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Research Paper No. 2008/62

Entrepreneurship and Structural Economic Transformation

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Abstract

A stylized fact of economic development is the structural transformation of countries from traditional, mainly agricultural societies to modern economies dominated by manufacturing and services. In this paper we provide an endogenous growth model to illuminate the role of entrepreneurial start-up firms in structural economic transformation. We follow the Lewis-model’s distinction between a traditional and modern sector, and underpin this with micro-foundations. We specify mature and start-up entrepreneurs and make a distinction between survivalist self-employment activities in the traditional sector, and opportunity-driven entrepreneurship in the modern sector. The model shows how opportunity-driven entrepreneurship can drive structural transformation through innovation, provision of intermediate inputs and services (which permits greater specialization in manufacturing), and by increasing employment and productivity in both the modern and traditional sectors.

Keywords: entrepreneurship, economic development, structural transformation, start-ups

JEL classification: M13, L26, O10, O14
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1 Introduction

Economic development entails changes to the quantity and quality, including the composition, of economic value added. It is generally characterized by economic growth, rising per capita incomes and a shift in the composition of value added and employment, first from agriculture to manufacturing, and finally to an economy dominated by the services sector. This dimension of the broader structural transformation that economies have been undergoing most dramatically since the Industrial Revolution is well known as a ‘stylized fact’ of economic development1 (Chenery 1960; Kuznets 1966; Syrquin 1988). A large research literature, both empirical as well as theoretical, has been devoted to describe measure and explain this pattern of structural change and its relation to economic growth. The earliest (classical) literature, in the immediate aftermath of the industrial revolution were much concerned to explain the ‘take-off’ in growth and development from stagnant agrarian societies, and herein following the contributions of Smith, Malthus and Ricardo, factor accumulation and productivity increases where accorded central place (e.g. Rostow 1960). It was much later, following the Second World War and the independence of most of the former colonies, that concerns with structural economic change and its relationship with economic growth more prominently came to the fore. One of the earliest contributions in this regard was the development of ‘dual economy’ models, first set out by Lewis (1954)2 and later by Ranis and Fei (1961).

The Lewis model, generally regarded as one of the most important contributions in the establishment of development economics (Kirkpatrick and Barrientos 2004: 2), described the duality in a typical developing economy between a traditional (agricultural or ‘stagnating’) sector and a modern (manufacturing or progressive) sector. Due to capital accumulation, technological progress and higher productivity in the modern (progressive) sector, surplus labour moves from the traditional sector to the modern sector, where they are paid according to their marginal productivity, and capitalist can earn a surplus which are reinvested in further productivity enhancing capital. This process will continue until all surplus labour has been transferred to the modern sector, and labour across the economy is paid according to their marginal productivity, with ‘duality’ in labour markets thus disappearing. At this stage most labour would be employed in the modern sector, which in the experience of the first industrializing countries in the West and Japan was the highly productive manufacturing sector.

Subsequent development in the industrialized countries have however shown that even if an economy pass the duality stage into so-called modern economic growth, a further phase of structural economic change would entail growth in the share of employment in the service sector (Bonatti and Felice 2007). As these changes, from manufacturing

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1 The structural transformation of economies since the industrial revolution has been described as ‘immense’, accompanied by ‘profound social changes’ (Bortis 2000: 185).

2 Other notable contributions in the early development economics literature were concerned with the relationship between various sectors of the economy and the implications thereof for growth take-off, for instance Rosenstein-Rodan (1943) argued for a ‘big push’ in all sectors to overcome co-ordination failures in demand between different sectors.
dominance to service sector dominance, mostly took place in advanced economies during the second half of the twentieth century, there was little analysis thereof in the development economics literature, and the Lewis model and its extensions are silent about structural changes in the modern sector once the essential features of duality had disappeared. Outside of development economics however, a substantial literature emerged since the late 1980s to explain this rise in service sector dominance in advanced economies. Three main strands in this literature attempt to explain the rise in the share of employment in the service sector (see Schettkat and Yocarini 2006). A first sees it as due to differences in productivity growth, with higher productivity growth in manufacturing and similar wage growth leading to prices and employment in services increasing (Baumol 1967; Baumol et al. 1985; Nordhaus 2006; Bonatti and Felice 2007). A second strand sees it as due to differences in the inter-industry division of labour. A third strand sees it as due to non-homothetic consumer preferences (the income elasticity of demand for services exceeds one and is less than one for manufacturing) which leads to changes in the composition of final demand as incomes rise (Echevarria 1997; Laitner 2000; Bonatti and Felice 2007). More recently Lopez et al. (2007: 318) modeled structural change as an endogenous response to resource constraints where productivity in the non-resource (modern) sector rises due to the accumulation of knowledge.

At roughly the same time that the Lewis-model and extensions were put forward to explain structural economic change in developing countries, neoclassical growth theory expanded following the contribution of Solow (1956) and others. In these models, where the emphasis was on the dynamics of steady-state growth and on convergence in per capita incomes between countries, there was no concern, nor any possibility in the steady-state framework, to focus on issues of structural change, despite the growing recognition that structure and growth are interdependent. In more recent times, the empirical inability of the Solow-model to explain patterns of productivity, capital accumulation and growth lead to endogenous growth theories, wherein human capital, and technological changes, play an important role in growth dynamics. This opened an important but relatively unexplored link between structural change and growth because the extent to which economic sectors differ in their human capital and technological requirements, and are differently affected by new technologies, will affect growth (Landesmann and Stehrer 2006). It also opened, more significantly for the purposes of this paper, a potential link between entrepreneurship and structural transformation, in that ‘entrepreneurial ability’ has come to be increasingly seen in economics as a vital form of human capital.

The development economics literature in general, and the literature on structural economic transformation inspired by the Lewis-model and its extensions specifically, has been largely silent on the role of entrepreneurship, or entrepreneurial ability in

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3 Bonatti and Felice (2007) discuss the fact that growth theory generally models growth as taking place against a backdrop of a constant economic structure, of a ‘balanced growth path’.

4 For purposes of this paper we will define entrepreneurship as the ‘process of starting and continuing to expand new businesses’ (Hart 2003: 5). We abstract from destructive or unproductive allocation of entrepreneurial ability (as in Baumol 1990). Entrepreneurs are thus make productive contributions to the economy as starters of new businesses. As we will illustrate, these entrepreneurs in our model innovate (as in Schumpeter 1961), they spot profitable opportunities (as in Kirzner 1973), and they reallocate resources (as in Schultz 1975).
economic development. We consider this to be a shortcoming for a number of reasons. First, from a theoretical point of view it is unsatisfactory to omit a potential important mechanism of economic dynamics. For instance, Lewis (1954) posited ‘capitalists’ who earns profits and save, akin to an ‘entrepreneur’ but fails to provide these capitalist with more micro-foundations. How for instance do economic agents choose between wage employment and being an entrepreneur? How do entrepreneurs overcome start-up obstacles, such as problems related to access to finance? Second, there is a theoretical disjunction, or scholarly disconnection, between a substantial literature on entrepreneurship and small business development, and the development economics literature. This is despite the valuable early contributions made by Schumpeter in stressing the role of the entrepreneur in innovation—a key activity in facilitating structural economic change. Also, in the entrepreneurship literature concerned with small firms there is, as we will show below, an increasing recognition that there is a strong empirical regularity between a country’s level of development and entrepreneurship. Indeed small businesses are widely seen to play an important, if sometimes disputed, role in economic development (Liedholm and Mead 1999). It has become a ‘stylized fact’ in the recent entrepreneurship and small business literature that start-up rates and rates of self-employment will decline as a country develops (Gollin 2008: 220). It is theoretically challenging to integrate this stylized fact into a model that is consistent with the stylized fact of structural economic change as explained for instance in the Lewis-model. Thus, is it entrepreneurs that drive structural changes, or vice versa? Third, as development economics is concerned with both economic growth and structural change, the interdependence between the two is of importance. Consequently striving for consistency between insights from Schumpeterian and endogenous growth theory, wherein entrepreneurial ability can be highlighted, and dual economy models, wherein structural change is endogenous, is an obvious though neglected research agenda. Fourth, the relative neglect in development economics of formal modeling of entrepreneurship, particular in structural economic change is a shortcoming as it contrast with the empirical evidence linking entrepreneurship with economic growth, and the almost universal adoption by governments and development agencies of policies to stimulate entrepreneurship as a way to further structural economic development and growth (Audretsch et al. 2007: 1).

This brings us to this paper, whose broad objective is to make a modest contribution to extend the formal modeling of entrepreneurship in development economics. In this regard it adds to recent contributions such as by Lazonick (2008), Naudé (2008), Gries and Naudé (2008), Naudé et al. (2008), Naudé (2007), Dias and McDermott (2006) and Nelson and Pack (1999). The more specific objective of this paper is to provide an

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5 This follows in the tradition of the classical economists (who like development economist in the twentieth century were most concerned about fundamental development issues) who with the exception of Cantillon (1755) generally omitted entrepreneurship from their analyses of economic development. According to Lewis (1988: 35) ‘Adam Smith detested business men’. Consider further that prominent development economics textbooks such as the four-volume Handbook of Development Economics and the Leading Issues in Development Economics do not contain a single chapter or any substantial section on entrepreneurship. Audretsch et al. (2007: 1-2) describes a ‘scholarly disconnection’ stating that ‘macroeconomics, has largely not considered the role that entrepreneurship plays in economic growth an employment’ and that ‘management—the academic discipline most squarely focused on entrepreneurship—has typically not considered the implications for the broader economic context’.

6 According to Murphy et al. (2006: 12) it was the ‘advent of entrepreneurship’ that allowed per capita income to grow exponentially in the West from the 1700s.
endogenous growth model of structural economic change that includes entrepreneurship, in the form of entrepreneurial start-ups, as the driver of both growth and structural change. This model, although it stands in the traditional of endogenous steady-state growth models, follows the basic ideas of the Lewis (1954) model in that it distinguishes between a traditional (or stagnant) sector and a modern (progressive) sector and allows for surplus labour to be transferred to start-up firms in the modern sector (see Section 2.2). We provide micro-foundations to the labour market outcomes in the model using a labour market matching framework, and explicitly model the financial sector in the start-up process. A novel element, in the context of occupational choice models of entrepreneurship in this model is that entrepreneurial ability is ‘matched’ in the modern sector to opportunities for starting up a new small firm. A further novel element of our model is the integration of endogenous growth and structural change models—specifically so by introducing entrepreneurship in our model—and of taking jointly into account growth rates, income levels and economic structure. Our model is thus a contribution to bridge the ‘scholar disconnect’ between the development economics and entrepreneurship literatures that were noted. We use this model to explain a number of the ‘stylized facts’ of economic development that were mentioned above. Thus our model shows how the economy can progress from a traditional-based economy to a modern economy based on manufacturing and eventually services, it explains how productivity can be lower in services than in manufacturing (an explanation for the recent rise in service sector employment in advanced economies), and it shows how structural changes, brought about by entrepreneurs, can affect the growth rate. We discuss some policy implications of the model, as well as some avenues for future extensions and elaborations of the model.

The paper is structured as follows. The next section provides an overview of the salient features of dual economy models and their relationship to entrepreneurship, as background. Section 3 describes our model, and illustrates its implications for various steady-state outcomes. Section 4 concludes with a discussion of the policy implications, shortcomings, and possible further extensions of the model.

2 The dual economy and entrepreneurship

Ranis (1988: 74) describes the concept of ‘dualism’ as it emerged from the Lewis model (1954) as the ‘co-existence of two sectors which are basically asymmetrical’. In the Lewis model and subsequent dual economy models (e.g. Ranis and Fei 1961; Ranis 1988) these two sectors are a traditional (often described as the agricultural, informal, or subsistence) sector and a modern (based on services and manufacturing) sector. Labour flow from the traditional sector to the modern sector due to the latter’s higher productivity (and wages)—underpinned by the Lewis assumption that labour’s marginal productivity in the traditional sector is zero—means that labour is in fact ‘surplus’. If the traditional sector is equated with the agricultural/rural sector, then this outflow results in the contribution of agriculture to total GDP to decline, the share of the population living in urban areas to increase, and average per capita incomes to rise.

