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Imperfect Information and the Meltzer-Richard Hypothesis

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Christian Bredemeier[1]

Imperfect Information and the Meltzer-Richard Hypothesis

Abstract

Despite a strong theoretical prediction that income skewness and redistribution should be positively linked, empirical evidence on this issue is mixed. This paper argues that it is important to distinguish between sources of changes in income skewness. Two sources of such changes are discussed: rising polarization and upward mobility, which both increase income skewness. Under imperfect information, these developments affect redistribution in different ways. While rising polarization increases redistribution, upward mobility can have the opposite effect. Reasonable degrees of informational imperfection are sufficient to generate increasing income skewness and decreasing redistribution in the presence of upward mobility.

JEL Classification: D72, H24, D83

Keywords: Voting; redistribution; imperfect information

October 2010

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1 Introduction

It is a common view that democratically implemented income redistribution should always favor the receiver of the median income. In this view, the individually optimal degree of redistribution is a downward-sloping function of one's income and the median income receiver is thus also the median voter. A clear-cut prediction that arises from such consideration is the Meltzer-Richard hypothesis (Meltzer and Richard 1981): the extent of redistribution rises when the mean-to-median ratio of the income distribution increases since the median voter will then gain more from redistribution. Income skewness and redistribution should thus be positively related. In this paper, I show that, in a model with imperfect information, redistribution can also decrease in response to a rise in income skewness depending on the source of such rise.

The Meltzer-Richard hypothesis is an important part of economic arguing. The negative link between income inequality and economic growth (Alesina and Rodrik 1994; Persson and Tabellini 1994) builds on this hypothesis. The argument that more unequal societies have slower growth relies on the disincentive effects caused by more pronounced income redistribution sought by the relatively poorer median voter.

However, empirical evidence regarding the link between income skewness and redistribution is anything as clear as the theoretical prediction. While a positive relation between income skewness and redistribution is indeed observed in some empirical studies, there are also studies which report the opposite. Cross-country studies find evidence supporting the Meltzer-Richard hypothesis (Easterly and Rebelo 1993; Lindert 1996; Milanovic 2000; Mohl and Pamp 2009) as well as contradictory results (Keefer and Knack 1995; Perotti 1996; Bassett, Burkett, and Putterman 1999). Cross-sectional studies within one country reveal evidence in favor of the hypothesis at the municipality level (Alesina, Baqir, and Easterly 2000 for the US, Borge and Rattsø 2004 for Norway) or comparing Brazilian states (Mattos and Rocha 2008) but also rejecting findings at the level of US states (Gouveia and Masia 1998; Rodríguez 1999).

Concerning time-series evidence, the study by Meltzer and Richard (1983) supports the theoretical prediction. The authors analyze US time series data of government spending and conclude that the spending level is positively related to the mean-to-median income ratio. Subsequent studies on similar questions arrive at the contrary (Rodríguez 1999; Ken-

worthy and McCall 2008). These studies report situations of increasing income skewness that are accompanied by cut-backs in the welfare state. Such episodes include e.g. the reductions in redistributive efforts implemented by the Reagan and Thatcher administrations around 1980 when mean-to-median income ratios were steadily increasing. Such developments are alien to a standard majority voting model.

The present paper argues that it is not sufficient to consider the skewness of the income distribution alone. When income skewness changes, it is important to distinguish between sources of such changes. I discuss two developments in the income distribution which have the same effect on skewness but may affect democratically implemented redistribution in different ways: rising polarization and upward mobility. Rising polarization is a development where those who are rich anyway become even richer. Thus differences between population groups become more severe and the population more polarized (see e.g. Esteban and Ray 1994).[1] In contrast to this, upward mobility describes a development where initially poor individuals catch up to richer population groups and move up in the income distribution (see e.g. Bénabou and Ok 2001).

These two developments are illustrated in Figure 1. Panel (a) of the figure shows a log-normal income distribution with its mean (dotted vertical line) exceeding its median (thin vertical line). In panel (b), the same distribution is represented by the thin curve. The thick curve represents the distribution after a rise in polarization, i.e. an income increase for some agents above mean income. Correspondingly, more mass lies at the very right tail of the distribution. Mean income rises while median income is not affected. Finally, panel (c) of the figure illustrates an example of upward mobility. Here, starting from the log-normal distribution (thin curve), some agents poorer than the median move towards richer population groups. Reflecting this, less mass lies at the very left tail of the distribution. Also in this scenario, mean income increases and median income is constant.

Both developments increase income skewness and therefore have the same effect on redistribution in a voting model with perfect information. However, in a model with imperfect information, these developments affect redistribution in different ways. While rising polarization generates

[1] Polarization would also increase if poor agents became poorer. However, this would decrease the mean-to-median income ratio. I will thus concentrate on the case where rich agents become richer and use the term 'rising polarization' accordingly.

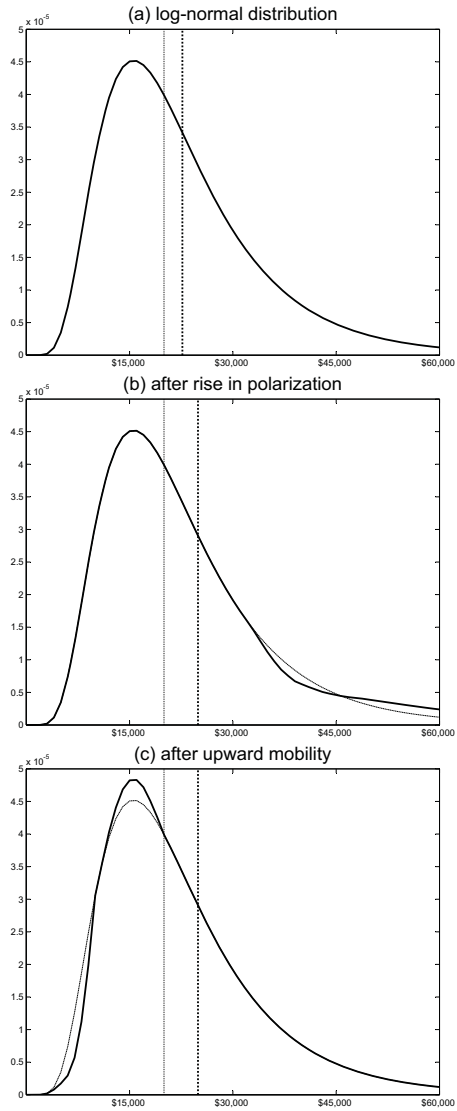

Figure 1: Three income distributions (thick curves) with mean (thick vertical line), median (thin vertical line), and baseline log-normal distribution (thin curve).

the standard effect, upward mobility can result in decreasing redistribution.

