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**Article**

## Is there a lack of public capital in the European Union?

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## ABSTRACT

*This paper investigates whether there is a lack of public capital in European Union countries. The analysis builds on a new database providing internationally comparable capital stock estimates for 22 OECD countries, among these 14 EU countries, for the period 1960-2001. A simple model of endogenous growth is used in order to highlight the inherently nonlinear relationship between public capital and economic growth and to derive a measure of the growth-maximising public capital stock. Against this background, the empirical analysis provides estimates of the growth-maximising level of public capital for EU countries. The empirical results suggest that there currently is no lack of public capital in most 'old' EU countries.*

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# Is there a lack of public capital in the European Union?

## 1. Introduction

The decline in government investment in many OECD countries has alerted policy makers and researchers alike. In the Broad Economic Policy Guidelines, the European Commission (2001) recommended the member countries of the European Union to draw “an appropriate balance and sequencing [...] between running down public debt, cutting taxes and financing public investment in key areas. To this end Member States should [...] redirect public expenditures towards physical and human capital accumulation.” Modigliani *et al.* (1998) and Blanchard and Giavazzi (2003), in turn, proposed to exclude net public investment from the measure of the budget deficit underlying the Stability and Growth Pact in order to prevent future fiscal consolidation in the euro area from negatively affecting public capital spending. Finally, at the end of the 1990s, the government in the United Kingdom launched a large multi-annual spending programme aimed at reversing the falling trend in the ratio of public capital spending to GDP, which had fallen to a very low level by 1997 (HM Treasury 2002).



**Christophe Kamps**

Figure 1 displays the ratios of government investment and government capital stock to GDP for a group of 22 OECD countries and for a group of 14 European Union member states.<sup>1</sup> The left-hand panel of the figure shows that in both groups of countries government investment in relation to GDP declined strongly in the 1970s and remained roughly stable in the 1980s. In the 1990s, government investment in relation to GDP again fell in the EU, probably reflecting in part fiscal consolidation efforts in the run-up to Economic and Monetary Union, while in other OECD countries—especially in Japan—government investment grew. Based on the evolution of public investment, the right-hand panel of Figure 1 depicts the evolution of public capital stocks.

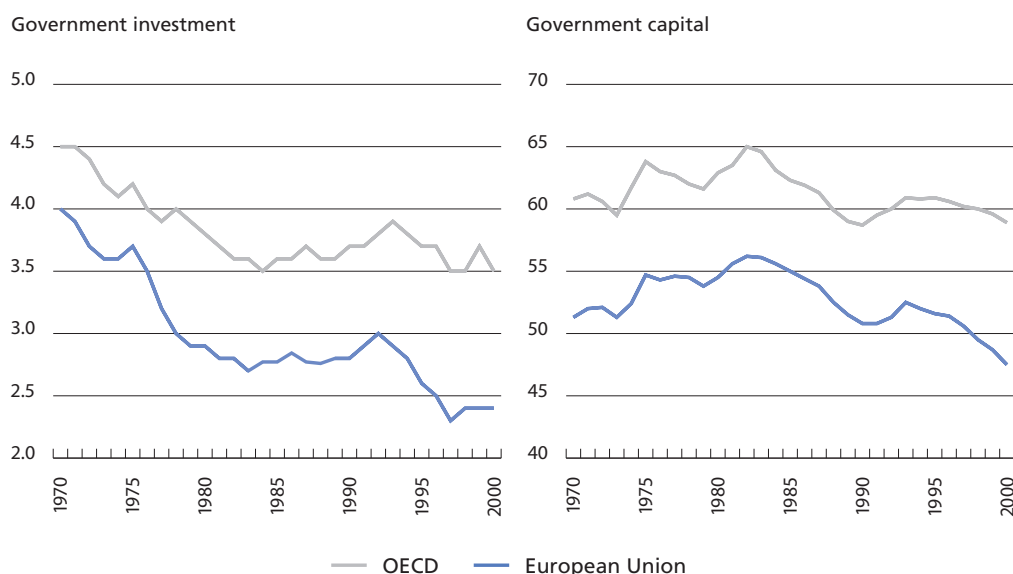
While government investment in relation to GDP has fallen strongly in OECD countries on average over the past three decades, the ratio of government capital to GDP has decreased only slightly. Had the high investment rates of the early 1970s been sustained, government capital to GDP ratios would even have increased. Whether the recent decline in the ratio of government capital to GDP is a reason for concern depends on the economic benefits and costs of additional government capital and is, thus, an empirical question.

The calls for an expansion of public capital spending, especially in European countries, share two important features: they build on the implicit assumptions that (i) public capital is productive and (ii) the decline in the ratios of public investment to GDP over the past decades has reduced the stock of public capital in many OECD countries below its optimal level.<sup>2</sup> The consensus view of the empirical literature is that public capital is indeed productive on average (see Romp and de Haan, this volume, for a survey). Yet, there is little empirical evidence showing that there is too little public capital in OECD countries. In fact, for most OECD countries no empirical evidence at all has been available so far. An exception is the study of Aschauer (2000), which suggests that the actual levels of public capital in the United States during the 1970s and 1980s were below the levels estimated to maximise economic growth.

<sup>1</sup> The European Union aggregate includes the following countries: Austria, Belgium, Denmark, France, Finland, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom. The OECD aggregate in addition includes also Australia, Canada, Iceland, Japan, New Zealand, Norway, Switzerland, and the United States. The data are drawn from Kamps (2005) who provides internationally comparable capital stock estimates for these 22 countries.

<sup>2</sup> Insofar as an increase in government spending is asked for, these calls are also based on the assumption that the capital goods concerned—e.g., additions to the stock of infrastructure—should be provided by the government rather than by the private sector.

**Figure 1. Real government gross fixed capital formation and real government net capital stock (in % of real GDP) in OECD and EU countries, 1970–2000**



Sources: OECD, Kamps (2005).

*This paper investigates whether there is a lack of public capital in EU countries.*

This paper adopts Aschauer's methodology in order to investigate whether there is a lack of public capital in EU countries. The analysis builds on a new database providing internationally comparable capital stock estimates for 22 OECD countries, among these 14 EU countries, for the period 1960–2001. A simple model of endogenous growth is used to highlight the inherently nonlinear relationship between public capital and economic growth. Against this background, the empirical analysis attempts to estimate the growth-maximising level of public capital for EU countries as a group.

The remainder of the paper is organised as follows. Section 2 presents a simple model of endogenous growth with public capital, which is used to derive the empirical model for estimation purposes. Section 3 discusses the data used in the empirical analysis. Section 4 estimates the relationship between public capital and economic growth based on linear and nonlinear specifications. Section 5 combines the implications of the theoretical model and the regression results to construct an estimate of the growth-maximising ratio of public capital to GDP and then compares actual public capital stocks in individual EU member states with the level that maximises aggregate growth. Section 6 concludes by discussing limitations of the methodology employed in this study and suggests avenues for further research.

## 2. Modelling public capital and economic growth

This section outlines the most important features of a simple endogenous growth model that is often used in theoretical analyses of the growth effects of public capital, and which motivates our empirical analysis.

In endogenous growth models, government policy has the potential to affect the long-run growth rate of the economy. The growth effects of the flow of productive government spending are analysed, e.g., in Barro (1990), Barro and Sala-i-Martin (1992), and Greiner (1996). In Barro's (1990)

model the growth rate is maximised when the share of productive government spending in output is equal to the elasticity of output with respect to productive government spending. The growth effects of the stock of public capital are analysed, e.g., in Turnovsky (1997) and Aschauer (2000).

An important benefit of using Aschauer's (2000) endogenous growth model is that it allows to explicitly test for growth effects of public capital without—as is shown in Section 4.2—sacrificing potential level effects of public capital if growth effects turn out to be absent. That said, it does not seem to be central whether an endogenous or an exogenous growth model is used. Ultimately, what matters is to have a theoretical framework that allows the derivation of a measure of optimal fiscal policy. This can be done in both model frameworks. In his survey of the empirical growth literature, Temple (1999) concludes that while the debate whether policy affects the long-run growth rate or just the steady-state level of income is impossible to resolve, the distinction between exogenous and endogenous growth models is less important than it might seem because even if policy does not affect growth, it still has large effects on the level of income and welfare.

