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Quality Distortions in Vertical Relations*

Pio Baake† Vanessa von Schlippenbach‡

April 2011

Abstract

This paper examines how delivery tariffs and private quality standards are determined in vertical relations that are subject to asymmetric information. We consider an infinitely repeated game where an upstream firm sells a product to a downstream firm. In each period, the firms negotiate a delivery contract comprising the quality of the good as well as a non-linear tariff. Assuming asymmetric information about the actual quality of the product and focusing on incentive compatible contracts, we show that from the firms’ perspective delivery contracts lead to more efficient contracts and thus higher overall profits the lower the firms’ outside options, i.e. the higher their mutual dependency. Buyer power driven by a reduced outside option of the upstream firm enhances the efficiency of vertical relations, while buyer power due to an improved outside option of the downstream firm implies less efficient outcomes.

JEL-Classification: D82, L14, L15

Keywords: Quality Uncertainty, Private Standards, Vertical Relations, Buyer Power

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1 Introduction

Consumer goods typically undergo an extended transformation process based on the use of numerous inputs before reaching final consumer markets. The quality of consumer goods, thus, relies upon the quality of all inputs used in the value chain. Besides faulty inputs, inappropriate handling at each transformation stage as well as careless storage and transportation can also cause product failures. Firms that can not effectively control for their suppliers’ quality run the risk of litigation\(^1\) as well as damage to brand names and reputations. Moreover, the costs for late failure elimination can be substantial. For example, DaimlerChrysler recalled about 1.3 million cars in order to check the battery control unit software for the electrical and braking systems as well as the voltage regulator in the alternator in 2005.\(^2\) Two years later, Mattel recalled about 18 million toys that were produced in China because of small dislodgeable magnets as well as toxic lead paint.\(^3\) More recently, the Chinese Melamine-scandal forced manufacturers such as Arla, Nestle and Cadbury to recall their products in a worldwide action.\(^4\) Quality agreements between the trading partners along the chain help to mitigate the risk of quality failures by complementing more basic public standards. \textit{Inter alia}, these clarify quality specifications and how these are met and define each trading partners’ responsibilities. These so called private quality standards can be either developed on a business-to-business (e.g. Global GAP, SQF100) or a business-to-consumer basis (for example, Tesco’s Nature’s Choice and Carrefour’s Filière

\(^{1}\)Several product safety regulations like the Consumer Product Safety Act in the US or the European Directive on General Product Safety (2001/95/EC) require manufacturers and retailers to only sell products that are safe. Furthermore, there are also specific requirements that apply to specific types of products (e.g. food, pharmaceuticals, and medical devices).

\(^{2}\)See http://www.dw-world.de/dw/article/0,1543346,00.html, November 2, 2010.

\(^{3}\)In particular, the recall of Mattel induced questions about the safety of products that were manufactured in China. However, Beamish and Bapuji (2008) find that most of the recalls were due to design flaws, although the importance of manufacturing flaws has increased over the last years.

Qualité), whereas business-to-consumer standards play an increasingly important role.\(^5\) Quality agreements are made in all manufacturing industries, and are especially common in the food sector.

In politics, just as in economics, private quality standards are receiving growing attention (e.g. OECD 2006). Although private standards can improve the efficiency of the food system, it may also establish entry barriers and may thus lead to exclusionary effects. It is heavily debated whether private standards may harm suppliers, in particular smallholders in developing countries.\(^6\) Against this background, we examine how delivery tariffs and private quality standards are determined in vertical relations that are subject to asymmetric information. In order to capture the implied incentive problems in a long-term buyer-seller relationship, we analyze an infinitely repeated game. At the beginning of each period, an upstream seller negotiates with a downstream buyer about a delivery contract that comprises delivery tariffs as well as quality requirements. The latter may refer to a quality agreement, which can be determined either jointly by both firms or unilaterally by the downstream firm. Given the negotiation outcome, the upstream firm starts to produce the input. Depending on various external factors like the workers’ mood, weather conditions or the infestation by pests in agricultural production, the product quality is randomly determined in each period. By spending effort, the upstream firm reduces the danger of product damage. For example, the more the upstream firm invests in smart and careful monitoring systems, the higher the probability of meeting the quality requirements specified in the delivery contract. However, despite the implementation of a monitoring system, the production process at the manufacturers’ level can run into failures. The actual quality can only be observed by the upstream firm. The downstream firm, in turn, neither observes the upstream firm’s effort nor can it verify the actual quality of the product. This is due to the fact, that it may be prohibitively expensive to fully control the complete batch even though new developed testing technologies will provide better information at lower cost and in

\(^5\)Tesco Nature’s Choice—the quality program of the UK’s largest retailer—was implemented in 1991 in order to ensure that fruits and vegetables are grown to high safety, quality and environmental standards. Likewise, Carrefour, Europe’s largest retailer, developed a quality supply chain label guaranteeing that products meet a specified level of quality at each production stage. This program is applied to about 250 supply chains, covering more than 35,000 producers.

\(^6\)For detailed case studies, see Balsevich et al. (2003) and Boselie et al. (2003).
a shorter time.

Given this framework of hidden action and hidden information, we analyze incentive compatible delivery contracts that guarantee that the upstream firm truthfully reveals the quality of the product and that firms continue the business relationship. We show that both the delivery tariffs and the quality negotiated between the firms are efficient, i.e. maximize the joint profit of both contracting parties, if the mutual dependency between the vertically related firms is high enough. The lower the firms’ outside options, i.e. the profits they could earn with alternative trading partners, the more firms depend upon each other. However, the greater the firms’ outside options, the more delivery tariffs and the target quality must be distorted in order to satisfy the upstream firm’s incentive constraints. While wholesale prices are monotonically increasing in the firms’ outside options, the negotiated quality is a u-shaped function of the firms’ outside options. Key for these results is that the upstream firm’s incentive constraint prevents efficient risk sharing between the firms. To reduce the implied efficiency losses and to increase the upstream firm’s investment incentives, wholesale prices must be distorted upwards. The higher the wholesale price the upstream firm gets when complying with the quality requirements, the more the upstream firm will invest. Furthermore, the lower the quality requirements the more likely the upstream firm can meet the requirements by increasing its investment. Hence, as long as wholesale prices are only little distorted, the quality requirements are inefficiently low. However, highly distorted wholesale prices may well induce the firms to increase the quality requirements as the upstream firm’s investment incentives are increasing in the wholesale price. Correspondingly, high investment incentives implied by high wholesale prices allow the firms to increase the quality requirements without reducing the probability of meeting these requirements too much.