The importance of informational imperfections in democratic decision making has been stressed by Downs (1957). Downs pointed out that even small information costs can lead voters to be rationally ignorant and cause pronounced uncertainty about issues important for the optimal vote. Understood broadly, imperfect information also comprises all differences between complete information and information that is reflected in behavior (Sims 2003). Such differences can arise from cognitive differences at any stage in the process between observing an information and the implementation of the appropriate response. Even with perfect information available, if voters choose not to use all information, have difficulties figuring out the appropriate response, or make mistakes while translating decisions into behavior, political decisions may appear as if voters had imperfect information in the first place. It is an important feature of the model presented in this paper that agents who are identical except for beliefs can vote differently. In the model, this is a result of imperfect information which may, however, not be the only reason for the existence of such differences in votes. They would also occur if voters had perfect information but made random mistakes in determining the optimal tax rate or, in the terminology of e.g. Shue and Luttmer (2009), misvoted. Recent papers use concepts related to imperfect information to study voting behavior, both theoretically (Gershkov and Szentes 2009; Hansen 2005; Dhami 2003) and empirically (Mullainathan and Washington 2009; Shue and Luttmer 2009).

This paper presents a model of direct democracy with selfish voters, perfect markets, and complete enforcement in which the relation between income skewness and redistribution depends on the causes of changes in income skewness. The model is a version of the Romer-Roberts-Meltzer-Richard model (Romer 1975; Roberts 1977; Meltzer and Richard 1981). Agents differ with respect to their productivity and, in consequence, income. The main difference to the standard model is imperfect information about the productivity distribution. Under perfect information, the extent of redistribution would be determined by the interest of the median-income earner with high-income agents wishing less redistribution and low-income agents more.

In this model, the optimal vote for an agent depends on her own productivity and the average productivity in the economy. While the agent

knows her own productivity for sure, she is assumed to be only imperfectly informed about productivities of others and thus about average productivity. Such assumption can be justified by the empirical findings of e.g. Betts (1996) and Ellison, Lusk, and Briggeman (2010). Betts (1996) find that people tend to misestimate average wages even in their own industry. Furthermore, perceptions differ across individuals with a variation coefficient of roughly 30%. Ellison, Lusk, and Briggeman (2010) report even higher variations of beliefs when people are asked to estimate the income of other population groups.

The model population is populated by three classes of agents who differ by productivity, a lower, a middle, and an upper class. Within classes, agents are identical except for beliefs about the productivity distribution. Votes on the degree of income redistribution are based on these beliefs and consequently even agents with the same productivity can vote differently. However, this does not happen in the lower and upper classes. In these classes, agents are sure to be at the bottom or the top of the distribution no matter how it is shaped. Independent of their beliefs about the shape of the distribution, these individuals will vote for either maximum redistribution or none at all.

In the middle class, the informational imperfection is relevant. Agents in this class are in the interior of the productivity distribution. And since votes depend on individuals' relative productivity, the exact shape of the distribution is important for these voters. Even though agents in the middle class have the same productivity, they differ in their beliefs about the average productivity of others. In the middle class, there is thus a distribution of votes around the optimal vote. The election outcome is determined by the vote of the economy-wide median voter. Voting powers of the upper and lower classes determine which vote from the distribution of middle-class votes is decisive.

In this set-up, it is important to distinguish between causes of changes in the mean-to-median income ratio when analyzing the effect on redistribution. When income skewness rises, the optimal rate of redistribution for the middle class increases. Under rising polarization, this unambiguously increases implemented redistribution. However, upward mobility has a second, counter-acting effect. When some agents catch up to richer population groups, shifts in voting power move the quantile of the median voter in the belief distribution towards voting for fewer redistribution. Depending on the magnitude of the informational imper-

fection, the second effect can dominate the first one and one can observe a negative relation between income skewness and redistribution.

Upward mobility is more likely to cause decreasing redistribution the more people disagree in their beliefs about average income in the economy. A quantification of model parameters shows that empirically reasonable degrees of disagreement are sufficient for the second effect to dominate. Empirical evidence on developments in the income distribution like Esteban and Ray (1994) and the key figures of the Luxembourg Income Study support the view that seemingly anomalous reductions in redistribution around 1980 were indeed preceded by upward mobility.

Some previous contributions on imperfect information in models of voting on redistribution have studied related questions. Dhami (2003) analyzes the effects of inequality on redistribution in a model of representative democracy where voters have asymmetric information about politicians' redistributive ambitions. Hansen (2005) and Laslier, Trannoy, and van der Straeten (2003) use similar models to the one presented in this paper. Hansen (2005) uses a Romer-Roberts-Meltzer-Richard type model with imperfect information about government efficiency and studies biases in the level of government size that can arise due to the information friction. Laslier, Trannoy, and van der Straeten (2003) address the topic of overtaxation in a model with uncertainty about the potential productivity of the unemployed. Both studies however do not cover the relation between changes in income skewness and changes in redistribution.

Bénabou and Ok (2001) and Alesina and La Ferrara (2005) have studied the influence of a prospect of upward mobility on voting decisions under perfect information. They demonstrate that such prospect can lead to less income redistribution. These studies however do not perform a comparative-static analysis of how election outcomes change once some agents have actually experienced upward mobility.

The remainder of the paper is organized as follows. Section 2 describes the set-up of the model and solves for individual decisions and collective choices. Section 3 presents a comparative-static analysis of changes in the mean-to-median ratio distinguishing between rising polarization and upward mobility. Section 4 concludes.

2 The Model

2.1 Model set-up

In this section, I describe the structure of the model. It is a version of the Romer-Roberts-Meltzer-Richard model (Romer 1975; Roberts 1977; Meltzer and Richard 1981). Agents differ with respect to their productivity and, in consequence, income. In contrast to the standard model, agents are imperfectly informed about the productivity distribution.

Preferences and Technology. I consider an economy that is populated by a mass-1 continuum of agents behaving according to the following preferences:

$$u_i = c_i - \frac{\phi}{2} n_i^2, \tag{1}$$

where c_i denotes agent i's consumption and n_i is the amount of hours worked. If working, agents produce consumption goods y_i with linear technology

$$y_i = a_i n_i, \tag{2}$$

where a_i is an agent-specific productivity.