***Policies that do not permanently raise the growth rate of an economy are nevertheless important if they raise the level of income.***

The model used by Aschauer is outlined briefly in Box 1, and the Annex provides further details and variations to this model. Here we discuss the model's main economic implications, which follow from equations (B6) to (B9) in Box 1. First, as equation (B6) shows, the model implies a nonlinear relationship between public capital and economic growth. Second, equation (B7) gives the growth-maximising ratio of public to private capital,  $\varphi^{\max}$ . It shows that the growth-maximising ratio of public to private capital is the larger, the larger the elasticity of output with respect to public capital,  $\alpha_{kg}$ . Third, equation (B8) captures a straightforward economic feature of  $\varphi^{\max}$ : the left-hand side of this equation is equal to the after-tax marginal product of private capital while the right-hand side shows the marginal product of public capital. Hence, the economic growth rate is maximised at the ratio of public to private capital for which the marginal product of public capital is equal to the after-tax marginal product of private capital. If the ratio of public to private capital is larger (smaller) than this value, the growth rate is smaller (larger) than the maximal growth rate. For completeness, we note that maximising economic growth also requires that the marginal product of public capital and the after-tax marginal product of private capital are equal to the interest rate,  $r$ , on government debt. Fourth, equation (B9) pictures a variation of the Aschauer model, with  $\varphi^*$  showing the welfare-maximising ratio of public to private capital. This ratio is smaller than the growth-maximising ratio for this model. The rationale for this result is the following: while growth maximisation implies that the marginal product of public capital should be equal to the after-tax marginal product of private capital, welfare maximisation implies that the marginal product of public capital should equal the gross marginal product of private capital. The distortions associated with taxation discourage the accumulation of private capital and, hence, the public to private capital ratio with growth maximisation is lower than in the first-best optimum. Maximising the growth rate involves a consumption loss and, thereby, lowers welfare.

In the empirical analysis, we follow Aschauer (2000) and focus on the growth-maximising public to private capital ratio. As it turns out, the main conclusions are unaffected by this choice. In Section 4, we present empirical results both for a linear specification of the relationship between public capital and economic growth and for a nonlinear specification, which is closely related to the growth model discussed above. The theoretical model implies that in the steady state the growth rate depends on the nonlinear term  $\varphi^{\alpha_{kg}} / [1 + (1 - \alpha_{kg}) \varphi]$ , i.e., there is a nonlinear relationship between the growth rate on the one hand and the public to private capital ratio,  $\varphi$ , and the elasticity of output with respect to public capital,  $\alpha_{kg}$ , on the other hand (see (A.10) in the Annex). This nonlinear term is used in the regressions reported in Section 4.2, providing estimates of the parameter  $\alpha_{kg}$ , which is the determinant of both the growth-maximising and the welfare-maximising public to private capital ratio.

### Box 1. Modelling the optimal stock of public capital

Consider an infinitively living representative agent who has preferences over consumption,  $c$ , as described by the lifetime utility function

$$(B1) \quad V = \int_0^{\infty} (c^{1-\sigma} - 1) / (1-\sigma) e^{-\rho t} dt,$$

where  $\sigma$  is the inverse of the elasticity of intertemporal substitution and  $\rho$  is the rate of time preference. The agent has access to a Cobb-Douglas production function

$$(B2) \quad y = k^{\alpha_k} kg^{\alpha_{kg}} \quad \text{with} \quad \alpha_k + \alpha_{kg} = 1,$$

where  $y$  is output,  $k$  is private capital, and  $kg$  is public capital. The production function specifies constant returns to scale in  $k$  and  $kg$ . For this reason, the economy is capable of endogenous growth. All variables are expressed in per worker terms. The model abstracts from a labour/leisure choice, from population growth, from technological progress, and from depreciation of both types of capital.

The government is assumed to maintain a particular ratio of public to private capital

$$(B3) \quad \varphi = kg/k,$$

which requires that the public capital stock be increased in line with the steady-state growth rate of the economy,  $\gamma$ , so that  $\dot{kg} = \gamma kg$ . The government levies a tax on output at rate  $\theta$ , which finances the interest payments on the stock of government debt,  $r \cdot b$ . The initial stock of public capital and public investment, instead, are financed by government debt. The flow government budget constraint is given by

$$(B4) \quad \dot{b} = r \cdot b + \dot{kg} - \theta \cdot y.$$

### 3. On the data

This section describes the data used in the empirical analysis. It first looks in some detail at the measures of public and private capital used. It then provides information about the other variables needed for the empirical analysis, like output and employment. Until recently, no information about the magnitude of public and private capital stocks was available for most OECD countries. A recent study (Kamps 2005) provides new estimates of real government and private net capital stocks for 22 OECD countries for the period 1960–2001.

The capital stocks are estimated using the so-called perpetual inventory method, based on historical data on gross investment provided by the OECD. These investment series cover capital spending of the general government, i.e., the central, local, and regional governments, including social security funds.<sup>3</sup> The perpetual inventory method builds on a standard capital accumulation equation.

3 These investment series do not cover gross investment by public enterprises. General government and non-financial public enterprises together form the public sector. In this paper the terms 'government capital' and 'public capital' are used interchangeably and refer to the general government only.

The agent maximises lifetime utility subject to the resource constraint

$$(B5) \quad \dot{k} + \dot{b} + c = (1 - \theta)y + r \cdot b.$$

The steady state of this economy involves a common growth rate for consumption, output, public and private capital given by

$$(B6) \quad \gamma = \frac{1}{\sigma} [(1 - \theta)(1 - \alpha_{kg})\varphi^{\alpha_{kg}} - \rho],$$

where the first term in square brackets is the after-tax marginal product of private capital. Equation (B6) shows that the growth rate of the economy depends positively on the ratio of public to private capital,  $\varphi$ , and negatively on the tax rate,  $\theta$ .

As shown in the Annex, the growth rate of per worker output is maximised at the value of the public to private capital ratio given by

$$(B7) \quad \varphi^{\max} = \alpha_{kg} / (1 - \alpha_{kg})^2.$$

As also shown in the Annex, the growth-maximising, steady-state public to private capital ratio is characterised by

$$(B8) \quad (1 - \theta)(1 - \alpha_{kg})\varphi^{\alpha_{kg}} = \alpha_{kg}\varphi^{\alpha_{kg} - 1}.$$

The economics of equation (B8) are discussed in the text.

The Annex also derives the welfare-maximising, as opposed to growth-maximising, steady-state growth path. Suffice to note here that the welfare-maximising public to private capital ratio,  $\varphi^*$ , is given by

$$(B9) \quad \varphi^* = \alpha_{kg} / (1 - \alpha_{kg}).$$

Bearing in mind that  $\alpha_{kg} > 0$ , a comparison of (B9) with (B7) shows that  $\varphi^* < \varphi^{\max}$ .

Its implementation requires making assumptions on the magnitude of an initial-year capital stock, the depreciation method, and on the magnitude and time profile of the depreciation rate. Kamps (2005) estimates his capital stock series based on the geometric depreciation method and a time-varying depreciation rate; the initial-year problem is solved through the construction of an artificial investment series for the pre-1960 period (for details see Kamps 2005). Estimates are available for the government capital stock, the private non-residential capital stock, and for the private residential capital stock.<sup>4</sup>

Table 1 displays the ratios of public capital to GDP for three years of reference: 1980, 1990, and 2000. The average ratio for the 14 EU countries considered has declined by 7 percentage points over the period 1980–2000, reaching around 48 percent in 2000. Remarkably, there has been considerable convergence regarding public capital to GDP ratios in the EU—the standard deviation of the ratio of public capital to GDP has fallen from 17 percent in 1980 to less than 7 percent in 2000. Public capital as a share of GDP has declined in 9 countries since the early 1980s; it has increased slightly in 4 countries, while it has risen strongly in Greece, Portugal, and Spain.

