

Roßbach, Peter; Karlow, Denis

**Working Paper**

## The stability of traditional measures of index tracking quality

Frankfurt School - Working Paper Series, No. 164

**Provided in Cooperation with:**

Frankfurt School of Finance and Management

*Suggested Citation:* Roßbach, Peter; Karlow, Denis (2011) : The stability of traditional measures of index tracking quality, Frankfurt School - Working Paper Series, No. 164, Frankfurt School of Finance & Management, Frankfurt a. M.

This Version is available at:

<https://hdl.handle.net/10419/45496>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

**Frankfurt School – Working Paper Series**

**No. 164**

**The Stability of Traditional Measures of Index Tracking Quality**

by Peter Roßbach and Denis Karlow

April 2011



**Frankfurt School of  
Finance & Management**  
Bankakademie | HfB

Sonnemannstr. 9–11 60314 Frankfurt am Main, Germany  
Phone: +49 (0) 69 154 008 0 Fax: +49 (0) 69 154 008 728  
Internet: [www.frankfurt-school.de](http://www.frankfurt-school.de)

## Abstract

Today, the investment into indices has become a widely used strategy in portfolio management. While Index Funds and ETFs try to represent the performance of a single index, other portfolio strategies use indices as portfolio components to concentrate on the allocation task. Because an index cannot be purchased directly, it has to be rebuilt. This is called index tracking. There exist different methods for index tracking. One of them is sampling and aims to reproduce the performance of an index by purchasing a smaller selection of its components. Therefore, the tracking portfolio with the highest tracking quality is searched using an optimization algorithm based on data of an estimation period. The assumption is that this portfolio does also have the highest tracking quality in the crucial investment period. Many approaches for optimizing a tracking portfolio have already been published in literature. Usually, one of three measures (mean absolute deviation, mean square error, and tracking error variance) is used in order to measure the tracking quality. An interesting research question is how stable these measures are. Do they produce tracking portfolios with the same tracking quality in the estimation period and the investment period? Are the tracking portfolios with a high tracking quality in the estimation period compared to the alternative tracking portfolios also those with a high relative tracking quality in the investment period? And finally, do the three traditional measures produce tracking portfolios which are of high value from the investor's point of view? We perform an empirical study using the HDAX in order to answer these questions. Applying the three measures on different time periods and different tracking portfolio sizes we analyze the absolute and the relative stability. Furthermore, we introduce a measure for the ex post tracking quality from an investor's point of view and compare the resulting tracking portfolios from the prior analyses using this measure. The results indicate a poor stability for every of the three traditional measures. Furthermore, the relation to the ex post tracking quality is weak. Thus, there is a need for further research in the area of sampling. It must be explored if there are alternative measures and/or alternative optimization algorithms to search for tracking portfolios which have the desired properties not only in the estimation period but also in the investment period.

Keywords: Portfolio Management, Index Tracking, Sampling, Tracking Error

JEL classification: G11

ISSN: 14369753

Contact:

Prof. Dr. Peter Roßbach  
Frankfurt School of Finance and Management  
Sonnemannstraße 9-11  
60314 Frankfurt am Main, Germany

Email [p.rossbach@fs.de](mailto:p.rossbach@fs.de)

Denis Karlow  
Frankfurt School of Finance and Management  
Sonnemannstraße 9-11  
60314 Frankfurt am Main, Germany

Email [d.karlow@fs.de](mailto:d.karlow@fs.de)

## Content

1	Introduction.....	4
2	Methods of Index Tracking.....	4
3	Variants of Sampling .....	5
4	Traditional Measures of Tracking Quality.....	7
5	Analysis of the Stability of the Traditional Measures of Tracking Quality.....	9
5.1	Research Method .....	9
5.2	Evaluation Method .....	14
5.3	Empirical Results.....	15
6	Summary and Outlook.....	34
	References .....	36

## 1 Introduction

In the recent years, the investment into indices has become more and more popular. There is a huge growth in the number and volumes of passively managed index funds and exchange-traded funds (ETFs) that match the market performance.<sup>1</sup> This is the consequence due to the empirical findings that it is very difficult for an actively managed portfolio to beat an index in the long term after cost of administration and analytics.<sup>2</sup> But also in active portfolio management there exist empirical results that show that the allocation over countries or industrial sectors provides a greater contribution to the portfolio performance than the selection of single assets.<sup>3</sup> The allocation task is usually done by using risk and return ratios of the indices representing the countries respectively sectors. As a consequence, one has to invest into these indices to achieve the desired risk-return-profile of the portfolio. In both cases, the investment into indices is a main task of the portfolio manager. But indices cannot be purchased directly. Instead, the portfolio manager has to rebuild the index he wants to invest into. This is called index tracking.

The term index tracking covers different methods which aim to reproduce an index of stocks, bonds, commodities, etc. Every method has its specific advantages and disadvantages. One of these methods, where the index is reproduced with a smaller number of assets, is called sampling. This so-called tracking portfolio is calculated using past data. The calculation is based on measures reflecting the tracking quality. It is assumed that a good tracking quality in the estimation period will also result in a good tracking quality in the following investment period.

The aim of this paper is to show that the traditional measures which are used to calculate the tracking portfolio have some weaknesses that are mainly subject to an overfitting effect. Hence, a good tracking quality in the estimation period does not guarantee a good tracking quality in the investment period.

In the next chapter, the methods of index tracking are discussed. Chapter 3 gives an overview over the variants of sampling. Chapter 4 introduces the traditional measures of tracking quality used in sampling. In chapter 5 the results of an empirical analysis about the tracking quality produced by these measures are presented. Finally, further research questions are discussed.

## 2 Methods of Index Tracking

There exist three main methods of index tracking: full replication, synthetic replication and sampling.

When applying full replication, the index is exactly replicated; this means that all the underlying assets of the specific index with the corresponding weights have to be purchased.<sup>4</sup> The advantage of this method is that the tracking portfolio is identical to the index. A disadvantage

---

<sup>1</sup> See Fuhr/Kelly (2010), p. 11.

<sup>2</sup> For example see Scharpe (1991), p. 7 and French (2008), p. 1538.

<sup>3</sup> See Hopkins/Miller (2001), pp. 63-64.

<sup>4</sup> See Rudd (1980), p. 58.

is that this method can be very capital intensive. Depending on the number of underlying assets, like the S&P 500 or the Wilshire 5000, a huge amount of capital may be needed to completely rebuild the index. If the amount is too small, e.g. if the tracking portfolio is just a part of an actively managed portfolio, the complete rebuild of the index is an impractical task. In many indices assets are replaced by others from time to time. In these cases the tracking portfolio must be revised in order to keep the full replication state, resulting in transaction costs.<sup>5</sup> The amount of the necessary transaction costs depends on the frequency of revisions and can be relatively high. Moreover, the replacements are predictable and pre-announced. This leads to abnormal high prices for new assets and low prices for removed assets.<sup>6</sup> In these cases, the goal of index tracking can not be accomplished.

Another popular method of index tracking is synthetic replication, which needs a counterparty. The issuer of the index fund or ETF constructs an in principle arbitrary portfolio and the counterparty agrees to pay the index return in exchange for a small fee and the returns on collateral held in the issuers portfolio.<sup>7</sup> Usually, this is done using a swap, covering the difference between the two returns. The advantage of this method is that one can achieve a perfect tracking at very low costs. Just the fee has to be paid. A disadvantage is the inherent counterparty risk, because only the counterparty's creditworthiness guarantees the return on the index. In the EU this risk is limited by regulations to a maximum of 10 percent exposure to the swap counterparty. That means that the fund manager has to have at least 90 percent collateral to back the swap. Otherwise, the collateral has to be revised, resulting in transaction costs. But in the event of the fund issuer's insolvency only the collateral acts as a liability.

The third method of index tracking is sampling.<sup>8</sup> Here, the index is reproduced by a tracking portfolio containing a smaller number of assets, mostly a subset of its components. As a consequence, the reproduction is imperfect. This imperfection can be measured by the so-called tracking error. This is the main disadvantage of the sampling method. The advantage is that a tracking portfolio can be built with only a small amount of capital with respect to an acceptable tracking error.

### 3 Variants of Sampling

When applying sampling, three questions have to be solved: (1) How many assets shall the tracking portfolio contain, (2) Which assets have to be chosen and (3) What weights should the chosen assets have? These questions result in a combinatorial problem with a huge number of combinations depending on the index.

In practice, usually simple heuristics are applied. The simplest one is stratified sampling.<sup>9</sup> Stratified sampling starts by dividing the index assets using one or a combination of characteristics. These could be the market capitalization and/or the industrial sectors, for example. In a very easy variant, the assets of the index are first ordered by their market capitalization. Then, the number of assets is chosen whereby a desired level of tracking quality is achieved. The weight of an asset in the tracking portfolio is calculated via the capitalization of the asset

<sup>5</sup> See Schioldager et al. (2004), pp. 391-393.

<sup>6</sup> See Chen et al. (2006), p. 31 and Blume/Edelen (2004), p. 37

<sup>7</sup> See Hill/Mueller (2004), pp. 522-523.

<sup>8</sup> See Montfort et al. (2008), p.144.

<sup>9</sup> See Meade/Salkin (1989), p. 872.

divided by the sum of the capitalizations of all selected assets. A more sophisticated variant of the stratified sampling first divides the index by its sectors. Then, the sectors are ordered by their market capitalization. In the tracking portfolio each sector is represented by the asset with the highest market capitalization. If there are more sectors than the desired number of assets in the tracking portfolio, the sectors with the lowest market capitalization are eliminated.

Another commonly used variant is optimized sampling.<sup>10</sup> Again, at first a preset number of assets is chosen, e.g. by their market capitalization. Subsequently, the weights of these assets are calculated using an optimization algorithm, minimizing the divergence of the tracking portfolios return to the index return. Depending on the measure of divergence (linear or quadratic), different optimization algorithms are used.

Both sampling methods rely on the assumption that the assets chosen by the above mentioned heuristics have the strongest influence on the index. If this is not the case, there must be other asset combinations which may have a better tracking quality.

Depending on the size of the index, the number of all possible combinations can be enormously huge. For example, tracking the DAX 30 index with 15 out of its 30 stocks for example results in 155,117,520 possible combinations. One of these possible combinations has the best tracking quality. To find this one, every possible combination can be optimized and finally the combination with the best tracking quality has to be chosen as the tracking portfolio. This procedure would require a huge amount of computing power and/or a huge amount of time. For bigger indices, the numbers of possible combinations are much larger.

To solve this combinatorial problem, usually meta-heuristics, like genetic algorithms<sup>11</sup>, threshold accepting<sup>12</sup> or differential evolution<sup>13</sup>, are applied.<sup>14</sup> These search methods are very efficient, even when large search spaces exist. They do not guarantee that the overall best combination will be found, but, with a proper setup, they are able to find solutions which are close to the overall optimum.

Several empirical studies show that the application of these methods results in tracking portfolios with a high tracking quality in the estimation period. Roßbach and Karlow have shown that a genetic algorithm clearly outperforms stratified sampling and optimized sampling.<sup>15</sup> They applied the three methods to the DAX 30 and an artificial index based on the underlying assets of the DAX 30 with no changes in weights. The analysis was repeated for different time intervals from 1998 to 2008. In the estimation period the genetic algorithm always outper-

---

<sup>10</sup> See Bonafede (2003), p. 5.

<sup>11</sup> See Ruiz-Torrubiano/Suarez (2009), p. 58.

<sup>12</sup> See Gilli/Kellezi (2001), p. 9.

<sup>13</sup> See Maringer (2008), pp. 12-14.

<sup>14</sup> In Literature, some additional approaches can be found. Alexander and Dimitriu (2004) select only stocks, which have a strong cointegration with the index. Canakgoz and Beasley (2009) and Stoyan and Kwon (2010) use a mixed integer programming approach where the selection of the assets and the calculation of their weights for the optimal tracking portfolio is done simultaneously. Focardi and Fabozzi (2004) and Dose and Cincotti (2005) use time series clustering to build groups of similar stocks for further selection. Corielli and Marcellino (2006) apply a factor analysis to the index stocks and select those stocks which are mostly correlated to the resulting factors. Gaivoronski et al. (2005) first build a tracking portfolio with all assets in the index and then select only the assets with the largest proportions.

<sup>15</sup> See Roßbach/Karlow (2009).

formed stratified sampling and optimized sampling. In the investment period a different result was achieved. Using the artificial index, the genetic algorithm was again able to sustainably find tracking portfolios of higher quality than the two other methods. However, using the real DAX 30, the genetic algorithm was worse in nearly half of the cases. Additionally, in all cases the tracking quality in the estimation period was clearly better than in the investment period. This indicates that the frequent changes of the weights of the DAX 30 performance index led to an overfitting effect when using the genetic algorithm. Thus, a high tracking quality in the estimation period does not guarantee a high tracking quality in the investment period.

This raises the question if the traditional measures of tracking quality are sufficient as an estimator for the future tracking quality. To answer this question is the purpose of this study.

## 4 Traditional Measures of Tracking Quality

The majority of measures of tracking quality are based on the tracking error. The tracking error (TE) measures the difference between the returns of the tracking portfolio and the returns of the index. It can be formulated as

$$TE_t = r_{I,t} - r_{P,t} = r_{I,t} - \sum_{j=1}^n w_j \cdot r_{j,t} \quad (1)$$

with  $r_p$  = return of the tracking portfolio

$r_I$  = return of the index

$r_j$  = return of asset  $j$  in the tracking portfolio

$w_j$  = weight of asset  $j$  in the tracking portfolio

$n$  = number of assets in the tracking portfolio

and 
$$\sum_{j=1}^n w_j = 1$$

The measures of tracking quality are using the tracking error in different ways. One commonly used measure is the mean absolute deviation (MAD), which measures the mean of the absolute differences between the returns over a given time interval

$$MAD = \frac{1}{T} \cdot \sum_{t=1}^T |r_{I,t} - r_{P,t}| \quad (2)$$

with  $t = 1, \dots, T$  time periods.

The mean square error (MSE) expresses the tracking quality by measuring the mean of the squared differences between the returns over a given time interval

$$MSE = \frac{1}{T} \cdot \sum_{t=1}^T (r_{I,t} - r_{P,t})^2 \quad (3)$$

Figure 1 illustrates the difference between the MAD and the MSE. It can be seen that the MSE considers greater return differences relatively more than smaller ones compared to the MAD.



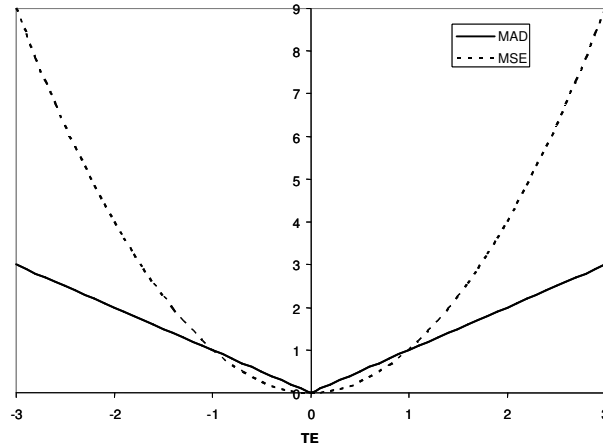


Figure 1: Difference between MAD and MSE

A variant of the MSE is the tracking error variance (TEV), which is defined as the variance of the return differences

$$TEV = \frac{1}{T} \cdot \sum_{t=1}^T (r_{I,t} - r_{P,t} - (\bar{r}_I - \bar{r}_P))^2 \quad (4)$$

with  $\bar{r}_P$  = mean of returns of the tracking portfolio

$\bar{r}_I$  = mean of returns of the index

Even though the TEV is very often used, it has the disadvantage of a shift.<sup>16</sup> That means, if the return of the tracking portfolio is always the same multiple to the return of the index, then the tracking error is a constant and the tracking error variance is zero, although there is a deviation to the index. This is illustrated in Figure 2. Portfolio 1 has an obviously worse tracking quality than portfolio 2 but the TEV of Portfolio 1 is equal to zero while the TEV of Portfolio 2 is greater than zero.

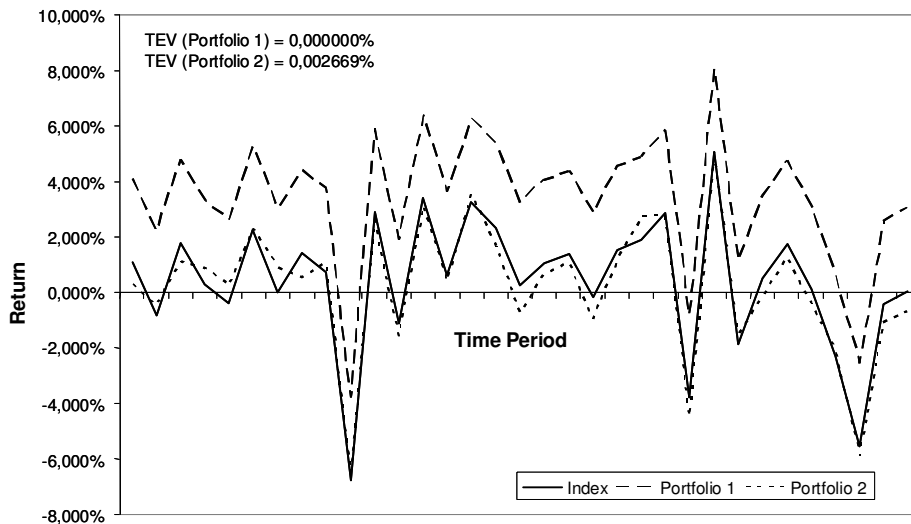


Figure 2: Weakness of the TEV

For a given selection of assets, the optimal tracking portfolio can be built by estimating the weights  $w_j$  that minimize the chosen measure. But which one is the appropriate measure?

<sup>16</sup> See Beasley et al. (2003), p. 629.

From a mathematical point of view, the tracking error  $TE$  is an error term, usually denoted by  $\varepsilon_t$ . Thus, we can rewrite (1) as

$$r_{I,t} = r_{P,t} + TE_t = r_{P,t} + \varepsilon_t = \sum_{j=1}^n w_j \cdot r_{j,t} + \varepsilon_t \quad (5)$$

The probabilistic characteristics of the error term have an essential influence on the estimation of the parameters  $w_j$ . Thus, it can be shown that, if the error terms are IID<sup>17</sup> and follow a normal distribution, the minimization of the MSE gives the best linear unbiased estimator.<sup>18</sup> But if the error terms are IID and follow a Laplace distribution, then it can be shown that the minimization of the MAD gives the most likely estimator. Rudolf et al. show that the different measures result in tracking portfolios with different risk/return properties.<sup>19</sup> Rudolf concludes that MAD produces tracking portfolios, which are closer to the index in the investment period.<sup>20</sup>

In the remainder of the paper the robustness of the MAD, MSE and TEV will be analyzed. The focus lays on the analysis on how stable the estimations of tracking quality that are based on the minimization of these measures are.

## 5 Analysis of the Stability of the Traditional Measures of Tracking Quality

### 5.1 Research Method

In contrast to other studies on tracking quality, the focus of our research lies on the analysis of the stability of the measures described in the previous chapter. That means we are not searching for a method to find the best tracking portfolio with the highest tracking quality. Instead, we want to compare the results for the estimation periods with those for the investment periods to prove the stability of the measures. Thus, the quality of the selection process in sampling does not play a role in this context.

We are interested in three topics. The first one is the comparison of the absolute tracking quality in the estimation period and the investment period. That means we want to examine the difference in the level of tracking quality. The second topic is the comparison of the relative positions of the tracking portfolios in both time periods ranked by their quality. If a tracking portfolio affiliates to the best ones in the estimation period, does it also do that in the investment period and vice versa? The third topic is a comparison of the ex post tracking qualities of the three traditional measures to evaluate if there are any significant differences in their predictive value.

For our analysis, we use the HDAX performance index having 110 underlying German stocks. To avoid conclusions caused by specific capital market situations, we select different time periods between 2004 and 2010. Every time period covers a different capital market situation (see figure 3).

<sup>17</sup> Independent and Identically Distributed.

<sup>18</sup> See Draper/Smith (1998), p. 567.

<sup>19</sup> See Rudolf/Wolter/Zimmermann (1999), p. 102.

<sup>20</sup> See Rudolf (2009).