The composition of the economy is characterized by discrete differences in productivity. There are three productivity levels. Agents either have a low productivity normalized to 0, a medium one, a_1, or a high one, a_2, where $a_2 = \sqrt{\alpha} \cdot a_1$, $\alpha > 1$. Agents with zero productivity will decide not to work. The population can thus be split up into three groups, which are labeled according to their productivities:

(i) an upper class with high-productivity agents, labeled "H" for high

(ii) a middle class with medium-productivity agents, labeled "M" for medium

(iii) a lower class with agents who do not work, labeled "L" for low

Group sizes are denoted by s_L, s_M, and s_H, respectively, $s_L + s_M + s_H = 1$. I assume that no group contains more than mass $\frac{1}{2}$ of agents. This assumption guarantees that the median gross income falls into group M.

Note that both, α and $\frac{s_H}{s_L}$ are determinants of the skewness of the productivity distribution, which is illustrated in Figure 2. With $\sqrt{\alpha} =$

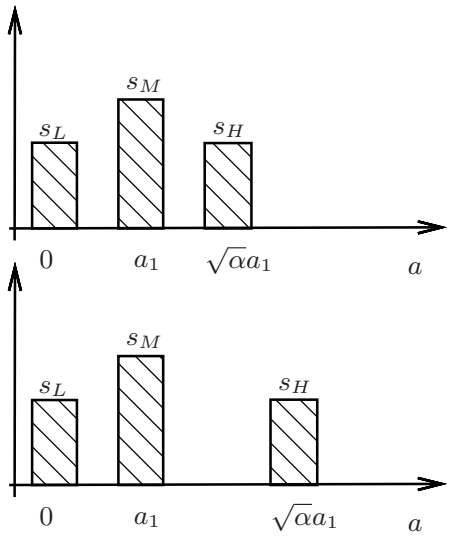

Figure 2: Two examples of a productivity distribution.

$\frac{s_H+s_L}{s_H}$ the distribution is symmetric (as in the upper part of the figure where $s_H = s_L$ and $\alpha = 4$). If $\sqrt{\alpha} > \frac{s_H+s_L}{s_H}$, the distribution is skewed to the right (as in the lower part of the figure where still $s_H = s_L$ but $\alpha > 4$) and vice versa. Skewness of the income distribution, which is key for the extent of redistribution sought by the middle class, jointly results from the skewness of the productivity distribution together with endogenous labor supply decisions.

Political Environment. The economy redistributes income through a linear income tax τ, the proceeds of which are to be distributed equally among the total population. Thus, an agent's net amount of consumption is a linear combination of his own gross earnings and the average earnings in the economy,

$$c_i = (1 - \tau) \cdot y_i + \tau \cdot y, \tag{3}$$

where y denotes the average gross income.[2]

The redistribution rate $\tau \in [0, 1]$ is determined in direct democracy by pairwise votes over proposals. All agents participate in this vote. Furthermore, I assume that agents vote truthfully in the sense that they vote for their individual expected-utility maximizing τ. Since any single voter has zero mass in this model, I abstain from analyzing strategic voting behavior and assume "sincere" (Bearse, Cardak, Glomm, and Ravikumar 2009) or "naive" (Feddersen and Pesendorfer 1997) voting.

Informational Environment. Agents are aware of the structure of the economy and know all parameter values except for other agents' productivities. The latter reflects the empirical evidence that there is disagreement about other people's wages (Betts 1996; Ellison, Lusk, and Briggeman 2010). For agents in groups M and H, this is tantamount to not knowing the productivity parameter α. This parameter measures the difference between the middle and the upper class and is one determinant of the skewness of the productivity distribution. From the perspective of agents, the parameter can take any value which exceeds 1. Agents cannot observe the draw of the parameter. After α is drawn, each agent receives an individual imperfect signal α_i^S about α, which equals the true value in expectation. Agents' signals are drawn independently from an identical uniform distribution on $[\alpha - \varepsilon, \alpha + \varepsilon]$. Any individual agent i only observes her own signal α_i^S and can not use other agents' signals to determine beliefs about α.

Even though agents do not know others' productivities with certainty, they know which group they themselves belong to and the ordering of productivities by groups. I.e. agents know that

$$a_2 > a_1 > 0. \tag{4}$$

For analytical simplicity, I will restrict the analysis to parameter constellations which fulfill

$$\alpha > 1 + 2\varepsilon \tag{5}$$

such that no signal contradicts condition (4). Furthermore, under this condition, beliefs will not be biased. For the main results of the paper,

[2]The literature on voting about redistribution usually studies voting on the parameterization of some given redistribution scheme. In more general set-ups, voting equilibria may not exist (see e.g. Mueller 2003).

this parameter restriction is innocuous since unbiasedness of beliefs is not crucial.

Time Structure. The timing of events is the following. First, the productivity parameter α is drawn. The draw is unobservable for agents. Second, agents receive signals $\left\{\alpha_i^S\right\}$ and update their beliefs. Third, the election over the redistribution parameter τ takes place and the median vote is implemented. Fourth, agents decide how much to work and produce gross income. Finally, redistribution is performed and goods are consumed.

2.2 Individual decisions

In this section, I present the optimal decisions of individual agents. Detailed derivations of decisions at the individual level can be found in Appendix A.

Labor supply decisions and income distribution. Agents have to decide on how much they want to work. When taking this decision, agents take into account the degree of income redistribution. For agents in group L, it is optimal not to work while agents in the other groups work positive hours. Equalizing marginal benefits and costs from working, given a tax rate τ, results in

$$n_i = \frac{(1-\tau)\, a_i}{\phi} \quad \forall i. \tag{6}$$

Redistribution reduces labor supply through a standard disincentive effect. It has the same effect on aggregate income, which is

$$y = (1-\tau) \cdot \frac{1}{\phi} \cdot \left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2\right] \tag{7}$$

as a result of individual labor supply decisions.

Labor supply decisions as described by equation (6) imply that income is a quadratic function of productivity as can be seen from Figure 3. The mean-to-median income ratio is

$$y/\left[(1-\tau) \cdot \phi^{-1} \cdot (a_1)^2\right] = s_M + s_H \alpha. \tag{8}$$

The income distribution is skewed to the right and the mean-to-median ratio greater than 1 if $\alpha > \frac{s_H + s_L}{s_H}$ and vice versa.