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7 The construction of ‘simple basic models’ to study structural transformation of complex systems is according to Domingo and Tonella (2000: 211) an ‘important approach’ to understand the salient features of the observed structural changes. Our model therefore follows in a tradition of basic models which reduces to complexity of structural change to a few basic features. As such it will abstract from many features of real economies and does not aim to explain all features of structural economic change.
Growth is due to higher productivity in the modern sector and an increase in aggregate demand due to higher wages that are paid in the modern sector (Rada 2007: 713). This transformation can also be consistent with an inverse-U relationship between per capita income and income inequality, as observed by Kuznets (1955). A number of extensions were made to address some of the simplifying assumptions of the Lewis model—such as the assumption of a closed economy (which we will follow for now in our model). Another relates to the requirement of ‘balanced growth’ implied by the model. Basically, as labour move out of agriculture, agricultural production might decline, food prices may rise, wages in the traditional sector and non-traditional sectors may start to increase, leading to reduced profits for investment and reduction in labour transfer to the modern sector, which may lead to a premature halt to the transformation process (see e.g. Dixit 1969).

Where is the entrepreneur in these dual economy models? Although Lewis did not use the term ‘entrepreneur’, he very much had the entrepreneur in mind in the modern sector agents described as ‘capitalists’. These agents, to be contrasted from workers who get paid a wage according to their marginal productivity, share in the surplus production in the modern sector. Moreover, in the Lewis model the assumption is that these capitalists (entrepreneurs) have much higher savings rates than workers, and that they reinvest their surpluses in expanding the modern sector. For Lewis (1954: 155) a rise in total savings in an economy is a prerequisite for economic development (we retain this feature in our model by explicitly modelling financial intermediation and savings according to the Ramsey rule). A weakness in dual economy models is that although there are ‘capitalists’, it is assumed that they are exogenously given as a constant proportion of the population. In our model described in Section 3 below, we model entrepreneurial start-up firms symmetrical to the role of capital in the Lewis-model; instead of capitalists in the Lewis model investing in capital accumulation from their own savings, we introduce a financial sector that intermediates access to savings of households by entrepreneurs who wish to establish a start-up firm in the modern sector. The increasing number of firms in the modern sector results in migration from the traditional to the modern sector. Whereas the Lewis-model has decreasing returns to investment in capital, in our model returns to ‘investment’ in start-up firms result in constant returns.

The popularity of dual economy models is due to their ability to explain in a simple manner the broad stylized facts of structural change. Given the discussion in Section 1, and the above remarks, the lack of formal modelling of an entrepreneur within models of structural economic change remains a shortcoming. Only a very few previous attempts have been made to address this shortcoming. Notable contributions in this regard include Nelson and Pack (1999), Ciccone and Matsuyama (1996), and Dias and McDermott (2006).

Nelson and Pack (1999) use a dual economy model to explain the structural transformation of economies such as Korea and Taiwan from being characterized by a

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8 Quadrini (2000) finds that entrepreneurs have much higher savings rates than workers. Henrekson (2007: 733) argues that the impact of savings on the start-up rate may depend on the form that savings take. If saving schemes restrict the owners’ control of their savings it may be of limited use as funds or collateral to start up a firm. We abstract from such concerns in our model, and assume that all lending takes place from banks which act as financial intermediaries between households/entrepreneurs which save, and entrepreneurs who need to borrow to start up a new firm.
‘craft’ sector to a ‘modern’ economy. They assign a key role to the ‘effectiveness of entrepreneurship’ (or entrepreneurial ability, which they see as a vital determinant of the rate of assimilation of technology (ibid.: 420). They stress the imitative role of entrepreneurship as well as its role in taking on uncertainty, given that the adoption of (mostly) foreign technology by entrepreneurs in these countries entails significant risk-taking (ibid.: 418). Entrepreneurs ‘trigger’ an investment in the modern sector once they have perceived profitable opportunities and facilitate the re-allocation of production factors from the traditional to the modern sector. Since the modern sector requires a higher level of skilled labour, entrepreneurs cause an increase in the demand for educated labour. This leads to an overall improvement in human capital in a country, in turn facilitating the imitation and adoption of foreign technology. Their model implies that a ‘rapid’ expansion of skilled labour can only be absorbed if entrepreneurial ability is high, and that without entrepreneurial ability the returns to physical and human capital is low (Nelson and Pack 1999: 423).

Dias and McDermott (2006) combine a dual economy model with an occupational choice model wherein people are born either as workers, or as managers. Workers can choose to work in the traditional sector, or they can migrate to the modern sector. For the latter a minimum level of human capital is needed. Managers, all of whom are in the modern sector, can choose between being productive entrepreneurs or to be rent-seekers. For the former they would need to cover start-up costs, and pay taxes. Their model shows that the better entrepreneurial ability are, the more workers will migrate to the modern sector, and the higher will be the overall levels of human capital accumulation in the economy. They support their model’s implications with panel data from Brazilian states.

The process of change involving the composition of goods produced in an economy has interesting implications for the development of entrepreneurship itself, so that entrepreneurship may be itself endogenous in the development process. Ciccone and Matsuyama (1996) explains this in a model where they make a distinction between consumer goods and intermediate goods. If a particular economy produces a limited range of intermediate goods, they show that the final (consumer) goods sector will use ‘primitive’ production methods and will have little demand for sophisticated, new inputs. This will lead to lower incentives for potential entrepreneurs to start-up new firms (ibid.: 34). The economy can get stuck in such an underdevelopment trap with primitive production in its (small) modern sector. They also point out that there might, in such an ‘underdevelopment trap’ be a case for assistance to new start-ups since these can provide both pecuniary and technological externalities if they start producing new intermediate goods—which will induce final good producers to demand more of these (in turn improving the incentives for other entrepreneurs to start-up firms due to greater demand and the example provided in the application new technology). In this model, start-ups face positive start-up costs that include R&D activities in bringing a new good to the market.

As we will show in the next section, our model contains most of these essential features of these few previous contributions towards integrating the entrepreneur into models of structural change. Thus, as in Nelson and Pack (1999) and Dias and McDermott (2006) we consider entrepreneurial ability as an important dimension of human capital, and we provide a novel approach whereby entrepreneurial abilities are ‘matched’ to profitable opportunities to start-up firms in the modern sector. Also, we model our start-ups to
provide intermediate goods and services to final-good producing firms in the modern sector, and moreover, that these start-ups innovate to provide unique intermediate goods and services, as in Ciccone and Matsuyama (1996). As in the latter we also have our start-ups face positive start-up costs that include amongst others R&D activities.

3  A Lewis-type model with entrepreneurship and finance

Having described the salient features of the Lewis-model in the previous section, in this section we set out an endogenous growth model based on these features. We first provide an intuitive explanation of the model in Section 3.1. Thereafter, in Sections 3.2 to 3.4 describe the model and illustrate its properties.

3.1 Intuitive explanation

In essence we extend the Lewis-model by an endogenously developing entrepreneurial start-up sector and a financial sector. As this model is an endogenous growth model we introduce some more sophisticated dynamics in the traditional sector as well. In particular, while the Lewis labour market is fully elastic with surplus labour supplying any labour needed to the modern sector the traditional sector in this approach is closed by introducing a richer labour market approach which may be more appropriate for the kind of problems considered in this paper, and an endogenous population dynamics.

The model mechanics is quite simple. In our model we assume a closed economy setting so as to abstract for the moment from international conditions which may affect a country’s economic structure. These conditions, which may include international technology spillovers and sources of demand, are important for structural transformation (see e.g. Lucas 2000); however we leave this as an avenue for future research (see also Section 4 below). In this closed economy there is, as in the Lewis-model, a modern sector and a traditional sector. The modern sector is host to final-good producing large firms. They use, through outsourcing services and purchasing goods for intermediate inputs, the services of small entrepreneurial firms, with each start-up firm providing a unique good or service.

The emphasis in the model is on the start-up of these firms as growth catalyst. Focusing on small firms is perhaps a good description of the initial stages of structural economic change, as this reflects the reality that small firms tend to dominate in developing countries (Gollin 2008: 219). Our small firm sector, by providing a unique intermediate good or service (i.e. they innovate), is consistent with Audretsch and Thurik’s (2000) view on the role of small firms in the entrepreneurial economy, wherein the establishment of a large number of small firms is argued to be good for economic growth by encouraging innovation. Ciccone and Matsuyama (1996) offer a model wherein new start-ups firm provide both pecuniary and technological externalities if they start producing new intermediate goods or services (as opposed to final goods). This they argue will induce final good producers to demand more of these, which in turn improves the incentives for other entrepreneurs to start-up their own firms.

Other research indeed suggests that start-up firms are the ones most likely to grow (Lingelbach et al. 2005; Johnson et al. 2000) and to create new jobs (Audretsch et al. 2006: 25; McMillan and Woodruff 2002: 166). In many transition countries, where there was no significant private sector to start out with, new firms often strengthened
reforms by improving economic conditions, as for instance in China (McMillan and Woodruff 2002: 153). New firms can be important in a transition context since they are ‘less encumbered with the historic influences of such a society’, as opposed to existing firms that may be undergoing reform, and that some form of private sector development could be a condition for successful privatization of inefficient state-owned firms (Estrin et al. 2006: 693).

Although we do not explicitly model innovation and technological change by small firms in the modern sector of our economy, the fact that each small start-up in essence innovate to bring a unique good or service to market imply that higher overall rates of innovation leads economic growth and structural change in our model.

Only the owners (which we term ‘mature’ entrepreneurs to contrast them with start-up entrepreneurs) of the final-good producing firms will save and accumulate according to an intertemporal optimal decision. As deposits are the only available asset in our model, savings are channelled through imperfect financial intermediaries to finance the only investment projects, which are the start-up firms. We show that the start-up rate will therefore be determined by the return on new firms, the savings decision and the efficiency of the financial market. Also, because (as we will model it) the start-up process is a matching process of entrepreneurial abilities (or business ideas) with opportunities given by market conditions, the risk of failure, entrepreneurial ability, as well as the opportunity costs of bringing a new firm to the market will be important for the growth of entrepreneurial start-ups sector. As each start-up is run by one agent the growth of the modern entrepreneurial start-up sector is absorbing people from other sectors, namely the traditional sector.

With entrepreneurs spotting opportunities for providing final-good manufacturing firms in the modern sector with inputs, including services, the overall incidence of outsourcing in our economy would increase. One result would be to raise the share of the services sector in employment and output, and to the extent that less productive services are outsourced, it would result in a less productive service sector. Both of these results are consistent with the stylized facts of structural economic change (see Section 1). It would also result in the increasing size of the service sector being accompanied by an increase in the contribution of small firms to the economy. This also seems to be consistent with empirical regularities. According to Audretsch and Thurik (2000: 30) small firms have become more important in the advanced economies in recent decades stating that ‘A series of empirical studies has identified that a pervasive shift in the industrial structure away from large corporations and towards small enterprises has taken place between the mid 1970s and early 1990s. This shift occurred

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9 Although start-up firms in our model only supply services, as intermediate inputs to firms producing for final demand, and not to final consumer demand itself, changes in final consumer demand in favour of more service and service-intensive goods as income rises (due to non-homothetic consumer preferences) will be consistent with more opportunities in the modern sector for providing service inputs. We leave it to future extensions and modifications of our model to explore the implications of changes in final demand on start-up rates and structural economic change.

