Beath, John; Poyago-Theotoky, Joanna; Ulph, David

Working Paper
University funding systems: impact on research and teaching

Economics Discussion Papers, No. 2011-1

Provided in Cooperation with:
Kiel Institute for the World Economy (IfW)

Suggested Citation: Beath, John; Poyago-Theotoky, Joanna; Ulph, David (2011) : University funding systems: impact on research and teaching, Economics Discussion Papers, No. 2011-1, Kiel Institute for the World Economy (IfW), Kiel

This Version is available at:
http://hdl.handle.net/10419/44431

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

http://creativecommons.org/licenses/by-nc/2.0/de/deed.en
University Funding Systems: Impact on Research and Teaching

John Beath
University of St Andrews

Joanna Poyago-Theotoky
La Trobe University, CRIEFF and RCEA

David Ulph
University of St Andrews

Abstract  We address the following question: how does a higher education funding system influence the trade-off that universities make between research and teaching? We do so by constructing a model that allows universities to choose actively the quality of their teaching and research when faced with different funding systems. In particular, we derive the feasible sets that face universities under such systems and show how, as the parameters of the system are varied, the nature of the university system itself changes. The culture of the university system thus becomes endogenous. This makes the model useful for the analysis of reforms in funding and also for international comparisons.

Paper submitted to the special issue
Quasi Markets in Education

JEL  I21, I22, I23

Keywords  University funding systems; higher education; research quality; teaching quality; university budget constraint

Correspondence  Joanna Poyago-Theotoky, School of Economics and Finance, La Trobe University, Melbourne, Victoria 3086, Australia; e-mail: j.poyago-theotoky@latrobe.edu.au

© Author(s) 2011. Licensed under a Creative Commons License - Attribution-NonCommercial 2.0 Germany
1 Introduction

The importance of higher education institutions, such as universities, as well as the role that these play in the knowledge economy, cannot be overemphasised especially in the current economic climate. Universities exist to teach and to perform research. Universities add to the stock of useful knowledge through their research and disseminate that stock through their teaching, but what determines the amounts of each that they do? We seek to answer that question in this paper and show how the 'culture' of a university system will systematically depend on the way that the higher education sector is funded (where 'culture' captures the emphasis placed on research and/or teaching). We do this by constructing a model in which the budget constraint facing the higher education sector plays a crucial role in determining the kind of research and teaching culture that will emerge. We use a generic type of funding model and, as we consider its parameters (specifically the premium for and the 'marginal cost' of research quality, as well as the threshold level of teaching quality), we find that one can obtain the emergence of cultural phenomena such as 'research elites' and the 'binary divide'.

Achieving quality in teaching and research takes time and as academics are time-limited, they face a stark choice. The more of their time that they spend on research, the higher is likely to be its quality. However this cuts back on the time that they can spend teaching students and, as this has implications for staff-student ratios, it can have a negative impact on teaching quality. Of course, in view of agencies such as the UK Quality Assurance Agency (QAA) – as well as the increasing 'voice' of the student consumers, there is going to be some quality threshold in teaching that all universities will need to attain. We take account of this in our analysis.

In publicly funded systems, financial resources come as grants for teaching

---

1 The 'binary divide' refers to the differentiation between 'polytechnic institutions' and 'universities' within the UK between 1965 and 1992, where only the latter could grant research degrees. This ended with the Further and Higher Education Act of 1992 which created a unified sector. A 'research elite' refers to groups of universities where a lot of emphasis is placed on the research function.
and grants for research. While there is as yet no quality-related component to the grant for teaching, this is not true of research – at least in the UK, Australia and New Zealand since the advent of the periodic research evaluation exercises. We have therefore allowed there to be a teaching grant proportional to the number of students that a university has on its books and a research grant with a fixed amount per staff member and a quality-related component. There is a minimum quality threshold above which the quality component kicks in and we explore what happens as the scale of this quality factor is varied.

