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## **The Cyclical Volatility of Labor Markets under Frictional Financial Markets**

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# The Cyclical Volatility of Labor Markets under Frictional Financial Markets

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## ABSTRACT

### The Cyclical Volatility of Labor Markets under Frictional Financial Markets<sup>\*</sup>

Financial frictions are known to raise the volatility of economies to shocks (e.g. Bernanke and Gertler 1989). We follow this line of research to the labor literature concerned by the volatility of labor market outcomes to productivity shocks initiated by Shimer (2005): in an economy with search on credit and labor markets, a financial multiplier raises the elasticity of labor market tightness to productivity shocks. This multiplier increases with total financial costs and is minimized under a credit market Hosios-Pissarides rule. Using a flexible calibration method based on small perturbations, we find the parameter values to match the US share of the financial sector. Those values are far away from Hosios and lead to a financial accelerator of about 3.6 (exogenous wages) to 4.5 (endogenous wages). Both match Shimer (2005)'s elasticity of labor market tightness to productivity shocks. Financial frictions are thus an alternative to the "small labor surplus" assumption in Hagedorn and Manovskii (2008): we keep the value of wages over productivity below 0.78. We conclude that financial frictions are a good candidate to solve the volatility puzzle and rejoin Pissarides (2009) in arguing that hiring costs must be partly non-proportional to congestion in the labor market, which is the case of financial costs.

JEL Classification: E44, J60

Keywords: search, financial imperfections, Shimer puzzle, macroeconomic volatility

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# 1 Introduction

Cole and Rogerson (1999) and Shimer (2005) have investigated the cyclical properties of the search matching models following Pissarides (1985) and Mortensen and Pissarides (1994). The celebrated Shimer's puzzle is the demonstration of the inability of the conventional matching model to replicate the US statistics regarding the volatility of job vacancies, unemployment and their ratio (called labor market tightness), in response to productivity shocks. Shimer's main finding is that the elasticity of labor market tightness to productivity shocks is around 20 in the data, and around 1 in a calibration of the Mortensen-Pissarides model. Several calibration improvements have been proposed. One of them, called the "small labor surplus" assumption, implies that the calibrated value of non-employment utility (Hagedorn and Manovskii 2008) becomes closer to market productivity, with only a few percentage points differences and very low values for the bargaining power of workers. This leads firms to also face a small surplus, of a few percentage points, after bargaining over the surplus. Firms are therefore more fragile to productivity shocks, leading the market to be overall more volatile. Other promising roads have been proposed, such as wage rigidity (Hall 2005) and on-the-job search (Mortensen and Nagypál 2007). The latter paper also makes the point that a large part of fluctuations in the unemployment/vacancy ratio is not due to productivity shocks. Since the partial correlation between tightness and productivity is around 40%, a lower value of the elasticity (approximately 7), needs to be matched. Pissarides (2009) retains a value of 7.56.<sup>1</sup>

One line of research that has so far been ignored but seems promising is the existence of credit market imperfections. Indeed, it has been known for a while that credit market imperfections generate additional volatility of the business cycle. Early papers such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and subsequent papers (such as Bernanke and Gertler 1995, Bernanke Gertler and Gilchrist 1996, and several others), have emphasized the amplification role of credit markets and the existence of a *financial accelerator*. Although part of this literature is centered on the role of credit shocks and the credit channel of monetary policy, the ingredients generating the amplification of credit shocks can very well be adapted to the amplification of business cycle shocks to labor markets.

In this paper we pursue this logic, in providing a dynamic extension of Wasmer and Weil (2004), who develop financial imperfections in a Mortensen-Pissarides economy with two matching functions (one in the labor market, one in the credit market). Firms arise from the result of the meeting of an

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<sup>1</sup>See Pissarides (2009) page 1351, footnote 15.

entrepreneur and a banker on a frictional credit market. The average cost of creating a firm is the sum of all prospecting costs on the credit market which, compared to the world with perfect credit markets in Mortensen and Pissarides (1994), imposes a lower limit on the value of a job vacancy to a firm.

Our results regarding the amplification of productivity shocks in this double matching economy can be summarized as follows. Consistent with Wasmer and Weil (2004) in a static context, financial imperfections raise the calibrated elasticity of labor market tightness to productivity shocks. We denote by  $M_f^D$  the dynamic financial accelerator (or hereafter, dynamic financial multiplier), which is an increasing function of total financial costs in the economy. This paper brings in addition several new results.

**First**, a Hosios-Pissarides rule exists in the credit market: the bargaining power of firms vis-à-vis banks is equal, at the social optimum, to the elasticity of the finding rate of banks with respect to credit market tightness. Under the Hosios rule, the search costs in the credit market are minimized, and so is the financial multiplier. Relaxing that condition leads to a larger financial multiplier, which can match or even overshoot the elasticity of market tightness in the data.

**Second**, using a flexible calibration method based on small perturbations (a trembling-hand calibration method), we find the parameter values that allow us to match the share of the financial sector in GDP in the US (3.3%), as well as much larger elasticities of labor market tightness to productivity shocks. The parameter values are generally far away from Hosios.

**Third**, with endogenous wages, we obtain a financial multiplier of 2.9 and an elasticity of labor market tightness to productivity shocks of 7 when bank's share of the surplus with the firm is 0.91 and the elasticity of the finding rate of banks with respect to credit market tightness is 0.55.

**Fourth**, this result is obtained keeping the share of wages to be around 0.78 of productivity and a bargaining share of workers of 0.10, thus quite far away from the "small labor surplus" assumption in Hagedorn and Manovskii (2008). Financial frictions are thus an alternative to be taken seriously. As Mortensen and Nagypal (2007) point out, the small labor surplus assumption implies that there is very little utility gain to accepting a job, nor does it fit well with estimates of the value of non-employment. Financial imperfections in our model enable us to partly relax this assumption in order to match the elasticity of market tightness to productivity found in the data.

**Fifth**, to obtain an elasticity of 20, we need a financial multiplier of 4.6 and for that, we need to reduce the bargaining power of workers over wages to 0.03, thus to a value close from Hagedorn and Manovskii (2008). Note however that we still keep the value of wages over productivity at 0.78.

This paper is organized as follows. In Section 2, we calculate the volatility of labor market tightness

to productivity shocks and show how the Hosios rule in the credit market affects the volatility of the labor market. In Section 3, we describe the stochastic extension of the model, with both endogenous and exogenous wages. In Section 4, we describe our calibration method. In Section 5, we derive our main results : deviating away from Hosios substantially raises the elasticity of labor market tightness with exogenous wages. The calibration with endogenous wages shows similar results. In addition, the “small labor surplus” assumption does not need to be maintained. In Section 6, we conclude that financial frictions are a good candidate to solve the volatility puzzle and rejoin Pissarides (2009) in arguing that hiring costs must be partly non-proportional to congestion in the labor market, which is the case of financial costs. We also suggest how to improve the calibration by extending financial imperfection to operating firms, along the lines of Petrosky-Nadeau (2009) who shows that in a costly state verification model with search, an amplification of the volatility of labor market tightness arises by a factor of 3.5.

## **2 Hosios-Pissarides in a continuous time economy with credit and labor market frictions and the elasticity of labor market tightness to shocks**

### **2.1 Model**

Time is continuous and there are three types of agents: entrepreneurs with no capital; banks with no ability to produce; and workers with no capital and no ability to start a business. The timing of events for entrepreneurs is as follows. They initially need to find a "banker" in order to start a business. This search process costs  $e$  units of effort per unit of time. Search is successful with probability  $p$ . The newly formed firm, from the successful meeting of entrepreneur and banker, then goes to the labor market. The bank finances the vacancy posting cost  $\gamma$  to attract workers (the so-called recruitment costs) for the firm. This search process succeeds with probability  $q$ . The firm is then able to produce and sell in the good market, which generates a flow profit  $y - w - \rho$  where  $y$  is the marginal product,  $w$  is the wage,  $r$  is the flow rate of discount, and  $\rho$  is the flow repayment to the bank (determined through bargaining). Jobs are subject to destruction shocks with Poisson parameter  $s$ . The steady-state asset values of the entrepreneurs are denoted by  $E_j$  with  $j = c, l$  or  $g$  the market in which the entrepreneur is operating, standing respectively for the credit, labor and good markets. We also assume free entry at the first stage,

that is  $E_c \equiv 0$ . We therefore have the following Bellman equations:

$$rE_c = 0 = -e + pE_l \quad (1)$$

$$rE_l = 0 + q(E_g - E_l) \quad (2)$$

$$rE_g = y - w - \rho + s(0 - E). \quad (3)$$

In the last line, it was assumed that job destruction also leads to the destruction of the firm and the lending relation with the bank.