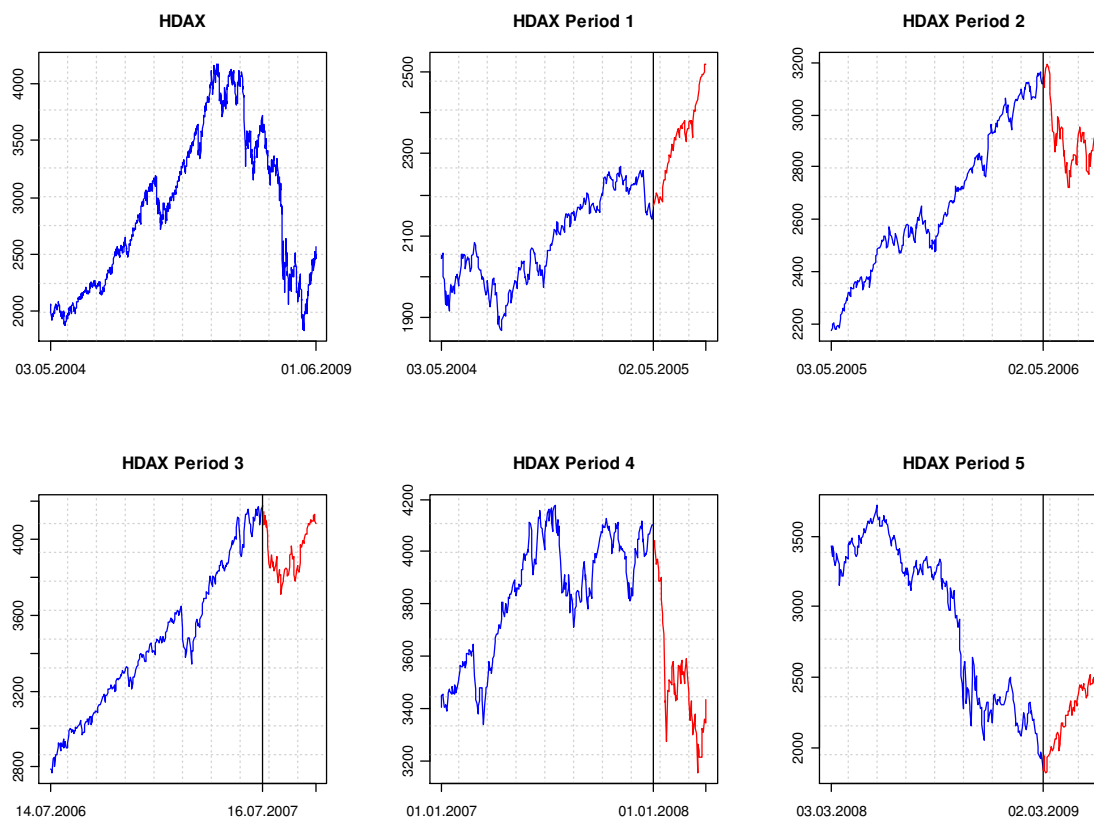


Figure 3: Price Curves of the HDAX

We divide every period into one estimation part with a length of 12 months and an investment part of 3 months. The data from the estimation part are used to calculate the tracking portfolios and in the investment part, the portfolios are virtually invested. We use daily data for our calculations.

To analyze the influence of the tracking portfolio size on the stability of the tracking quality we select portfolio sizes between 10 and 50 with steps of 5, resulting in tracking portfolio sizes of 10, 15, 20, ..., 50.

For every time period and every portfolio size, tracking portfolios are optimized using the three measures of tracking quality (MAD, MSE and TEV) as objective.

For every time period, every portfolio size and every measure, 5000 tracking portfolios are calculated. For every portfolio, the stocks are randomly selected out of the 110 HDAX stocks. In total,  $(5000(\text{portfolios}) \cdot 9(\text{sizes}) \cdot 3(\text{objectives}) \cdot 5(\text{periods}))$  675000 tracking portfolios are calculated. These portfolios are used for three analyses.

In the first analysis, we examine the absolute values of the three traditional measures of tracking quality. Thus, we want to analyze, if there are sustainable differences in the quality level of the estimation period compared to the investment period. If there does exist such a difference, the tracking quality in the estimation period would not be a good indicator for the tracking quality in the investment period. For this purpose, we compare the value of the measure used as objective for the estimation period with the value of the same measure in the invest-

ment period. We calculate means and variances for every combination of portfolio size, time period and objective.

In the second analysis, we evaluate the stability of the relative tracking quality of the measures used as objective. Therefore, we use each of the 5000 tracking portfolios for every combination of portfolio size, time period and objective, rank them according to their tracking quality in the estimation and in the investment period and finally allocate the portfolios into deciles for both time periods according to the ranks. The resulting tables indicate a bad stability if a high percentage of the tracking portfolios do not remain in the same deciles in the estimation and the investment period.

The third analysis is a comparison of the ex post tracking qualities of the measures used as objectives. One important question is how to measure the quality in this context. First, it must be a measure to evaluate the ex post tracking quality in the view of an investor. Second, it does not seem to be advisable using a measure, which is used in parts of the optimizations when creating the tracking portfolios. We assume that such an ex post measure prefers the tracking portfolios built with the same measure used as objective.

For an investor, a tracking portfolio is of high quality if it performs similarly to the index. Similarity means in this context, that the tracking portfolio and the index do not really deviate from each other in the growth of value. The more similar a tracking portfolio is the higher quality it has for the investor.

Unfortunately, both the MAD and the MSE do not reflect this definition of quality when regarding the deviations of growth in value. This can be demonstrated easily. Figure 4 shows the returns of the DAX and two artificially constructed tracking portfolios. Apart from period 6, where the return has a surplus of 6.6 percent, the tracking portfolio 1 has exactly the same returns as the index. In opposite, tracking portfolio 2 has small deviations to the index in every period. The MAD of both tracking portfolios is equal. That means, according to the MAD both tracking portfolios have an identical tracking quality.

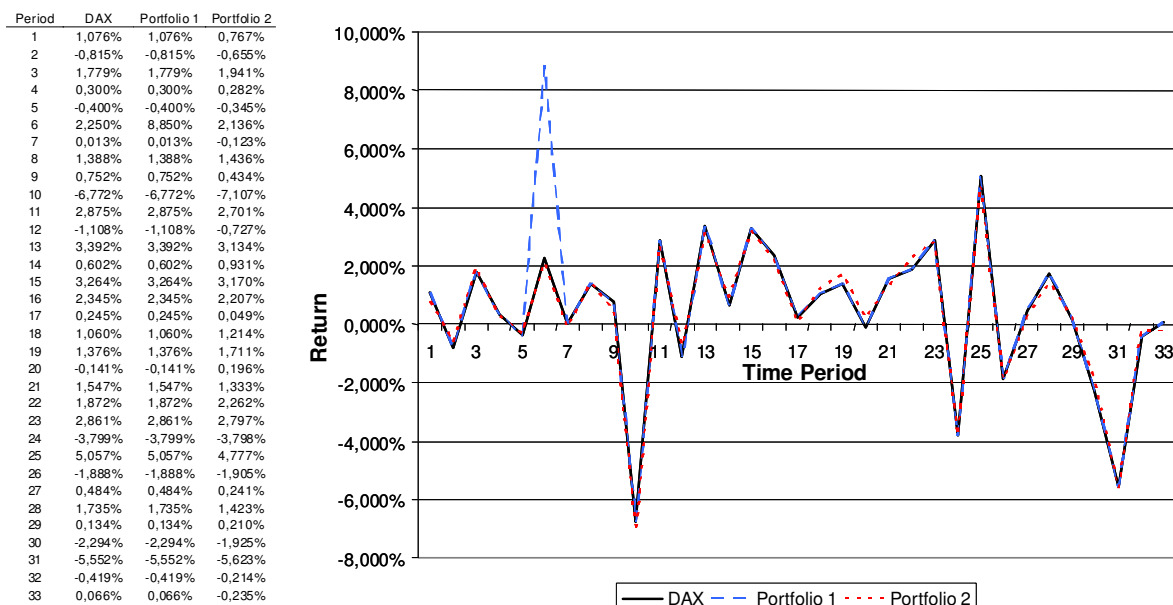


Figure 4: Returns of the DAX and two tracking portfolios with an equal MAD

Assuming, one would invest 1,000 Euro in the index and in every tracking portfolio in period 0, the growth of the three portfolios would be like it is illustrated in figure 5. It is obvious that tracking portfolio 1 is much worse than tracking portfolio 2. Thus, the MAD is not feasible to measure the ex post tracking quality.

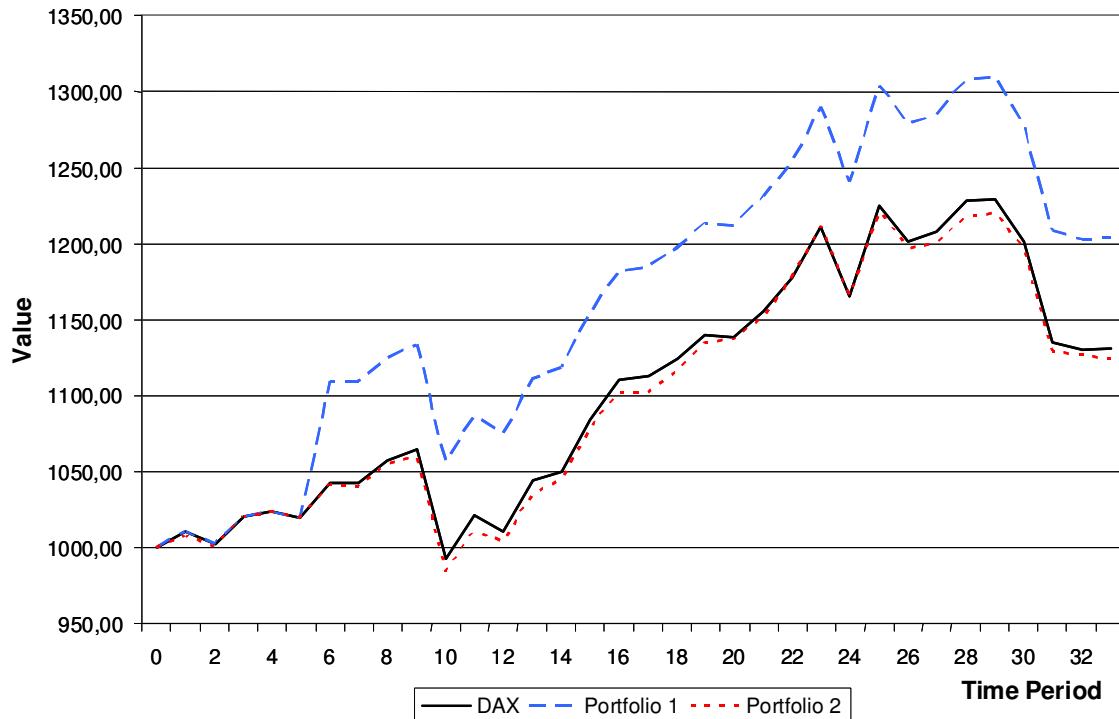


Figure 5: Values of a DAX and two tracking portfolios with an equal MAD

Repeating this experiment with the MSE shows similar results. Again, the index and tracking portfolio 1 are used. Then a tracking portfolio 2 with the same MSE as tracking portfolio 1 is generated (see figure 6).

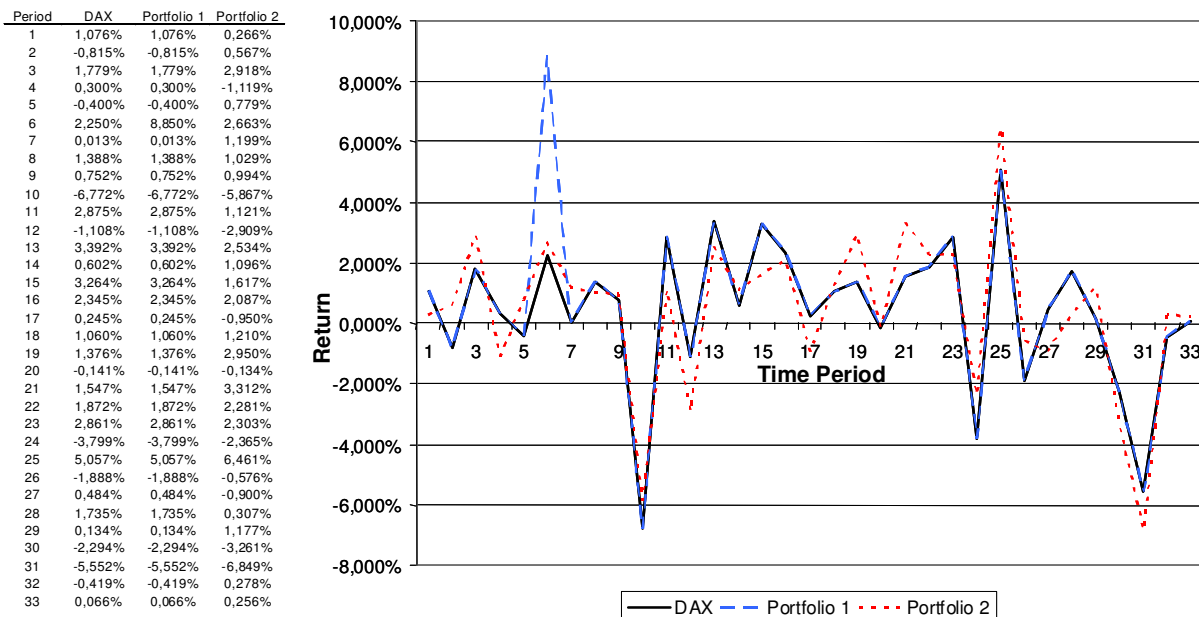


Figure 6: Returns of the DAX and two tracking portfolios with an equal MSE

Figure 7 shows, that tracking portfolio 1 is again worse than tracking portfolio 2. Thus, also the MSE is not feasible to measure the ex post tracking quality.

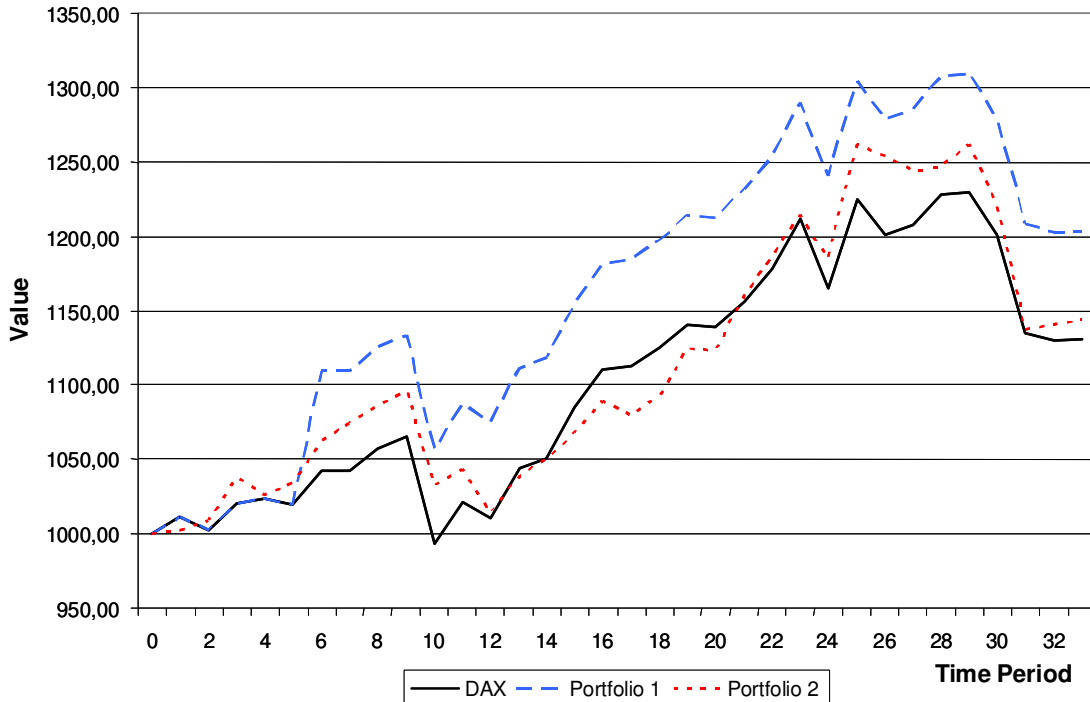


Figure 7: Values of a DAX and two tracking portfolios with an equal MSE

This raises the question which measure should be used. Taking the fact into account that the closer the tracking portfolio to the index, the better it is from the investors view, a measure which calculates the deviations between the values seems to be appropriate. Just the subtraction of the terminal values of the tracking portfolio and the index portfolio at the end of the time interval is too much influenced by coincidence. Thus, a useful measure of the ex post tracking quality should measure the mean absolute differences over all the periods of the time interval.

This ex post tracking quality  $TQ_{ep}$  can be calculated using the formula<sup>21</sup>

$$TQ_{ep} = \frac{1}{T} \sum_{t=1}^T \left| \frac{V_{I,t}}{V_{I,0}} - \frac{V_{P,t}}{V_{P,0}} \right| \quad (6)$$

with  $V_{P,t}$  = value of the tracking portfolio at time  $t$

$V_{I,t}$  = value of the index at time  $t$

This measure expresses the average difference of the returns of the tracking portfolio and the index related to their initial values at time  $t_0$ . Its application clearly shows that tracking portfolio 1 is worse in both of the cases illustrated in the figures 5 and 7. For tracking portfolio 1 it has a value of 6.149 percent, for tracking portfolio 2 in figure 5 the value is 0.468 percent and for tracking portfolio 2 in figure 7 the value is 1.884 percent. Thus, the measure meets the requirements stated above. Using this measure, we are able to evaluate the tracking quality of MAD, MSE and TEV as objective in the tracking portfolio generation process.

<sup>21</sup> A similar measure is used by Gaivoronski et al. (2005), p. 120.

## 5.2 Evaluation Method

The explanations above show that for our analysis much data has to be evaluated which is structured into several dimensions. To analyze the data we use OLAP (On-Line Analytical Processing) technology. OLAP is a software technology that focuses on multidimensional data structures. The aim is to give the user a flexible way to access those data structures and to analyze them from multiple perspectives.

In OLAP the data is organized in a multidimensional cube (hypercube). Every edge of this cube represents a dimension. The cells in the cube contain data (facts), e.g. sales or turnover, which is categorized by the dimensions, e.g. time, product, and location.

The user interface of OLAP tools usually gives access onto two-dimensional tables, where different dimensions are chosen while others are fixed. Therewith, the user is able to surf through the cube by cutting two-dimensional slices. This technique is called slicing. Furthermore, the data within the cube can be aggregated ad hoc by merging values within the dimensions, called drilling. With these techniques, the user has nearly any opportunity to analyze the data provided by the hypercube.

	A	B	C	D	E	F
1	localhost/result					
2	cube					
3	estimation_period					
4	all					
5	mad					
6	mean					
7	ex_post_tq					
8						
9		period1	period2	period3	period4	period5
10	tp_size_10	0.035588	0.039921	0.042325	0.044046	0.049990
11	tp_size_15	0.028062	0.030742	0.033953	0.035337	0.039991
12	tp_size_20	0.023158	0.026538	0.029318	0.029978	0.034554
13	tp_size_25	0.019465	0.022889	0.026781	0.026529	0.030262
14	tp_size_30	0.016945	0.020532	0.024285	0.023853	0.027417
15	tp_size_35	0.015134	0.018082	0.022866	0.021742	0.024426
16	tp_size_40	0.013283	0.016638	0.021976	0.019972	0.022106
17	tp_size_45	0.011789	0.015149	0.020706	0.018606	0.020284
18	tp_size_50	0.010566	0.013855	0.019719	0.017420	0.018779

Figure 8: Screenshot of Palo

For our research we use Palo ([www.palo.net](http://www.palo.net)) which is an open source product providing its functions as an Excel Add-In (see figure 8 for a screenshot). The advantage of this solution is that one can use all the Excel functionality simultaneously.

Formally, a hypercube  $H$  can be described by its  $n$  dimensions  $D_i$  which categorize  $m$  facts  $F_j$  as the data content of the cells

$$H = ((D_1, D_2, \dots, D_n), (F_1, F_2, \dots, F_n))$$

In our analysis, we have six dimensions and four facts as shown in figure 9.

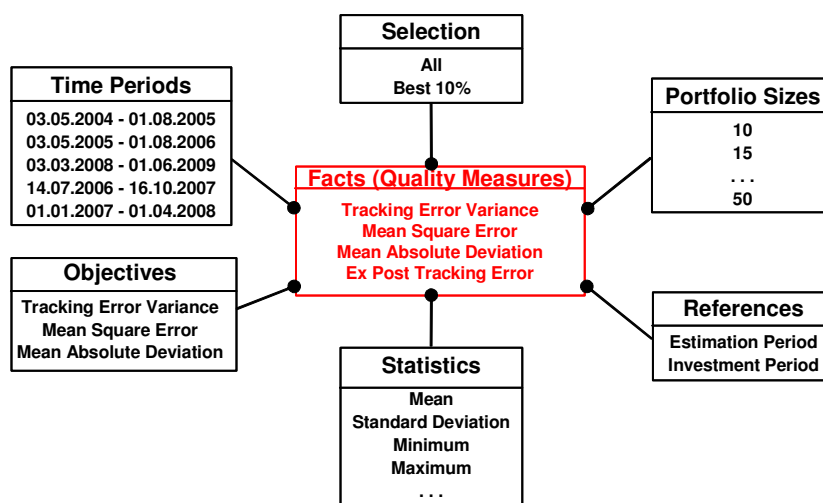


Figure 9: Model of the OLAP Cube

### 5.3 Empirical Results

The goal of the first analysis is to examine the differences in the level of tracking quality by comparing the tracking qualities of the estimation and the investment period. Therefore, we calculate the tracking portfolios for the different measures used as objectives, tracking portfolio sizes and time periods using daily prices. Subsequently, we calculate the means and standard deviations for the 5000 tracking portfolios of every combination. The results are shown in figure 10. The values in the tables represent the means and standard deviations of the different combinations, i.e. the 0.316559 represents the mean of the MAD of the 5000 tracking portfolios calculated for the tracking portfolio size of 10 stocks of the estimation period in time period 1 using the MAD as objective and the 0.001781 in the same row represents the mean of the MSEs of the 5000 tracking portfolios of the same size in the same time period using the MSE as objective. For presentation reasons, all values in the following are expressed in percent and therefore multiplied by 100.