Our paper contributes to the wide literature on buyer power by analyzing its sources and consequences for the overall efficiency of vertical relations. Particularly, the size of retailers and the degree of concentration in retail markets constitutes a common source of buyer power. If a retailer is large enough, it can credibly threaten to integrate backwards and, thereby, render the supplier redundant (Katz 1987, Sheffman and Spiller 1992). Inderst and Wey (2007) argue that when bargaining breaks down with a large buyer, the supplier finds it difficult to unload

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7 For a survey of the sources and consequences of buyer power, see Inderst and Mazzarotto (2008) as well as Inderst and Shaffer (2008).
the cancelled demand onto the remaining buyers as this involves “marching down” the declining marginal surplus functions. Considering the consequences of buyer power, Inderst and Shaffer (2007) argue that a retail merger makes single sourcing strategies more profitable and can thus lead to lower product differentiation (see also Chen 2004). Battigalli et al. (2007) focus on a supplier’s incentive to invest in quality improvements and show that buyer power leads to lower investments. Analyzing capacity investments, Montez (2008) finds that upstream firms may choose higher capacities when buyers merge as long as the costs of capacity are sufficiently low. In Inderst and Wey (2009) suppliers invest in order to improve their efficiency such that a large buyer—threatening to integrate backwards—meets stronger competitors. Inderst and Wey (2003, 2007) show that suppliers can respond to the emergence of large buyers by incremental innovations in order to cope better with losing a large fraction of the market in the case of negotiation break-down. In contrast to the existing literature we analyze buyer power allowing for asymmetric information in vertical relations. Investigating the quality requirements of a downstream firm, our results point out that delivery contracts are more efficient the lower the firms’ outside options. Thus, buyer power based on credible threats to vertically integrate or on global sourcing strategies induces less efficient contracts, while buyer power due to lower outside options of the suppliers leads to more efficient contracts.

We also contribute to the literature on quality standards. So far, most of the economic literature refers to public minimum quality standards. Only a small strand of literature focuses on private standards. For example, Giraud-Héraud et al. (2003) show that the retailer’s incentive to differentiate their businesses by premium private labels are higher, the lower the public minimum standard. While they ignore asymmetric information as well as quality uncertainties, we take them explicitly into account when analyzing the vertical negotiations. Thereby, we refer to the standard literature on contracting with imperfect information. Our assumption that the upstream firm’s investment are not observable corresponds to the classical principal

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8So far, however, most economic literature refers to public minimum quality standards. Ronnen (1991) shows that a minimum quality standard that is set by the government reduces differentiation among competitors and, thus, leads to more intense competition. Crampes and Hollander (1995) approve these results for variable costs of differentiation. Furthermore, Ecchia and Lambertini (1997) show that minimum quality standards impede collusion. These results are, however, only robust for price competition. If firms compete in quantities, minimum quality standards decrease overall welfare (Valletti 2000).
agent models with moral hazard.\(^9\) Furthermore, we assume that the downstream firm can not observe the actual quality of the upstream firm’s product. Therefore, delivery contracts must be based on the upstream firm’s announcement about the realized quality of the product. The implied incentive constraint for truthful revelation rests on repeated interactions and is similar to the incentive constraints analyzed in hidden information models with observable output (see for example Baron and Myerson 1982). Considering the dynamic structure of our model, we assume that periods are independent and focus on stationary contracts (see Fudenberg et al. 1990). Our model thus combines the classical moral hazard problem with deviation incentives analyzed in the literature on reputation and collusion (see for example Milgrom and Roberts 1982, Schmidt 1993, and Feuerstein 2005).

The remainder of the paper is organized as follows: In Section 2 we specify our model. Section 3 focuses on the downstream firm’s pricing decisions as well as the upstream firm’s incentives to invest and to truthfully announce the quality of the product. In Section 4, we analyze the bargaining process in the intermediate good market and consider the quality choice of the downstream firm. Section 5 presents an example that illustrates our results. Finally, we conclude and discuss our findings.

2 The Model

We analyze an infinitely repeated game where an upstream firm, \(U\), sells an input to a downstream firm, \(D\), that transforms the input on a one-to-one basis into a final consumer good, \(x\). The quality of good \(x\) relies on the quality, \(\theta\), of the input as well as on some effort of the downstream firm in handling, transportation and storage of the good. For simplicity, we focus on \(\theta\) and take the downstream firm’s effort to secure the quality of good \(x\) as given. At the beginning of each period firms \(U\) and \(D\) negotiate a delivery contract that is supposed to be binding. The contract comprises a target quality, \(\bar{\theta}\), for the input as well as a two-part delivery tariff \(T\) consisting of a wholesale price, \(w\), and a fixed fee, \(F\). Focusing on quality uncertainty at the upstream level, we assume that the input’s quality, \(\theta\), is stochastically determined in each period. The upstream firm, \(U\), can increase the probability of complying with the target quality by investing some effort, \(e\). We further assume that the downstream firm, \(D\), neither observes

\(^9\)See for example Holmström (1979 a and b) and the comprehensive analysis in Bolton and Dewatripont (2005).
the realization of \( \theta \) nor the effort spent by the upstream firm. In order to avoid potential efficiency losses due to this kind of asymmetric information, firms must rely on incentive compatible delivery contracts. Furthermore, we assume that both firms are risk neutral, which allows us to focus on the effects which buyer power has on the negotiated target quality.\(^{10}\)

**Demand and Quality.** We assume that product \( x \) is an experience good. In each period, consumers learn the product’s actual quality only after having consumed it. Consumers buy sequentially and immediately share information. Accordingly, in each period, demand adjusts to the quality of the good in that period. We denote the demand for good \( x \) by \( X(p, \theta) \) where \( p \) indicates the price. Demand is increasing in \( \theta \), decreasing in \( p \) and concave in both arguments, i.e. \( X_p < 0 < X_\theta \) and \( X_{pp}, X_{\theta\theta} < 0 \).\(^{11}\) In each period the input’s quality, \( \theta \), is stochastically determined, where \( \theta \) can be either high or low, i.e. \( \theta \in \{\hat{\theta}, \bar{\theta}\} \) with \( \hat{\theta} < \bar{\theta} \). The probability that the input is of quality \( \bar{\theta} \) is decreasing in \( \bar{\theta} \), while it is increasing in the effort, \( e \), that firm \( U \) invests in careful production techniques, monitoring and/or quality assurance systems. However, the higher the target quality level, the less the probability of meeting the quality requirements is increasing in the effort spent by the upstream firm. Denoting \( \rho(e, \bar{\theta}) \) the probability that \( \bar{\theta} \) realizes, we have

\[
\Pr(\theta = \bar{\theta}|e) = \rho(e, \bar{\theta}); \quad \Pr(\theta = \hat{\theta}|e) = 1 - \rho(e, \bar{\theta}) \tag{1}
\]

with : \( \rho(0, \bar{\theta}) = 0 \) and \( \rho_{\bar{\theta}} < 0 < \rho_e, \rho_{e\bar{\theta}} < 0 \) for all \( e > 0 \).

Additionally, we assume that effort induces increasing and convex costs of \( c(e) \) per period with \( c', c'' > 0 \). After exerting effort, the upstream firm privately learns the realization of \( \theta \).