10 Baumol (1967) growth asymptotically approaches zero as a result of growth in services, which are less productive than manufacturing, in our model this does not happen. This does not imply that our results are inconsistent with that of Baumol (1967), since Oulton (2001) has shown that when services are provided as intermediate inputs Baumol’s result will not hold.
not just in one or a few of the developed countries but rather in virtually every single leading industrial country'.

From a long-run development point of view, our entrepreneurial start-ups thus contribute to modern structural economic change by increasing the specialization of manufacturing firms by allowing them to outsource intermediate input supply. Consequently in countries with higher levels of entrepreneurial ability and less constraints on start-ups, there should be more opportunity-driven start-ups and more specialized manufacturing firms. This could perhaps explain why the USA, with a higher start-up rate than in the EU, manufacturing firms are more specialized and tend to rely more on outsourcing than manufacturing firms in the EU (Schettkat and Yocarini 2006: 133). In comparison to the Lewis-model, start-up firms in our model function in broad terms as capital does in Lewis-model. However, in contrast to decreasing returns to capital in the Lewis-model, our new start-up firms face constant returns. This together with savings which in our model are determined by the Ramsey Rule, allow us to switch to an endogenous growth setting.\textsuperscript{11}

As in the Lewis-model, the modern sector in our model absorbs labour from the traditional sector. Also, just as in the Lewis-model the traditional sector here is characterized by surplus labour. The traditional sector can be associated with a rural agriculture sector. Even if each unit of employed labour (wage employment or self employment) has a given constant productivity, labour profiles and employment opportunities will lead to large fluctuations and frictions in the search and matching process in this labour market segment. As a result the surplus labour rate is determined by a lack of easy-to-find employment opportunities. While the modern sector is endogenously growing through new start-ups, the traditional sector is growing by an endogenous population growth. If the growth rate of the modern sector, via entrepreneurial start-up growth, exceeds the growth rate of the traditional sector, we encounter structural transformation to a modern economy. If, however, growth in the start-up rate is too slow, then we will encounter a stagnating rural economy. We begin to describe this model in the next sub-section by detailing the modern sector, wherein the start-up activities of entrepreneurs are at the heart of the model.

3.2 The modern sector

Final good production

In our modern sector there are, at any given point in time, a number $N$ of small entrepreneurial firms. Each produce a specific and differentiated good or service as intermediate inputs for large firms in the final output sector which produce aggregate final output $Y_M$. These large final-good producing firms are owned by mature entrepreneurs. Mature entrepreneurs produce with their entrepreneurial and organizational human capital $H$ and $N$ intermediate inputs $x_j$ outsourced to the $N$ small supplying firms. Because we place the emphasis on start-ups and obstacles to their growth, the final good industry of the modern sector is modelled rather simply.

\textsuperscript{11} In fact the formal endogenous growth setting is a closed ‘Romer model’. A frequently discussed problem of using endogenous growth models to study transformation is their steady state assumption. In our model we show that this need not be a constraint.
Specifically, we propose a continuum of final good producing firms supplying to a competitive final goods market. The production function\textsuperscript{12} for the representative final product producing firm\textsuperscript{13} can then be written as
\[
Y_M = AH^{1-\alpha} \sum_{j=1}^{N} (x_j)^{\alpha} = AH^{1-\alpha} N x^{\alpha}
\]
(1)

In equation (1) \(A\) is a scaling and efficiency parameter. Mature entrepreneurs producing the final good maximize profits according to the profit function
\[
\pi_j = Y_M - w_H H - N p_j x_j
\]
with \(p_j\) denoting the price of intermediate service \(x_j\) and \(w_H\) denoting the income compensation for the entrepreneurial and organizational abilities of the mature entrepreneur. In this model the mature entrepreneur is an organizer of production processes, more a manager rather than a risk taker or innovator. For simplicity we also assume the market for entrepreneurial human capital to be competitive. Using the first order conditions we can derive\textsuperscript{14} the demand for each intermediate (service) input, namely
\[
x_j = H \left( \frac{A \alpha}{p_j} \right)^{\frac{1}{\alpha - 1}}
\]
(2)

The assumption of perfect competition in the final goods market or in the market for entrepreneurial abilities is just a reference system. We leave it for future work to analyse the effects of different market structures on sectoral transition and the start-up rates of entrepreneurial firms.

\textit{Households in the modern sector}

Only households connected to modern final goods production will be able to make explicit intertemporal decisions about savings and investments. Households in the traditional sector and new start-up firms in the modern sector are not able to save. The representative household in the modern sector owns the modern sector firm and receives rental income from entrepreneurial activities \(w_H H\) and accumulated wealth. As for a household deposits are the only capital asset the aggregate capital income flow consists of interest income from deposits \(D r_d\). Hence the budget constraint is given by
\[
Dr_d + w_H = C + S = C + \dot{D}
\]
Total income can be consumed or saved in terms of deposits.

The intertemporal household decision problem is standard. The representative household maximizes a utility function with constant relative risk aversion. The objective function is
\[
\max_{C(t)} U(C(t)) = \int_{0}^{\infty} U(C(t)) e^{-\rho t} dt
\]
with \(\rho\) the rate of time preference. We assume a constant intertemporal elasticity of substitution (CIES) utility function, i.e., \(u'(c) > 0\), \(u''(c) < 0\), with \(\Theta = -u''(c)c/u'(c)\)

\textsuperscript{12} This specification of a production function originates from Ethier (1982). Similarly, Romer (1987) and (1990) used this specification to model technological change and growth, driven by newly invented variations of productive inputs.

\textsuperscript{13} Growth is driven by an expansion in \(N\), denoting the number of small firms in the market and hence the number of different intermediate goods available.

\textsuperscript{14} See Appendix 1.
denoting the constant relative risk aversion or the reciprocal of the intertemporal
elasticity of substitution. Optimization results in the Ramsey rule
\[ \gamma_C = \frac{r_d - \rho}{\Theta} \]  
(3)
where \( \gamma_C \) is the growth rate of consumption. In what follows \( \gamma \) generally denotes a
growth rate of the variable indicated by the subscript. As the rate of start-ups determines
income growth in the production process, the household can achieve the desired growth
rate by financing start-ups in the required way. Therefore
\[ \gamma_N = \gamma_C \]  
(4)

**Start-up firms**

We see entrepreneurs as individuals who recognize new opportunities, similar to
descriptions of the entrepreneur by Kirzner (1973) and Schultz (1975). In the present
case, they may recognize opportunities in the modern sector to produce new variants of
services or intermediate inputs to large final-goods producing firms. Each product or
service variation has certain properties that make the variation unique compared to other
already existing variations. This is consistent with the view that ‘entrepreneurship can
be seen as a continual quest for economic rents’ (Henrekson 2007: 19).

**Start-up ideas and matching of business opportunities.** There exists in the modern sector
opportunities for successful firm start-ups. Potential entrepreneurs need to be able to
perceive these opportunities, and be willing to try and exploit them. This depends on
their entrepreneurial ability. The start-up entrepreneur is the visionary with an idea how
to define a product or service to take advantage of business opportunities in the market.
With these start-up product profiles a new start-up firm may match the requirements and
conditions in the modern sector market. A useful approach to model this situation is the
matching approach. With the matching approach we can address the problem of
constantly evolving start-up opportunities, a high exit rate of new start-ups, and
heterogenous business ideas (which makes for innovation). In this approach activities
are described by a failure of present activities, search activities for new opportunities,
and the matching process leading to new firm start-ups. The match between start-up
profiles (reflecting entrepreneurial ability) and the requirements given by the market
determine a start-up.

**Contract separation, business ideas and opportunities.** For the total number of start-up
firms that exist at any given moment \( N \), the continuous adjustment of entrepreneurial
ability/perceptions and opportunities lead to a separation of outsourcing contracts to
new start-up firms at the rate \( \vartheta \). Even if there is a potential demand for other varieties
of intermediate services the fraction \( \vartheta \) of recent start-up firms will not survive due to a
wrong match between ability and opportunity. The rate of separating contracts is also
assumed to be determined by competitive business ideas offered. Defining \( \Delta_N \) as the
number of offered but yet not realized business ideas and \( \delta_N = \Delta_N / N \) as the surplus
rate of business ideas, final good producing firms will more likely separate (end)

15 See Appendix 2.

16 Cressy (2006: 103) notes the high failure rate of most new firms citing the finding that up to 50 per
cent of new firms exit the market after 18 to 24 months.
outsourcing contracts with existing start-up firms if the surplus of alternative business ideas is high. Hence separation of contracts is a positive function of the surplus rate of business ideas

\[ \vartheta = \vartheta(\delta) = \delta^\zeta_N, \quad \frac{d\vartheta}{d\delta} = \zeta \delta^{\zeta - 1} > 0 \]  

(5)

A separation of an existing contract is a start-up opportunity for another small firm. Therefore, the number of open start-up opportunities in the modern sector \( O \) is

\[ O = \vartheta N = \delta^\zeta N \]  

(6)

**Contract tightness and the matching process.** At every moment potential start-up entrepreneurs are looking for opportunities to start new firms. They try to find a match between their business ideas (determined by their entrepreneurial capital) and the profile of business (outsourcing) opportunities in the modern sector. For the sake of simplicity we reduce this search and matching process to a pure random process. Hence, the individual probability to successfully start a new firm (i.e. find a match between ability and opportunity) \( P_N \) is described by a Poisson distribution\(^{17} \) and given by

\[ P_N = \lambda_N e^{-\lambda_N} \]  

(7)

Further, we define \( \theta_N \) as contract tightness, given by

\[ \theta_N = \Delta / O = \vartheta^{-1} \delta \]  

(8)

Contract tightness indicates how difficult it is for a start-up to sign a new outsourcing contract with a final output producing firm. The size of \( \theta_N \) is determined by the surplus rate of business ideas \( \delta_N \). The higher \( \delta_N \) the more difficult it is for a potential new entrepreneur to start up a new firm offering one of the relatively large number of new business ideas.

A core element of the matching approach is the matching function. The matching function describes the efficiency and determinants of the matching process. In order to keep the model simple we assume that the expected matching rate is negatively related to the contract tightness \( \theta_N \), and positively related to the entrepreneurial ability of the start-up entrepreneur \( h \), namely

\[ \lambda_N = \lambda_N(\theta_N, h) = \theta_N^{-\varepsilon_N} h^{\nu_N} \quad 0 < \varepsilon_N, \mu_N < 1 \]  

(9)

with \( \nu_N \) being the effectiveness of entrepreneurial human capital to match business ideas or perceived opportunities with market conditions.

\(^{17}\) In many matching models the matching process is covered by a linear homogeneous matching function. There is empirical evidence that the assumption of a linear homogeneous matching function is reasonable (See Pissarides 2000, and the references therein; Petrongolo and Pissarides 2001). Nevertheless, Diamond (1982), Howitt (1985), and Mortensen (1989) allow for increasing returns and obtain more interesting results including multiple equilibria and co-ordination failures. Given the purpose of this paper we try to keep matters simple and cover the idea of a labour market matching process by a pure random process.
Business opportunities and start-up matching equilibrium. The equilibrium matching flow process is determined by the flow of opportunities \((O = \vartheta N)\) due to changing product properties, and a flow of opening new businesses due to a match of these properties \(\lambda_N \Delta_N\). As we assume that matching takes place instantaneously, all business opportunities are realized on average

\[
\lambda_N(\theta_N, h)\Delta_N = \vartheta N
\]

(10)

**Proposition 1.** The equilibrium surplus rate of business ideas \(\delta_N\) and equilibrium rate of business failure (contract separation rate) \(\vartheta\) is determined by entrepreneurial human capital \(h\), and the effectiveness of this entrepreneurial human capital in finding a market niche \(\upsilon_N\), with

\[
\delta_N = -\frac{\upsilon_N}{h^{(1-\varepsilon_N)(1-\varepsilon)}} \frac{d\delta_N}{dh} < 0 \quad \text{The surplus rate of business ideas} \tag{11}
\]

and

\[
\vartheta = -\frac{(\upsilon_N)\zeta}{h^{(1-\varepsilon_N)(1-\varepsilon)}} \frac{d\vartheta}{dh} < 0 \quad \text{The business failure rate} \tag{12}
\]

**Proof:** See Appendix 3.

Among other parameters of the matching process the failure rate of start-ups depend on the start-up entrepreneur’s ability, \(h\). The higher this entrepreneurial ability the lower the surplus rate of business ideas and the lower the expected rate of separation.

