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Fixed to VoIP Interconnection: Regulation with Asymmetric Termination Costs

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Abstract

Typically, incumbent providers enjoy a demand-side advantage over any entrant. However, market entrants may enjoy a supply-side advantage in costs over the incumbent, since they are more efficient or operate on innovative technologies, such as the voice of internet protocol (VoIP) telephony. Regulation with a supply-side asymmetry has rarely been addressed. Considering both a supply-side and a demand-side asymmetry, the present model analyzes the effects different regulation regimes. Regulation may have adverse effects on subscribers, market shares, and profits. If providers can discriminate between on-net and off-net prices, asymmetric regulation has no local effect on market shares, independent of any demand- and supply-side asymmetry. Otherwise, with reciprocal termination charges, price discrimination leads to qualitatively same effects than nondiscriminatory pricing.

Keywords: Termination charges; Interconnection; Regulation; Price Discrimination; Voice over Internet Protocol (VoIP)

JEL-Classification: L13, L51, L96

1 Introduction

The emergence of voice telephony based on IP networks (VoIP) leads to fundamental changes in the telecommunications markets and disrupts the position of fixed-line incumbents. The VoIP adoption of US households has been steadily increasing from 28 % in 2008 to expected 50 % in 2010.¹ The same holds for Germany, where the share of calls placed on IP networks increased from 10 % in 2006 to 34 % in 2009², whereas the share of calls placed on traditional fixed-line is accordingly decreasing.

²See http://www.bundesnetzagentur.de/media/archive/17897.pdf.
VoIP providers offer their service based on the Internet Protocol (IP), where access to end consumers is often controlled by fixed-line network operators. By regulatory requirements to offer local loop unbundling and bitstream access at the wholesale level, regulatory authorities have facilitated market entry of alternative providers into telecommunications markets. However, there are still open questions with respect to call termination between traditional fixed-line and IP-based networks. With interconnection between both networks, calls from one network to the other are delivered through an interconnection point, or gateway, often controlled by the traditional fixed-line network. In this case, the fixed-line operator meters calls and sets a termination charge to the VoIP provider for calls terminating in its network. Otherwise, calls to the VoIP provider are terminated on the Internet, where costs of providing access are significantly lower than in traditional fixed-line networks (see Monopolkommission (2006), p. 25). Apart from this cost-asymmetry, other asymmetries can be observed in telecommunications markets, e.g., asymmetries in size of the customer base of providers. The question now arises whether VoIP providers should be regulated in the same way or if some other regulation is more adequate. To answer this question, an analytical model explores the effect of asymmetric and reciprocal regulation of termination charges in a setting of nondiscriminatory pricing and in a setting where providers can discriminate between on-net and off-net calls.

The ability to take advantage of lower termination costs for VoIP providers depends on the regulatory regime. In May 2009, the European Commission (EU Commission (2009a)) issued a “Recommendation on the Regulatory Treatment of Fixed and Mobile Termination Rates in the EU” which sets out its views on how national regulators in Europe should approach this issue in the future. The recommendation basically is (i) to set termination charges to the long-run incremental cost level, and (ii) to require reciprocity with networks. In addition the European Commission recommends (iii) to adopt “bill-and-keep” (i.e. zero termination charges), which would effectively abolish termination charges. Each of these alternatives is considered in the European Commission’s recommendation (see in particular (EU Commission (2009b), p. 29), where it is noted that, “a significant reduction of termination rates from current levels might create appropriate incentives for voluntary inter-operator agreements and consequently Bill and Keep type arrangements could evolve naturally”.

This recommendation does not explicitly address termination issues between traditional fixed-line and VoIP networks. Thus, it is an open question whether terms of regulation designed for fixed-line networks are adequate in the emerging market of VoIP telephony. The present paper explicitly addresses the proposals by the European Commission and analyzes: i) whether the VoIP provider should be allowed to charge a markup on termination costs to take advantage of its lower termination costs and ii) whether reciprocal termination costs should be set below the costs of the fixed-line network.

3For a discussion on various regulatory instruments concerning wholesale regulation see, e.g., Vogelsang (2003), Foros (2004), and De Bijl and Peitz (2007).

The model introduces a supply-side asymmetry, namely the asymmetry in termination costs. For calls from a VoIP to a fixed-line network, a VoIP provider has to pay higher charges for off-net termination than it otherwise receives for calls terminated on its network. Additionally, the model introduces demand-side asymmetry to capture the recent structure of European telecommunications markets where incumbent fixed-line providers still capture a larger share of the market than alternative providers. Given supply- and demand-side asymmetries, there are open policy questions concerning regulation of interconnection between both types of networks, which may not be optimally solved by the recommendation of the European Commission.

Both regimes induce direct and indirect effects on competition in the market. An increase in the VoIP termination charge directly increases marginal costs of calls for the fixed-line provider and, thus, its per-minute price. But this indirectly triggers changes in market shares, which feed back into changes in perceived marginal costs. This leads to the ambiguous result that the VoIP network does not necessarily benefit from above cost termination charges as total income in the termination market and retail profits may be reduced. Otherwise, a higher termination charge on the VoIP network does not generally harm the fixed-line provider. The effects on profits are not necessarily monotone in the termination charge of the VoIP provider. The effects crucially depend on the degree of competition in the market and on the extent of both kinds of asymmetry. Qualitatively, both providers face the same effects with reciprocal termination charges. A marginal decrease of the reciprocal termination charge decreases the per-minute interconnection profit but may increase total profit.

Serving providers involves fixed and marginal cost. Marginal costs are technologically determined for a call that terminates in the same network (“on-net”), but it depends on the termination charge if the call is terminated in a rival network (“off-net”). If the providers can charge their customers different prices for on-net and off-net calls it can be shown that asymmetric regulation has no local effects on market shares, independent of any demand-and supply-side asymmetry. The VoIP provider locally benefits from a marginal increase in its termination charge, whereas the fixed-line provider suffers. If providers set a reciprocal termination charge, the same holds given a demand- or a supply-side symmetry.

Regulatory concerns between asymmetric networks have already been addressed in recent literature, where the asymmetry stems from several advantages of an established incumbent network vis-à-vis an entrant. In the models of Carter and Wright (1999, 2003) an entrant has to undercut the incumbent’s price to gain market shares due to a reputation advantage of the incumbent. The same holds in the model of Peitz (2005) where an entrant provides a lower fixed utility to consumers due to reliability and reputation advantages of an incumbent network. Carter and Wright (2003) and Peitz (2005) show that a provider benefits from a marginal increase of its termination charge. Kocsis (2007) confirms the result of Peitz (2005) in a model with supply-side asymmetry. In the case of reciprocal termination charges Carter and Wright (2003) show that the incumbent provider favors a cost-based termination charge, which coincides with the social optimal charge. If the incumbent’s advantage is sufficiently strong, the entrant has the
same preferences. Common to these papers is that the incumbent’s advantage results in
a larger installed base, but does not affect calling patterns, which drives their results.
Additionally to the incumbent’s advantage, the present model introduces an entrant’s
advantage, the cost-advantage, which has rarely been analyzed in the literature, yet. In
contrast to the demand-side asymmetry, this supply-side asymmetry directly affects call
prices and thus the demand for calls. A change in termination charges feeds back into
market shares. It will be shown that a unilateral increase of the termination charge may
be unprofitable if the negative feedback effect on market shares dominates generated
termination income. This effect is not present in the models of asymmetric regulation of
Carter and Wright (2003) and Peitz (2005). In their models, termination charges have
a neutral effect on market shares (locally around cost-based regulation), so unilaterally
increasing the termination charge is unambiguously in favor of the respective network.

The paper is in line with the wide literature on interconnection terms between com-
munication networks such as Armstrong (1998) and Laffont et al. (1998a) and Laffont
et al. (1998b), which focus on mobile communication. Asymmetries in network size
have also been addressed by Gans and King (2001) who show that networks maximize
joint profits by setting off-net prices below the efficient level and therefore termination
rates below the true cost of termination. Dewenter and Haucap (2005) also analyze mo-
 bile termination rates when networks are of asymmetric size. They show that a mobile
network’s termination charge is the higher the smaller the network’s size and that asym-
metric regulation of only the larger network will induce the smaller networks to increase
their termination rates. They support their results by empirical evidence. Hoernig (2009)
calibrates a model of competition between an arbitrary number of telecommunications
networks in the presence of tariff-mediated network externalities, call externalities, and
cost and surplus asymmetries. He shows that a reduction in the mobile-to-mobile termi-
nation rate still mitigates network effects, and hence relaxes competition between mobile
networks for market shares, the reduction in competition may or may not be sufficient to
reduce consumer surplus in equilibrium, and it is less likely to do so the more significant
call externalities are, and the larger the number of competing networks. Harbord and
Hoernig (2010) run simulations based on the model of Hoernig (2009) to show that a
‘bill-and-keep’ regime increases social welfare, consumer surplus, and networks’ profits.
First research on competition between traditional fixed-line and VoIP networks is con-
ducted by De Bijl and Peitz (2009). As the fixed-line incumbent also controls the IP
network, it also has the opportunity to offer IP-based service, so their model deals with
endogenous consumer migration between both technologies.

The rest of the paper is organized as follows: Section 2 provides the base model.
Section 3 allows for raising the VoIP termination charge above marginal cost. Section
4 discusses the effect of reciprocity of termination charges for both networks. Section 5
allows providers to discriminate between on-net and off-net prices. Section 6 concludes.

\(^5\)For an overview on the literature see Armstrong (2002) and for aspects on call externalities and
network effects Harbord and Pagnozzi (2010)).
2 The Model

Across Europe former state-owned incumbent fixed-line operators compete with alternative telecommunications providers. In the present model, it is assumed that a fixed-line provider (firm 1) competes with an entrant (firm 2), which operates at lower termination costs. Henceforth, this provider is labeled as a VoIP provider, although, it could be any provider which operates at lower termination costs than an established incumbent network. The providers compete for customers in the retail market. VoIP customers completely substitute the fixed-line service. The VoIP provider needs access to an IP based network to offer voice services. To abstract from any regulatory issues on access regulation at the wholesale level and to focus on termination charges between networks, all costs and charges at the wholesale level are set to zero. The VoIP provider may use local loop unbundling to reach end users, which means that the VoIP provider makes use of the incumbent network through so called “bitstream access”. Hence, the framework captures “naked DSL” a service provision in which the VoIP provider provides a broadband Internet connection based on DSL by leasing only the broadband part of the frequency spectrum of the copper wire. The model follows Laffont et al. (1998a). It assumes that both networks are interconnected and provide full local coverage.

2.1 Cost Structure

For calls from the VoIP network to the traditional fixed-line operator the VoIP provider has to pay a termination charge of $a_1$. For calls to the VoIP provider the traditional fixed-line network has to pay a termination charge of $a_2$. It is assumed that termination charges are set by a regulator prior to competition in the retail market.

