the previous section is based on the assumption that the intensity of workfare, in terms of utility, is identical for everyone. This is, admittedly, an extreme simplification. In this section, we propose two alternative specifications of the intensity of workfare. The first alternatives posits a third dimension of heterogeneity among individuals, namely in how they experience workfare. We show that this alternative is of no consequence to our qualitative results. The second alternative allows the disutility of workfare to be a deterministic function of the parameters $m$ and $n$, so that, for example the distaste for workfare might varies systematically with the distaste for work. In this variant, it is possible (but not necessary) that workfare might actually reduce participation in the workforce. If this turns out to be the case, workfare and an earned income tax credit can coexist because, due to falling participation, the introduction of workfare saves on the EITC.

3.1 Three-Dimensional Heterogeneity Among Individuals

Just as individuals may differ in their costs of market work, they might also differ in their distaste for publicly-required work. To account for this possibility, we re-interpret the workfare variable $r$ as an objective measure of required work (for example, its duration). We then posit a preference parameter $k$ that measures the intensity of distaste
for this activity. The utility level for a person that is unemployed — and, therefore, participating in workfare — is given by $u(b, kr)$.\(^5\) Individuals are endowed with an ordered triple of characteristics $(m, n, k)$. The joint distribution of these three characteristics is denoted $F(m, n, k)$ and the associated density, $f(m, n, k)$. Apart from continuity, we make no assumptions on the joint distribution function. Thus, the model admits an arbitrary correlation structure among individual characteristics. For ease of notation only, we assume that the support of the distribution is a cube $[m, \bar{m}] \times [n, \bar{n}] \times [k, \bar{k}]$. Individuals that are indifferent between market work and public welfare benefits are described by the equation

$$u(c(n), m) = u(b, kr)$$

(9)

For a fixed skill level $n$ and tax-benefit system, (9) determines an upward-sloping locus in $(k, m)$-space. We parameterize this locus by $m = \varphi(k, r)$. We suppress the dependence of this locus on the parameters of the tax-transfer system $c(n)$ and $b$, but, given our focus on workfare, make explicit the dependence of this locus on $r$.\(^6\) Individuals above and to the left of this locus remain out of the labor market, because these people either find market work relatively more costly in terms of utility or find workfare relatively less onerous. The mass of workers of skill type $n$ that choose market work is given by\(^7\)

$$\int_{\bar{k}}^{k} \int_{m}^{\varphi(k,r)} f(m, n, k) dm \, dk.$$  

(10)

An increase in $r$ shifts the locus $m = \varphi(k, r)$ upward, leading to increased participation in market work. To see this, applying the Implicit Function Theorem to (9) yields

$$\frac{\partial \varphi}{\partial r} = \frac{ku(b, kr)}{u_1(c(n), m)} > 0.$$  

(11)

\(^5\)The model of Section 2 corresponds to the special case of $k = 1$ for all individuals.

\(^6\)By the Implicit Function Theorem, $\frac{\partial \varphi}{\partial k} = \frac{ru(b, kr)}{u_1(c(n), m)} > 0$.

\(^7\)We are implicitly assuming that the locus $\varphi(k, r)$ intersects the line $k = \bar{k}$ above $m = m$, so that for each skill type there is some sufficiently low value of $m$ that induces market work. If this is not the case, we can reformulate our analysis by parameterizing the locus as $k = \eta(m, r)$ and reversing the order of integration in (10) and all subsequent integrals. Our results do not change.
From this point, the analysis of Section 2 carries forward, with slightly more cumbersome notation. The utilitarian social welfare function can be written

\[ \mathcal{W} = \int_{n}^{\bar{n}} \int_{k}^{\bar{k}} \left[ \int_{m}^{\bar{m}} u(c(n), m) f(m, n, k) dm + \int_{\varphi(k,r)}^{\bar{m}} u(b, kr) f(m, n, k) dm \right] dk \, dn. \]  

(12)

The budget constraint is

\[ \int_{n}^{\bar{n}} \int_{k}^{\bar{k}} \left[ \int_{m}^{\bar{m}} [n - c(n)] f(m, n, k) dm - \int_{\varphi(k,r)}^{\bar{m}} b f(m, n, k) dm \right] dk \, dn = R. \]  

(13)

Apart from the appearance of \( \varphi \) in place of \( m^* \) and integration over the variable \( k \), equations (12) and (13) are identical to the corresponding equations (3) and (4) in Section 2. Starting from a optimal tax-transfer scheme with no workfare, the effect of a marginal increase in \( r \) is

\[ \frac{d\mathcal{W}}{dr} = \frac{\partial u(b, 0)}{\partial r} \int_{n}^{\bar{n}} \int_{k}^{\bar{k}} \int_{\varphi(k,0)}^{\bar{m}} f(m, n, k) dm \, dk \, dn + \lambda \int_{n}^{\bar{n}} \int_{k}^{\bar{k}} \tau(n) \frac{\partial \varphi(k,0)}{\partial r} f(\varphi(k,0), n) dm \, dk \, dn. \]  

(14)

Equation (14) is a direct analogue of (8) and can be interpreted in exactly the same way. Thus, Proposition 2 carries over in this extended version of the model.