The results for time period 1 show that the means and standard deviations of the tracking portfolios in the estimation period and in the investment period are very similar. This holds for every of the three measures. The characteristic of this time period is that there is a clear upward trend in the index curve with some small oscillations in the estimation period. The investment period in contrast is clearly stable. As expected, the tracking quality is enhancing with an increasing tracking portfolio size.

Period 2 shows a different picture. Here, we have a clear upward trend in the estimation period and in the investment period at first a downward trend followed by a sideward motion with oscillation. Consequently, the means and especially the standard deviations are larger in the investment period compared to the estimation period. This holds for every measure.

In period 3, we have a clear upward trend in the estimation period with one short discontinuity, but the investment period looks like a U-curve with a downward motion at first followed by an upward motion. Again, the means and standard deviations are clearly larger in the investment period compared to the estimation period.





Figure 10: Comparison of the Levels of Tracking Quality over the Time Periods

In period 4, there is at first an upward tendency switching to a sideward tendency in the estimation period. In the investment period, we have a downward trend. Again, there are clear differences in the means and standard deviations in the investment period compared to the estimation period.

Period 5 is characterized by a downward trend in the estimation period and an upward trend in the investment period. This period covers the beginning of the finance market crisis in 2008. Here, the differences between the means of the measures are small compared to the periods 2, 3 and 4, but the values are on the highest level over all time periods.

		MAD				
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	0.316559	0.331421	0.341161	0.401510	0.708254	
15	0.250145	0.265700	0.278402	0.324430	0.585735	
20	0.211823	0.226038	0.238069	0.279543	0.511248	
25	0.183199	0.194570	0.210672	0.245775	0.460945	
30	0.159706	0.172091	0.188454	0.220274	0.422982	
35	0.143394	0.153887	0.170613	0.201188	0.392581	
40	0.127029	0.138613	0.154559	0.181580	0.367129	
45	0.114520	0.124090	0.141217	0.168103	0.345751	
50	0.102935	0.112141	0.129449	0.153866	0.327630	

		MSE				
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	0.001781	0.001953	0.002036	0.002882	0.010222	
15	0.001104	0.001249	0.001350	0.001853	0.007062	
20	0.000789	0.000904	0.000989	0.001363	0.005446	
25	0.000591	0.000673	0.000778	0.001050	0.004453	
30	0.000449	0.000527	0.000625	0.000842	0.003779	
35	0.000363	0.000422	0.000517	0.000703	0.003282	
40	0.000286	0.000344	0.000428	0.000574	0.002886	
45	0.000233	0.000278	0.000361	0.000492	0.002578	
50	0.000188	0.000228	0.000305	0.000413	0.002330	

		TEV				
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	0.001783	0.001955	0.002037	0.002864	0.010245	
15	0.001104	0.001252	0.001352	0.001837	0.007082	
20	0.000789	0.000905	0.000990	0.001349	0.005464	
25	0.000591	0.000674	0.000778	0.001038	0.004469	
30	0.000449	0.000529	0.000625	0.000832	0.003793	
35	0.000363	0.000424	0.000517	0.000693	0.003296	
40	0.000286	0.000346	0.000427	0.000566	0.002898	
45	0.000232	0.000279	0.000360	0.000484	0.002590	
50	0.000188	0.000229	0.000305	0.000407	0.002342	

Figure 11: Comparison of the Levels of Tracking Quality in the Estimation Periods

Figure 11 shows a comparison of the tracking quality levels of the estimation periods over the time periods. It can be seen that the tracking quality is decreasing from period to period. This leads to the conclusion that the degree of fluctuations on the stock market has an impact on the tracking quality, when applying sampling methods based on the used measures. The stronger the fluctuations, especially when combined with changes in trend, the inferior the tracking quality due to changing price relations of the stocks to the index. Especially in period 5, the turbulences on the stock markets led to a poor tracking quality.

Next, we examine the relationships between the means of the estimation periods and the corresponding values of the investment periods. Because the MAD is a linear measure and MSE and TEV are quadratic measures neither the absolute difference nor the ratio between the estimation period and the investment period can be directly used for comparison. To overcome this problem, we first calculate the square root of the MSEs and TEVs. After this, we calculate the ratios between the values of the estimation period and the investment period.

Figure 12 shows that in the majority of cases the values in the investment period are on a higher level compared to the estimation period indicating a lower tracking quality in the investment period. This is a normal case because the calculations of the tracking portfolios are based on the data of the estimation period reflecting the specific situation in this time interval. Special focus lies on the amount of difference because the bigger the discrepancy the more we have a so called overfitting effect caused by changes in the price relations of the stocks to the index. A strong overfitting effect can foil the intension of sampling because the calculated tracking quality in the estimation period is not a good predictor for the tracking quality in the investment period in this case.

<b>MAD</b>					
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5
10	0.961279	1.309659	1.323544	1.523446	1.127529
15	0.995816	1.332093	1.340843	1.553510	1.138888
20	1.021035	1.354352	1.351655	1.571740	1.144666
25	1.047637	1.369411	1.364162	1.582862	1.154122
30	1.072424	1.392365	1.376902	1.593809	1.159328
35	1.093883	1.415466	1.387714	1.596838	1.165137
40	1.115857	1.433836	1.394301	1.605317	1.172248
45	1.135181	1.460759	1.402251	1.605935	1.175213
50	1.157183	1.481409	1.407333	1.605136	1.174954

<b>MSE</b>					
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5
10	0.970483	1.351634	1.313268	1.553600	1.054303
15	1.003479	1.374476	1.314085	1.585787	1.057713
20	1.024469	1.394192	1.312337	1.603708	1.060462
25	1.049819	1.407971	1.312006	1.611070	1.069074
30	1.074208	1.436345	1.319287	1.617954	1.072847
35	1.093396	1.460641	1.320457	1.615835	1.078343
40	1.112982	1.482841	1.321737	1.621103	1.085187
45	1.130076	1.520337	1.328683	1.617228	1.089900
50	1.150783	1.549022	1.328453	1.614004	1.091881

<b>TEV</b>					
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5
10	0.973595	1.357823	1.317272	1.560093	1.056148
15	1.007232	1.380235	1.317478	1.593192	1.060036
20	1.028091	1.399676	1.315301	1.611424	1.062733
25	1.053716	1.412610	1.314213	1.619718	1.070998
30	1.078401	1.439929	1.321180	1.627371	1.074373
35	1.097665	1.463286	1.322160	1.626265	1.079536
40	1.117141	1.483753	1.322887	1.632551	1.085966
45	1.133860	1.519595	1.330165	1.629228	1.090366
50	1.154856	1.545283	1.329445	1.627642	1.091573

Figure 12: Comparison of the Ratios between Estimation and Investment Period

The results in figure 12 show, that there are big discrepancies in the relationships depending on the time period and on the portfolio size. The smallest discrepancies can be found in period 1. The biggest discrepancies occur in periods 2, 3 and 4 where we have changes in trend from upward to downward. In period 4 where the discrepancies are the highest we have additionally the strongest oscillations. This leads to the conclusion, that oscillating stock market movements and trend changes have a negative impact on the predictability of the traditional tracking quality measures. Interestingly, for period 5 we have the smallest discrepancies even though the beginning of the financial crisis is covered, accompanying a trend change and oscillations. A feasible explanation for this fact is that the tracking quality in this time period is not only poor in the investment period but has already been poor in the estimation period.

This is supported by the interesting fact that the values in figure 12 raise with the tracking portfolio size. Thus, the more stocks are used to track the index, the bigger the discrepancy between the tracking quality level in the estimation period and the investment period. Again,, the tracking quality of small tracking portfolio sizes is already worse compared to the bigger tracking portfolio sizes, so that the degree of deterioration turns out to be smaller.

The comments and conclusions from above hold for all the three measures. Comparing the measures themselves, there is no clear picture. In periods 2 and 4 MAD clearly shows better results and in 3 and 5 MSE followed by TEV. MSE and TEV are again very close. In period 1 all measures are very close.

The previous findings are based on averages with over 5000 tracking portfolios per analysis which covers good tracking portfolios to poor tracking portfolios as a consequence of the random stock selection process. To find out if the results are influenced by an averaging effect,

we repeat the analysis with only the 500 best portfolios. That means, for every measure we select those 10 percent of the tracking portfolios with the lowest value in the estimation period. We apply this for every period and tracking portfolio size. Thus, the particular 500 best tracking portfolios could be composed by different stocks for every measure.



Figure 13: Levels of Tracking Quality over the Time Periods (Best 10%)

The figures 13, 14 and 15 show the results. They confirm the findings stated above but on a higher level of tracking quality.

<b>MAD</b>						
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	0.217879	0.235708	0.255895	0.295158	0.537448	
15	0.176001	0.189965	0.209956	0.239095	0.452627	
20	0.148878	0.160013	0.177406	0.203682	0.399023	
25	0.128262	0.138632	0.156333	0.179679	0.366214	
30	0.112835	0.120313	0.137091	0.159176	0.336879	
35	0.100538	0.107254	0.122526	0.142584	0.315152	
40	0.088391	0.094776	0.108332	0.125628	0.295928	
45	0.079411	0.084826	0.095879	0.116138	0.280207	
50	0.071713	0.074712	0.087309	0.102886	0.268114	

<b>MSE</b>						
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	0.000821	0.000956	0.001137	0.001508	0.005780	
15	0.000532	0.000621	0.000766	0.000988	0.004187	
20	0.000380	0.000444	0.000551	0.000717	0.003321	
25	0.000281	0.000334	0.000430	0.000556	0.002810	
30	0.000218	0.000253	0.000334	0.000439	0.002437	
35	0.000173	0.000202	0.000268	0.000351	0.002161	
40	0.000135	0.000159	0.000211	0.000274	0.001936	
45	0.000109	0.000128	0.000167	0.000233	0.001751	
50	0.000089	0.000100	0.000140	0.000185	0.001631	

<b>TEV</b>						
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	0.000820	0.000956	0.001137	0.001499	0.005797	
15	0.000532	0.000621	0.000766	0.000980	0.004201	
20	0.000379	0.000445	0.000551	0.000712	0.003334	
25	0.000281	0.000334	0.000430	0.000551	0.002821	
30	0.000217	0.000253	0.000333	0.000435	0.002448	
35	0.000173	0.000202	0.000268	0.000348	0.002171	
40	0.000134	0.000159	0.000211	0.000272	0.001945	
45	0.000109	0.000129	0.000167	0.000232	0.001761	
50	0.000089	0.000100	0.000140	0.000184	0.001640	

Figure 14: Levels of Tracking Quality in the Estimation Periods (Best 10%)

<b>MAD</b>						
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	1.058316	1.242835	1.328486	1.519636	1.154987	
15	1.081369	1.294410	1.344185	1.524121	1.166784	
20	1.097228	1.337174	1.349124	1.536891	1.165318	
25	1.117923	1.371179	1.353797	1.557043	1.165405	
30	1.137297	1.418850	1.371516	1.573887	1.174136	
35	1.148267	1.469220	1.385218	1.588473	1.162332	
40	1.160461	1.510576	1.400355	1.584126	1.167223	
45	1.185795	1.548207	1.413297	1.584778	1.151705	
50	1.201002	1.576731	1.421258	1.614941	1.141279	

<b>MSE</b>						
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	1.085826	1.311197	1.264828	1.534278	1.098314	
15	1.095088	1.367815	1.277627	1.541910	1.092874	
20	1.105694	1.419750	1.288606	1.540882	1.085954	
25	1.115006	1.446490	1.285261	1.552175	1.094517	
30	1.139014	1.544595	1.293446	1.576382	1.100648	
35	1.142409	1.596892	1.299644	1.579497	1.097816	
40	1.150148	1.659510	1.305084	1.566532	1.106398	
45	1.167026	1.718302	1.325732	1.553600	1.096394	
50	1.177158	1.808996	1.326104	1.573991	1.096600	

<b>TEV</b>						
TP Size	Period 1	Period 2	Period 3	Period 4	Period 5	
10	1.090292	1.314786	1.262822	1.535565	1.099261	
15	1.101320	1.370765	1.274363	1.551352	1.095067	
20	1.109562	1.419917	1.286442	1.544550	1.086793	
25	1.116917	1.446720	1.286630	1.559854	1.095670	
30	1.141357	1.543101	1.292218	1.584025	1.100972	
35	1.142694	1.591752	1.301197	1.590922	1.098274	
40	1.152365	1.652181	1.305250	1.575417	1.103921	
45	1.169300	1.706000	1.327747	1.565187	1.095208	
50	1.178770	1.794945	1.326030	1.583828	1.092630	

Figure 15: Relationships between Estimation and Investment Period (Best 10%)

Compared to figure 12, figure 15 shows a more nebulous result. Here, we cannot even say in some periods which measure is the more stable. It is remarkable that the values are on a higher level in figure 15. That means that the stability of the best 10 percent is lower on average than the stability over all portfolios. In other words, the discrepancy between estimation period and investment period is bigger on average.

In order to analyze this in detail, we perform a second analysis. Here, for every measure, every tracking portfolio size and every time period we rank the 5000 tracking portfolios according to their tracking quality for the estimation period and for the investment period. Then, we allocate the tracking portfolios for both periods into deciles according to their ranks. Finally, we build a matrix with the deciles of both periods as dimensions.

MAD		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	54.60%	25.00%	13.00%	4.80%	1.80%	0.40%	0.20%	0.20%	0.00%	0.00%
	2	21.60%	28.60%	21.60%	14.00%	7.60%	3.60%	1.80%	1.00%	0.20%	0.00%
	3	9.60%	16.00%	18.80%	20.20%	16.00%	10.60%	5.80%	1.60%	1.40%	0.00%
	4	5.20%	13.60%	14.40%	17.20%	17.40%	14.40%	8.60%	6.80%	1.80%	0.60%
	5	4.40%	7.60%	11.60%	14.80%	16.40%	16.80%	12.20%	7.60%	8.00%	0.60%
	6	2.20%	4.40%	9.80%	10.80%	14.20%	19.20%	16.40%	12.20%	8.40%	2.40%
	7	1.60%	2.60%	6.00%	8.40%	10.60%	14.40%	18.60%	18.20%	14.00%	5.60%
	8	0.80%	1.80%	3.40%	5.40%	9.00%	10.60%	14.40%	21.80%	19.60%	13.20%
	9	0.00%	0.40%	1.00%	4.00%	6.00%	7.60%	14.20%	17.20%	23.80%	25.80%
	10	0.00%	0.00%	0.40%	0.40%	1.00%	2.40%	7.80%	13.40%	22.80%	51.80%

MSE		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	50.40%	24.40%	13.80%	6.60%	3.80%	0.80%	0.20%	0.00%	0.00%	0.00%
	2	22.80%	29.40%	17.80%	13.40%	9.20%	4.60%	1.60%	1.00%	0.20%	0.00%
	3	10.20%	14.00%	18.40%	17.20%	15.60%	13.60%	6.40%	2.80%	1.60%	0.20%
	4	6.40%	12.40%	16.80%	15.80%	13.20%	15.40%	10.00%	6.40%	2.80%	0.80%
	5	5.20%	7.80%	11.20%	13.20%	15.60%	16.80%	12.00%	9.80%	6.40%	2.00%
	6	2.60%	6.00%	7.60%	13.00%	16.20%	13.00%	13.80%	13.20%	9.80%	4.80%
	7	1.60%	4.00%	7.00%	6.40%	9.20%	15.20%	17.40%	16.40%	14.00%	8.80%
	8	0.80%	1.60%	5.20%	8.20%	8.00%	8.80%	16.40%	19.40%	17.60%	14.00%
	9	0.00%	0.20%	1.80%	5.20%	5.60%	7.60%	14.20%	16.80%	22.00%	26.60%
	10	0.00%	0.20%	0.40%	1.00%	3.60%	4.20%	8.00%	14.20%	25.60%	42.80%

TEV		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	49.40%	26.00%	14.00%	6.20%	3.60%	0.60%	0.20%	0.00%	0.00%	0.00%
	2	23.40%	28.80%	18.00%	13.60%	9.00%	4.60%	1.60%	0.80%	0.20%	0.00%
	3	10.20%	13.00%	19.20%	17.20%	15.20%	13.00%	7.60%	2.80%	1.60%	0.20%
	4	6.80%	12.40%	15.60%	15.60%	15.00%	15.00%	8.60%	7.00%	3.20%	0.80%
	5	5.00%	7.80%	12.00%	14.00%	14.20%	17.00%	11.80%	9.40%	6.40%	2.40%
	6	3.00%	5.80%	7.60%	12.00%	16.60%	13.60%	13.00%	14.00%	9.60%	4.80%
	7	1.40%	4.20%	6.40%	6.80%	9.40%	15.00%	17.80%	15.80%	15.00%	8.20%
	8	0.80%	1.60%	4.80%	8.60%	7.40%	9.40%	17.20%	18.60%	16.60%	15.00%
	9	0.00%	0.20%	2.00%	4.80%	6.40%	7.20%	14.00%	17.00%	21.80%	26.60%
	10	0.00%	0.20%	0.40%	1.20%	3.20%	4.60%	8.20%	14.60%	25.60%	42.00%

Figure 16: Stability of the Relative Tracking Quality (Period 1)

Figure 16 shows the matrices for the three measures for period 1 and a tracking portfolio size of 30. On the main diagonal of the matrices one can find the percentages of those portfolios which are allocated in the same decile in the estimation period and in the investment period. The values on this diagonal indicate the stability of the relative tracking quality. That means for example, that a tracking portfolio which belongs to the best 10 percent in the estimation period also belongs to the best 10 percent in the investment period. The other values in the rows of the matrices show the percentages of those tracking portfolios which change their decile from the estimation period to the investment period. The 4.8% in the first row of the MAD matrix in figure 16 expresses that 24 tracking portfolios out of the 500 from the best 10% moved from the 1<sup>st</sup> decile in the estimation period to the 4<sup>th</sup> decile in the investment period.