Start-up of firms. A start-up in the modern sector produces a services or good as intermediate input for the large firms in the sector. Each product or service variation is unique and different compared to other already existing variations. In order to get the new service or product to the market, an entrepreneurial venture, or start-up firm, needs to be created. This is however, subject to start-up costs. Start-up costs are a barrier to entry to the market and by overcoming these, the entrepreneur open up a new market segment. Start-up costs include all costs such as initial capital endowment information and organization and management costs, administrative costs, costs of learning, cost of acquiring and developing a business idea and business plan suitable to obtain finance from a financial intermediation. We assume that start-up costs are determined by the density \(N/Y\) of already existing small firms. With increasing density more ingenuity is needed are more resources are needed (for instance in R&D) before a new firm can be started. Start-up costs are denoted by \(\chi\) and hence

\[
\chi(N/Y) = \varepsilon \frac{N}{Y}, \text{with} \quad \varepsilon > 0
\]

(13)

Operating the new firm. In addition to start-up costs there are permanent costs to operate the business. These costs are denoted by \(c_x\). These periodic operating costs per
unit output of the intermediate good may have two components. First, there are given costs related to the specific start-up project $\bar{c}_x$. Second, the start-up entrepreneur relates their entrepreneurial income to the income a representative agent $i$ could expect alternatively if employed or self-employed in the traditional sector $E_{y_{i,t}}$. This income is the minimum income the entrepreneur would like to earn from their firm, hence as soon as the firm is started they take this income from the revenues earned

$$c_{sj} = \bar{c}_{sj} + E_{y_{i,t}} \tag{14}$$

Due to the start-up costs in (13), we take it that once a small firm is set up it will remain monopolistic for the specific service/product variation. As a result, each period’s profits are determined by the price of the product variation $p_j$ and the periodic costs $c_x$. Hence net periodic profits are given by $\pi^* = (p_j - c_{sj})x_j$. The expected net present value of such a monopoly is

$$EV_m(\tau) = (1 - \vartheta)\int_0^\infty (p_j - c_{sj})x_je^{-r_d(t,\tau)\vartheta(t-\tau)}dt$$

where $\vartheta$ represents the expected rate of business failure, $(1-\vartheta)$ is expected rate of success. Monopoly profits are maximized by the optimal choice of the intermediate good price $p_j$, as

$$p_j = \frac{c_{sj}}{\alpha} \tag{15}$$

Where $\alpha$ is the elasticity of production of intermediates in the final goods sector. With the optimal price rule we can also determine periodic profits. Each period profits are determined by the price of the optimal product variation (15) and the periodic costs (14). Net periodic profits are given by $\pi^* = (p_j - c_{sj})x_j$, and hence

$$\pi^* = (1-\alpha)HA^{\frac{1}{1-\alpha}}c_{sj}^{\frac{1}{\alpha}}$$

As a result, the expected maximum net present value of a new firm is

$$EV_m = \frac{(1 - \vartheta)}{r_d}(1 - \alpha)H\left(\frac{A^{\frac{1}{1-\alpha}}}{c_{sj}^{\frac{1}{\alpha}}}\right)^{\frac{1}{\alpha}}$$

**Financing the new firm.** Since the prospective entrepreneur is assumed to have no immediate income or accumulated savings, the start-up costs $\chi$ must be financed at a loan rate, which we denote by $r_l$. We assume that external financing is the only viable option. To simplify, we assume a firm that revolves loans infinitely and services interest only (i.e. Ponzi finance is excluded). Denoting the deposit rate $r_d$ the present value of setup costs ($V_s$) including finance is

$$V_s = \frac{r_l^{r_l}}{r_d}$$

As long as there is no steady state equilibrium, start-up entrepreneurs realize a net rent. However, in steady state equilibrium the net present value of the new firm will just cover total start-up costs which, $V_s = V_m$. Thus periodic monopoly rents are eventually fully distributed as income of the entrepreneur and under competition used to finance start-up costs. We can extend this to take into account non-pecuniary benefits of

19 See Appendix 4b.
20 See Appendix 4c.
21 See Appendix 4d.
entrepreneurship (since entrepreneurial rents are often found to be less than returns from wage employment; see e.g. Hamilton 2000) but for the sake of simplicity leaves this for future elaboration. With respect to financial markets, start-up activities by entrepreneurs lead to a perfectly elastic loan demand\(^{22}\)

\[
    r_i = \frac{(1-\vartheta)(1-\alpha)\alpha H^2 \alpha^2 \varphi c_{\alpha} A^{\frac{1}{\alpha+1}}}{\epsilon(A)}
\]  

(17)

**Financial intermediation**

Given that the financial sector is often not very well developed in a developing economy, especially an economy where there is a large traditional sector, we allow for imperfections in financial markets to affect credit availability to prospective start-ups. Following the contribution of Stiglitz and Weiss (1981) it is widely accepted that informational asymmetries and agency problems can result in newer, smaller firms finding it difficult to access sufficient external finance, i.e. being credit rationed (Bonnet et al. 2005). The problem of small firms being credit rationed can be more severe if the modern sector is characterized by a high concentration of market power by financial intermediaries/banks. In order to model concentration of market power in the financial market we assume a historical given number of banks \(B\). Each bank \(b\) offers deposits \(D_b\) to households and loans \(K_b\) to potential start-up firms. Banks have symmetric monitoring costs \(c_B\). The expected profit function of a bank \(b\) is given as

\[
    \pi_b = (1 - \vartheta) r_i K_b - r_d(D) D_b - c_B \vartheta K_b - \bar{c}_b
\]

where \(r_d(D)\) is the deposit demand function and \(D\) denotes total deposits in the region, and \(\vartheta\) is the expected default rate of the loans given to start-up firms. As the bank is a pure intermediary its balance sheet can be represented as

\[
    K_b = D_b
\]

(19)

When the bank maximizes profits subject to the balance sheet constraint, the first order condition (FOC) is

\[
    \frac{d\pi_b}{dD_b} = (1 - \vartheta) r_i - r_d(D) + \frac{\partial r_d}{\partial D} \frac{\partial D}{\partial D_b} D_b - c_B = 0
\]

Rearranging the FOC and using the definition of total deposits \((D = BD_b)\) and the definition of the elasticity of the deposit demand function

\[
    \eta = -\frac{\partial D}{\partial r_d} \frac{r_d}{D}
\]

(where \(\eta\) is assumed constant) we obtain an optimal deposit rate for banks offered to the public\(^{23}\)

\[
    r_d = \left[ (1-\vartheta)r_i - c_B \right] \left( 1 + \frac{1}{B\eta} \right)
\]

(20)

The solution to the banks’ optimization problem results in a loan-deposit rate spread. As can be seen from equation (20) the spread is determined by two factors, namely the costs of monitoring \(c_B\) and the concentration of banks measured by the index \(1 + \frac{1}{\eta}\). A lower number of banks will increase the concentration of financial intermediaries and widen the interest spread. In principle we can extend this to full market equilibrium. We

\(^{22}\) See Appendix 4e.

\(^{23}\) See Appendix 5.
would endogenously determine the number of banks and apply the zero profit condition to the banking sector. This would take away the pressure for market entry by additional banks. However, as we would like to focus on bank concentration and the number of banks, we end our modelling of the financial intermediation at this stage and rather proceed to the traditional sector as other sector in this economy.

### 3.3 The traditional sector

The traditional sector in our model can be described by its population dynamics and labour market frictions.

#### Population dynamics

Population in the traditional sector at any given moment consists of \( L \) number of people earning an income \( w \) and \( \Delta_T \) earning no income (i.e. surplus labour). While the income earning proportion is employed (this can either be in formal agriculture, or informally in agricultural or off-farm activities), the proportion of the population with no income is either not searching for a job, has not found a job, or is not successful in informal self-employment in the traditional sector.\(^{24}\) Hence total population in the traditional sector is

\[
Pop_T = L + \Delta_T = L(1 + \delta_T), \quad \text{with} \quad \delta_T = \Delta_T / L
\]

where \( \delta_T \) is the ratio of no-income earning surplus labour to income earners in the traditional sector. With the definition of the surplus labour rate we can also determine the unemployment rate \( u \) and the probability of being employed

\[
u = \frac{\Delta_T}{L + \Delta_T} = \frac{\delta_T}{1 + \delta_T}\quad (22)
\]

As labour is the only factor of production in this sector, we assume a given constant marginal productivity \( a_T \) per unit of income-earning labor and hence \( w_T = a_T \). Income earned by the fraction of the population employed is partially redistributed to the fraction of population not employed. Most often this redistribution process is organized within households. However, with a \((1-u)\) probability of employment, expected per capita income for those in the traditional sector is given by

\[
Ey_T = y_T = \frac{a_T}{1 + \delta_T}\quad (23)
\]

To keep the model simple population growth in the traditional sector is assumed to be a function of per capita income in the sector, \( y_T \). Thus the lower the average income per capita in the traditional sector the lower the rate of survivors of a potentially growing population. Population dynamics in the traditional sector can then be given as

\[
\gamma_L \equiv \frac{\dot{L}}{L} = g_L(y_T) = y_T^\varphi \quad \text{with} \quad \varphi > 0\quad (24)
\]

where \( \varphi \) is the elasticity of net population growth with respect to per capita income in the traditional sector. A higher per capita income in the traditional sector will allow for a higher survival rate and hence growth rate of population in the traditional sector. From the ‘demographic transition’ we know however, that continued increases in GDP per capita growth will lead to a decrease in the birth rate and hence population growth, but these effects are not included in our model.

\(^{24}\) We assume a distinction between the start-up of firms in the modern sector, and informal self-employment in traditional sector. The former takes place due to opportunities being spotted, whilst the latter reflect survivalist, necessity actions. Our concern in this paper is with the impact of opportunity-driven start-ups on economic development. This is consistent with empirical evidence based on data from the Global Entrepreneurship Monitor (GEM) which finds that only opportunity-driven entrepreneurship is associated with per capita GDP growth (Wong et al. 2005: 341).
capita are associated with a reduction in the population growth rate after time. Hence with this assumption we define the economy being in a certain stage of development or maturity. Analysing the alternative assumption will be left for future research.

Labour market frictions and surplus labour

Matching job and self-employment opportunities. The point of departure of the Lewis-model is that high unemployment (surplus labour) is a common feature in most developing countries, particularly in the traditional/rural sector. Thus, at any given moment a proportion of the population in the traditional sector will not be earning any income. Short-term employment or self-employment of heterogeneous labour and heterogeneous earning opportunities need to be covered in a model of the traditional and informal sector. Here we do not distinguish between employment and (survivalist) self-employment in the traditional sector. Also, there are no opportunities for formal entrepreneurial ventures in the traditional sector. At any given moment, the unemployed will be searching for either wage-employment or self-employment in the traditional sector, or for opportunities to start-up an entrepreneurial venture as a formal small firm in the modern sector. If the start-up is successful the respective agent will leave the traditional sector and migrate to the modern formal sector.

As discussed earlier, in the modern sector opportunities for entrepreneurial ventures are attached to outsourcing of intermediate input requirements and support services by large firms producing a final output. The identification and utilization of opportunities by entrepreneurs have generated a substantial literature (see McMullen et al. 2007). In this paper we solved the problem of how entrepreneurs identify opportunities that they then explore in a novel way by assuming that entrepreneurs and opportunities are ‘matched’ in a similar process that job-seekers and vacant job opportunities are matched in well-known labour matching models (e.g. Pissarides 2000). We discussed this matching process above. We use a similar matching process to model the ‘match’ between job seekers and vacancies in the traditional sector.