There is a substantial literature in the economics of higher education (e.g., see Clotfelter (1999)). However, this has tended to focus on the costs of and returns to higher education, often concentrating on issues associated with various financing/funding systems and their effects on student participation as well as equity and welfare aspects (Barr and Crawford (1998), Chapman (1997), García-Peñalosa and Wälde (2000), Gary-Bobo and Trannoy (2004), Greenaway and Hayes (2003), Kaiser et. al. (1992), Kemnitz (2007)). There has also been a significant amount of work on the organisation of the university (e.g., Borooah (1994)), on the link between the quality of educational provision, mobility costs and student choice (de Fraja and Iossa (2002), del Rey (2001)), on the allocation of academics’ time (Beath et al. (2003), Gautier and Wauthy (2007), Hare (2002)) and on the efficiency of universities (Glass et al. (2006)).

Despite this flurry of research, relatively little attention appears to have been paid to the question of the link between what universities actually do, in terms of both teaching and research quality, and the way in which they are funded. In view of the important role envisaged for universities in the “knowledge economy”, particularly where they are supported by public funding, it

\[\text{2In the UK research excellence has been evaluated until recently by the Research Assessment Exercises (RAE) of 1986, 1989, 1992, 1996, 2001 and 2008 while in future this will be done within the 2014 Research Excellence Framework (REF). New Zealand has operated a similar exercise, the Performance Based Research Fund (PBRF) in 2003, 2006 and the forthcoming one in 2012. Australia operates the Excellence for Research in Australia (ERA) while several European countries have been moving their HE funding mechanisms in this direction.}\]

\[\text{3European universities are heavily reliant on the public purse; e.g., Germany spends 1% of GDP on higher education yet only 0.1% is funded by the private sector (The Economist, September 25, 2004).}\]
seems surprising that the link between the type of funding system and the mix of activities that universities undertake has not been explored in greater detail, with the exception of Del Rey (2001). This paper analyses a stylised game between two universities that are competing for students in a Hotelling-like fashion and spend their publicly provided budgets on teaching and research. The universities maximise an objective function which depends on the quality of their student output and expenditure on research. Del Rey (2001) characterises the subgame perfect equilibria and explores how these vary as the parameters of the funding system are changed and in particular, the balance between research and teaching effort as a function of the funding rules. However, research is treated as a residual item in the universities’ budgets and no attention is paid to its quality. More recently, Gautier and Wauthy (2007) in a complementary paper to the present one, have explored the potential implications of incentive schemes as a tool to promote efficiency within a single university and contrast two governance modes, a multi-department university with a single-unit one, paying particular attention to multi-tasking issues regarding the choice of teaching and research efforts. In a recent empirical study Glass et al.(2006), combining data envelopment analysis with stochastic frontier analysis, estimate the profit efficiency (composed of a technical and allocative element) for the population of UK universities during the 1996 Research Assessment Exercise (RAE) and establish that the top traditional universities (strong in research culture) are generally more efficient than the ‘new’ universities (former polytechnics, with a strong teaching-emphasis culture).

What we seek to do in the present paper is to incorporate research quality directly into a university’s budget constraint (a pivotal element of our analysis) and to provide a rather general modelling framework that allows universities to actively choose the quality of their teaching and research when faced with different funding systems. In particular, we derive feasible sets that face universities under different funding systems and show how, as the parameters of the funding system are varied, the nature of the university system changes. Thus we
endogenise the ‘culture’ of the university system. We believe that in the current climate of the higher education sector, this is important if one is concerned with making comparisons with actual systems across different countries, especially in the UK, Europe, Australia and New Zealand.

The paper is organised as follows. Section 2 introduces the model and sets out the generic characteristics of a university funding system. Section 3 uses that framework to analyse how a typical university, operating under the funding limits described in Section 2 chooses teaching and research quality. Section 4 discusses the results of the analysis and Section 5 concludes.

2 The Model

We describe a higher education system in which there is a multitude of universities

The characteristics of this system are as follows:

1. The minimum teaching quality is specified by the funding authority. Rather than specifying this directly, we capture this by the fraction of time, \( t < 1 \), that academics have to spend on teaching in order to meet this minimum requirement. Each academic is endowed with one unit of time to be used in research and/or teaching.