The networks incur a marginal cost $c_i$ per minute for originating and terminating a call, so total marginal costs of a call are assumed to be $2c_i$, where the model abstracts from any additional costs, e.g. transmission costs. Since the VoIP network provides its service on the Internet, its costs are assumed to be lower than on fixed-line, hence $c_2 < c_1$. As De Bijl and Peitz (2009) state the “true” marginal costs of electronic communications are virtually zero. Also the German Monopolies Commission states that there should be no termination costs on IP based networks in general (Monopolkommission (2006)).

The model analyzes two regulatory regimes. In the first part, it evaluates the effects of a marginal increase of the VoIP provider’s termination charge above its marginal costs. In the second part it analyzes the effects of a marginal decrease of a reciprocal termination charge below the costs of the fixed-line incumbent.

2.2 Demand Structure

Consider a market where an incumbent fixed-line provider has a larger installed subscriber base than a VoIP provider, which has recently entered the market. To model the

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6For issues on wholesale regulation in telecommunications markets see, e.g., Foros (2004).
7Nevertheless, in practice, operators allocate fixed costs to traffic, and hence may partly treat these costs as marginal costs when setting prices.
demand-side asymmetry the present model follows the framework of Carter and Wright (2003). The utility derived by a consumer for subscribing to either network $i$ is given as

$$U_i = v_0 + \theta_i + u(q(p_i)),$$

where $q(p_i)$ is the number of calls placed on network $i$, depending on the price $p_i$. $v_0$ represents a fixed surplus (“option value”) from being connected to either network and is assumed sufficiently large so that all subscribers choose to be connected to a network. Subscribers receive a network specific benefit of subscribing to network $i$ of

$$\theta_1 = \frac{\beta}{2\sigma} + \frac{1-x}{2\sigma}$$

and

$$\theta_2 = \frac{x}{2\sigma}.$$

Customers are endowed with a value of $x$ drawn from a uniform distribution on the $[0, 1]$ interval, with the networks 1 and 2 located at either end of the interval. The parameter $\sigma$ expresses the degree of substitution between both providers, where lower values correspond to a lower degree, so that providers can charge higher prices without loosing all their market shares. Hence, $\sigma$ can be interpreted to reflect the degree of competition in the market, with higher values corresponding to more intense competition.

As in the models of Carter and Wright (1999, 2003) the present model introduces an incumbency advantage of $\beta > 0$. An incumbency advantage results from a variety of factors. It might capture reputation effects of an established network, whereas there is uncertainty about the quality and service of the new network. Alternatively, it can proxy for switching costs (see De Bijl and Peitz (2002)) due to consumers’ inertia or due to technical reasons. In either case it is assumed that the initial advantage is such that the fixed-line network has a larger installed base, which mirrors present market structures in most fixed-line telecommunications markets in Europe. Given equal prices, the fixed-line network can attract more consumers than its rival, hence the VoIP provider has to offset the fixed-line network’s advantage by undercutting the fixed-line network’s tariff.

Given that all consumers’ marginal willingness to pay for calls is the same and known, networks can do no better than offering two-part tariffs. Each network charges a per-minute price $p_i$ and a fixed fee $F_i$. Therefore, the two-part tariff is given as $T_i(q) = F_i + p_i q(p_i)$.

The function

$$v(p_i) = \max_q\{u(q) - p_i q\}$$

denotes the indirect utility derived from making calls at a price $p$, so $v'(q) = -q(p)$ gives the associated demand function. For example, a linear demand function of $q(p) = 1 - p$ is represented by an indirect utility of $v(p) = \frac{1}{2}(1 - p)^2$. A consumer’s net surplus of belonging to network $i$ is $\omega_i = v(p_i) - F_i$. Subscribers are assumed to be identical in terms of their demand for calls to other subscribers.
Solving for the indifferent consumer with $U_1 = U_2$, the market share of the fixed-line provider is

$$s_1 = \frac{1}{2} + \beta \sigma(\omega_1 - \omega_2)$$  \hspace{1cm} (2)$$

and $s_2 = 1 - s_1$ for the VoIP provider.

### 3 Asymmetric Regulation

In the following analysis the VoIP provider may charge a termination fee above marginal costs.\(^8\) This assumption captures the policy concerns about call termination from fixed-line to VoIP networks. For calls terminated in the Internet termination costs are generally assumed to be lower than in fixed-line networks. Now, with cost based regulation the VoIP provider receives less for calls from rival subscribers than it pays for calls, which are terminated in the traditional fixed-line network. Hence, a relevant policy question is whether to allow VoIP networks to charge a termination fee above their marginal costs of termination.

Since market shares $s_i$ are directly determined by the net surplus $\omega_i$, it is more convenient to consider networks to compete over $p_i$ and $\omega_i$ rather than in $p_i$ and $F_i$. Substituting $F_i = \upsilon(p_i) - \omega_i$ the profit function of provider $i$ is denoted as

$$\Pi_i = s_i(p_i - 2c_i)q(p_i) + s_i\upsilon(p_i) - \omega_i + s_is_j((a_i - c_i)q(p_j) - (a_j - c_i)q(p_i)).$$  \hspace{1cm} (3)$$

The first two parts denote the profits in the retail market due to per-minute prices and fixed fees. Calling patterns are assumed to be balanced, with a share of $s_is_j$ requiring interconnection.\(^9\) The third part represents the profit in the interconnection market. Provider $i$ charges a termination rate of $a_i$, but incurs costs of $c_i$ for rival subscribers' calls terminated in its network. Otherwise, for off-net calls by fellow subscribers the provider has to pay a termination charge of $a_j$ but saves the termination costs.

The first order conditions for network $i$ with respect to $p_i$ and $\omega_i$ are

$$\frac{\partial \Pi_i}{\partial p_i} = s_i(q(p_i) + (p_i - 2c_i)q'(p_i)) + s_i\upsilon'(p_i) + s_is_jc_iq'(p_i) = 0$$

and

$$\frac{\partial \Pi_i}{\partial \omega_i} = \sigma((p_i - 2c_i)q(p_i) + (\upsilon(p_i) - \omega_i)) + (s_j - s_i)((a_i - c_i)q(p_j) - (a_j - c_j)q(p_i)) - s_i = 0,$$

\(^8\)In a different model setup De Bijl and Peitz (2009) analyze the effects of charging termination fees at the fixed-line network, assuming bill-and-keep pricing at the VoIP network.

\(^9\)This is the standard assumption in the literature (see, e.g., Laffont et al. (1998a) or Valetti and Cambini (2005)). Gabrielsen and Vagstad (2008) deviate and assume that people tend to place more calls in “calling clubs” i.e. to family and friends, independent of the market share of the providers.
where \( q_i' = \frac{dq_i(p_i)}{dp} \). Using \( v'(q) \equiv -q(p) \), the FOCs with respect to \( p_i \) yield equilibrium prices corresponding to “the perceived marginal costs” of a call of

\[
p_i^* = 2c_i + s^*_j(a_j - c_i),
\]

which is the standard result in the symmetric setup of Laffont et al. (1998a) and asymmetric setups of Carter and Wright (2003), Peitz (2005) and Valletti and Cambini (2005). By setting prices equal to the perceived marginal costs the networks can extract consumers’ surplus by the fixed fee. The providers incur costs of \( 2c_i \) for originating and terminating calls on-net but save costs of \( s_jc_i \) for calls terminated off-net. Rearranging \( F_i = v(p_i) - \omega_i \), the fixed fee at the equilibrium per-minute price is given as

\[
F_i^* = \frac{s^*_i}{\sigma} - s^*_i(a_j - c_i)q^*_i + (s^*_i - s^*_j)(a_i - c_i)q^*_j.
\]

A first insight into the effects of increasing termination charges can be gained by inspection of equations (4) and (5). An increase of termination charge \( a_j \) only directly affects the per-minute price of the rival firm \( i \) but is offset by a reduction in its fixed fee. The total effect on profit is ambiguous and depends on the asymmetry between operators. The first order effect of allowing the VoIP provider to charge a termination fee \( a_2 > c_2 \) is straightforward. It increases the marginal cost of a call for the traditional fixed-line network and thus the per-minute price \( p_1 \). As the termination fee on the VoIP network pushes prices for customers of the fixed-line network, this implies a lower indirect utility from calls. At the margin this effect is equal to \( -\frac{\partial p_1}{\partial a_2} q(p_1) \). Given lower indirect utility of calls, the fixed-line network lowers the fixed fee by the second term in equation (5) of \( s^*_i(a_2 - c_1)q(p_1) \). Observe now from the equilibrium tariff of

\[
T_i^* = \frac{s^*_i}{\sigma} + (s^*_i - s^*_j)(a_j - c_i)q^*_j
\]

that for equal market shares \( s_1 = s_2 \) both effects just offset each other, leading to a neutral result on market shares, as net surplus of calls \( \omega_i \) is unaffected. This does not hold any longer for asymmetric termination costs, which is shown in the following section.

### 3.1 Subscribers’ Net Surplus

Each provider sets its per-minute price equal to the perceived marginal cost and, thus, makes no profit from the amount of off-net and on-net traffic by fellow subscribers. The only source of income stems from subscription and inbound calls from rival subscribers. Accordingly, each operator makes a profit terms of net surplus of

\[
\Pi_i^* = s^*_i(v(p^*_i) - \omega^*_i) + s^*_i s^*_j(a_j - c_i)q^*_j.
\]

**Proposition 1** For symmetric termination costs subscribers of both networks benefit from a marginal increase of the VoIP termination charge. For asymmetric termination costs net utilities may increase or decrease. Subscribers of both networks will likely benefit if providers are not too differentiated and termination costs are not too asymmetric.
The complete technical proof is relegated to Appendix 4A and goes along the line originated by Peitz (2005). Assume a larger installed base of the fixed-line network, i.e. $s_1 > s_2$. The first-order conditions of $\frac{\partial \Pi}{\partial a_i} = 0$ at equilibrium per-minute prices define the best-response functions in terms of net utilities for each provider, labeled as pseudo best-response functions. Providers offer pseudo best-response functions that are either strategic complements or substitutes, depending on the degree of competition between providers and the difference in termination costs. The cross derivative of the fixed-line provider’s pseudo best-response function is denoted as

$$\frac{\partial^2 \Pi}{\partial \omega_1 \partial \omega_2} |_{a_i = c_i} = \sigma - 2\sigma^2(c_2 - c_1)q(p_1^*) + \sigma^2(c_2 - c_1)^2 s_1^* q' \leq 0.$$  

This implies that the traditional fixed-line network’s pseudo best-response function is upwards sloping if providers are hardly differentiated and the difference in termination costs $(c_1 - c_2)$ is not too large. An increase in the VoIP termination charge $a_2$ shifts the pseudo best-response function outwards, as

$$\frac{\partial^2 \Pi}{\partial \omega_1 \partial a_2} |_{a_i = c_i} = \sigma(s_1^* - s_2^*)q(p_1^*) + s_1^* s_2^* \sigma(c_2 - c_1)q' > 0.$$  

This term is strictly positive for $s_1^* > s_2^*$, $c_2 < c_1$, and $q' < 0$ which has been assumed. Consider the VoIP provider’s profit. Applying same technique, we obtain that

$$\frac{\partial^2 \Pi}{\partial \omega_2 \partial \omega_1} |_{a_i = c_i} = \sigma - 2\sigma^2(c_1 - c_2)q(p_2^*) + \sigma^2(c_1 - c_2)^2 s_2^* q' \leq 0.$$  

A marginal increase of the VoIP termination charge $a_2$ shifts the function outwards as

$$\frac{\partial^2 \Pi}{\partial \omega_2 \partial a_2} |_{a_i = c_i} = \sigma(s_1^* - s_2^*)q(p_2^*) + s_1^* s_2^* \sigma(c_2 - c_1)q' > 0.$$  

Observe that for symmetric termination costs $(c_1 = c_2)$, the pseudo best-response functions are upward sloping, hence are strategic complements. The pseudo best-response functions are shifted outwards in response of a marginal increase in the VoIP termination charge. This confirms the positive effect on subscribers of both providers, obtained in the model of demand-side asymmetry and supply-side symmetry of Peitz (2005). However, for any $c_1 > c_2$ the pseudo best-response functions are either strategic complements are substitutes, since $\frac{\partial^2 \Pi}{\partial a_i \partial a_j} \leq 0$, depending on the parameters. Subscribers of both providers may benefit or suffer from a marginal increase of the VoIP termination charge above marginal costs.