Separation of earning activities. For the total number of successful employment or survivalist self-employment activities \(L\), a continuous adjustment of job specification and permanent changing conditions for self-employment leads to separation of earning opportunities at the rate \(\sigma\). Hence, the number of available earning opportunities in the traditional sector \(V\) can be presented by

\[
V = \sigma L
\]

(25)

Search for jobs and matching. At every moment unemployed agents (unemployed due to recent separation of employment or changing opportunities for self-employment) are searching for an activity to earn an income. They try to find a match between their personal profiles (abilities) offered to the market and the profile required for an income earning activity. To keep the model simple we reduce the search and matching process of workers to a pure random process. Hence, the individual probability to find a job or successful self-employment (find a match) \(P_T\) is also described by a Poisson distribution, which in this case can be written as

\[
P_T = \lambda e^{-\lambda}.
\]

Further, labour market tightness in the traditional sector, \(\theta_T\), is defined as the ratio of job seekers to vacancies

\[
\theta_T = \Delta_T / V = \sigma^{-1} \delta_T
\]

(26)
A central element of the matching approach is the matching function. We assume that the expected matching rate is negatively related to labour market tightness $\theta_T$ in the traditional sector.

We assume a link between formal start-up firms in the modern sector and ‘survivalist’ or informal self-employment in the modern sector. With an increasing growth of entrepreneurial start-ups the variety and diversity of economic activities (since each start-up ‘innovates’ in bringing a unique product or service to market) in the modern sector increases. As a result, the variety of survivalist self-employment opportunities in the informal traditional sector might also increase. Hence the matching rate in the traditional sector reacts positively (with the elasticity $\nu$) to an increase in the growth of entrepreneurial start-ups, $\gamma_N = \frac{\dot{N}}{N}$. Therefore, $\gamma_N$ enters the matching process positively. Further, in this simplifying model the matching process is driven by technical parameters of the search process. The expected rate of matching is

$$
\lambda = \lambda(\theta_T, \gamma_N) = \theta_T^\nu \gamma_N
$$

$$
= \lambda(\sigma, \delta_T, \gamma_N) = \sigma^\nu \delta_T^\nu \gamma_N \quad 0 < \nu, \mu < 1
$$

Labour market equilibrium. The equilibrium labour market flow process is defined by a simultaneous inflow ($V = \sigma L$) of workers into the market and an equally simultaneous outflow out of the market into employment and self-employment determined by the matching process $\lambda \Delta_T$. We assume that the labour market instantaneously adjusts. In labour market equilibrium, on average all vacancies and opportunities to earn income are filled

$$
\lambda(\sigma, \delta_T, \gamma_N) \Delta_T = \sigma L
$$

### 3.4 Model solution and comparative static effects

Having provided a detailed description of the various sectors and elements of the model, we now summarize the essence of the model in three propositions, and solve the resulting system.

Summarizing the traditional sector. The discussion of the traditional sector condenses to a description of the population dynamics and the determinants of the surplus labour rate.

**PROPOSITION 2.** Population growth in the traditional sector is a negative function of the labour surplus rate $\delta_T$ and a positive function of the marginal and average labour productivity in the traditional sector.

$$
\gamma_L = \left( \frac{\alpha_T}{(1 + \delta_T)} \right)^\phi \quad \text{population dynamics}
$$

$$
= g_L(\delta_T, a_T, \phi), \quad \text{with} \quad \frac{d g_L}{d \delta_T} < 0, \quad \frac{d g_L}{d a_T} > 0, \quad \frac{d g_L}{d \phi} > 0.
$$

Proof: See Appendix 6  ■
Proposition 2 suggests that generally improving conditions in the traditional sector tend to push population growth.

PROPOSITION 3. The equilibrium surplus labour rate $\delta_t$ is a function the endogenous growth rate of entrepreneurial start-ups $\gamma_N$ and a set of parameters

$\delta_t = \sigma \gamma_N^{\frac{\alpha}{\gamma}}$, Surplus labour rate

$\delta_t = \delta_t(\gamma_N, \sigma)$, with $\frac{d\delta_t}{d\gamma_N} < 0, \frac{d\delta_t}{d\sigma} > 0$ (30)

Proof: See Appendix 7

From proposition 3 follows that the higher the entrepreneurial start-up rate, the more people from the traditional sector will find employment and self-employment even if they remain in the traditional sector. This mechanics will reduce not only unemployment but also raise average per capita income in the traditional sector.

Summarizing the modern sector. The discussion of the modern sector lead to the loan market equilibrium and eventually to the start-up rate of entrepreneurial firms in the modern sector. From (17) and (20) we obtain the loan market equilibrium with an optimal loan-deposite rate spread, channeling households savings to loans financing start-ups. (14) and (23) describes the costs and opportunity costs of running the own business, and the Ramsey-rule (3) gives the optimal intertemporal choice of households in the modern sector and the respective savings decision. Combining all these elements leads to the growth rate of start-ups depending on the conditions in the traditional sector depicted by the endogenous surplus labour rate $\delta_t$ and the exogenous productivity $a_t$ and a variety of exogenous determinants in the modern sector.

PROPOSITION 4. The growth rate of the number of start-ups and hence the steady state growth rate of the modern sector $\gamma_N$ is function of the endogenous surplus labour rate $\delta_t$, and number of other parameters in the traditional and modern sector, specifically $a_t$ and $H$, $\vartheta(h)$, $B$

$$\gamma_N = \frac{1}{\Theta} \left[ \left( \frac{(1-\vartheta(h))(1-a)}{a_t} H \right)^2 \left( \frac{\vartheta(h)}{\vartheta(h)+a_t} \right)^{2-\vartheta(h)} \right] - c_h(\vartheta(h)) \left( 1 + \frac{1}{B\eta} \right) - \rho$$

$$= g_N \left( \delta_t, a_t, H, h, \vartheta, B, \Theta, \rho \right)$$

start - up rate, modern sector

$$\frac{d\gamma_N}{d\delta_t} > 0, \quad \frac{d\gamma_N}{da_t} < 0, \quad \frac{d\gamma_N}{dH} > 0, \quad \frac{d\gamma_N}{dh} > 0$$

Proof: See Appendix 8
Solving the full system. As given by propositions 2, 3 and 4 we have a system of three equations describing the interdependent mechanics between growth of the modern sector by the start-up rate of small firms $\gamma_N$ and the surplus labour rate $\delta_T$ in the traditional sector, as well as the population growth $\gamma_L$ in the economy. With the surplus labour condition (30), the condition for the start-up growth rate (31), and the condition describing the population dynamics (29) we obtain three conditions to solve for the three remaining endogenous variables $\gamma_N, \delta_T, \gamma_L$

\[
0 = F_1(\delta_T, \gamma_N, \sigma) = \delta_T(\gamma_N, \sigma) - \delta_T \quad \text{(surplus labour rate)}
\]

\[
0 = F_2(\delta_T, \gamma_N, a_T, H, ..., \rho) = g_N(\delta_T, a_T, H, ..., \rho) - \gamma_N \quad \text{(start-up rate)}
\]

\[
0 = F_3(\delta_T, \gamma_L, a_T, \varphi) = g_L(\delta_T, a_T, \varphi) - \gamma_L \quad \text{(population dynamics)}
\]

As additional labour is always absorbed by the traditional sector the growth of the start-up sector must be large enough to develop the modern sector relatively to the traditional sector. The speed of modernization must exceed the speed of expansion of the traditional sector.

In order to illustrate the role of entrepreneurial start-ups in structural transformation we will first solve the model (32) and then discuss the conditions that are favourable or unfavourable for structural economic transformation.

**PROPOSITION 5.** System (32) implicitly defines functions for the equilibrium surplus labour rate $\bar{\delta}_T$, the equilibrium start-up rate $\bar{\gamma}_N$ and the equilibrium population growth rate $\bar{\gamma}_L$ as functions of a vector of exogenous variables and parameters $(a_T, ..., H, h, B, \bar{c}_x, A, \Theta, \rho)$.\(^{25}\)

\[
\bar{\delta}_T = \bar{\delta}_T(a_T, ..., H, h, B, \varphi, \bar{c}_x, A, \Theta, \rho) \quad \text{surplus labour rate}
\]

\[
\bar{\gamma}_N = \bar{\gamma}_N(a_T, ..., H, h, B, \varphi, \bar{c}_x, A, \Theta, \rho) \quad \text{start-up rate}
\]

\[
\bar{\gamma}_L = \bar{\gamma}_L(a_T, ..., H, h, B, \varphi, \bar{c}_x, A, \Theta, \rho) \quad \text{population dynamics}
\]

A graphical illustration of the model solution and comparative static effects is given in the $\gamma_N - \gamma_L - \delta_T$ diagram system of Figure 1 and Figure 2.

In Figure 1 the $\gamma_N - \delta_T$ diagram depicts the growth-surplus labour relation for the traditional sector as well as for the modern sector. In the north-east quadrant of Figure 1 the initial steady-state equilibrium values of $\gamma_N$ and $\delta_T$ is shown at point $A$. The $\gamma_L - \delta_T$ quadrant links the surplus labour rate with the growth rate of the traditional sector and the $\gamma_L - \gamma_N$ diagram (north-west and south-west quadrants) identifies the degree of structural economic transformation and the effect on growth per capita. The surplus labour curve has a negative slope and the entrepreneurial start-up curve has a positive slope.

If the steady-state equilibrium moves from $A$ to $A'$ (for reasons we will explore below) the economy’s structure changes from being in rural stagnation (at point $\gamma$) to being in the region of ‘modern transformation’ at point $\gamma'$. The interdependence between structural change and growth in our model is evident from the fact that it can be seen that this structural transformation has been accompanied by an increase in per capita

\(^{25}\) For proof see Appendix 9.
GDP. Indeed, there is no per capita growth without turning the economy into a modern economy. As rural productivity is constant, transformation to a modern economy and per capita growth are two aspects of the same phenomenon. Furthermore, to the extent that modern sector growth is driven by growth in entrepreneurial start-ups that supply services, the service sector assumes an increasingly important role in the economy, consistent with the stylized facts as described in Section 1. Even more, while the productivity of each intermediate service remains constant, total factor productivity permanently increases in the final goods producing sector (manufacturing). Over time this can account for a further stylized fact of modern structural change, namely the higher productivity of the manufacturing sector relative to the services sector.

In this structural economic transformation the vehicle is entrepreneurial start-up firms. The expansion of entrepreneurial start-ups is driving the process of development.

With this model we can analyse a variety of aspects concerning entrepreneurial start-ups, sectoral transformation and growth. In this paper, for illustration purposes, we restrict ourselves to discuss three topics in structural change and entrepreneurship. These are the effects on structural change and growth of (i) entrepreneurial ability, (ii) bank concentration and access to finance, and finally (iii) increasing productivity in the traditional sector.

**Entrepreneurial ability and structural change**

There are no ‘entrepreneurs’ as such in the Lewis-model. Thus it is silent on the role of ‘entrepreneurial ability’ in the structural transformation process. In the economics of entrepreneurship literature however, entrepreneurial ability plays an important role. For instance, entrepreneurial ability is a core element of occupational choice models (e.g. Lucas 1978; Evans and Jovanovic 1989). As recognized by Kanniainen and Poutvaara (2007: 676) ‘people differ substantially in terms of their ability to produce a business idea, elaborate their idea, and make its way to a marketable product or service’. Baptista et al. (2007) consider entrepreneurial ability to consist of human capital, social capital, and cognition. It has also been treated as an important determinant of firm survival (Cagetti and De Nardi 2005a, b). Fonseca et al. (2007: 648) make a distinction between entrepreneurial ability and working ability. The former they define as the capacity to ‘invest capital productivity’, and the latter as the capacity to ‘produce income out of labour’. Thus an individual with high entrepreneurial ability might have even higher working ability, and will be less likely to enter into entrepreneurship (Fonseca et al. 2007: 655). Empirical evidence that measures entrepreneurial ability by educational level and/or age finds that entrepreneurial ability influences the probability that the entrepreneur will start a firm in the formal (as opposed to the informal or survivalist) sector of the economy (De Paula and Scheinkman 2007). Dias and McDermott (2006) provides a model that shows that the better entrepreneurial ability are, the more workers will migrate to the modern sector, and the higher will be the overall levels of human capital accumulation in the economy.