2. Universities are funded under the mechanism, \( I = pS + AR(q) \), where \( I \) is a university’s income, \( p \) is the unit of resource delivered by the system for teaching a student, \( S \) is the number of students, \( A \) is the number of academics, \( R(q) \) is the research funding per academic, and \( q \) is the quality of research produced by academics. Notice that we have chosen here not to relate

\footnote{In this paper we stay away from inter-university competition and related issues of imperfect competition in higher education. These are not without interest but our focus here is on how the choice of teaching and research quality is affected by various funding systems in the absence of competition.}

\footnote{In the UK this would be the sum of the teaching resource provided by the funding council through its TR grant and the tuition fee that a student pays. In other systems, this could be entirely funded by the student fee.}

\footnote{Note that we treat the population of students as a homogeneous group, i.e., we do not distinguish undergraduates from postgraduates. However, in later work, it would be interesting to consider separately how these two groups of students respond to changes in the funding mechanism and also on the quality of teaching and research provided.}
funding to teaching quality.\footnote{The reason is that our primary aim is to focus on the effects of incentivising universities to perform research, so it seems useful in the first instance to ignore teaching quality incentives. Moreover, while it may be possible to specify and measure minimum teaching quality (and we allow for that possibility), measuring actual teaching quality is far more controversial and resource intensive. We could also argue for this approach on grounds of realism.}

\[ R(q) = \alpha + \rho \max[0, q - q_0] \]

where $\alpha \geq 0$ is the lump-sum payment per academic, $\rho \geq 0$ is the research quality premium, and $q \geq 0$ is the research quality threshold. This is quite general so that $\alpha > 0, \rho = 0$ corresponds to a funding system without incentives while $\alpha \geq 0, \rho > 0$ corresponds to an incentivised system. A university funding system is then defined by the vector $(t, p, \alpha, \rho, q)$. In the analysis that follows we shall treat $t$ and $p$ as exogenous and will examine how different values of the remaining parameters determine the choice a university makes with respect to the teaching and research quality it offers.

\[ A/S = g(q, t), \quad \frac{\partial g}{\partial q} > 0, \quad \frac{\partial g}{\partial t} > 0. \quad (1) \]

As each academic has one unit of time to spend on teaching and/or research, and, from above, it costs $t$ units of academic time per student to achieve the specified teaching quality, $t$. Thus, if a university has $A$ academics and $S$ students with $A \geq tS$ then the amount of time each academic can devote on

\footnote{This assumption is made to simplify the analysis. Moral hazard and/or adverse selection issues are outside the scope of the present paper but not without interest.}
research while achieving the minimum teaching quality is

\[ r = 1 - t(S/A). \]

The quality of research, \( q \), is related to the time devoted to research, \( r \), via the simple function \( q = r^\gamma \), \( 0 < \gamma < 1 \), indicating diminishing returns to time spent on research. Then \( 1 - t(S/A) = q^\beta \), where \( \beta = (1/\gamma) > 1 \). As a result, equation (1) becomes

\[
g(q, t) = \frac{t}{1 - q^\beta}, \quad 0 \leq q \leq 1. \quad (1')
\]

[6] Academics are paid a fixed salary, \( w > 0 \). This salary \( w \) is independent of \( q \) thus enabling universities to enforce a target level of quality on teaching.

[7] There are no other sources of income for universities so that the salary bill for academics is the only cost. Consequently a university faces a budget constraint

\[
wA \leq pS + A \left( \alpha + \rho \max\{0, q - q_s\} \right). \quad (2)
\]

Notice that using the relationship in expression (1) we can re-write this as:

\[
wSg(q, t) \leq pS + Sg(q, t) \left( \alpha + \rho \max\{0, q - q_s\} \right),
\]

or (per student),

\[
w - \frac{p}{g(q, t)} \leq \alpha + \rho \max\{0, q - q_s\}. \quad (3)
\]

For the particular form given in (1') above this becomes:
Finally, we assume that all universities have as their mission the creation (research) and dissemination (teaching) of fundamental knowledge. Thus universities care about two issues: the quality-weighted volume of research they produce, $qA$, and the quality-weighted number of graduates, $\tau(t)S$, where $\tau(t)$ is a function that determines the quality of teaching when a fraction $t$ of academic time is devoted to it. Thus each university’s objective function can take the general form $U[qA, \tau(t)S]$, where $U$ is strictly increasing in both arguments.