Symmetrically, the bank's asset values are denoted by  $B_j$ ,  $j = c, l$  or  $g$  for each of the stages. We also assume free entry of the banking relationship:  $B_c = 0$ . We denote by  $\kappa$  the screening cost per unit of time of banks in the first stage, and by  $\hat{p}$  the Poisson rate at which a bank finds a firm to be financed. We have:

$$rB_c = 0 = -\kappa + \hat{p}B_l \quad (4)$$

$$rB_l = -\gamma + q(B_g - B_l) \quad (5)$$

$$rB_g = \rho + s(0 - B_g). \quad (6)$$

The matching rates  $p$  and  $\hat{p}$  are made mutually consistent by the existence of a matching function  $M_c(\mathcal{B}, \mathcal{E})$ , where  $\mathcal{B}$  and  $\mathcal{E}$  are respectively the number of bankers and of entrepreneurs in stage  $c$ . This function is assumed to have constant returns to scale. Hence, denoting by  $\phi$  the ratio  $\mathcal{E}/\mathcal{B}$ , which is a reflection of the tension in the credit market and that we shall call credit market tightness from the point of view of entrepreneurs, we have

$$p = \frac{M_c(\mathcal{B}, \mathcal{E})}{\mathcal{E}} = p(\phi) \text{ with } p'(\phi) < 0.$$

$$\hat{p} = \phi p(\phi) \text{ with } \hat{p}'(\phi) > 0.$$

After the contact, the bank and the entrepreneur engage in bargaining about  $\rho$  which is such that

$$(1 - \beta)B_l = \beta E_l \quad (7)$$



where  $\beta$  is the bargaining power of the bank relative to the entrepreneur. With  $\beta = 0$  the bank leaves all the surplus to the entrepreneur.

Combining (1), (4) and (7), we obtain the equilibrium value of  $\phi$  denoted by  $\phi^*$  with

$$\phi^* = \frac{\kappa}{e} \frac{1 - \beta}{\beta}.$$

Matching in the labor market is denoted by  $M_l(\mathcal{V}, u)$  where  $u$  is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1.  $\mathcal{V}$  is the number of "vacancies", that is the number of firms in stage  $l$ . The function is also assumed to be constant return to scale, hence the rate at which firms fill vacancies is a function of the ratio  $\mathcal{V}/u$ , that is tightness of the labor market. We have

$$q(\theta) = \frac{M_l(\mathcal{V}, u)}{\mathcal{V}} \text{ with } q'(\theta) < 0.$$

Further using (2), (3) and (5), (6), we simultaneously solve for  $\rho$  :

$$\frac{\rho}{r+s} = \beta \frac{y-w}{r+s} + (1-\beta) \frac{\gamma}{q(\theta)}$$

and obtain the two main equations of the model :

$$\text{(EE)} : \frac{e}{p(\phi)} = \frac{q(\theta)}{r+q(\theta)} \left( \frac{y-w}{r+s} - \frac{\gamma}{q(\theta)} \right) (1-\beta) \quad (8)$$

$$\text{(BB)} : \frac{\kappa}{\phi p(\phi)} = \frac{q(\theta)}{r+q(\theta)} \left( \frac{y-w}{r+s} - \frac{\gamma}{q(\theta)} \right) \beta \quad (9)$$

Each equation provides a link between  $\theta$  and  $\phi$  that is of opposite sign. There is therefore at most one equilibrium set of  $(\theta^*, \phi^*)$ .<sup>2</sup> Finally, summing up (EE) and (BB), one obtains a single market equation denoted by (CC) for  $\theta^*$  describing a job creation condition for this double matching economy:

$$\text{(CC):} \frac{e}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)} = \frac{q(\theta)}{r+q(\theta)} \left( \frac{y-w}{r+s} - \frac{\gamma}{q(\theta)} \right) \quad (10)$$

where the left-hand side is a measure of the total amount of search costs in financial markets. These are

<sup>2</sup>Wasmer and Weil (2004) provide a condition for existence.

the total financial costs associated with the creation of a firm and that we shall denote by

$$K \equiv \frac{e}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)} \quad (11)$$

## 2.2 Steady-state volatility of $\theta$ to shocks

For the moment, to keep the analysis simple, we fix wages at some exogenous value. Endogenous wages are introduced only in the stochastic extension, in next Section. We now want to calculate the elasticity of  $\theta$  to profit shocks, denoted by  $\Lambda_{\theta/\pi}$ . Let  $\theta^P$  be the value of tightness solving for

$$\frac{y-w}{r+s} = \frac{\gamma}{q(\theta^P)} \quad (12)$$

The value of  $\theta^P$  defined here is the credit frictionless world in Pissarides (1985), which one would obtain from (10) when  $K = 0$ . In using (CC), one has:

$$\frac{\gamma}{q(\theta^P)} - \frac{\gamma}{q(\theta^*)} = K \left( \frac{r+q(\theta^*)}{q(\theta^*)} \right) > 0$$

Hence, given that  $q'$  is downward sloping, we have that  $\theta^* < \theta^P$ , as was shown in Wasmer and Weil (2004) and arises in Petrosky-Nadeau (2009), and the difference is precisely due to the existence of search costs in the credit market. Posing  $r = 0$  to marginally simplify the analysis, we have an equilibrium job creation condition under frictional credit markets which states that the profit flows from a job net of the total financial costs to creating a firm must equal the average cost of filling a job vacancy:

$$\frac{y-w}{r+s} = K + \frac{\gamma}{q(\theta^*)}. \quad (13)$$

The presence of frictional credit markets adds a new component in the entry costs for firms that, in the special case  $r = 0$ , is independent of labor market tightness. As we will see, this will raise the volatility of the economy, an insight already brought by Pissarides (2009).<sup>3</sup>

Let  $\pi = (y-w)/(r+s)$  be the present discounted value of profits. Taking logs and differentiating,

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<sup>3</sup>A stated in Pissarides (2009, page 1341) : “(...) a simple remodeling of the [matching] costs from proportional to partly fixed and partly proportional can increase the volatility of tightness and job finding, virtually matching the observed magnitudes, without violating wage flexibility.”

we have

$$-\frac{q'(\theta^*)\theta^*}{q(\theta)^*} \frac{d\theta}{\theta^*} = \frac{d\pi}{\pi} \frac{\pi}{\pi - K}$$

or, reusing (12) and (13) and where  $\eta = -q'(\theta)\theta/q(\theta)$  is the (non-necessarily constant) elasticity of  $q$  to  $\theta$ , we have

$$\Lambda_{\theta/\pi} = \frac{d\ln \theta}{d\ln \pi} = \frac{1}{\eta} \frac{\frac{\gamma}{q(\theta^P)}}{\frac{\gamma}{q(\theta^*)}} = \frac{1}{\eta} \frac{q(\theta^*)}{q(\theta^P)}$$

Two remarks are in order. First, in the (credit) frictionless world in Pissarides, the elasticity is simply the inverse of the elasticity of  $q$  to  $\theta$ , that is  $1/\eta$ . Second, the existence of credit market imperfections reduces  $\theta^*$  relative to  $\theta^P$ , and therefore raise the volatility  $\Lambda_{\theta/\pi}$  by a factor due to the financial multiplier identified in Wasmer and Weil (2004): higher profits raise the entry of firms, hence banks make faster profits, which in turn benefits firms, and so on. Denote by

$$M_f^S = \frac{q(\theta^*)}{q(\theta^P)}$$

the value of the financial multiplier where the superscript  $S$  reflects that this is calculated in a static context. The multiplier can more generically be defined as the ratio of the elasticity in a world with credit frictions and the elasticity in a world where credit frictions disappear.

Under the assumption of an exogenous wage, the response of this economy to productivity shocks on  $y$  is therefore :

$$\Lambda_{\theta/y} = \frac{d\ln \theta}{d\ln y} = \frac{d\ln \theta}{d\ln \pi} \frac{d\ln \pi}{d\ln y} = \frac{1}{\eta} \frac{y}{y-w} M_f^S$$

The first component of this elasticity is the amplification due to the existence of search frictions on the labor market. The second component is the gap between wages and marginal product - the smaller the gap, the more responsive job creation is to productivity shocks; and finally, the third is the financial multiplier.

The labor literature has attempted to raise the elasticity of market tightness to productivity with either wage rigidities (Hall 2005) or by making what we will call hereafter the "small labor surplus" assumption by choosing higher values of non-employment utility and lower values for the bargaining power of workers (Hagedorn and Manovskii 2008), reducing the gap between wages and marginal product. While acknowledging the interest of these approaches, we pursue another avenue here and attempt to understand the determinants of  $M_f^S$ .