In our model, entrepreneurial ability has been emphasized in the matching of opportunities in the modern sector with entrepreneurs’ ideas (or business plans). Entrepreneurs with better ability are thus better matched with opportunities and will, as in De Paula and Scheinkman (2007), be more likely to migrate to the modern sector and start a firm rather than remain in wage employment or be self-employed in the informal, survivalist sector.
How does this affect structural transformation in our model? We can illustrate this with the help of Figure 1, wherein entrepreneurial ability, both of the mature entrepreneur as well as of the start-up entrepreneur, will shift the modern sector curve upwards. Entrepreneurial ability of the mature entrepreneur will improve the productivity of intermediate services and hence the return on investment in start-up firms. Therefore, savings and financing of start-up firms will increase and a larger number of new start-ups will enter the modern sector in each period. Further, an increasing growth rate of entrepreneurial start-ups will also spill over to traditional sector, as additional start-up firms will not only absorb labour from the traditional sector, but also improve employment opportunities in the traditional sector (see Proposition 1). Consequently, surplus labour will decrease. Decreasing surplus labour will increase per capita income in the traditional sector. If the effect on population growth is small enough the growth rate of the traditional sector will be relatively lower than the growth of the modern sector and the economy will develop structurally from the point of rural stagnation—at $\gamma$ in Figure 1, to modern transformation at point $\gamma'$.

Figure 1: Growth in start-ups and the structural transformation
A similar process can be described with respect to improved entrepreneurial ability of start-up entrepreneurs. In this case start-up entrepreneurs have more realistic expectations and ideas of what might be a successful business plan, and hence the match of start-up ideas and opportunities improves. As a result the failure rate of new firms decreases and financing of new firms becomes more profitable. This again channels more resources to start-ups and improves the growth rate \( (A', \gamma') \) as indicated in Figure 1 by the move from \( A \) to \( A' \) and \( \gamma \) to \( \gamma' \).

**Bank concentration and access to finance**

The observation that entrepreneurs are wealthier than wage-earners (e.g. Cagetti and De Nardi 2005a, 2005b; Hurst and Lusardi 2006: 3) has been taken as evidence of capital constraints on start-ups. The view is that under constrained capital markets, individual wealth and informal credit markets will be a determinant of the start-up rate. Following Stiglitz and Weiss (1981) it has been realized that capital markets could provide inadequate finance to entrepreneurs due to moral hazard and limited liability problems (Paulson et al. 2006: 102). The key initial insight in the context of start-ups has been formalized by Evans and Jovanovic (1989). In their model, the significance of wealth as a determinant of start-ups is interpreted to signify that potential entrepreneurs are credit-constrained.

In our model we ruled out the possibility that entrepreneurs can finance start-ups out of their own wealth or savings. However, being dependent on the formal bank sector to obtain credit may affect the start-up rate. One way in which potential entrepreneurs in a developing country context can be credit constrained is due to the underdeveloped nature of the formal financial system, which is often characterized by a high degree of bank concentration (Naudé et al. 2008). Assuming that as the economy develops, the financial sector also develops, we can use our model to show the impact of financial sector development on economic structural change via the start-up activities of entrepreneurs.

A decrease in bank concentration as a result of financial sector development can be shown with the help of Figure 1. In Figure 1 a decrease in bank concentration \( (B \uparrow) \) reduces the loan deposit interest spread and hence makes the investment in entrepreneurial start-ups more profitable. As a result the modern sector curve will shift upwards again. Improved access to finance will increase the start-up rate \( (A', \gamma') \). All resulting effects are as described earlier in the paper. As improved access to finance increase the start-up rate, and the latter again raises GDP, financial development and deepening leads as well as follows GDP in our model.

**Rural development**

Our model can be seen to question the common view that rural development strategies—raising agricultural productivity for instance—may be an optimal development strategy in developing countries. As was discussed in Section 2, the balanced-growth requirements of dual economy models imply that higher productivity in agricultural sector may be necessary to allow surplus food production and labour to benefit the modern sector. In contrast, in our model the implication is that growth in productivity in the agricultural (traditional) sector may not necessarily be in the best interest of modern sector and in economic structural transformation. Although in contrast with the large literature and policy initiatives advocating rural development in
developing countries as a central pillar of development strategies, this implication of our model is not novel, and is not inconsistent with the growth experiences of many countries. Matsuyama (1992) points out that empirical evidence and historical experience would suggest that countries with higher agricultural productivity were slower to industrialize.\textsuperscript{26} He writes (1992: 318):

Why were Belgium and Switzerland the first to become leading industrial countries in continental Europe, while the Netherlands lagged behind and did not take off until the last decades of the nineteenth century? Or why did industrialization of the United States during the antebellum period, mainly in the cotton textile industry, occur in New England, not in the South? Economic historians who studied these experiences found their answer in the Law of Comparative Advantage, which implies a negative link between agricultural productivity and industrialization.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Effects of productivity growth in the traditional sector}
\end{figure}

\textsuperscript{26} Matsuyama (1992) provides an endogenous growth model with an open economy to show how higher agricultural productivity to harm industrialization. Our model shows that this result can also be obtained in a closed economy setting.
We can illustrate, with the help of Figure 2, the implication for structural economic change when agricultural productivity increases faster than that of the modern sector. The figure shows two simultaneous effects. In the modern sector increasing productivity will increase opportunity costs of being an entrepreneur in the modern sector. Therefore, the growth rate of new entrepreneurial start-ups starts to slow down. In the traditional sector higher productivity will increase average income and hence drive up population growth in the traditional sector. As a result productivity growth in the modern sector will reduce the chance for modern transformation and the economy move from $A$ to $A'$ and from $\gamma$ to $\gamma'$ in Figure 2.

4 Concluding remarks

A stylized fact of economic development is the structural transformation of countries from traditional, mainly agricultural societies to a modern economies dominated by manufacturing and services. The Lewis-model has been influential in providing theoretical foundations for this structural transformation. However, in the Lewis-model, as in much of development economics, the entrepreneur is never formally modelled. Our contribution in this paper has been to provide an endogenous growth model to illuminate the role of entrepreneurial start-up firms in structural economic transformation.

We followed the Lewis-model distinction between a traditional and modern sector, and underpinned this with micro-foundations (optimizing households, firms, and labour market matching). In introducing the entrepreneur into our Lewis-type model, we followed some of the ‘stylized facts’ from the entrepreneurship and small business literatures. Thus we distinguished between mature and start-up entrepreneurs, between large firms and small firms. We also distinguished between survivalist self-employment activities in the traditional sector, and opportunity-driven entrepreneurship in the modern. Furthermore, mature entrepreneurs provide final consumption goods while start-up entrepreneurs innovate by providing unique intermediate goods and services.

In essence the transformation from a low-income, traditional economy to a modern economy also involves significant changes to production methods, a process of change where our modelled showed that entrepreneurs provide essential roles: first, in creating new firms outside of the household, second by absorbing surplus labour from the traditional sector, third by providing innovative intermediate inputs to final-goods producing firms, fourth by permitting greater specialization in manufacturing, and fifth by raising productivity and employment in both the modern and traditional sectors.

The model results are consistent with the stylized facts of labour migration from the traditional to the modern sector, a rise in the share of services in output and employment over time, and greater total productivity in manufacturing over services. We illustrated how entrepreneurial ability, financial access, and rural development may determine entrepreneurial start-ups, and the consequences for structural transformation.

As far as the policy implications from our model are concerned, it is both in support of current orthodoxy as well as questioning. As was discussed earlier, it questions the general approach towards improving traditional sector productivity. However, it is supportive of the widely held notion that small firms will dominate employment during the transition from traditional to modern growth, and that repression of productive
entrepreneurship and small firm formation may be a cause of stagnation because countries cannot make the transformation out of the traditional sector to a growth take-off in the modern sector (see also Fiaschi and Lavezzi 2007: 272). Consequently our model suggested that policies to improve entrepreneurial ability, such as education and training and a supportive environment that allows for entrepreneurial learning (and experimenting) to take place, as well as policies to reduce obstacles to start-ups, in particular providing for access to finance, could be justified.

While pro small firm, our model stressed the link between firms, through the outsourcing of services or sourcing of intermediate inputs by established firms from new firms. To the extent that entrepreneurs need to innovate to be able to prove these and therefore need to incur ‘R&D’ costs as part of the general costs of starting up a new firm, support for such ‘R&D’ activities is a further policy implication from our model. In fact the link between established firms and innovation by small firms is central in our model in getting growth and transformation started in the first place.

As a model of structural economic change, our model has a number of weaknesses when compared to the empirics of actual structural economic changes. For one, in the tradition of Lewis (1954) and others our model takes structural changes as gradual, continues changes within the context of eventual steady-state growth. In reality, structural changes may take place more drastically, even be revolutionary and can either propel the economy into a trajectory of higher growth, or shove the economy into a period of fast growth reversal. These sudden or extreme cases of structural change also calls for the role of the entrepreneur—and the impact of these on the entrepreneur—to be better understood. Naudé (2007) for instance considers a subset of such experiences, namely of countries in conflict, where entrepreneurs can play a catalyst role in either promoting conflict, perpetuating conflict, or facilitating the transition from conflict to peace and post-conflict reconstruction. Two, as was noted, our model assumes a closed economy, while it was acknowledged that technological innovation and trade in open economy models have important implications for structural transformation in the real world. Three, we assumed a perfectly competitive modern final goods sector, an assumption which may not hold in the early stages of economic development.