We allow the possibility that universities may differ in their views as to the relative importance of teaching and research and so may have differing objective functions within this class. Notice that, by substituting (1) we can write this as

$$V(q, t, S) = U[qg(q, t)S, \tau(t)S]$$

which, for given, $S$, is a strictly increasing function of $t$ and $q$. Indeed, in the special case where $U(.)$ is homothetic, this can be written:

$$V(q, t, S) = \eta(t, q)\sigma(S).$$

In the interest of analytical tractability, in what follows we will use the homothetic functional form and moreover will restrict our attention to the case where

$$\eta(t, q) = \omega q + (1 - \omega)t$$

where $\omega, 0 \leq \omega \leq 1$ is the relative weight that a university places on research. Note that $\omega$ is the characteristic that differentiates universities.
3 Analysing the Budget Constraint

We now examine what options are open to a university that is constrained by the budget constraint as defined by (3). To do this, suppose for the moment that a university is delivering the minimum teaching quality, and consider what research quality it can achieve. Then (3) becomes

\[ \left( w - \frac{p}{\ell} \right) + \frac{p}{\ell} q^2 \leq \alpha + \rho \max[0, q - \frac{1}{2}] \]  

and represents the funding constraint faced by a university when it offers the minimum teaching quality, i.e., \( t = \ell \) (see Figure 1 for an illustration of the constraint).

Figure 1: A university delivering the minimum teaching quality: two cases
Notice that the LHS of (4) is a strictly increasing and strictly convex function of research quality, \( q \), that takes the value \( w - \frac{p}{t} \) when \( q = 0 \) and the value \( w \) when \( q = 1 \). It has a simple interpretation: it is the resource per academic that is needed to deliver research of quality \( q \) when the quality of teaching is at its minimum threshold level. The RHS of (4) is a piecewise linear function that takes the value \( \alpha \) when \( 0 \leq q \leq \frac{\alpha}{\rho} \) and the value \( \alpha + \rho(1 - q) \) when \( q = 1 \). It also has a simple interpretation: the resource per academic that is actually delivered by the funding system for research of quality \( q \). Clearly, if research of any given quality is to be achieved, the resources must be at least sufficient to meet the needs. In fact we will make two further assumptions:

**Assumption 1 (a1).** The university funding system is such that there exist some \( q \in [0, 1] \) such that \( (w - \frac{p}{t}) + \frac{\rho q^2}{t} > \alpha + \rho \max[0, q - \frac{\alpha}{\rho}] \).

**Assumption 2 (a2).** The university funding system is such that there exist some \( q \in [0, 1] \) such that (4) is satisfied.

If (a1) were not satisfied then the range of values of \( q \) that satisfy (4) is the entire interval \([0, 1]\), and so universities would face no effective restriction on the quality of research they can achieve. In other words by invoking (a1) we are ruling out the possibility that universities are so generously funded that they face no constraints on research quality! One immediate implication of (a1) is that \( \alpha \leq w \). This is inherently plausible – university funding systems do not provide universities a minimum amount of funding per academic for research that exceeds the average academic salary. If (a2) were not true then effectively universities are so badly funded that no university could deliver even the lowest quality research while meeting the minimum teaching quality threshold.

An implication of assumptions (a1) and (a2) is that we need to partition the analysis into two sets of cases of funding (see Figure 1). Set A represents situations where \( w - \frac{p}{t} < \alpha < w \), while set B comprises the cases where \( \alpha < w - \frac{p}{t} \). The interpretation of these two conditions is as follows: \( 1/t \) is the number of students an academic can teach while achieving minimum quality, so

\[9\] The set of cases where \( \alpha = w - (p/t) \) can be ignored since this set is of measure zero.
$p/t$ is the amount of money the university receives per academic for teaching at minimum quality. Hence, cases belonging to set A arise when the money for teaching is more than sufficient to cover the gap between academic salaries and the minimum payment per academic for research ($p/t > w$); set B arises when the funding for teaching is not sufficient to cover the gap between academic salaries and the required funds for research. In the Appendix we provide a detailed characterisation of these cases, while in the next section we discuss their implications.  