## 2.3 Hosios-Pissarides in the credit market and the entry costs for firms

We start by noting that frictions in the credit market may lead to a second best efficiency condition similar to that in Hosios (1990) and Pissarides (1990).

### 2.3.1 The efficiency of financial markets in a search-economy

To see this, we can calculate the social welfare function as output net of all search costs. We have :

$$\Omega = y(1 - u) + zu - \gamma\theta u - \kappa\mathcal{B} - e\mathcal{E}$$

where  $z$  is the value of non-employment utility and  $\theta u = \mathcal{V}$  is the number of firms prospecting in the labor market. To obtain a simpler expression for  $\Omega$ , we can note that in a steady-state, we have  $\mathcal{E}p(\phi) = q(\theta)\mathcal{V}$  which states that inflows into the financing stage are compensated by outflows out of that stage. It follows that

$$\mathcal{E} = \frac{q(\theta)\theta u}{p(\phi)} \text{ and } \mathcal{B} = \frac{\mathcal{E}}{\phi} = \frac{q(\theta)\theta u}{\phi p(\phi)}$$

Therefore, the social planner's program can be rewritten as

$$\begin{aligned} \max_{u, \theta, \phi} \Omega &= y(1 - u) + zu - \gamma\theta u - \left( \frac{\kappa}{\phi p(\phi)} + \frac{e}{p(\phi)} \right) q(\theta)\theta u \\ \text{s.t. } u &= s / (s + \theta q(\theta)) \end{aligned}$$

Relative to the choice of the optimal  $\phi$  denoted by  $\phi^{opt}$ , the problem is simple and block-recursive in  $\phi$  and then in  $u$  and  $\theta$ . For the first block that we only consider here, the optimal choice of  $\phi$  amounts to minimizing total search costs  $K(\phi) = \frac{\kappa}{\phi p(\phi)} + \frac{e}{p(\phi)}$ .<sup>4</sup>

$$\begin{aligned} \frac{\partial \Omega}{\partial \phi} &= q(\theta)\theta u \frac{\partial}{\partial \phi} K(\phi) = 0 \\ \Leftrightarrow \phi^{opt} &= \frac{1 - \varepsilon}{\varepsilon} \frac{\kappa}{e} \text{ where } \varepsilon = - \frac{\phi p'(\phi)}{p(\phi)} \end{aligned}$$

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<sup>4</sup>Intermediate steps are :

$$\begin{aligned} \frac{\partial \Omega}{\partial \phi} &= 0 \Leftrightarrow \frac{k}{\phi p(\phi)} \frac{\phi p'(\phi) + p(\phi)}{\phi p(\phi)} + \frac{e}{p(\phi)} \frac{p'(\phi)}{p(\phi)} = 0 \\ \Leftrightarrow \frac{k}{\phi p(\phi)} (1 - \varepsilon) &= \frac{e}{p(\phi)} \varepsilon \end{aligned}$$

Hence, since  $\frac{\partial^2}{\partial \phi^2} K(\phi) > 0$ , the socially optimal value of credit market tightness is the one that minimizes search costs on credit markets. The Hosios-Pissarides rule, which states that there is a value of the bargaining parameter over  $\rho$  that internalizes the matching externalities due to the search frictions, applies here :

$$\begin{aligned}\phi^* &= \phi^{opt} \\ \Leftrightarrow \beta &= \varepsilon : \text{Hosios condition in the credit market}\end{aligned}$$

### 2.3.2 Minimizing the financial costs and the gap between $\theta^*$ and $\theta^P$

One may think that the Hosios condition is the one that minimizes entry costs in the credit market. One can check this formally. The left-hand side of job creation condition (CC) is a function of  $\beta$  and  $\varepsilon$  denoted by  $K(\beta, \varepsilon)$  ; the right-hand side is increasing in  $\theta$ . It is therefore enough to show that  $K(\beta, \varepsilon)$  is minimized in  $\beta = \varepsilon$ . Before doing so, we can use two intermediate steps. First, note that  $K(\beta, \varepsilon) = \frac{e}{1-\beta}$  from equation (EE) divided by  $(1-\beta)$ . Second, we have  $\frac{\partial \phi^*}{\partial \beta} = \frac{-1}{\beta^2} \frac{\kappa}{e}$  hence

$$\begin{aligned}\frac{\partial K}{\partial \beta} &= \frac{\frac{-e\rho'}{p^2(\phi^*)} \frac{\partial \phi^*}{\partial \beta}}{1-\beta} + \frac{\frac{e}{p(\phi^*)}}{(1-\beta)^2} = 0 \\ \Leftrightarrow \varepsilon &= \beta\end{aligned}$$

Given that  $M_f^S$ , and hence  $\Lambda_{\theta/y}$ , is increasing in the gap between  $\theta^*$  and  $\theta^P$ , at any  $\phi^*$ , the Hosios condition in the credit market is the one minimizing the volatility induced by financial imperfections. The calibration in Wasmer and Weil (2004) thus implied a minimized financial multiplier of  $M_f^S = 1.74$  by setting  $\beta = \varepsilon$ . Away from this equation, one has a larger financial multiplier.

## 3 A stochastic extension

### 3.1 Dynamic setup

In this Section, we study a dynamic stochastic model with first exogenous and then endogenous wages, and offer a flexible – that is, easy-to-implement– calibration method to obtain both a set of first order

moments (unemployment and financial sector's share of GDP) and then second moments (the volatility of labor market tightness to productivity shocks).

We make the following assumptions for convenience. First, time is discrete and labor productivity is assumed to follow a stationary AR(1) process  $y_t = \rho_y y_{t-1} + v_t$ , where  $0 < \rho_y < 1$  and  $v_t$  is white noise. Second, we relax the assumption that  $r = 0$ . Third, an entrepreneur meeting a banker begins the recruiting process within the period. A successful meeting between a firm and worker begins production the following period. Maintaining our assumption of free entry on both sides of the credit market and bargaining over  $\rho$ , we find that the equilibrium credit market tightness  $\phi^*$  is time invariant and of the same form as earlier.<sup>5</sup> Moreover,  $\rho$  is assumed to be determined when a banker and an entrepreneur meet and is solved as

$$E_t [\rho_{t+1}] = \beta \mathbb{E}_t [y_{t+1} - w_t] + (1 - \beta) \mathbb{E}_t \left[ \frac{(1+r)\gamma}{q(\theta_t)} - \frac{(1-s)\gamma}{q(\theta_{t+1})} \right] \quad (14)$$

where  $\mathbb{E}_t$  is an expectations operator over productivity and  $w_t$  is a wage determined later on.

From the constant values of being in the recruiting stage,  $B_{l,t} = \frac{\kappa}{\phi^* p(\phi^*)}$  and  $E_{l,t} = \frac{e}{p(\phi^*)}$ , we can combine the (EE) and (BB) curves in this stochastic environment,

$$\begin{aligned} \frac{e}{p(\phi)} &= \frac{q(\theta_t)}{1+r} \mathbb{E}_t [E_{g,t+1}] + \frac{(1-q(\theta_t))}{1+r} \frac{e}{p(\phi)} \\ \frac{\kappa}{\phi p(\phi)} &= -\gamma + \frac{q(\theta_t)}{1+r} \mathbb{E}_t [B_{g,t+1}] + \frac{(1-q(\theta_t))}{1+r} \frac{\kappa}{\phi p(\phi)} \end{aligned}$$

to obtain a job creation condition in the presence of frictional credit markets

$$\frac{\Gamma_t}{q(\theta_t^*)} = \frac{1}{1+r} \mathbb{E}_t \left[ y_{t+1} - w_t + (1-s) \frac{\Gamma_{t+1}}{q(\theta_{t+1}^*)} \right] \quad (15)$$

where

$$\Gamma_t \equiv \gamma + K \left( 1 - \frac{1}{1+r} (1 - q(\theta_t^*)) \right) \quad (16)$$

are vacancy costs augmented for frictional credit markets and  $K = \frac{e}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)}$  is once again total search costs on the credit market.