As a result there are a number of possible extensions that the future development of this model may take. The first could be to consider the ‘misallocation’ of entrepreneurial activities to activities that are not productive, as for instance in Mehlum et al. (2003) and to allow for the impact of ‘destructive’ entrepreneurship to be modelled in an endogenous growth context. The second could be to extend the model to the case of an open economy. This could allow for sudden external shock’s effects to be investigated, and also allow one to model the interaction between a country, and its entrepreneurial start-up rate, with other countries in a multi-country setting. As such the model might be used to provide insights into changes in the structural of the global economy (structural transformation is after all not only taking place on a country-level, but also on a global level). Ranis (1988) describes a single-country open dual economy model. More recent models focusing on structural change in multi-country settings (but without the entrepreneur) are contained in Landesmann and Stehrer (2006, 2001). Finally further research using our model could explore the implications on steady-state growth of changes in the market structure for final goods, as a less competitive structure than the one assumed in our paper may have an important impact on the start-up rate in our modern sector.
Appendix 1

Production in the modern final output sector

\[ Y_i = AH^{1-\alpha} \sum_{j=1}^{N} (x_j)^{\alpha} . \]
\[ Y = AH^{1-\alpha} Nx^{\alpha} \]

Demand for intermediate services:

\[ \pi = ANH^{1-\alpha} x^{\alpha} - wH - Np_j x_j \]
\[ \frac{d\pi}{dx} = \alpha ANH^{1-\alpha} x^{\alpha-1} - Np_j = 0 \]
\[ p_j = \alpha AH^{1-\alpha} x^{-(1-\alpha)} \]
\[ x^{(1-\alpha)} = \frac{\alpha}{p_j} AH^{1-\alpha} \]
\[ x = H \left( \frac{A\alpha}{p_j} \right)^{\frac{1}{\alpha}} \]

Local factor price:

\[ w = (1-\alpha) \left( \frac{Y}{H} \right) \]
Appendix 2

Household’s optimal intertemporal choice with (CIES) utility function, i.e., \( u'(c) > 0 \), \( u''(c) < 0 \), and \( \Theta \equiv -u''(c)c/u'(c) \)

\[
\max_{C(t)} U(C(t)) = \int_0^\infty U(C(t))e^{-\gamma dt},
\]

s.t. \( Dr_d + wH = C + S = C + D \)

Ramsey Rule:

\[
\gamma_c = \frac{r_d - \rho}{\Theta}
\]
Appendix 3: Proof of Proposition 1

Start-up sector: matching of ideas and opportunities:

Separation of contracts is a positive function of the surplus rate of business ideas

\[ \vartheta = \vartheta(\delta_N) = \delta_N^{\zeta}, \quad \frac{d \vartheta}{d \delta_N} = \zeta \delta_N^{\zeta-1} > 0. \]

Start-up opportunities in modern sector \( O \):

\[ O = \vartheta N = \delta_N^{\zeta} N. \]

Probability of successful start-up

\[ P_N = \lambda_N e^{-\lambda_N \vartheta}. \]

Contract tightness

\[ \theta_N = \Delta_N / O = \theta^{\zeta-1} \delta_N. \]

\[ \theta_N = \Delta_N / O = \left( \delta_N^{\zeta} \right)^{-1} \delta_N = \delta_N^{(1-\zeta)\vartheta}. \]

Matching function

\[ \lambda_N = \lambda_N (\theta_N, h) = \theta_N^{\epsilon_N} h^{\nu_N} \quad 0 < \epsilon_N, \mu_N < 1. \]

Business opportunities and start-up matching equilibrium:

\[ O = \vartheta N \]

\[ \lambda_N (\theta_N, h) \Delta_N = O. \]

\[ \vartheta^{\epsilon_N} \delta_N^{(1-\zeta)\vartheta} \delta_N = \vartheta \]

\[ \delta_N^{(1-\zeta)\vartheta} \epsilon_N h^{\nu_N} = 1. \]

Equilibrium surplus rate of business ideas:

\[ \delta_N = h^{-\nu_N (1-\zeta)/(1+\zeta)} \]

\[ \frac{d \delta_N}{dh} = -\nu_N h^{-\frac{\nu_N}{(1-\epsilon_N)(1-\zeta)}(1+\zeta)} < 0 \]

Equilibrium contract separation rate

\[ \vartheta(\delta_N) = \delta_N^{\zeta} = h^{-\nu_N (1-\zeta)/(1+\zeta)}, \]

\[ \frac{d \vartheta}{dh} = -\nu_N \zeta h^{-\frac{\nu_N}{(1-\epsilon_N)(1-\zeta)}(1+\zeta)} < 0 \]
Appendix 4

a) Start-up costs: agglomeration $\varepsilon$ and firm density $N/Y$ effects and start up costs $\chi$:

$$
\chi(N/Y) = \varepsilon \frac{N}{Y}, \text{ with } \varepsilon > 0, \text{ and } \frac{Y}{N} = AH^{1-\alpha} X^\alpha
$$

$$
= \varepsilon \left( AH^{1-\alpha} X^\alpha \right)^{-1}
$$

$$
= \varepsilon \left( H \left( \frac{A \alpha}{p_j} \right)^{1-\alpha} \right)^{-1} = \varepsilon \left[ \left( \frac{X}{H} \right)^{\alpha} HA \right]^{-1}
$$

b) Start-up firms’ decisions

Present value of the cash flow, of the new firm including probability of success $(1-\vartheta)$ and failure $\vartheta$:

$$
EV_n(t) = (1-\vartheta) \int_0^\infty (p_j - c_{sj} - E_{yr,i}) x_j e^{-r_d(t+t_r-t)} dt + \vartheta 0,
$$

with : $c_{sj} = \bar{c}_{sj} + E_{yr,i}$

$$
= \frac{(1-\vartheta)}{r_d} (p_j - c_{sj}) x_j
$$

$$
= \frac{(1-\vartheta)}{r_d} (p_j - c_{sj}) H_j \left( \frac{A \alpha}{p_j} \right)^{1-\alpha}
$$

Maximizing the present value of the firm:

$$
\frac{dEV_n}{dp_j} = \frac{(1-\vartheta)}{r_d} H_j \left( \frac{A \alpha}{p_j} \right)^{1-\alpha} - \frac{1}{1-\alpha} \frac{(1-\vartheta)}{r_d} (p_j - c_{sj}) H_j \left( \frac{A \alpha}{p_j} \right)^{1-\alpha} \frac{\alpha}{(p_j)^2} = 0
$$

$$
= \frac{(1-\vartheta)}{r_d} H_j \left( \frac{A \alpha}{p_j} \right)^{1-\alpha} - \frac{1}{1-\alpha} (p_j - c_{sj}) \left( \frac{A \alpha}{p_j} \right)^{1-\alpha} \frac{\alpha}{(p_j)^2} = 0
$$

$$
= \frac{(1-\vartheta)}{r_d} H_j \left( \frac{A \alpha}{p_j} \right)^{1-\alpha} \left[ 1 - \frac{(p_j - c_{sj}) 1}{1-\alpha p_j} \right] = 0
$$

$$
0 = 1 - \frac{(p_j - c_{sj}) 1}{1-\alpha p_j}
$$

$$
(1-\alpha) p_j = (p_j - c_{sj})
$$

$$
p_j = \frac{c_{sj}}{\alpha}
$$
c) Optimal periodic profit:

\[ c_{s_j} = \overline{c}_{s_j} + E y_{r,i} \quad \text{and} \quad p_j = \frac{c_{s_j}}{\alpha} \]

\[ x_j = H \left( \frac{A \alpha}{p_j} \right)^{\frac{1}{\gamma}} = H \left( \frac{A \alpha^2}{c_{s_j}} \right)^{\frac{1}{\gamma}} \]

\[ \pi_j = (p_j - c_{s_j}) x_j \]

\[ = \left( \frac{1}{\alpha} - 1 \right) c_{s_j} H \left( \frac{A \alpha^2}{c_{s_j}} \right)^{\frac{1}{\gamma}} \]

\[ = (1 - \alpha) \frac{c_{s_j}}{\alpha} \left( \frac{A \alpha^2}{c_{s_j}} \right)^{\frac{1}{\gamma}} \]

\[ = (1 - \alpha) \frac{c_{s_j}}{\alpha} HA^{\frac{1}{\gamma}} \alpha^{\frac{1}{\gamma}} c_{s_j}^{-\frac{1}{\gamma}} \]

\[ = (1 - \alpha) HA^{\frac{1}{\gamma}} \alpha^{\frac{1}{\gamma}} c_{s_j}^{-\frac{1}{\gamma}} \]

\[ d) \text{Expected Maximum net present value of cash flows is} \]

\[ EV_m = \frac{1 - \vartheta}{r_d} (p_j - c_{s_j}) H \left( \frac{A \alpha}{p_j} \right)^{\frac{1}{\gamma}} \]

\[ = \frac{1 - \vartheta}{r_d} c_{s_j} \left( \frac{1}{\alpha} - 1 \right) H \left( \frac{A \alpha^2}{c_{s_j}} \right)^{\frac{1}{\gamma}} \]

\[ = \frac{1 - \vartheta}{r_d} c_{s_j} \left( \frac{1}{\alpha} - 1 \right) H \left( A \alpha^2 \right)^{\frac{1}{\gamma}} c_{s_j}^{-\frac{1}{\gamma}} \]

\[ = \frac{1 - \vartheta}{r_d} c_{s_j} \left( \frac{1}{\alpha} - 1 \right) H c_{s_j}^{\frac{1}{\gamma}} A^{\frac{1}{\gamma}} \alpha^{-\frac{1}{\gamma}} \]

\[ = \frac{1 - \vartheta}{r_d} c_{s_j}^{\frac{1}{\gamma}} (1 - \alpha) \alpha^{-1} H \left( \alpha^2 \right)^{\frac{1}{\gamma}} A^{\frac{1}{\gamma}} c_{s_j}^{-\frac{1}{\gamma}} \alpha^{-\frac{1}{\gamma}} \]

\[ = \frac{1 - \vartheta}{r_d} c_{s_j}^{\frac{1}{\gamma}} (1 - \alpha) HA^{\frac{1}{\gamma}} \alpha^{-\frac{1}{\gamma}} \]

\[ = \frac{1 - \vartheta}{r_d} c_{s_j}^{\frac{1}{\gamma}} (1 - \alpha) HA^{\frac{1}{\gamma}} \alpha^{\frac{1}{\gamma}} \]

\[ = (1 - \vartheta) \frac{1 - \alpha}{r_d} H \left( \frac{A \alpha^{\frac{1}{\gamma}}}{c_{s_j}^{\frac{1}{\gamma}}} \right)^{\frac{1}{\gamma}} \]
e) Solving for the loan rate:
Present value of start up costs including borrowing:

\[ V_s = \chi \frac{r_l}{r_d} \]

No arbitrage condition: present value of start up costs = present value of cash flows

\[ V_s = V_m \]

Plugging in everything to solve for the loan rate:

\[ \chi (N/Y) \frac{r_l}{r_d} = (1-\theta)(1-\alpha)H \left( A \frac{1+\alpha}{c_{ij}} \right) \]

\[ \varepsilon \left( \frac{x}{H} \right)^{\alpha} \frac{r_l}{r_d} = (1-\theta)(1-\alpha)H \left( A \frac{1+\alpha}{c_{ij}} \right) \]

\[ \varepsilon \left( \frac{H \left( \frac{1+\alpha}{c_{ij}} \right)}{H} \right)^{\alpha} \frac{r_l}{r_d} = (1-\theta)(1-\alpha)H \left( A \frac{1+\alpha}{c_{ij}} \right) \]

\[ \varepsilon \left( \frac{1+\alpha}{c_{ij}} \right)^{\alpha} \frac{r_l}{r_d} = (1-\theta)(1-\alpha)H \left( A \frac{1+\alpha}{c_{ij}} \right) \]

\[ \varepsilon \left( \frac{1+\alpha}{c_{ij}} \right)^{\alpha} \frac{1}{r_l} = (1-\theta)(1-\alpha)H^2 \frac{1+\alpha}{c_{ij}} A \frac{1+\alpha}{c_{ij}} A \]

\[ \varepsilon \frac{1+\alpha}{c_{ij}} \frac{r_l}{r_d} = (1-\theta)(1-\alpha)H^2 \frac{1+\alpha}{c_{ij}} \frac{1+\alpha}{c_{ij}} A \frac{1+\alpha}{c_{ij}} \]

\[ \varepsilon r_l = (1-\theta)(1-\alpha)H^2 \frac{1+\alpha}{c_{ij}} \frac{1+\alpha}{c_{ij}} A \frac{1+\alpha}{c_{ij}} A \frac{1+\alpha}{c_{ij}} \]

\[ r_l = \frac{(1-\theta)(1-\alpha)}{\varepsilon(A)} H^2 \frac{1+\alpha}{c_{ij}} \frac{1+\alpha}{c_{ij}} A \frac{1+\alpha}{c_{ij}} A \frac{1+\alpha}{c_{ij}} \]
Appendix 5

Financial Sector and Imperfect Financial Intermediation with default risk of start-up firms and optimal activities of banks: maximize expected bank profits:

\[ E\pi_b = r_l(1 - \vartheta)K_b - r_d(D)D_b - c_b(\vartheta)K_b - c_b \quad \text{see (18)} \]