4 The Trade-off between Teaching and Research

Consider what happens when the budget constraint, (4), holds as a strict inequality. This happens when the research quality $q$ offered by a university lies in the interior of the relevant quality intervals; in other words, there is a potential surplus of funding (see Figure 1 and also relevant figures in the Appendix). There are two possibilities:

(i) A university is achieving a given quality $q$ of research, is teaching at minimum quality $t$, but is accumulating a surplus that it is using to build up resources.

(ii) A university is achieving a given quality $q$ of research but could be teaching at above minimum quality $t$, so as to just break even. In fact we define $\bar{t}(q) \geq t$ as the maximum teaching quality achievable by a university when its research quality is $q$ and it is just breaking even. This is given by

$$\bar{t}(q) \equiv \frac{p(1-q^2)}{w - \left\{ \alpha + \rho \max[0, q - q]\right\}}, \quad (EF)$$

and describes an efficiency frontier (EF) in $(t, q)$ space that can be plotted for each of the cases we have identified. In what follows, we graph this frontier and discuss its implications. In the discussion that follows we assume that

10 We do not discuss the implications of case B as this is not very realistic but we included it in the Appendix for completeness.
universities can freely choose where to locate on the efficiency frontier depending on their specific $\omega$.

**The Research Elite**[^11] The efficiency frontier that this funding case generates is shown in Figure 2. This case is interesting because there is a unique value of $\omega$, $\omega^0$, say, such that a university with this specific characteristic maximising its objective[^12] will produce a double tangency at, say $q_0$ and $\bar{q}_0$, where $q_0$ lies on first hump of the efficiency frontier (EF) and so $\bar{q}_0 < q$, and $\bar{q}_0$ lies on second hump and so $\bar{q}_0 > q$. No university will operate with $q$ between $q_0$ and $\bar{q}_0$. Those universities with lower weight to research than $\omega^0$ will choose $q < q_0$, while those with higher weight to research than $\omega^0$ will choose $q > q_0$. So this funding case produces two discretely different groups of university – one group below the funding threshold, $q$, and one above it (the latter is the ‘research elite’). Surprisingly there will be no universities close to the quality funding threshold. The explanation for the existence/sorting of the two groups lies entirely in differences in preferences over $\omega$.

![Figure 2: The Research Elite](image-url)

[^11]: For details see case A1 in the Appendix.
[^12]: Given homothetic utility functions of the form we have assumed, indifference curves are straight lines with slope $-\omega/(1-\omega).$
The efficiency frontier that this case produces is shown in Figure 3. This case is also interesting because this is precisely the frontier that is produced if there are no research incentives ($\rho = 0$). In this instance universities are expected to spread themselves across the frontier (EF). The only reason for bunching would be if preferences were bunched – say there was a kind of ‘binary divide’ with some institutions ordered to give a high weight to teaching and the others to research.

Figure 3: A Flat System

The efficiency frontier that this produces is shown in Figure 4. To see the implications of this, consider the convex hull of the efficiency frontier. There are two instances. The first one is where the teaching quality when $q = 0$ is higher than the maximum on the right hand portion of the frontier. In this instance the convex hull will consist of most of the downward sloping part of right hand portion plus a little bit of the left hand portion. Essentially the convex hull is very similar to the case of the ‘research elite’ above. Once again two discrete groups of universities will form: those that do no research at all and those that do, i.e., a sort of stark ‘binary divide’ emerges across institutions.

---

13 For details see case A2 in the Appendix.
14 For details see case A3 in the Appendix.
The second case (not shown) is where the teaching quality at \( q = 0 \) is no higher than the maximum on the right hand portion of the frontier. Here the convex hull is just all of the downward sloping part of the right-hand side of the frontier plus a horizontal line at the maximum. Now all universities would be spread around the right hand portion of the frontier, and there would be no discretely different groups.

Figure 4: A highly-incentivised system (the binary divide)

We are now in a position to address the question of what happens when the funding mechanism increasingly rewards research quality. This is an interesting issue for two reasons. The first is that it allows us to compare university systems in general across countries; the second is that it allows us to examine what has happened (and may continue to happen) over time within any one country.

