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On educational risk and the quality of education

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Shutting the Stable Door after the Horse Has Bolted? On Educational Risk and the Quality of Education

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Abstract

We analyze whether a redistributive government should provide ex ante insurance against unfortunate outcomes or whether it should instead rely on transfers for redistributing income ex post. To this end, we develop a model of education in which individuals face educational risk and wage dispersion across two types of skills. Successful graduation and working as a skilled worker depends on individual effort in education and on public resources, but educational risk still causes (income) inequality. We show that in a second-best setting, in which learning effort is not observable, improving the quality of education by public funding of the educational sector has a significant effect and that this increases efficiency in comparison to a pure (linear) income tax with income transfers from skilled to unskilled workers. Compared to a first-best solution, providing ex ante insurance significantly gains importance relative to traditional ex post redistribution, because it simultaneously alleviates moral hazard in education. These results are strengthened when a (distortionary) skill-specific tax can be implemented.


Keywords: human capital investment, endogenous risk, learning effort, optimal taxation, public education.
1 Introduction

One of the oldest questions in welfare economics is whether one should achieve equity by equalizing incomes or by equalizing chances. With respect to risky human capital investment of individuals and wage inequality between skill groups, an important aspect of this question is: Should social insurance rely on ex post redistribution (e.g., by income taxation and transfers), or should a government instead enhance ex ante the quality of (public) education to increase the probability of all individuals attaining higher education?

This paper demonstrates that even in a world with complete equality of opportunity, providing ex ante insurance by investing public resources in the educational sector is less costly than ex post redistribution via a distortive tax system. A higher quality of education is therefore a crucial insurance device. Our theoretical findings are highly policy-relevant, because they contribute to the political debate over whether the current EU-strategy for growth and jobs is preferable to traditional welfare policies. The latter focus on ex post redistribution via taxation and direct income transfers such as, e.g., a negative income tax (or earned income tax credits, as in the USA). We provide a rationale for the EU strategy, better known as the ‘renewed Lisbon-agenda,’ which was announced by the European Union in 2005 (EU-Council, 2005) and will be extended and strengthened in the upcoming “EU2020-strategy” (EU-Commission, 2009). The EU aims to foster success in education (i.e., to decrease drop-out rates, especially in secondary schools) and to ensure that the majority of the population successfully acquires higher education.

Education, like any other investment activity, is a risky activity; therefore, the educational system can be regarded as a (imperfect) ‘filter,’ preventing some individuals from entering the labor market as skilled workers and thereby generating ex post heterogeneity, although individuals might be identical ex ante (Arrow, 1973; Konrad, 2004). Educational investment suffers from significant ‘input risk,’ i.e., the risk of failure to graduate and of loss of most of the resources invested. For instance, drop-out rates in tertiary education are on average about 30% both in OECD and in EU19 countries (OECD, 2009, pp. 63 and Table A3.6). In addition, educational investment is exposed to ‘output risk,’ since both returns on education (Carneiro et al., 2003) and (increasing) wage differentials (Katz and Autor, 1999; Chen, 2008) are difficult to predict, and unemployment rates are quite substantial across all levels of education (OECD, 2009). However, attaining higher education provides insurance against wage risks and the risk of unemployment. For workers with less than upper-secondary education, the risk of being unemployed is on average (OECD and EU19) three times higher than for workers with a tertiary education (OECD, 2009, Tables A6.2a to A6.4a). Empirical studies provide further evidence that human capital investment cuts both ways with respect to the exposure to risk (see, e.g., Palacios-Huerta, 2003, 2006; Belzil and Hansen, 2004).\footnote{The theoretical analysis of ‘input risk’ and ‘output risk’ dates back to Levhari and Weiss (1974).}

In summary, education is a risky activity and it is intertwined with (increasing) wage in-
equality, but also by itself provides a degree of insurance. Consequently, the OECD correctly regards upper-secondary education as the “minimum level needed to obtain a satisfactory, competitive position in the labor market” (OECD, 2007, p. 128f). Furthermore, private insurance markets are incomplete due to moral hazard, adverse selection, and legal limitations (Sinn, 1996), resulting in demand for an adequate welfare policy to alleviate educational risks and the resulting wage inequality.

Most of the literature focuses on ex post insurance devices such as redistribution using traditional income taxation and income transfers. These papers apply the modeling framework of Levhari and Weiss (1974), in which education either increases or decreases the exposure to an exogenous income shock. In order to reduce efficiency losses, they complement the trade-off between distortive income taxation and a welfare-increasing insurance effect à la Eaton and Rosen (1980) and Varian (1980) with educational policy. See, for example, Hamilton (1987) and Jacobs et al. (2010) for linear tax instruments and da Costa and Maestri (2007) and Anderberg (2009) for non-linear ones. If a government can dictate mandatory private educational investment levels, Anderberg and Andersson (2003) show that education should be overprovided (underprovided) compared to the first-best level, if education decreases (increases) exposure to income risk. In a decentralized economy, however, this insurance effect is fully exploited by self-insurance of individuals, and (labor) taxation causes moral hazard in educational investment. This results in inefficient educational investment and fiscal externalities, calling for supplementary direct or indirect (e.g., capital taxation) education subsidies in order to mitigate distortions. However, it is important to note that neither capital taxation nor direct education subsidies provide any insurance in these models (Jacobs et al., 2010).

If the government can condition its policy mix on income and on factors like the level of completed education, a graduate tax, accompanied by some direct education subsidies, would be optimal in order to insure individuals against income risks and would outperform income-contingent loans (e.g., García-Peñalosa and Wälde, 2000; Wigger and von Weizsäcker, 2001; Jacobs and van Wijnbergen, 2007). If individuals are heterogenous ex ante, this finding is strengthened, since a combination of scholarships and graduate taxation superiorly allows to level the playing field and to select the gifted students as well (Cigno and Luporini, 2009).

To the best of our knowledge, however, there has not yet been an (theoretical) analysis of the trade-off between improved quality of education and ex post redistribution. This is not an issue of equality of opportunities; improving the quality of education constitutes by itself an insurance device in a risky environment, namely, decreasing income risk for all individuals by increasing their probability of success. For our analysis, we extend the Andersson and Konrad (2003a,b) two-period model in which endogenous educational risk arises from an imperfect learning technology. In the first period, ex ante homogenous individuals enter a

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2Andersson and Konrad (2003a) analyzes potential private insurance instead of governmental instruments, while Andersson and Konrad (2003b) considers hold-up problems and time-consistent taxation in the context of a Leviathan government and individual mobility. These papers neither consider the quality of education and direct public spending in the educational sector nor endogenous labor supply. Although the mobility of their skilled
publicly-financed educational system of certain quality and decide on their learning effort. This
decision determines an individual’s probability to graduate and to become a skilled worker in
the second period. In the second period, uncertainty is resolved, (un)successful students enter
the competitive labor market as (un)skilled workers and each worker chooses its labor supply.
A benevolent government can use a linear wage tax and may have access to a skill-specific
tax in order to finance both a general lump-sum transfer and public funding of the educational
system. Public educational spending is assumed to increase the probability of success, because
enhanced equipment in the educational sector (e.g., an increased number of university profes-
sors) improves the learning technology. The primary idea behind our outline is that learning ef-
fort (how individuals spend their time at university) and resource investment in the educational
sector open an additional channel for governmental policy by endogenizing the efficiency of
the filter ‘educational system.’

In this set-up, we derive two main results for a second-best world, in which learning effort
cannot be controlled by the government and collecting tax revenue induces distortions (e.g.,
moral hazard in learning effort). First, we show in comparison to the first-best allocation that
ex ante improving the quality of education in order to increase the probability of successful
graduation (i.e., to enhance the ‘filter mechanism’), significantly gains importance relative to
providing ex post income insurance via a distortive linear income tax and transfers from the
skilled to the unskilled. Second, we show that this result is strengthened when the government
can apply a skill-specific tax (viz., if ‘tagging’ is possible). Skill-specific taxes strongly distort
the private contribution to education and require an even higher degree of public spending in
order to mitigate this moral hazard effect. In short, the main message is that society should
first implement a proper educational system to enhance the ‘filter technology.’ The intuition for
this is: Collecting revenue in a second-best world is costly due to efficiency losses. However,
spending the revenue in the educational sector instead of granting income transfers not only
decreases exposure to risk, but, additionally, alleviates distortions in learning effort and decreases
the marginal costs of taxation.

The paper is organized in six sections. In Section 2, we present the model; we examine
household behavior in the third section. Section 4 then establishes the first-best allocation as
a benchmark case, whilst Section 5 introduces public policy and determines optimal tax and
educational policies. Section 6 concludes.