It is worth noting two special cases. First, when  $r = 0$ ,  $\Gamma_t$  is simply the sum of all prospection costs

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<sup>5</sup>Time invariance follows from the sharing rule  $(1 - \beta)B_{l,t} = \beta E_{l,t}$  which implies a constant ratio  $\frac{E_{l,t}}{B_{l,t}} = \frac{1-\beta}{\beta}$ .

in credit and labor markets, unadjusted for discounting: we obtain in particular

$$\frac{\Gamma_t}{q(\theta_t^*)} \equiv \frac{\gamma}{q(\theta_t^*)} + K \quad (17)$$

which indicates that set-up costs of firms now include a part unrelated to labor market tightness, with therefore a potential for raising volatility as setup costs will be less procyclical. Second, when credit markets are perfect,  $\Gamma_t$  boils down to  $\gamma$ , and the job creation condition reduces to

$$\frac{\gamma}{q(\theta_t^P)} = \frac{1}{1+r} \mathbb{E}_t \left[ y_{t+1} - w_t + (1-s) \frac{\gamma}{q(\theta_{t+1}^P)} \right] \quad (18)$$

### 3.2 Elasticity of $\theta_t$ to productivity shocks, fixed wage

Define period profits from labor as  $\Pi_t = y_t - \bar{w}$  where  $w_t = \bar{w}$  is a fixed wage. We can compute two elasticities of labor market tightness to productivity innovations, first in the absence of financial imperfections, second with financial imperfections, and compare them.

Taking log-linear deviations around a steady state of equation (18), deviations in labor market tightness in the credit frictionless world can be expressed as a discounted sum of deviations in future expected profits

$$\widehat{\theta}_t^P = \frac{q(\theta^P)\Pi}{\eta\gamma(1+r)} \mathbb{E}_t \sum_{i=0}^{\infty} \left( \frac{1-s}{1+r} \right)^i \widehat{\Pi}_{t+1+i}$$

Given a fixed wage and the assumption on productivity, this is simply  $\widehat{\theta}_t^P = \frac{q(\theta^P)}{\eta\gamma(1+r)} \sum_{i=0}^{\infty} \rho_y^{i+1} v_t$  such that the elasticity of market tightness to a productivity shock in the Pissarides world with a fixed wage is denoted by  $\Lambda^P$  with

$$\Lambda^P = \frac{\partial \widehat{\theta}_t^P}{\partial v_t} = \frac{q(\theta^P)\rho_y}{\eta\gamma[(1+r) - (1-s)\rho_y]} \quad (19)$$

By the same steps, the elasticity in the presence of credit frictions is given by  $\Lambda$  with

$$\Lambda = \frac{\partial \widehat{\theta}_t^*}{\partial v_t} = \frac{q(\theta^*)\rho_y}{\eta\gamma(r)[(1+r) - (1-s)\rho_y]} \quad (20)$$

where  $\gamma(r) \equiv \left[ \gamma + K \left( \frac{r}{1+r} \right) \right] > \gamma(0) = \gamma$  is a measure of total frictional costs in both credit and labor markets.

The financial multiplier in this dynamic setting is thus denoted by a superscript  $D$  and is:

$$M_f^D \equiv \frac{\Lambda}{\Lambda^P} = \frac{q(\theta^*)}{q(\theta^P)} \frac{\gamma}{\gamma(r)} \quad (21)$$

which boils down to the static financial multiplier  $M_f^S$  derived in Section 2 when  $r = 0$ . We thus provide here a dynamic generalization of this multiplier.

### 3.3 Elasticity of $\theta_t$ to productivity shocks, endogenous wages

Endogenous wages strongly reduce the elasticity of labor market tightness to productivity shocks. We thus expect that the financial multiplier will need to be higher to generate the volatility obtained in the economy with a fixed wage.

The wage determination we select is as follows. We assume that the worker bargains the wage with a firm, defined as the entrepreneur-banker block, at the time of meeting, instead of a bilateral bargaining between the worker and the entrepreneur (leaving the bank aside).<sup>6</sup>

Define the values of employment and unemployment in a discrete time stochastic setting as

$$\begin{aligned} U_t &= z + f(\theta_t)\beta\mathbb{E}_t W_{t+1} + (1 - f(\theta_t))\beta\mathbb{E}_t U_{t+1} \\ W_t &= w_t + \beta\mathbb{E}_t [(1 - s)W_{t+1} + sU_{t+1}] \end{aligned}$$

where  $z$  is the value of non-employment activities and  $f(\theta) = \theta q(\theta)$  the job finding rate. The Pissarides wage is  $w_t^P = \alpha (y_t + \gamma\theta_t^P) + (1 - \alpha)z$  where  $\alpha$  is the bargaining power of workers vis-à-vis the firm. Taking log-deviations, movements in labor market tightness to future productivity in the credit frictionless world are given by

$$\widehat{\theta}_t^P = \frac{q(\theta^P)(1 - \alpha)}{\eta\gamma(1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Psi^i \widehat{y}_{t+1+i}$$

where the second term in  $\Psi = \left(\frac{1-s}{1+r}\right) - \frac{\alpha\theta^P q(\theta^P)}{\eta(1+r)}$  reflects the share of the change in productivity accruing

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<sup>6</sup>There are two related reasons for this choice. The first one is that the natural alternative, bargaining between the entrepreneur and the worker, leads to complex strategic interactions illustrated in Wasmer and Weil (2004, Section IV-A): the entrepreneur and the bank wish to raise the debt of the firm above what is needed in order to reduce the size of total surplus to be shared between the firm and the worker at a later time. Hence, wages are driven down to the reservation wage of workers and do not vary with the firm's productivity, which is counterfactual. This leads to the second reason, which is that we want our endogenous wage extension to be comparable to the classical wage solution in the labor search literature in order to compare the volatility in the model to other elasticities found in the literature.



to the worker through the wage. The latter strongly reduces the elasticity of labor market tightness to productivity shocks which, with our specification, is<sup>7</sup>

$$\Lambda^P = \frac{\partial \hat{\theta}_t^P}{\partial v_t} = \frac{q(\theta^P)(1-\alpha)\rho_y}{\eta\gamma(1+r) - \gamma[\eta(1-s) - \alpha f(\theta^P)]\rho_y} \quad (22)$$

Compared to the elasticity when wages are fixed, only a share  $(1-\alpha)$  of the rise in productivity accrues to the firm. In addition, the equilibrium rise in labor market tightness following a positive productivity shock improves the outside option of the worker and his bargaining position in the wage determination. This appears in the denominator as the term  $\alpha f(\theta^P)$ , further reducing the elasticity of labor market tightness to productivity shocks.

Turning now to the responsiveness of labor market tightness under frictional credit markets, we begin by detailing the determination of the wage. As discussed earlier, we assume that the wage is negotiated in a worker-firm pair and, in the presence of credit market frictions, it must satisfy a sharing rule  $\alpha F_{g,t} = (1-\alpha)(W_t - U_t)$ , where  $F_{g,t} = E_{g,t} + B_{g,t}$  is the joint value of the firm to the entrepreneur-banker pair. Under this assumption the wage is

$$w_t = \alpha [y_t + \Gamma_t \theta_t^*] + (1-\alpha)z$$

and differs from the Pissarides wage by the coefficient  $\Gamma_t$  on market tightness. To the extent that this term is negatively correlated with productivity, credit market frictions induce a certain degree of wage rigidity by limiting the effect of a rise in market tightness on wages, a feature also present in Petrosky-Nadeau (2009). To see why this is the case, recall that  $\Gamma_t \equiv \gamma + K \left(1 - \frac{1}{1+r}(1 - q(\theta_t^*))\right)$  are the set-up costs augmented for frictional credit markets. Since  $q$  is decreasing in market tightness, so is  $\Gamma$ . This effect, however, will be marginal in the quantitative results.

Finally, the elasticity of labor market tightness under frictional credit markets and an endogenous wage is

$$\Lambda = \frac{\partial \hat{\theta}_t^*}{\partial v_t} = \frac{q(\theta^*)(1-\alpha)\rho_y}{\eta\gamma^T(1+r) - [\eta\gamma^T(1-\delta) - \alpha f(\theta^*)(\gamma^T + (1-\eta)\tilde{\kappa})]\rho_y} \quad (23)$$

where  $\tilde{\kappa} \equiv K \frac{q(\theta^*)}{1+r}$ . The dynamic financial multiplier is then equal to

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<sup>7</sup>To check the result, note that if  $\rho_y = 1$  this is the elasticity obtained when comparing steady states, or to a permanent productivity shock, as in Shimer (2005), i.e.  $\varepsilon_{\theta,y} = \frac{(1-\alpha)}{\gamma \left[ \frac{\eta(r+s)}{q(\theta^P)} + \alpha \theta^P \right]}$ . The details for deriving the elasticities can be found in the appendix.

$$M_f^D = \frac{q(\theta^*)}{q(\theta^P)} \left( \frac{\eta\gamma - \gamma\rho_y [\eta(1-s) - \alpha f(\theta^P)]}{\eta\gamma(r) - \gamma(r)\rho_y [\eta(1-s) - \alpha f(\theta^*)(1 + (1-\eta)K^{\frac{q(\theta^P)}{\gamma(r)})}] } \right) \quad (24)$$

and we provide here a generalization of the static multiplier in Wasmer and Weil (2004) along two dimensions: stochastic dynamics and endogenous wages.