The Stability of Traditional Measures of Index Tracking Quality

MAD		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	58.20%	24.40%	9.20%	5.00%	1.40%	1.20%	0.60%	0.00%	0.00%	0.00%
	2	21.80%	26.60%	23.80%	15.40%	8.80%	2.80%	0.40%	0.40%	0.00%	0.00%
	3	12.00%	23.00%	20.80%	16.20%	15.00%	7.60%	3.60%	1.60%	0.20%	0.00%
	4	4.60%	13.00%	19.80%	21.20%	17.00%	13.40%	6.60%	3.60%	0.80%	0.00%
	5	2.80%	6.80%	13.80%	17.80%	18.40%	16.00%	13.60%	8.00%	2.80%	0.00%
	6	0.60%	4.60%	7.80%	13.40%	18.80%	21.20%	15.20%	13.00%	4.40%	1.00%
	7	0.00%	1.00%	3.40%	6.80%	10.60%	18.60%	21.60%	23.00%	12.40%	2.60%
	8	0.00%	0.60%	1.20%	3.20%	7.80%	10.00%	23.60%	23.00%	24.00%	6.60%
	9	0.00%	0.00%	0.20%	1.00%	1.80%	8.00%	10.80%	20.00%	34.40%	23.80%
	10	0.00%	0.00%	0.00%	0.00%	0.40%	1.20%	4.00%	7.40%	21.00%	66.00%

MSE		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	46.60%	23.20%	13.20%	6.80%	6.40%	2.40%	1.00%	0.20%	0.20%	0.00%
	2	22.80%	26.60%	18.60%	11.80%	9.40%	6.00%	2.60%	1.60%	0.60%	0.00%
	3	16.40%	17.40%	20.00%	16.60%	13.40%	8.40%	5.40%	1.80%	0.60%	0.00%
	4	9.20%	14.00%	17.60%	17.00%	15.60%	11.40%	7.60%	5.20%	1.80%	0.60%
	5	3.40%	10.00%	14.60%	17.20%	14.40%	13.40%	13.00%	8.80%	4.40%	0.80%
	6	1.40%	5.80%	9.00%	13.40%	16.20%	16.80%	15.60%	12.00%	8.40%	1.40%
	7	0.20%	2.20%	4.80%	10.20%	11.40%	15.40%	18.60%	18.20%	12.60%	6.40%
	8	0.00%	0.80%	1.40%	5.80%	8.60%	15.00%	17.60%	22.80%	18.60%	9.40%
	9	0.00%	0.00%	0.80%	1.20%	3.60%	9.60%	12.80%	21.80%	29.00%	21.20%
	10	0.00%	0.00%	0.00%	0.00%	1.00%	1.60%	5.80%	7.60%	23.80%	60.20%

TEV		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	47.00%	21.00%	15.00%	7.20%	6.20%	2.60%	0.80%	0.00%	0.20%	0.00%
	2	23.60%	26.40%	18.20%	11.60%	9.40%	5.80%	2.60%	1.80%	0.60%	0.00%
	3	15.60%	18.80%	19.80%	15.60%	14.00%	9.20%	4.40%	2.20%	0.40%	0.00%
	4	8.40%	15.00%	17.00%	17.60%	15.80%	11.00%	7.20%	5.80%	1.80%	0.40%
	5	3.80%	10.00%	13.00%	17.40%	15.20%	13.60%	13.00%	9.00%	4.20%	0.80%
	6	1.40%	5.40%	10.40%	12.80%	15.40%	17.40%	15.60%	12.40%	7.80%	1.40%
	7	0.20%	2.60%	4.40%	10.00%	11.80%	14.80%	19.20%	16.60%	13.80%	6.60%
	8	0.00%	0.80%	1.40%	6.40%	7.20%	15.00%	18.80%	22.00%	18.80%	9.60%
	9	0.00%	0.00%	0.80%	1.20%	4.20%	9.00%	13.20%	21.80%	28.40%	21.40%
	10	0.00%	0.00%	0.00%	0.20%	0.80%	1.60%	5.20%	8.40%	24.00%	59.80%

Figure 17: Stability of the Relative Tracking Quality (Period 2)

MAD		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	57.40%	22.80%	11.20%	6.00%	1.80%	0.60%	0.20%	0.00%	0.00%	0.00%
	2	20.80%	26.40%	21.00%	13.80%	8.20%	5.60%	3.20%	0.60%	0.40%	0.00%
	3	11.80%	23.00%	17.60%	16.80%	14.00%	8.60%	4.80%	2.80%	0.60%	0.00%
	4	4.80%	12.20%	16.80%	16.80%	17.20%	11.60%	12.20%	5.60%	2.40%	0.40%
	5	3.40%	8.40%	13.40%	15.80%	19.20%	12.60%	13.60%	8.20%	4.80%	0.60%
	6	1.40%	5.00%	8.20%	11.80%	14.00%	18.60%	14.40%	16.00%	8.00%	2.60%
	7	0.20%	1.60%	7.20%	12.20%	12.00%	17.80%	18.20%	14.60%	12.40%	3.80%
	8	0.20%	0.40%	3.40%	5.20%	9.40%	13.60%	15.60%	21.20%	21.60%	9.40%
	9	0.00%	0.20%	1.20%	1.60%	4.00%	8.20%	13.20%	22.60%	27.40%	21.60%
	10	0.00%	0.00%	0.00%	0.00%	0.20%	2.80%	4.60%	8.40%	22.40%	61.60%

MSE		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	57.60%	24.40%	10.80%	4.40%	2.00%	0.60%	0.20%	0.00%	0.00%	0.00%
	2	22.40%	26.60%	21.20%	12.80%	9.20%	5.60%	1.40%	0.40%	0.40%	0.00%
	3	10.80%	21.20%	20.80%	16.20%	14.60%	9.80%	4.60%	1.60%	0.40%	0.00%
	4	5.00%	12.80%	17.40%	19.80%	16.00%	14.00%	7.00%	6.00%	2.00%	0.00%
	5	2.40%	6.80%	16.00%	17.00%	15.80%	14.80%	12.60%	8.20%	5.80%	0.60%
	6	1.80%	5.40%	6.60%	14.00%	15.40%	16.60%	16.60%	14.80%	7.80%	1.00%
	7	0.00%	2.20%	4.80%	11.40%	11.60%	17.60%	19.20%	18.00%	12.40%	2.80%
	8	0.00%	0.60%	2.20%	2.40%	10.80%	13.60%	18.40%	21.60%	21.60%	8.80%
	9	0.00%	0.00%	0.20%	2.00%	4.00%	6.00%	16.20%	21.20%	26.40%	24.00%
	10	0.00%	0.00%	0.00%	0.00%	0.60%	1.40%	3.80%	8.20%	23.20%	62.80%

TEV		Investment Period									
		1	2	3	4	5	6	7	8	9	10
Esti- mation Period	1	58.20%	24.60%	10.60%	4.00%	2.00%	0.40%	0.20%	0.00%	0.00%	0.00%
	2	22.20%	26.40%	22.00%	12.40%	9.80%	5.20%	1.00%	0.80%	0.20%	0.00%
	3	10.60%	21.60%	21.40%	15.80%	14.60%	9.60%	4.40%	1.60%	0.40%	0.00%
	4	4.80%	13.60%	17.60%	19.40%	17.20%	12.00%	8.40%	5.60%	1.40%	0.00%
	5	2.60%	5.80%	15.80%	16.80%	16.40%	15.40%	12.00%	8.40%	6.00%	0.80%
	6	1.60%	5.20%	7.20%	13.60%	15.40%	17.00%	17.60%	15.20%	6.40%	0.80%
	7	0.00%	2.20%	3.60%	12.00%	11.40%	18.60%	18.40%	18.00%	13.20%	2.60%
	8	0.00%	0.60%	1.60%	3.80%	9.20%	13.80%	18.00%	22.40%	22.20%	8.40%
	9	0.00%	0.00%	0.20%	2.20%	3.80%	6.60%	16.00%	20.00%	26.60%	24.60%
	10	0.00%	0.00%	0.00%	0.00%	0.20%	1.40%	4.00%	8.00%	23.60%	62.80%

Figure 18: Stability of the Relative Tracking Quality (Period 3)

		Investment Period									
		1	2	3	4	5	6	7	8	9	10
<b>MAD</b>	1	58.40%	23.80%	11.20%	4.80%	1.00%	0.60%	0.20%	0.00%	0.00%	0.00%
	2	21.60%	30.20%	20.80%	13.60%	9.20%	2.60%	1.60%	0.40%	0.00%	0.00%
	3	9.40%	20.80%	21.80%	17.00%	13.00%	10.60%	4.40%	2.20%	0.80%	0.00%
	4	5.60%	13.40%	17.00%	18.20%	17.40%	12.80%	9.00%	4.60%	2.00%	0.00%
	5	3.00%	5.80%	13.00%	17.20%	17.80%	16.20%	13.00%	9.40%	4.00%	0.60%
	6	1.00%	3.40%	9.00%	13.80%	15.40%	18.20%	15.60%	12.20%	9.40%	2.00%
	7	0.60%	1.60%	4.40%	8.80%	10.80%	18.80%	17.80%	20.80%	12.80%	3.60%
	8	0.40%	0.60%	1.40%	3.80%	10.40%	11.20%	19.20%	19.80%	23.40%	9.80%
	9	0.00%	0.20%	0.80%	2.40%	4.00%	7.60%	14.40%	22.00%	25.60%	23.00%
	10	0.00%	0.20%	0.60%	0.40%	1.00%	1.40%	4.80%	8.60%	22.00%	61.00%
<b>MSE</b>	1	58.20%	23.60%	12.20%	4.20%	1.00%	0.80%	0.00%	0.00%	0.00%	0.00%
	2	24.40%	26.60%	25.60%	12.00%	7.40%	2.60%	1.00%	0.40%	0.00%	0.00%
	3	9.20%	20.20%	20.00%	20.00%	13.80%	8.80%	5.40%	2.40%	0.20%	0.00%
	4	4.80%	13.20%	20.00%	17.80%	18.80%	11.20%	7.80%	5.00%	1.40%	0.00%
	5	2.00%	9.00%	9.80%	18.80%	17.60%	18.00%	13.80%	7.60%	3.00%	0.40%
	6	1.00%	4.20%	7.00%	13.20%	16.20%	17.60%	16.80%	14.60%	8.60%	0.80%
	7	0.40%	2.20%	3.20%	8.40%	12.60%	18.20%	22.80%	19.00%	9.40%	3.80%
	8	0.00%	0.60%	1.80%	4.40%	8.80%	13.60%	16.20%	22.80%	23.80%	8.00%
	9	0.00%	0.00%	0.40%	1.20%	3.20%	7.60%	13.60%	21.60%	30.00%	22.40%
	10	0.00%	0.40%	0.00%	0.00%	0.60%	1.60%	2.60%	6.60%	23.60%	64.60%
<b>TEV</b>	1	57.20%	23.60%	13.40%	3.60%	1.20%	0.80%	0.20%	0.00%	0.00%	0.00%
	2	24.20%	27.40%	24.00%	12.80%	7.60%	2.40%	1.20%	0.40%	0.00%	0.00%
	3	9.40%	20.60%	21.80%	18.60%	13.20%	8.40%	5.20%	2.20%	0.60%	0.00%
	4	5.60%	13.00%	18.80%	19.80%	18.00%	11.80%	6.80%	4.40%	1.80%	0.00%
	5	2.20%	7.20%	9.60%	17.80%	17.80%	19.40%	14.20%	7.80%	3.60%	0.40%
	6	0.60%	5.40%	7.80%	13.40%	16.60%	16.60%	17.60%	13.80%	7.40%	0.80%
	7	0.60%	1.40%	3.20%	9.00%	13.80%	17.00%	19.60%	20.60%	10.60%	4.20%
	8	0.20%	0.80%	1.20%	4.60%	8.20%	13.20%	19.20%	23.80%	21.60%	7.20%
	9	0.00%	0.20%	0.20%	0.40%	3.20%	9.00%	13.80%	20.00%	30.40%	22.80%
	10	0.00%	0.40%	0.00%	0.00%	0.40%	1.40%	2.20%	7.00%	24.00%	64.60%

Figure 19: Stability of the Relative Tracking Quality (Period 4)

		Investment Period									
		1	2	3	4	5	6	7	8	9	10
<b>MAD</b>	1	45.80%	21.80%	16.40%	6.80%	3.60%	3.00%	1.00%	0.80%	0.60%	0.20%
	2	25.80%	22.20%	17.20%	11.60%	8.00%	8.20%	3.00%	1.60%	2.00%	0.40%
	3	12.40%	18.00%	16.80%	16.00%	11.20%	11.20%	7.60%	4.80%	1.60%	0.40%
	4	7.80%	13.60%	15.20%	14.40%	14.40%	12.60%	9.40%	7.40%	3.80%	1.40%
	5	4.40%	10.20%	12.20%	14.40%	11.20%	13.00%	11.40%	12.80%	6.80%	3.60%
	6	2.20%	6.00%	10.00%	11.60%	16.60%	13.80%	14.00%	11.00%	10.20%	4.60%
	7	0.60%	4.00%	4.60%	11.20%	14.80%	14.60%	13.00%	13.40%	14.20%	9.60%
	8	0.60%	3.00%	4.80%	6.60%	10.00%	11.00%	17.80%	19.60%	17.40%	9.20%
	9	0.20%	1.20%	2.40%	5.20%	8.00%	9.40%	13.60%	18.00%	20.40%	21.60%
	10	0.20%	0.00%	0.40%	2.20%	2.20%	3.20%	9.20%	10.60%	23.00%	49.00%
<b>MSE</b>	1	41.80%	24.60%	13.00%	10.00%	4.40%	1.80%	2.00%	1.60%	0.80%	0.00%
	2	24.40%	20.00%	17.60%	12.80%	10.00%	5.40%	4.80%	2.80%	1.80%	0.40%
	3	16.00%	17.60%	16.40%	12.60%	11.20%	12.60%	6.60%	4.00%	2.40%	0.60%
	4	9.00%	14.20%	14.20%	14.20%	15.40%	9.00%	11.40%	6.00%	4.80%	1.80%
	5	5.00%	10.80%	10.80%	15.60%	13.00%	13.80%	10.00%	10.60%	6.60%	3.80%
	6	2.60%	6.00%	10.80%	12.60%	14.40%	12.80%	13.80%	11.40%	10.60%	5.00%
	7	1.00%	4.20%	8.20%	11.00%	11.80%	14.40%	13.80%	16.60%	11.00%	8.00%
	8	0.00%	1.80%	4.40%	5.20%	10.80%	14.40%	14.80%	16.60%	20.00%	12.00%
	9	0.20%	0.80%	3.80%	4.40%	6.40%	12.00%	13.60%	17.20%	17.80%	23.80%
	10	0.00%	0.00%	0.80%	1.60%	2.60%	3.80%	9.20%	13.20%	24.20%	44.60%
<b>TEV</b>	1	41.40%	25.00%	12.80%	10.40%	3.60%	2.40%	2.00%	1.60%	0.80%	0.00%
	2	24.60%	19.60%	18.20%	11.60%	10.80%	5.20%	5.00%	2.80%	1.80%	0.40%
	3	15.40%	18.20%	16.60%	12.80%	10.60%	12.60%	6.60%	4.00%	2.40%	0.80%
	4	9.20%	13.00%	15.60%	14.40%	14.80%	10.20%	9.80%	7.00%	4.40%	1.60%
	5	5.20%	10.80%	10.20%	15.20%	13.20%	12.80%	11.00%	9.80%	8.20%	3.60%
	6	2.60%	6.80%	10.00%	12.80%	14.00%	13.80%	13.60%	12.00%	9.60%	4.80%
	7	1.00%	4.00%	7.80%	10.60%	13.20%	14.40%	13.80%	15.20%	11.60%	8.40%
	8	0.40%	1.80%	4.20%	5.80%	11.20%	13.40%	14.60%	17.00%	20.00%	11.60%
	9	0.20%	0.80%	3.60%	4.80%	6.60%	11.60%	14.20%	16.80%	18.20%	23.20%
	10	0.00%	0.00%	1.00%	1.60%	2.00%	3.60%	9.40%	13.80%	23.00%	45.60%

Figure 20: Stability of the Relative Tracking Quality (Period 5)



Because of the amount of matrices, we select a tracking portfolio size of 30 for the further presentation. They are representative in the way that the conclusions for the other tracking portfolio sizes are similar. There only exist small differences in the results as the values on the main diagonal increase with an increasing tracking portfolio size. Regarding the results in the figures 16 to 20 we can see that the stability of the relative tracking quality is very poor for all three measures. On the ends of the diagonals, the values range between 40 und 60 percent and the inner diagonal values lay around 20 percent. The higher values on the ends result from the fact that the portfolios can only move in one direction. Again, period 5 shows the worse results.

Summarizing, the values in the matrices show that the majority of the tracking portfolios do not remain in the same decile and instead have a broad distribution. Thus, we can conclude that on average we have neither a high absolute stability as yielded from the first analysis nor we have a high relative stability.

Comparing the relative stability of the three measures, the results indicate that the MAD has a marginal higher stability as the MSE or TEV. The two last-mentioned are again very close. To proof this, we calculated the mean of the main diagonal for every matrix. Figure 21 shows the results in a bar chart for tracking portfolio sizes of 10, 30 and 50 stocks. In the majority of cases, the MAD has the higher relative stability, while MSE and TEV alternate.

So far, we worked with aggregated values. To get an impression about the distribution of the portfolios we map them using scatter plots with the tracking quality of the estimation period on the x-axis and the tracking quality of the investment period on the y-axis.

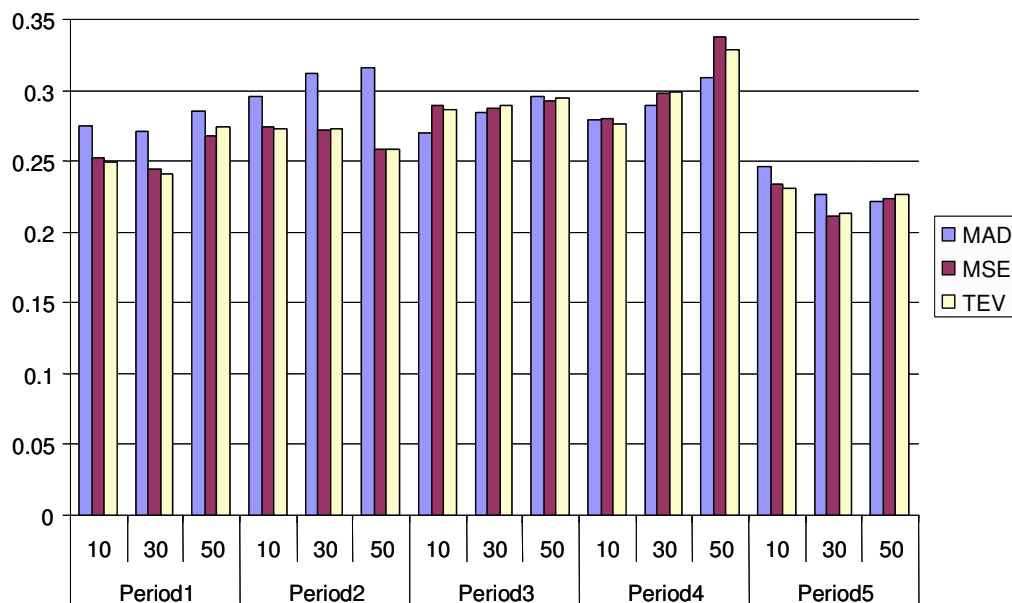


Figure 21: Comparison of the Relative Stability

Figure 22 exemplarily shows the scatter plot of the 5000 tracking portfolios optimized using MAD for period 1 and a tracking portfolio size of 30. It can clearly be seen that there exist some tracking portfolios having a good tracking quality in the estimation period and in the investment period. But it can also be seen that the portfolio with the best tracking quality in

the estimation period (marked with ①) is definitively not the best in the investment period.<sup>22</sup> There exist many portfolios with a higher tracking quality in the investment period. This even holds in the opposite case, where the best portfolio in the investment period is marked with ②.

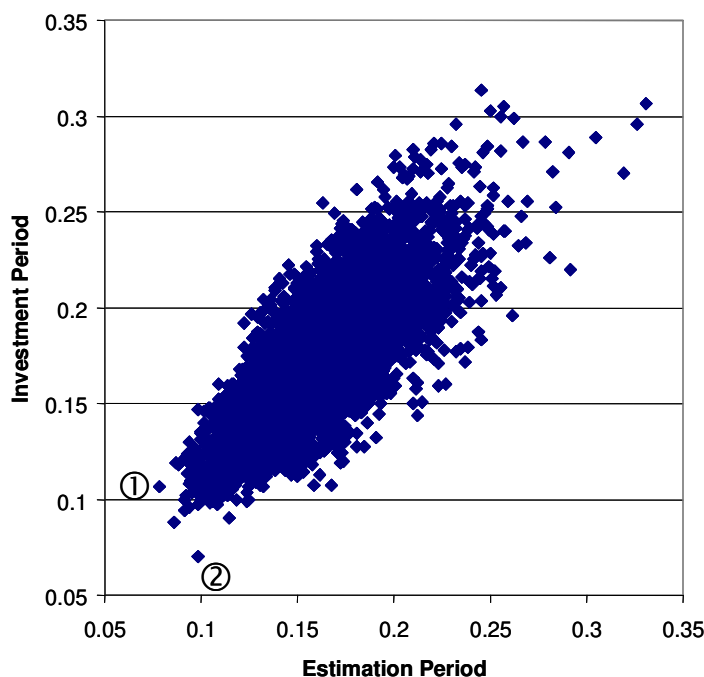


Figure 22: Scatter plot of the MAD in Period 1

Figure 23 contains the corresponding scatter plots for the other measures and time periods for a tracking portfolio size of 30. Again, we use the square roots of MSE and TEV to make the results comparable to the linear MAD. In the ideal case, the points would lay on the main diagonal. The more the dispersion, the smaller the relative stability, and the more there exists a rotation of the main axis of the cloud, the smaller the absolute stability.

The scatter plots show that the portfolio clouds have similar shapes within the time periods. A remarkable difference between the three measures is visually not detectable except for period 5. The diffusion of the clouds is not directly comparable between the time periods because of the different scales of the axis.

The best result can be stated for period 1. Here, we have the smallest scale and a nearly diagonal distributed portfolio cloud. In periods 2, 3 and 4, we can identify a considerable left rotation of the cloud which indicates that the tracking qualities in the investment periods are clearly worse than in the estimation periods. In period 5, we have a high dispersion combined with the broadest scale which indicates a poor tracking quality in the estimation and the investment period. The clouds of MSE and TEV have just a barely rotation while MAD shows a clear rotation.

<sup>22</sup> It should be remarked that the 5000 tracking portfolios in figure 22 are just a fraction of the complete amount of all possible combinations of 30 out of 110 stocks. Thus, it is very unlikely that the marked tracking portfolios are the overall best in their period. At this point, we just intend to demonstrate that finding a good tracking portfolio is not a guarantee for the same relative quality level in the investment period.