2 The Model

We consider a two-period model in which individuals of each generation are active for two peri-
ods of time. In the second period, each individual gives birth to one child so that the population
remains constant over time; each cohort is normalized to one, adding up to a total population

individuals could be seen as an extreme form of skilled labor supply elasticity, their unskilled individuals still
cannot react to, e.g., tax rate changes.
of two. In each period, individuals are endowed with one divisible unit of time. Implicitly assuming that individuals have already attended compulsory schooling, ex ante homogenous individuals invest in higher education at the beginning of the first period and start working in the second period. Both education in the first period and working in the second period are time-consuming activities generating disutility. When entering the higher-educational system (‘universities’), individuals must decide on how much time effort \( e \in [0, 1] \) to devote to learning; the remainder, \((1 - e)\), is consumed as first-period leisure. At the beginning of the second period, individuals decide on their individual labor supply.

Education is a risky investment that when successful alters the quality of labor from unskilled to skilled labor. Each successful graduate is supplied with one unit of human capital.\(^3\)

Thus, the human capital production function is given by a probability function \( p \) for entering the skilled sector that depends on learning effort \( e \) and on public funding \( E \):\(^4\)

\[
p = p(e, E) \in [0, 1].
\] (1)

The probability function is considered to be a concave function of both learning effort \( e \) and public funding \( E \), consequently showing positive but diminishing marginal productivities. Assuming some basic quality \( p(e, 0) > 0 \), public funding \( E \) as real investment determines the quality of education. Increased public spending enhances the quality of education, by improving the teacher-student ratio or library services, for example. Moreover, we assume that such an increase in the quality of education also increases the marginal productivity of each time unit invested.\(^5\)

We further assume that without any private effort, \( e = 0 \), a successful education is impossible. The properties of the probability function in equation (1) are summarized in:

**Assumption 1.** The probability function for successful entrance into the skilled sector has the following properties:

\[
\frac{\partial p}{\partial e} = p_e, \quad \frac{\partial p}{\partial E} = p_E > 0; \quad \frac{\partial^2 p}{\partial e^2} = p_{ee}, \quad \frac{\partial^2 p}{\partial E^2} = p_{EE} < 0; \quad \frac{\partial^2 p}{\partial e \partial E} = p_{eE} > 0; \quad p(0, E) = 0, \quad p(e, 0) > 0.
\]

The human capital production fits the stylized facts summarized in the introduction. Education is risky, but a higher investment and a better quality of education allow better-paid jobs and

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\(^3\)Disregarding the effect of investment on human capital levels is done out of convenience in order to concentrate on educational risk. A different formulation for the human capital production function would not change our qualitative results, since individuals decide about their working time in the second period.

\(^4\)We neglect private real investment in the process of education in order to simplify the formal analysis. Analytically, this assumption also guarantees that the first-order conditions are necessary and sufficient (otherwise cf. Booth and Coles, 2010). Our main results do not change, if we apply private real investment and public subsidies for these investments. Furthermore, the quality of education in a society ultimately relies on government investment. In fact, public investment accounts for about 80% (85%) of overall spending on all educational institutions and for about 75% (80%) of overall expenditure in tertiary education on OECD (EU19) average in recent years (OECD, 2009, B3 and Tables B3.1, B3.3).

\(^5\)This can be seen as an analog to complementarity between ability and educational investment, as used, e.g., in Jacobs and Bovenberg (2010).
reduced income risk. Therefore, we endogenize educational risk, in the sense that it emerges from individual decisions and that educational investment has both advantages and disadvantages. This distinguishes our approach from papers in the tradition of Levhari and Weiss (1974), in which an exogenous shock is either mitigated or enforced by educational investment.

Our mechanism can be rationalized as follows: According to Arrow (1973), the educational system acts as an imperfect ‘filter technology.’ Hence, it prevents some individuals from entering the skilled sector and so produces ex post (income) inequality, see Konrad (2004, p. 68f). This holds true even if the playing field is leveled, i.e., if there is equality of opportunity. Although individuals are identical with respect to (innate) ability, productivity, endowments, etc., and although they invest the same (learning) effort, some of them will fail because of differences in imponderable soft skills (e.g., behavior in job interviews, exam nerves) or due to an imperfect matching technology or bad luck in final exams. Learning effort and public spending have an ameliorating effect on the magnitude of this educational risk by directly affecting the probability of success. The former decreases drop-out risk; the latter improves the efficiency of the educational system as a filter technology, making the ‘filter’ more penetrable and allowing more students to pass into the skilled sector.

Based on these ideas, we interpret the probability of success as a measure of the efficiency of the filter technology and public investment as the quality of education. Increased public spending improves the quality of education, leading to an increase in the probability of success for all individuals. This is another instrument for ex ante insurance (redistribution) beyond the concept of ‘equality of opportunity’. Thus, in a risky economy, the quality of education creates a further dimension, which has to the best of our knowledge so far been neglected. To analyze this dimension, it is sufficient to focus on ex ante homogenous households, differing ex post due to idiosyncratic risk. Improving the filter technology by increasing the quality of education opens another channel of influence apart from ex post income transfers – a channel by which the government can directly affect the magnitude of risk in the economy – and it embeds an ex ante insurance device against ex post income inequality. This would even be the case, if we allowed for private resource investment, as long as there is no perfect crowding-out.

At the beginning of the second period, those individuals who successfully graduated and passed the application process start working as skilled workers, while those who failed enter the labor market as unskilled workers. In the second period, households divide their time endowment between second-period leisure and labor supply. Total wage income is spent on total family consumption. In accordance with the majority of the literature, we assume that private insurance against educational risk is not available. This might be because of market failure due to moral hazard, adverse selection, or the fact that individuals are too young to write insurance contracts, when they decide on their human capital investment (see, e.g., Sinn, 1996).

Equality of opportunity implies that heterogenous individuals should not be held responsible for circumstances outside of their sphere of influence that affect their capability to earn, i.e., innate ability to learn, socio-economic status, etc. Governmental intervention, e.g., by financing education, should level the playing field for all individuals investing the same level of effort, see Roemer (1998).
All individuals have identical preferences, which are defined over leisure in period one and two, \( l_1 \) and \( l_2 \), and over total family consumption \( C \) in period two. In order to simplify the formal analysis without qualitatively affecting the results, we suppress a consumption choice in the first period of life. Instead, we assume that the parent generation decides on total consumption; consequently, family consumption includes (good) consumption by the child, and young individuals decide on their effective leisure time only. Formally, the preferences are described by a von Neumann-Morgenstern expected utility function that is additively separable in its intertemporal components. Thus, we have

\[
E[U] = U_1(1 - e) + p(e, E) \cdot U_2(C_H, 1 - H) + [1 - p(e, E)] \cdot U_2(C_L, 1 - L),
\]

where \( H = 1 - l_2H \) denotes labor supplied by a skilled worker in the second period, and \( L = 1 - l_2L \) denotes labor supplied by an unskilled worker in the second period,\(^7\) and where \( \frac{\partial U_1}{\partial l_1}, \frac{\partial U_2}{\partial C} > 0; \frac{\partial^2 U_1}{\partial l_1^2}, \frac{\partial^2 U_2}{\partial C^2} < 0, \ i = 1, 2. \) In order to ensure an interior solution, especially for learning effort \( e = 1 - l_1 \), we assume that the utility function meets the Inada conditions \( \lim_{l_1 \to 0} \frac{\partial U_1}{\partial l_1} = \lim_{C \to 0} \frac{\partial U_2}{\partial C} \to \infty \) and \( \lim_{l_1 \to 1} \frac{\partial U_1}{\partial l_1} = \lim_{C \to \infty} \frac{\partial U_2}{\partial C} = 0, \ i = 1, 2. \)

Wages for both skill groups are exogenously given, denoted by \( w_H \) and \( w_L \), respectively, and the skill premium in wages equals \( w_H - w_L > 0.\(^8\)\) Financing public expenditure depends on the information available to the government: it can always use a standard linear income tax scheme consisting of a constant tax rate \( t \) and a lump-sum transfer \( T \in \mathbb{R} \).

Furthermore, if the government possesses information on skill status (i.e., if ‘tagging’ is possible), the government can raise a skill-specific tax \( f_B \). This tax is a fixed amount of money that must be paid by individuals, who have successfully entered the skilled sector. Analogously to Cigno and Luporini (2009), this tax can be seen as a combination of a scholarship and a graduate tax. The scholarship is granted in order to finance tuition fees in the first period. Then, the graduate tax re-collections the money in the second period (viz., implementing a compulsory credit-repayment scheme), but the tax payment will exceed the scholarship (plus interest), if a student is successful. There are no real payments in the first period of life. In our model, only skilled workers will pay the tax \( f_B \), and the difference to real-world graduate taxes (e.g., to the Australian HECS-system) is that it is not proportional to income.

The respective budget constraints of a skilled and an unskilled household can be written as

\[
C_H = (1 - t) \cdot w_H \cdot H - f_B + T, \quad (3)
\]

\[
C_L = (1 - t) \cdot w_L \cdot L + T. \quad (4)
\]

The educational risk is assumed to be idiosyncratic; thus, there are ex post \( p(e, E) \) skilled workers and \( 1 - p(e, E) \) unskilled workers in each generation. The government uses its in-

\(^7\)Subscripts \( H \) and \( L \) denote the respective values for the different skill groups.