## 4 Calibration

### 4.1 Targets: first and second moments

Our first objective is to find a precise measure of the share of the financial sector in GDP and try to reproduce it in the steady-state of the model's stochastic extension. Theoretically, the share of the financial sector in the value added is

$$\Sigma = \frac{(1-u)\rho - \mathcal{V}\gamma - \mathcal{B}\kappa}{1-u} \quad (25)$$

where in the numerator, the first term represents total bank gross profits  $\rho$  times the number of banks in the profit state, which is equal to the number of firms  $1-u$ ; the second term represents the negative cash flows of banks financing vacancies times the number of job vacancies  $\mathcal{V}$ , where  $\mathcal{V} = \theta u$ ; and the last term represents the financial intermediation costs paid by banks. Note that we assumed the costs  $e$  paid by entrepreneurs don't enter GDP as they are effort costs. The denominator is total production at  $y = 1$ .

US national economic accounts data (various tables in the NIPA<sup>8</sup>) allow us to calculate the gross value added of financial services. For that, we use one of the seven components of value added entitled financial business gross value added, from which we subtract, from the expenditure account, Households Consumption on Expenditures in Insurance and Financial Services. Over the period 1985-2008, this represents approximately 3.0% of GDP and will be the target for  $\Sigma$ . Our second target for first moments will be the rate of unemployment that we try to keep in the neighborhood of 7%. Our third target is to find an elasticity of  $\theta$  to productivity shocks around 20.

Our approach can be summarized in three stages, described in the following sub-sections.

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<sup>8</sup>e.g. [http://www.econstats.com/nipa/NIPA1\\_1\\_14\\_.htm](http://www.econstats.com/nipa/NIPA1_1_14_.htm)

## 4.2 Initial values of parameters under Hosios-Pissarides

We first find, both for the fixed wage model and the endogenous wage model, a set of parameters that reaches the target unemployment level. The calibration of the credit market requires choosing parameters of the credit matching function, assumed to be of the form  $M_c(\mathcal{B}, \mathcal{E}) = \zeta \mathcal{E}^{1-\varepsilon} \mathcal{B}^\varepsilon$ , the costs of prospecting on credit markets and the bargaining weight  $\beta$ . We start agnostically from a Hosios-Pissarides rule in the credit market and proceed as follows.

We start from an initial, informed guess on parameters, where we choose in particular a symmetric set of parameters regarding prospecting costs and the matching function in the credit market. We include these parameters in a vector  $X$  using as a starting point a "balanced" credit matching function and the credit market Hosios condition; i.e.,  $\beta = \varepsilon = 0.5$ , symmetry in prospecting costs  $\kappa = e = 0.05$  and set the remaining parameter,  $\zeta$ , to 0.05. On the labor market, we include the unit recruitment costs of  $\gamma$  and the level parameter  $\chi$  of the matching function  $M_l(\mathcal{V}, u) = \chi \mathcal{V}^{1-\eta} u^\eta$  in the vector of parameters  $X$  to achieve a desired level of unemployment. For the exogenous wage specification it is assumed to equal three quarters of labor productivity. The steady state rate of job separation is set to  $s = 0.06$ , corresponding to the value reported in Davis et al. (2006). We assume an elasticity of the labor matching function with respect to unemployment of  $\eta = 0.5$ <sup>9</sup> and, with endogenous wages, we set the flow value of non-employment  $z = 0.4$  as suggested by Shimer (2005). Finally, the risk free rate is set to 4% annually, corresponding to a 3-month treasury bill, and the persistence coefficient in the process for productivity is set to 0.975, a commonly used value in the real business cycle literature.

In this initial calibration, the value of  $\Sigma$  (share of financial sector in GDP in equation 25) happens to be too low and relatively stable to parameters. The parameter space is large and it is difficult to find the "right" parameter values. For example, the natural idea of raising screening costs  $\kappa$  does not raise sufficiently the share of the financial sector because the free-entry condition reduces the number of banks entering, so that  $\Sigma$  remains fairly constant. Raising  $\beta$  reduces instead the number of firms and thus affect the value of unemployment.

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<sup>9</sup>See Petrongolo and Pissarides (2001) for a survey of estimates of the labor matching function.

### 4.3 A “trembling hand” calibration method

We will therefore propose a transparent calibration method, inspired from the “simulated annealing method.”<sup>10</sup>The procedure consists of perturbing each element of an initial vector of parameters  $X$  by a random shock drawn from a normal distribution in a 7-dimensional parameter space for exogenous wages:  $\gamma, \beta, \varepsilon, e, \kappa$  and the two scale parameters in the matching functions denoted by  $\chi$  in the labor market and  $\varsigma$  in the credit market. With endogenous wages, the parameter space is 8-dimensional with the inclusion of the worker’s bargaining weight  $\alpha$ .

We run perturbations of the set of parameters  $X$  in a neighborhood of the starting values of parameters, where perturbations are small : each parameter receives a multiplicative normal shock of variance  $1/60$  (exogenous wages) or  $1/80$  (endogenous wages). We obtain a corresponding value of the equilibrium variables  $\theta, u$  and  $\phi$  as well as a value of the credit market share in GDP  $\Sigma$ . We only retain values of the parameters for which  $u$  is between 7 and 8% and for which  $q(\theta)$  is between 0 and 1. After we reach 100 “acceptable” draws, we pick up the set of parameters where  $\Sigma$  is maximal and denote the corresponding new vector of parameters  $X'$ .

We then iterate on the same procedure, where the initial value of parameters is  $X'$  of the previous iteration. We stop the iterations when the value of  $\Sigma$  exceeds 3.0%, generally slightly above this threshold. The convergence occurs relatively fast, in about 10 to 20 steps. We call this first procedure Step 1, and it aims at matching the credit market share in GDP  $\Sigma$ . Given that we have an underidentified system, since there are several new parameters compared to traditional calibrations, in particular  $\beta, \varepsilon, e, \kappa$ , we believe that our method is a fairly transparent one, more transparent than using a GMM method where the multiplicity of parameters would leave a lot of discretion.

Next, we replicate this procedure to progressively raise the elasticity of tightness of the labor market to productivity shocks. In particular, with endogenous wages, we also shock the bargaining power of workers and the value of leisure. We stop when we obtain an elasticity  $\Lambda$  of labor market tightness to productivity shocks larger than, respectively, 7, 15 and 20 in the case of endogenous wages, since the first one corresponds to the value suggested by Pissarides (2009), but also show that it is possible to obtain an elasticity of 20 without making the small labor surplus assumption.

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<sup>10</sup>The difference between this method and ours is that the annealing method accepts all perturbation raising an objective function, but also accepts some perturbations reducing the objective function, with a probability which is exponential in the variation. Our method, as explained below, pools a large number of perturbations and chooses the one maximizing the criterion.

## 5 Results of the calibration

### 5.1 Endogenous wages

Table 1 summarizes the baseline parameter values, both the starting point and the results of the numerical search procedure. It also presents the steady state values of a series of endogenous quantities that are part of the constraint set. The first process, matching the share of the credit market, is stopped after 19 iterations. The values of the credit matching function's elasticity  $\varepsilon$  and the bargaining weight  $\beta$  evolve quite smoothly at each iteration, as seen in Figure 1 of the appendix. Both parameters start at 0.5 and the model diverges away from Hosios-Pissarides: the matching elasticity remains around 0.5 while the bank bargaining weight  $\beta$  increases to 0.78. Note also the all other parameters included in the vector  $X$  change very little during this procedure.

Although value of  $\Sigma$  is matched, the elasticity of labor market tightness to productivity shocks is still low, with a value of 4.47 and a credit multiplier of 2.30. We thus launch the second step in the calibration procedure that aims at raising the value this elasticity, keeping the value of  $\Sigma$  below 3.1%. We reach the values 7, 15 and 20 (respectively in the columns labeled 2a, 2b and 2c) in a few iterations (56 iterations for the last case) and the financial multiplier reaches a factor of 4.6. During this second step we deviate marginally more away from Hosios :  $\beta$  reaches 0.92 and  $\varepsilon$  is still approximately 0.5. The appendix plots the evolution of all the parameters in the vector  $X$  and shows that they all converge quickly to their final values reported in Table 1. The duration of search for credit is also reduced to a year when the elasticity of labor market tightness to productivity is large enough (columns 2b and 2c).