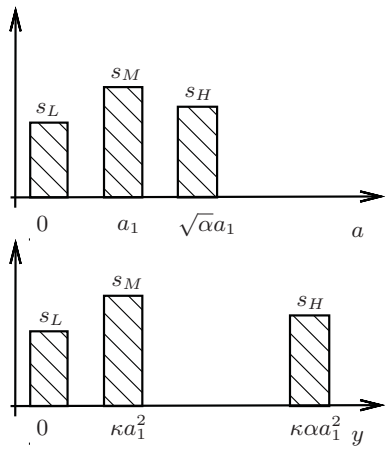

Figure 3: Example of a productivity and the corresponding income distribution ($\kappa = \frac{1-\tau}{\phi}$), relative income difference: $\frac{y_H}{y_M} = \alpha$.

Preferred tax rates. When agents vote for a certain tax rate τ, they form rational expectations about the disincentive effects of redistribution. Rationally anticipating subsequent labor supply decisions of all agents, an agent votes for the tax rate which maximizes her expected indirect utility.

For a non-working agent, $i \in L$, transfers are the only source of income. Since she does not work, expected indirect utility is

$$E_i u_i = \tau \cdot (1-\tau) \cdot \frac{1}{\phi} \cdot \left[s_M E_i \left(a_1\right)^2 + s_H E_i \left(a_2\right)^2 \right] \quad \forall i \in L. \tag{9}$$

The tax rate which maximizes expected indirect utility for this agent is independent of expected productivities and

$$\tau_i = \frac{1}{2} \quad \forall i \in L, \tag{10}$$

which is the Laffer-curve maximizer in this model. For agents in this group, the optimal rate of redistribution does not depend on the shape

of the income distribution. They are transfer receivers independent of the exact shape of the distribution.

In contrast to this, the skewness of the distribution matters for the preferred tax rate of agents in the middle class. Agents in this group receive their own net income as well as transfers and incur utility losses from working. Their expected indirect utility is

$$\begin{aligned} E_i u_i = &(1-\tau)^2 \frac{(a_1)^2}{\phi} + \tau \cdot (1-\tau) \cdot \frac{s_M (a_1)^2 + s_H E_i\alpha (a_1)^2}{\phi} \\ &- \frac{\phi}{2} \cdot \frac{(1-\tau)^2 (a_1)^2}{\phi^2} \quad \forall i \in M, \end{aligned} \tag{11}$$

which is maximized by

$$\tau_i = \max \left[\frac{1 - (s_M + s_H E_i\alpha)}{1 - 2(s_M + s_H E_i\alpha)}, 0 \right] \forall i \in M. \tag{12}$$

Equation (12) is the belief-vote mapping for middle-class agents. The term in the round brackets is the expected mean-to-median income ratio. All agents determine their vote according to their perceived position relative to mean income. For agents in the middle class, this coincides to the perceived mean-to-median income ratio. The preferred tax rate of a middle-class agent is a (weakly) upward sloping function of her expectation of productivity differences, $E_i\alpha$. When $E_i\alpha$ is relatively high, the agent believes that income differences between the upper and the middle class are pronounced and that she can gain much from taxing the members of the upper class. In the opposite case, when $E_i\alpha$ is relatively low, the agents believes to pay more taxes in order to finance transfers to the lower class. A middle-class agent i votes for positive redistribution only if the expected mean-to-median ratio is above 1, i.e. if she believes the income distribution is skewed to the right.[3]

Finally, members of the upper class can only loose from redistribu-

[3]The expression on the right hand side of equation (12) is also positive if the expected mean-to-median ratio is less than one half and would then describe a minimizer outside $[0, 1]$. However, this case is excluded by the assumed restrictions on group sizes.

tion. Their expected indirect utility,

$$E_i u_i = (1-\tau)^2 \frac{(a_2)^2}{\phi} + \tau \cdot (1-\tau) \cdot \frac{s_M E_i \left(\alpha^{-1/2} a_2\right)^2 + s_H (a_2)^2}{\phi} - \frac{\phi}{2} \cdot \frac{(1-\tau)^2 (a_2)^2}{\phi^2} \quad \forall i \in H\,, \tag{13}$$

is a strictly downward sloping function of the tax rate τ, since agents know for sure that $\alpha > 1$. Agents in group H therefore vote for

$$\tau_i = 0 \quad \forall i \in H. \tag{14}$$

Considering the expected indirect utility functions (9), (11), and (13), one can see that all have a unique maximizer on $[0, 1]$. Thus, preferences over τ are single-peaked for all agents. When determining election outcomes, the median-voter theorem is therefore applicable.

2.3 Belief formation and belief distribution

To determine the median voter, the distribution of votes has to be considered. Since agents in group M vote based on subjective beliefs, the distribution of beliefs needs to be determined first. This distribution arises as a result of agents' belief formation based on their individual signals.

The structure of the economy is common knowledge. However, before receiving the signal, agents only know that α is not less than 1. All values above 1 are equally likely from the perspective of agents. The signal α_i^S carries additional information about α. The signal α_i^s is the only relevant piece of information for agent i for two reasons. First, under condition (5), the knowledge that $\alpha > 1$ is redundant to the signal. Second, the agent does not observe other agents' signals and can not use them. Therefore, after receiving the signal, the agent's expectation of the productivity parameter α is identical to the signal,

$$\mu_i = E_i [\alpha] = \alpha_i^S. \tag{15}$$

Since there is a continuum of agents, each possible signal realization on $[\alpha - \varepsilon, \alpha + \varepsilon]$ is drawn by an equal mass of agents. According to (15), all agents build mean beliefs equal to their received signals. The

distribution of mean beliefs is thus identical to the distribution of signals and given by

$$g(\mu) = \begin{cases} \frac{1}{2\varepsilon}, & \alpha - \varepsilon \leq \mu \leq \alpha + \varepsilon \\ 0, & \text{else} \end{cases}, \tag{16}$$

where $g(\mu)$ denotes the density of a specific belief μ. The distribution of beliefs is the same across all groups and equal to the economy-wide distribution described by equation (16). Beliefs are unbiased, i.e. the economy-wide mean belief is true, $\bar{\mu} = \int \mu g(\mu) \, d\mu = \alpha$. Nevertheless, almost every agent misestimates α in one or the other direction.

2.4 Election outcomes

Since preferences over the tax rate τ are single peaked, the median vote is the unique Condorcet winner. The cumulative density F of votes and the determination of the median voter are illustrated in Figure 4. The distribution of votes in the economy is as follows: On the one hand, there is a mass of people at both extremes. Fraction s_H of agents (the upper class H) vote for zero redistribution, $F(0) = s_H$. Fraction s_L of agents (the lower class L) vote for the Laffer-curve maximizing tax rate, $F\left(\frac{1}{2}\right) = 1 - s_L$. On the other hand, there is non-degenerate distribution of votes between these two extremes. Since beliefs about α differ across agents, votes of agents in the M group differ from one another. Beliefs are unbiased within groups, therefore the optimal tax rate for the middle class, τ_M^{opt}, is always the median of the vote distribution *within the M group*.