\(^8\)Assuming exogenous wages can be justified by focusing on a small open economy with two sectors.
Instruments in order to maximize the utility of a representative steady-state generation. Consequently, the government faces a trade-off between efficient financing of public expenditure and optimal redistribution between successful and unsuccessful students, as well as optimal insurance against the risk of education.

In short, the timing structure and the model can be summarized as follows: First, the benevolent government decides on public funding of the educational sector and on the tax instruments. Second, the younger generation will choose learning effort given the wages and the governmental decisions. This in turn determines the probability of success \( p(e, E) \) and with it the fraction of skilled and unskilled workers. At the beginning of the second period, each individual knows whether it graduated into the skilled sector or failed and will then decide on its labor supply. In the following analysis, we will solve the model by backward induction.

### 3 Household Behavior

The decision problem of a representative household can be described by the following maximization problem:

\[
\max_{\{e,H,C_H,L,C_L\}} E[U] = U_1(1 - e) + p(e, E) \cdot U_2(C_H, 1 - H) + [1 - p(e, E)] \cdot U_2(C_L, 1 - L) \quad \text{s.t. (3) and (4)}
\]

Substitution of (3) and (4) for \( C_H \) and \( C_L \) in (5) yields the following first-order conditions:

\[
\frac{\partial E[U]}{\partial H} = U_{2C}(C_H, 1 - H) \cdot (1 - t)w_H - U_{2l_2}(C_H, 1 - H) = 0, \\
\frac{\partial E[U]}{\partial L} = U_{2C}(C_L, 1 - L) \cdot (1 - t)w_L - U_{2l_2}(C_L, 1 - L) = 0, \\
\frac{\partial E[U]}{\partial e} = -U_{1l_1}(1 - e) + p_e \cdot [U_2(C_H, 1 - H) - U_2(C_L, 1 - L)] = 0.
\]

The system of first-order conditions (6) to (8) is block recursive, such that optimal labor supply \( H^*, L^* \) and correspondingly optimal consumption \( C_H^*, C_L^* \) are separately defined by (6) and (7), respectively. Note that optimal consumption and labor supply of the respective skill groups are conditional on the policy mix used by the government \((t, T)\) as well as on the respective wage rate \( w_H, w_L \). Additionally, the skill-specific tax \( f_B \) is only relevant for labor supply and consumption of skilled workers. Inserting optimal labor supply and consumption into the second-period utility function gives the indirect utility function for both types of workers:

\[
V_H = U_2(C_H^*, 1 - H^*), \quad V_L = U_2(C_L^*, 1 - L^*). \quad \text{Using } V_H \text{ and } V_L \text{ in (8) yields the optimal}
\]

\[\text{We thereby assume that the government can credibly commit to its chosen tax instruments, and we do not consider any hold-up or time-consistency problems. Moreover, we do not focus on extortionary Leviathan governments. See Andersson and Konrad (2003b) for these issues in a related context.}\]

\[\text{Throughout the paper, asterisks denote optimal values. To simplify the notation, we drop the functional arguments } t, T, f_B, w_H, w_L, \text{ when this does not cause confusion.}\]
effort $e^* = e(t, T, f_B, E, w_H, w_L)$. Finally, evaluating first-period utility at the optimal effort $e^*$ results in the first-period indirect utility function $V = U_1(1-e^*)$.

The second-order conditions $SOC(i) < 0$, $i = H, L$, $e$ are fulfilled under standard assumptions together with the fact that, compared to an unskilled worker a skilled worker must have higher utility, $V^H > V^L$, because otherwise no one would invest in education and $e^* = 0$.

In the following sections, we derive the optimal policy mix. For that reason, we need to derive comparative statics of the individual choice variables with respect to the different instruments. The effects on the labor supply of both skill groups are standard, but note that the lump-sum transfer $T$ and the skill-specific tax $f_B$ have analogous income effects on skilled labor supply:

$$\frac{\partial H^*}{\partial T} = -\frac{\partial H^*}{\partial f_B} = -\frac{U_{2CC}(1-t)w_H - U_{2Cl2}}{SOC(H)} < 0,$$

where we have assumed that leisure is a normal good.

Learning effort $e^*$ depends negatively on the lump-sum transfer $T$:

$$\frac{\partial e^*}{\partial T} = -\frac{p_e \cdot (\alpha^H - \alpha^L)}{SOC(e)} < 0,$$

with $\alpha^j = \frac{\partial V^j}{\partial C} > 0$, $j = H, L$ denoting the marginal utility of income. The inequality in equation (9) stems from the fact that we assume agent monotonicity (Mirrlees, 1976) to hold. This implies that a skilled worker always commands a higher income than an unskilled worker, and hence $\alpha^H < \alpha^L$. The intuition is straightforward: any increase in lump-sum income $T$ decreases the learning intensity $e$, because an educational degree becomes marginally less attractive.

An increase in the skill-specific tax affects learning effort according to

$$\frac{\partial e^*}{\partial f_B} = \frac{p_e \cdot \alpha^H}{SOC(e)} < 0,$$

while increased public spending in education $E$ changes the effort according to

$$\frac{\partial e^*}{\partial E} = -\frac{p_eE \cdot (V^H - V^L)}{SOC(e)} > 0.$$

Learning effort is unambiguously reduced if the skill-specific tax rises, because this directly reduces the return on education and creates a negative substitution effect. Instead, an improved quality of education unambiguously fosters learning effort, since it increases the productivity of learning, $p_eE > 0$. Unlike labor supply, there are no offsetting substitution and ‘income’ effects, and the sign of the effect only depends on whether learning effort and the quality of education are technically complements in production.

Contrary to these effects, the effect of an increase in the wage tax $t$ is ambiguous. Increasing
the tax burden on skilled wage income decreases learning effort, because the returns on education decrease. Increasing ceteris paribus the wage tax on unskilled workers increases the returns on education, which boosts the learning intensity. Combining both effects, we end up with

\[
\frac{\partial e^*}{\partial t} = - \frac{p_e \left[ \alpha^L \cdot w_L L^* - \alpha^H \cdot w_H H^* \right]}{SOC(e)} \geq 0.
\] (12)

The total effect is ambiguous: On the one hand, wage taxation makes learning effort a less risky investment, because it reduces the variation in income (viz., income risk). On the other hand, wage taxation lowers the skill premium in wages, making education ceteris paribus less attractive.

In summary, the skill-specific tax only has an income effect on the skilled labor supply, but strongly distorts learning effort. Wage taxation, in contrast, distorts both the skilled and unskilled labor supply, but affects learning effort rather modestly.

Evaluating the expected utility function (5) at the optimal labor supplies \(H^*, \quad L^*\) and the optimal learning effort \(e^*\), the indirect expected utility function of the household can be written as

\[
E[V^*(t, T, f_B, E)] = V(t, T, f_B, E) + p(e^*, E) \cdot V^H(t, T, f_B) + [1 - p(e^*, E)] \cdot V^L(t, T).
\] (13)

It is important to note that \(E[V^*]\) is a function of the policy mix chosen by the government. This policy mix is exogenously given for the households. Using the envelope theorem, we can derive the marginal impact of a policy change on the expected utility of household as

\[
\frac{\partial E[V^*]}{\partial f_B} = -p^* \cdot \alpha^H < 0, \quad \frac{\partial E[V^*]}{\partial t} = p^* \cdot \alpha^H + (1 - p^*) \cdot \alpha^L > 0, \quad \frac{\partial E[V^*]}{\partial E} = p^*_E \cdot [V^H - V^L] > 0.
\]

4 First-Best as Benchmark

Before we analyze the optimal public policy in a second-best setting, as described in Section 2, we establish the first-best solution as a benchmark. This allows later examination of potential shifts in optimal insurance strategies and allows us to determine when income insurance or improved quality of education have more importance.

The first-best allocation can be characterized by

\[
\max_{e,E,C_H,C_L} U_1(1 - e) + p(e, E) \cdot V^H(C_H, 1 - H) + [1 - p(e, E)] \cdot V^L(C_L, 1 - L),
\] (14)

subject to the resource constraint

\[
E + p(e, E) \cdot C_H + [1 - p(e, E)] \cdot C_L = p(e, E) \cdot w_H H + [1 - p(e, E)] \cdot w_L L.
\] (15)
Note that in a first-best solution the government not only chooses consumption $C_j$ and labor supply $Z_j$, $j = H, L$, for skilled and unskilled households, but also fully controls learning effort $e$ and real educational investment $E$.