4 The capital stock estimates are available on request to the author or at the website <http://www.ifw-kiel.de/staff/kampsc.htm>

**Table 1. Government capital stock in % of GDP**

Country	1980		1990		2000	
	Ratio	Rank	Ratio	Rank	Ratio	Rank
Austria	75.4	4	69.3	1	57.0	1
Belgium	40.2	12	45.5	11	37.9	13
Denmark	76.4	2	60.8	4	45.9	9
Finland	43.7	10	47.1	10	46.9	8
France	55.0	7	53.0	5	54.0	3
Germany	58.4	6	52.0	6	47.1	7
Greece	44.4	9	51.9	7	51.0	4
Ireland	75.9	3	66.8	3	35.2	14
Italy	44.7	8	49.0	8	47.9	6
Netherlands	80.2	1	68.9	2	56.4	2
Portugal	27.9	14	32.0	14	43.3	10
Spain	35.8	13	40.9	12	48.0	5
Sweden	42.1	11	40.2	13	42.0	11
United Kingdom	63.9	5	48.5	9	40.3	12
EU average	54.5		50.8		47.5	
Standard deviation	17.3		11.3		6.6	

Note: (1) 'Ratio' gives the ratio of the real government net capital stock and real GDP. (2) 'Rank' gives the ranking of the countries according to the size of their capital to GDP ratio. (3) The EU average is a weighted average, computed on the basis of 1995 GDP weights expressed in 1995 purchasing power parities, taken from OECD (2002). (4) Capital stock and GDP in 1995 prices.

Source: Kamps (2005) and own calculations.

According to the estimates, in 2000, Austria had the highest ratio of public capital to GDP among the EU countries considered in this paper, while Ireland had the lowest.<sup>5</sup> The large decline in the ratio of public capital to GDP in Ireland during the 1990s mirrors the strong fall in public investment as a share of GDP during this period. Notably, the Irish government views the lack of infrastructure as one of the major impediments to growth (see Fitz Gerald *et al.* 1999 for details).

***The public capital stock (relative to GDP and private capital) has declined, on average, in pre-enlargement EU countries, as has its variation across countries.***

Another way to compare the government capital stock across countries is to look at the ratio of government capital to private capital. This is especially interesting in view of the theoretical model presented in the previous section, which suggests a relationship between the growth rate,  $\gamma$ , and the government to private capital ratio,  $\phi$ . Table 2 gives the ratio of public capital to private non-residential capital for the years 1980, 1990, and 2000. The average ratio for the 14 EU countries is somewhat smaller than the average ratio of public capital to GDP: it reached around 40 percent in 2000, down from about 44 percent in 1980. As in the case of public capital to GDP, there is evidence of substantial convergence, as the standard deviation of public to private capital has declined strongly over the period 1980–2000. As regards the ranking of countries, it is striking that Ireland ranks considerably higher than in Table 1. This discrepancy can be explained by the different dynamics of private capital and output in this country. Output growth in Ireland was very fast and much faster than growth in private capital in the 1990s, leading not only to a decreasing ratio of public capital to GDP but also to a decreasing ratio of private capital to GDP.

<sup>5</sup> It has to be kept in mind, though, that the public capital to GDP ratio in Austria is much lower than that in Japan, which amounted to 117 percent in 2000 (see Kamps 2005).



**Table 2. Government capital in % of private non-residential capital**

Country	1980		1990		2000	
	Ratio	Rank	Ratio	Rank	Ratio	Rank
Austria	58.4	3	53.0	2	41.9	5
Belgium	33.0	12	38.8	10	30.5	13
Denmark	55.4	5	42.8	7	34.2	12
Finland	23.3	13	28.3	13	37.9	9
France	47.6	6	45.9	5	47.9	1
Germany	41.0	7	39.5	8	36.6	11
Greece	38.3	8	43.3	6	46.1	3
Ireland	65.6	1	60.2	1	44.4	4
Italy	33.6	11	38.9	9	39.1	7
Netherlands	57.0	4	52.3	3	47.8	2
Portugal	13.0	14	17.0	14	23.3	14
Spain	33.7	10	37.8	11	41.3	6
Sweden	36.5	9	34.9	12	37.9	8
United Kingdom	61.5	2	47.2	4	37.0	10
EU average	43.9		42.0		39.7	
Standard deviation	15.5		10.8		6.9	

Note: (1) 'Ratio' gives the ratio of the real government net capital stock and the real private non-residential net capital stock. (2) 'Rank' gives the ranking of countries. (3) For calculation of EU average, see Table 1. (4) Government and non-residential capital stocks in 1995 prices.

Source: Kamps (2005) and own calculations.

The empirical analysis in the next section is based on a panel consisting of the 22 OECD countries and on a smaller panel consisting of 14 EU countries. Figure 2 shows the dynamics of selected variables that will be used in the empirical analysis for both panels for the period 1970–2000. While the empirical analysis builds on data for the individual countries, this aggregate figure conveys some interesting summary findings. It shows that the volatility of growth rates has declined considerably in both panels. Another interesting finding is that in the EU, the ratio of public capital to private capital and the ratio of public capital to GDP have dropped only moderately over the sample period, whereas the same is not true for the ratio of public capital to private capital for the OECD aggregate. The constancy of the so-called great ratios (consumption-output, investment-output and capital-output ratios as well as labor and capital shares in income), which is an important concept in growth theory, can, thus, only be confirmed for the EU panel. The figure also shows the dynamics of the unemployment rate, which will be used as a variable controlling for business cycle effects in the empirical analysis.

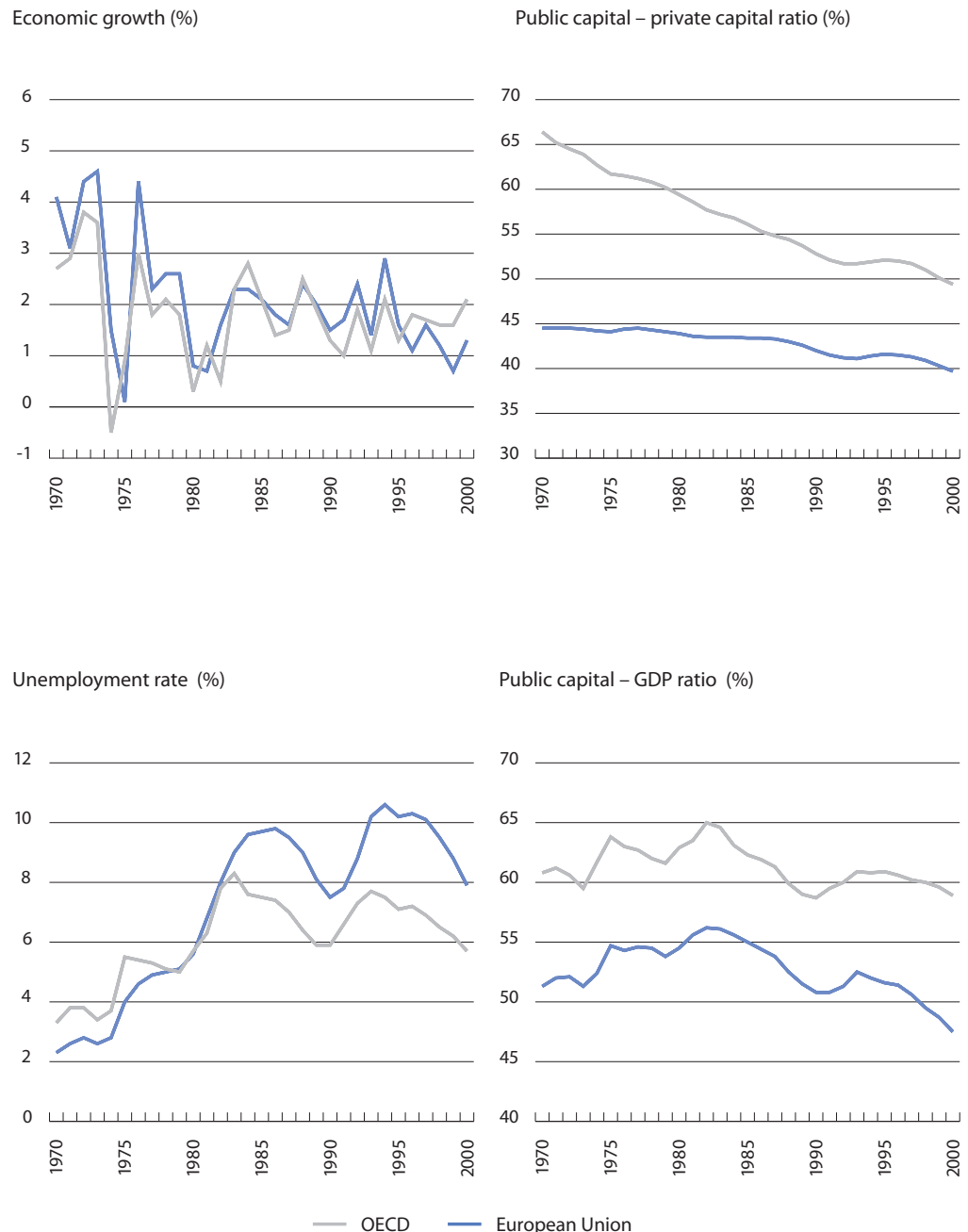
Table 3 displays summary statistics for the variables used in the regressions in Section 4. The figures reported in the table are based on ten-year averages for the sub-periods 1971–1980, 1981–1990, and 1991–2000. The growth rate,  $\gamma$ , is calculated as the average growth rate of real GDP per employed person,  $y$ , over a ten-year period.<sup>6</sup> The average growth rate for the period 1971–2000 amounted to 1.9 percent for the OECD panel and to 2.2 percent for the EU panel. The maximum growth rate in the sample was observed in Spain over the 1971–1980 period; the minimum growth rate for the OECD panel was observed in New Zealand over the 1971–1980 period; the minimum growth rate