**Tariffs.** Using the revelation principle, we focus on delivery tariffs which are contingent on the quality level, \( \hat{\theta} \), that the upstream firm, \( U \), announces after having observed the realized quality of the input produced. The delivery tariff \( T(w, F, \hat{\theta}) \) is given by

\[
T(w, F, \hat{\theta}) = \begin{cases} 
(w, F) & \text{if } \hat{\theta} = \bar{\theta} \\
(w, E) & \text{if } \hat{\theta} = \hat{\theta} 
\end{cases} \tag{2}
\]

---

\(^{10}\)Allowing for risk-aversion at the upstream level would complicate the analysis as risk-aversion induces an additional trade-off between the provision of incentives and the efficient allocation of risk.

\(^{11}\)Subscripts denote partial derivatives. In order to simplify the notation, we omit the arguments of the functions when there is no resulting ambiguity.
Given the upstream firm’s quality announcement, the appropriate delivery tariff is selected. Due to prohibitively high costs, the downstream firm cannot fully control the entire delivery. Furthermore, potential side effects are unknown at the moment of control as these may only occur some time after consumption. Although the downstream firm can not directly observe the actual quality of the input, it is able to infer the actual quality using actual demand as consumers learn the quality and adjust demand accordingly. Thus, the downstream firm can detect untruthful announcements by observing actual demand. Nevertheless it cannot verify untruthful announcements to a third party, as low qualities can also be caused by misconduct such as improper shipping or handling at the downstream level. Actual demand is therefore only an imperfect signal of whether the upstream firm’s announcement was truthful or not.\footnote{This “observable but unverifiable” assumption is common in the incomplete contracting literature even though “the most reasonable assumption is that the principal’s information about the agent’s action is worse than the agent’s but, because of the principal’s expertise and relation-specific knowledge, better than the judge’s” (Hermalin and Katz 1991).}

As untruthful announcements cannot be verified, delivery contracts cannot be contingent on the actual demand. Furthermore, untruthful announcements cannot be punished contractually. However, we assume that the downstream firm refrains from continuing the relation with the upstream firm, $U$, once untruthful announcements are detected and that both firms revert to its outside option forever.

**Expected profits.** To simplify the analysis, we normalize production costs at the upstream level as well as the distribution and transformation costs at the downstream level to zero. Employing $T(w, F, \bar{\theta})$ and assuming truthful announcement, the expected per period profit of the upstream firm $E\pi^U$ is given by

$$E\pi^U = \rho(e, \bar{\theta})\pi^U + (1 - \rho(e, \bar{\theta}))\bar{\pi}^U - c(e)$$  \hspace{1cm} (3)$$

with$$\pi^U = wX(p, \bar{\theta}) + F$$

$$\bar{\pi}^U = wX(p, \bar{\theta}) + F.$$

$\rho(e, \bar{\theta})$ is the probability of the quality announcement being true.
In turn, the downstream firm’s expected per period profit $E\pi^D$ can be written as

$$E\pi^D = \rho(e, \bar{\theta})\bar{\pi}^D + (1 - \rho(e, \bar{\theta}))\bar{\pi}^D$$

(4)

with:

$$\bar{\pi}^D = (p - \bar{w})X(p, \bar{\theta}) - E$$

$$\bar{\pi}^D = (p - w)X(p, \bar{\theta}) - E$$

**Outside options.** The upstream and the downstream firms interact as long as there is a benefit from the relationship, i.e. as long as the joint profit exceeds the surplus that could be earned with alternative counterparts and as long as no untruthful announcement has been observed. Denoting $\Gamma^U$ and $\Gamma^D$ as the outside options of the upstream and the downstream firm, respectively, we assume that $\Gamma^U, \Gamma^D \geq 0$ and that $\Gamma^U$ as well as $\Gamma^D$ are exogenously given. The less alternative counterparts fit, the lower the values of the firms’ outside options. For example, the more that the downstream firm depends on the input supplied by the upstream firm, the lower $\Gamma^D$. In turn, the more the upstream firm depends on the downstream firm in order to get its products distributed to final consumers, the lower $\Gamma^U$. A strong gatekeeper position of the downstream firm can thus imply a low $\Gamma^U$. Note further, that $\Gamma^U$ does not depend on the realized quality since delivery contracts are negotiated before the quality realizes and contracts are binding. We exclude the possibility that the upstream or the downstream firm can quit the contract after the quality has been realized.\(^{13}\)

In summary, we analyze an infinitely repeated game where the following four-stage game takes place in every period: First, firms $U$ and $D$ negotiate a target quality $\bar{\theta}$ and a menu of two-part tariffs which are contingent on the quality $\hat{\theta}$ the upstream firm will announce. The upstream firm decides how much effort to spend and observes the realized quality in the second stage. Subsequently, it decides whether or not to truthfully report the actual quality. Given the upstream firm’s announcement, the appropriate delivery tariff is selected. In the last stage of the game, the downstream firm chooses its prices and, finally, demand as well as profits realize. The interaction between firms $D$ and $U$ ends if the downstream firm detects an untruthful announcement by observing that actual demand differs from expected demand. In this case,\(^{13}\)

\(^{13}\)We consider explicit contracts and do not analyze the possibility that the negotiating parties can break the contractual agreement on the (contingent) delivery tariffs. This setting is in line with the traditional literature on incentive contracts but differs from relational incentive contracts in repeated games with moral hazard and adverse selection as analyzed in Levin (2003).
both firms get their outside option in all future periods. We focus on stationary contracts and solve the game by backward induction.

3 Prices, Announcement, and Effort

We begin our analysis by characterizing the optimal downstream prices for given two-part tariffs and an announced quality, $\bar{\theta}$. We then solve the third stage of the game, where we determine the upstream firm’s incentives to announce the true realization of $\theta$. Subsequently, we consider the effort the upstream firm spends in order to enhance the probability of achieving the target quality. Delivery tariffs, as well as target qualities, negotiated by the firms will be analyzed in the next section.

**Downstream Prices.** In the final stage of the game, the downstream firm sets the price $p$ for product $x$. This decision is based on the quality the upstream firm announces and the implied delivery tariff. Although the downstream firm can infer the actual quality based on demand ex post, we assume that price adjustments induce physical costs like reprogramming cash registers, reproducing leaflets, as well as decision making and communication costs. We assume that these costs are high enough to discourage the downstream firm from changing its prices if actual demand does not correspond to expected demand.\(^\text{14}\) The downstream firm’s optimal prices are, therefore, determined by

$$\max_p \left[ (p - w)X(\cdot, \hat{\theta}) - F \right]_{T(w, F, \bar{\theta})}. \quad (5)$$

Let $\overline{p}(w, \bar{\theta})$ and $\underline{p}(w, \bar{\theta})$ denote the solutions of (5) for $\hat{\theta} = \bar{\theta}$ and $\hat{\theta} = \theta$ respectively.