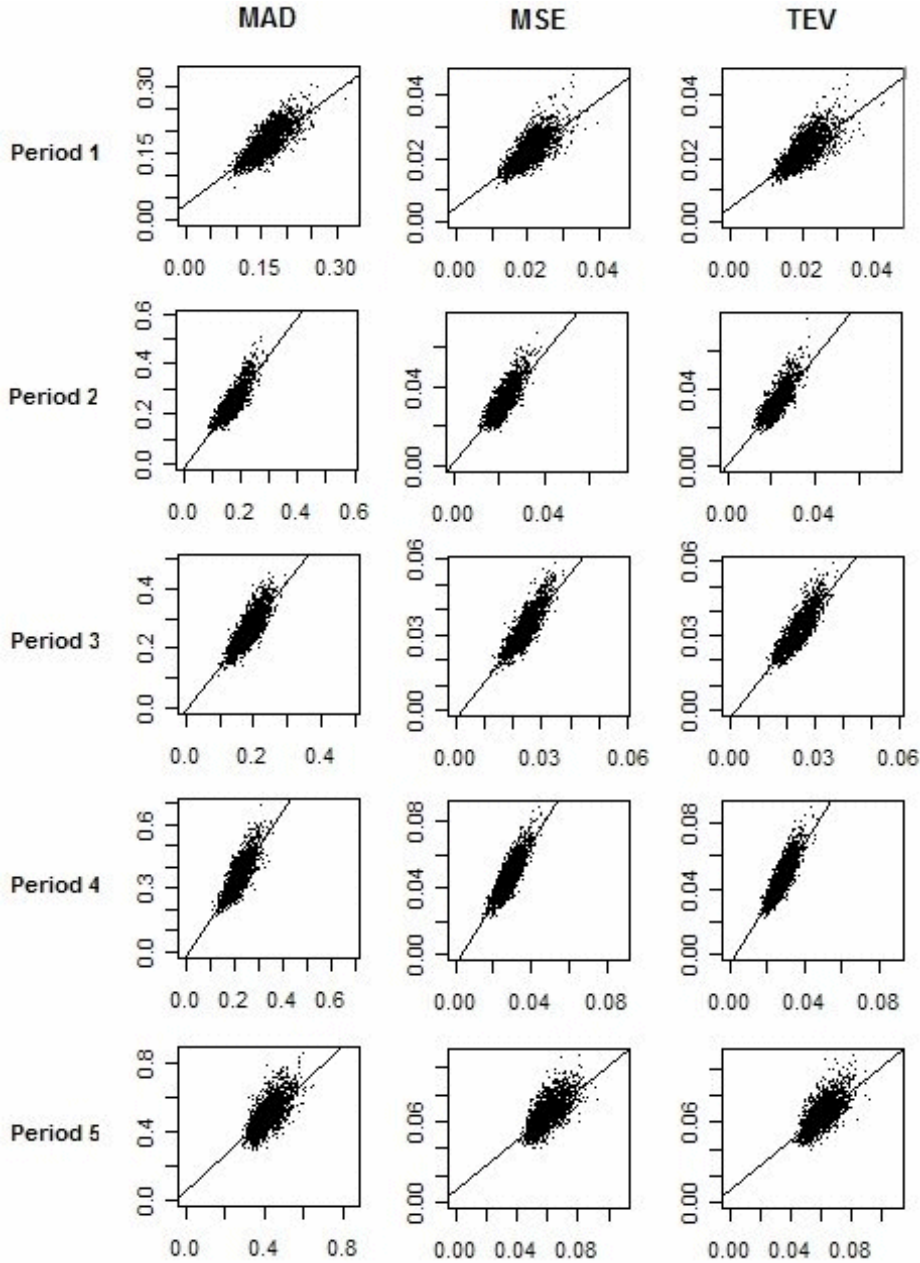


Figure 23: Scatter plots for a Tracking Portfolio Size of 30

To further analyze the results, we apply a regression analysis to every scatter plot using the equation

$$Measure_{Investment,i} = \alpha + \beta \cdot Measure_{Estimation,i} + \varepsilon_i \quad (7)$$

where *Measure* denotes MAD or the square roots of MSE and TEV. The *i* is used as an index for the tracking portfolios. The slope  $\beta$  is an indicator for the absolute stability. For  $\beta$  equal to 1, the tracking quality in the investment period would on average be equal to the tracking quality in the estimation period. Otherwise, if  $\beta$  is smaller or greater 1 there exist differences in the tracking quality in both periods. If the deviation to 1 is greater, the differences of the tracking quality between the estimation and the investment period will be greater.

	TP Size	MAD					MSE					TEV				
		alpha	t-stat	beta	t-stat	R <sup>2</sup>	alpha	t-stat	beta	t-stat	R <sup>2</sup>	alpha	t-stat	beta	t-stat	R <sup>2</sup>
Period 1	10	0.082	(32.7)*	0.704	(91.4)*	0.63	0.012	(26.8)*	0.688	(67.2)*	0.47	0.012	(27.3)*	0.687	(67.1)*	0.47
	15	0.065	(31.2)*	0.734	(89.5)*	0.62	0.009	(26.0)*	0.719	(67.5)*	0.48	0.009	(26.2)*	0.719	(67.3)*	0.48
	20	0.056	(29.1)*	0.757	(84.8)*	0.59	0.008	(25.1)*	0.747	(69.2)*	0.49	0.008	(25.3)*	0.747	(68.8)*	0.49
	25	0.046	(27.0)*	0.798	(87.8)*	0.61	0.006	(23.0)*	0.795	(73.5)*	0.52	0.006	(23.1)*	0.796	(73.2)*	0.52
	30	0.034	(21.4)*	0.860	(88.1)*	0.61	0.004	(18.0)*	0.863	(76.0)*	0.54	0.004	(17.9)*	0.866	(75.7)*	0.53
	35	0.027	(19.2)*	0.904	(92.7)*	0.63	0.003	(15.7)*	0.912	(82.7)*	0.58	0.003	(15.6)*	0.917	(82.5)*	0.58
	40	0.023	(18.3)*	0.938	(98.7)*	0.66	0.003	(14.2)*	0.953	(89.0)*	0.61	0.003	(14.2)*	0.957	(88.9)*	0.61
	50	0.018	(15.2)*	0.981	(99.1)*	0.66	0.002	(10.8)*	1.005	(92.3)*	0.63	0.002	(10.4)*	1.012	(92.7)*	0.63
Period 2	10	-0.091	-(18.5)*	1.585	(108.6)*	0.70	-0.009	-(11.4)*	1.524	(89.1)*	0.61	-0.009	-(11.7)*	1.536	(89.0)*	0.61
	15	-0.070	-(17.7)*	1.595	(109.3)*	0.70	-0.006	-(10.6)*	1.530	(92.2)*	0.63	-0.007	-(11.0)*	1.544	(92.1)*	0.63
	20	-0.052	-(16.4)*	1.585	(114.9)*	0.73	-0.004	-(8.1)*	1.501	(98.0)*	0.66	-0.004	-(8.7)*	1.518	(97.8)*	0.66
	25	-0.038	-(14.0)*	1.563	(114.8)*	0.73	-0.002	-(4.9)*	1.465	(98.4)*	0.66	-0.002	-(5.6)*	1.482	(97.8)*	0.66
	30	-0.021	-(9.0)*	1.512	(115.7)*	0.73	0.001	(2.6)	1.386	(93.9)*	0.64	0.000	1.4	1.405	(93.6)*	0.64
	35	-0.007	-(3.2)*	1.458	(113.7)*	0.72	0.002	(8.0)*	1.334	(92.2)*	0.63	0.002	(6.8)*	1.353	(92.1)*	0.63
	40	0.006	(3.5)*	1.391	(113.1)*	0.72	0.004	(16.5)*	1.244	(89.1)*	0.61	0.004	(14.9)*	1.264	(89.0)*	0.61
	50	0.008	(5.2)*	1.396	(115.5)*	0.73	0.005	(20.9)*	1.219	(86.8)*	0.60	0.005	(19.0)*	1.240	(87.0)*	0.60
Period 3	10	-0.060	-(10.8)*	1.500	(93.4)*	0.64	-0.011	-(15.8)*	1.542	(99.9)*	0.67	-0.012	-(16.8)*	1.562	(101.0)*	0.67
	15	-0.040	-(8.7)*	1.483	(91.4)*	0.63	-0.008	-(14.1)*	1.515	(99.7)*	0.67	-0.008	-(15.1)*	1.533	(101.4)*	0.67
	20	-0.031	-(8.1)*	1.482	(93.6)*	0.64	-0.006	-(13.1)*	1.495	(100.8)*	0.67	-0.007	-(14.2)*	1.513	(102.4)*	0.68
	25	-0.028	-(8.3)*	1.497	(94.7)*	0.64	-0.005	-(11.7)*	1.470	(101.5)*	0.67	-0.005	-(12.5)*	1.483	(102.7)*	0.68
	30	-0.015	-(5.4)*	1.459	(97.7)*	0.66	-0.004	-(10.4)*	1.454	(104.2)*	0.68	-0.004	-(11.2)*	1.466	(105.6)*	0.69
	35	-0.018	-(7.1)*	1.494	(101.4)*	0.67	-0.003	-(10.2)*	1.449	(105.9)*	0.69	-0.003	-(10.6)*	1.456	(106.4)*	0.69
	40	-0.012	-(5.4)*	1.471	(104.5)*	0.69	-0.002	-(7.3)*	1.408	(107.1)*	0.70	-0.002	-(7.6)*	1.413	(107.3)*	0.70
	50	-0.009	-(4.5)*	1.463	(110.6)*	0.71	-0.002	-(6.4)*	1.399	(110.2)*	0.71	-0.002	-(6.5)*	1.402	(110.2)*	0.71
Period 4	10	-0.017	-(2.5)	1.566	(92.3)*	0.63	-0.002	-(1.8)	1.569	(93.2)*	0.63	-0.001	-1.6	1.571	(92.4)*	0.63
	15	-0.038	-(6.9)*	1.671	(99.2)*	0.66	-0.007	-(9.9)*	1.737	(103.7)*	0.68	-0.007	-(9.8)*	1.744	(102.7)*	0.68
	20	-0.037	-(8.0)*	1.705	(104.0)*	0.68	-0.008	-(12.9)*	1.802	(109.0)*	0.70	-0.008	-(12.9)*	1.812	(108.2)*	0.70
	25	-0.032	-(7.7)*	1.713	(102.7)*	0.68	-0.008	-(14.8)*	1.843	(110.5)*	0.71	-0.008	-(14.9)*	1.855	(110.2)*	0.71
	30	-0.026	-(7.0)*	1.712	(102.7)*	0.68	-0.006	-(13.5)*	1.825	(110.9)*	0.71	-0.007	-(13.6)*	1.839	(110.6)*	0.71
	35	-0.016	-(4.9)*	1.678	(103.0)*	0.68	-0.005	-(12.4)*	1.799	(113.1)*	0.72	-0.005	-(12.9)*	1.818	(113.2)*	0.72
	40	-0.015	-(5.0)*	1.686	(106.8)*	0.70	-0.004	-(12.0)*	1.790	(117.5)*	0.73	-0.005	-(12.4)*	1.809	(117.2)*	0.73
	50	-0.010	-(3.8)*	1.668	(105.0)*	0.69	-0.004	-(10.8)*	1.766	(117.2)*	0.73	-0.004	-(11.4)*	1.788	(118.2)*	0.74
Period 5	10	0.078	(8.0)*	1.017	(74.5)*	0.53	0.016	(12.6)*	0.886	(69.3)*	0.49	0.017	(12.7)*	0.884	(68.2)*	0.48
	15	0.090	(10.9)*	0.984	(70.3)*	0.50	0.016	(14.5)*	0.861	(65.5)*	0.46	0.016	(14.2)*	0.863	(64.7)*	0.46
	20	0.076	(10.3)*	0.997	(70.3)*	0.50	0.012	(12.9)*	0.885	(67.3)*	0.48	0.012	(12.4)*	0.891	(67.0)*	0.47
	25	0.035	(5.1)*	1.078	(71.9)*	0.51	0.009	(9.4)*	0.933	(67.9)*	0.48	0.008	(9.1)*	0.938	(67.6)*	0.48
	30	0.043	(6.6)*	1.058	(69.7)*	0.49	0.008	(9.8)*	0.929	(66.4)*	0.47	0.008	(9.4)*	0.935	(66.1)*	0.47
	35	0.028	(4.5)*	1.093	(69.0)*	0.49	0.006	(7.5)*	0.964	(67.3)*	0.48	0.006	(7.0)*	0.971	(67.3)*	0.48
	40	0.016	(2.7)	1.128	(69.2)*	0.49	0.004	(4.7)*	1.010	(68.8)*	0.49	0.003	(4.2)*	1.018	(68.9)*	0.49
	50	-0.004	-(0.7)	1.188	(69.3)*	0.49	0.003	(3.8)*	1.025	(67.0)*	0.47	0.002	(3.2)*	1.035	(67.3)*	0.48
50	-0.020	-(3.3)*	1.234	(67.9)*	0.48	0.001	(1.8)	1.056	(64.7)*	0.46	0.001	1.3	1.064	(65.0)*	0.46	

Figure 24: Regressing Estimation vs. Investment Period

The coefficient of determination  $R^2$  gives the proportion of the variance of the dependent variable that is predictable from the independent variable. Therefore, it can be used as an indicator to determine how reliable the predictions using the regression equation are. The closer the portfolios lay around the regression line the higher the  $R^2$  and vice versa. Thus,  $R^2$  gives some information about the relative stability.

The lines plotted in figure 23 represent the regression lines. The detailed results of the regression analysis are shown in figure 24. The asterisks in the columns of the t-stat values indicate that the corresponding parameters are significant with an alpha of 0.005.

Comparing the betas in figure 24, we can state that the values are very high in the periods 2, 3 and 4 indicating that there is a strong overfitting effect within these estimation periods. This

results in a poor absolute stability of the measures. In periods 1 and 5 the betas are much lower. In period 5 there is additionally a remarkable difference between the beta of MAD and the betas of MSE and TEV. This confirms the findings from the first analysis especially the results expressed in figure 12. In this period MSE and TEV show a higher absolute stability. In all other periods the differences between the measures are very small and winners and looser alternate.

The  $R^2$  values in figure 24 confirm the findings of the analyses about the relative stability of the measures. In periods 1, 2, and 5 MAD has higher  $R^2$  values and in periods 3 and 4 the values of the measures are very close. The range from 60% to 75% in periods 1 to 4 indicate that there are further factors influencing the values of the measures in the investment period than just the values of the estimation period. Summarizing, the results of the regression analysis are consistent with the results of the previous analyses.

The third analysis has the aim to compare the ex post tracking quality of MAD, MSE and TEV from an investors view. In chapter 5.1 we elaborate an adequate measure to evaluate the ex post tracking quality, stated in formula 6. We calculate this measure for every tracking portfolio for the investment period assuming that we have an initial portfolio value  $V_{P,0}$  of 1. For the index we also assume an imaginary portfolio containing the real index with an initial portfolio value  $V_{I,0}$  of 1. In this case, the resulting ex post tracking quality value  $TQ_{ep}$  can be interpreted as the mean difference in the values of both portfolios. For example, a value of 0.01 expresses that on average the absolute daily difference between the tracking and the index portfolio is 0.01 of the monetary unit. Thus, the smaller the value of the measure is the better because the tracking portfolio is closer to the index and vice versa.

For every combination of objective, time period, and tracking portfolio size, we calculate the mean and the standard deviation. Finally, for every time period and tracking portfolio size, we compare the ex post tracking qualities of the three measures by ranking them using a pair wise t-test. The null hypothesis  $H_0$  and its alternative  $H_1$  are defined as

$$H_0 : \mu_1 \geq \mu_2$$

$$H_1 : \mu_1 < \mu_2$$

Where  $\mu_i$  = mean of measure i. We apply this test by calculating the p-value, which can be interpreted as the probability that measure 1 is worse to measure 2.

The results are shown in figure 25. In period 1, MAD is clearly the worst of the three measures. MSE and TEV are very close but TEV is slightly better. In period 2 we have nearly the opposite picture. Here MAD clearly outperforms MSE which outperforms TEV. In period 3 MAD is again worse, but MSE is slightly better than TEV. In period 4 there is no clear order observable. MAD and MSE show better values than TEV but they alternate. For small tracking portfolio sizes MAD shows better results and vice versa. In period 5 MSE clearly outperforms TEV which outperforms MAD.

Even if the results of the t-test often indicate significant differences between the measures, the values in the rows of figure 25 are very close. Comparing the results of different tracking portfolio sizes, in the majority of the cases the tracking quality increases with an increasing portfolio size. The only exception is period 2 where the values of portfolios with more than 30 stocks remain on the same level independent of the tracking portfolio size. This is clearly dif-

ferent to the results in figure 10 based on the original measures. A plausible reason is that the effect described in chapter 5.1 occurred in this period.

Period 1									
TP Size	MAD		MSE		TEV		p-Value MAD>=MSE	p-Value MAD>=TEV	p-Value MSE>=TEV
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.			
10	0.015792	0.008709	0.015574	0.008610	0.015570	0.008614	0.90	0.90	0.51
15	0.013036	0.007086	0.012802	0.006955	0.012797	0.006955	0.95	0.96	0.52
20	0.011307	0.006069	0.011029	0.005946	0.011023	0.005940	0.99	0.99	0.52
25	0.010271	0.005449	0.009962	0.005253	0.009955	0.005246	1.00	1.00	0.52
30	0.009203	0.004848	0.008965	0.004713	0.008960	0.004711	0.99	0.99	0.52
35	0.008559	0.004550	0.008282	0.004373	0.008276	0.004363	1.00	1.00	0.53
40	0.008019	0.004222	0.007786	0.004030	0.007781	0.004026	1.00	1.00	0.52
45	0.007571	0.004006	0.007352	0.003722	0.007345	0.003718	1.00	1.00	0.54
50	0.007132	0.003722	0.007000	0.003482	0.006990	0.003479	0.97	0.98	0.56

Period 2									
TP Size	MAD		MSE		TEV		p-Value MAD>=MSE	p-Value MAD>=TEV	p-Value MSE>=TEV
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.			
10	0.022476	0.012761	0.022689	0.012858	0.022748	0.012891	0.20	0.14	0.41
15	0.020552	0.010796	0.020784	0.010778	0.020845	0.010806	0.14	0.09	0.39
20	0.019396	0.009356	0.019703	0.009240	0.019769	0.009258	0.05	0.02	0.36
25	0.018462	0.008229	0.018809	0.008076	0.018883	0.008085	0.02	0.00	0.32
30	0.018136	0.007208	0.018469	0.007127	0.018557	0.007127	0.01	0.00	0.27
35	0.017724	0.006762	0.017999	0.006610	0.018080	0.006605	0.02	0.00	0.27
40	0.017614	0.006226	0.017879	0.006044	0.017969	0.006035	0.02	0.00	0.23
45	0.017459	0.005689	0.017739	0.005538	0.017826	0.005514	0.01	0.00	0.22
50	0.017434	0.005206	0.017717	0.005050	0.017801	0.005024	0.00	0.00	0.20

Period 3									
TP Size	MAD		MSE		TEV		p-Value MAD>=MSE	p-Value MAD>=TEV	p-Value MSE>=TEV
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.			
10	0.020465	0.011573	0.020044	0.011329	0.020059	0.011337	0.97	0.96	0.47
15	0.017149	0.009862	0.016588	0.009466	0.016598	0.009460	1.00	1.00	0.48
20	0.014808	0.008490	0.014210	0.008114	0.014218	0.008114	1.00	1.00	0.48
25	0.013551	0.007753	0.012947	0.007275	0.012954	0.007274	1.00	1.00	0.48
30	0.012424	0.007195	0.011823	0.006729	0.011830	0.006726	1.00	1.00	0.48
35	0.011358	0.006503	0.010811	0.006049	0.010816	0.006046	1.00	1.00	0.48
40	0.010548	0.006101	0.010043	0.005683	0.010049	0.005679	1.00	1.00	0.48
45	0.009624	0.005661	0.009149	0.005307	0.009153	0.005301	1.00	1.00	0.48
50	0.008969	0.005224	0.008597	0.004952	0.008603	0.004944	1.00	1.00	0.48

Period 4									
TP Size	MAD		MSE		TEV		p-Value MAD>=MSE	p-Value MAD>=TEV	p-Value MSE>=TEV
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.			
10	0.030922	0.017873	0.031156	0.017945	0.031217	0.018016	0.26	0.21	0.43
15	0.026081	0.015223	0.026218	0.015136	0.026307	0.015221	0.33	0.23	0.38
20	0.022938	0.013562	0.023037	0.013503	0.023126	0.013584	0.36	0.24	0.37
25	0.020050	0.011846	0.020098	0.011852	0.020171	0.011909	0.42	0.30	0.38
30	0.018136	0.010910	0.018127	0.010914	0.018206	0.010963	0.52	0.38	0.36
35	0.016266	0.009784	0.016191	0.009726	0.016263	0.009771	0.65	0.51	0.36
40	0.014395	0.008661	0.014341	0.008673	0.014387	0.008694	0.62	0.52	0.40
45	0.013193	0.007917	0.012993	0.007903	0.013039	0.007929	0.90	0.83	0.38
50	0.011772	0.007269	0.011562	0.007257	0.011598	0.007281	0.93	0.88	0.40

Period 5									
TP Size	MAD		MSE		TEV		p-Value MAD>=MSE	p-Value MAD>=TEV	p-Value MSE>=TEV
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.			
10	0.048902	0.029820	0.047194	0.028288	0.047616	0.028612	1.00	0.99	0.23
15	0.038024	0.020775	0.036991	0.020091	0.037450	0.020447	0.99	0.92	0.13
20	0.031811	0.016666	0.031121	0.016461	0.031597	0.016860	0.98	0.74	0.08
25	0.028530	0.014522	0.028039	0.014608	0.028552	0.015030	0.95	0.47	0.04
30	0.025860	0.012832	0.025550	0.012998	0.026082	0.013452	0.88	0.20	0.02
35	0.024111	0.011592	0.023556	0.011575	0.024096	0.012014	0.99	0.53	0.01
40	0.022467	0.010300	0.021937	0.010501	0.022475	0.010942	0.99	0.48	0.01
45	0.021120	0.009655	0.020582	0.009776	0.021128	0.010224	1.00	0.49	0.00
50	0.020228	0.008995	0.019605	0.008916	0.020148	0.009357	1.00	0.67	0.00

Figure 25: Comparison of the Ex Post Tracking Quality

Comparing the tracking quality levels between the time periods we can state, that the values are different depending on the stock market situation. For period 1 with a clear trend and few

fluctuations we have better results and for periods with changes in trend and price fluctuations we have lower levels of tracking quality. With an increasing tracking portfolio size the tracking quality levels are assimilating over the time periods except for periods 2 and 5. Especially period 5 is quite different in the tracking quality level. This confirms the findings from above that the classical procedures of sampling using the traditional measures do not produce results of good quality in times of financial market crisis and uncertainty.