### 3.2 Market Shares

After substitution of $\omega_i = v(p_i) - F_i$ in equation (2), the market share of firm 2 in equilibrium is

$$s_2^* = \frac{1}{2} - \frac{\beta}{2} - \frac{\sigma}{2} (v(p_1^*) - v(p_2^*) + s_2^* q(p_1^*) (a_2 - c_2) + s_1^* q(p_1^*) (c_2 - c_1) - s_2^* q(p_2^*) (a_1 - c_1) + s_2^* q(p_2^*) (c_2 - c_1)).$$  

(8)
Inserting equilibrium per-minute prices and total differentiation of equation (8) (locally around cost-based regulation of \( a_i = c_i \)) yields:

\[
\frac{ds_2^*}{da_2} \bigg|_{a_i = c_i} = \frac{q'^*s_1^*s_2^*(c_2 - c_1)}{2(c_1 - c_2)(q(p_2^* - q(p_1^*)) - (c_2 - c_1)^2q' - \frac{3}{\sigma})}
\]

(9)

and of \( \frac{ds_1^*}{da_2} \bigg|_{a_i = c_i} = -\frac{ds_2^*}{da_2} \bigg|_{a_i = c_i} \) for the traditional fixed-line provider. Hence, there is a local effect on market shares for any asymmetry in termination costs \( (c_1 \neq c_2) \). Given that \( c_2 < c_1 \) the numerator is positive, as \( q' < 0 \). The sign of \( \frac{ds_2^*}{da_2} \) is thus determined by the sign of the denominator.

**Proposition 2** For symmetric termination costs, there is no local effect on market shares. For asymmetric termination costs an increase of the VoIP provider’s termination charge has a positive local effect on its market share if i) the degree of substitution between both networks is sufficiently low (i.e., \( \sigma \) is sufficiently large), ii) if termination costs are sufficiently asymmetric, and iii) the demand for calls is sufficiently inelastic.

**Proof** See Appendix 4A.

**Example 1:** To illustrate the above propositions assume an indirect utility of calls of \( v(p_i) = \frac{1}{2} \frac{(A-p_i)^2}{b} \) for \( A, b > 0 \), which leads to a linear demand of calls of \( q(p_i) = \frac{A-p_i}{b} \) and set \( A = b = 1 \). From evaluation of equation (9) at cost-based regulation it follows that there is a positive effect on the VoIP market share if

\[
(c_1 - c_2)^2 > \frac{1}{\sigma}.
\]

(10)

Given that providers are hardly differentiated, i.e., competition is intense, and given that termination costs are sufficiently asymmetric, an increase of the VoIP termination charge has a positive local effect on its market share. Otherwise, if competition is sufficiently weak, this may be reversed. The intuition behind the result is as follows: The fixed-line provider suffers from higher termination charges at the VoIP network. Therefore, it is in its interest to decrease the outflow of calls. As the per-minute prices are set to marginal cost, a larger termination fee directly increases those. Hence, the fixed-line network can only attract subscribers by lowering the fixed fee. Due to the intense competition, the VoIP provider in turn sets a lower fixed fee itself in order not to lose subscribers in the retail market. Otherwise, for less intense competition, subscribers are less flexible and so the VoIP responds less fiercely to a decreasing fixed-line network’s fixed fee in order to obtain a higher profit in the interconnection market. Therefore, both providers balance their profits in both the retail and the interconnection market.

### 3.3 Profits

Since providers set per-minute prices equal to perceived marginal cost, the equilibrium profits are denoted by equation (7). Since regulation affects market shares, it affects both the retail market (the first part of equation (7)), and the interconnection market
(the second part of the equation). Differentiation of the profit functions with respect to $a_2$ locally around cost-based regulation of $a_i = c_i$, yields

$$\frac{\partial \Pi_1^*}{\partial a_2}|_{a_i = c_i} = 2s_1^* \frac{ds_1^*}{da_2} \left( \frac{1}{\sigma} + (c_1 - c_2)q(p_1^*) \right) + s_1^2 \left( (c_1 - c_2)q \frac{dp_1^*}{da_2} - q(p_1^*) \right)$$

(11)

and

$$\frac{\partial \Pi_2^*}{\partial a_2}|_{a_i = c_i} = 2s_2^* \frac{ds_2^*}{da_2} \left( \frac{1}{\sigma} - (c_1 - c_2)q(p_2^*) \right) + s_2^2 \left( q(p_2^*) - (c_1 - c_2)q \frac{dp_2^*}{da_2} \right).$$

(12)

**Proposition 3** With symmetric termination costs a marginal increase of the VoIP provider’s termination charge positively (negatively) affects the profit of the VoIP provider (fixed-line provider) locally around cost-based regulation. With asymmetric termination costs this may be reversed, so both providers may benefit or suffer. If competition becomes too intense both providers prefer cost-based regulation of termination charges.

Given symmetric termination costs of $c_1 = c_2$ it has been shown in equation (9) that there is no local effect on market shares, hence $\frac{ds_i^*}{da_2}|_{a_i = c_i} = 0$. Applying the neutrality of market shares simplifies the effect of a marginal increase of the VoIP termination charge on providers’ profits denoted as

$$\frac{\partial \Pi_1^*}{\partial a_2}|_{c_1 = c_2} = -s_1^2 q(p_1) < 0$$

and

$$\frac{\partial \Pi_2^*}{\partial a_2}|_{c_1 = c_2} = s_2^2 q(p_1) > 0.$$  

This confirms the non-neutrality result on profits obtained by Peitz (2005) in a model of demand-side asymmetry and by Kocsis (2007) in a model of supply-side asymmetry for symmetric termination costs. However, in the present model the cost asymmetry additionally affects calling patterns, so that the effect on profits is less straightforward and the results of Peitz (2005) and Kocsis (2007) may be reversed. The VoIP provider may suffer and the traditional fixed-line provider may benefit from a markup on the VoIP provider’s termination cost.

Let us decompose the effects on profits in the retail and in the interconnection market and assume the VoIP provider captures market shares from the fixed-line provider, i.e. $\frac{ds_i^*}{da_2} > 0$. An increase in the termination fee above marginal termination cost of the VoIP provider affects i) the per-minute profit of rival subscribers making off-net calls $(a_i - c_i)$, ii) the demand for off-net calls per rival subscriber $(q(p_j^*))$, and iii) the total amount of off-net calls $(s_i^* s_j^*)$. Obviously, a termination markup increases the per-minute profit per rival subscriber unit. Calling patterns are assumed to be balanced. Starting from the asymmetric situation of $s_2 < s_1$, an increase in $s_2$ increases the number of off-net calls, which is maximized at $s_1 = s_2$. Both effects are to the benefit of the VoIP provider. Total interconnection profit is determined by $s_i^* s_j^*(a_i - c_i)q(p_j^*)$. Hence, it is further
necessary to determine the impact on rival subscriber’s demand, given as $dq(p_1) = q'(p_1)$ with $q' < 0$. It holds that
\[
\frac{\partial p_2^*}{\partial a_2}\bigg|_{a_1=c_i} = s_2 - (c_1 - c_2)\frac{ds_2^*}{da_2} \leq 0.
\]
Thus the effect on rival subscribers’ demand is ad hoc unclear. If the difference in termination costs is large the incumbent has an incentive to push the demand for off-net calls to save its termination costs and thereby, to decrease its per-minute price. Otherwise, if the VoIP provider’s subscriber base is too large, an increase in the VoIP termination charge may be to the detriment of its termination profit. This is due to providers’ perceived marginal costs. If the VoIP subscriber base is sufficiently large, there are many off-net calls. Now, an increase in $a_2$ has a larger impact on rival’s per-minute prices for a larger VoIP market share. Given the difference in termination costs, the fixed-line provider increase its per-minute prices for a larger market share of the VoIP provider, reducing the demand of the fixed-line subscribers which may overturn the cost-saving effect. This cost-saving effect is new and not present in the current literature. The extent of this cost-saving effect essentially determines many of the results.

Let us now consider the retail market. It follows from equation (7) that the effect of a termination markup of the VoIP provider’s retail profit is determined by market shares and the fixed fee, determined by subscribers net surplus as $F_i = v(p_i) - \omega_i$. Assume again that the VoIP provider’s market share is increasing in $a_2$. Locally evaluating the derivative of the fixed fee with respect to the termination fee of the VoIP provider around cost-based termination charges yields
\[
\frac{\partial F_2^*}{\partial a_2}\bigg|_{a_1=c_i} = \frac{ds_2^*}{da_2} \left( -\frac{1}{\sigma} - (c_1 - c_2)q(p_2^*) - (c_1 - c_2)q' \right) - (s_1^* - s_2^*)q(p_1^*).
\]
It has been stated above that for symmetric termination cost subscribers’ net surplus increases. Hence, since market share are locally unaffected for $c_i = c_j$, fixed fees decrease, leading to lower profits in the retail market. Otherwise, for asymmetric termination costs, the fixed fee may increase or decrease. If providers are sufficiently differentiated ($\sigma$ is small) the VoIP provider will likely benefit in the retail market, otherwise it may be harmed. Notice that the fixed-line provider compensates its subscribers for paying higher per-minute prices by increasing subscribers’ net surplus. In order not to lose market shares, the VoIP provider has to respond by offering a higher net surplus itself. If providers are rarely differentiated, competition on net surplus is intense. Otherwise, if they are sufficiently differentiated, competition is relatively weak and the VoIP provider responds less fiercely to the fixed-line provider and can maintain a larger fixed fee. The effects in the retail and interconnection market may be countervailing, leading to a non-monotone relationship between the termination charge and profits. This will be illustrated in example 2.