**Announcement.** After having observed the realized quality, the upstream firm announces the quality, $\hat{\theta}$, which also determines the actual delivery tariff. Deciding whether or not to announce the realized quality truthfully, the upstream firm trades off its potential gains from deviating in the current period against the losses resulting from not trading with the downstream firm in all future periods. Denoting $\overline{E\pi^U}$ as firm $U$’s expected continuation profit and $\delta > 0$ as the discount factor, firm $U$’s incentive constraints for truthful announcements can be written

\(^{14}\)Using a data set of weekly retail prices, Slade (1998) finds significant fixed costs of changing prices in retailing. For a discussion of potential costs induced by changing prices see also Zbaracki et al. (2004).
The upstream firm reports the actual quality of product $x$ truthfully if the incentive constraints (6) and (7) are satisfied.

**Effort.** Turning to the effort chosen by the upstream firm, we focus on the case where the incentive constraints (6) and (7) are satisfied.\(^{16}\) Employing (3), the supplier’s optimal effort $e^*(\cdot)$ is implicitly given by

$$
\rho_e^{\pi^U} - \rho_e^{\pi^U} = c'(e) \Leftrightarrow \rho_e = \frac{c'(e)}{\Delta \pi^U} \text{ with } \Delta \pi^U := \pi^U - \pi^U. 
$$

For later reference, note further that (8) and simple comparative statics lead to

$$
sign \frac{\partial e^*}{\partial w} = sign \frac{d \Delta \pi^U}{d \pi^U}. 
$$

Inspection of (9) shows that for given $F$ and $\bar{F}$ as well as $\bar{w}$ sufficiently low, the effort level chosen by the upstream firm will increase in $\bar{w}$, i.e. $\partial e^*/\partial \bar{w} > 0$ as long as $\bar{w} < \bar{w}^k := \arg \max \bar{w}X$. In turn, the effort level reacts ambiguously in the target quality level $\bar{\theta}$. Since we have

$$
sign \frac{\partial e^*}{\partial \bar{\theta}} = sign \left[ \rho_e \Delta \pi^U + \rho_e \frac{d \Delta \pi^U}{d \bar{\theta}} \right],
$$

with $\rho_e \bar{\theta} < 0$, (10) implies that $\partial e^*/\partial \bar{\theta}$ is negative as long as $\Delta \pi^U$ is sufficiently high.

### 4 Delivery Tariffs and Quality

In the first stage of the game, both firms negotiate over a non-linear delivery tariff and a target quality. We apply the Nash bargaining solution to specify the negotiation outcome between the

\(^{15}\) For simplicity, we assume that the outside option, the upstream firm reverts to after the breaking off of interaction with the downstream firm, does not depend on whether the upstream firm previously told the truth. Thus, the outside options are assumed not to be path-dependent.

\(^{16}\) Since the equilibrium delivery conditions will be such that the incentive constraints are satisfied, we do not analyze the optimal effort when either IC\(_1\) or IC\(_2\) are violated.
retailer and the supplier. In the case of disagreement both firms revert to their outside options, i.e. \( \Gamma^U \) and \( \Gamma^D \).\(^{17}\) Thus, the equilibrium bargaining outcome in each period can be characterized by the solution

\[
\max_{T(w,F,\theta,\bar{\theta})} N = \left[ E\pi^D - \Gamma^D \right] \left[ E\pi^U - \Gamma^U \right].
\] (11)

Wholesale prices as well as the target quality, \( \bar{\theta} \), are determined in order to maximize the joint profit of both firms. The joint profit is divided such that each party gets its disagreement payoff plus a share of the incremental gains from trade. Additionally, (11) implies that the upstream (downstream) firm’s bargaining position is better the higher its outside option \( \Gamma^U \) (\( \Gamma^D \)). In order to analyze both the negotiated target quality \( \bar{\theta} \) and the delivery tariff \( T(w,F,\bar{\theta}) \), we first assume that the incentive constraints (6) and (7) are not binding. We use this solution as a benchmark for the more complicated case where contracts must ensure truthful announcement.

### 4.1 Unconstrained Bargaining

Assuming that (6) and (7) are not binding, we maximize (11) with respect to the tariffs \( (F, w) \) and \( (\bar{F}, \bar{w}) \) as well as \( \bar{\theta} \). The optimal tariffs \( (w^*, \bar{w}^*) \) and \( (\bar{w}^*, \bar{F}^*) \) satisfy\(^ {18}\)

\[
w^* = \bar{w}^* = 0; \quad \bar{F}^* - F^* = \Delta \quad \text{and} \quad E\pi^D - \Gamma^D = E\pi^U - \Gamma^U,
\] (12)

where \( \Delta \) is defined as \( \bar{p}X - p\bar{X} \). Furthermore, (12) and the envelope theorem imply \( d\Delta / d\bar{\theta} = \bar{p}X_{\bar{\theta}} \). Hence, the first order condition for the optimal target quality \( \bar{\theta}^* \) can be written as

\[
\frac{\partial N}{\partial \bar{\theta}} = \rho_\theta \Delta + \rho \frac{d\Delta}{d\bar{\theta}} = \rho \bar{p}X_{\bar{\theta}} + \rho \rho_\theta \Delta = 0.
\] (13)

As expected, wholesale prices equal marginal costs thus implying undistorted monopoly prices in the downstream market. In turn, the fixed fees are used to divide the joint profit and to efficiently allocate the risk of getting low quality. Since (12) leads to \( \bar{\pi}^D - \bar{\pi}^D = 0 \), any risk is fully borne by the upstream firm, which also implies that the upstream firm’s decision with respect to \( e \) maximizes the expected joint profit of both firms (see (8)). Using these results,

\(^{17}\)The cooperative Nash bargaining solution can be interpreted in terms of the non-cooperative alternating-offer game proposed by Rubinstein (1982). If the time interval between offers becomes relatively small, the solution of the dynamic non-cooperative process converges to the symmetric Nash bargaining (Binmore et al. 1986).

\(^{18}\)These results are derived in the proof of Proposition 1.
the optimal target quality $\overline{\theta}^*$, implicitly given by (13), maximizes the overall expected profits of both firms. Thus, we get:

**Proposition 1** If the incentive constraints are not binding, the bargaining outcome is efficient.

**Proof.** See Appendix. ■

Proposition 1 confirms the well-known result that non-linear tariffs ensure an efficient outcome in a bilateral bargaining framework. That is, the negotiating firms maximize the joint surplus by determining the delivery tariffs in order to guarantee optimal investment and pricing decisions. However, taking imperfect information about the upstream firm’s investments and the actual quality into account, efficiency can only be ensured if the upstream firm’s incentives constraints are satisfied. Employing (12), note first that (7) is satisfied as long as (6) holds. Moreover, focusing on the stationary solution, i.e. on $E\pi^U = E\pi^U$ and $E\pi^D = E\pi^D$, it is easy to show that (6) is equivalent to\(^{19}\)

$$
\Gamma \leq (\rho - 2\delta)\Delta + pX - c(e^*)
$$

with $\Gamma := \Gamma^U + \Gamma^D$.