Period 1									
TP Size	MAD		MSE		TEV		p-Value	p-Value	p-Value
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	MAD>=MSE	MAD>=TEV	MSE>=TEV
10	0.012943	0.007007	0.012737	0.006866	0.012720	0.006858	0.68	0.69	0.52
15	0.010501	0.005781	0.010574	0.005671	0.010552	0.005656	0.42	0.44	0.52
20	0.009542	0.005153	0.009262	0.005033	0.009246	0.005035	0.81	0.82	0.52
25	0.009057	0.004746	0.008893	0.004661	0.008876	0.004661	0.71	0.73	0.52
30	0.008180	0.004287	0.008077	0.004147	0.008103	0.004150	0.65	0.61	0.46
35	0.007454	0.003886	0.007216	0.003708	0.007214	0.003695	0.84	0.84	0.50
40	0.007163	0.003390	0.007078	0.003218	0.007071	0.003227	0.66	0.67	0.51
45	0.007032	0.003394	0.006968	0.003150	0.006929	0.003158	0.62	0.69	0.58
50	0.006764	0.003278	0.006798	0.002972	0.006734	0.002980	0.43	0.56	0.63

Period 2									
TP Size	MAD		MSE		TEV		p-Value	p-Value	p-Value
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	MAD>=MSE	MAD>=TEV	MSE>=TEV
10	0.017884	0.008568	0.018289	0.008782	0.018344	0.008822	0.23	0.20	0.46
15	0.017969	0.007984	0.018046	0.007844	0.017976	0.007927	0.44	0.49	0.56
20	0.018270	0.007275	0.018163	0.007054	0.018198	0.007043	0.59	0.56	0.47
25	0.017379	0.006438	0.017450	0.006373	0.017497	0.006312	0.43	0.38	0.45
30	0.018240	0.005790	0.018120	0.005831	0.018232	0.005819	0.63	0.51	0.38
35	0.018338	0.005269	0.018364	0.005187	0.018398	0.005164	0.47	0.43	0.46
40	0.018611	0.004849	0.018425	0.004701	0.018507	0.004630	0.73	0.64	0.39
45	0.018388	0.004410	0.018529	0.004137	0.018517	0.004131	0.30	0.32	0.52
50	0.018162	0.003825	0.018189	0.003639	0.018231	0.003603	0.45	0.38	0.43

Period 3									
TP Size	MAD		MSE		TEV		p-Value	p-Value	p-Value
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	MAD>=MSE	MAD>=TEV	MSE>=TEV
10	0.019022	0.011219	0.017720	0.010621	0.017712	0.010604	0.97	0.97	0.51
15	0.015312	0.009454	0.014333	0.008566	0.014363	0.008569	0.96	0.95	0.48
20	0.013095	0.007681	0.012126	0.007146	0.012123	0.007126	0.98	0.98	0.50
25	0.011152	0.006831	0.010718	0.006440	0.010711	0.006441	0.85	0.85	0.51
30	0.009727	0.005647	0.009512	0.005497	0.009491	0.005480	0.73	0.75	0.52
35	0.009174	0.005101	0.008804	0.004830	0.008762	0.004785	0.88	0.91	0.56
40	0.008001	0.004663	0.008133	0.004560	0.008120	0.004538	0.32	0.34	0.52
45	0.007286	0.004437	0.007313	0.004358	0.007339	0.004364	0.46	0.43	0.46
50	0.006866	0.004039	0.007242	0.004156	0.007208	0.004129	0.07	0.09	0.55

Period 4									
TP Size	MAD		MSE		TEV		p-Value	p-Value	p-Value
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	MAD>=MSE	MAD>=TEV	MSE>=TEV
10	0.019068	0.010380	0.019909	0.011207	0.019909	0.011397	0.11	0.11	0.50
15	0.015113	0.008592	0.015260	0.008820	0.015402	0.008887	0.39	0.30	0.40
20	0.013205	0.007475	0.013238	0.007972	0.013268	0.008015	0.47	0.45	0.48
25	0.011351	0.006492	0.011105	0.006226	0.011215	0.006292	0.73	0.63	0.39
30	0.010340	0.005378	0.010656	0.005957	0.010845	0.006166	0.19	0.08	0.31
35	0.009162	0.004917	0.009192	0.004921	0.009199	0.004944	0.46	0.45	0.49
40	0.007878	0.004209	0.008111	0.004232	0.008056	0.004106	0.19	0.25	0.58
45	0.007506	0.004040	0.007237	0.004004	0.007198	0.003948	0.85	0.89	0.56
50	0.006653	0.003526	0.006556	0.003486	0.006524	0.003447	0.67	0.72	0.56

Period 5									
TP Size	MAD		MSE		TEV		p-Value	p-Value	p-Value
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	MAD>=MSE	MAD>=TEV	MSE>=TEV
10	0.036186	0.019104	0.035220	0.018402	0.035901	0.018894	0.79	0.59	0.28
15	0.031317	0.016731	0.029730	0.015760	0.030107	0.015942	0.94	0.88	0.35
20	0.026716	0.012742	0.026054	0.013235	0.026423	0.013530	0.79	0.64	0.33
25	0.023064	0.010625	0.023357	0.012053	0.023921	0.012492	0.34	0.12	0.23
30	0.022385	0.010783	0.021505	0.010488	0.021959	0.010844	0.90	0.73	0.25
35	0.021318	0.009612	0.020304	0.009098	0.020754	0.009493	0.96	0.82	0.22
40	0.020136	0.008690	0.020378	0.009401	0.020835	0.009816	0.34	0.12	0.23
45	0.019506	0.008449	0.018569	0.008364	0.018993	0.008686	0.96	0.83	0.22
50	0.018369	0.008162	0.018072	0.007259	0.018459	0.007605	0.73	0.43	0.21

Figure 26: Comparison of the Ex Post Tracking Quality (Best 10%)

In order to check if this finding also holds for the best 10% of the portfolios, we recalculate the tables in figure 25 with the same portfolios we used in figure 13 (see figure 26). Compared to the results of the complete sample, we can not find a clear structure for the assessment and ranking of the measures. The winners and the loser do not only alternate between the periods but also within the periods. Despite of this, the general findings from above also apply for the best 10%.

Period 1						
TP Size	MAD		MSE		TEV	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
10	1.220087	1.242864	1.222693	1.253957	1.224068	1.256062
15	1.241314	1.225636	1.210718	1.226443	1.212744	1.229672
20	1.184968	1.177866	1.190808	1.181304	1.192181	1.179733
25	1.134007	1.148084	1.120156	1.126880	1.121619	1.125583
30	1.125134	1.130661	1.109890	1.136302	1.105793	1.135028
35	1.148208	1.171109	1.147720	1.179466	1.147172	1.180842
40	1.119443	1.245518	1.100091	1.252222	1.100535	1.247731
45	1.076732	1.180499	1.055119	1.181578	1.060094	1.177327
50	1.054431	1.135358	1.029784	1.171866	1.037999	1.167515

Period 2						
TP Size	MAD		MSE		TEV	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
10	1.256777	1.489343	1.240604	1.464073	1.240083	1.461256
15	1.143736	1.352115	1.151704	1.373993	1.159616	1.363187
20	1.061648	1.286047	1.084790	1.309933	1.086361	1.314420
25	1.062346	1.278193	1.077864	1.267229	1.079208	1.280896
30	0.994257	1.244952	1.019251	1.222261	1.017834	1.224791
35	0.966555	1.283320	0.980123	1.274470	0.982742	1.279052
40	0.946458	1.284040	0.970396	1.285678	0.970905	1.303305
45	0.949477	1.290264	0.957365	1.338717	0.962660	1.334844
50	0.959924	1.361062	0.973999	1.387821	0.976398	1.394373

Period 3						
TP Size	MAD		MSE		TEV	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
10	1.075860	1.031565	1.131124	1.066671	1.132535	1.069061
15	1.120019	1.043108	1.157313	1.105005	1.155566	1.104055
20	1.130813	1.105233	1.171865	1.135517	1.172837	1.138567
25	1.215193	1.134924	1.207977	1.129637	1.209366	1.129322
30	1.277349	1.274214	1.242981	1.224038	1.246420	1.227530
35	1.238106	1.274750	1.227987	1.252424	1.234522	1.263399
40	1.318356	1.308459	1.234829	1.246366	1.237496	1.251411
45	1.320751	1.275824	1.251078	1.217643	1.247155	1.214491
50	1.306430	1.293448	1.187105	1.191608	1.193609	1.197458

Period 4						
TP Size	MAD		MSE		TEV	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
10	1.621712	1.721913	1.564962	1.601196	1.567986	1.580782
15	1.725695	1.771824	1.718110	1.716032	1.708074	1.712823
20	1.737138	1.814223	1.740220	1.693686	1.742989	1.694831
25	1.766361	1.824628	1.809740	1.903654	1.798597	1.892740
30	1.753968	2.028702	1.701067	1.832079	1.678731	1.777804
35	1.775284	1.989984	1.761360	1.976534	1.767966	1.976380
40	1.827317	2.057576	1.768125	2.049361	1.785922	2.117554
45	1.757655	1.959497	1.795206	1.973733	1.811574	2.008280
50	1.769585	2.061481	1.763554	2.081684	1.777678	2.112352

Period 5						
TP Size	MAD		MSE		TEV	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
10	1.351409	1.560930	1.339974	1.537165	1.326297	1.514329
15	1.214167	1.241669	1.244229	1.274847	1.243887	1.282592
20	1.190702	1.307920	1.194505	1.243798	1.195798	1.246142
25	1.236958	1.366770	1.200470	1.211988	1.193570	1.203191
30	1.155209	1.190030	1.188084	1.239266	1.187754	1.240477
35	1.130998	1.205977	1.160159	1.272240	1.161033	1.265591
40	1.115725	1.185303	1.076473	1.117037	1.078704	1.114683
45	1.082773	1.142740	1.108408	1.168796	1.112389	1.176992
50	1.101239	1.102043	1.084814	1.228356	1.091509	1.230369

Figure 27: Ratios of the Ex Post Tracking Qualities (All/Best 10%)

To evaluate the differences between the complete sample and the best 10% we calculate the ratios (see figure 27). In every case, the standard deviation of the complete sample is higher. In most of the cases this does also apply to the means. An exception can be found in period 2



where the values of the bigger tracking portfolio sizes show that the mean ex post tracking quality of the best 10% is higher than the corresponding mean of the complete sample. Again, we can not find a clear structure. Sometimes the ratio is increasing with an increasing portfolio size (periods 3 and 4) and sometimes it is decreasing (periods 1, 2 and 5). We find the highest differences in period 4. This is consistent with the previous findings.

To inspect the relations between the measures and the ex post tracking quality on a portfolio level, we create scatter plots with the values of the original measures used as objectives in the estimation period on the x-axis and the corresponding ex post tracking quality value in the investment period on the y-axis.

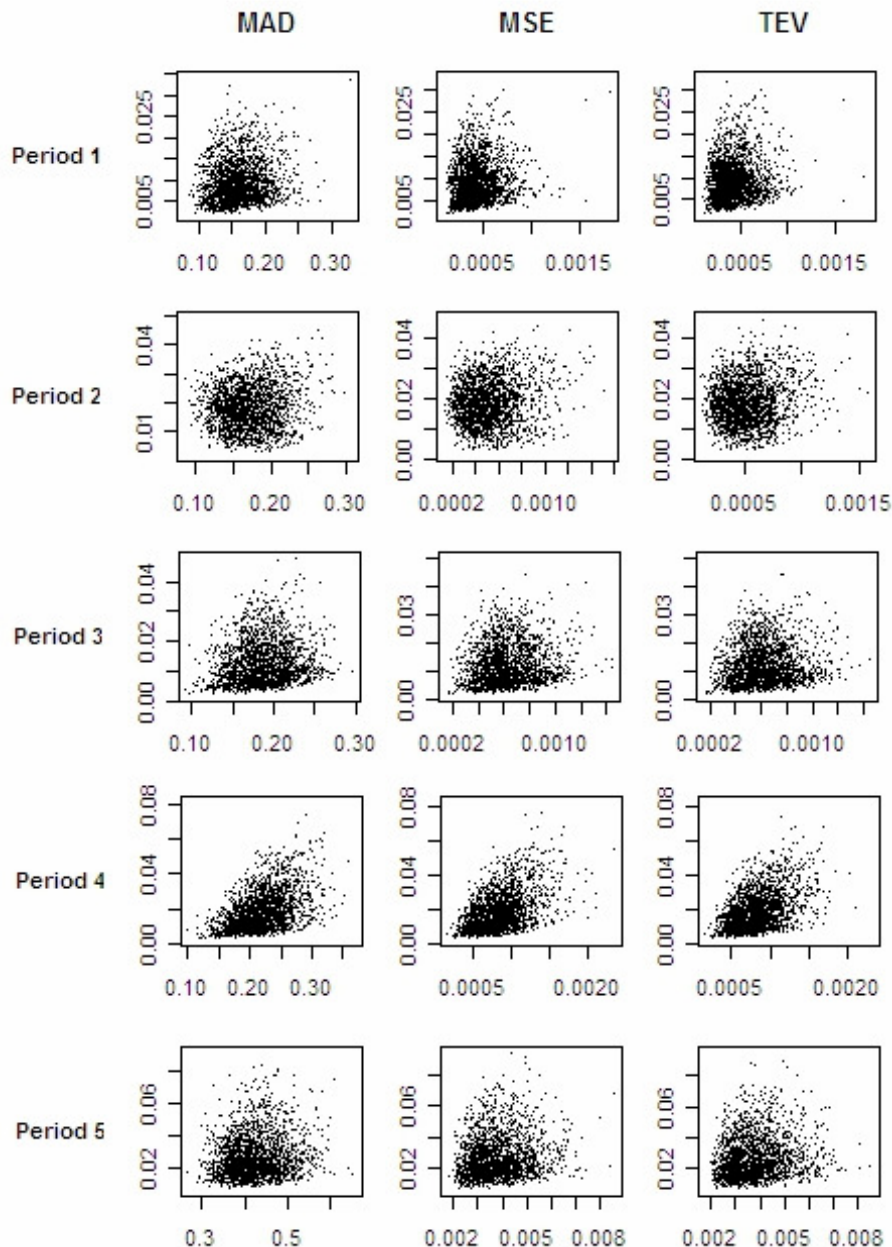


Figure 28: Comparing Traditional Measures and Ex Post Tracking Qualities

Figure 28 shows that the shapes of the portfolio clouds are again very similar within the time periods. Thus, there is no remarkable difference detectable between the three measures. But compared to the scatter plots in figure 23, we can state a much higher dispersion without an

exception. This indicates that the measures alone are not good predictors for the ex post tracking quality.

TP Size	MAD					MSE					TEV					
	alpha	t-stat	beta	t-stat	R^2	alpha	t-stat	beta	t-stat	R^2	alpha	t-stat	beta	t-stat	R^2	
Period 1	10	0.007	(12.6)*	0.027	(15.3)*	0.04	0.011	(37.7)*	2.390	(15.4)*	0.05	0.011	(37.6)*	2.407	(15.6)*	0.05
	15	0.006	(12.1)*	0.029	(15.0)*	0.04	0.009	(37.6)*	3.109	(14.9)*	0.04	0.009	(37.5)*	3.129	(15.0)*	0.04
	20	0.005	(11.8)*	0.029	(14.8)*	0.04	0.008	(36.9)*	3.702	(14.3)*	0.04	0.008	(36.9)*	3.724	(14.5)*	0.04
	25	0.006	(15.9)*	0.022	(10.8)*	0.02	0.008	(41.3)*	3.238	(10.6)*	0.02	0.008	(41.4)*	3.253	(10.7)*	0.02
	30	0.006	(16.3)*	0.021	(9.6)*	0.02	0.008	(41.8)*	3.126	(8.3)*	0.01	0.008	(41.7)*	3.177	(8.5)*	0.01
	35	0.005	(14.8)*	0.026	(11.3)*	0.03	0.007	(40.7)*	4.225	(9.9)*	0.02	0.007	(40.8)*	4.243	(10.0)*	0.02
	40	0.005	(15.6)*	0.027	(11.7)*	0.03	0.006	(43.3)*	4.930	(10.4)*	0.02	0.006	(43.2)*	5.038	(10.6)*	0.02
	45	0.005	(18.7)*	0.020	(8.3)*	0.01	0.007	(47.9)*	3.368	(6.2)*	0.01	0.007	(47.8)*	3.512	(6.5)*	0.01
	50	0.006	(20.5)*	0.016	(6.1)*	0.01	0.007	(49.8)*	2.458	(3.8)*	0.00	0.006	(49.6)*	2.689	(4.2)*	0.00
Period 2	10	0.002	(2.2)	0.062	(23.0)*	0.10	0.012	(25.6)	5.340	(23.3)*	0.10	0.012	(25.7)*	5.329	(23.2)*	0.10
	15	0.004	(5.2)*	0.062	(21.0)*	0.08	0.013	(30.2)	6.622	(21.3)*	0.08	0.013	(30.2)*	6.638	(21.4)*	0.08
	20	0.009	(13.4)*	0.044	(14.6)*	0.04	0.014	(40.1)	5.793	(15.5)*	0.05	0.014	(40.1)*	5.839	(15.7)*	0.05
	25	0.011	(17.9)*	0.038	(12.4)*	0.03	0.015	(47.9)	5.784	(13.4)*	0.03	0.015	(48.0)*	5.854	(13.6)*	0.04
	30	0.015	(26.9)*	0.021	(6.8)*	0.01	0.016	(57.6)	4.543	(9.2)*	0.02	0.016	(57.8)*	4.628	(9.4)*	0.02
	35	0.016	(32.3)*	0.010	(3.2)	0.00	0.017	(64.9)	2.943	(5.2)*	0.01	0.017	(65.1)*	3.055	(5.4)*	0.01
	40	0.017	(38.0)*	0.003	(0.9)	0.00	0.017	(74.2)	1.499	(2.4)	0.00	0.017	(74.4)*	1.642	(2.6)	0.00
	45	0.018	(45.3)*	-0.006	-(1.9)	0.00	0.018	(85.9)	-0.308	-(0.4)	0.00	0.018	(86.5)*	-0.176	-(0.3)	0.00
	50	0.020	(55.0)*	-0.018	-(6.0)*	0.01	0.018	(98.6)	-2.628	-(3.5)*	0.00	0.018	(99.4)*	-2.460	-(3.3)	0.00
Period 3	10	0.012	(11.3)*	0.026	(8.8)*	0.02	0.014	(27.9)*	2.755	(11.4)*	0.03	0.014	(28.0)*	2.745	(11.4)*	0.03
	15	0.010	(11.0)*	0.026	(8.1)*	0.01	0.012	(26.8)*	3.345	(10.5)*	0.02	0.012	(26.9)*	3.338	(10.5)*	0.02
	20	0.010	(12.8)*	0.020	(6.3)*	0.01	0.011	(27.9)*	3.291	(8.7)*	0.01	0.011	(28.0)*	3.271	(8.6)*	0.01
	25	0.008	(11.7)*	0.025	(7.5)*	0.01	0.010	(28.8)*	3.539	(8.1)*	0.01	0.010	(29.0)*	3.515	(8.1)*	0.01
	30	0.007	(10.8)*	0.030	(9.1)*	0.02	0.009	(28.5)*	4.449	(9.2)*	0.02	0.009	(28.6)*	4.435	(9.2)*	0.02
	35	0.004	(7.6)*	0.042	(13.5)*	0.04	0.007	(27.0)*	6.643	(13.2)*	0.03	0.007	(27.2)*	6.611	(13.2)*	0.03
	40	0.004	(7.6)*	0.044	(14.2)*	0.04	0.007	(28.8)*	6.744	(12.2)*	0.03	0.007	(28.8)*	6.761	(12.3)*	0.03
	45	0.003	(6.3)*	0.050	(17.0)*	0.05	0.006	(28.7)*	8.025	(14.1)*	0.04	0.006	(28.7)*	8.121	(14.3)*	0.04
	50	0.002	(6.5)*	0.050	(17.4)*	0.06	0.006	(29.7)*	8.526	(13.7)*	0.04	0.006	(29.7)*	8.624	(13.9)*	0.04
Period 4	10	-0.006	-(4.4)*	0.092	(27.6)*	0.13	0.015	(21.3)*	5.740	(25.7)*	0.12	0.015	(21.5)*	5.738	(25.5)*	0.11
	15	-0.012	-(10.2)*	0.117	(33.0)*	0.18	0.009	(14.5)*	9.492	(31.4)*	0.16	0.009	(14.7)*	9.555	(31.1)*	0.16
	20	-0.013	-(12.6)*	0.128	(35.4)*	0.20	0.005	(10.1)*	12.925	(34.8)*	0.19	0.005	(10.2)*	13.091	(34.6)*	0.19
	25	-0.014	-(15.2)*	0.137	(38.3)*	0.23	0.003	(6.3)*	16.331	(38.7)*	0.23	0.003	(6.2)*	16.633	(38.7)*	0.23
	30	-0.013	-(16.3)*	0.142	(39.5)*	0.24	0.002	(4.5)*	19.263	(40.7)*	0.25	0.002	(4.4)*	19.676	(40.8)*	0.25
	35	-0.012	-(17.4)*	0.140	(41.9)*	0.26	0.001	(3.7)*	21.172	(44.1)*	0.28	0.001	(3.2)	21.795	(44.4)*	0.28
	40	-0.010	-(17.1)*	0.134	(42.6)*	0.27	0.001	(3.7)*	23.004	(45.5)*	0.29	0.001	(3.2)	23.692	(45.9)*	0.30
	45	-0.009	-(16.6)*	0.131	(42.2)*	0.26	0.001	(3.4)*	24.513	(45.8)*	0.30	0.001	(2.8)	25.303	(46.3)*	0.30
	50	-0.007	-(15.2)*	0.122	(41.4)*	0.26	0.001	(5.0)*	24.974	(44.4)*	0.28	0.001	(4.1)*	25.991	(45.1)*	0.29
Period 5	10	-0.011	-(4.5)*	0.085	(23.9)*	0.10	0.019	(15.2)*	2.748	(23.5)*	0.10	0.020	(15.5)*	2.720	(22.9)*	0.10
	15	0.002	(1.2)	0.061	(18.4)*	0.06	0.020	(20.2)*	2.410	(17.9)*	0.06	0.020	(20.2)*	2.408	(17.6)*	0.06
	20	0.008	(5.1)*	0.046	(14.4)*	0.04	0.020	(23.9)*	2.030	(13.7)*	0.04	0.020	(23.8)*	2.032	(13.4)*	0.03
	25	0.005	(3.5)*	0.050	(15.6)*	0.05	0.017	(21.6)*	2.546	(15.2)*	0.04	0.017	(21.5)*	2.540	(14.7)*	0.04
	30	0.011	(8.0)*	0.035	(11.0)*	0.02	0.017	(23.3)*	2.314	(12.5)*	0.03	0.017	(23.0)*	2.332	(12.2)*	0.03
	35	0.012	(9.2)*	0.032	(10.0)*	0.02	0.016	(25.0)*	2.157	(11.1)*	0.02	0.017	(24.6)*	2.195	(10.9)*	0.02
	40	0.011	(9.2)*	0.032	(10.5)*	0.02	0.015	(24.2)*	2.439	(11.8)*	0.03	0.015	(24.0)*	2.441	(11.3)*	0.03
	45	0.011	(10.1)*	0.029	(9.2)*	0.02	0.015	(25.2)*	2.222	(10.0)*	0.02	0.015	(24.7)*	2.270	(9.8)*	0.02
	50	0.009	(8.4)*	0.034	(10.7)*	0.02	0.013	(23.7)*	2.683	(11.4)*	0.03	0.014	(23.1)*	2.745	(11.1)*	0.02