The first-order conditions are

$$\frac{\partial L}{\partial e} = -U_{H}(1 - e) + p_e \cdot (V^H - V^L) + \lambda \cdot p_e \cdot (T_H - T_L) = 0, \quad (16)$$

$$\frac{\partial L}{\partial E} = p_E \cdot (V^H - V^L) - \lambda + \lambda \cdot p_E \cdot (T_H - T_L) = 0, \quad (17)$$

$$\frac{\partial L}{\partial C_H} = p(e, E) \cdot (\alpha^H - \lambda) = 0, \quad (18)$$

$$\frac{\partial L}{\partial H} = p(e, E) \cdot (\lambda w_H - U_{H}^2 l^2_H) = 0, \quad (19)$$

$$\frac{\partial L}{\partial C_L} = [1 - p(e, E)] \cdot (\alpha^L - \lambda) = 0, \quad (20)$$

$$\frac{\partial L}{\partial L} = [1 - p(e, E)] \cdot (\lambda w_L - U_{L}^2 l^2_L) = 0, \quad (21)$$

where $T_j = w_j Z_j - C_j$ with $Z_j$, $j = H, L$, can be interpreted as lump-sum tax payment of a household of skill group $j$, where $\lambda$ represents the Lagrangian multiplier and where, according to Section 3, $\alpha^j$ equals marginal utility of income in the respective skill groups $j = H, L$.

From equations (18) and (20), it follows that

$$\alpha^H = \lambda = \alpha^L = \alpha, \quad (22)$$

thus all households have the same marginal utility of income. Combining (19) and (21) results in

$$\frac{U_{H}^2}{U_{L}^2} = \frac{w_H}{w_L} > 1, \quad (23)$$

implying $U_{H}^2 > U_{L}^2$. Skilled households have a higher marginal utility of leisure in the second period; together with (22), this implies that skilled workers work more than the unskilled, $H^F > L^F$.\textsuperscript{11} This is reasonable from an efficiency point of view, because the skilled are more productive. These results then suggest that the government provides full income insurance, in the sense of equalized marginal utilities of consumption/income, but that the skill premium, measured in utility, $V^H - V^L$ turns negative. These are the most important differences to a laissez-faire economy or to a second-best solution, and they are driven by the fact, that the social planer (the government) can control learning effort perfectly in a first-best approach. If there is moral hazard in learning, a positive skill premium in utility is absolutely necessary in order to induce learning effort (see equation (8)) – else there would be no skilled worker in the economy, because $p(0, E) = 0$.\textsuperscript{12}

\textsuperscript{11}Throughout the paper, the superscript $FB$ will characterize the value of a variable in the first-best solution.

\textsuperscript{12}All these results become immediately clear in the special case of an additively separable second-period utility function. Note as well that all these conditions are independent of learning effort and the quality of the educational
Given the results for optimal consumption and labor supply in the second period, first-best efficient learning effort balances marginal disutility of forgone first-period leisure \( (U_l) \) and the second-period welfare loss by an increased number of skilled households (due to \( V^H - V^L < 0 \)) on the one hand and gains in tax revenue by an increased number of skilled households on the other hand, see equation (16). As the first two terms in (16) are negative and \( \lambda = \alpha > 0 \) as well as \( p_e > 0 \), the bracket in the third term must be positive. Accordingly, a first-best optimum implies \( T_H > T_L \).

Optimal public spending \( E \) on the quality of education is determined by a similar trade-off: Public spending needs resources. Furthermore, improving the efficiency of the filter technology \( (p_E > 0) \), i.e., increasing the number of skilled households, ceteris paribus decreases welfare, because \( V^H - V^L < 0 \). However, an increase in skilled workers also increases the resources available for redistribution since \( T_H > T_L \).

By applying the resource constraint (15) and \( T_H > T_L \), we can derive

\[
T_H > E^{FB} > T_L. \tag{24}
\]

Therefore, there will be direct cash transfers from the skilled to the unskilled when the tax revenue from skilled workers is large and optimal spending on the quality of education is sufficiently low, namely, if \( p^* \cdot T_H > E^{FB} \).

From rearranging equation (17), we obtain the first-best optimal quality of education as

\[
E^{FB} = p^{FB} \cdot \epsilon_{pE} \cdot \frac{V^H - V^L}{\alpha} + p^{FB} \cdot \epsilon_{pE} \cdot [T_H - T_L], \tag{25}
\]

where \( \epsilon_{pE} = \frac{E}{p'(e,E)} \cdot p^*_E > 0 \) is the elasticity of the probability function \( p(e,E) \) with respect to variation in \( E \). \( E^{FB} \) decreases with the negative skill premium \( V^H - V^L \), but increases with the additional resources available for redistribution \( T_H - T_L \).\(^{13} \) We conclude:

**Proposition 1.** *In a first-best solution, the government always provides full income insurance by ensuring equalized marginal utility of income across skill types. If the optimal quality of education is relatively low \( (E < p^{FB} \cdot T_H) \), there are direct resource (income) transfers from skilled to unskilled households \( (T_L < 0) \).*

In short, income insurance is of major importance relative to improving the quality of education, and by it the probability of success \( p(e,E) \). In the following section, we will characterize second-best efficient policies and finally compare the results to the first-best benchmark.

\(^{13} \)In principle, there can be a corner solution \( E^{FB} = 0 \), in which the government would like to have a negative resource investment in education, if either the negative utility premium \( V^H - V^L \) is too large or the positive gain in tax revenue \( T_H - T_L \) is too small. However, we focus on interior solutions, where \( E^{FB} \geq 0 \) is optimal.
5 Public Policy in a Second-Best World

A benevolent government aims to maximize social welfare. To this end, it can select the quality of the educational system by choosing public spending on education $E$, and it can grant a lump-sum transfer $T$, but it cannot directly control private learning effort. If the government has complete information about the skill and employment status of a worker, overall expenditure $E + T$ can be financed by a skill-specific tax $f_B$ and by a proportional wage tax at rate $t$. If taxes cannot be based on a person’s level of education (i.e., if tagging is impossible) the government must rely on the wage tax only (see Subsection 5.1). We stress again that the educational risk is idiosyncratic; consequently, there is no aggregate risk. From the government’s perspective, there are $p(e^*, E)$ skilled workers supplying $p^* \cdot H^*$ efficiency units of skilled labor and $[1 - p(e^*, E)]$ unskilled workers supplying $(1 - p^*) \cdot L^*$ efficiency units of unskilled labor. Thus, the governmental budget constraint can be written as

$$E + T = p^* \cdot [tw_H H^* + f_B] + (1 - p^*) \cdot tw_L L^*.$$  \hspace{1cm} (26)

Using $E$, the government can directly influence the percentage of skilled workers. Using the tax instruments, it can redistribute income between skilled and unskilled households, also indirectly affecting the proportions of skilled and unskilled workers via incentives for learning effort.

The questions we now seek to answer are: (i) In a second-best world, is income insurance still more important than improving the quality of education? (ii) Does this result depend on the informational assumptions and the (non-)availability of skill-specific taxes? (iii) What is the optimal combination of wage taxes, lump-sum elements, and skill-specific taxes (i.e., graduation taxes) in such an environment?

Formally, the problem can be written as:

$$\max_{\{E, f_B, t, T\}} E[V^*(E, f_B, t, T)] \ \text{s.t.} \ [t \cdot w_H H^* + f_B] p + t \cdot w_L L^* (1 - p) = E + T$$  \hspace{1cm} (27)

Note that the government anticipates the reaction of households when making its choice of the policy mix. Forming the Lagrangian $L$, introducing the Lagrange multiplier $\lambda$, and relying on
envelope effects, first-order conditions are:

\[
\frac{\partial L}{\partial f_B} = -p^* \alpha H + \lambda \left( p^* + p^* t_w H \frac{\partial H^*}{\partial f_B} + [t_w H^* + f_B - t_w L^*] p_e \frac{\partial e^*}{\partial f_B} \right) = 0, \quad (28)
\]

\[
\frac{\partial L}{\partial T} = p^* \alpha H + (1 - p^*) \alpha L + \lambda \left( t \left[ p^* w_H \frac{\partial H^*}{\partial T} + (1 - p^*) w_L \frac{\partial L^*}{\partial T} \right] - 1 \right)
+ \lambda [t_w H^* + f_B - t_w L^*] p_e \frac{\partial e^*}{\partial T} = 0, \quad (29)
\]

\[
\frac{\partial L}{\partial t} = -p^* \alpha H \cdot w_H H^* - (1 - p^*) \alpha L \cdot w_L L^* + \lambda \cdot p^* t_w H \frac{\partial H^*}{\partial t}
+ \lambda \left( (1 - p^*) t w_L \frac{\partial L^*}{\partial T} + [t_w H^* + f_B - t_w L^*] p_e \frac{\partial e^*}{\partial T} \right)
+ \lambda \left( p^* \cdot w_H H^* + [1 - p^*] \cdot w_L L^* \right) = 0, \quad (30)
\]

\[
\frac{\partial L}{\partial E} = p^* \left[ V_H - V_L \right] + \lambda \left( [t_w H^* + f_B - t_w L^*] \left[ p_e \frac{\partial e^*}{\partial E} + p^* \right] - 1 \right) = 0. \quad (31)
\]

In Subsection 5.1, we first derive the optimal tax and education policy when the skill-specific tax is not available. In Subsection 5.2, we then broaden the analysis by allowing for tagging by introducing a skill-specific tax. As we will show, allowing a more accurate redistributive instrument strengthens the overall importance of improving the quality of education.