Consider the effects for the fixed-line provider. Notably, as per-minute prices are set equal to perceived marginal cost, an increase of $a_2$ does not affect the interconnection profit of equation (7) locally around $a_1 = c_1$. So the local effect on total profit is given
as
\[
\frac{\partial \Pi_1^*}{\partial a_2} \bigg|_{a_1=c_1} = \frac{ds_1^*}{da_2} F_1 + s_1^* \frac{\partial F_1^*}{\partial a_2}
\]

Remember that the fixed-line provider may offer a higher net surplus to its subscribers in response to an increase in \(a_2\). In order to determine the effect on the fixed fee it is necessary to additionally determine the effect on the indirect utility from making calls, as \(F_i = \nu(p_i) - \omega_i\). Given the indirect utility \(\nu(p_i)\) the fixed fee is the lower the higher the net utility \(\omega_i\). The effect on the indirect utility from making calls is affected by the per-minute price, which may increase or decrease in \(a_2\) as \(\frac{dp_1^*}{\partial a_2} |_{a_1=c_1} = s_2^* - (c_1 - c_2) \frac{ds_2^*}{da_2} \leq 0\).

Now if competition is sufficiently weak, it follows that \(ds_2^* > 0\), and the per-minute price for fixed-line customers decrease. The fixed-line provider saves the higher termination cost on its network for every call terminated in the VoIP network. For \(s_2 < s_1\) an increase in the VoIP provider’s market share increases the number of off-net calls, which is maximized at \(s_2 = s_1\). Now, the perceived marginal cost is the lower the higher the VoIP provider’s market share. It can even be in the interest of the fixed-line provider to give up market share to the rival. This enables the fixed-line provider to increase the indirect utility and set a higher fixed fee to remaining subscribers. This positive effect on indirect utility vanishes if termination costs become symmetric. This positive effect holds if the share of off-net calls, determined by rival’s market share is small, otherwise for large \(s_2\) the total loss in market shares might become too large compared to the cost saving effect. Thus, the effects on profits crucially depend on the demand- and supply-side asymmetry and on the degree of competition in the market.

The following example illustrates that both positive and negative effects on profits are possible for both providers and the relationship between profits and the VoIP termination charge is non-monotone for a more global deviation from cost-based regulation.

**Example 2:** Consider a linear demand of \(q(p) = 1 - p\) and set parameters at \(a_1 = c_1 = 0.5\) and \(c_2 = 0\). Table 2 illustrates the impact of a small increase of the VoIP provider’s termination charge from \(a_2 = 0\) to \(a_2 = 0.05\) on profits and market shares, depending on the degree of competition and the traditional fixed-line provider’s initial advantage, which determines the installed providers’ subscriber base.

<table>
<thead>
<tr>
<th>(\sigma = 0.01)</th>
<th>(\sigma = 0.1)</th>
<th>(\sigma = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta = 1)</td>
<td>(\beta = 4)</td>
<td>(\beta = 1)</td>
</tr>
<tr>
<td>(\Delta s_2)</td>
<td>+0.03 %</td>
<td>+0.40 %</td>
</tr>
<tr>
<td>(\Delta \Pi_1)</td>
<td>+0.20 %</td>
<td>-0.10 %</td>
</tr>
<tr>
<td>(\Delta \Pi_2)</td>
<td>+0.05 %</td>
<td>+0.80 %</td>
</tr>
</tbody>
</table>

An increase in the termination charge is not necessarily beneficial for the VoIP provider and not necessarily detrimental for the traditional fixed-line provider. If com-
petition is very soft ($\sigma = 0.01$) and the fixed-line incumbency advantage is large ($\beta = 4$), the VoIP provider benefits in terms of market shares and profits, whereas the fixed-line provider loses. Otherwise, if the VoIP installed base is already sufficiently large (i.e., $\beta = 1$), this is reversed. Given competition is intense and the VoIP installed base is sufficiently small, both providers prefer cost-based regulation, as an increase of the VoIP provider’s termination charge is to the detriment of both providers’ profits.

Figure 1 plots the profit functions of the VoIP provider for a larger deviation from cost-based regulation in the above example for $\sigma = 0.5$ and $\beta = 1$. The VoIP provider prefers an above, but close to marginal cost termination charge, whereas the traditional fixed-line provider prefers the VoIP provider to be regulated at marginal costs. To conclude, there are opposing effects a regulatory authority has to consider when regulating termination charges for VoIP networks. Regulation of termination fees may have a non-monotone effect on profits for asymmetric termination costs. This reverses the results of Peitz (2005) and Kocsis (2007).\footnote{Kocsis (2007) also considers asymmetric termination costs, but obtains similar results as Peitz (2005). Her model implicitly assumes that providers set termination charges at different stages of the game: At a first stage the more efficient firm sets its termination charge, assuming that the less efficient provider is regulated to marginal costs at the second stage. Instead, the present paper assumes that termination charges are set simultaneously. This contradicts her results.} If termination costs become more symmetric, the market share effect becomes less effective, moving towards the results of the previous literature, otherwise for a more dominant market share effect, their results less likely hold and virtually anything is possible.
4 Reciprocal Regulation

For fixed-line networks the European Commission proposes to set termination charges on a reciprocal basis. With mobile telecommunications the European Commission allows for temporary higher termination charges for entrants until they have reached an efficient size of firm. In the fixed-line telecommunications markets, though, the European Commission does not propose any temporary asymmetries of termination charges. Any asymmetries have to be explicitly justified to the national regulatory authorities. Communications providers shall set the same termination charge as the fixed-line network to ensure efficient market entry and to avoid price squeezing vis--vis smaller operators. According to the European Commission entrants would not have any significant disadvantages in cost as they would primarily offer services in regional conurbations and may lease access to the incumbents’ networks. The European Commission states that an entrant may not face any disadvantages in costs, but does not consider that it might face advantages.

The following section analyzes the effects of reciprocity of termination charges, \( a_1 = a_2 = a \), in the previous model of cost-asymmetries. Since “bill-and-keep” pricing is proposed in the long run, the model analyzes the effect of marginally decreasing the reciprocal termination charge below the fixed-line network’s costs.

For reciprocal termination charges equilibrium per-minute-prices are set to

\[ p_i^* = 2c_i + s_j(a - c_i). \]

(13)

4.1 Subscribers’ Net Surplus

Considering reciprocal termination fees it can be shown that subscribers may be again adversely affected by regulation. The technical proof goes along the line of section 3.1 and is relegated to Appendix 4A.

**Proposition 4** For symmetric termination costs there is no local effect on subscribers’ net utilities. Otherwise, for asymmetric termination costs, locally decreasing the reciprocal termination charge below the costs of the fixed-line provider is unambiguously beneficial for fixed-line subscribers. VoIP subscribers may benefit or suffer.

**Proof** See Appendix 4A. ■

Since termination charges are set below costs of the fixed-line provider, it faces a deficit from interconnection. Hence, it has an incentive to reduce the number of off-net calls. As the number of off-net calls is determined by the market shares, the fixed-line provider should increase the net utility to the subscribers in order to increase its market share and to reduce the number of off-net calls. Locally around \( a = c_1 \) and \( c_2 < c_1 \) the VoIP provider may still benefit from interconnection, though. Therefore, it might respond to the fixed-line provider less fiercely by itself not increasing the net-utility of subscribers. It balances income streams from subscription and interconnection. If income from interconnection is still sufficiently large, there are fewer incentives to decrease fixed fees in the retail market. Otherwise, if income from interconnection decreases, profits
from subscription become more important, which also increases the incentives of the VoIP provider to gain market shares in the retail market. This will be analyzed in the following section.

4.2 Market Shares

The equilibrium market shares with reciprocal termination charges is given as

\[ s_2^* = \frac{1}{2} - \frac{\beta}{6} - \frac{2}{3} \left( (v(p_1^*) - v(p_2^*) + s_2^* q(p_1^*) (a - c_2) + s_1^* q(p_1^*) (c_2 - c_1) - s_1^* q(p_2^*) (a - c_1) + s_2^* q(p_2^*) (c_2 - c_1) \right) \]

for the VoIP provider and

\[ s_1^* = 1 - s_2 \]

for the fixed-line provider. By total differentiation of equation (14) with respect to \( a \) locally around cost-based regulation of the fixed-line network \( (a = c_1) \) it follows that

\[ \frac{ds_2^*}{da} \bigg|_{a=c_1} = \frac{(c_1 - c_2)(s_2^* - 2s_1^* s_2^*)q'}{2(c_1 - c_2)(q(p_2^*) - q(p_1^*) - (c_1 - c_2)^2 s_2^* q' - \frac{3}{\sigma}).} \]

Proposition 5 For symmetric termination costs there is no local effect on market shares. Otherwise, for asymmetric termination costs, a marginal decrease of the reciprocal termination charge below the cost of the fixed-line network increases the VoIP provider’s market share if i) providers are sufficiently differentiated, ii) the difference in termination costs is not too large, and iii) the VoIP provider’s market share is not too large.

The analysis shows that the “neutrality result” on market shares by Carter and Wright (2003) only holds for symmetric termination costs. Otherwise, there is a local effect of regulation on market shares, determined by the sign of the denominator. Comparison with equation (9) shows that the VoIP network qualitatively has to consider the same effects as with asymmetric termination charges. With asymmetric termination charges the fixed-line provider could save its higher termination costs by terminating calls in the VoIP network. As with asymmetric termination charges \( \left( \frac{dp_1^*}{da} \bigg|_{a=c_1} = s_2^* - (c_1 - c_2) \frac{ds_2^*}{da} \right) \) the fixed-line provider offers a higher net surplus to its subscribers. Given symmetric termination charges there is no cost saving and the positive effect on fixed-line subscriber’s indirect utility, and in turn the market share for the fixed-line provider, vanishes. Now, if providers are sufficiently differentiated, i.e., \( \sigma \) is small, the VoIP provider will likely gain market shares. The fixed-line provider has to offset the advantage in termination costs of the VoIP provider by reducing the fixed fee, but if subscribers find it costly to switch this does not offset the higher per-minute price.

Example 3: Consider a linear demand of \( q(p_i) = 1 - p_i \) again. The VoIP provider gains market shares by reducing the reciprocal termination charge below the cost of the traditional fixed-line network, i.e. \( \frac{ds_2^*}{da} \bigg|_{a=c_1} < 0 \) if

\[ (c_1 - c_2)^2 < \frac{3}{\sigma} \frac{1}{2 + 3s_2^*}. \]
This holds if the VoIP provider’s initial cost-advantage is sufficiently low, competition in the market is sufficiently soft, and the VoIP provider’s market share is sufficiently small. Consider from the per-minute price of the fixed-line provider of \( p_1^* = 2c_1 + s_2^* (a - c_1) \) that a reciprocal termination charge of \( a < c_1 \) decreases the price and thus increases the indirect utility of calls \( u(p_1^*) \). Given a larger market share of the VoIP provider this effect is intensified and the VoIP provider has to offset the increase of fixed-line subscribers’ net surplus in order not to lose market shares.