Note that (14) does not only rely on $\Gamma^U$. Given that the total surplus is divided between both firms, incentive compatibility requires that the sum of both firms’ outside options is low enough. Efficient bargaining outcomes, thus, require that the total gains from trade, i.e. $E\pi^U + E\pi^D - (\Gamma^U + \Gamma^D) = \rho\Delta + pX - c(e^*) - (\Gamma^U + \Gamma^D)$, are high enough while the discount rate $\delta$ must be low enough.

### 4.2 Constrained Bargaining

When (14) is binding, the negotiated delivery tariffs as well as the target quality $\overline{\theta}$ have to be distorted in order to meet the upstream firm’s incentive constraints. More precisely, delivery tariffs and the target quality must induce the upstream firm to reveal the actual quality of the product.

Analyzing the constrained bargaining problem, we get that the wholesale price $\overline{w}$ is strictly positive. Furthermore, the target quality is distorted in order to reduce efficiency losses due to inefficient effort decisions by the upstream firm.

[^19]: See (28) and (29) in the Appendix.
Starting with (6), $F$ leads to truthful announcements as long as
\[ F = wX + E - \bar{w}X + \frac{1}{\delta} \left( E\pi^U - \Gamma^U \right) \] with $\bar{X} := X(\bar{p}, \bar{q})$. (15)

Employing (15), maximizing (11) with respect to $\bar{w}$, $w$ and $E$ and focusing again on the stationary solution, i.e. on $E\pi^D(\cdot) = E\pi^D$ and $E\pi^U(\cdot) = E\pi^U$, the optimal tariffs $(w^c, E^c)$ and $(\bar{w}^c, \bar{E}^c)$ satisfy $w^c = 0$, $E\pi^D = E\pi^U - \Gamma$ and
\[ \rho \frac{d}{d\bar{w}} \left[ \bar{p}\bar{X} \right] + \rho_* \frac{\partial e^*}{\partial \bar{w}} (\Delta - \Delta \pi^U) = 0. \] (16)

The optimal target quality $\bar{\theta}^c$ is implicitly given by
\[ \left[ \rho_\bar{\theta} \Delta + \rho \frac{d\Delta}{d\bar{\theta}} \right] + \rho_* \frac{\partial e^*}{\partial \bar{\theta}} (\Delta - \Delta \pi^U) = 0. \] (17)

Compared to (12) and (13), (16) and (17) show that the optimal choice of $\bar{w}$ and $\bar{\theta}$ balances the efficiency losses due to inefficient wholesale prices along with target qualities and its impact on the upstream firm’s investment decision. The implications for the upstream firm’s investment are captured by the second term on the RHS of (16) and (17), respectively, where $\rho_* (\Delta - \Delta \pi^U)$ measures the difference between joint marginal profits and the upstream firm’s marginal profits from increasing $e$. Furthermore, we get that the optimal delivery contract is such that the following inequalities
\[ [\Delta - \Delta \pi^U] > 0, \quad [\rho_\bar{\theta} \Delta + \rho \frac{d\Delta}{d\bar{\theta}}] > 0 \text{ and } \frac{\partial e^*}{\partial \bar{\theta}} < 0 \] (18)

hold. Combining (16)—(18) and restricting the analysis to $\bar{w}^c < \bar{w}^k$, we obtain:

**Proposition 2** If the incentive constraint is binding, the bargaining solution is characterized by inefficiently low upstream investments and $w^c = 0 < \bar{w}^c$. Furthermore, for given $\bar{w}^c$ and $e^*$, the optimal target quality $\bar{\theta}^c$ is inefficiently low.

**Proof.** See Appendix. ■

Under unconstrained bargaining, wholesale prices and the target quality maximize the joint surplus, while the fixed fees are used to divide the joint surplus between the firms and to ensure efficient effort decisions by shifting all risk to the upstream firm. If the upstream firm’s incentive

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20 These results are derived in the proof of the next proposition.
constraint is binding, one of these instruments must be used to ensure truthful announcements. Hence, delivery tariffs, the allocation of risk, and, thus, the upstream firm’s effort decisions become inefficient. In order to alleviate the implied inefficiencies and to increase the upstream firm’s effort, the optimal \( \bar{w}^c \) as well as the optimal target quality, \( \bar{\theta}^c \), are distorted. While a positive \( \bar{w}^c \) induces higher effort (see (9)), the optimal target quality, \( \bar{\theta}^c \), must be distorted downwards for given \( \bar{w}^c \) and \( e^* \) (see (10) and (18)). Note that \( \bar{w}^c > 0 \) causes a double mark-up problem inducing further efficiency losses.

Although Proposition 2 shows that both \( \bar{w}^c \) and \( \bar{\theta}^c \) are inefficient, the overall effect of the incentive constraint (14) on the target quality \( \bar{\theta}^c \) can be ambiguous. Restricting the analysis to linear demand \( X(p, \theta) \), with \( X_{pp} = X_{p\theta} = 0 \) and defining \( \Gamma^c \) as the highest \( \Gamma \) where the upstream firm’s incentive constraint is not binding, we obtain:

**Proposition 3** Assuming linear demand and starting with \( \Gamma = \Gamma^c \), an increase in \( \Gamma \) leads to a lower target quality, i.e. \( \bar{\theta}^c < \bar{\theta}^* \), as long as \( \rho_0/\rho \) is decreasing in \( e \). With \( \Gamma > \Gamma^c \) an increase in \( \Gamma \) is the more likely to increase \( \bar{\theta}^c \) the more the wholesale price \( \bar{w}^c \) increases in \( \Gamma \).

**Proof.** See Appendix. ■

The first part of Proposition 3 confirms Proposition 2 as \( \Gamma \) close to \( \Gamma^c \) leads to relatively low distortions with respect to \( \bar{w}^c \) and \( e^* \). Thus, in order to increase the upstream firm’s effort, the target quality must be lower than in the case of unconstrained bargaining. A further increase in the firms’ outside options, i.e. \( \Gamma > \Gamma^c \), results in higher distortions of the wholesale price and greater effort spent by the upstream firm. This allows the firms to increase the target quality, \( \bar{\theta}^c \), without reducing the probability of achieving \( \bar{\theta}^* \) too much. In fact, the example analyzed in the next section shows that a high outside option \( \Gamma \) may well lead to \( \bar{\theta}^c > \bar{\theta}^* \).

So far, we limit our analysis to the case where the downstream and the upstream firm jointly decide the quality requirements. However, our results do not change if either the downstream firm or the upstream firm unilaterally sets the quality requirements. When deciding about the target quality, either firm anticipates that delivery tariffs are negotiated in order to maximize total industry profit. As this profit is split between the firms according to the Nash bargaining solution, each firm has an incentive to choose the target quality such that total industry profit is maximized. Hence, we have:
Corollary 1  The choice of the target quality is the same irrespective of whether firms negotiate the target quality or whether the target quality is chosen unilaterally by one of the firms before negotiations about delivery tariffs take place.