Most other parameter values remain stable. In particular, the value of the bargaining parameter of workers  $\alpha$  remains at 0.10, a value that is in line with recent papers, when we target the elasticity of labor market tightness suggested by Pissarides (2009).<sup>11</sup>

Finally, we calculate the elasticity of unemployment to unemployment benefits. In Costain and Reiter (2008), this elasticity was around 14.3 for the Hagedorn and Manovskii calibration, therefore leading to the criticism that it is difficult to match both the elasticity of labor market tightness to productivity and the elasticity of unemployment to policy variables such as the replacement ratio. What we show here is that, at an elasticity of 20, we can reduce the elasticity of unemployment to  $z$  6.8.

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<sup>11</sup>Cahuc, Postel-Vinay and Robin (2006) find a bargaining power of low skilled workers in this range, and Delacroix (2006) obtains needs a value of 6 to 8% to replicate the union premium.

Table 1: Baseline parameter and steady state values, endogenous wage

	Initial	Step1	Step 2a	Step2b	Step 2c	Steady state values	Initial	Step 1	Step 2a	Step2b	Step2c
<b>Labor market</b>											
matching elasticity	$\eta$	0.5	0.5 (*)	0.5 (*)	0.5 (*)	$q(\theta)$	0.27	0.25	0.55	0.52	0.64
worker's share	$\alpha$	0.15	0.13	0.10	0.03	$f(\theta)$	0.93	0.70	0.70	0.75	0.73
value of leisure	$z$	0.4	0.4 (*)	0.52	0.70	$w/y$	0.83	0.78	0.78	0.78	0.78
job separation rate	$s$	0.06	0.06 (*)	0.06 (*)	0.06 (*)	Unmp. rate in (7%;8%)	6.0%	8% (*)	8% (*)	7.5% (*)	7.6% (*)
vacancy cost	$\gamma$	0.1	0.10	0.10	0.15	Average recruiting cost	2.43	3.26	3.10	3.16	3.07
matching level param.	$\chi$	0.5	0.42	0.62	0.68	Credit market share $\Sigma$	1.58%	3.04%	2.91%	3.10%	2.98%
<b>Credit market</b>						Elasticity of $\theta$ to $y$ : $\Lambda$	3.82	4.47	7.03	15.34	20.08
bank's barg. weight	$\beta$	0.5	0.78	0.88	0.92	Financial multiplier $M_f^D$	2.05	2.30	2.86	3.94	4.58
matching elasticity	$\varepsilon$	0.5	0.49	0.55	0.48	Duration, search for credit	20.0	11.32	5.87	4.62	4.72
matching level param.	$\zeta$	0.05	0.045	0.049	0.059	Elasticity of $u$ to $z$		1.75	5.08	6.8	
Bank search costs	$\kappa$	0.05	0.048	0.045	0.038						
Firm search costs	$e$	0.05	0.054	0.058	0.47						

Note: Costain-Reiter elasticity with Hagedorn-Manovskii parameters was 14.3

Match  $\Sigma$  after 19 iterations, match resp.  $\Lambda = 7, 15, 20$  after 22, 37 and 56 iterations.

(\*): constrained

## 5.2 Exogenous wages

Table 2 summarizes the results from the equivalent procedure for fixed wages. The process of Step 1 is stopped after 10 iterations. Again, the values of the credit matching function's elasticity  $\varepsilon$  and the bargaining weight  $\beta$  evolve quite smoothly at each iteration. The model also diverges away from Hosios-Pissarides and the bank bargaining weight  $\beta$  increases to 0.85. The value of  $\Sigma$  is matched, but the elasticity of tightness to productivity innovations is still a bit low, with a value of 19.5 and a credit multiplier of 3.07. We thus launch the second step calibration procedure that aims at raising the value the elasticity, keeping the value of  $\Sigma$  below 3.1%. We end up fast to the required value of 20, overshooting a little at 23. The financial multiplier reaches 3.64, deviating marginally more away from Hosios:  $\beta$  reaches 0.86 and  $\varepsilon$  is lower at 0.51. The duration of search for credit is a bit large, around 11 quarters.

Table 2: Baseline parameter and steady state values, fixed wages

	Initial	Step1	Final	Steady state values	Initial	Step 1	Final
<b>Labor market</b>							
matching elasticity	$\eta$	0.5	0.5 (*)	$q(\theta)$	0.17	0.21	0.26
fixed wage	$\bar{w}$	0.77	0.77 (*)	$f(\theta)$	0.99	0.86	0.81
				$w/y = 0.77$ (*)			
job separation rate	$s$	0.06	0.06 (*)	Unmp. rate in (6%;7%)	5.4%	6.5% (*)	6.9% (*)
vacancy cost	$\gamma$	0.2	0.198	Average recruiting cost	3.28	3.28	3.28
matching level param.	$\chi$	0.42	0.422	Credit market share $\Sigma$	2.55%	3.05%	3.04% (*)
<b>Credit market</b>				Elasticity of $\theta$ to $y:\Lambda$	16.0	19.5	23.0
bank's barg. weight	$\beta$	0.5	0.85	Financial multiplier $M_f^D$	2.51	3.07	3.64
matching elasticity	$\varepsilon$	0.5	0.53	Duration, search for credit	20.0	11.9	11.3
matching level param.	$\zeta$	0.05	0.056				
Bank search costs	$\kappa$	0.05	0.045				
Firm search costs	$e$	0.05	0.049				
Match $\Sigma$ after 10 iterations, match $\Lambda$ after 2 iterations							
(*) : constrained							



## 6 Conclusion

Financial imperfections raise the calibrated elasticity of labor market tightness to productivity shocks by a factor  $M_f$  called the financial multiplier. With exogenous wages, it is easy to generate a plausible large elasticity of labor market tightness to productivity shocks, if one relaxes the Hosios-Pissarides rule in the credit market. Under the assumption of a large enough difference between the bargaining power of banks vis-à-vis entrepreneurs ( $\beta$ ) with the elasticity of the rate at which entrepreneurs meet bankers with respect to credit market tightness ( $\varepsilon$ ), one can obtain an elasticity around 20 or even larger.

Under endogenous wages with bargaining power  $\alpha$  of workers relative to firms, defined as the joint bank-entrepreneur entity, all elasticities are divided by a factor 4 to 5, as was established by Shimer (2005) and Hall (2005). Hence, the model requires a higher financial multiplier. However we manage to keep the wage/productivity ratio around 0.78, thus relaxing the “small labor surplus assumption” and obtain large values of the elasticity, ranging from 7 when the bargaining power of workers over wages is 0.10 to 21 when it is allowed to go down to 0.03.

Our results are in fact a generalization of the “small labor surplus” assumption: when the credit market is either very tight or very slack for firms, one side of the market has a very small *total* surplus to entering the relationship. Consequently, the entry of that side of the credit market is restricted and even small productivity shocks can generate large relative increases in the number of agents on the restricted side of the market. Here, the small surplus is on firms in the credit-prospection stage.

In addition, we can go back to the intuitions of equation (13) or its discrete time equivalents combined in (15) and (17). These entry equations for firms have a common denominator: they introduce a new element to hiring costs, which is not strictly proportional to the duration of a vacancy  $1/q(\theta)$ . As pointed out in Pissarides (2009), this leads to a greater volatility. An interpretation of our paper, fully consistent with Pissarides (2009), is that this non-proportional part is a financial cost arising from frictions in the credit market. Our paper thus provides a set of parameters allowing for an interpretation of this fixed part in entry costs, linked to financial market imperfections.

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# Appendix: The Cyclical Volatility of Labor Markets under Frictional Financial Markets

## A Introduction

This appendix details the derivation of the various equations and elasticities presented in the main text. We begin by fully describing the stochastic model in discrete time.

### A.1 Asset values of an entrepreneur

$$\begin{aligned}
 E_{c,t} &= -e + p_t E_{l,t} + (1 - p_t) \frac{1}{1+r} \mathbb{E}_t E_{c,t+1} \\
 E_{l,t} &= -\gamma + \gamma + \frac{1}{1+r} \mathbb{E}_t [q_t E_{g,t+1} + (1 - q_t) E_{l,t+1}] \\
 E_{g,t} &= y_t - w_t - \rho_t + \frac{1}{1+r} \mathbb{E}_t [s E_{c,t+1} + (1 - s) E_{g,t+1}]
 \end{aligned}$$

The cost of convincing a bank to fund future negative cash flows is  $e$ , and with probability  $0 < p_t < 1$  this results in a successful match within the period. During the second stage, the bank covers the cost of recruiting a worker,  $\gamma$ , who is met with probability  $0 < q_t < 1$ . During the production stage,  $y$  goods are produced which must cover both the wage rate  $w$  and interest payments  $\rho$ . During the last stage, firms are subject to death shocks with probability  $s$ .