*The ratio of public to private capital has fallen considerably in the average OECD country, contrasting with the experience in the EU.*

6 More specifically:  $\gamma_{it} = 1/9[\log(y_{it+9}) - \log(y_{it})]$ ,  $t = 1971, 1981, 1991, i = 1, \dots, N$ .

for the EU panel was observed in Greece over the 1981–1990 period. Finally, the table also reports descriptive statistics for the ten-year averages of the ratio of public to private capital,  $kg/k$ , and of the unemployment rate,  $u$ .

**Figure 2. Selected macroeconomic variables for OECD Countries, 1970–2000**



Note: (1) The OECD and EU aggregate comprises 22 and 14 countries, respectively (see Footnote 1); aggregate measures are computed on the basis of 1995 GDP weights expressed in 1995 purchasing power parities. (2) Economic growth reflects annual growth rates of real GDP per employed person. (3) Public capital-GDP ratio and public-private capital ratio as defined in Table 1 and 2, respectively.

Source: OECD Analytical Database (Version June 2002), OECD (2002), and Kamps (2005).

**Table 3. Descriptive statistics of the variables employed (in %)**

Variable	Region	Mean	Standard deviation	Minimum	Maximum
Economic growth rate $y$	OECD	1.9	0.9	-0.2	3.9
	EU	2.2	0.8	0.3	3.9
Public-private capital ratio $\varphi = kg / k$	OECD	45.6	19.4	12.4	106.3
	EU	42.0	12.9	12.4	68.1
Unemployment rate $u$	OECD	6.0	3.6	0.2	16.2
	EU	7.0	3.6	1.3	16.2

Note: (1) Statistics are based on 10-year averages for individual countries for the respective variable; the sample period (1971–2000) is divided into three 10-year sub-periods (1971–1980, 1981–1990 and 1991–2000); the OECD panel comprises 22 countries and the EU panel comprises 14 countries. (2) Economic growth reflects real GDP growth per employed person. (3) Public-private capital ratio,  $\varphi$ , as defined in Table 2, with  $kg$  and  $k$  indicating public capital and private capital, respectively.

Source: Kamps (2005), OECD (2002), own calculations and estimations.

#### 4. Empirical evidence on the growth effects of public capital

This section presents regression results on the relationship between public capital and economic growth. Regression results are presented both for a panel of 22 OECD countries and for a sub-panel of 14 EU countries. The regressions are based on averages of all variables for three ten-year periods in order to minimise the impact of the business cycle. This procedure considerably reduces the degrees of freedom in the regressions. However, regression results based on five-year averages—not reported here, but available on request—show that the regression results based on ten-year averages are robust. The regressions in Section 4.1 build on the assumption of a linear relationship between public capital and economic growth. As was shown in Section 2, economic theory suggests that the relationship between public capital and economic growth may be nonlinear. Section 4.2 takes this into account and presents regression results for a nonlinear specification of the empirical model.

##### 4.1 Evidence from a linear model

The empirical approach in this section follows Aschauer (2000), who has carried out equivalent regressions for US states for the 1980s and 1990s. The first regression builds on the following specification:

$$(1) \quad y_{it} = a + b \cdot \varphi_{it} + c \cdot z_{it} + \varepsilon_{it}, \quad t = 1971, 1981, 1991, \quad i = 1, \dots, N,$$

where  $\varphi = kg / k$  is the public to private capital ratio (with  $kg$  and  $k$  indicating public capital and private capital, respectively),  $z$  is a vector of control variables such as the initial-year natural logarithm of real GDP per employed person,  $y$ , and the average unemployment rate,  $u$ . The number of cross sections,  $N$ , is 22 in the case of the OECD panel and 14 in the EU panel. Equation (1) does not allow for any individual-country effects by imposing a common constant and common slope coefficients. Table 4 presents regression results for this pooled specification for the two panels, with and without control variables. The results do not support the hypothesis that public capital affects economic growth. In neither specification is the coefficient of public capital statistically significant. As regards the effect of initial output per employed person,  $y$ , there is evidence for (conditional) convergence for the EU panel, but not for the larger OECD panel. Yet, the empirical fit of the pooled regressions is quite weak as witnessed by the low centred  $R^2$ . This might indicate that the assumed degree of homogeneity across countries is too high.

**Empirical evidence from a linear growth model does not support the hypothesis that public capital affects economic growth.**

**Table 4. Government capital and economic growth – OLS regressions**

	EU		OECD	
$\varphi$	0.001	0.004	–0.006	–0.006
$y$	—	–0.003**	—	–0.000
$u$	—	–0.000	—	–0.000
<i>Constant</i>	0.021***	0.014*	0.022***	0.022***
$R^2$	0.873	0.888	0.829	0.829
<i>Centred <math>R^2</math></i>	0.000	0.042	0.018	0.020
<i>N·T</i>	42	42	66	66

Note: The independent variable is the economic growth rate  $\gamma$ . \*\*\* (\*\*, \*) denotes statistical significance at the 1 (5, 10) percent level. For additional information see text.

The regressions in Table 5 therefore introduce separate fixed effects for the individual countries. In addition, these regressions include separate time effects for the decades of the 1970s (*d70s*) and 1980s (*d80s*). The regressions, thus, build on the following specification:

$$(2) \quad \gamma_{it} = a_{it} + b \cdot \varphi_{it} + c \cdot z_{it} + d \cdot d70s_{it} + e \cdot d80s_{it} + \varepsilon_{it}, \quad t = 1971, 1981, 1991, i = 1, \dots, N.$$

Table 5 shows that the main results remain unaffected. In the case of the EU panel, the coefficient of public capital is still statistically insignificant. In the case of the OECD panel, the coefficient of public capital is only (marginally) significant if all control variables are excluded from the regression (see the first OECD column in Table 5). However, initial-year output per employee,  $y$ , clearly should be included in the regressions (see the last column in Table 5).

**Table 5. Government capital and economic growth – fixed effects regressions**

	EU		OECD	
$\varphi$	–0.022	–0.018	–0.026*	–0.016
$y$	—	–0.031*	—	–0.053***
$u$	—	–0.000	—	–0.000
<i>d70s</i>	0.008***	0.014*	0.006***	–0.014**
<i>d80s</i>	0.001	–0.005	0.001	–0.008***
$R^2$	0.945	0.951	0.921	0.947
<i>Centred <math>R^2</math></i>	0.565	0.615	0.546	0.696
<i>N·T</i>	42	42	66	66

Note: The independent variable is the economic growth rate  $\gamma$ . \*\*\* (\*\*, \*) denotes statistical significance at the 1 (5, 10) percent level. The regressions allow both for individual country and time period effects. For additional information see text.