Corollary 1 shows that our analysis can be directly applied to private standards chosen by either the downstream or the upstream firm. In particular, private standards implemented by downstream retailers in order to ensure the quality assurance along the value chain tend to be more efficient the lower the upstream firm’s outside option. Note that this may be caused by the ongoing consolidation process in the retail markets and thus the increasing retailer’s gatekeeper control towards final consumer markets. Conversely, private standards tend to be inefficiently high if upstream firms have various alternative trading possibilities.

5  Example

In order to illustrate our results and to characterize the potential inefficiencies due to imperfect information in more detail, we now present a simple example. We use a standard quasi-linear utility function and focus on the comparative statics with respect to the firms’ outside options.

Consumers’ utility is given by

\[ u(x, \theta) = (1 + \frac{1}{4} \sqrt{\theta})x - \frac{1}{2} x^2 - px. \]  
(19)

Differentiating \( u(x, \theta) \) with respect to \( x \), we obtain

\[ X(p, \theta) = \begin{cases} 1 + \sqrt{\theta}/4 - p & \text{for } p \leq 1 + \sqrt{\theta}/4 \\ 0 & \text{otherwise} \end{cases}. \]  
(20)

Furthermore, we normalize \( \theta \) to zero and assume that the probability \( \rho(e, \theta) \), the upstream firm’s costs \( c(e) \) and the discount factor \( \delta \) are given by

\[ \rho(e, \theta) = \min \left\{ \frac{e}{1+\theta}, 1 \right\}, \quad c(e) = \frac{e^2}{2} \quad \text{and} \quad \delta = 0.1. \]  
(21)

Note that (21) implies that \( \rho_\theta/\rho \) does not depend on \( e \).

Calculating \( w^* \) and \( \overline{\theta}^* \) (see (12) and (13)) and analyzing the upstream firm’s incentive constraint (6), it is easy to show that there exists a critical value \( \Gamma^c \approx 0.22 \) such that (6) is binding for all \( \Gamma = \Gamma^U + \Gamma^D > \Gamma^c \). We also find a value of \( \Gamma \) above which total industry surplus is lower
than the firms’ outside options. This corresponds to $\Gamma^m \approx 0.25$ indicating that no trade occurs for all $\Gamma = \Gamma^U + \Gamma^D > \Gamma^m$. Using (15)–(17), we obtain the graphs shown in Figure 1. If the incentive constraint is binding, i.e. $\Gamma > \Gamma^c$, a higher $\Gamma$ unambiguously increases the optimal wholesale price $\bar{w}^c$. However, the relation between $\Gamma$ and the optimal target quality $\bar{\theta}^c$ is not monotone. Starting from $\Gamma = \Gamma^c$, an increase in $\Gamma$ first reduces $\bar{\theta}^c$, while for higher values of $\Gamma$ the target quality $\bar{\theta}^c$ is finally increasing in $\Gamma$.

![Figure 1: Optimal $\bar{w}^c(\Gamma)$ and $\theta^c(\Gamma)$.](image)

This non-monotonic relation confirms Proposition 3. It also shows that rather high outside options may well lead to inefficiently high target qualities, i.e. to $\bar{\theta}^c > \bar{\theta}^s$. Additionally, while higher outside options lead to more severe distortions and thus to lower overall industry profits, it is easy to show that each firm’s expected profit is increasing in its own outside option but decreasing in the other firm’s outside option. Finally, extending our results to the analysis of social welfare, we define expected social welfare $EW$ as the sum of expected consumer surplus and firms’ profits, i.e.

$$EW = \rho [u(X, \bar{\theta}) + pX] + (1 - \rho) [u(X, \bar{\theta}) + pX] - c(e^*).$$

Figure 2 shows that $EW$ is unambiguously decreasing in $\Gamma$ for all $\Gamma > \Gamma^c$. Obviously, positive wholesale prices $\bar{w}^c$ as well as low target qualities $\bar{\theta}^c$ reduce the firms’ expected profits as well the expected consumer surplus. Although relatively high values of $\Gamma$ may lead to higher target qualities, the implied distortions due to a high wholesale price $\bar{w}^c$ and an inefficiently low effort level lead to lower expected social welfare. Thus, business strategies that enhance the firms’ trading alternatives may well reduce social welfare. This holds for global sourcing strategies that allow buyers to better replace (established) suppliers as well as for certification decisions of
suppliers. The latter is true as certification may increase the outside option a supplier has vis-
à-vis its buyers by lowering its transactions costs when delivering alternative trading partners.

Our model also allows us to evaluate the welfare consequences of retailers’ increasing buyer power (OECD 1998, EC 1999). Relating retailer’s buyer power to its share in overall profits, buyer power in our model can be either caused by a high value of the buyer’s outside option or by a low value of the supplier’s outside option.21 We find that the welfare effects of increasing buyer power crucially depend on its sources. If higher buyer power is caused by a diminished outside option of the supplier, contracts become more efficient and social welfare raises. Thus, as long as downstream consolidation reduces the supplier’s outside option, i.e. \( \Gamma^U \), by limiting its trading alternatives, downstream consolidation and the implied increase in buyer power lead to more efficient contracts. In turn, if higher buyer power is based on an improved outside option of the retailer, i.e. \( \Gamma^D \), contracts become less efficient and social welfare decreases. Additionally, buyer power tracing back to an improved outside option of the retailer may well lead to more stringent quality requirements as currently observed in the retail industry (OECD 2006).

So far, our discussion neglects potential interactions between markets and, thus, competition effects on the different layers of the vertical structure. For example, globalization generally does not only affect the outside option of the downstream firms. It can also affect the upstream firms’ outside options as global sourcing may lead to stronger competition between upstream firms and may thus decrease their outside options. \textit{A priori}, it is, therefore, not clear which effects dominate and how globalization and downstream consolidation affects negotiations in intermediate goods markets in terms of quality and tariffs.

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21We do not consider the exogenously given bargaining power of the negotiating parties.
6 Conclusion

We analyze a simple vertical structure with one upstream firm selling a good to a downstream firm over an infinite number of periods. Considering a framework of hidden action and hidden information and using the Nash bargaining solution, we show that high gains from trade lead to efficient qualities and delivery tariffs. With high outside options of the firms, incentive compatible contracts must be distorted. While wholesale prices are inefficiently high, the negotiated target qualities depend non-monotonically on the firms’ outside options. Inefficiently high qualities can result whenever the firms’ outside options are high enough.

Applying these results to the analysis of buyer power, we find that large buyer power resulting from the low outside options of suppliers leads to reduced wholesale and retail prices as well as to more efficient qualities. Similarly, relation specific investments by upstream firms cannot only enhance the bargaining position of the downstream firm, it can also increase the efficiency of the firms’ interaction. Conversely, a better outside option of the retailer—for example due to global sourcing (or private label) strategies—implies less efficient contracts in intermediate good markets. Thus, the impact of buyer power on the efficiency of delivery contracts crucially depends on the sources of buyer power.