An assumption of free entry for entrepreneurs leads  $\frac{e}{p_t} = E_{l,t}$  such that the final stage may be simplified to

$$E_{g,t} = y_t - w_t - \rho_t + (1 - s) \frac{1}{1+r} \mathbb{E}_t E_{g,t+1}$$

### A.2 Matching on credit markets

We follow the matching literature and assume that the total number of matches is governed by a matching technology associating the total number of banks in stage 0, denoted by  $\mathcal{B}$ , and the total number of entrepreneurs in stage 0, denoted by  $\mathcal{E}$ . Let  $M_C(\mathcal{E}, \mathcal{B})$  be the matching process in the credit market. We have that  $p = M_C(\mathcal{E}, \mathcal{B})/\mathcal{E}$ . Symmetrically, the rate at which banks find a project they are willing to finance is  $M_C(\mathcal{E}, \mathcal{B})/\mathcal{B} = \phi p$  where  $\phi = \mathcal{E}/\mathcal{B}$ . Under the assumption of constant returns to scale of

$M_C(\mathcal{E}, \mathcal{B})$ , we have that  $p = p(\phi)$  with  $p'(\phi) < 0$ , elasticity  $\varepsilon(\phi) = -\phi p'(\phi)/p(\phi)$ , and it follows that  $\phi$  is a natural measure of the tightness of the credit market. We also make the assumptions

$$\begin{aligned}\lim_{\phi \rightarrow 0} p(\phi) &= 1 \\ \lim_{\phi \rightarrow +\infty} p(\phi) &= 0\end{aligned}$$

The first line states that in the relative scarcity of competing firms relative to banks, matching with a banker is instantaneous, and the second line states that in the relative abundance of competing firms relative to banks, matching with a banker is infinitely slow.

### A.3 Asset values for a banker

$$\begin{aligned}B_{c,t} &= -\kappa + \phi_t p(\phi_t) B_{l,t} + (1 - \phi_t p(\phi_t)) \frac{1}{1+r} \mathbb{E}_t B_{c,t+1} \\ B_{l,t} &= -\gamma + \frac{1}{1+r} \mathbb{E}_t [q_t B_{g,t+1} + (1 - q_t) B_{l,t+1}] \\ B_{g,t} &= \rho_t + \frac{1}{1+r} \mathbb{E}_t [s B_{c,t+1} + (1 - s) B_{g,t+1}]\end{aligned}$$

Bankers search for a suitable investment at a cost of  $\kappa$  and enter the recruiting stage with probability  $\phi_t p(\phi_t)$  during which the vacancy cost  $\gamma$  must be disbursed. Meeting a worker occurs at the rate  $q_t$ , at which point a banker enters the production stage and the remuneration  $\rho$  is received. An assumption of free entry for bankers leads  $\frac{\kappa}{\phi_t p(\phi_t)} = B_{l,t}$ ,

### A.4 Time invariant credit market tightness

Free entry on both sides of the credit market, along with Nash bargaining over the surplus of a credit relationship, results in a time invariant credit market tightness. To show this, note first that we had under free entry

$$B_{l,t} = \frac{\kappa}{\phi_t p(\phi_t)}; \text{ and } E_{l,t} = \frac{e}{p(\phi_t)}$$

Denoting the banker's bargaining weight by  $\beta$ , and defining the credit relationship surplus as  $S_{C,t} = (E_{l,t} - E_{c,t}) + (B_{l,t} - B_{c,t})$ , results in  $\frac{E_{l,t}}{B_{l,t}} = \frac{1-\beta}{\beta}$  and

$$\phi^* = \frac{1-\beta}{\beta} \frac{\kappa}{e}$$

## A.5 Deriving a job creation condition:

It will be convenient at this stage to express the joint value of recruiting a worker to banker and entrepreneur as  $F_{l,t} = E_{l,t} + B_{l,t}$ , which corresponds to the surplus from the credit relationship, as

$$\frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} = -\gamma + q_t \frac{1}{1+r} \mathbb{E}_t [E_{g,t+1} + B_{g,t+1}] + (1-q_t) \frac{1}{1+r} \left[ \frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} \right]$$

Define total costs on the credit market as  $K(\phi) = \frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)}$  and  $\Gamma_t \equiv \gamma + K(\phi) \left(1 - \frac{1}{1+r}(1-q_t)\right)$ , then

$$\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t [E_{g,t+1} + B_{g,t+1}]$$

Using the Bellman equations for entrepreneur and banker during production to define  $[E_{g,t} + B_{g,t}] = F_{g,t} = y_t - w_t + (1-s) \frac{1}{1+r} \mathbb{E}_t [F_{g,t+1}]$ , we obtain a job creation condition in the presence of frictional credit and labor markets

$$\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1-s) \frac{\Gamma_{t+1}}{q_{t+1}} \right]$$

Note that when the credit market is perfect  $K(\phi) = 0$  and  $\Gamma_t = \gamma$ , such that the job creation condition collapses to the familiar

$$\frac{\gamma}{q_t} = \frac{1}{1+r} \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1-s) \frac{\gamma}{q_{t+1}} \right]$$

## A.6 Rental rate

This section provides the details in deriving the rental rate

$$\mathbb{E}_t [\rho_{t+1}] = \beta \mathbb{E}_t [y_{t+1} - w_{t+1}] + (1-\beta) \mathbb{E}_t \left[ \frac{(1+r)\gamma}{q(\theta_t)} - \frac{(1-s)\gamma}{q(\theta_{t+1})} \right]$$

Define the surplus to the credit relationship as  $S_{C,t} = E_{l,t} + B_{l,t}$ . The sharing rule under Nash bargaining implies  $B_{l,t} = \beta S_{C,t}$  and  $E_{l,t} = (1-\beta) S_{C,t}$ . Expanding on the former,

$$\begin{aligned} -\gamma + \frac{1}{1+r} \mathbb{E}_t [q_t B_{g,t+1} + (1-q_t) B_{l,t+1}] &= -\beta \gamma + \beta q_t \frac{1}{1+r} \mathbb{E}_t [E_{g,t+1} + B_{g,t+1}] \\ &\quad + \beta (1-q_t) \frac{1}{1+r} \mathbb{E}_t [E_{l,t+1} + B_{l,t+1}] \end{aligned}$$

Rearranging terms,

$$\begin{aligned}
\mathbb{E}_t B_{g,t+1} + \frac{(1-q_t)}{q_t} \mathbb{E}_t B_{l,t+1} &= (1-\beta) \frac{\gamma(1+r)}{q_t} \\
&\quad + \beta \mathbb{E}_t \left( [E_{g,t+1} + B_{g,t+1}] + \frac{(1-q_t)}{q_t} [E_{l,t+1} + B_{l,t+1}] \right) \\
\mathbb{E}_t \left[ \rho_{t+1} + \frac{1}{1+r} (1-s) B_{g,t+2} \right] &= (1-\beta) \frac{\gamma(1+r)}{q_t} \\
&\quad + \beta \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1-s) \frac{1}{1+r} [B_{g,t+2} + E_{g,t+2}] \right] \\
&\quad + \beta \frac{(1-q_t)}{q_t} \mathbb{E}_t [E_{l,t+1} + B_{l,t+1}] - \frac{(1-q_t)}{q_t} \mathbb{E}_t B_{l,t+1}
\end{aligned}$$

Since  $B_{l,t} = \beta [E_{l,t} + B_{l,t}]$ ,  $\mathbb{E}_t B_{g,t+1} = (1-\beta) \frac{\gamma(1+r)}{q_t} + \beta \mathbb{E}_t [E_{g,t+1} + B_{g,t+1}]$ , or

$$\mathbb{E}_t [(1-\beta) B_{g,t+1} - \beta E_{g,t+1}] = (1-\beta) \frac{\gamma(1+r)}{q_t}$$

then

$$\mathbb{E}_t \left[ \rho_{t+1} + \frac{1}{1+r} (1-s) B_{g,t+2} \right] = (1-\beta) \frac{\gamma(1+r)}{q_t} + \beta \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1-s) \frac{1}{1+r} [B_{g,t+2} + E_{g,t+2}] \right]$$

and

$$\mathbb{E}_t [\rho_{t+1}] = \beta \mathbb{E}_t [y_{t+1} - w_{t+1}] + (1-\beta) \mathbb{E}_t \left[ \frac{(1+r)\gamma}{q(\theta_t)} - \frac{(1-s)\gamma}{q(\theta_{t+1})} \right]$$

## A.7 Workers and wages

An individual may be unemployed and with a flow value of non-employment  $z < y$ . The unemployed meet job offers at rate  $f(\theta) = \theta q$ . Once employed, workers earn wage  $w$  until separation, which occurs with probability  $s$  per unit of time. The Bellman equations describing each of these stages are

$$\begin{aligned}
U_t &= z + \frac{1}{1+r} \mathbb{E}_t [f(\theta_t) W_{t+1} + (1-f(\theta_t)) U_{t+1}] \\
W_t &= w_t + \frac{1}{1+r} \mathbb{E}_t [(1-s) W_{t+1} + s U_{t+1}]
\end{aligned}$$

We assume that the wage is negotiated between a worker-firm pair, with surplus  $S_{L,t} = F_{g,t} + W_t - U_t$ , and satisfies the sharing rule  $\alpha F_{g,t} = (1-\alpha)(W_t - U_t)$ , where  $F_{g,t} = E_{g,t} + B_{g,t}$  is the joint value of the

firm to the entrepreneur-banker pair and  $\alpha \in (0, 1)$  is the relative Nash bargaining weight of workers.