To sum up, the linear regression models do not support the hypothesis that public capital affects economic growth. This finding can be interpreted in two ways: (i) public capital and economic growth are unrelated in general, or (ii) public capital is at or near its optimal level so that the linear specification fails to detect any effect of public capital on economic growth even though there is a relationship—albeit a nonlinear one. The next section investigates which of these interpretations is supported by the data.

## 4.2 Evidence from a nonlinear model

This section builds on a regression equation of the form

$$(3) \quad y_{it} = a_{it} + b \cdot f_{it} + c \cdot z_{it} + d \cdot d70s_{it} + e \cdot d80s_{it} + \varepsilon_{it}, \quad \text{with } t = 1971, 1981, 1991 \text{ and } i = 1, \dots, N.$$

In (3), the variable  $f$  denotes the transformed ratio of public to private capital given by

$$(4) \quad f_{it} = \frac{\varphi_{it}^{\alpha_{kg}}}{1 + (1 - \alpha_{kg})\varphi_{it}},$$

and  $z$  again represents control variables. The regressions allow for individual-country fixed effects as well as for decade effects. In the estimation of equation (3), we choose a different approach than Aschauer (2000). While his approach combines calibration and estimation by ordinary least squares (he estimates (3) for various assumed values of  $\alpha_{kg}$ ), we estimate the parameters of equation (3) by nonlinear least squares. This allows us to recover not only a point estimate for the elasticity of output with respect to public capital,  $\alpha_{kg}$ , but also the standard error of this coefficient. This, in turn, allows us to calculate a point estimate and a confidence interval for the growth-maximising ratio of public to private capital.

Table 6 summarises the results of the nonlinear-least-squares regressions. It shows that the elasticity of output with respect to public capital,  $\alpha_{kg}$ , is highly statistically significant across all specifications. It also suggests that initial-year output per employed person,  $y$ , has to be included as control variable. According to these estimates,  $\alpha_{kg}$  varies between 0.21 for the EU panel and 0.20 for the OECD panel. These results are very similar to those obtained by Kamps (2005), who reported an elasticity of 0.22 for a panel of OECD countries based on the production function approach. The results on the growth effects of public capital are less conclusive. The coefficient of  $f$  is positive and statistically significant for the EU panel for the specification including control variables (see the second EU column in Table 6), thus indicating that public to private capital ratios are lower than their growth-maximising levels in the EU. However, this coefficient is insignificant in the case of the OECD panel. That said, the significant result for the elasticity of output with respect to public capital suggests that public capital has important effects on the level of output, if not on its growth rate.

*Empirical evidence from a nonlinear growth model suggests a positive impact of public capital on economic growth for the panel of EU countries but not for the OECD panel.*

**Table 6. Government capital and economic growth – nonlinear least squares**

	EU		OECD	
$\alpha_{kg}$	0.201***	0.208***	0.157**	0.202***
$f$	0.383	0.546**	0.188	0.280
$y$	—	−0.041**	—	−0.055***
$u$	—	−0.000	—	−0.000
$d70s$	0.011***	−0.003	0.007**	−0.012*
$d80s$	0.002	−0.004	0.001	−0.007***
$R^2$	0.948	0.957	0.922	0.949
Centred $R^2$	0.591	0.658	0.551	0.708
$N \cdot T$	42	42	66	66

Note: The independent variable is the economic growth rate  $y$ . \*\*\* (\*\*, \*) denotes statistical significance at the 1 (5, 10) percent level. The regressions allow both for individual country and time period effects. For additional information see text.

Using the results reported in Table 6 and insights from Box 1, we can provide estimates for the optimal stock of public capital. Box 2 sets out details of the underlying calculations. We report here the main findings for the panel of EU countries. First, the point estimate for the growth-maximising ratio of public to private capital is about 34 percent. The associated 95-percent confidence interval for this estimate ranges from around 25 to 42 percent.

### Box 2. Estimating the optimal stock of public capital

In this Box, we use the results reported in Table 6 and insights from Box 1 to provide estimates for the optimal stock of public capital. We limit ourselves to the panel of EU countries.

First, using equation (B7) and the estimate for the elasticity of output with respect to public capital,  $\hat{a}_{kg}$ , the point estimate for the growth-maximising public to private capital ratio can be calculated as

$$(B10) \quad \hat{\varphi}^{\max} = \frac{\hat{a}_{kg}}{(1 - \hat{a}_{kg})^2} = \frac{0.21}{0.79^2} = 0.3365.$$

The growth-maximising ratio of public to private capital for the EU panel is, thus, estimated to amount to 33.7 percent.

Second, using the production function together with the assumption of constant returns to scale across the capital stocks (equation (B2)), the ratio of public capital to GDP can be expressed as  $kg/y = \varphi^{1-\hat{a}_{kg}}$ . The point estimate of the growth-maximising ratio of public capital to GDP can then be easily calculated; for the EU panel, it amounts to 42.3 percent.

Third, using equation (B9) and  $\hat{a}_{kg}$ , the point estimate for the welfare-maximising public to private capital ratio is 26.6 percent, and the welfare-maximising ratio of public capital to GDP is 35.1 percent. Welfare-maximising public capital stocks are thus some 17 to 21 percent lower than growth-maximising public capital stocks.

Let us then assess the uncertainty surrounding these point estimates. The starting point is the standard deviation of the elasticity of output with respect to public capital,  $\hat{\sigma}_{\hat{a}_{kg}}$ , which is estimated to be 0.017. A 95-percent confidence interval for  $\hat{a}_{kg}$  thus ranges from 0.175 to 0.245. Standard errors for the growth-maximising ratios can be derived using the so-called delta method (see, e.g., Greene 2000). The standard error for the growth-maximising ratio of public to private capital is given by

$$(B11) \quad \hat{\sigma}_{\hat{\varphi}^{\max}} = \frac{1 + \hat{a}_{kg}}{(1 - \hat{a}_{kg})^3} \cdot \hat{\sigma}_{\hat{a}_{kg}}.$$

With (B11) we arrive at our fourth result: a 95-percent confidence interval for  $\hat{\varphi}^{\max}$  that ranges from 25.3 to 42.0 percent.

Finally, the standard error for the growth-maximising ratio of public capital to GDP is given by

$$(B12) \quad \hat{\sigma}_{\frac{kg}{y}^{\max}} = \left[ \frac{1 - \hat{a}_{kg}}{(\hat{\varphi}^{\max})^{\hat{a}_{kg}}} \cdot \frac{1 + \hat{a}_{kg}}{(1 - \hat{a}_{kg})^3} - \left( \frac{kg}{y} \right)^{\max} \cdot \log(\varphi^{\max}) \right] \cdot \hat{\sigma}_{\hat{a}_{kg}},$$

so that a 95-percent confidence interval for public capital to GDP ratio ranges from 32.4 to 52.1 percent.

Second, the point estimate for the growth-maximising ratio of public capital to GDP amounts to some 42 percent, with a 95-percent confidence interval for this ratio ranging from about 32 percent to 52 percent of GDP.

Finally, we note that – as expected – the welfare-maximising stock of public capital is lower than the growth-maximising stock of public capital. Specifically, the welfare-maximising public to private capital ratio is estimated at about 27 percent (which is around 21 percent lower than the growth-maximising ratio) and the welfare-maximising public capital to GDP ratio is estimated to amount to 35 percent (around 17 percent lower than the growth-maximising ratio).

Given these estimates for the optimal stock of public capital, a natural question to ask is whether there is a lack of public capital in the EU. We address this question next.

## 5. Is there a lack of public capital in the European Union?

This section compares actual public capital stocks in individual EU member states with the growth-maximising level estimated in the previous section. It should be kept in mind that this study takes a shortcut by assuming that the determinant of the optimal public capital stock – the elasticity of output with respect to public capital – takes on the same value across countries. This study thereby does not account for differences among the panel units as regards their individual optimal capital stocks. However, as the literature has documented, it would not be an attractive alternative to estimate individual-country models because estimates based on such models tend to be unreliable. In contrast, the panel approach promises more robust results since it also makes use of the cross-section dimension of the data. Also note that in the present context there are only three observations (three ten-year averages) per country, making it impossible to test for the growth effects of public capital in an individual-country setting.