4.3 Profits

For symmetric termination costs a marginal reduction of the reciprocal termination charge does not affect providers’ profits. This no longer holds for asymmetric termination costs. From the previous section it follows that providers can both gain or lose market shares in response to a marginal reduction of the reciprocal termination charge below costs of the fixed-line network. Then, both providers’ profits may be positively or negatively affected. The effect on providers’ profit crucially depends on the degree of competition in the market and the demand- and supply-side asymmetry.

**Proposition 6** For symmetric termination costs a marginal reduction of the reciprocal termination charge does not affect providers’ profits. For asymmetric termination costs providers can both gain or suffer. If competition is sufficiently soft a marginal reduction of the reciprocal termination charge is generally to the detriment of the fixed-line provider and to the benefit of the VoIP provider. If competition is intense and the demand-side asymmetry is sufficiently large, the fixed-line provider may benefit. Both providers prefer cost-based regulation at termination costs of the fixed-line provider if competition is intense and the asymmetries are sufficiently large.

**Proof** See Appendix 4A.

Consider the effects for the VoIP provider in both the interconnection and the retail market. Marginally decreasing the reciprocal termination charge induces countervailing effects in the interconnection market, where the termination charge affects i) the per-minute profit per rival subscriber \( (a - c_2) \), ii) the total off-net traffic by rival subscribers \( (q(p_1^*)) \), and iii) the amount of off-net traffic \( s_1^* s_2^* \). The first effect is clearly negative. The second effect is positive. Marginally reducing the termination fee leads to a decrease in the fixed-line provider’s per-minute price, notably \( \frac{\partial p_1^*}{\partial a} \big|_{a = c_1} = s_2^* > 0 \). From \( q' < 0 \) it follows that off-net traffic per fixed-line subscriber is increasing, which is to the benefit of the VoIP provider as long as \( a > c_2 \). Total off-net traffic \( (s_1^* s_2^* q(p_1^*)) \) depends on the sign of the market shares effect. Given soft competition, the VoIP provider gains market shares, and thus, the number of off-net traffic is increasing for any \( s_2 < s_1 \). Hence, the total effect on interconnection profit is ambiguous.

Consider the effects in the retail market. The effect on retail profit is determined by the fixed fee, given by

\[
F_2^* = u(p_2^*) - \omega_2.
\]
The effect on the fixed fee is determined by the indirect utility from making calls and the subscribers’ net utility. Notice from section 4.1. that the fixed-line provider offers a larger net surplus to its subscribers. This implies a tendency towards a lower fixed fee for the VoIP provider, too, in order not to lose (too much) market share. However, a marginal reduction of the reciprocal termination charge decreases the per-minute price for VoIP provider, if the provider gains market shares, as \[\frac{\partial p^*}{\partial a} = - \frac{ds^*}{da} (c_1 - c_2) + s^*_1 > 0\] for \(\frac{ds^*}{da} < 0\). The per-minute price decreases, as, on the one hand, the termination charge decreases and, on the other hand, fewer calls are terminated off-net. This translates into a larger indirect utility from marking calls and, thus, to an opposing effect on the fixed fee.

Now, the effect on total profit is ambiguous. Suppose competition is sufficiently soft, i.e., \(\sigma\) is low, so that according to condition (15) the market share of the VoIP provider is increasing, i.e. \(\frac{ds^*}{da} < 0\). In this case, effects on the retail market are relatively weak and the positive effects in the interconnection markets dominate. For more intense competition the reduction in fixed fees to gain market share can become too large, so that total profit is decreasing. This especially holds for a large demand-side asymmetry, so that the VoIP provider has a small installed subscriber base. In this case relatively few calls are terminated in its network and a marginal benefit from interconnection becomes relatively unimportant for total profits. Thus, a gain in market shares is not necessarily sufficient for the profit to increase.

Let us consider competition to be intense, so that the VoIP provider loses market share. This may not necessarily profit reducing either. For any \(s_2 < s_1\) the VoIP network has a net outflow of calls to the fixed-line network and pays a per-minute price of \(a\) for every call. A reduction of \(a\) decreases the price the VoIP provider has to pay, but also the total number of off-net calls. The two effects oppose each other. Given that the provider initially captures only a small installed subscriber base the demand effect is negligible so that the provider is harmed. Given a larger subscriber base the demand effect becomes more important and the VoIP provider may benefit although it loses market shares. In total, positive as well as negative effect are possible, which is shown in example 4 below. More generally, the VoIP provider will likely benefit from a reduction of the reciprocal termination charge. The effects on profits seem to be more clean-cut than with asymmetric regulation, as illustrated in figure 2.

Consider the profit of the fixed-line provider. It will be shown in the Appendix that whenever its market share is decreasing, its total profit is decreasing. Note that fixed line subscribers’ net surplus is unambiguously increasing. Hence, if market shares are not increasing, the fixed fee and thus the retail profit is decreasing. In the interconnection market it faces a loss per rival subscriber. If the fixed-line provider gives market shares to the rival, total off-net traffic of a VoIP provider increases, leading to a larger loss from interconnection. Moreover, since \[\frac{\partial p^*}{\partial a} |_{a=c_1} = s^*_1 + (c_1 - c_2) \frac{ds^*}{da} > 0\] for \(\frac{ds^*}{da} > 0\) VoIP subscribers’ calling demand increases, leading to loss in the interconnection market, too, which leads total profit to decrease.

Otherwise, for increasing markets shares the fixed-line provider may benefit. Decompose the effects of the retail and the interconnection market. The effect in the
interconnection market depends on the effects on the revenue per rival subscriber and total off-net traffic. Clearly, as termination fees are regulated below the costs of the fixed-line provider there is an unambiguous loss from interconnection of $s_1 s_2 q(p_2^*)$ per rival subscriber. Starting from the asymmetric situation of $s_1 > s_2$, off-net traffic to the fixed-line network is reduced. The effect on the demand for off-net calls depends on the fixed-line provider’s market share, as $\frac{\partial p_2^*}{\partial a} \mid_{a = c_1} = s_1^* + (c_1 - c_2) \frac{ds_1^*}{da} \leq 0$. Consider $\frac{ds_1^*}{da} < 0$, so the fixed-line provider gains market shares. The per-minute price of the VoIP provider will increase if $s_1$ is sufficiently low, i.e. the demand-side asymmetry is sufficiently low. This benefits the fixed-line network as VoIP total off-net traffic, and thus, the loss from interconnection is reduced. Otherwise, for a higher $s_1$ the per-minute price of VoIP subscribers might decrease, so subscriber’s demand for calls is increasing, which in turn harms the fixed-line network. Now, the total effect on the fixed-line profit depends on the demand-asymmetry. For a large asymmetry it may be harmed, for lower values it benefits.

**Example 4:** Consider a linear demand of calls of $q(p_i) = A - p_i b$ and set $A = 2, b = 5, c_1 = 0.5$, and $c_2 = 0$. The following table illustrates the sign of the marginal derivatives of providers’ profits and the VoIP provider’s market share, depending on the degree of competition and on the fixed-line provider’s initial advantage.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma = 0.01$</th>
<th>$\sigma = 0.5$</th>
<th>$\sigma = 0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 1$</td>
<td>$\beta = 4$</td>
<td>$\beta = 1$</td>
<td>$\beta = 4$</td>
</tr>
<tr>
<td>$\frac{ds_1^*}{da} \mid_{a = c_1}$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\frac{dd_{\Pi}^*}{da} \mid_{a = c_1}$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\frac{dd_{\Pi}^*}{da} \mid_{a = c_1}$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

A positive sign indicates that the variable is decreasing in response to a reduction of the reciprocal termination charge.

In this example the VoIP provider gains market share and profit, whereas the fixed-line provider is harmed by a reduction of the reciprocal termination charge. Only if competition is very intense and the fixed-line advantage very high, both provider suffer from a marginal decrease of the reciprocal termination charge. In this case the VoIP provider captures a small installed subscriber base, so interconnection is relatively unimportant for total profits. If competition in the retail market is intense, competition on net surplus is intense. As the fixed-line subscribers’ net surplus is increasing in a marginal reduction of the reciprocal termination charge, the VoIP provider has to offset the increase, in order to be competitive at the retail level. If competition becomes too intense, both providers would prefer keeping the reciprocal termination charge at the cost level of the traditional fixed-line provider, which is in line with Carter and Wright (2003).
Also the global effect on profits seem to very clean-cut, which is shown in figure 2.\textsuperscript{11} A decrease of the reciprocal termination charge is beneficial for the VoIP provider and detrimental for the fixed-line provider. In this sense, a reduction of the reciprocal termination charge towards a “bill-and-keep” regime of zero termination charges may serve as an instrument to encourage market entry into the fixed-line telecommunications market.

5 Price Discrimination

The following section allows providers to charge different prices for calls terminated on the subscriber’s network (“on-net”) and for those terminated on the rival’s network (“off-net”). Denote provider $i$’s on-net price as $p_i$ and its off-net price as $\hat{p}_i$. If a provider’s market share is $s_i$, its subscribers make a fraction $s_i$ of their calls on-net and the remaining $1 - s_i$ calls off-net. Then, subscribers’ net surplus $\omega(p_i, \hat{p}_i)$ is

$$\omega(p_i, \hat{p}_i) = s_i \nu(p_i) + s_j \nu(\hat{p}_i) - F_i. \quad (17)$$

Following the analysis of section 2, solving for the indifferent subscriber yields a market share for the fixed-line provider of

$$s_1 = \frac{1}{2} + \frac{\beta}{2} + \sigma(\omega(p_1, \hat{p}_1) - \omega(p_2, \hat{p}_2)) \quad (18)$$

and of $s_2 = 1 - s_1$ for the VoIP provider.

\textsuperscript{11}Parameter values are set to: $A = 1, b = 1, \sigma = 0.5, \beta = 1, c_1 = 0.5$, and $c_2 = 0.$
5.1 Asymmetric Regulation

Provider $i$’s profit is denoted as
\[
\Pi_i = s_i(s_i(p_i - 2c_i)q(p_i) + s_j(p_i - c_i - a_j)q(p_j)) + s_i(s_i\upsilon(p_i) + s_j\upsilon(p_j) - \omega(p_i, \hat{p}_i)) + s_i s_j(a_i - c_i)q(\hat{p}_j).
\]  
(19)

The first two parts denote the profit in the retail market from setting on-net and off-net per-minute prices net the costs of calls. The third part denotes the profit from the fixed fee. The fourth part denotes the income in the interconnection market.