3.2 Distaste for Workfare as a Function of the Other Characteristics

As a further test of the robustness of the basic model, we now consider an economy in which the distaste for required work is a function of an individual’s other characteristics. We return to a world in which individuals vary only with respect to \( m \) and \( n \). The utility of an individual on workfare is given by \( u(b, \psi(r, m, n)) \), where, as in the previous subsection, \( r \) is some objective measure of required work. The function \( \psi \) is increasing in \( r \), but there are no \textit{a priori} reasons to restrict how \( \psi \) varies with \( m \) and \( n \). In this framework, an individual is indifferent between market work and remaining out of the labor force if

\[ u(c(n), m) - u(b, \psi(r, m, n)) = 0. \]  

(15)
The left-hand side of (15) is the net benefit of being in the workforce. In the model of Section 2, this net benefit is everywhere decreasing in $m$ for a given $n$ and a fixed tax-benefit system. This is why a single critical value $m^*(c(n), b, r)$ is sufficient to separate market workers from non-participants. In the more general model sketched here, it is possible that the distaste for workfare increases with $m$, rendering the net benefit of market work non-monotonic in $m$. In this case, participants (or non-participants) may be described as a union of disjoint intervals in $[m, \bar{m}]$, rather than as a single interval. The analysis of Section 2 can be repeated, as long as care is taken to break the domains of integration into appropriate pieces.\footnote{In the extreme, the net benefit to market work might increase with $m$, and individuals with the highest values of $m$ might be those who choose to work, while those with lower distastes for market work might actually choose to be on workfare.} It remains true that workfare increases the net gains to employment and encourages workforce participation. This is illustrated in Figure 4, where the upward shift net benefit schedule shrinks the interval over which net-benefits are negative, which is exactly the region of non-participation.

The qualitative effects of workfare, therefore, remain the same as those outlined in Section 2: participants are made worse off than when public benefits are offered for free, and increased participation provides an extra source of (draw on) public funds if there is a participation tax (subsidy).

4 Conclusion

The informal arguments for workfare and earned income tax credits are quite similar. Both policy instruments are designed to provide additional incentives for individuals to enter the labor force. As such, one might expect these two policies to be complementary. Indeed, in the work of Cuff (2000), there is some support for this idea in the existing literature on workfare when labor supply decisions are made along the intensive margin. In this article, we have shown, contrary to these notions, that there is a natural antipathy between workfare and earned income tax credits when labor supply decisions are along
the extensive margin. In so doing, we have highlighted the need to carefully examine the margin along which policy operates when making policy recommendations. Additionally, when deciding whether workfare ought to be part of the policy mix one needs to take due account of what other policies are part of the mix.

Our results are robust to the specification of the costs of workfare. What is crucially important in our analysis is that workfare raises the net benefits of working and discourages enrollment in public welfare programs. Solow (1998) points out three forces that might counterbalance the effects highlighted in this article: workfare might be productive in itself; utility might be increasing in workfare rather than decreasing due to an increased sense of self-reliance among the recipients of public welfare benefits; and workfare might increase the number of welfare claimants as any stigma attached to welfare receipt might be reduced. In Section 2, we have already suggested how our results can be adjusted to account for productive workfare. The other two of Solow’s forces have the potential to change the sign of the welfare and revenue effects highlighted in this article, but we conjecture that the basic logic of policy design problem is unaltered.

Models of labor supply along the extensive margin may yield further results into workfare. For example, adapting to the extensive margin the model of Blumkin et al. (2010) in which workfare serves a deterrent to income misreporting and welfare fraud may provide extra incites. In addition, Brett (2005) showed that a role for unproductive workfare can emerge in an intensive margin model when second-order incentive compatibility conditions of the optimal income tax problem bind so that workers of different productivities are bunched. Under some circumstance, workfare can help to separate types within the bunch. It is possible that adding workfare to the extensive margin analysis of Choné and Laroque (2011) that allows workers to earn less than their full potential might uncover interactions between workfare and the monotonicity constraints that arise in their model. As in any short article, we leave open some interesting questions.
Appendix

Proof of Proposition 1. As in Diamond (1980), the Lagrangian associated with the optimal tax problem is

\[ L(c(n), b, \lambda; r) = \int_\bar{m} \left[ \int_\bar{m}(n) u(c(n), m)f(m, n)dm + \int_{m^*(c(n), b, r)}^{\bar{m}(n)} u(b, r)f(m, n)dm \right] dn \]

A.1

The first-order condition with respect to \( c(n) \) yields:

\[ \int_{m^*(c(n), b, r)}^{\bar{m}(n)} \left[ 1 - \frac{1}{\lambda} \frac{\partial u(c(n), m)}{\partial c(n)} \right] f(m, n)dm = [n - c(n) + b] \frac{\partial m^*(c(n), b, r)}{\partial c(n)} f(m^*, n). \]

A.2

Using (5) and (6), and dividing both sides by \( F(m^*, n) \), (A.2) can be rewritten as

\[ 1 - g(n) = \tau(n) \frac{\partial m^*(c(n), b, r)}{\partial c(n)} f(m^*, n). \]

A.3

From (A.3) we see that \( \tau(n) \) takes the sign of \( 1 - g(n) \) as stated in Proposition 1. \( \square \)

The first-order condition with respect to \( r \) is

\[ \frac{\partial L}{\partial r} \leq 0, \quad r \geq 0 \quad \text{and} \quad r \frac{\partial L}{\partial r} = 0. \]

A.4

Differentiating (A.1) yields:

\[ \frac{\partial L}{\partial r} = \frac{\partial u(b, r)}{\partial r} \int_\bar{m} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n)dmdn \]

\[ + \lambda \int_\bar{m} \frac{\partial m^*(c(n), b, r)}{\partial r} [n - c(n) + b] f(m^*, n)dn. \]

A.5

By the Envelope Theorem, (A.5) provides the \( dW/dr \) that we display in (8).
References


Figure 1: The Effect of Workfare on Participation