### 5.1 Optimal Public Investment and Linear Wage Taxation

If the government has limited information, it cannot implement a skill-specific tax and must rely on the linear income tax. FOC (28) is canceled and the parameter \( f_B \) is equal to zero throughout equations (29) to (31).

In this case, we define the net social marginal value of income (including the income effects on the tax base) of a household of type \( j \) as

\[
b^j = \frac{\alpha^j}{\lambda} + t \cdot w_j \cdot Z_j^* \cdot \frac{\partial Z_j}{\partial T} + t \cdot (w_H \cdot H^* - w_L \cdot L^*) \cdot p_e \cdot \frac{\partial e}{\partial T}, \quad j = H, L, \quad (32)
\]

where \( Z_j = H, L \) for \( j = H, L \). The second summand on the RHS of equation (32) represents the loss in tax revenue due to an income-effect induced decrease in labor supply, and the third summand incorporates the revenue effect from taxing the skill premium as the household adjusts its learning effort and thereby its probability of finding employment as a skilled worker. The expected net social marginal value of income is given from (32) by

\[
\bar{b} = \frac{p^* \cdot \alpha H + (1 - p^*) \cdot \alpha L}{\lambda} + p^* \cdot t \cdot w_H \cdot H^* \cdot \frac{\partial H}{\partial T}
+ (1 - p^*) \cdot t \cdot w_L \cdot L^* \cdot \frac{\partial L}{\partial T} + t \cdot (w_H \cdot H^* - w_L \cdot L^*) \cdot p_e \cdot \frac{\partial e}{\partial T}. \quad (33)
\]

Slightly rearranging FOC (29) and inserting the definition of \( \bar{b} \) from equation (33), it is straight-
forward to demonstrate that the expected net social marginal value of income ($\bar{b}$) must equal resource costs (unity), i.e.,

$$\bar{b} = 1.$$  \hfill (34)

In the next step, we define the insurance characteristic as the negatively normalized covariance $\chi$ of net social marginal value of income $b^j$ and labor income $w_j \cdot Z_j$, analogous to Feldstein’s distributational characteristic, measuring society’s concern for risk avoidance. Thus, the insurance effect is given by

$$\chi = - \frac{\text{Cov}(b^j, w_j \cdot Z_j)}{\mathbb{E}[b_j] \cdot \mathbb{E}[w_j \cdot Z_j]} = - \frac{\text{Cov}(b^j, w_j \cdot Z_j)}{b^j \cdot (p^r \cdot w^H \cdot H^* + (1 - p^r) \cdot w^L \cdot L^*)} > 0. \hfill (35)$$

$\chi$ is positive because the net social marginal value of income decreases with increasing income. Moreover, we define

$$\bar{\tau}_{HH} = \frac{p^r \cdot w^H \cdot H^*}{p^r \cdot w^H \cdot H^* + (1 - p^r) \cdot w^L \cdot L^*} \cdot \frac{(1 - t)w^H}{H^*} \cdot S_{HH} > 0 \quad \text{and} \quad (36)$$

$$\bar{\tau}_{LL} = \frac{(1 - p^r) \cdot w^L \cdot L^*}{p^r \cdot w^H \cdot H^* + (1 - p^r) \cdot w^L \cdot L^*} \cdot \frac{(1 - t)w^L}{L^*} \cdot S_{LL} > 0 \hfill (37)$$

as weighted compensated elasticities of labor supply with respect to its net wage, where $S_{jj} > 0$ represents the substitution effect in labor supply $Z_j$. The weights are the proportion of skilled and unskilled labor income, respectively, in aggregate labor income.

The compensated elasticity of learning effort with respect to a change in the expected net wage $(1 - t)\bar{w}$ is calculated as

$$\bar{\epsilon}_{e\bar{w}} = \frac{(1 - t)\bar{w}}{e} \cdot S_{e\bar{w}}, \hfill (38)$$

where $\bar{w} = p^r \cdot w^H + (1 - p^r) \cdot w^L$.

Applying equations (34) to (38), some covariance rules, and the Slutsky-decomposition, FOC (30) can be transformed in order to receive (see Appendix A.1)

$$\frac{t}{1 - t} = \frac{\chi}{\bar{\tau}_{HH} + \bar{\tau}_{LL} + \frac{p^r(w^H - H^* - w^L - L^*)}{p^r \cdot w^H \cdot H^* + (1 - p^r) \cdot w^L \cdot L^*} \cdot \epsilon_{pe} \cdot \left[ \epsilon_{e\bar{w}} - \psi \cdot \eta_{eT} \right]} > 0, \hfill (39)$$

where $\epsilon_{pe} = \frac{e}{p^r(e, E)} \cdot p^e \cdot \eta_{eT} > 0$ is the elasticity of the probability function $p(e, E)$ concerning a change in learning effort $e$; $\psi = \frac{\text{Cov}(w_j, Z_j)}{\bar{w}^j \cdot p^r \cdot H^* + (1 - p^r) \cdot L^*} > 0$, $j = H, L$, is the coefficient of correlation between labor supply and wages, positive as long as labor supply is not backward-bending; and $\eta_{eT} = \frac{\bar{w}^0 - \bar{w}^0}{\epsilon_{e\bar{w}} - \psi \cdot \eta_{eT}} < 0$ is the income elasticity of learning effort with respect to a change in expected net wage income.

As expected, the optimal wage tax rate increases with society’s concern for insurance $\chi$ and the only role of taxation is to insure against educational risk and ex post inequality, because the tax rate would be zero if the insurance characteristic vanishes ($\chi = 0$). Furthermore, the tax rate decreases with induced distortions in skilled and unskilled labor supply, measured by the
elasticities $\varepsilon_{HH}$ and $\varepsilon_{LL}$.

Finally, wage taxation has a negative effect on compensated investment in learning, as

$$
\epsilon_{eH} - \psi \cdot \eta_{eT} > 0. \quad (40)
$$

This effect translates via $\epsilon_{pe}$ in a change in the efficiency of learning technology (that is, in the probability of success). Thus, weighted by the expected skill premium in wages (relative to expected income), the third summand in the denominator of the RHS in equation (39) measures the income- (or revenue-) relevant effect of wage taxation on learning effort. Accordingly, allowing for moral hazard in learning effort and endogenizing the efficiency of the educational system reduce optimal insurance by wage taxation.

Turning to optimal resource investment, the optimal quality of education can be derived from multiplying FOC (31) by $E$, rearranging and recognizing that $t^* > 0$ from (39):

$$
E^* = p^* \cdot \left( \frac{V^H - V^L}{\lambda} \cdot \epsilon_{pE} + t^* \cdot [w_H \cdot H^* - w_L \cdot L^*] \cdot [\epsilon_{pe} + \epsilon_{pe} \cdot \eta_{eE}] \right) > 0, \quad (41)
$$

where $\eta_{eE} = \frac{E}{e} \cdot \frac{\partial e}{\partial E} > 0$ is the elasticity of learning effort with respect to public educational expenditure.

There are three effects, that determine the optimal quality: (i) The first term in the bracket on the RHS of (41) represents both the need for ex ante insurance and the net welfare increase (net of financing costs $\lambda$) by adding a household in the skilled sector due to improved quality of education. Note that, contrary to the first-best solution, the skill premium in (indirect) utility, $V^H - V^L$, must be positive in order to have a positive effort investment $e > 0$ by households. (ii) The second term, $t^* \cdot [w_H \cdot H^* - w_L \cdot L^*] \epsilon_{pE}$, measures increased tax revenue and, accordingly, the self-financing effect, because higher educational investment will increase the number of skilled taxpayers by $\epsilon_{pe}$, taxpayers who earn higher incomes, and also pay additional taxes. (iii) The third term, $t^* \cdot [w_H \cdot H^* - w_L \cdot L^*] \cdot [\epsilon_{pe} \cdot \eta_{eE}]$, illustrates that investing in the quality of education is another way of fostering learning effort $e$ (due to the complementarity in $\eta_{eE}$) and of increasing the probability of success. Again, this increases ceteris paribus the number of skilled taxpayers. Additionally – and more importantly – improved quality of education mitigates the distortions in learning effort caused by implementing a wage tax $t^* > 0$.

Summarizing our discussion in this subsection, we can conclude:

**Proposition 2.** (i) If skill-specific taxes are not available ($f_B = 0$), the government implements a positive wage tax rate $t^* > 0$, balancing income insurance on the one hand and distortions in labor supply and learning effort on the other hand. Moral hazard in learning effort decreases (ex post) insurance by the wage tax. (ii) Optimally improving the quality of education by public spending $E^* > 0$ provides ex ante insurance. Furthermore, mitigation of tax-induced

\footnote{See Appendix A.2 for a proof of the inequality in equation (40).}
distortions in learning effort has a positive effect on selecting the quality of the educational sector.