By solving $\frac{\partial \Pi_i}{\partial p_i} = 0$ and $\frac{\partial \Pi_i}{\partial \hat{p}_i} = 0$ providers set per-minute prices equal to the true marginal costs, i.e.

\[
p_i^* = 2c_i
\]  
(20)

and

\[
\hat{p}_i^* = c_i + a_j.
\]  
(21)

Without price discrimination, the first-order conditions with respect to call prices weights the optimal per-minute prices with price discrimination of equations (20) and (21) by their market shares, which gives equation (4). Since termination costs differ for both providers, a uniform per-minute price is the average of marginal on-net and off-net costs, which reflects a weighted average of true marginal costs.

The equilibrium fixed fee is set to
\[
F_i^* = \frac{s_i^*}{s} + s_i^*(\upsilon(\hat{p}_i^*) - \upsilon(p_i^*)) + (s_i^* - s_j^*)(a_i - c_i)q(\hat{p}_j^*).
\]  
(22)

If providers are unable to discriminate between on-net and off-net prices, the analysis of section 3 explores that both providers’ market shares are positively or negatively locally affected by a marginal increase in the VoIP provider’s termination charge $a_2$ above marginal costs. However, if providers can price discriminate it can be shown that market shares are locally unaffected, i.e.

\[
\frac{ds_i^*}{da_2} |_{a_2 = c_i} = 0.
\]  
(23)

This restores the result of Carter and Wright (2003) and Peitz (2005) in a model with cost-asymmetries and price discrimination. At the point of cost-based regulation, equilibrium market shares do not respond to an increase in the VoIP provider’s termination fee, independent of any asymmetry in size or termination costs. With price discrimination regulation of termination fees leaves on-net per-minute prices (locally) unaffected. As in the models of Carter and Wright (2003) and Peitz (2005) the asymmetries only determine the decision to subscribe to either network, but once subscribed, the asymmetry does not affect subscribers’ calling demand.

A termination markup generates income from inbound calls from rival subscribers for the VoIP provider. Locally around cost-based regulation, the VoIP provider benefits from a marginal increase in its termination charge. Otherwise, the fixed-line provider...
has to pay a higher termination charge for outbound calls, and hence, it suffers from the increase. Technically,

$$\frac{\partial \Pi^*_i}{\partial a_2}|_{a_i=\bar{c}_i} = -s^*_1 q(p^*_1) < 0$$

and

$$\frac{\partial \Pi^*_2}{\partial a_2}|_{a_i=\bar{c}_i} = s^*_2 q(p^*_2) > 0.$$ (25)

**Proposition 7** If provider can discriminate between on-net and off-net prices for calls, a marginal increase in the VoIP provider’s termination charge does not affect equilibrium market shares (locally around cost-based regulation). At this point, a marginal increase in the VoIP provider’s termination charge gives rise to higher (lower) profits for the VoIP (fixed-line) provider. This holds independent of any demand- and supply-side asymmetry.

**Proof** See Appendix 4A. ■

Hence, price discrimination can restore the results of the previous literature in a model of asymmetric termination costs.

### 5.2 Reciprocal Regulation

Now consider that providers set a reciprocal termination charge of $a$. Providers set an on-net price of

$$p^*_i = 2c_i$$ (26)

and an off-net price of

$$\hat{p}^*_i = c_i + a.$$ (27)

The equilibrium fixed fee is denoted as

$$F^*_i = \frac{s^*_i}{\sigma} + s^*_i \left(v(\hat{p}^*_i) - v(p^*_i)\right) + (s^*_i - s^*_j)(a - c_i)q(\hat{p}^*_j).$$ (28)

Consider the effect of a marginal decrease of the reciprocal termination charge below the costs of the fixed-line provider on the market shares, given as

$$\frac{ds^*_2}{da}|_{a=\bar{c}_1} = \frac{(s^*_2 - s^*_1)(c_1 - c_2)q(\hat{p}^*_1)'}{2((c_2 - c_1)q(p^*_1) - v(\hat{p}^*_1) - v(p^*_1) + v(\hat{p}^*_2) - v(p^*_2)) - \frac{3}{\sigma}}.$$ (29)

Hence, in the case of a reciprocal termination charge, the effects on market shares depends on both the demand- and the supply-side asymmetry. For symmetric market shares or symmetric termination cost market shares do not locally respond to a marginal decrease of the reciprocal termination charge below the marginal cost of the fixed-line provider. Otherwise, for both a demand- and supply side asymmetry, market shares do locally respond. Interestingly, with price discrimination, the neutrality result on market shares can also be restored with asymmetric termination costs. Compare the market share equations in the regime of reciprocal regulation without price discrimination (14)
and with price discrimination (29). Observe that without price discrimination only symmetric termination costs lead to a neutral effect on market shares, whereas with price discrimination symmetric termination costs or symmetric market shares are sufficient. With price discrimination regulation of termination charges only affects providers’ off-net but not the on-net prices. With nondiscriminatory pricing, the uniform per-minute price weights on-net and off-net prices with market shares of the providers, so also the on-net part of the nondiscriminatory price is affected by regulation. Observe from the per-minute prices of equations (13) and (27) that with non-discriminatory pricing the per-minute prices of providers are only identical with both a demand- and a supply-side symmetry, whereas with discriminatory pricing they are identical for a supply-side symmetry, independent of any demand-side asymmetry.

Now, observe from equation (17) that subscribers’ net surplus is an average of surplus from on-net and off-net calls. With price discrimination the surplus from on-net calls remains unaffected. The marginal effect on surplus is determined by the effects on net-surplus from off-net calls and the adjustment of the fixed fee. If providers can price discriminate, they can extract every extra surplus by adjusting the fixed fee accordingly. Compare equations (17) and equation (28). It holds that the marginal effect on net surplus is given by

\[ \frac{\partial \omega}{\partial a} = s_j \frac{\partial (\hat{p}_j^*)}{\partial a} - s_i \frac{\partial (\hat{p}_i^*)}{\partial a} - (s_i^* - s_j^*) (a - c_i) q(\hat{p}_j^*) \]

and

\[ \frac{\partial \omega}{\partial a} = 2 \frac{ds_1}{da} s_1' \left( \frac{1}{\sigma} + v(\hat{p}_1^*) - v(p_1^*) \right) + \frac{ds_2}{da} s_2' \left( \frac{1}{\sigma} + v(\hat{p}_2^*) - v(p_2^*) + (c_1 - c_2) q(\hat{p}_1^*) \right) \]

As has been stated above, market shares are locally unaffected for any demand- or supply-side symmetry. However observe, that profit are unaffected only for a supply-side symmetry, but not for a demand side symmetry. Given a supply-side symmetry of

Proposition 8 If provider can discriminate between on-net and off-net prices for calls, a marginal decrease of the reciprocal termination charge does not affect equilibrium market shares (locally around cost-based regulation), given a demand- or a supply-side symmetry.

The first order conditions of the profit functions with respect to a marginal decrease of the reciprocal termination charge are denoted as

\[ \frac{\partial \Pi_1^*}{\partial a} |_{a=c_1} = 2 \frac{ds_1}{da} s_1' \left( \frac{1}{\sigma} + v(\hat{p}_1^*) - v(p_1^*) \right) + \frac{ds_2}{da} s_2' \left( \frac{1}{\sigma} + v(\hat{p}_2^*) - v(p_2^*) + (c_1 - c_2) q(\hat{p}_1^*) \right) + s_1^* q(\hat{p}_1^*) - q(\hat{p}_1^*) \]

and

\[ \frac{\partial \Pi_2^*}{\partial a} |_{a=c_1} = 2 \frac{ds_1}{da} s_1' \left( \frac{1}{\sigma} + v(\hat{p}_1^*) - v(p_1^*) \right) + \frac{ds_2}{da} s_2' \left( \frac{1}{\sigma} + v(\hat{p}_2^*) - v(p_2^*) + (c_1 - c_2) q(\hat{p}_1^*) \right) \]

As has been stated above, market shares are locally unaffected for any demand- or supply-side symmetry. However observe, that profit are unaffected only for a supply-side symmetry, but not for a demand side symmetry. Given a supply-side symmetry of
$c_1 = c_2$ providers’ profits are locally unaffected by regulation, i.e. $\frac{\partial \Pi_i^*}{\partial a} |_{a = c_1} = 0$. This directly follows from the neutral market share effect and the fact that on-net and off-net prices are identical for both providers. The total effect on profits then depends on the net traffic of off-net calls (inbound calls from rival subscribers vs. outbound calls from fellow subscribers). If cost are identical, it directly follows that $q(\hat{p}_2^*) = q(\hat{p}_1^*)$ and regulation has no (local) effect on profits. In this case the reduction in interconnection profit from rival off-net calls is just balanced by the reduction in the payment for off-net calls by fellow subscribers. Otherwise, if costs differ, the fixed-line provider is locally harmed by regulation and the VoIP provider locally benefits. In case of a cost asymmetry, VoIP customers place more off-net calls than fixed-line customers, i.e. $q(\hat{p}_2^*) > q(\hat{p}_1^*)$. Thus, even for symmetric market shares, the fixed-line provider terminates more off-net traffic than the VoIP provider (which is even intensified if market shares are asymmetric). Hence, it faces a net deficit from interconnection, whereas the VoIP provider earns a net profit from interconnection (at least locally if the reciprocal termination charge is not even set below its marginal costs).

**Proposition 9** If provider can discriminate between on-net and off-net prices for calls, a marginal decrease of the reciprocal termination charge does not affect equilibrium profits for any supply-side symmetry. For any demand-side symmetry, the fixed-line (VoIP) provider’s profit locally decreases (increases).

### 6 Conclusion

This paper has explored the ramification of interconnection terms in telecommunications networks with asymmetries in termination costs. Traditional fixed-line networks usually face a positive marginal cost of terminating calls, whereas for calls terminated in IP networks, the termination cost should generally be lower and close to zero. In its “Recommendation on the Regulatory Treatment of Fixed and Mobile Termination Rates in the EU” the European Commission recently set out its views on how national regulators in Europe should approach termination charges in the future. The present paper has discussed whether these terms of regulation, originally designed for fixed-line networks, should be applied in the presence of asymmetries of termination costs between networks. With the proposed cost-based regulation, a VoIP network will receive less for rival calls terminated on its network, than it has to pay for calls by fellow subscribers terminated in the fixed-line network. This does not seem to be in line with efforts to encourage market entry of alternative telecommunications providers in the market of fixed line telephony.

Thus it is a relevant policy question, whether to deviate from the cost-based regulation in VoIP networks and allow for termination fees above marginal cost. The model shows that unilaterally increasing the VoIP provider’s termination charge may or may not increase its profit, as feedback effects into market shares have to be taken into account. A unilateral increase in the termination charge of the VoIP network increases the marginal cost for the traditional fixed-line network, which increases its per-minute
price, which in turn decreases the demand for calls. This has adverse consequences for total interconnection profit, which may decrease by deviating from cost-based regulation. Hence, regulation of termination charges has an effect on calling patterns and market shares.

An increase in the VoIP provider’s termination charge has an impact on net surplus of both providers’ subscribers. The fixed-line network compensates the increase in per-minute prices by lowering fixed fees for their subscribers. This may even lead the fixed-line provider’s market share to increase in response to the higher marginal termination cost it faces. This will be to the detriment of efforts to enable VoIP providers to catch up with traditional fixed-line providers.