Suppose we start with a completely flat system in which universities are funded for teaching students and receive a block grant per academic to support research and scholarship (Figure 3, 'the flat system'). The analysis predicts that, while there may be the odd university that focuses almost wholly on teaching and whose research quality is modest, the vast majority will be moderately good at both teaching and research, but there will be few doing world-class quality
research. In such a higher education system academics are absorbers of ideas rather than their creators. If we then introduce a premium for research quality, this can only be funded, given the overall fiscal balance, by a reduction in the block grant element. It may also require a university to achieve some threshold level of research quality before the premium is paid. What results now is a university system in which there is a bifurcation: a small research elite emerges while the bulk of institutions are strong in teaching and solid, if uninspiring, research (‘the research elite’, figure 2); this can delineate a sort of emerging ‘culture’ where universities on the one side of the frontier cannot move easily to the other, they are rather confined to their primary role of teaching or research not being allowed to run a significant deficit to cross over. If we further increase the steepness of the reward function for research quality, we end up with the kind of system that existed in the UK prior to 1992. In other words, the ‘binary divide’ is restored and we observe one set of the higher education institutions concentrating on teaching and doing minimal research and the remainder doing high-quality (most likely internationally-rated) research (‘the binary divide’, figure 4). Between these two groups a gap in the research quality spectrum opens up in which there are no institutions present. This lack of research spectrum might be problematic depending on the type and sort of research it represents. Indeed, the ‘lacking’ research might be extremely valuable for policy say, but is not much valued by researchers in terms of its quality. Thus in one group of universities, academics are so busy teaching, they do not have the time to think about policy and, in the other group, the academics are so busy trying to deliver research at the frontiers of knowledge, they have neither the time for nor the interest in it.

In summary, both the ‘research elite’ and the ‘binary divide’ cases describe incentivised education systems that generate multiple outcomes in the sense of two discretely different types of university emerging. In all three cases the funds available for teaching per academic (p/t) are more than sufficient to cover the

15 This simple discussion points out to the need of further work to assess research quality issues and the breadth of research coverage but is outside the scope of the present paper.
difference between salary, \( w \), and block grant, \( \alpha \), received. The non-incentivised flat system arises when the research quality threshold is above the research quality associated with the minimum teaching quality and a binding budget constraint with incentives absent \( (q > \hat{q}) \). The incentivised systems obtain: (i) when the research funding scheme is relatively strong \( (\rho > \rho^0) \) and \( q > \hat{q} \) or (ii) for any research funding scheme when the research quality threshold is below the research quality associated with the minimum teaching quality and a binding budget constraint were incentives absent \( (q < \hat{q}) \). Hence, the design and characteristics of the university funding system (as captured by the budget constraint) are determining in the manner that we have described a ‘culture’: an incentivised system gives rise to a ‘research elite’ co-existing with universities performing no (or minimal) research but all universities are providing at least the minimum teaching quality; a non-incentivised system by its nature leads to less polarisation.\(^{16}\)

5 Concluding Remarks

In this paper we have taken some first steps in modelling the way in which higher education funding systems can generate university ‘cultures’. The important elements in the modelling framework are as follows: (1) we have recognised that universities are principally concerned about the quality of teaching and research; (2) we have endogenised the choice by a university of its actual selection of teaching and research quality; (3) we have taken explicit account of the fact that research and teaching has to be performed by academics who face a time constraint; and (4) we have explicitly modelled the quality of teaching and research. Understanding how these interact matters if we are to be able to assess the implications of making higher education funding systems depend on indicators of teaching and research quality. What we have shown is that, by

\(^{16}\)We note here that one somehow unsatisfactory aspect with both incentivised systems is that no university is very close to the critical research funding threshold. Essentially, what drives the outcomes is the diversity of views within universities as to their objectives as captured by the weight placed on research/teaching.
varying the key parameters of the public funding system, a range of university 'cultures' can be generated and this seems to offer a theoretical framework for empirical cross-country comparisons and for policy advice.
References


6 Appendix

6.1 Set A Cases: \( w - \frac{p}{t} < \alpha \)

Define \( \hat{q} \) as the research quality such that teaching quality is at the minimum threshold and the budget constraint is binding in the absence of research incentives, that is,

\[
\hat{q} \equiv \left\{ q \mid \left( w - \frac{p}{t} \right) + \frac{p}{t} q^a = \alpha \right\}.
\]

Notice that given the definition for set A cases, there is a unique \( \hat{q}, 0 < \hat{q} < 1 \) that satisfies the above equation. There are then 3 individual sub-cases to consider.