In order to describe the complete tax policy, we must determine the optimal lump-sum transfer $T^\ast$. From the governmental budget constraint (26), it follows

$$T^\ast = t^\ast \cdot w_L \cdot L^\ast + t^\ast \cdot p^\ast \cdot [w_H \cdot H^\ast - w_L \cdot L^\ast] - E^\ast,$$

(42)

when $f_B = 0$. Inserting the optimal value of $E^\ast$ from (41), we end up with

$$T^\ast = t^\ast \cdot w_L \cdot L^\ast - p^\ast \cdot \frac{V_H - V_L}{\lambda} - t^\ast \cdot p^\ast \left(w_H H^\ast - w_L L^\ast\right)(\epsilon_{pE} + \epsilon_{pe} \cdot \eta_{eE} - 1)$$

(43)

and it turns out that unskilled workers are also net taxpayers, as long as the learning technology is sufficiently elastic with respect to public spending (i.e., as long as $\epsilon_{pE} + \epsilon_{pe} \cdot \eta_{eE} > 1$). In this case, all households must pay for the educational system ($T^\ast < t^\ast \cdot w_L \cdot L^\ast$ or $T^\ast < 0$ even) and unskilled workers are made better off ex ante by increasing the quality of education due to (strong) investment in universities, rather than by providing income transfers ex post.

In all, the optimal tax and education policy when skill-specific taxes are unavailable can be summarized as decreasing the variance in income by implementing a wage tax and attempting to get as many households as possible employed in the skilled sector – constraint by direct resource costs and excess burden caused. This provides ex ante insurance by increasing the probability of success and ex post insurance by decreasing income variation. Both effects increase the expected utility of each household. However, as long as the learning technology is (sufficiently) imperfect, (ex post) net cash transfers from skilled to unskilled workers are (very) expensive compared to ex ante improving the quality of education, since the latter also reduces distortions.

However, if it does not hold that learning effort and public spending are complementary inputs in the production function $p = p(e, E)$, this finding will change. If $p_{eE} < 0$, then $\eta_{eE} < 0$ and an enhanced quality of education will reduce private learning effort. The government then has an incentive to reduce its spending in order to encourage more learning effort $e$. In this case, not only insurance by wage taxation but also the quality of education are ceteris paribus reduced due to moral hazard in learning effort. Nevertheless, providing ex ante insurance still calls (ceteris paribus) for an increase in the quality of the educational sector.

Comparing the results to standard Levhari-Weiss models, the main difference in our setting is that the government can directly influence probability of success, viz., the magnitude of risk. In papers based on Levhari and Weiss (1974), risk is either alleviated or boosted by educational investment, and this insurance effect is exploited by households. Consequently, the government can provide social insurance by taxation and (ex post) transfers, and it can affect self-insurance by households – either by implementing direct subsidies (Anderberg, 2009), by indirect subsidies via capital taxation (Hamilton, 1987), or by direct control of education.
(Anderberg and Andersson, 2003). However, subsidizing education does not have a (stand-alone) insurance effect and can only mitigate efficiency costs in these models (see Jacobs et al., 2010). Our results demonstrate a dramatic difference, if the government can affect the quality of education and if this quality enhances the probability of positive economic outcomes.

In the next subsection, we will extend the instruments of the government to include a skill-specific tax, and we will show that this encourages the focus on the quality of education even more.

5.2 The Case of Tagging: Skill-Specific Taxes

If the government can utilize information on the outcome of the educational process, households can be “tagged” (Akerlof, 1978) and tax instruments can be based on the skill status. In our framework, this implies that the skill-specific tax \( f_B \) can be used and the full set of first-order conditions (28) to (31) is now relevant. However, the government is still unable to control learning effort. The optimal wage tax rate can be inferred from (see Appendix A.3)

\[
\frac{t}{1-t} \cdot [\epsilon_{HH} + \epsilon_{LL}] = 0. \tag{44}
\]

and we derive the following lemma:

**Lemma 1.** If the government can apply the skill-specific tax \( f_B \) and if the government simultaneously has access to an unconstrained lump-sum transfer \( T \), the use of a proportional wage tax is not optimal to; consequently, \( t^* = 0 \).

**Proof.** Unconstrained lump-sum transfer implies that this transfer can become negative and can be used in order to finance public educational spending. In this case, we can apply the above calculations, and obtain \( t^* = 0 \) directly from (44), because the income-weighted compensated elasticities \( \epsilon_{jj}, j = H, L \), are positive, thus the squared bracket in (44) is positive. \( \square \)

Not utilizing from the wage tax avoids labor supply distortions, but also forgoes income insurance. Thus, insurance must be provided by combining skill-specific taxes and investment in the quality of education. This can be shown by making use of Lemma 1 in order to first define the net social marginal value of income (including income effects on the tax base) as

\[
b^L = \frac{\alpha_L}{\lambda} + f_B \cdot p_e \cdot \frac{\partial e}{\partial T} \tag{45}
\]

for unskilled workers and

\[
b^H = \frac{\alpha_H}{\lambda} + f_B \cdot p_e \cdot \frac{\partial e}{\partial T} \tag{46}
\]

for skilled workers. The expected net social marginal value of income is

\[
\tilde{b} = p^* \cdot \frac{\alpha^H}{\lambda} + (1 - p^*) \cdot \frac{\alpha^L}{\lambda} + f_B \cdot p_e \cdot \frac{\partial e}{\partial T}. \tag{47}
\]
From these definitions and FOC (29), it follows that the expected net social marginal value of income must still be
\[ \bar{b} = 1. \] (48)

Applying \( t^* = 0 \) in the first-order condition (28) results in
\[ p^* \cdot \left[ \frac{\alpha^H}{\lambda} - 1 \right] = f_B \cdot p_e \cdot \frac{\partial \varrho}{\partial f_B}. \] (49)

Inserting equations (48) and (47) on the LHS of equation (49), this implies (after some rearrangements)
\[ f_B^* = (1 - p) \cdot \frac{b^L - b^H}{(-\epsilon_{eB}) \cdot \epsilon_{pe}} > 0. \] (50)

where we defined \( \epsilon_{eB} = \frac{\partial e}{\partial f_B} + p \cdot \frac{\partial e}{\partial f} < 0 \) as the compensated elasticity of learning effort with respect to skill-specific taxes.\(^{15}\) The optimal skill-specific tax increases with income inequality, measured by \( b^L - b^H \). It decreases with the excess burden caused by distorting learning effort and by reducing the probability of success.

Relying again on the definitions of the elasticities \( \epsilon_{pe}, \epsilon_{pE} \) and \( \eta_{eE} \), we find from rearranging equation (31)
\[ E^* = p^* \cdot \left[ \epsilon_{pE} \cdot \frac{V^H - V^L}{\lambda} + f_B^* \cdot (\epsilon_{pE} + \epsilon_{pe} \cdot \eta_{eE}) \right], \] (51)
and can state

**Proposition 3.** If tagging is possible, social insurance is provided ex post by a skill-specific tax \( f_B^* > 0 \) and ex ante by improving the quality of education \( (E^* > 0) \). The latter is also inevitably necessary in order to alleviate severe distortions in learning effort caused by skill-specific taxation.

**Proof.** See Appendix A.4.\( \square \)

Income redistribution is implemented by skill-specific taxes that must be paid by successful workers only. Contrary to standard models featuring risky human capital and taxation (e.g., Eaton and Rosen, 1980; Hamilton, 1987; Anderberg and Andersson, 2003) the wage tax is not used, as the possibility of tagging via skill-specific taxes provides a very efficient instrument to redistribute from high income to low income groups that does not cause any excess burden in labor supply. Since there is no intra-sector heterogeneity in our framework, there is no need for intra-sector taxation; skill-specific taxes ensure balanced resources across both sectors in order to equalize the marginal social value of additional resources. These results are fully in line with the literature on tagging (e.g., Akerlof, 1978; Immonen et al., 1998), but their application is less trivial than it seems at first glance: Via learning effort, households can influence the probability

\(^{15}\)Note that we implicitly assume the optimal \( f_B \) to be within the limits of \( V^H > V^L \); this is necessary for \( e^* > 0 \) to occur. If this condition does not hold, there would be a corner solution.
of success (i.e., the category to which they are assigned by the government) and skill-specific taxes induce a strong substitution effect in learning effort, since graduation becomes less attractive. The latter becomes very clear in inspection of equation (50), in which gains from redistribution (the numerator) are traded against distortions in education (the denominator).

The standard intuition for second-best policies would tell us to balance distortions and overall excess burden on all margins by using several distorting instruments (cf. Lipsey and Lancaster, 1956). However, this is not the case in our setting. The skill-specific tax narrows the income gap and raises tax revenue. The distortions in learning effort caused by the skill-specific tax are alleviated by increased public funding of the educational sector. This mitigating effect is similar to the result in Bovenberg and Jacobs (2005) and Jacobs and Bovenberg (2010). In order to avoid major inefficiencies when redistributing from skilled to unskilled workers, subsidies are necessary. In these previous papers, direct subsidies are granted; however, in our model, the government subsidizes education indirectly via improved learning technologies and an increased quality of education. Indeed, the more the government spends on education, the higher a) the private learning effort (ceteris paribus) and b) the probability for each household of graduation into the skilled sector. Accordingly, spending tax revenue on the quality of education provides (ex ante) insurance against educational risk by increasing the probability of success for all households.