Bank balance sheet:

\[
K_b = D_b \quad \text{see (19)} \\
\frac{\partial E\pi_b}{\partial D_b} = r_l(1 - \vartheta) - r_d(D) + \frac{dr_d}{dD} \frac{\partial D}{\partial D_b} D_b - c_b(\vartheta) = 0 \\
= r_l(1 - \vartheta) + \left( \frac{dr_d}{dD} \frac{D}{r_d} \frac{1}{B} - 1 \right) r_d(D) - c_b(\vartheta) \\
= r_l(1 - \vartheta) - \left( \frac{1}{\eta B} + 1 \right) r_d(D) - c_b(\vartheta)
\]

Optimal loan deposite spread:

\[
r_d = \left( r_l(1 - \vartheta) - c_b(\vartheta) \right) \left( 1 + \frac{1}{B\eta D,r_d} \right) \quad \text{see (20)}
\]
Appendix 6: Proof of Proposition 2

Traditional sector: wages per capita income and population dynamics:

a) wage and per capita income in the traditional sector:

\[ Y_T = \bar{w}L, \quad \bar{w} = a_T \]

\[ y_T = y_T(\overline{w}, \delta_T) = \frac{Y_T}{Pop} = \frac{a_T L}{L(1 + \delta_T)} = \frac{a_T}{(1 + \delta_T)} \]

\[ Ey_T = a_T \frac{L}{L + \delta_T} + 0 \frac{\Delta_T}{L + \Delta_T} = y_T = \frac{a_T}{(1 + \delta_T)} \]

b) Population dynamics:

\[ \gamma_L = \frac{\dot{L}}{L} = g_L(y_T) = g_L(\overline{w}, \delta_T) = \left( \frac{a_T}{(1 + \delta_T)} \right)^\phi \]

c) Properties of the population dynamics curve in the \( \gamma_L - \delta_T - \text{diagram} \) slope:

\[ \frac{d\gamma_L}{d\delta_T} = -\varphi(a_T(1 + \delta_T)^{-1})^{-\phi} a_T(1 + \delta_T)^{-2} \]

\[ = -\varphi a_T^\phi (1 + \delta_T)^{-\phi-1} < 0 \]

Location:

\[ \frac{d\gamma_L}{da_T} = \varphi a_T^{\phi-1} (1 + \delta_T)^{-\phi} > 0 \]

\[ \ln \gamma_L = \varphi \ln \left( \frac{a_T}{1 + \delta_T} \right) \]

\[ \frac{d\gamma_L}{d\varphi} \gamma_L = \ln \left( \frac{a_T}{1 + \delta_T} \right) > 0 \quad \text{for} \quad \frac{a_T}{(1 + \delta_T)} > 1 \]
Appendix 7: Proof of Proposition 3

a) Labour market frictions and surplus labour:

separation rate:
\[ V = \sigma L \]

matching rate:
\[ \lambda = \lambda(\theta_T^m, \gamma_L^m) = \theta_T^m \gamma_N^u, \quad \theta_T = \sigma^{-1} \delta_T \]
\[ = \lambda(\sigma, \delta_T, \gamma_N) = \sigma^{-1} \delta_T^\varepsilon \gamma_N^u \quad 1 < \varepsilon, 0 < \nu < 1. \]

search and matching equilibrium and equilibrium surplus labour rate:
\[ \lambda \Delta_T = \sigma L \]
\[ \lambda(\sigma, \delta_T, \gamma_L) \Delta_T = \sigma L \]
\[ \sigma^\varepsilon \delta_T^{-\varepsilon} \gamma_N^u \delta_T = \sigma. \]
\[ \sigma^\varepsilon \delta_T^{-\varepsilon} \gamma_N^u = \sigma \]
\[ \delta_T^{-\varepsilon} = \sigma^{1-\varepsilon} \gamma_N^{1-\nu} \]
\[ \delta_T^{1-\varepsilon} = \sigma^{1-\varepsilon} \gamma_N^{-\nu} \]
\[ \delta_T = \delta_T(\sigma, \gamma_N) = \sigma \gamma_N^{\frac{\varepsilon}{1-\nu}} \]

b) Properties of the surplus labour curve in the \( \gamma_N - \delta_T - \text{diagram} \) slope:
\[ \frac{d \delta_T}{d \gamma_N} = -\frac{\nu}{1-\varepsilon} \sigma \gamma_N^{\frac{\varepsilon}{1-\nu}} < 0 \]

Location:
\[ \frac{d \delta_T}{d \sigma} = \gamma_N^{\frac{\varepsilon}{1-\nu}} > 0 \]
Appendix 8: Proof of Proposition 4

a) Deriving the start-up curve:
Combining (20) and (17) gives the deposite rate offered to the households

\[ r_y = \left( \frac{(1-\varphi(h))(1-\alpha)}{\epsilon} H^2 \alpha^{\frac{1-\eta}{\alpha}} \xi_x c_x^{\frac{2-\alpha}{\alpha}} A^{\frac{1}{1-\alpha}+1} \right) \left( 1 - \varphi(h) \right) - c_y(\varphi(h)) \left( 1 + \frac{1}{B\eta_{D,c_y}} \right) \]

With this deposite rate, and (14), (23) and the Ramsey rule (3) we can determine the optimal growth rate of the consumption path and the entrepreneurial start-up growth rate \( \gamma_N \) of the modern sector

\[ \gamma_N = \frac{1}{\Theta} \left[ \left( \frac{(1-\varphi(h))^2(1-\alpha)}{\epsilon (A)} \right) H^2 \left( \frac{\alpha^{\frac{1-\eta}{\alpha}}}{\xi_x + \frac{\alpha\varphi_y}{\alpha+\varphi_y}} \right)^{\frac{2-\alpha}{\alpha}} A^{\frac{1}{1-\alpha}+1} \right] - c_y(\varphi(h)) \left( 1 + \frac{1}{B\eta} \right) - \rho \]

\[ = g_N(\frac{\delta_T, \alpha_T}{\text{traditional sector}}, H, h, \xi_x, B, \Theta, \rho). \]

b) Properties of the start-up curve in the \( \gamma_N - \delta_T - \text{diagram} \) slope:

\[ \frac{d\gamma_N}{d\delta_T} = \frac{2\alpha \epsilon \varphi_y (1-\varphi(h))^2 (1-\alpha)}{\Theta (1-\alpha) \epsilon \xi_x c_x^{\frac{2-\alpha}{\alpha}} A^{\frac{1}{1-\alpha}+1} \left( 1 + \frac{1}{B\eta} \right)} > 0 \]

location:

\[ \frac{d\gamma_N}{dH} = \frac{2(1-\varphi(h))^2 (1-\alpha)}{\Theta \epsilon} H^2 \alpha^{\frac{1-\eta}{\alpha}} \xi_x c_x^{\frac{2-\alpha}{\alpha}} A^{\frac{1}{1-\alpha}+1} \left( 1 + \frac{1}{B\eta} \right) > 0 \]

\[ \frac{d\gamma_N}{d\alpha_T} = -\frac{2\alpha (1-\delta_T)(1-\varphi(h))(1-\alpha)}{\Theta (1-\alpha) \epsilon \xi_x c_x^{\frac{2-\alpha}{\alpha}} A^{\frac{1}{1-\alpha}+1}} \left( 1 + \frac{1}{B\eta} \right) < 0 \]

\[ \frac{d\gamma_N}{dh} = -\frac{2(1-\varphi(h))(1-\alpha)}{\epsilon} H^2 \xi_x c_x^{\frac{2-\alpha}{\alpha}} \alpha^{\frac{1-\eta}{\alpha}} A^{\frac{1}{1-\alpha}+1} \left( 1 + \frac{1}{B\eta} \right) \frac{d \varphi(h)}{dh} - \frac{dc_y}{dh} \frac{d \varphi(h)}{dh} > 0 \]
Appendix 9: Proof of Proposition 5

Given the system

\[ 0 = F_1(\delta_T, \gamma_N, \sigma) = \delta_T(\gamma_N, \sigma) - \delta_T \quad \text{(surplus labour rate)} \]

\[ 0 = F_2(\delta_T, \gamma_N, a_T, H, ..., \rho) = g_N(\delta_T, a_T, H, ..., \rho) - \gamma_N \quad \text{(start-up rate)} \]

\[ 0 = F_3(\delta_T, \gamma_L, a_T, \varphi) = g_L(\delta_T, a_T, \varphi) - \gamma_L \quad \text{(population dynamics)} \]

we want to solve for the set of implicit functions

\[ \tilde{\delta}_T = \tilde{\delta}_T(a_T, H, h, B, \varphi, \bar{c}, ..., A, \Theta, \rho) \quad \text{surplus labour rate} \]

\[ \tilde{\gamma}_N = \tilde{\gamma}_N(a_T, H, h, B, \varphi, \bar{c}, ..., A, \Theta, \rho) \quad \text{start-up rate} \]

\[ \tilde{\gamma}_L = \tilde{\gamma}_L(a_T, H, h, B, \varphi, \bar{c}, ..., A, \Theta, \rho) \quad \text{population dynamics} \]

a) Partial derivatives of \( F_1, F_2, F_3 \) with respect to all relevant variables exist and are continuous:

For \( F_1 \) we obtain:

\[ \frac{dF_1}{d\delta_T} = -1, \quad \frac{dF_1}{d\gamma_N} = -\frac{\nu}{1 - \epsilon} \sigma \gamma_N^{\tau-1} < 0 \]

\[ \frac{dF_1}{d\sigma} = \gamma_N^{- \tau}, \quad \frac{dF_1}{dH} = ... = \frac{dF_1}{d\rho} = 0. \]

For \( F_2 \) we obtain:

\[ \frac{dF_2}{d\delta_T} = \frac{dg_N}{d\delta_T}, \quad \frac{dF_2}{d\gamma_N} = -1 \]

\[ \frac{dF_2}{da_T} < 0, \quad \frac{dF_2}{dH} = \frac{dg_N}{dH} > 0, \quad \frac{dF_2}{dh} = \frac{dg_N}{dh} > 0. \]

For \( F_3 \) we obtain:

\[ \frac{dF_3}{d\delta_T} = -\varphi \left( \frac{a_T}{1 + \delta_T} \right)^\varphi < 0, \quad \frac{dF_3}{d\gamma_L} = -1 \]

\[ \frac{dF_3}{da_T} = \frac{\varphi}{(1 + \delta_T)} \left( \frac{a_T}{1 + \delta_T} \right)^{\varphi-1}, \quad \frac{dF_3}{d\varphi} = \left( \frac{a_T}{(1 + \delta_T)} \right)^\varphi \ln \left( \frac{a_T}{(1 + \delta_T)} \right) > 0 \]

\[ \frac{dF_3}{dH} = ... = \frac{dF_3}{d\rho} = 0 \]

b) Jakobian of \( (32) \) system can be determined by:

\[ J' = \begin{bmatrix}
-1 & \frac{dF_1}{d\gamma_N} & 0 \\
\frac{dF_2}{d\delta_T} & -1 & 0 \\
\frac{dF_3}{d\delta_T} & 0 & -1
\end{bmatrix} \neq 0 \]
Considering a) and b) than there exists an m-dimensional neighbourhood ($m$ is the number of exogenous variables), in which the variables $\tilde{\delta}_r, \tilde{\gamma}_N, \tilde{\gamma}_L$ are functions of the form

$$\tilde{\delta}_r = \tilde{\delta}_r (a_r, H, h, B, \varphi, \bar{e}_x, ..., A, \Theta, \rho) \quad \text{surplus labour rate}$$

$$\tilde{\gamma}_N = \tilde{\gamma}_N (a_r, H, h, B, \varphi, \bar{e}_x, ..., A, \Theta, \rho) \quad \text{start-up rate}$$

$$\tilde{\gamma}_L = \tilde{\gamma}_L (a_r, H, h, B, \varphi, \bar{e}_x, ..., A, \Theta, \rho) \quad \text{population dynamics}$$
References