Where the *economy-wide* median voter is located depends on relative group sizes. As neither group L nor group H contains at least 50% of the population, the median voter is surely a member of the M group. The economy-wide median voter, m, is the agent whose preferred tax rate τ_m is such that M-group voters of mass $\frac{1}{2} - s_H$ opt for less redistribution than herself since mass s_H of voters vote for zero redistribution anyway. Thus the median voter is at the lower $\frac{\frac{1}{2} - s_H}{s_M}$ quantile of the vote distribution within the M group. For agents in the middle class, τ_i is an upward sloping function of $E_i\alpha$, see equation (12). Therefore, the median voter is also at the lower $q_m = \frac{\frac{1}{2} - s_H}{s_M}$ quantile of subjective expectations of income differences α.

The distribution of subjective expectations is characterized by equa-

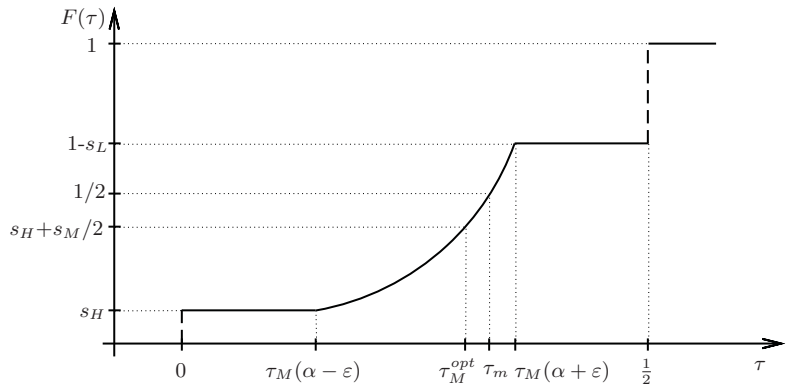

Figure 4: The distribution of votes and the election outcome.

tion (16). Since the distribution is uniform on $[\alpha-\varepsilon, \alpha+\varepsilon]$, the lower q_m quantile can be calculated as $(1-q_m)\cdot(\alpha-\varepsilon)+q_m\cdot(\alpha+\varepsilon)$. The median voter's expectation of α is thus

$$\mu_m = \alpha + \frac{s_L - s_H}{s_M}\varepsilon. \tag{17}$$

If and in which direction the median voter's belief differs from the truth, depends on the relative sizes of the upper and lower class.[4] Applying the belief-vote mapping of the middle class (12) to this belief, the preferred redistribution rate of the median voter and thus the implemented rate of redistribution is

$$\tau_m = \max\left[\frac{1-(s_M+s_H\mu_m)}{1-2\,(s_M+s_H\mu_m)}, 0\right]. \tag{18}$$

Positive redistribution occurs when

$$\alpha + \frac{s_L - s_H}{s_M}\varepsilon > \frac{s_L}{s_H} + 1, \tag{19}$$

i.e. when the median voter believes the income distribution to be right skewed and, equivalently, the mean-to-median income ratio to exceed 1.

[4]To obtain the expression for μ_m in equation (17), use that $s_L + s_M + s_H = 1$.

In this framework with imperfect information, implemented redistribution does not necessarily have to be optimal for the median-income receiver as it would have to be under perfect information. The implemented tax rate given by equation (18) coincides with the optimal tax rate for agents in the middle class only if the lower and the upper class are of equal size. Then, by coincidence, the median voter will be exactly in the center of the belief distribution of the middle class and have unbiased beliefs about the productivity distribution.

If upper and lower class are of unequal size, however, the median voter will be someone who misestimates the skewness of the productivity distribution and therefore votes for a potentially suboptimal redistribution rate. The larger of the two other groups forms a majority together with a minority of the middle class which misestimates productivity skewness. This majority can prevent any lower or higher tax rate even if it would improve the situation of some of its members (who, however, are not aware of this).

3 Changes in Income Skewness

Standard models of voting on redistribution predict that the extent of redistribution increases in the mean-to-median income ratio. The empirical evidence on this prediction is mixed (see Section 1). In this section, I analyze how changes in the mean-to-median income ratio affect election outcomes in this model.

Here, the mean-to-median income ratio can change due to two developments. Skewness can change because of rising polarization, i.e. by an income increase of the rich relative to the middle class (captured by an increase in the parameter α) or by upward mobility, i.e. by some agents catching up to richer ones (captured by an increase in s_M or s_H). While these two scenarios have similar impact on the skewness of the income distribution, their effect on the vote distribution differs. The reason is that while the first scenario simply moves earnings shares to the right of the distribution, the second also moves voting power.

3.1 Rising polarization

Consider first the case of rising polarization, i.e. an increase in the relative productivity of the upper class, and assume the parameter α rises from $\bar{\alpha}$ to α^{pol}, with $\alpha^{pol} > \bar{\alpha}$. Figure 5 illustrates the effects of ris-

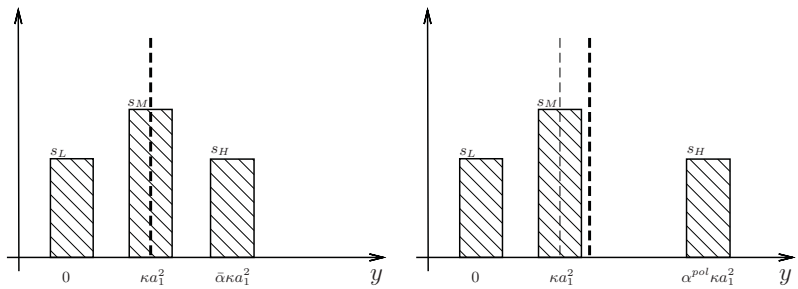

Figure 5: Income Distribution, mean (thick dashed line), and median (thin dashed line) income before and after polarization ($\alpha^{pol} > \bar{\alpha}$, $\kappa = \frac{1-\tau}{\phi}$).

ing polarization on income skewness. The left panel shows a symmetric distribution where mean and median income are identical. A symmetric distribution is chosen only for illustrational purposes in the figure, subsequent results do not require symmetry. The right part shows the income distribution after a rise in polarization. Due to income growth of the upper class, mean income (thick dashed line) has risen while median income (thin dashed line) has remained constant. The rise in polarization has thus led to an increased mean-to-median ratio.