The combination of both instruments (skill-specific taxation and public spending) consequently ensures an optimal social insurance package. Since the outlined policy results in lower costs than a wage tax (linear income tax), efficiency in labor supply need not be sacrificed. Public spending in the educational sector is increasingly important, the more it improves learning technology and the more elastic learning effort is. In particular, as long as the filter technology is very imperfect (i.e., as long as \( p(e, E) \) is low), providing sufficient quality of education and sufficient learning technology is of significant importance. Note, however, that the same caution from the previous subsection applies if learning effort and public spending are substitutes \((\eta_e E < 0)\).

To complete the model, we must determine the optimal lump-sum transfer. For \( t^* = 0 \), the governmental budget constraint reduces to \( E^* + T = p^* \cdot f_B^* \); by substituting for \( p^* \cdot f_B^* \) in equation (51), we end up with

\[
T = \frac{1 - (\epsilon_p E + \epsilon_p e \cdot \eta_e E)}{\epsilon_p E + \epsilon_p e \cdot \eta_e E} \cdot E^* - p^* \cdot \epsilon_p E \cdot \frac{V^H - V^L}{\lambda}. \tag{52}
\]

Redistributing ex ante requires ceteris paribus a negative lump-sum transfer, since \( p^* \cdot \epsilon_p E \cdot \frac{V^H - V^L}{\lambda} > 0 \). This effect is strengthened if the education system is (very) imperfect, that is, if learning technology is elastic \((\epsilon_p E + \epsilon_p e \cdot \eta_e E > 1)\). Both skilled and unskilled workers are then taxed and it follows again that it is first and foremost important to establish a proper education quality (i.e., to improve the ‘filter technology,’ as Konrad (2004) puts it). Thus, even if more information is available to the government and if it can apply skill-specific taxes, the quality of
education plays a major role. This strengthens the corresponding result in Subsection 5.1.

In our model, a negative lump-sum transfer $T^* < 0$ can be interpreted as a general tuition fee that must be paid irrespective of whether a student is successful in educational investment. Overall tuition fees are then given by $F = -T + f_B$, but these will only be paid in full by the skilled workers. Our model and our results thus argue in favor of general tuition fees that are only partially insured by a skill-specific (graduation) tax.

Our graduate-tax structure complements findings in Cigno and Luporini (2009), which examined a self-financing educational sector. This analysis showed that a scholarship scheme financed by a graduate tax is optimal for insurance and for overcoming borrowing constraints in imperfect insurance and capital markets. Both in this study and in our study, moral hazard prevents full insurance. Note, however, that tagging places high demands on the commitment technology. If full commitment were weakened, skill-specific taxation could lead to a severe hold-up problem and adverse welfare effects. Consequently, incomplete information (in our model, linear income taxation without tagging) can be beneficial in order to overcome potential underinvestment in education, see Konrad (2001).

5.3 Comparison to the First-Best: Quality of Education Matters

Finally, we address the question of direct income insurance versus insurance via improved quality of education. We have seen in the first-best analysis that direct income insurance is always guaranteed by equalized (ex post) marginal utilities of income. Resource investment in order to improve the quality of the educational sector fosters tax revenue by increasing the number of skilled workers ($T^H > T^L$), but this comes at the cost of adverse redistributive effects (i.e., of increasing the exposure to risk), since $V^H - V^L < 0$ in the first-best solution; see equation (25).

All this changes dramatically if the government has only limited instruments available, as it cannot control private learning effort. In such a second-best setting, any direct income redistribution has negative incentive effects on learning effort and causes moral hazard, because the government cannot control, how students spent their time at university. Therefore, endogenous probabilities of success render taxation more expensive, because not only labor supply may be distorted. Taken together, these effects reduce both the wage tax and skill-specific taxes (see Propositions 2 and 3) and weaken the case for standard income insurance à la Eaton and Rosen (1980).

At the same time, new effects emerge that call for an improved quality of education by increased public spending in the educational sector. First of all, the quality of education turns into an ex ante insurance device. The skill premium, measured in utility $V^H - V^L$, turns positive in a second-best optimum, and improved learning technology reduces the risk of adverse outcomes in the educational process. Ceteris paribus, this calls for an improved quality of education, as an additional graduate increases social welfare.
Secondly, increased public spending on education still increases tax revenue (see $p^* \cdot t^* \cdot \left[ w_H H^* - w_L L^* \right] \cdot \epsilon_p > 0$ in equation (41) and $p^* \cdot f_B \cdot \epsilon_p > 0$ in equation (51)). Thirdly, it is necessary to alleviate the moral hazard effect caused in learning by the tax instruments; see $p^* \cdot t^* \cdot \left[ w_H H^* - w_L L^* \right] \cdot \epsilon_p \cdot \eta > 0$ in equation (41) and $p^* \cdot f_B \cdot \epsilon_p \cdot \eta > 0$ in equation (51). The more private investment in learning is sensitive to tax instruments and public investment, the stronger the call for better quality of education.16

Combining all the effects described above, we can state:

**Proposition 4.** Compared to the first-best solution, increasing the quality of education in a second-best world significantly gains in importance relative to providing income insurance, because providing (full) insurance via taxation and transfers becomes (too) expensive, and because the quality of education becomes an insurance device, additionally correcting for inefficiencies in private learning effort (moral hazard).

This result is robust and independent of informational assumptions. It holds even if the government can implement skill-specific taxes via $f_B$. Indeed, the call for improving the quality of education is at the heart of all our results, as it reduces efficiency losses from distorting learning effort and simultaneously provides ex ante insurance.

### 6 Conclusions

This paper has demonstrated that to insure against educational risk in a second-best world, improving the quality of education in order to increase the probability of success in graduation is of major importance. Compared to a first-best solution, redistribution in an ex ante sense significantly gains in importance relative to spending resources for ‘healing’ bad outcomes ex post. The primary message from this analysis is that a society should set up a proper educational system and enhance the ‘filtering mechanism’ of the educational system before it engages in income transfers ex post. This is particularly true when ex post redistribution becomes costly due to distortions and in particular if there is moral hazard (as the government cannot control how time at school or at university is spent). As the proverb says, “It is too late to shut the stable door after the horse has bolted.”

Our results have strong policy relevance, as they provide a theoretical underpinning and justification for the EU’s Lisbon agenda and its successors (including, “EU-2020”). These results hold as long as the government cannot control private learning effort and they are independent of the availability of skill-specific taxes. Indeed, if the government can utilize information on the skill status, the call for an improved quality of education is strengthened.

16By comparing the first-best level of learning effort in equation (16) with the household’s first-order condition (8), the last effect can have a second interpretation: Households neglect the resource-increasing effect on behalf of the government (i.e., the last term in equation (16)). Accordingly, this underinvestment must be countered by increased public spending. This need would persist even in a setting with private resource investment in the quality of education.
Our results should also be of relevance in light of concern over the rising wage dispersion between skilled and unskilled workers driven by globalization (Krugman, 1995) and skill-biased technological change (Katz and Autor, 1999). These factors are seen as putting pressure on low-skilled wages and favoring skilled labor (the ‘American way’) or as creating unemployment in the low-skilled sector when there are labor market rigidities (the ‘European way’), see Krugman (1995). Instead of focusing on pure ex post redistribution measures, the government could intervene in these cases as well by first improving the educational sector.

\[ \text{A Appendix} \]

\[ \text{A.1 Derivation of Equation (39)} \]

Inserting \( f_B = 0 \) and rearranging FOC (30), we obtain

\[
\frac{p^* \alpha^H w_H H^* + (1 - p^*) \alpha^L w_L L^*}{\lambda} - p^* w_H H^* + (1 - p^*) w_L L^* = p^* \ t \ w_H \ \frac{\partial H}{\partial t} + (1 - p^*) \ t \ w_L \ \frac{\partial L}{\partial t} + t \ (w_H H^* - w_L L^*) \ p_e \ \frac{\partial e}{\partial t} \quad (53)
\]

Steiner’s rule implies

\[
\text{Cov}(b^i, w_j Z_j) = E[b^i \cdot w_j Z_j] - E[b^i] \cdot E[w_j Z_j] = \frac{p^* \alpha^H w_H H^* + (1 - p^*) \alpha^L w_L L^*}{\lambda} + p^* \ t \ w_H \ \frac{\partial H}{\partial T} \ w_H H^* + (1 - p^*) \ t \ w_L \ \frac{\partial L}{\partial T} \ w_L L^* + t \ (w_H H^* - w_L L^*) \ p_e \ \frac{\partial e}{\partial T} \quad (54)
\]

because \( E[b^i] = 1 \) from equation (34).