Our model does not reproduce quality failures and thus food scandals in equilibrium. Although there is a positive probability that the product’s quality undercuts the target quality level, product recalls never occur as the upstream firm always informs the downstream firm truthfully about the actual quality. Our analysis relies on the fact that the upstream firm is perfectly informed about the quality of its product. Consider, for example, the case where the product quality also depends on exogenous factors that the upstream firm is not aware of. Then, quality failures become possible and the analysis would have to be amended by allowing for cases in which the actual quality of the good is low although the upstream firm truthfully announced a high quality of its input. The implied analysis is more involved as the belief system of the downstream firm must be explicitly taken into account.\textsuperscript{22}

Finally, we limit our analysis to the case of binding contracts where firms are committed to adhere to the negotiated delivery tariffs. Turning to informal or relational contracts, one would have to consider the additional constraint that contracts must be self-enforcing. More

\textsuperscript{22}See for example Cabral (2009) who considers umbrella branding with imperfect monitoring.
specifically, the downstream firm must have an incentive to pay the negotiated payments. As this may restrict the gap between the payments for high and low qualities, relational contracts tend to limit incentives and may thus induce further inefficiencies and distortions.

Appendix

Proof of Proposition 1

Maximizing (11) with respect to the tariff \( w, \overline{F} \) and \( \overline{w}, \overline{F} \), using the envelope theorem with respect to \( e^*(\cdot) \) and analyzing the respective first order conditions, we first obtain

\[
\frac{\partial N}{\partial \overline{F}} = 0 \Leftrightarrow \frac{\rho e^* \Delta \pi^D - \rho}{\rho} = -\frac{E\pi^D(\cdot) - \Gamma^D}{E\pi^U(\cdot) - \Gamma^U} \quad (23)
\]

\[
\frac{\partial N}{\partial F} = 0 \Leftrightarrow \frac{-\rho e^* \Delta \pi^D - (1 - \rho)}{(1 - \rho)} = -\frac{E\pi^D(\cdot) - \Gamma^D}{E\pi^U(\cdot) - \Gamma^U} \quad (24)
\]

with \( \Delta \pi^D := \pi^D - \pi^D \). (23), (24) and \( \partial e^*/\partial \overline{F} = -\partial e^*/\partial F \) imply that we must have

\[
\Delta \pi^D = 0 \quad \text{and} \quad E\pi^D(\cdot) - \Gamma^D = E\pi^U(\cdot) - \Gamma^U. \quad (25)
\]

Using (25), the first order conditions with respect to \( \overline{w} \) and \( \overline{w} \) can be written as

\[
\frac{\partial N}{\partial \overline{w}} = \rho \overline{w} \frac{\partial X}{\partial p} \frac{\partial p}{\partial \overline{w}} = 0; \quad \frac{\partial N}{\partial \overline{w}} = (1 - \rho) \overline{w} \frac{\partial X}{\partial p} \frac{\partial p}{\partial \overline{w}} = 0, \quad (26)
\]

which leads to \( \overline{w}^* = \overline{w}^* = 0 \). Employing these results, we also get

\[
\frac{\partial N}{\partial \overline{\theta}} = \rho \frac{d\Delta}{d\overline{\theta}} + \rho \overline{\theta} \Delta = \rho \overline{\theta} \overline{X} + \rho \overline{\theta} \Delta = 0 \quad (27)
\]

where the last equality follows from using the envelope theorem with respect to \( \overline{\theta} \). Finally, solving for \( F^* \) and \( \overline{F}^* \), we obtain

\[
F^* = \frac{1}{2} [(2 - \rho) \overline{\theta} \overline{X} - (1 - \rho) \rho \overline{X} + c(e^*) + \Gamma^U - \Gamma^D] \quad (28)
\]

\[
\overline{F}^* = \frac{1}{2} [(1 + \rho) \overline{\theta} \overline{X} - \rho \overline{\theta} \overline{X} + c(e^*) + \Gamma^U - \Gamma^D]. \quad (29)
\]

Proof of Proposition 2

Using (15) note first that the firms’ profits for \( \theta = \overline{\theta} \) and \( \theta = \overline{\theta} \) can be written as

\[
\pi^D = (\overline{\theta} - \overline{w}) X - \overline{w} X - \overline{E} + \overline{w} \overline{X} - \frac{1}{\delta} (E\pi^U - \Gamma^U) \quad (30)
\]

\[
\rho \pi^D = (p - w) X - \overline{F} \quad (31)
\]

\[
\pi^U = \overline{w} X + \overline{w} X + \overline{F} - \overline{w} \overline{X} + \frac{1}{\delta} (E\pi^U - \Gamma^U) \quad (32)
\]

\[
\overline{\pi}^U = \overline{w} X + \overline{F}. \quad (33)
\]
Employing (30)—(33) and considering first $F$ and $w$, we obtain

\[
\frac{\partial N}{\partial F} = 0 \Leftrightarrow E\pi^D(\cdot) - \Gamma^D = E\pi^U(\cdot) - \Gamma^U \tag{34}
\]

\[
\frac{\partial N}{\partial w} = (1 - \rho) \frac{d}{dw} [p\bar{X}] = 0 \Rightarrow \bar{w}^* = 0. \tag{35}
\]

Turning to $w$, solving (34) for $F^c$, using $E\pi^D(\cdot) = E\pi^D$ and $E\pi^U(\cdot) = E\pi^U$ as well as (8), the first order condition for $\bar{w}^c$ can be written as

\[
\frac{\partial N}{\partial w} = \rho_e \frac{\partial e^*}{\partial \bar{w}} [\Delta - \Delta\pi^U] + \rho \frac{d\bar{X}}{dw} = 0, \tag{36}
\]

where $\Delta\pi^U$ is given by (recall $\Gamma := \Gamma^U - \Gamma^D$)

\[
\Delta\pi^U = \bar{w}(\bar{X} - \hat{X}) + \frac{1}{2\delta} \left[ \rho \Delta + p\bar{X} - c(e^*) - \Gamma \right]. \tag{37}
\]

To prove the proposition, note first that $d/\bar{w} [p\bar{X}] = 0$ for $\bar{w} = 0$ and $d/\bar{w} [p\bar{X}] < 0$ for all $\bar{w} > 0$. Furthermore, using (9), we obtain

\[
sign \frac{\partial e^*}{\partial \bar{w}} = sign \frac{d}{d\bar{w}} \left[ \bar{w}(\bar{X} - \hat{X}) \right] > 0 \text{ as long as } \bar{w} < \bar{w}^k. \tag{38}
\]

Considering the sign of $\Delta - \Delta\pi^U$ and assuming $\Delta - \Delta\pi^U \leq 0$, (36) and (38) imply $\bar{w} = 0$. Additionally, we get

\[
\Delta - \Delta\pi^U \leq 0 \Leftrightarrow \Gamma \leq (\rho - 2\delta)\Delta + p\bar{X} - c(e^*) \text{ for } \bar{w} = 0, \tag{39}
\]

which contradicts the assumption that (14) is binding. Hence we must have $\Delta - \Delta\pi^U > 0$ and therefore inefficient risk sharing as well as $\bar{w}^c > 0$.