Figure 29: Regressing Traditional Measures and Ex Post Tracking Qualities

To proof this, we apply a regression analysis using the equation

$$TQ_{EP,Investment,i} = \alpha + \beta \cdot Measure_{Estimation,i} + \varepsilon_i \quad (8)$$

Here, the  $R^2$  is especially of interest because it expresses the quality of the measure as a predictor for the ex post tracking quality. The results shown in Figure 29 confirm that the measures alone are not good predictors for the ex post tracking quality. The majority of the  $R^2$  values range below 5%. Only in period 4 we have about 20%. But even these values are very low.

The compounding of the deviations between the tracking portfolio and index from day to day can explain the poor tracking quality by the ex post tracking quality measure. For example, if there exists one large deviation at the beginning of the investment period, it rolls up in the next days and can significantly reduce the quality of tracking. This can be seen if we rewrite formula (6) in an equivalent form

$$TQep = \frac{1}{T} \sum_{t=1}^T \left| \prod_{i=1}^t (1 + r_{I,i}) - \prod_{i=1}^t (1 + r_{P,i}) \right| \quad (9)$$

Fortunately, looking at the scatter plots, we can see that there exist tracking portfolios with high tracking qualities even in the estimation and the investment period. This raises the question, how they can be found.

## 6 Summary and Outlook

The purpose of this paper was to analyze the stability and performance of the traditional measures of tracking quality when used as objective in sampling. In the theoretical part in chapter 4, we first compared the concepts of the three measures MAD, MSE and TEV and elaborated some weaknesses when using them as a measure to evaluate the ex post tracking quality.

In the empirical part in chapter 5.3, we achieved several findings. One main finding is that the stock market phase has a sustainable influence on the tracking quality. The more there are turbulences like trend changes or strong price oscillations on the market the worse is the tracking quality when applying sampling via minimizing the specific measure. In these cases, the differences between the tracking quality calculated in the estimation period and the tracking quality really achieved in the investment period are increasing. The greater this difference the less reliable is the tracking quality calculated in the estimation period as a predictor for the future tracking quality. Thus, the latter is underestimated due to an overfitting effect. This applies for all the three measures.

Another finding is that the differences in tracking quality between the different market phases decrease disproportional with increasing tracking portfolio sizes. This leads to the conclusion that the tracking portfolio size should be increased in turbulent markets to achieve a target tracking quality.

On the other hand, we find increasing differences in tracking quality between the estimation period and the investment period with an increasing tracking portfolio size. This indicates an increasing overfitting effect meaning that the tracking quality enhancement in the estimation period resulting from a greater portfolio size does not proportionally apply to the investment period.

Our analyses show, that the traditional measures used in sampling not only have weaknesses in their absolute stability but also have a poor relative stability on average. Independent from the stock market phase, the relative tracking quality in the estimation period compared to alternative tracking portfolios is not a good predictor for the relative tracking quality in the investment period. Thus, the tracking portfolio with the best tracking quality in the estimation period might not necessarily belong to the best ones in the investment period and vice versa. As a consequence, the commonly used sampling procedure of searching the tracking portfolio with the minimal value of the used tracking quality measure is not leading to the desired result independent of the measure used in the optimization process.

According to the definition of the ex post tracking quality, we used in this paper, all three measures show very poor results when used as a predictor for this ex post tracking quality. On average, there is just a marginal coherence. This raises the question if the traditional measures are really suitable for the sampling process.

The empirical results, which are also confirmed by another analysis with the S&P1500, also show that there exist tracking portfolios with high tracking qualities even in the estimation period and the investment period. This leads to several questions for further research. As stated before, one important question is, if an alternative measure can be found with a better ability for the prediction of the future tracking quality and therefore has a better ability against overfitting.

In addition, it seems worth to search for methods to calculate the tracking portfolios with a focus on robustness as an alternative to the commonly used optimization methods. A promising approach with an alternative tracking quality measure based on Support Vector Regression can be found in Karlow and Rossbach.<sup>23</sup>

Finally, there is some evidence that also the process of selecting the stocks has an important influence on the robustness of the tracking portfolio and therefore on the ex post tracking quality. The commonly used procedures which are searching for the one combination of stocks providing a minimal value of the used measure does not seem to be appropriate as the results indicate. Instead, there is a need for research about the specific properties of stocks being suitable for sampling and how they can be identified.

---

<sup>23</sup> Karlow/Rossbach (2011).

## References

- Alexander, C.; Dimitriu, A. (2004): A Comparison of Cointegration and Tracking Error Models for Mutual Funds and Hedge Funds. ISMA Centre Discussion Papers in Finance, No. 4.
- Bamberg, G.; Wagner, N. (2000): Equity index replication with standard and robust regression estimators, in: *OR Spectrum*, 22 (4), pp. 525-543.
- Beasley, J.E.; Meade, N.; Chang, T.J. (2003): An evolutionary heuristic for the index tracking problem, in: *European Journal of Operational Research*, 148 (3), pp. 621-643.
- Blume, M.E.; Edelen, R.M. (2004): S&P 500 indexers, tracking error, and liquidity, in: *The Journal of Portfolio Management*, 30, pp. 37-47.
- Bogle, J.C. (1992): Selecting equity mutual funds, in: *The Journal of Portfolio Management*, 18 (2), pp. 94-100.
- Bonafede, J. K. (2003): The Wilshire 5000 Total Market Index: The Logistics Behind Managing the U.S. Stock Market, in: *Journal of Indexes*, Third Quarter.
- Canakgoz, N.A.; Beasley, J.E. (2009): Mixed-integer programming approaches for index tracking and enhanced indexation, in: *European Journal of Operational Research*, 196 (1), pp. 384-399.
- Corielli, F.; Marcellino, M. (2006): Factor based index tracking in: *Journal of Banking & Finance*, 30 (8), pp. 2215-2233.
- Chen, H.; Noronha, G.; Singal, V. (2006) Index changes and losses to index fund investors, in: *Financial Analysts Journal*, 62 (4), pp. 31-47
- Dose, C.; Cincotti, S. (2005): Clustering of financial time series with application to index and enhanced index tracking portfolio, in: *Physica A: Statistical Mechanics and its Applications*, 355 (1), pp. 145-151
- Focardi, S.M.; Fabozzi, F.J. (2004): A methodology for index tracking based on time-series clustering, in: *Quantitative Finance*, 4 (4), pp. 417-425.
- French, K.R. (2008): Presidential address: The cost of active investing, in: *The Journal of Finance*, 63 (4), pp. 1537-1573.
- Fuhr, D.; Kelly, S. (2010) ETF landscape industry review, August 2010, BlackRock.
- Gaivoronski, A.A.; Krylov, S.; van der Wijst, N, (2005): Optimal portfolio selection and dynamic benchmark tracking in: *European journal of operational research*, 163 (1), pp. 115-131.
- Gilli, M.; Kellezi, M.G. (2001): Threshold Accepting for Index Tracking, Working Paper, Department of Econometrics, University of Geneva.
- Hill, J.; Mueller, B. (2004): Choosing among Index Vehicles: How Does an Institutional Investor Select an Investment Product? in: Schoenfeld, S.A. (ed.): *Active Index Investing*, Wiley, New Jersey, pp. 503-534.
- Hopkins, P.J.B.; Miller, C.H. (2001): Country, sector and company factors in global equity portfolios, The Research Foundation of AIMR, Charlottesville.

- Karlow, D.; Roßbach, P. (2011) A method for robust index tracking, To appear in: Hu, B.; Morasch, K.; Siegle, M.; Pickl, S. (eds.): *Operations Research Proceedings 2010*, Springer.
- Maringer, D. (2008): Constrained index tracking under loss aversion using differential evolution, in: Brabazon, A.; Neill, M. O. (eds.): *Natural Computing in Computational Economics and Finance*, Springer, pp. 9-29.
- Meade, N.; Salkin, G.R. (1989): Index funds-construction and performance measurement, in: *Journal of the Operational Research Society*, 40 (10), pp. 871-879.
- Montfort, K.; Visser, E.; van Draat, L.F. (2008): Index tracking by means of optimized sampling, in: *The Journal of Portfolio Management*, 34 (2), pp. 143-152.
- Roll, R. (1992) A mean/variance analysis of tracking error, in: *The Journal of Portfolio Management*, 18 (4), pp. 13-22.
- Roßbach, P.; Karlow, D. (2009): *Index Tracking with Genetic Algorithms*, Euro 2009, Bonn.
- Rudd, A. (1980): Optimal selection of passive portfolios, in: *Financial Management*, 9 (1), pp. 57-66.
- Rudolf, M.; Wolter, H.J.; Zimmermann, H (1999): A linear model for tracking error minimization, in: *Journal of Banking & Finance*, 23 (1), pp. 85-103.
- Rudolf, M (2009): *Innovative Index Tracking Methoden*, Vortragsfolien, Tagung: *Finanz- und Vermögensberatung im Private Banking und Wealth Management*, 10.06.2009, Frankfurt am Main.
- Ruiz-Torrubiano, R.; Suarez, A. (2009): *A hybrid optimization approach to index tracking*, Springer.
- Stoyan, S.J.; Kwon, R.H (2010): A two-stage stochastic mixed-integer programming approach to the index tracking problem, in: *Optimization and Engineering*, 11 (2), pp. 247-275
- Sharpe, W.F (1991): The arithmetic of active management, in: *Financial Analysts Journal*, 47 (1), pp. 7-9.
- Schioldager, A.; Hahn, W.; Hoyt, E.; Leung, J (2004): The Unique Challenges of U.S. Equity Index Management, in: Schoenfeld, S.A. (ed.): *Active Index Investing*, Wiley, New Jersey, pp. 389-404.

FRANKFURT SCHOOL / HFB – WORKING PAPER SERIES

No.	Author/Title	Year
163.	Libman, Alexander / Herrmann-Pillath, Carsten / Yarav, Gaudav Are Human Rights and Economic Well-Being Substitutes? Evidence from Migration Patterns across the Indian States	2011
162.	Herrmann-Pillath, Carsten / Andriani, Pierpaolo Transactional Innovation and the De-commoditization of the Brazilian Coffee Trade	2011
161.	Christian Büchler, Marius Buxkaemper, Christoph Schalast, Gregor Wedell Incentivierung des Managements bei Unternehmenskäufen/Buy-Outs mit Private Equity Investoren – eine empirische Untersuchung –	2011
160.	Herrmann-Pillath, Carsten Revisiting the Gaia Hypothesis: Maximum Entropy, Kauffman’s “Fourth Law” and Physiosemeiosis	2011
159.	Herrmann-Pillath, Carsten A ‘Third Culture’ in Economics? An Essay on Smith, Confucius and the Rise of China	2011
158.	Boeing, Philipp / Sandner, Philipp The Innovative Performance of China’s National Innovation System	2011
157.	Herrmann-Pillath, Carsten Institutions, Distributed Cognition and Agency: Rule-following as Performative Action	2011
156.	Wagner, Charlotte From Boom to Bust: How different has microfinance been from traditional banking?	2010
155.	Libman Alexander / Vinokurov, Evgeny Is it really different? Patterns of Regionalisation in the Post-Soviet Central Asia	2010
154.	Libman, Alexander Subnational Resource Curse: Do Economic or Political Institutions Matter?	2010
153.	Herrmann-Pillath, Carsten Meaning and Function in the Theory of Consumer Choice: Dual Selves in Evolving Networks	2010
152.	Kostka, Genia / Hobbs, William Embedded Interests and the Managerial Local State: Methanol Fuel-Switching in China	2010
151.	Kostka, Genia / Hobbs, William Energy Efficiency in China: The Local Bundling of Interests and Policies	2010
150.	Umber, Marc P. / Grote, Michael H. / Frey, Rainer Europe Integrates Less Than You Think. Evidence from the Market for Corporate Control in Europe and the US	2010
149.	Vogel, Ursula / Winkler, Adalbert Foreign banks and financial stability in emerging markets: evidence from the global financial crisis	2010
148.	Libman, Alexander Words or Deeds – What Matters? Experience of Decentralization in Russian Security Agencies	2010
147.	Kostka, Genia / Zhou, Jianghua Chinese firms entering China’s low-income market: Gaining competitive advantage by partnering governments	2010
146.	Herrmann-Pillath, Carsten Rethinking Evolution, Entropy and Economics: A triadic conceptual framework for the Maximum Entropy Principle as applied to the growth of knowledge	2010
145.	Heidorn, Thomas / Kahlert, Dennis Implied Correlations of iTraxx Tranches during the Financial Crisis	2010
144.	Fritz-Morgenthal, Sebastian G. / Hach, Sebastian T. / Schalast, Christoph M&A im Bereich Erneuerbarer Energien	2010
143.	Birkmeyer, Jörg / Heidorn, Thomas / Rogalski, André Determinanten von Banken-Spreads während der Finanzmarktkrise	2010
142.	Bannier, Christina E. / Metz, Sabrina Are SMEs large firms en miniature? Evidence from a growth analysis	2010
141.	Heidorn, Thomas / Kaiser, Dieter G. / Voinea, André The Value-Added of Investable Hedge Fund Indices	2010
140.	Herrmann-Pillath, Carsten The Evolutionary Approach to Entropy: Reconciling Georgescu-Roegen’s Natural Philosophy with the Maximum Entropy Framework	2010
139.	Heidorn, Thomas / Löw, Christian / Winker, Michael Funktionsweise und Replikationstil europäischer Exchange Traded Funds auf Aktienindizes	2010
138.	Libman, Alexander Constitutions, Regulations, and Taxes: Contradictions of Different Aspects of Decentralization	2010

137.	Herrmann-Pillath, Carsten / Libman, Alexander / Yu, Xiaofan State and market integration in China: A spatial econometrics approach to 'local protectionism'	2010
136.	Lang, Michael / Cremers, Heinz / Hentze, Rainald Ratingmodell zur Quantifizierung des Ausfallrisikos von LBO-Finanzierungen	2010
135.	Bannier, Christina / Feess, Eberhard When high-powered incentive contracts reduce performance: Choking under pressure as a screening device	2010
134.	Herrmann-Pillath, Carsten Entropy, Function and Evolution: Naturalizing Peircian Semiosis	2010
133.	Bannier, Christina E. / Behr, Patrick / Güttler, Andre Rating opaque borrowers: why are unsolicited ratings lower?	2009
132.	Herrmann-Pillath, Carsten Social Capital, Chinese Style: Individualism, Relational Collectivism and the Cultural Embeddedness of the Institutions-Performance Link	2009
131.	Schäffler, Christian / Schmaltz, Christian Market Liquidity: An Introduction for Practitioners	2009
130.	Herrmann-Pillath, Carsten Dimensionen des Wissens: Ein kognitiv-evolutionärer Ansatz auf der Grundlage von F.A. von Hayeks Theorie der „Sensory Order“	2009
129.	Hankir, Yassin / Rauch, Christian / Umber, Marc It's the Market Power, Stupid! – Stock Return Patterns in International Bank M&A	2009
128.	Herrmann-Pillath, Carsten Outline of a Darwinian Theory of Money	2009
127.	Cremers, Heinz / Walzner, Jens Modellierung des Kreditrisikos im Portfoliofall	2009
126.	Cremers, Heinz / Walzner, Jens Modellierung des Kreditrisikos im Einwertpapierfall	2009
125.	Heidorn, Thomas / Schmaltz, Christian Interne Transferpreise für Liquidität	2009
124.	Bannier, Christina E. / Hirsch, Christian The economic function of credit rating agencies - What does the watchlist tell us?	2009
123.	Herrmann-Pillath, Carsten A Neurolinguistic Approach to Performativity in Economics	2009
122.	Winkler, Adalbert / Vogel, Ursula Finanzierungsstrukturen und makroökonomische Stabilität in den Ländern Südosteuropas, der Türkei und in den GUS-Staaten	2009
121.	Heidorn, Thomas / Rupprecht, Stephan Einführung in das Kapitalstrukturmanagement bei Banken	2009
120.	Roszbach, Peter Die Rolle des Internets als Informationsbeschaffungsmedium in Banken	2009
119.	Herrmann-Pillath, Carsten Diversity Management und diversitätsbasiertes Controlling: Von der „Diversity Scorecard“ zur „Open Balanced Scorecard“	2009
118.	Hölscher, Luise / Clasen, Sven Erfolgsfaktoren von Private Equity Fonds	2009
117.	Bannier, Christina E. Is there a hold-up benefit in heterogeneous multiple bank financing?	2009
116.	Roßbach, Peter / Gießamer, Dirk Ein eLearning-System zur Unterstützung der Wissensvermittlung von Web-Entwicklern in Sicherheitsthemen	2009
115.	Herrmann-Pillath, Carsten Kulturelle Hybridisierung und Wirtschaftstransformation in China	2009
114.	Schalast, Christoph: Staatsfonds – „neue“ Akteure an den Finanzmärkten?	2009
113.	Schalast, Christoph / Alram, Johannes Konstruktion einer Anleihe mit hypothekarischer Besicherung	2009
112.	Schalast, Christoph / Bolder, Markus / Radünz, Claus / Siepmann, Stephanie / Weber, Thorsten Transaktionen und Servicing in der Finanzkrise: Berichte und Referate des Frankfurt School NPL Forums 2008	2009
111.	Werner, Karl / Moormann, Jürgen Efficiency and Profitability of European Banks – How Important Is Operational Efficiency?	2009