Figure 3 plots the ratio of public to private capital in the year 2000 for the 14 EU countries as well as the EU average. The plot also contains three horizontal lines representing the growth-maximising ratio of public to private capital as well as the lower and upper bounds of a 95-percent confidence interval estimated in the previous section. The figure reveals that in the year 2000, all countries but two had ratios of public to private capital that exceeded the growth-maximising value. The same is true for the EU average. However, if one accounts for the uncertainty surrounding the point estimate of the growth-maximising ratio, only four countries (France, Greece, Ireland and the Netherlands) have a ratio of public to private capital that is significantly above the growth-maximising ratio, while there is only one country (Portugal) with a suboptimally low ratio.<sup>7</sup> This suggests that in most EU countries there is currently neither a shortage nor an excess of public capital—judged relative to the stock of private capital.

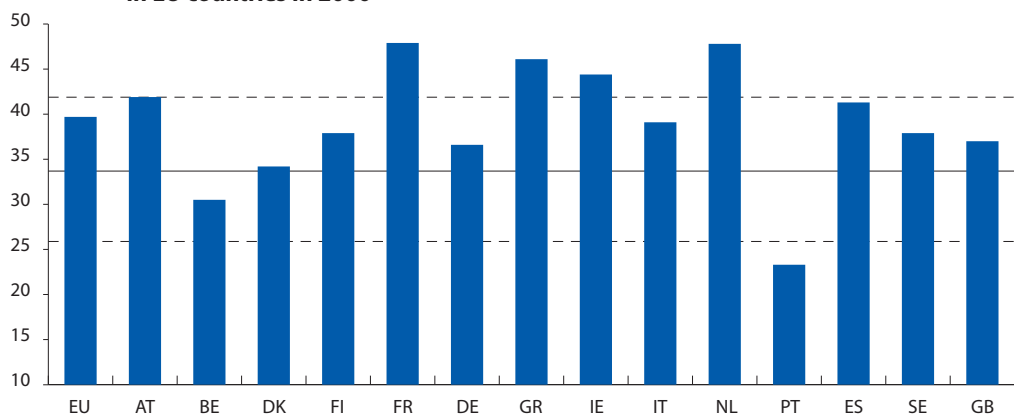
An alternative way of looking at the evidence is to compare the actual and growth-maximising values of the ratio of public capital to GDP for the same group of countries. From a theoretical perspective, it does not make a difference which ratios one considers. However, any theoretical model is inevitably only an approximation of reality, and it would be surprising if both ways of

*In most pre-enlargement EU countries, there is currently neither a shortage nor an excess of public capital.*

<sup>7</sup> However, the figures for Portugal are not directly comparable to those for the other countries. The private investment series used to construct capital stocks for Portugal do not allow a breakdown of private investment into its non-residential and residential components. In the OECD Analytic Database, the series for non-residential and total private investment coincide in the case of Portugal. As a consequence, the private capital stock used to calculate the public to private capital ratio is the total private capital stock in the case of Portugal. Thus, the public to private capital ratio reported for Portugal in Figure 3 understates the public to private non-residential capital ratio.

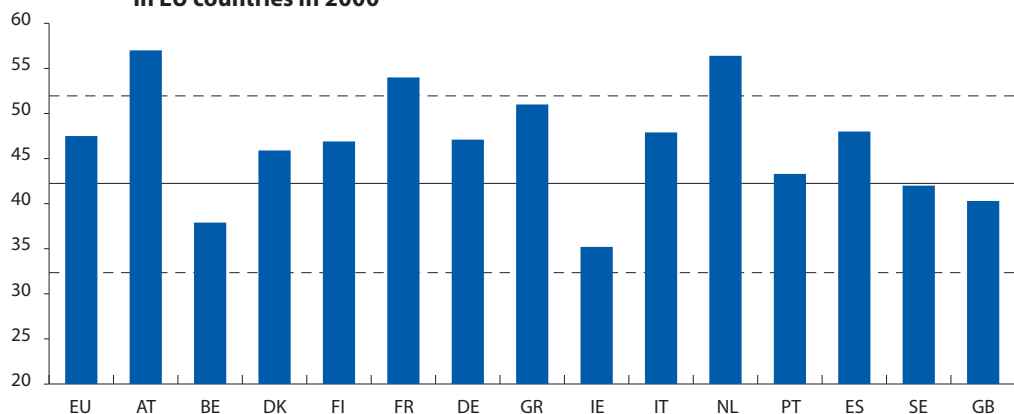
looking at the evidence gave exactly the same results. Figure 4 plots the ratios of public capital to GDP in the year 2000 for the 14 EU countries and the EU average as well as a measure of the growth-maximising ratio. As in Figure 3, most countries as well as the EU average exhibit a ratio above the growth-maximising value. And again, for most countries the deviation from the optimum is not statistically significant. The exceptions are Austria, France, and the Netherlands, which in 2000 had a ratio of public capital to GDP significantly above the growth-maximising value.

**Figure 3. Actual and growth-maximising ratios of public-to-private capital (in %) in EU countries in 2000**



Note: The solid line shows the growth-maximising government capital to private capital ratio; the dotted lines mark the boundaries of the 95-percent confidence interval for the growth-maximising ratio. The bars show the actual year-2000 government capital to private capital ratio for a EU aggregate as well as for 14 individual countries. The capital stock series for the individual countries are taken from Kamps (2005). The EU aggregate is computed on the basis of 1995 GDP weights expressed in 1995 purchasing power parities, taken from OECD (2002).

**Figure 4. Actual and growth-maximising ratios of public capital to GDP (in %) in EU countries in 2000**



Note: The explanations for this figure are similar to those of Figure 3.

A comparison of Figures 3 and 4 reveals some striking differences for two individual countries. While the evidence based on ratios of public to private capital suggests a shortage of public capital in Portugal and an excess of public capital in Ireland, the evidence based on public capital to GDP ratios suggests that there is a shortage of public capital in Ireland, while the ratio is roughly equal to



the growth-maximising level in the case of Portugal. However, for both countries, the public capital to GDP ratio seems to be the more appropriate indicator.<sup>8</sup>

The results reported above suggest that for most 'old' individual EU countries as well as for the 'old' EU as a whole there is currently evidence for neither a shortage nor an excess of public capital.<sup>9</sup> Now, an interesting question is whether a shortage of public capital might arise if current trends in public investment continue. As mentioned in the introduction, many policymakers and researchers fear that the current levels of government investment are too low.

Against this background, we use some simple arithmetics to explore two questions: first, how big is the gap between actual and optimal public investment and, second, what would happen to the public capital-GDP ratio in individual countries in the long run if these countries maintained their government investment to GDP at current levels? Box 3 explains these calculations, and Table 7 presents their results – the gist of which can be summarised as follows. To start with the first question, the growth-maximising ratio of public investment to GDP (shown in column 5 of Table 7) varies between 2.3 percent in Germany and Italy and 4.0 percent in Ireland. Comparing growth-maximising with actual public investment reveals that the actual level seems roughly appropriate for the 'old' EU as a whole (the gap in column 7 of Table 7 is close to zero). However, there are large discrepancies for some individual countries: Austria, Belgium, and the United Kingdom currently seem to grossly under-invest in public capital, while there is evidence for over-investment in the case of Portugal.

*In most pre-enlargement EU countries, the current level of public investment is roughly in line with the growth-maximising level.*

Turning to the second question, column 3 of Table 7 shows the long-run public capital to GDP ratio for individual countries and for the EU average if public investment (as a ratio to GDP) remained at its 2001 level. With 39 percent, the implied long-run ratio for the EU average is very close to the growth-maximising level of about 42 percent. Moreover, for most countries the long-run public capital to GDP ratio implied by current policy remains inside the confidence interval for the growth-maximising ratio; to recall from the previous section, this interval ranges from about 32 to 52 percent. Exceptions are Austria, Belgium, Denmark and the United Kingdom, for which the current level of public investment implies a shortage of public capital in the long run, as well as the Netherlands and Portugal, for which the current level of public investment implies an excess of public capital in the long run. Of course, there is no reason to believe that these countries will maintain their current levels of public investment indefinitely if they turn out to be suboptimal.

<sup>8</sup> The previous footnote describes why the public to private capital ratio might be misleading in the case of Portugal. In the case of Ireland, the results taken together indicate that there might be a shortage of both public and private capital. During the 1990s, both public and private capital accumulation did not keep pace with the very strong output growth.