The European Commission generally favors reciprocal termination charges for fixed-line networks. Hence, in a second step, the paper has analyzed the effects of reciprocity in termination charges. The model shows that fixed-line subscribers benefit from a marginal reduction of the reciprocal termination fee, whereas VoIP subscriber may or may not benefit, depending on the degree of substitution of providers and the difference in termination costs. The local effects on providers’ profits are also ambiguous but more clean-cut than with an unilateral increase of the VoIP termination charge. For larger deviations from cost-based regulation the fixed-line provider more generally suffers from a decrease of the reciprocal termination charge, whereas the VoIP provider more generally benefits.

If providers can discriminate between on-net and off-net prices, asymmetric regulation has no local effect on market shares, independent of a demand- and supply-side asymmetry. The VoIP provider locally benefits from an increase in its termination charge and the fixed-line provider suffers. This restores the result of the previous literature in a model of demand-and supply-side asymmetry. If provider can discriminate between on-net and off-net prices for calls, a marginal decrease of the reciprocal termination charge does not affect equilibrium profits for any supply-side asymmetry. For any demand-side symmetry, the fixed-line provider is locally harmed, whereas the VoIP provider locally benefits from a marginal reduction of the reciprocal termination charge.

To conclude, a regulatory authority has to consider (positive or negative) feedback effects on market shares and on the demand for calls, when determining the most adequate regulation for fixed to VoIP interconnection.

A Appendix

Asymmetric Regulation

Proof of Proposition 1:

Profit functions of both providers are given as

$$\Pi^*_1 = s^*_1(p^*_1 - 2c_1)q(p^*_1) + s_1(v(p^*_1) - \omega_1) + s^*_1s^*_2\{(a_1 - c_1)q(p^*_2) - (a_2 - c_1)q(p^*_1)\}$$
which cost-based regulation of termination charges this simplifies to

\[ \Pi_i^* = s_i^*(p_j^s - 2c_2)q(p_j^s) + s_2(v(p_j^s) - \omega_j^s) + s_1s_2^* \{(a_2 - c_2)q(p_i^1) - (a_1 - c_1)q(p_i^2)\}, \]

where market shares of \( s_1 = \frac{1}{2} + \frac{\beta}{2} + \sigma(\omega_1 - \omega_2) \) and \( s_2 = \frac{1}{2} - \frac{\beta}{2} + \sigma(\omega_2 - \omega_1) \) depend on consumer net surplus \( \omega_i \). Along its best-response function each operator sets per-minute prices to perceived marginal costs. Thus the only income source stems from subscription and off-net traffic, leading to profit in terms of net surplus of

\[ \Pi_i^* = s_i^*(v(p_i^s) - \omega_i^*) + s_i^* s_j^*(a_i - c_i)q(p_j^*). \]

The first order condition of the fixed-line provider with respect to consumer net surplus \( \omega_1 \) is given as

\[ \frac{\partial \Pi_i^*}{\partial \omega_1} = \sigma(v_i^* - \omega_i^*) + s_1^*(\frac{\partial v_i^*}{\partial p_1} \frac{\partial p_1^*}{\partial \omega_1} - 1) + (a_1 - c_1)(\sigma(s_2^* - s_1^*)q(p_2^*) + s_1^* s_2^* \frac{\partial q(p_2^*)}{\partial p_2} \frac{\partial p_2^*}{\partial \omega_1}). \]

For convenience label \( v(p_i) = v_i \), \( q(p_i) = q_i \), and \( \frac{\partial v_i}{\partial p_i} = q_i' \). Taking account for \( \frac{\partial q_i}{\partial p_i} = -q_i \) and for per-minute prices of equation (4) it follows that

\[ \frac{\partial \Pi_i^*}{\partial \omega_1} = \sigma(v_i^* - \omega_i^*) + s_1^*(\sigma q_1^*(a_2 - c_1) - 1) + \sigma(a_1 - c_1)((s_2^* - s_1^*)q_2^* + s_1^* s_2^* q'(a_1 - c_2)). \]

The cross-derivative is

\[ \frac{\partial^2 \Pi_i^*}{\partial \omega_1 \partial \omega_2} = \sigma(\frac{\partial v_i^*}{\partial p_1} \frac{\partial p_1^*}{\partial \omega_1}) + \sigma(a_2 - c_1)(-\sigma q_1^* + s_1^* q' \frac{\partial p_1^*}{\partial \omega_2}) + \sigma \]

which cost-based regulation of termination charges this simplifies to

\[ \frac{\partial^2 \Pi_i^*}{\partial \omega_1 \partial \omega_2} \bigg|_{a_i = c_i} = \sigma - 2\sigma^2(c_2 - c_1)q_1^* + \sigma^2(c_2 - c_1)^2 s_1^* q', \]

which implies that the fixed-line network’s pseudo best-response functions is upwards sloping if competition is not too weak and the difference in termination costs \( (c_1 - c_2) \) is not too large. One obtains that an increase in the VoIP termination charge \( a_2 \) shifts the pseudo best-response function outwards, as

\[ \frac{\partial^2 \Pi_i^*}{\partial \omega_1 \partial a_2} = \sigma(\frac{\partial v_i^*}{\partial p_1} \frac{\partial p_1^*}{\partial a_2}) + s_1 \sigma((a_2 - c_1)q_i \frac{\partial p_1^*}{\partial a_2} + q_i) \]

which reduces to

\[ \frac{\partial^2 \Pi_i^*}{\partial \omega_1 \partial a_2} \bigg|_{a_i = c_i} = \sigma(s_1^* - s_2^*) q_1^* + s_1^* s_2^* \sigma(c_2 - c_1) q' > 0. \]

This term is strictly positive for \( s_1 > s_2 \) and \( c_2 < c_1 \), which has been assumed.
Consider the VoIP provider’s profit. Applying same technique, the marginal profit is
\[
\frac{\partial \Pi^*_2}{\partial \omega^*_2} = \sigma(v_2^* - \omega_2^*) + s_2^*(\sigma q_2^*(a_1 - c_2) - 1) + \sigma(a_2 - c_2)((s_1^* - s_2^*)q_1^* + s_1^*s_2^*q'(a_2 - c_1)).
\]

The cross derivative is denoted as
\[
\frac{\partial^2 \Pi^*_2}{\partial \omega^*_2 \partial a_2}|_{a_i = c_i} = \sigma - 2\sigma^2(c_1 - c_2)q_2^* + \sigma^2(c_1 - c_2)^2s_2^*q'.
\]

From
\[
\frac{\partial^2 \Pi^*_2}{\partial \omega^*_2 \partial a_2} = \sigma(\frac{\partial v_1^*}{\partial p_1^*} \frac{\partial s_1^*}{\partial a_2} + \sigma s_2^*(a_1 - c_2)q' \frac{\partial p_2^*}{\partial a_2}) + \sigma(s_1^* - s_2^*)q_1^* + s_1^*s_2^*q' \sigma(a_2 - c_1)
\]
\[
+ \sigma(a_2 - c_2)((s_1^* - s_2^*)q_1^* + s_1^*s_2^*q').
\]
As per-minute prices are only affected by rival’s termination charges it follows that \(\frac{\partial p_2^*}{\partial a_2} = 0\) and thus
\[
\frac{\partial^2 \Pi^*_2}{\partial \omega^*_2 \partial a_2}|_{a_i = c_i} = \sigma(s_1^* - s_2^*)q_1^* + s_1^*s_2^*\sigma(c_2 - c_1)q' > 0.
\]

Hence, also the VoIP provider’s pseudo best-response is shifted outwards. For identical termination costs, effects of both provider’s pseudo best-response function are positive. This confirms the neutrality result on market shares for symmetric termination costs. ■

**Proof** of Proposition 2:

Total differentiation of equation (8) locally around cost-based regulation of \(a_i = c_i\) leads to
\[
\frac{ds_2^*}{da_2}|_{a_i = c_i} = \frac{\sigma}{3} \left\{ \frac{\partial v_1^*}{\partial p_1^*} \frac{\partial p_1^*}{\partial a_2} - \frac{\partial v_1^*}{\partial p_2^*} \frac{\partial p_2^*}{\partial a_2} + s_1^*q_1^* \right\}.
\]

Using \(\frac{ds_2^*}{da_2} = -\frac{ds_1^*}{da_2}, v'(p_i) = -q_i\), inserting optimal per-minute prices and rearranging yields that
\[
\frac{ds_2^*}{da_2}|_{a_i = c_i} = \frac{q's_1^*s_2^*(c_2 - c_1)}{2(c_1 - c_2)(q_2^* - q_1^*) - (c_2 - c_1)^2(s_1^*q' + s_2^*q')} - \frac{3}{\sigma}.
\]

**Reciprocal Regulation**

**Proof** of Proposition 4:
To show that subscribers benefit from a marginal decrease of the reciprocal termination charge apply the same steps as in the proof of proposition 1. First consider the fixed-line provider’s marginal profit of

\[
\frac{\partial \Pi_1^*}{\partial \omega_1} = \sigma(v_1^* - \omega_1^*) + s_1(\sigma q_1^*(a - c_1) - 1) + \sigma(a - c_1)((s_2^* - s_1^*)q_2^* + s_1^*s_2^*q'(a - c_2)).
\]

The cross derivative is denoted as

\[
\frac{\partial^2 \Pi_1^*}{\partial \omega_1 \partial \omega_2} = \sigma - 2\sigma^2(a - c_1)q_1^*.
\]

where at \( a = c_1 \) it holds that

\[
\frac{\partial^2 \Pi_1^*}{\partial \omega_1 \partial \omega_2}|_{a=c_1} = \sigma > 0.
\]

A marginal decrease of the reciprocal termination charge shifts the fixed-line network’s pseudo best-response function outwards as

\[
\frac{\partial^2 \Pi_1^*}{\partial \omega_1 \partial \omega_2} = \sigma(s_1^* - s_2^*)(q_1^* - q_2^*) + \sigma(c_1 - c_2)s_1^*s_2^*q' < 0.
\]

First, for \( a = c_1 \) and from

\[
sign(q_2^* - q_1^*)|_{c_1=c_2} = sign(p_1^* - p_2^*)|_{a=c_1} = (c_1 - c_2)(2 - s_1^*) = 0.
\]

follows that \( \frac{\partial^2 \Pi_1^*}{\partial \omega_1 \partial a} = 0 \). Otherwise, for \( c_1 > c_2 \) the second part is negative, since \( q_1^* < 0 \). The sign of the first part is determined by \( sign(q_1^* - q_2^*) = sign(p_2^* - p_1^*) \). At \( a = c_1 \) it holds that \( sign(p_2^* - p_1^*) = (c_2 - c_1)(2 + s_2^*) < 0 \). From this it follows that the term is clearly negative and the pseudo best-response functions shifts outwards.