110.	Herrmann-Pillath, Carsten Moralische Gefühle als Grundlage einer wohlstandschaffenden Wettbewerbsordnung: Ein neuer Ansatz zur erforschung von Sozialkapital und seine Anwendung auf China	2009
109.	Heidorn, Thomas / Kaiser, Dieter G. / Roder, Christoph Empirische Analyse der Drawdowns von Dach-Hedgefonds	2009
108.	Herrmann-Pillath, Carsten Neuroeconomics, Naturalism and Language	2008
107.	Schalast, Christoph / Benita, Barten Private Equity und Familienunternehmen – eine Untersuchung unter besonderer Berücksichtigung deutscher Maschinen- und Anlagenbauunternehmen	2008
106.	Bannier, Christina E. / Grote, Michael H. Equity Gap? – Which Equity Gap? On the Financing Structure of Germany's Mittelstand	2008
105.	Herrmann-Pillath, Carsten The Naturalistic Turn in Economics: Implications for the Theory of Finance	2008
104.	Schalast, Christoph (Hrsg.) / Schanz, Kay-Michael / Scholl, Wolfgang Aktionärschutz in der AG falsch verstanden? Die Leica-Entscheidung des LG Frankfurt am Main	2008
103.	Bannier, Christina E./ Müsch, Stefan Die Auswirkungen der Subprime-Krise auf den deutschen LBO-Markt für Small- und MidCaps	2008
102.	Cremers, Heinz / Vetter, Michael Das IRB-Modell des Kreditrisikos im Vergleich zum Modell einer logarithmisch normalverteilten Verlustfunktion	2008
101.	Heidorn, Thomas / Pleißner, Mathias Determinanten Europäischer CMBS Spreads. Ein empirisches Modell zur Bestimmung der Risikoaufschläge von Commercial Mortgage-Backed Securities (CMBS)	2008
100.	Schalast, Christoph (Hrsg.) / Schanz, Kay-Michael Schaeffler KG/Continental AG im Lichte der CSX Corp.-Entscheidung des US District Court for the Southern District of New York	2008
99.	Hölscher, Luise / Haug, Michael / Schweinberger, Andreas Analyse von Steueramnestiedaten	2008
98.	Heimer, Thomas / Arend, Sebastian The Genesis of the Black-Scholes Option Pricing Formula	2008
97.	Heimer, Thomas / Hölscher, Luise / Werner, Matthias Ralf Access to Finance and Venture Capital for Industrial SMEs	2008
96.	Böttger, Marc / Guthoff, Anja / Heidorn, Thomas Loss Given Default Modelle zur Schätzung von Recovery Rates	2008
95.	Almer, Thomas / Heidorn, Thomas / Schmaltz, Christian The Dynamics of Short- and Long-Term CDS-spreads of Banks	2008
94.	Barthel, Erich / Wollersheim, Jutta Kulturunterschiede bei Mergers & Acquisitions: Entwicklung eines Konzeptes zur Durchführung einer Cultural Due Diligence	2008
93.	Heidorn, Thomas / Kunze, Wolfgang / Schmaltz, Christian Liquiditätsmodellierung von Kreditzusagen (Term Facilities and Revolver)	2008
92.	Burger, Andreas Produktivität und Effizienz in Banken – Terminologie, Methoden und Status quo	2008
91.	Löchel, Horst / Pecher, Florian The Strategic Value of Investments in Chinese Banks by Foreign Financial Insitutions	2008
90.	Schalast, Christoph / Morgenschweis, Bernd / Sprengetter, Hans Otto / Ockens, Klaas / Stachuletz, Rainer / Safran, Robert Der deutsche NPL Markt 2007: Aktuelle Entwicklungen, Verkauf und Bewertung – Berichte und Referate des NPL Forums 2007	2008
89.	Schalast, Christoph / Stralkowski, Ingo 10 Jahre deutsche Buyouts	2008
88.	Bannier, Christina E./ Hirsch, Christian The Economics of Rating Watchlists: Evidence from Rating Changes	2007
87.	Demidova-Menzel, Nadeshda / Heidorn, Thomas Gold in the Investment Portfolio	2007
86.	Hölscher, Luise / Rosenthal, Johannes Leistungsmessung der Internen Revision	2007
85.	Bannier, Christina / Hänsel, Dennis Determinants of banks' engagement in loan securitization	2007

84.	Bannier, Christina "Smoothing" versus "Timeliness" - Wann sind stabile Ratings optimal und welche Anforderungen sind an optimale Berichtsregeln zu stellen?	2007
83.	Bannier, Christina E. Heterogeneous Multiple Bank Financing: Does it Reduce Inefficient Credit-Renegotiation Incidences?	2007
82.	Cremers, Heinz / Löhner, Andreas Deskription und Bewertung strukturierter Produkte unter besonderer Berücksichtigung verschiedener Marktszenarien	2007
81.	Demidova-Menzel, Nadeshda / Heidorn, Thomas Commodities in Asset Management	2007
80.	Cremers, Heinz / Walzner, Jens Risikosteuerung mit Kreditderivaten unter besonderer Berücksichtigung von Credit Default Swaps	2007
79.	Cremers, Heinz / Traughber, Patrick Handlungsalternativen einer Genossenschaftsbank im Investmentprozess unter Berücksichtigung der Risikotragfähigkeit	2007
78.	Gerdemesmer, Dieter / Roffia, Barbara Monetary Analysis: A VAR Perspective	2007
77.	Heidorn, Thomas / Kaiser, Dieter G. / Muschiol, Andrea Portfoliooptimierung mit Hedgefonds unter Berücksichtigung höherer Momente der Verteilung	2007
76.	Jobe, Clemens J. / Ockens, Klaas / Safran, Robert / Schalast, Christoph Work-Out und Servicing von notleidenden Krediten – Berichte und Referate des HfB-NPL Servicing Forums 2006	2006
75.	Abrar, Kamyar / Schalast, Christoph Fusionskontrolle in dynamischen Netzsektoren am Beispiel des Breitbandkabelsektors	2006
74.	Schalast, Christoph / Schanz, Kay-Michael Wertpapierprospekte: Markteinführungspublizität nach EU-Prospektverordnung und Wertpapierprospektgesetz 2005	2006
73.	Dickler, Robert A. / Schalast, Christoph Distressed Debt in Germany: What's Next? Possible Innovative Exit Strategies	2006
72.	Belke, Ansgar / Polleit, Thorsten How the ECB and the US Fed set interest rates	2006
71.	Heidorn, Thomas / Hoppe, Christian / Kaiser, Dieter G. Heterogenität von Hedgefondsindizes	2006
70.	Baumann, Stefan / Löchel, Horst The Endogeneity Approach of the Theory of Optimum Currency Areas - What does it mean for ASEAN + 3?	2006
69.	Heidorn, Thomas / Trautmann, Alexandra Niederschlagsderivate	2005
68.	Heidorn, Thomas / Hoppe, Christian / Kaiser, Dieter G. Möglichkeiten der Strukturierung von Hedgefondsportfolios	2005
67.	Belke, Ansgar / Polleit, Thorsten (How) Do Stock Market Returns React to Monetary Policy ? An ARDL Cointegration Analysis for Germany	2005
66.	Daynes, Christian / Schalast, Christoph Aktuelle Rechtsfragen des Bank- und Kapitalmarktsrechts II: Distressed Debt - Investing in Deutschland	2005
65.	Gerdemesmer, Dieter / Polleit, Thorsten Measures of excess liquidity	2005
64.	Becker, Gernot M. / Harding, Perham / Hölscher, Luise Financing the Embedded Value of Life Insurance Portfolios	2005
63.	Schalast, Christoph Modernisierung der Wasserwirtschaft im Spannungsfeld von Umweltschutz und Wettbewerb – Braucht Deutschland eine Rechtsgrundlage für die Vergabe von Wasserversorgungskonzessionen? –	2005
62.	Bayer, Marcus / Cremers, Heinz / Kluß, Norbert Wertsicherungsstrategien für das Asset Management	2005
61.	Löchel, Horst / Polleit, Thorsten A case for money in the ECB monetary policy strategy	2005
60.	Richard, Jörg / Schalast, Christoph / Schanz, Kay-Michael Unternehmen im Prime Standard - „Staying Public“ oder „Going Private“? - Nutzenanalyse der Börsennotiz -	2004
59.	Heun, Michael / Schlink, Torsten Early Warning Systems of Financial Crises - Implementation of a currency crisis model for Uganda	2004
58.	Heimer, Thomas / Köhler, Thomas Auswirkungen des Basel II Akkords auf österreichische KMU	2004
57.	Heidorn, Thomas / Meyer, Bernd / Pietrowiak, Alexander Performanceeffekte nach Directors Dealings in Deutschland, Italien und den Niederlanden	2004

56.	Gerdemesier, Dieter / Roffia, Barbara The Relevance of real-time data in estimating reaction functions for the euro area	2004
55.	Barthel, Erich / Gierig, Rauno / Kühn, Ilmhart-Wolfram Unterschiedliche Ansätze zur Messung des Humankapitals	2004
54.	Anders, Dietmar / Binder, Andreas / Hesdahl, Ralf / Schalast, Christoph / Thöne, Thomas Aktuelle Rechtsfragen des Bank- und Kapitalmarktrechts I : Non-Performing-Loans / Faule Kredite - Handel, Work-Out, Outsourcing und Securitisation	2004
53.	Polleit, Thorsten The Slowdown in German Bank Lending – Revisited	2004
52.	Heidorn, Thomas / Siragusano, Tindaro Die Anwendbarkeit der Behavioral Finance im Devisenmarkt	2004
51.	Schütze, Daniel / Schalast, Christoph (Hrsg.) Wider die Verschleuderung von Unternehmen durch Pfandversteigerung	2004
50.	Gerhold, Mirko / Heidorn, Thomas Investitionen und Emissionen von Convertible Bonds (Wandelanleihen)	2004
49.	Chevalier, Pierre / Heidorn, Thomas / Krieger, Christian Temperaturderivate zur strategischen Absicherung von Beschaffungs- und Absatzrisiken	2003
48.	Becker, Gernot M. / Seeger, Norbert Internationale Cash Flow-Rechnungen aus Eigner- und Gläubigersicht	2003
47.	Boenkost, Wolfram / Schmidt, Wolfgang M. Notes on convexity and quanto adjustments for interest rates and related options	2003
46.	Hess, Dieter Determinants of the relative price impact of unanticipated Information in U.S. macroeconomic releases	2003
45.	Cremers, Heinz / Kluß, Norbert / König, Markus Incentive Fees. Erfolgsabhängige Vergütungsmodelle deutscher Publikumsfonds	2003
44.	Heidorn, Thomas / König, Lars Investitionen in Collateralized Debt Obligations	2003
43.	Kahlert, Holger / Seeger, Norbert Bilanzierung von Unternehmenszusammenschlüssen nach US-GAAP	2003
42.	Beiträge von Studierenden des Studiengangs BBA 012 unter Begleitung von Prof. Dr. Norbert Seeger Rechnungslegung im Umbruch - HGB-Bilanzierung im Wettbewerb mit den internationalen Standards nach IAS und US-GAAP	2003
41.	Overbeck, Ludger / Schmidt, Wolfgang Modeling Default Dependence with Threshold Models	2003
40.	Balthasar, Daniel / Cremers, Heinz / Schmidt, Michael Portfoliooptimierung mit Hedge Fonds unter besonderer Berücksichtigung der Risikokomponente	2002
39.	Heidorn, Thomas / Kantwill, Jens Eine empirische Analyse der Spreadunterschiede von Festsatzanleihen zu Floatern im Euroraum und deren Zusammenhang zum Preis eines Credit Default Swaps	2002
38.	Böttcher, Henner / Seeger, Norbert Bilanzierung von Finanzderivaten nach HGB, EstG, IAS und US-GAAP	2003
37.	Moormann, Jürgen Terminologie und Glossar der Bankinformatik	2002
36.	Heidorn, Thomas Bewertung von Kreditprodukten und Credit Default Swaps	2001
35.	Heidorn, Thomas / Weier, Sven Einführung in die fundamentale Aktienanalyse	2001
34.	Seeger, Norbert International Accounting Standards (IAS)	2001
33.	Moormann, Jürgen / Stehling, Frank Strategic Positioning of E-Commerce Business Models in the Portfolio of Corporate Banking	2001
32.	Sokolovsky, Zbynek / Strohhecker, Jürgen Fit für den Euro, Simulationsbasierte Euro-Maßnahmenplanung für Dresdner-Bank-Geschäftsstellen	2001
31.	Roßbach, Peter Behavioral Finance - Eine Alternative zur vorherrschenden Kapitalmarkttheorie?	2001
30.	Heidorn, Thomas / Jaster, Oliver / Willeitner, Ulrich Event Risk Covenants	2001
29.	Biswas, Rita / Löchel, Horst Recent Trends in U.S. and German Banking: Convergence or Divergence?	2001

28.	Eberle, Günter Georg / Löchel, Horst Die Auswirkungen des Übergangs zum Kapitaldeckungsverfahren in der Rentenversicherung auf die Kapitalmärkte	2001
27.	Heidorn, Thomas / Klein, Hans-Dieter / Siebrecht, Frank Economic Value Added zur Prognose der Performance europäischer Aktien	2000
26.	Cremers, Heinz Konvergenz der binomialen Optionspreismodelle gegen das Modell von Black/Scholes/Merton	2000
25.	Löchel, Horst Die ökonomischen Dimensionen der ‚New Economy‘	2000
24.	Frank, Axel / Moormann, Jürgen Grenzen des Outsourcing: Eine Exploration am Beispiel von Direktbanken	2000
23.	Heidorn, Thomas / Schmidt, Peter / Seiler, Stefan Neue Möglichkeiten durch die Namensaktie	2000
22.	Böger, Andreas / Heidorn, Thomas / Graf Waldstein, Philipp Hybrides Kernkapital für Kreditinstitute	2000
21.	Heidorn, Thomas Entscheidungsorientierte Mindestmargenkalkulation	2000
20.	Wolf, Birgit Die Eigenmittelkonzeption des § 10 KWG	2000
19.	Cremers, Heinz / Robé, Sophie / Thiele, Dirk Beta als Risikomaß - Eine Untersuchung am europäischen Aktienmarkt	2000
18.	Cremers, Heinz Optionspreisbestimmung	1999
17.	Cremers, Heinz Value at Risk-Konzepte für Marktrisiken	1999
16.	Chevalier, Pierre / Heidorn, Thomas / Rütze, Merle Gründung einer deutschen Strombörse für Elektrizitätsderivate	1999
15.	Deister, Daniel / Ehrlicher, Sven / Heidorn, Thomas CatBonds	1999
14.	Jochum, Eduard Hoshin Kanri / Management by Policy (MbP)	1999
13.	Heidorn, Thomas Kreditderivate	1999
12.	Heidorn, Thomas Kreditrisiko (CreditMetrics)	1999
11.	Moormann, Jürgen Terminologie und Glossar der Bankinformatik	1999
10.	Löchel, Horst The EMU and the Theory of Optimum Currency Areas	1998
09.	Löchel, Horst Die Geldpolitik im Währungsraum des Euro	1998
08.	Heidorn, Thomas / Hund, Jürgen Die Umstellung auf die Stückaktie für deutsche Aktiengesellschaften	1998
07.	Moormann, Jürgen Stand und Perspektiven der Informationsverarbeitung in Banken	1998
06.	Heidorn, Thomas / Schmidt, Wolfgang LIBOR in Arrears	1998
05.	Jahresbericht 1997	1998
04.	Ecker, Thomas / Moormann, Jürgen Die Bank als Betreiberin einer elektronischen Shopping-Mall	1997
03.	Jahresbericht 1996	1997
02.	Cremers, Heinz / Schwarz, Willi Interpolation of Discount Factors	1996
01.	Moormann, Jürgen Lean Reporting und Führungsinformationssysteme bei deutschen Finanzdienstleistern	1995

**FRANKFURT SCHOOL / HFB – WORKING PAPER SERIES**  
**CENTRE FOR PRACTICAL QUANTITATIVE FINANCE**

No.	Author/Title	Year
27.	Detering, Nils / Weber, Andreas / Wystup, Uwe Return distributions of equity-linked retirement plans	2010
26.	Veiga, Carlos / Wystup, Uwe Ratings of Structured Products and Issuers' Commitments	2010
25.	Beyna, Ingo / Wystup, Uwe On the Calibration of the Cheyette. Interest Rate Model	2010
24.	Scholz, Peter / Walther, Ursula Investment Certificates under German Taxation. Benefit or Burden for Structured Products' Performance	2010
23.	Esquivel, Manuel L. / Veiga, Carlos / Wystup, Uwe Unifying Exotic Option Closed Formulas	2010
22.	Packham, Natalie / Schlögl, Lutz / Schmidt, Wolfgang M. Credit gap risk in a first passage time model with jumps	2009
21.	Packham, Natalie / Schlögl, Lutz / Schmidt, Wolfgang M. Credit dynamics in a first passage time model with jumps	2009
20.	Reiswich, Dimitri / Wystup, Uwe FX Volatility Smile Construction	2009
19.	Reiswich, Dimitri / Tompkins, Robert Potential PCA Interpretation Problems for Volatility Smile Dynamics	2009
18.	Keller-Ressel, Martin / Kilin, Fiodar Forward-Start Options in the Barndorff-Nielsen-Shephard Model	2008
17.	Griebisch, Susanne / Wystup, Uwe On the Valuation of Fader and Discrete Barrier Options in Heston's Stochastic Volatility Model	2008
16.	Veiga, Carlos / Wystup, Uwe Closed Formula for Options with Discrete Dividends and its Derivatives	2008
15.	Packham, Natalie / Schmidt, Wolfgang Latin hypercube sampling with dependence and applications in finance	2008
14.	Hakala, Jürgen / Wystup, Uwe FX Basket Options	2008
13.	Weber, Andreas / Wystup, Uwe Vergleich von Anlagestrategien bei Riesterrenten ohne Berücksichtigung von Gebühren. Eine Simulationsstudie zur Verteilung der Renditen	2008
12.	Weber, Andreas / Wystup, Uwe Riesterrente im Vergleich. Eine Simulationsstudie zur Verteilung der Renditen	2008
11.	Wystup, Uwe Vanna-Volga Pricing	2008
10.	Wystup, Uwe Foreign Exchange Quanto Options	2008
09.	Wystup, Uwe Foreign Exchange Symmetries	2008
08.	Becker, Christoph / Wystup, Uwe Was kostet eine Garantie? Ein statistischer Vergleich der Rendite von langfristigen Anlagen	2008
07.	Schmidt, Wolfgang Default Swaps and Hedging Credit Baskets	2007
06.	Kilin, Fiodar Accelerating the Calibration of Stochastic Volatility Models	2007
05.	Griebisch, Susanne / Kühn, Christoph / Wystup, Uwe Instalment Options: A Closed-Form Solution and the Limiting Case	2007
04.	Boenkost, Wolfram / Schmidt, Wolfgang M. Interest Rate Convexity and the Volatility Smile	2006
03.	Becker, Christoph / Wystup, Uwe On the Cost of Delayed Currency Fixing Announcements	2005
02.	Boenkost, Wolfram / Schmidt, Wolfgang M. Cross currency swap valuation	2004

01.	Wallner, Christian / Wystup, Uwe Efficient Computation of Option Price Sensitivities for Options of American Style	2004
-----	---	------

**HFB – SONDERARBEITSBERICHTE DER HFB - BUSINESS SCHOOL OF FINANCE & MANAGEMENT**

No.	Author/Title	Year
01.	Nicole Kahmer / Jürgen Moormann Studie zur Ausrichtung von Banken an Kundenprozessen am Beispiel des Internet (Preis: € 120,-)	2003

Printed edition: € 25.00 + € 2.50 shipping

Download:

Working Paper: [http://www.frankfurt-school.de/content/de/research/publications/list\\_of\\_publication/list\\_of\\_publication](http://www.frankfurt-school.de/content/de/research/publications/list_of_publication/list_of_publication)  
CPQF: [http://www.frankfurt-school.de/content/de/cpqf/research\\_publications.html](http://www.frankfurt-school.de/content/de/cpqf/research_publications.html)

**Order address / contact**

Frankfurt School of Finance & Management  
Sonnemannstr. 9–11 ▪ D–60314 Frankfurt/M. ▪ Germany  
Phone: +49 (0) 69 154 008–734 ▪ Fax: +49 (0) 69 154 008–728  
eMail: [e.lahdensuu@fs.de](mailto:e.lahdensuu@fs.de)

Further information about Frankfurt School of Finance & Management  
may be obtained at: <http://www.fs.de>