<sup>9</sup> The term 'old' here characterises countries that were EU member states before the latest EU enlargement, which took effect on May 1, 2004.

### Box 3. Calculating deviations of actual from optimal public capital stocks and investment

#### How big is the gap between actual and optimal public investment?

To illustrate an answer to this question, we calculate the constant country-specific public investment to GDP ratio that stabilises the ratio of public capital to GDP at its growth-maximising level. This investment ratio can be expressed as

$$(B13) \quad \left(\frac{ig}{y}\right)_i^{\max} = (\delta_g + \gamma_i) \left(\frac{kg}{y}\right)^{\max}, \quad i=1, \dots, N.$$

where  $ig$  is public investment and  $\delta_g$  is the depreciation rate of public capital. Following Kamps (2005), we assume that the depreciation rate of public capital amounts to 4 percent for the countries considered. The long-run growth rate  $\gamma_i$  is assumed to be country-specific and approximated by the growth rate of potential output in the year 2003 according to OECD (2004) estimates. Equation (B13) then determines the public investment to GDP ratio that, in the long run, stabilises the ratio of public capital to GDP at its growth-maximising level (which – as shown in the previous section – is 42.3 percent for the panel of EU countries). These investment ratios are shown in column 5 of Table 7. Column 6 shows actual (2001) investment ratios and, consequently, column 7 shows the gap between actual and the optimal level of public investment.

#### What if public investment-to-GDP ratios remained unchanged?

To shed light on this question, we need to calculate the long-run ratio of public capital to GDP implied by the current level of government investment. This long-run ratio (denoted by the superscript  $LR$ ) can be expressed as

$$(B14) \quad \left(\frac{kg}{y}\right)_i^{LR} = \frac{1}{\delta_g + \gamma_i} \left(\frac{ig}{y}\right)_i, \quad t = 2001, i=1, \dots, N.$$

To calculate these ratios for individual countries, we make the following assumptions: we use again a depreciation rate of public capital of 4 percent; assume that government investment ratios in individual countries remain constant at their current levels; and approximate long-run growth rates of individual countries by the respective growth rate of potential output. Column 3 of Table 7 shows the results of these calculations for individual countries and the EU average. Column 2 shows the optimal public capital stock to GDP ratio and, thus, column 4 shows the gap between optimal values and those resulting from unchanged public investment ratios.

**Table 7. Public capital stock and investment ratios: comparing actual with 'optimal' figures**

Country	Growth rate(in %)	Public capital to GDP (in %)			Public investment to GDP (in %)		
		'Optimal'	'Long-run' with actual (2001) public investment ratio	Gap	'Optimal'	Actual (in 2001)	Gap
	(1)	(2)	(3)	(4)=(2)-(3)	(5)	(6)	(7)=(5)-(6)
Austria	2.3	42.3	19.7	22.6	2.7	1.3	1.4
Belgium	2.1	42.3	24.6	17.7	2.6	1.5	1.1
Denmark	2.0	42.3	31.4	10.9	2.5	1.9	0.7
Finland	2.4	42.3	39.3	3.0	2.7	2.5	0.2
France	2.1	42.3	49.3	-7.0	2.6	3.0	-0.4
Germany	1.5	42.3	34.7	7.6	2.3	1.9	0.4
Greece	4.2	42.3	48.1	-5.8	3.5	3.9	-0.5
Ireland	5.5	42.3	33.5	8.8	4.0	3.2	0.8
Italy	1.4	42.3	42.9	-0.6	2.3	2.3	0.0
Netherlands	2.1	42.3	55.2	-12.9	2.6	3.3	-0.8
Portugal	1.9	42.3	70.1	-27.8	2.5	4.1	-1.6
Spain	3.0	42.3	45.5	-3.2	3.0	3.2	-0.2
Sweden	2.6	42.3	33.4	8.9	2.8	2.2	0.6
United Kingdom	2.4	42.3	25.7	16.6	2.7	1.7	1.1
EU average	2.0	42.3	39.1	3.2	2.6	2.4	0.2

Note: (i) growth rate reflects growth rate of potential output; (ii) public capital stock assumed to depreciate at 4% p.a.; (iii) 'optimal' in the sense of growth maximising; (iv) EU average: for country weighting see Table 1.

Source: Kamps (2005), OECD (2002, 2004) and own calculations.

## 6. Limitations of the study and avenues for further research

The analysis presented in this paper builds on a specific theoretical model and uses specific econometric methods as well as a small set of economic variables in the empirical application. This raises the question whether the results obtained are specific to the chosen setup or robust to alternative setups. This section discusses therefore modifications of the basic setup that could help to assess the robustness of the main results.

Regarding the theoretical model, a number of alternative setups could be considered, especially with respect to their implications for optimal public capital spending. Three modelling devices deserve special attention. First, the model considered in this study is an endogenous growth model. It would be of interest to compare optimal fiscal policy in this model with optimal fiscal policy in an exogenous growth model. Second, the model considered here assumes an infinitely lived representative agent, abstracting from questions relating to intergenerational redistribution. Yet, the benefits of public capital accumulation might be unevenly distributed across generations. The welfare implications could be addressed in an overlapping-generations model. Third, the financing decision plays an important role for the long-run effects of public capital spending. The model used here assumes that the government has access to government debt and to a distortionary output tax. It would be interesting to study the model's implications for alternative financing choices.

*The findings of this paper rest on a specific theoretical model, using a limited number of variables. Future research could consider different specifications and/or more variables.*

As regards the empirical analysis, three complementary approaches could be followed in order to explore the robustness of the results presented here. First, the empirical analysis presented in this study includes only a limited number of control variables. A number of studies in the empirical growth literature check for the robustness of the growth effects of specific explanatory variables using the so-called extreme bounds test (see, e.g., Levine and Renelt 1992 and Sala-i-Martin 1997). These studies consider in general 50 or more variables, such as the stock of human capital, and run regressions for many combinations of explanatory variables. Second, the empirical model used here estimates the steady-state relationship between public capital and economic growth. One could relax the assumption that the countries are in the steady state and account for transitional dynamics. Third, the empirical model used here does not explicitly take the government budget constraint into account. In their study on the growth effects of fiscal policy Bleaney *et al.* (1999) show that it is important to control for the way in which public spending is financed. It would therefore be interesting to explore whether the empirical growth effects of public capital in the sample of OECD countries vary across financing alternatives.

Finally, it should be kept in mind that this study makes a shortcut by assuming that the determinant of the optimal public capital stock – the elasticity of output with respect to public capital – takes on the same value across countries. At first sight it might, thus, be an interesting next step to estimate country-specific optimal public capital stocks. However, one conclusion to be drawn from the large body of empirical literature on the productivity of public capital is that estimates based on individual-country regressions often turn out to be unreliable, one important reason being that such regressions tend to be subject to multicollinearity problems. One way to deal with these problems is to use the cross-section dimension of the data and to carry out panel regressions. This is the approach followed in this study. An alternative could be to assemble regional data on capital stocks for individual countries and then carry out regressions using a regional panel. So far, however, regional data on public and private capital stocks are unavailable for the vast majority of OECD countries.

## Annex

### The growth model

#### A.1 Decentralised economy

The objective of the representative agent is to maximise his lifetime utility (B1) subject to his flow budget constraint (B5) and the production function (B2). To solve this optimisation problem we use the current-value Hamiltonian

$$(A.1) \quad H = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda \left[ (1-\theta) k^\alpha kg^{\alpha_{kg}} + rb - c \right].$$

The representative agent takes  $kg$  and  $\theta$  as given and maximises (A.1) with respect to  $c$ ,  $k$  and  $b$ . The optimality conditions are

$$(A.2) \quad c^{-\sigma} = \lambda,$$

$$(A.3) \quad (1-\theta) (1-\alpha_{kg}) \varphi^{\alpha_{kg}} = \rho - \frac{\dot{\lambda}}{\lambda},$$

$$(A.4) \quad r = \rho - \frac{\dot{\lambda}}{\lambda}.$$

Equation (A.2) equates the marginal utility of consumption to the shadow value of wealth,  $\lambda$ . Equation (A.3) asserts that the after-tax marginal product of private capital must equal the rate of return on consumption, while equation (A.4) equates the interest rate with the rate of return on consumption. (A.3) and (A.4) taken together, imply that in equilibrium the real interest rate must equal the after-tax marginal product of private capital,

$$(A.5) \quad r = (1-\theta) (1-\alpha_{kg}) \varphi^{\alpha_{kg}}.$$

In addition, the following transversality conditions must be met

$$(A.6) \quad \lim_{t \rightarrow \infty} \lambda k e^{-\rho t} = 0,$$

$$(A.7) \quad \lim_{t \rightarrow \infty} \lambda b e^{-\rho t} = 0,$$

thereby ruling out explosive equilibria.