The effects of this change on the implemented redistribution rate are illustrated graphically in Figure 6. The thin dashed line represents the initial vote distribution with $\alpha = \bar{\alpha}$ whereas the thick solid line stands for the new vote distribution associated with $\alpha = \alpha^{pol}$.

Since group sizes are constant, the median voter's quantile in the belief distribution remains unchanged. As the economy-wide mean belief shifts to α^{pol}, the median voter's belief, as given by equation (17), increases as well,

$$\mu_m^{pol} = \alpha^{pol} + \frac{s_L - s_H}{s_M}\varepsilon > \bar{\mu}_m. \tag{20}$$

According to the belief-vote mapping in the middle class (12), the im-

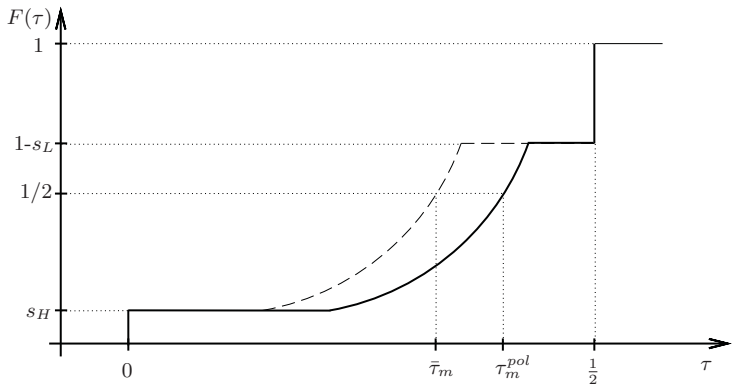

Figure 6: Vote distribution before (thin dashed line) and after (thick solid line) polarization ($\alpha^{pol} > \bar{\alpha}$).

plemented redistribution rate is now

$$\tau_m^{pol} = \max\left[\frac{1-\left(s_M + s_H\mu_m^{pol}\right)}{1-2\left(s_M + s_H\mu_m^{pol}\right)}, 0\right] \geq \bar{\tau}_m$$

and either larger or equal than with the lower difference in productivities, $\bar{\alpha}$.

The intuition behind this result is the following. Since the mean-to-median income ratio increases, the optimal rate of redistribution for the middle class rises. Group sizes are constant and thus the median voter misestimates income skewness by the same absolute deviation. The median voter's belief about the mean-to-median income ratio thus increases and so does her vote. Consequently, the winning tax rate rises.

Thus, the model predicts that, in reaction to an income increase for the rich, one observes indeed a positive correlation between the mean-to-median income ratio and redistribution. This prediction is equivalent to the standard model's one and corresponds to the Meltzer-Richard hypothesis.

3.2 Upward mobility

Effects are not as clear if the mean-to-median income ratio changes due to changes in relative group sizes. Consider a scenario where the upper class grows at the expense of the lower class (for simplicity with constant size of the middle class). Assume that group sizes change from $\bar{s}_L$, s_M, $\bar{s}_H$ to s_L^{um}, s_M, s_H^{um}, with $s_L^{um} < \bar{s}_L$ and $s_H^{um} > \bar{s}_H$. Figure 7 illustrates the effects of upward mobility on income skewness. Starting from a symmetric distribution (left panel), upward mobility increases mean income (thick dashed line) while median income (thin dashed line) is not affected since it lies still in group M. Thus the mean-to-median ratio is larger after upward mobility (right panel).

The consequences of this scenario on redistribution are illustrated in Figure 8. Again, the thin dashed line stands for the initial vote distribution and the thick solid line represents the vote distribution after upward mobility.

Since the compositional change affects the skewness of the income distribution, it alters the belief-vote mapping of the middle class. For a given belief about productivity differences, μ_i, the agent's expected mean-to-median income ratio, $s_M + s_H\mu_i$, increases with s_H and so does her vote. Agent $i \in M$ with belief μ_i now votes for

$$\tau_i^{um} = \max\left[\frac{1-(s_M + s_H^{um}\mu_i)}{1-2(s_M + s_H^{um}\mu_i)}, 0\right] \geq \bar{\tau}_i.$$

In the figure, this effect is manifested in the movement of the non-degenerate part of the distribution to the right. This increase in redistribution sought by the middle class does, however, not imply that the winning tax rate necessarily increases as well.

When the upper class increases in size, voting power shifts towards this group as well. In the figure, this is associated with an upward movement of the non-degenerate part of the distribution, since more mass lies at zero redistribution. As a consequence, the position of the economy-wide median voter within the belief distribution moves to the left. The median voter's belief about productivity differences, as given by equation (17), is now

$$\mu_m^{um} = \alpha + \frac{s_L^{um} - s_H^{um}}{s_M}\varepsilon < \bar{\mu}_m.$$

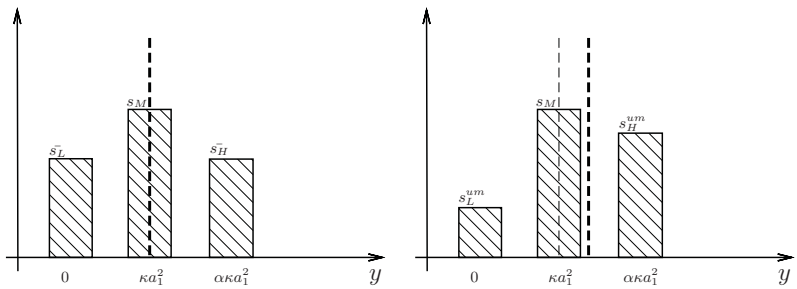

Figure 7: Income Distribution, mean (thick dashed line), and median (thin dashed line) income before and after upward mobility ($s_L^{um} < \bar{s}_L$, $s_H^{um} > \bar{s}_H$, $\kappa = \frac{1-\tau}{\phi}$).

These two developments result in an ambiguous effect on the implemented redistribution rate. While the increase in redistribution sought by the middle class tends to increase implemented redistribution, the shift in voting power has the opposite effect.