Turning to $\bar{\theta}$ and using the envelope theorem with respect to $e^*$, the optimal target quality $\bar{\theta}^c$ is implicitly given by

\[
\frac{dN}{d\bar{\theta}} = \rho_c \Delta + \rho_e \frac{\partial e^*}{\partial \bar{\theta}} [\Delta - \Delta\pi^U] + \rho \frac{d\Delta}{d\bar{\theta}} = 0. \tag{40}
\]

Employing (40) and $\Delta - \Delta\pi^U > 0$, we get

\[
sign \frac{\partial e^*}{\partial \bar{\theta}} = -sign \left( \rho_c \Delta + \rho \frac{d\Delta}{d\bar{\theta}} \right). \tag{41}
\]

To determine the sign of the RHS of (41), note first that (40) leads to

\[
\frac{\partial e^*}{\partial \bar{\theta}} = \frac{\rho_c \Delta + \rho \frac{d\Delta}{d\bar{\theta}}}{\rho_e (\Delta - \Delta\pi^U)}. \tag{42}
\]
Substituting $\partial e^* / \partial \tilde{\theta}$ and solving (42) for $\rho_\theta$, we get

$$\rho_\theta \leq 0 \Rightarrow \rho_\theta^0 \leq \Psi := -\frac{1}{\rho_e \Delta \pi^U} \left[ -\rho \frac{d\Delta / d\tilde{\theta}}{\Delta - \Delta \pi^U} \Theta + \rho_e^2 \frac{p\tilde{X}_\theta}{p\tilde{X}} \tilde{X} \pi^U \right]$$

(43)

with $\Theta := \frac{d}{de} \left[ \rho_e \pi^U - \rho_e \pi^U - c'(e) \right] < 0$.

Furthermore, differentiating $\partial e^* / \partial \tilde{\theta}$ partially with respect to $\rho_\theta^0$, we get

$$\frac{\partial}{\partial \rho_\theta^0} \left[ \frac{\partial e^*}{\partial \tilde{\theta}} \right] = -\frac{\Delta \pi^U}{\Theta} > 0.$$

(44)

Finally, evaluating $\partial e^* / \partial \tilde{\theta}$ at the critical level $\Psi$, we have

$$\left. \frac{\partial e^*}{\partial \tilde{\theta}} \right|_{\rho_\theta^0 = \Psi} = -\frac{\rho \frac{d\Delta / d\tilde{\theta}}{\Delta - \Delta \pi^U}}{\rho_e (\Delta - \Delta \pi^U)} < 0.$$

(45)

Turning back to (41), we thus have $\rho_\theta \Delta + \rho \frac{d\Delta / d\tilde{\theta}}{\Delta - \Delta \pi^U} > 0$, which completes the proof.

**Proof of Proposition 3** To prove the first part of the proposition, note first that $\Gamma = \Gamma^c$ implies $\tilde{w}^c = 0$ and that (17) reduces to (13). Furthermore, we have

$$\frac{\partial^2 N}{\partial \tilde{\theta} \partial \Gamma} = \rho_e \frac{\partial e^*}{\partial \tilde{\theta}} < 0 \leq \rho_e \frac{\partial e^*}{\partial \tilde{\theta}} = \frac{\partial^2 N}{\partial \tilde{\theta} \partial \Gamma},$$

where the first inequality in (46) follows from the continuity of $\partial e^* / \partial \tilde{\theta}$ and (43)—(45). Using (46) and linear demand, simple but tedious calculations yield

$$\left. \text{sign} \frac{\partial \tilde{\theta}^c}{\partial \Gamma} \right|_{\Gamma = \Gamma^c} = \text{sign} \left[ \frac{\partial^2 N}{\partial \tilde{\theta} \partial \Gamma} \frac{\partial^2 N}{\partial \tilde{\theta} \partial \Gamma} - \frac{\partial^2 N}{\partial \tilde{\theta} \partial \Gamma} \frac{\partial^2 N}{\partial \tilde{\theta} \partial \Gamma} \right]_{\Gamma = \Gamma^c}$$

$$= \text{sign} \left[ 2 \rho_e \tilde{X}_{\pi} \left( \frac{\partial e^*}{\partial \tilde{\theta}} \right)^2 + \frac{\partial e^*}{\partial \tilde{\theta}} \left( 2 \frac{\partial e^*}{\partial \tilde{\theta}} \left[ \tilde{X} - \tilde{X}^c \right] - \rho \tilde{X}_{\pi} \right) \right]_{\Gamma = \Gamma^c}.$$

(47)

Substituting $\partial e^* / \partial \tilde{\theta}$ and $\partial e^* / \partial \tilde{\theta}$ into (48) and evaluating at $\Gamma = \Gamma^c$, we obtain

$$\rho_\theta \Delta + \rho_e \Delta \theta \leq 0 \Rightarrow \text{sign} \frac{\partial \tilde{\theta}^c}{\partial \Gamma} \bigg|_{\Gamma = \Gamma^c} < 0,$$

(49)

which together with $\rho_\theta \Delta + \rho \Delta_\theta = 0$ leads to $\tilde{\theta}^c < \tilde{\theta}^*$ for $\Gamma > \Gamma^c$ but $\Gamma$ small enough. To prove the second part, assume that the incentive constraint (14) is binding. Then we must have

$$\Gamma = \rho \Delta + \tilde{p} \tilde{X} + 2 \delta \tilde{w}^c (\tilde{X} - \tilde{X}) - c(e^*) - 2 \delta \frac{c'(e^*)}{\rho_e}.$$

(50)

Defining

$$\Phi(\tilde{\theta}(\Gamma), \tilde{w}^c(\Gamma), \Gamma) := \rho \Delta + \rho \tilde{p} \tilde{X} + 2 \delta \tilde{w}^c (\tilde{X} - \tilde{X}) - c(e^*) - 2 \delta \frac{c'(e^*)}{\rho_e},$$

(51)
applying the implicit function theorem, differentiating (51) totally with respect to \( \Gamma \), substituting the equilibrium values of \( \partial e^* / \partial \pi^e \) and \( \partial e^* / \partial \theta \) (see (40) and (36)), we obtain

\[
\frac{d \theta^e}{d \Gamma} = \frac{1}{\Phi} \left[ \rho \frac{d \left( pX \right) / d \bar{\pi}}{d \bar{\pi}} - \frac{\rho d \left( \rho \Delta + \rho d \bar{\theta} / d \bar{\theta} \right)}{d \bar{\theta}} \right] d \pi^e \tag{52}
\]

with \( \Phi := \frac{\partial \Phi}{\partial \theta} + \frac{\partial \Phi}{\partial e^*} \).

where we have again used \( X_{pp} = X_{p\theta} = 0 \). Since \( d \left( pX \right) / d \bar{\pi} < 0 < \rho p \Delta + \rho d \bar{\theta} / d \bar{\theta} \), (52) shows that \( \theta^e \) is more likely to increase in \( \Gamma \) the higher \( d \pi^e / d \Gamma \).

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