Applying same technique for the VoIP provider it follows that

\[
\frac{\partial \Pi_2^*}{\partial \omega_2} = \sigma(v_2^* - \omega_2^*) + s_2^*(\sigma q_2^*(a - c_2) - 1) + \sigma(a - c_2)((s_1^* - s_2^*)q_1^* + s_1^*s_2^*q'(a - c_1)).
\]

The cross derivative is given as

\[
\frac{\partial^2 \Pi_2^*}{\partial \omega_2 \partial \omega_1}|_{a=c_1} = \sigma + 2\sigma^2(c_1 - c_2)(q_1^* - q_2^*) \leq 0
\]

which again follows from \( sign(q_1^* - q_2^*) = sign(c_2 - c_1)(2 + s_2^*) < 0 \) and \( c_1 > c_2 \). The pseudo best-response function is shifted outwards, as

\[
\frac{\partial^2 \Pi_2^*}{\partial \omega_2 \partial a}|_{a=c_1} = \sigma(s_1^* - s_2^*)(q_1^* - q_2^*) + s_1^*s_2^*\sigma(c_1 - c_2)q' + \sigma(c_1 - c_2)((s_1^* - s_2^*)q's_2 + s_1^*s_2^*q') < 0,
\]

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which holds for \( s_1 > s_2 \). Observe again, that for symmetric termination costs of \( c_1 = c_2 \) are identical, confirming the neutrality result on market shares of the literature.

**Proof** of Proposition 5:

The VoIP provider’s market share with reciprocal access regulation is given as

\[
s_{2}^{*} = \frac{1}{2} - \frac{\beta}{6} - \frac{\sigma}{3}(v_1^{*} - v_2^{*} + s_{1}^{*}q_{1}^{*}(a - c_2) - s_{2}^{*}q_{2}^{*}(a - c_1) + (c_2 - c_1)(s_{1}^{*}q_{1}^{*} + s_{2}^{*}q_{2}^{*})).
\]

Total differentiation of \( \frac{ds_{2}^{*}}{da} \) yields

\[
\frac{ds_{2}^{*}}{da}_{|a=c_1} = -\frac{\sigma}{3} \left\{ \frac{dv_{1}^{*}}{dp_{1}^{*}} dp_{1}^{*} - \frac{dv_{2}^{*}}{dp_{2}^{*}} dp_{2}^{*} + (a - c_2)(s_{1}^{*}q_{1}^{*} + s_{2}^{*}q_{2}^{*} dp_{2}^{*}) + s_{2}^{*}q_{2}^{*} - (a - c_1)(s_{1}^{*}q_{2}^{*} + s_{1}^{*}q_{1}^{*}) - s_{2}^{*}q_{1}^{*} + (c_2 - c_1)(s_{1}^{*}q_{1}^{*} + s_{1}^{*}q_{2}^{*} dp_{2}^{*})
\right. \\
\left. + s_{2}^{*}q_{2}^{*} + s_{2}^{*}q_{2}^{*} dp_{2}^{*} \right\}.
\]

Using \( v'(p) \equiv -q(p) \), \( \frac{ds_{2}^{*}}{da} = -\frac{ds_{1}^{*}}{da} \) and evaluation locally around \( a = c_1 \), this reduces to

\[
\frac{ds_{2}^{*}}{da} = \frac{(c_1 - c_2)q'(s_{2}^{*2} - 2s_{1}^{*}s_{2}^{*})}{2(c_1 - c_2)(q_{2}^{*} - q_{1}^{*}) - (c_1 - c_2)^2s_{2}^{*}q' - \frac{\sigma}{3}}.
\]

As \( c_1 > c_2, s_1 > s_2 \) and \( q' < 0 \) the numerator is always positive, so the sign of \( \frac{ds_{2}^{*}}{da} \) is determined by the denominator. **Proof** of proposition 6:

The effect on total profits is decomposed in effects in the retail market and in the interconnection market as

\[
\Pi_{1}^{*} = s_{1}^{*}F_{1}^{*} + s_{1}^{*}s_{2}^{*}(a - c_1)q(p_{2}^{*}).
\]

Total resulting effects on profits are depicted by evaluating the derivatives of the profit functions with respect to a marginal change in the reciprocal termination charge locally around \( a = c_1 \). Consider the marginal change of the fixed-line network’s profit of

\[
\frac{\partial \Pi_{1}^{*}}{\partial a} \bigg|_{a=c_1} = s_{1}^{*}(\frac{2}{\sigma} \frac{ds_{1}^{*}}{da} + s_{1}^{*}(q(p_{2}^{*}) - q(p_{1}^{*}))).
\]

and of

\[
\frac{\partial \Pi_{2}^{*}}{\partial a} \bigg|_{a=c_1} = 2s_{2}^{*} \left( \frac{1}{\sigma} + (c_1 - c_2)(q(p_{1}^{*}) - q(p_{2}^{*})) \right)
\]

\[
+ s_{2}^{*2}(q(p_{1}^{*}) - q(p_{2}^{*}) + (c_1 - c_2)(q_{2}^{*} - q_{1}^{*}) + (c_1 - c_2)^2 \frac{ds_{2}^{*}}{da}).
\]

for the VoIP provider.

Remind from equation (10) that there is no local effect on market shares for symmetric termination cost. Secondly notice that \( (q_{2}^{*} - q_{1}^{*})|_{a=c_1} = sign(p_{1}^{*} - p_{2}^{*}) = (c_1 - c_2)/(2 - s_{1}^{*}) = 0 \) for \( c_1 = c_2 \). From both follows that

\[
\frac{\partial \Pi_{1}^{*}}{\partial a} \bigg|_{c_1=c_2} = 0.
\]

\( \blacksquare \)
Price Discrimination

Providers set optimal on-net, off-net prices and the fixed fee by maximizing the profit function of equation (19) with respect to $p_i$, $\hat{p}_i$, and $\omega(p_i, \hat{p}_i)$.

From
$$\frac{\partial \Pi_i}{\partial p_i} = s_i(s_i q_i + s_i (p_i - 2c_i)q_i') + s_i^2 v'_i = 0$$
and using $v'_i = -q_i$ follows that
$$p_i^* = 2c_i.$$

By solving
$$\frac{\partial \Pi_i}{\partial \hat{p}_i} = s_i s_j \hat{q}_i + s_i s_j (\hat{p}_i - c_i - a_j)\hat{q}_i' + s_i s_j \hat{v}_i' = 0$$
follows that
$$\hat{p}_i^* = c_i + a_j.$$

To derive the optimal fixed fee it is again convenient to consider providers to compete on net-surplus rather than on the fixed fee directly. From evaluation the FOC at equilibrium per-minute prices it follows that
$$\frac{\partial \Pi_i}{\partial \omega_i} = \sigma(s_i v_i + s_j \hat{v}_i - \omega_i) + s_i(\sigma(v_i - \hat{v}_i) - 1) + \sigma(a_i - c_i)\hat{q}_i(s_i - s_j).$$

From setting this equal to zero follows that the optimal net-surplus is given as
$$2s_i v_i + (s_i - s_j)\hat{v}_i - \frac{s_i^2}{\sigma} + (a_i - c_i)(s_j - s_i)\hat{q}_i.$$

After re-substituting $F_i = s_i v_i + s_j \hat{v}_i - \omega_i$ follows that
$$F_i^* = \frac{s_i^*}{\sigma} + s_i^*(\hat{v}_i^* - v_i^*) + (s_i^* - s_j^*) (a_i - c_i)\hat{q}_i^*.$$

**Proof** of proposition 9:

The equilibrium market share of the fixed-line provider implicitly determined by
$$s_i^* = \frac{1}{2} + \frac{\beta}{6} + \frac{\sigma}{3}(2(s_i^* v_1^* - s_2 v_2^*) + (s_2^* - s_1^*) (\hat{v}_1^* + \hat{v}_2^*) + (a_1 - c_1)q_2^* + (a_2 - c_2)q_1^*))$$
and by $s_2 = 1 - s_1$ for the VoIP provider.

Total differentiation locally around cost-based regulation yields
$$\frac{ds_i^*}{da_2}
|_{a_i=c_i} = \frac{\sigma}{3} (2(\frac{ds_i^*}{da_2} v_1^* - \frac{ds_i^*}{da_2} v_2^*) + (\frac{ds_i^*}{da_2} - \frac{ds_i^*}{da_2}) (\hat{v}_1^* + \hat{v}_2^*)
+ (s_2^* - s_1^*) (\frac{\partial \hat{v}_1^*}{\partial \hat{p}_1^*} \frac{\partial \hat{p}_1^*}{\partial a_2} + \hat{q}_1^*))$$
After rearranging and using $\frac{dq_i}{dp_i} = -q_i$ follows that

$$\frac{ds_i^*}{da_2} \bigg|_{a_i=c_i} = \frac{(s_i^* - s_i)(\hat{q}_i^* - q_i^*)}{\frac{3}{\sigma} - 2(v_i^* - v_1^* + v_2^* - \hat{v}_i^*)}.$$ 

Hence, it follows that

$$\frac{ds_1^*}{da_2} \bigg|_{a_i=c_i} = -\frac{ds_2^*}{da_2} \bigg|_{a_i=c_i} = 0.$$

Since equilibrium per-minute prices are set to the marginal cost, provider earn profits from the fixed fee and inbound calls from rival subscribers, leading to profits of

$$\Pi_1^* = \frac{s_2^2}{\sigma} + s_2^2(\hat{v}_1^* - v_1^* + (a_1 - c_1)\hat{q}_2^*).$$

The FOC with respect to $a_2$ yields

$$\frac{\partial \Pi_1^*}{\partial a_2} \bigg|_{a_i=c_i} = \frac{ds_2^*}{da_2} \left(\frac{2}{\sigma} + 2s_1^*(\hat{v}_1^* - v_1^* + (a_1 - c_1)\hat{q}_2^*)\right)$$

$$+ s_1^2 \left(\frac{\partial v_1}{\partial p_1} \frac{\partial \hat{p}_1^*}{\partial a_2} - \frac{\partial v_1}{\partial p_1} \frac{\partial p_1^*}{\partial a_2} + (a_1 - c_1)\frac{\partial q}{\partial p_2} \frac{\partial p_2}{\partial a_2}\right).$$

Since $\frac{ds_2^*}{da_2} = 0$ and only the off-net price $\hat{p}_1$ responds to $a_2$ it follows that

$$\frac{\partial \Pi_1^*}{\partial a_2} \bigg|_{a_i=c_i} = -s_1^2 \hat{q}_1 < 0.$$

The VoIP providers profit is denoted as

$$\Pi_2 = \frac{s_2^2}{\sigma} + s_2^2(\hat{v}_2^* - v_2^* + (a_2 - c_2)\hat{q}_1^*).$$

Since per-minute prices and market shares do not (locally) respond to $a_2$ it simply follows that

$$\frac{\partial \Pi_2}{\partial a_2} \bigg|_{a_i=c_i} = s_2^2 \hat{q}_1 > 0.$$

\[\blacksquare\]

**References**