Which effect is dominant depends on the degree of informational imperfections as measured by the dispersion of the signal, ε. To determine a threshold for ε, it is useful to consider the median voter's expected mean-to-median income ratio, $s_M + s_H \cdot \mu_m$, which is positively linked to implemented redistribution, see equation (18). Using the median voter's belief μ_m from equation (17) and eliminating the size of the lower class, the median voter's expected mean-to-median income ratio is

$$E_m y / y_m = s_M + s_H \left(\alpha + \frac{1 - s_M - 2s_H}{s_M} \cdot \varepsilon \right). \tag{21}$$

The effects of upward mobility can be studied by considering the marginal derivatives of (21) to group sizes. When some agents move from group L to group H, this leads to decreasing redistribution if $\varepsilon > \alpha \cdot \frac{s_M}{s_M + 4s_H - 1}$. Another, potentially more realistic, case of upward mobility is a movement of agents from group L to group M. Such development decreases redistribution if

$$\varepsilon > \frac{(s_M)^2}{s_H \cdot (1 - 2s_H)}, \tag{22}$$

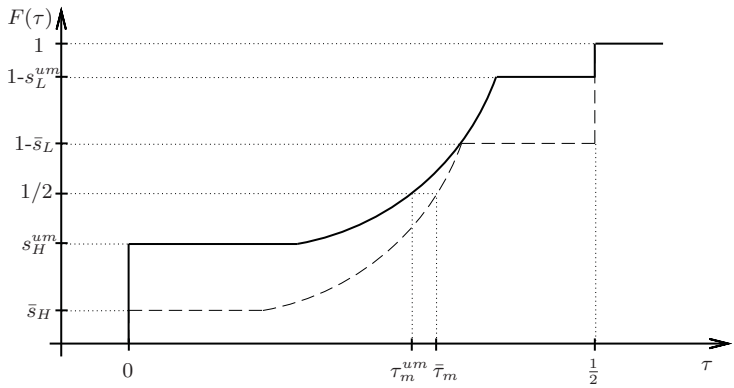

Figure 8: Vote distribution before (thin dashed line) and after (thick solid line) upward mobility ($s_L^{um} < \bar{s}_L$, $s_H^{um} > \bar{s}_H$).

i.e. if informational imperfections are pronounced enough.

In other words, upward mobility is more likely to cause decreasing redistribution the more people disagree in their beliefs about the income distribution in the economy. If condition (22) is fulfilled, an increase in s_M leads to the mean-to-median income ratio and redistribution moving into opposite directions. The Meltzer-Richard hypothesis is then turned upside-down.

How likely is it that condition (22) is fulfilled, i.e. that informational imperfections are strong enough for upward mobility to decrease redistribution? Suppose that group sizes are equal, i.e. $s_H = s_M = s_L = \frac{1}{3}$. Under this quantification, condition (22) gives a threshold value for ε of 1. To put this number into perspective, it is useful to calculate the variation coefficient of perceived average income for which an empirical counterpart is reported by Betts (1996). Doing this requires a quantification for α which I derive by matching the empirical US mean-to-median income ratio. In 2008, the mean-to-median income ratio among US households was 1.37.[5] With equal group sizes, this implies $\alpha = 3.12$ in this model, according to equation (8). Under the belief distribution (16), this quantification for α, ε, and group sizes gives a variation coefficient

[5]Source: U.S. Census Bureau, 2008 American Community Survey.

of perceived average income of $\sqrt{\int (E_i y - y)^2 \, di}/y \approx 14\%$. Since Betts (1996) finds that peoples' beliefs even about average wages in their own industry differ by a variation coefficient of roughly 30%, this magnitude does not seem implausibly high.

The model's prediction concerning the consequences of a rise in income skewness is in general not clear. If income skewness increases due to increased polarization, redistribution does unambiguously increase. If, however, an increase in income skewness is caused by upward mobility, redistribution may decrease. This ambiguity can be seen as a reason for why, in reality, one sometimes observes positive relationships between redistribution and income skewness and sometimes the opposite.

3.3 Empirical episodes

In this section, I consider some empirical evidence on developments in income distributions in the time around the year 1980 which form a major anomaly to the Meltzer-Richard hypothesis. The Reagan administration in the US and the government of Thatcher in the UK were massively reducing redistributive spending although mean-to-median ratios of pretax income distributions were steadily increasing (see e.g. Rodríguez 1999). The model evaluation above proposes to determine the drivers of the increasing income skewness in that time. I will focus on the late 1970s, the time preceding Reagan's and Thatcher's first elections into office (1980 and 1979, respectively). Major reductions in redistribution occurred shortly after, e.g. in the first Reagan tax cut (1981) and in Thatcher's first budget (1979).

Esteban, Gradín, and Ray (2007) study developments in certain measures of income polarization for different countries including the US and the UK. They e.g. report the average income of certain population groups such as the top 20% or the bottom 40% of the distribution relative to mean income. These measures are suitable to distinguish between drivers of increasing income skewness. Rising polarization as discussed in Section 3.1 would increase top relative to medium incomes. Opposed to this, upward mobility as discussed in Section 3.2 would lead to a rise in the lowest relative incomes.

For the time period of interest, the results of Esteban, Gradín, and Ray (2007) point toward upward mobility as the source of increasing income skewness. The relative income of the top 20% of the distribution

decreased from 1974 to 1979 while the relative income of the bottom 40% increased in both, the US and the UK.

A second source of useful information is the key figures of the Luxembourg Income Survey (LIS) which provide some summary statistics on the income distributions in the US and the UK for different waves of the survey. I will focus on two measures of the distribution, the 90/50 percentile ratio and the 50/10 percentile ratio, in the wave years 1974 and 1979.[6] The two developments discussed in this paper, rising polarization and upward mobility, affect the mean-to-median ratio in the same way but the two percentile ratios are affected differently. Rising polarization would increase the 90/50 ratio but would have no effect on the 50/10 ratio while upward mobility could decrease the 50/10 ratio but not affect the 90/50 ratio.

Also the LIS data suggests that rising polarization did not take place between 1974 and 1979 since the 90/50 ratio actually decreased between these two years in both, the US and the UK. Concerning upward mobility, the evidence is supportive. From 1974 to 1979, the 50/10 percentile income ratio indeed decreased in both countries.

This evidence suggests the view that increases in income skewness in the late 1970s have been caused by upward mobility rather than rising polarization. The succeeding cuts in redistribution can thus indeed be explained by the model presented in this paper.

4 Conclusion

Despite a sharp theoretical prediction, empirical evidence on the relationship between the mean-to-median income ratio and redistribution is mixed. Some empirical studies find a positive relationship, some studies find a negative one. Changes in income skewness are often accompanied by developments in redistribution into the opposite direction.

This paper has argued that it is important to distinguish between sources of changes in income skewness. In a model with imperfect information, rising polarization and upward mobility, though having the same effect on income skewness, affect redistribution in different ways.