Adding \( p^* \cdot t \cdot w_H \cdot \frac{\partial H}{\partial T} \cdot w_H H^* + (1 - p^*) \cdot t \cdot w_L \cdot \frac{\partial L}{\partial T} \cdot w_L L^* + t \cdot (w_H H^* - w_L L^*) \cdot p_e \cdot \frac{\partial e}{\partial T} \ [p^* \cdot w_H H^* + (1 - p^*) \cdot w_L L^*] \) on both sides of (53), we can make use of Steiner’s rule (54) on the LHS of (53) in order to obtain

\[
\text{Cov}(b^i, w_j Z_j) = p^* \ t \ w_H \left[ \frac{\partial H}{\partial t} + \frac{\partial H}{\partial T} \cdot w_H H^* \right] \quad (55)
\]

where \( E[w_j Z_j] = [p^* \ w_H H^* + (1 - p^*) \ w_L L^*] \).
The Slutsky decompositions imply

\[
\frac{\partial H}{\partial t} = \left[ S_{HH} + H^* \cdot \frac{\partial H}{\partial T} \right] \cdot (-w_H), \tag{56}
\]

\[
\frac{\partial L}{\partial t} = \left[ S_{LL} + L^* \cdot \frac{\partial L}{\partial T} \right] \cdot (-w_L), \tag{57}
\]

\[
\frac{\partial e}{\partial t} = \left[ S_{e\bar{w}} + \bar{Z} \cdot \frac{\partial e}{\partial T} \right] \cdot (-\bar{w}), \tag{58}
\]

where \( S_{jj} > 0 \) represents the substitution effect with regard to the own wage and \( S_{e\bar{w}} \) is the substitution effect of a change in the expected wage \( \bar{w} \) on learning effort. Moreover, \( \bar{Z} = \mathbb{E}[Z_j] = p^* H^* + (1 - p^*) L^* \) and \( \bar{w} = \mathbb{E}[w_j] = p^* w_H + (1 - p^*) w_L \).

When we revert to equations (56) to (58) and cancel income effects in equation (55) where possible, we are left with

\[
\text{Cov}(b^j, w_j Z_j) = p^* \cdot t \cdot w_H \cdot S_{HH} \cdot (-w_H) + (1 - p^*) \cdot t \cdot w_L \cdot S_{LL} \cdot (-w_L) \tag{59}
\]

\[
+ \cdot t \cdot (w_H H^* - w_L L^*) \cdot p_e \cdot \left[ S_{e\bar{w}} \cdot (-\bar{w}) + \left( \mathbb{E}[w_j Z_j] - \bar{w} \cdot \bar{Z} \right) \cdot \frac{\partial e}{\partial T} \right].
\]

Utilizing Steiner’s rule again, the parenthesis in the last line of equation (59) turns into

\[
\mathbb{E}[w_j Z_j] - \bar{w} \cdot \bar{Z} = \text{Cov}(w_j, Z_j),
\]

and dividing equation (59) on both sides by expected wage income \( \mathbb{E}[w_j Z_j] = p^* \cdot w_H \cdot H^* + (1 - p^*) \cdot w_L \cdot L^* \), multiplying by (-1), and applying the definitions in equations (35) to (38) as well as

\[
\psi = \frac{\text{Cov}(w_j, Z_j)}{\bar{w} \cdot [p^* \cdot H + (1 - p^*) \cdot L]} > 0 \quad \text{and} \quad \eta_{eT} = \frac{(1 - t) \cdot \bar{w} \cdot \bar{Z}}{e} \cdot \frac{\partial e}{\partial T} < 0
\]

results in

\[
\chi = \frac{t}{1 - t} \cdot \left\{ \epsilon_{HH} + \epsilon_{LL} + \frac{p^* \cdot (w_H H^* - w_L L^*)}{p^* \cdot w_H H^* + (1 - p^*) \cdot w_L L^*} \cdot \epsilon_{pe} \cdot [\epsilon_{e\bar{w}} - \psi \cdot \eta_{eT}] \right\}. \tag{60}
\]

As the squared bracket on the RHS is positive from equation (40) and all other terms are positive by definition, the optimal tax rate \( t^* \) must be positive as well. This then proves equation (39).

**A.2 Derivation of Equation (40)**

From the steps in Appendix A.1 and equations (60) and (55), it is inferred that

\[
\epsilon_{e\bar{w}} - \psi \cdot \eta_{eT} = - \left[ \frac{\partial e}{\partial t} + \frac{\partial e}{\partial T} \cdot \mathbb{E}[w_j Z_j] \right]. \tag{61}
\]
By inserting comparative statics results from (12) and (9), this transforms into
\[
\epsilon_{e\theta} - \psi \eta_{eT} = \frac{p_e \cdot \left\{ \alpha^L w_LL^* - \alpha^H w_HH^* + \left( \alpha^H - \alpha^L \right) E[w_jZ_j] \right\}}{SOC(e)}
\]
\[
= - \frac{p_e \cdot \left\{ \left( (1 - p^*) \alpha^H + p \alpha^L \right) (w_HH^* - w_LL^*) \right\}}{SOC(e)} > 0, \quad (62)
\]
where \(E[w_jZ_j] = p w_HH^* + (1 - p^*) w_LL^*\) and \(SOC(e)\) represents the second-order condition of optimal household learning effort.

The inequality in (62) stems from the fact that \(SOC(e) < 0\), \(w_HH^* > w_LL^*\) due to assuming agent monotonicity, \(p \in [0, 1]\) and \(p_e, \alpha^H, \alpha^L > 0\). This proves equation (40).

### A.3 Derivation of Equation (44)

We solve FOC (28) for \(p^* \cdot \alpha^H\) and respectively solve (29) for \(-(1 - p^*) \cdot \alpha^L\). Substituting both rearranged expressions into FOC (30) results in
\[
(w_LL^* - w_HH^*) \cdot \lambda \cdot \left\{ p^* + p_e \cdot A \cdot \frac{\partial e}{\partial f_B} + p^* \cdot tw_H \cdot \frac{\partial H^*}{\partial t} \right\}
\]
\[
+ w_LL^* \cdot \lambda \cdot \left\{ (1 - p^*) \cdot tw_H \cdot \frac{\partial H^*}{\partial t} + (1 - p^*) \cdot tw_L \cdot \frac{\partial L^*}{\partial t} \right\} \quad (63)
\]
\[
= 0,
\]
where \(A = t \cdot [w_HH^* - w_LL^*] > 0\). After collecting terms and simplifying, we apply the Slutsky equations for the derivatives of labor supplies \(H^*\) and \(L^*\). Note that the derivatives of decision variables for the lump-sum transfer \(T\) are pure income effects, and that \(\frac{\partial H^*}{\partial f_B} = -\frac{\partial H^*}{\partial \lambda}\).

Canceling income effects and rearranging leads to
\[
p_e \cdot A \cdot \left\{ (w_LL^* - w_HH^*) \cdot \frac{\partial e}{\partial f_B} + w_LL^* \cdot \frac{\partial e}{\partial t} + \frac{\partial e}{\partial t} \right\}
\]
\[
- t \cdot \left\{ p^* \cdot w_H^2 \cdot S_{HH} + (1 - p^*) w_L^2 \cdot S_{LL} \right\} = 0, \quad (64)
\]
with \(S_{jj}, j = H, L\) as the substitution effect of labor supply when wage change.

Applying the comparative-static results from equations (9), (10), and (12), we find that
\[
(w_LL^* - w_HH^*) \cdot \frac{\partial e}{\partial f_B} + w_LL^* \cdot \frac{\partial e}{\partial t} + \frac{\partial e}{\partial t} = 0. \quad (65)
\]

Utilizing this relationship in equation (64), extending the LHS by \(\frac{1}{1 - t}\), dividing the entire equation by aggregate income \(p^* w_HH^* + (1 - p^*) w_LL^*\), and relying on the definitions of compensated wage elasticities in equations (36) and (37), (64) reduces to equation (44) in the text.
A.4 Proof of Proposition 3

As \( p \in [0, 1) \), \( \epsilon_{pe} > 0 \) by definition and \( b^L - b^H > 0 \) from agent monotonicity, it follows that

\[
\text{sign} (f_B) = \text{sign} (-\epsilon_{efB}) \tag{66}
\]

from equation (50). Examining the results of comparative-statics, we find

\[
\epsilon_{efB} = \frac{\partial e}{\partial f} + p^* \cdot \frac{\partial e}{\partial T} < 0, \tag{67}
\]

because \( \frac{\partial e}{\partial f}, \frac{\partial e}{\partial T} < 0 \) from (10) and (9), respectively. Hence, \( f_B > 0 \).

When \( f_B > 0 \), it follows from equation (51) that the optimal public spending in the educational sector must be positive, because all elasticities and the marginal costs of tax revenue \( \lambda \) are positive, and an interior solution for learning effort \( e \) requires \( V^H > V^L \).\(^{17}\) Furthermore, \( E^* \) increases with \( f^*_B \cdot (\epsilon_{pE} + \epsilon_{pe} \cdot \eta_e) \), the distortions caused by skill-specific taxation.

References


\(^{17}\)Recall that \( V^H \leq V^L \) cannot occur as long as households choose learning effort, because this would imply \( e = 0 \), and \( p^*(0, E) = 0 \), which cannot be socially optimal as long as \( w_H > w_L \).