**Case A1.** \( q \leq \hat{q} \)

Assumption (a1) can only be satisfied if \( \alpha + \rho(1 - q) < w \), in which case the set of values of \( q \) that satisfy the budget constraint, see (4), is \([0, \hat{q}_1]\) where \( \hat{q}_1 \) is the unique solution to

\[
\left( w - \frac{p}{t} \right) + \frac{p}{t} q^a = \alpha + \rho(q - \hat{q}).
\] (5)

This is illustrated in Figure A1.

![Figure A1](image-url)
To understand the next two cases let $\rho^0$ and $q^0 \geq \frac{q}{\lambda}$ be the unique solutions to the equation (5) above and
d

\[ \beta \frac{P}{L}(q^0)^{\beta-1} = \rho^0, \tag{6} \]

where (6) is just the slope of the LHS of (4) evaluated at $q^0$ and set equal to $\rho^0$. Figures A2 and A3 illustrate.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig_a2.png}
\caption{Case A2. $q > \hat{q}$ and $\rho < \rho^0$}
\end{figure}

The only set of values of $q$ that satisfy equation (4) is $[0, \hat{q}]$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig_a3.png}
\caption{Case A2. $q > \hat{q}$ and $\rho < \rho^0$}
\end{figure}
Case A3. $q > \hat{q}$ and $\rho > \rho^0$

In this case equation (5) has two solutions: $\hat{q}_2, \hat{q}_3$, with $\hat{q} < \hat{q}_2 < \hat{q}_3$. This subdivides further into two sub-cases:

Case A3(a). In addition to the two conditions above suppose that $\alpha + \rho(1 - q) < w$. Then $\hat{q}_3 < 1$. Thus the set of values of $q$ that satisfy (4) comprises the union of two disjoint intervals $[0, \hat{q}] \cup [\hat{q}_2, \hat{q}_3]$.

Case A3(b). If $\alpha + \rho(1 - q) \geq w$ then $\hat{q}_3 \geq 1$. Therefore, the set of values of $q$ that satisfy (4) comprises the union of the disjoint intervals $[0, \hat{q}] \cup [\hat{q}_2, 1]$.

6.2 Set B Cases: $a < w - p/t$

It turns out that there is just one general case, though, as in case A3 above, this divides into two sub-cases. We can once again define $\rho^0$ and $q^0$ as the solutions to equations (5) and (6). In order to ensure that assumption (a2) is satisfied we need to impose that $\rho > \rho^0$. It is still true that equation (5) has two solutions: $\hat{q}_2, \hat{q}_3$, with $\hat{q} < \hat{q}_2 < \hat{q}_3$. So there are just two sub-cases:

Case B(a) Here $\hat{q}_3 < 1$. This arises when $\rho^0 < \rho < \frac{w - a}{1 - q}$. Then the set of values of $q$ that satisfy (4) comprises the intervals $[\hat{q}_2, \hat{q}_3]$. Figure A4 illustrates.

Case B(b) Here $\hat{q}_3 \geq 1$. This arises when $\rho \geq \frac{w - a}{1 - q} > \rho^0$. Then the set of values of $q$ that satisfy (4) comprises the interval $[\hat{q}_2, 1]$.

Figure A4

The case where $\hat{q} > \hat{q}$ and $\rho = \rho^0$ is of no significance since this arises on a set of measure zero.
*Acknowledgment: We would like to thank Bernard Bellock, Axel Gauthier, Yannis Katsoulacos, Louis Lévy-Garboua, Robert Owen, Xavier Wauthy for helpful discussions, seminar audiences at ANU, Antwerp, Dundee, Edinburgh, La Trobe, Manchester, Massey, St Andrews, Sheffield, Stirling, JMA Conference (Hammamet), EARIE Conference (Oporto), RES Conference (Nottingham), ASSET Meeting (Rethymno) and Conference on Higher Education (Mons) for useful comments and, MAICH (Crete) for the hospitality provided while Beath and Ulph were working on the initial ideas leading to this paper. Beath and Poyago-Theotoky acknowledge the ESRC for financial support (grant no. RES-000-22-0340).
Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

Please go to:


The Editor