I presented a model of direct democracy under imperfect information in which the relation between the mean-to-median income ratio

[6] Data source: Luxembourg Income Study (LIS) Key Figures, http://www.lisproject.org/keyfigures.htm, data downloaded on May 31, 2010.

and redistribution depends on the sources of changes in income skewness. While rising polarization generates a positive relation between income skewness and redistribution, upward mobility can have the opposite effect.

The mechanism leading to this non-standard result model works through the existence of extreme voter groups that can lead to a median voter with biased beliefs. Increases in income skewness lead to stronger redistribution sought by the middle class. However, if voting power is shifted to richer population groups, the position of the median voter moves towards voting for fewer redistribution. For informational imperfections pronounced enough, the second effect dominates. Then, the model generates a relationship between the mean-to-median income ratio and the extent of redistribution that would seem anomalous in light of standard voting models.

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Appendix

A Individual decisions and aggregate income

Individual labor supply. At this stage, an agent i choose consumption c_i and labor supply n_i to maximizes utility (1) subject to the budget constraint

$$c_i = (1-\tau) \cdot a_i \cdot n_i + \tau \cdot y, \tag{23}$$

which is a combination of (2) and (3). Denoting the Lagrange multiplier on the budget constraint as λ_i, the first-order conditions are

$$\begin{aligned} 1 - \lambda_1 &= 0 \\ -\phi n_i + \lambda_1 \cdot (1-\tau) \cdot a_i &= 0, \end{aligned}$$

respectively. Combining the two conditions gives optimal labor supply $n_i = \phi^{-1} \cdot (1-\tau) \cdot a_i$ as in equation (6).

Aggregate income. To determine aggregate income (7), individual labor-supply decisions (6) are aggregated in the following way:

$$\begin{aligned} y &= \int a_i n_i \, \mathrm{d}i \\ &= \phi^{-1} \cdot (1-\tau) \cdot \int (a_i)^2 \, \mathrm{d}i \\ &= \phi^{-1} \cdot (1-\tau) \cdot \left[s_L \cdot 0^2 + s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2 \right] \\ &= (1-\tau) \cdot \frac{1}{\phi} \cdot \left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2 \right] \end{aligned}$$

Voting decision. At this stage, an agent i chooses a tax rate τ_i to maximize utility (1) subject to the buget constraint (23), (6) and (7) capturing optimal subsequent behavior of all agents leading, $\tau = \tau_i$ capturing the sincerity of the voting decision, and $0 \leq \tau_i \leq 1$. Substituting the equality constraints into the problem, the Lagrangean reads as

$$\begin{aligned}\mathcal{L}_i = &(1-\tau_i)^2 \cdot (a_i)^2 \cdot \phi^{-1} + \tau_i \cdot (1-\tau_i) \cdot \phi^{-1} \cdot \left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2\right] \\ &-\frac{\phi}{2}\left(\phi^{-1}(1-\tau_i)\, a_i\right)^2 + \eta_i \cdot \tau_i + \nu_i \cdot [1-\tau_i],\end{aligned}$$

where η_i and ν_i are the Lagrange multipliers on the inequality constraints. The derivative with respect to τ_i is

$$\begin{aligned}\frac{\partial \mathcal{L}_i}{\partial \tau_i} = &-2 \cdot (1-\tau_i) \cdot (a_i)^2 \cdot \phi^{-1} \\ &+ (1-2\tau_i) \cdot \phi^{-1} \cdot \left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2\right] \\ &+ \phi^{-1} \cdot (1-\tau_i) \cdot (a_i)^2 + \eta_i - \nu_i.\end{aligned}$$

First note that the inequality constraint $\tau_i \leq 1$ is never binding. If it were binding, $\tau_i = 1$ and $\nu_i > 0$, then the derivative of the Lagrangean would evaluate as $\partial \mathcal{L}_i / \partial \tau_i = -\phi^{-1} \cdot \left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2\right] - \nu_i < 0$, i.e. this can not be an optimizer. Therefore it holds that $\nu_i = 0$ in the optimum.

The second inequality constraint, $\tau_i \geq 0$, can be binding. If it is binding, $\tau_i = 0$ and $\eta_i > 0$, then the derivative of the Lagrangean evaluates as $-2 \cdot (a_i)^2 \cdot \phi^{-1} + \phi^{-1} \cdot \left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2\right] + \phi^{-1} \cdot (a_i)^2 + \eta_i = \phi^{-1} \left\{\left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2\right] - (a_i)^2\right\} + \eta_i$. With $\eta_i > 0$, this expression can only be zero when

$$\left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2\right] < (a_i)^2. \tag{24}$$

Since $a_2 > a_1$ and $s_M, s_H \in (0, 1/2)$, this is fulfilled for agents in group H. Agents in this group therefore vote for zero redistribution, $\tau_i = 0$ as in equation (14).

Condition (24) is not fulfilled for agents in group L having zero productivity. These agents thus vote for positive redistribution. With $a_i = 0$ and $\nu_i = 0$, the derivative of the Lagrangean simplifies to

$$(1-2\tau_i) \cdot \phi^{-1} \cdot \left[s_M \cdot (a_1)^2 + s_H \cdot (a_2)^2\right],$$

which is zero for $\tau_i = 1/2$ which is therefore the optimal vote for agents in this group as stated in equation (10).

For agents in group M having productivity $a_1 = a_2/\sqrt{\alpha}$, condition (24) is fulfilled if and only if $s_M + s_H\alpha < 1$. If this is the case, agents in this group vote for zero redistribution, otherwise they vote for positive redistribution. Therefore the voting decision of agents in this group contains a case distinction. If $s_M + s_H\alpha \geq 1$, the inequality constraint $\tau_i \geq 0$ is not binding for agents in group M. Then, with $a_2 = \sqrt{\alpha}a_1$, the derivative of the Lagrangean simplifies to

$$(a_i)^2 \left\{(1-\tau_i)\cdot\phi^{-1} + (1-2\tau_i)\cdot\phi^{-1}\cdot[s_M + s_H\alpha]\right\},$$

which is zero for $\tau_i = \frac{1-(s_M+s_H E_i\alpha)}{1-2(s_M+s_H E_i\alpha)}$. Since this expression is negative when the inequality constraint $\tau_i \geq 0$ is binding, the optimal vote for agents in group M can be expressed by $\tau_i = \max\left[\frac{1-(s_M+s_H E_i\alpha)}{1-2(s_M+s_H E_i\alpha)}, 0\right]$ as stated in equation (12).