Recalling that the government finances public capital with government debt ( $kg=b$ ), the government budget constraint (B4) and the production function (B2) can be used to express the tax rate as  $\theta = r \varphi^{\alpha_k}$ . Substituting this expression for the tax rate in equation (A.5) yields

$$(A.8) \quad r = \frac{(1-\alpha_{kg}) \varphi^{\alpha_{kg}}}{1 + (1-\alpha_{kg}) \varphi}.$$

In the following, we focus on the steady state of the model economy. Along the balanced growth path all variables grow at the same rate. Differentiating (A.2) with respect to time, and using (A.4), the common growth rate can be expressed as

$$(A.9) \quad \gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} [r - \rho].$$

Now, using (A.8), the growth rate can be expressed as a nonlinear function of the public to private capital ratio,

$$(A.10) \quad \gamma = \frac{1}{\sigma} \left[ \frac{(1-\alpha_{kg}) \varphi^{\alpha_{kg}}}{1 + (1-\alpha_{kg}) \varphi} - \rho \right].$$

Setting the derivative of  $y$  with respect to  $\varphi$  to zero, yields the growth-maximising public to private capital ratio

$$(A.11) \quad \varphi^{\max} = \frac{\alpha_{kg}}{(1 - \alpha_{kg})^2}.$$

In order to derive the tax rate associated with the growth-maximising public to private capital ratio, we use the expression  $\theta = r\varphi^{\alpha_k}$ , substituting (A.8) for the interest rate and (A.11) for  $\varphi$ . After some transformations this yields  $\theta^{\max} = \alpha_{kg}$ , i.e., the tax rate should be set equal to the elasticity of output with respect to public capital. Substituting  $\alpha_{kg}$  for the tax rate in the expression  $\theta = r\varphi^{\alpha_k}$  shows that, in the growth-maximising steady state, the marginal product of public capital is equal to the interest rate,  $r = \alpha_{kg}\varphi^{\alpha_{kg}-1}$ . Comparing this expression with (A.5) shows that in the growth-maximising steady state the after-tax marginal product of private capital is equal to the marginal product of public capital,  $(1 - \theta)(1 - \alpha_{kg})\varphi^{\alpha_{kg}} = \alpha_{kg}\varphi^{\alpha_{kg}-1}$ .

## A.2 Optimal policy

This section derives the welfare-maximising public to private capital ratio, drawing on Turnovsky (1997). The objective of the social planner is to maximise the agent's utility (B1) subject to the production function (B2), the public capital accumulation equation  $\dot{k}g = gy$ , where  $g$  denotes the share of output devoted to public investment, and the economy-wide resource constraint  $\dot{k} = y - \dot{k}g - c$ . The current-value Hamiltonian now is

$$(A.12) \quad H = \frac{c^{1-\sigma} - 1}{1-\sigma} + \nu \left[ (1 - g) k^{\alpha_k} kg^{\alpha_{kg}} - c \right] + \mu g k^{\alpha_k} kg^{\alpha_{kg}}.$$

The social planner maximises (A.12) with respect to  $c$ ,  $k$ ,  $kg$  and  $g$ . The optimality conditions are

$$(A.13) \quad c^{-\sigma} = \nu,$$

$$(A.14) \quad (1 - g)(1 - \alpha_{kg})\varphi^{\alpha_{kg}} + qg(1 - \alpha_{kg})\varphi^{\alpha_{kg}} = \rho - \frac{\dot{\nu}}{\nu},$$

$$(A.15) \quad \frac{1}{q}(1 - g)\alpha_{kg}\varphi^{\alpha_{kg}-1} + g\alpha_{kg}\varphi^{\alpha_{kg}-1} + \frac{\dot{q}}{q} = \rho - \frac{\dot{\nu}}{\nu},$$

$$(A.16) \quad q = 1,$$

where  $q = \mu/\nu$  denotes the shadow price of public capital in terms of private capital. According to equation (A.16), the shadow values of public and private capital should be equal. Equating (A.14) and (A.15), using (A.16), yields the welfare-maximising public to private capital ratio ( $\varphi^*$ ), which—as can easily be seen—for  $\alpha_{kg} > 0$  is smaller than the growth-maximising ratio ( $\varphi^{\max}$ )

$$(A.17) \quad \varphi^* = \frac{\alpha_{kg}}{1 - \alpha_{kg}} < \varphi^{\max}.$$

The rationale for this result is the following: while growth maximisation implies that the marginal product of public capital should be equal to the *after-tax* marginal product of private capital, welfare maximisation implies that the marginal product of public capital should equal the *gross* marginal product of private capital. The distortions associated with taxation discourage the accumulation of private capital. As a result the public to private capital ratio with growth maximisation is lower than in the first-best optimum. Maximising the growth rate involves a consumption loss and, thereby, lowers welfare.

## References

- Aschauer, D.A. (2000). "Do states optimize? Public capital and economic growth". *Annals of Regional Science*, (34:(3), pp. 343–363.
- Barro, R.J. (1990). "Government spending in a simple model of endogenous growth". *Journal of Political Economy*, (98:5), pp. 103–125.
- Barro, R.J., and Sala-i-Martin, X. (1992). "Public finance in models of economic growth". *Review of Economic Studies*, (59:4), pp. 645–661.
- Baxter, M., and King, R.G. (1993). "Fiscal policy in general equilibrium". *American Economic Review*, (83:3), pp. 315–333.
- Blanchard, O.J., and Giavazzi, F. (2003). "Improving the SGP through a proper accounting of public investment". European Economic Perspective newsletter, CEPR, No 1. February. .
- Bleaney, M., Gemmell, N., and Kneller, R. (1999). "Fiscal policy and growth: Evidence from OECD countries". *Journal of Public Economics*, (74:2), pp. 171–190.
- European Commission (2001). "2001 broad economic policy guidelines". *European Economy* No. 72. D.G. for Economic and Financial Affairs, Brussels.
- Fitz Gerald, J., Kearney, I., Morgenroth, E., and Smyth, D. (1999). "National investment priorities for the period 2000–2006". ESRI Policy Research Series 33.
- Greene, W.H. (2000). *Econometric Analysis*. 4<sup>th</sup> edition. Upper Saddle River, NJ: Prentice Hall. USA.
- Greiner, A. (1996). *Fiscal policy and economic growth*. Aldershot: Avebury. England.
- HM Treasury (2002). Departmental investment strategies: A summary. 2002 Spending Review. London, UK.
- Kamps, C. (2005). "New estimates of government net capital stocks for 22 OECD countries 1960–2001". IMF Staff Papers (forthcoming). An earlier version is available as IMF Working Paper 04/67.
- Levine, R., and Renelt, D. (1992). "A sensitivity analysis of cross-country growth regressions". *American Economic Review*, (82:4), pp. 942–963.
- Modigliani, F., Fitoussi, J.-P., Moro, B., Snower, D.J., Solow, R.M., Steinherr, A., and Labini, S.S.(1998). "An economists' manifesto on unemployment in the European Union". *Banca Nazionale del Lavoro Quarterly Review*, (51:206), pp. 327–361.
- OECD (2002). Economic Outlook No. 71. June.
- OECD (2004). Economic Outlook No. 76. December.
- Sala-i-Martin, X. (1997). "I just ran two million regressions". *American Economic Review*, (87:2), pp. 178–193.
- Temple, J. (1999). "The new growth evidence". *Journal of Economic Literature*, (37:1), pp. 112–156.
- Turnovsky, S.J. (1997). "Public and private capital in an endogenously growing economy". *Macroeconomic Dynamics*, (1:3), pp. 615–639.