Applying this sharing rule to the worker-firm surplus, first we have

$$\begin{aligned}
S_{L,t} &= y_t - w_t + (1-s) \frac{1}{1+r} \mathbb{E}_t F_{g,t+1} \\
&\quad w_t + \frac{1}{1+r} \mathbb{E}_t [(1-s)W_{t+1} + sU_{t+1}] - z - \frac{1}{1+r} \mathbb{E}_t [\theta_t q_t W_{t+1} - (1-\theta_t q_t)U_{t+1}] \\
S_{L,t} &= y_t - z + (1-s) \frac{1}{1+r} \mathbb{E}_t [F_{g,t+1} + W_{t+1} - U_{t+1}] - \theta_t q_t \frac{1}{1+r} \mathbb{E}_t [W_{t+1} - U_{t+1}] \\
S_{L,t} &= w_t - z + (1-s) \frac{1}{1+r} \mathbb{E}_t S_{L,t+1} - \alpha \theta_t q_t \frac{1}{1+r} \mathbb{E}_t S_{L,t+1}
\end{aligned}$$

and second, using  $F_{g,t} = (1-\alpha)S_{L,t}$  and  $\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t (1-\alpha)F_{g,t+1}$ ,

$$y_t - w_t + (1-s) \frac{1}{1+r} \mathbb{E}_t F_{g,t+1} = (1-\alpha) \left( y_t - z + (1-s) \frac{1}{1+r} \mathbb{E}_t S_{L,t+1} \right) - \alpha \theta_t \Gamma_t$$

Rearranging terms yield the wage rule under frictional labor and credit markets:

$$w_t = \alpha(y_t + \theta_t \Gamma_t) + (1-\alpha)z$$

## B Deriving the elasticity of market tightness to a productivity shock

### B.1 Canonical framework

We assume that the matching function is Cobb-Douglas, such that  $q(\theta_t) = \chi \theta_t^{-\eta}$ , and define period profit flows as  $\Pi = y - w$ . Taking log-linear deviations of the job creation condition (18) around a stationary steady state, yields a relationship between current and future deviations of labor market tightness and future changes in profit flow,  $\eta \frac{\gamma(1+r)}{q(\theta^P)} \hat{\theta}_t^P = \Pi \mathbb{E}_t \hat{\Pi}_{t+1} + \eta \frac{\gamma(1-s)}{q(\theta^P)} \mathbb{E}_t \hat{\theta}_{t+1}^P$ . Making use of a forward operator,  $x_{t+1} = L^{-1}x_t$ , we have that  $[1 - \frac{1-s}{1+r} \mathbb{E}_t L^{-1}] \hat{\theta}_t^P = \frac{q(\theta^P)\Pi}{\eta\gamma \times (1+r)} \mathbb{E}_t \hat{\Pi}_{t+1}$  and can express current deviations of labor market tightness as

$$\hat{\theta}_t^P = \frac{q(\theta^P)\Pi}{\eta\gamma \times (1+r)} \mathbb{E}_t \sum_{i=0}^{\infty} \left( \frac{1-s}{1+r} \right)^i \hat{\Pi}_{t+1+i}$$



a forward looking expression, discounting future deviations of profits. Using the definition of the wage  $w_t = \alpha (y_t + \gamma \theta_t) + (1 - \alpha)z$  we can further express the deviations of labor market tightness as a discounted sum of the expected future path of productivity:

$$\widehat{\theta}_t^P = \frac{q(\theta^P)(1 - \alpha)}{\eta \gamma \times (1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Psi^i \widehat{y}_{t+1+i}$$

where  $\Psi = \left(\frac{1-s}{1+r}\right) - \frac{\alpha \theta^P q(\theta^P)}{\eta(1+r)}$ . Assuming that productivity follows an AR(1) with persistence parameter  $0 < \rho_y < 1$ , and innovation  $v_t$  as white noise, then

$$\widehat{\theta}_t^P = \frac{q(\theta^P)(1 - \alpha)}{\eta \gamma \times (1 + r)} \sum_{i=0}^{\infty} \Psi^i \rho_y^{i+1} v_t$$

so that  $\widehat{\theta}_t^P = \frac{q(\theta^P)(1-\alpha)}{\eta \gamma \times (1+r)} \left(\frac{1}{1-\Psi \rho_y}\right) \rho_y v_t$ , and the elasticity of labor market tightness to productivity shocks is

$$\frac{\partial \widehat{\theta}_t^P}{\partial v_t} = \frac{(1 - \alpha) q(\theta^P) \rho_y}{\eta \gamma \times (1 + r) - \gamma \times [\eta(1 - s) - \alpha f(\theta)] \rho_y} \quad (26)$$

As a note, if  $\rho_y = 1$ , this expression correspond to the elasticity obtained when comparing steady states, or to a permanent productivity shock, as in Shimer (2005), i.e.  $\frac{(1-\alpha)}{\gamma \left[ \frac{\eta(r+\delta)}{q(\theta)} + \alpha \theta \right]}$ .

## B.2 Frictional credit markets - fixed wage

We derive the elasticity of labor market tightness to productivity shocks in a model with frictional credit markets and a fixed wage. Recall that the job creation condition in this setting is  $\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t \left[ y_{t+1} - \bar{w} + (1-s) \frac{\Gamma_{t+1}}{q_{t+1}} \right]$ , with  $\Gamma_t \equiv \gamma + K \left(1 - \frac{1}{1+r} (1 - q_t)\right)$  and, again,  $q(\theta_t) = \chi \theta_t^{-\eta}$ . Taking the log-linear deviations around a stationary steady state, we have:  $\frac{\gamma(1+r)}{q(\theta)} \left[ \gamma + K \left(\frac{r}{1+r}\right) \right] \widehat{\theta}_t = \mathbb{E}_t \widehat{y}_{t+1} + \frac{\eta(1-s)}{q(\theta)} \left[ \gamma + K \left(\frac{r}{1+r}\right) \right] \mathbb{E}_t \widehat{\theta}_{t+1}$ . Call  $\gamma(r) \equiv \left[ \gamma + K \left(\frac{r}{1+r}\right) \right]$  with  $\gamma(0) = \gamma$ , then making use of the forward operator,  $\left[1 - \frac{1-s}{1+r} \mathbb{E}_t L^{-1}\right] \widehat{\theta}_t = \frac{q(\theta)}{\eta \gamma(r) \times (1+r)} \mathbb{E}_t \widehat{y}_{t+1}$ , and we can express the current deviations of labor market tightness as the discounted expected future path of productivity:

$$\widehat{\theta}_t = \frac{q(\theta)}{\eta \gamma(r) \times (1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \left(\frac{1-s}{1+r}\right)^i \widehat{y}_{t+1+i}$$

If productivity follows the same AR(1) process, then  $\widehat{\theta}_t = \frac{q(\theta)}{\eta \gamma(r) \times (1+r)} \mathbb{E}_t \sum_{i=0}^{\infty} \left(\frac{1-s}{1+r}\right)^i \rho_y^{i+1} v_t$ , or  $\widehat{\theta}_t = \frac{q(\theta)}{\eta \gamma(r) \times (1+r)} \left(\frac{\rho_y}{1 - \frac{1-s}{1+r} \rho_y}\right) v_t$  and the expression for the elasticity of labor market tightness to productivity shocks under frictional

credit markets and a fixed wage is:

$$\frac{\partial \hat{\theta}_t}{\partial v_t} = \frac{q(\theta)\rho_y}{\eta\gamma(r) \times [(1+r) - (1-s)]\rho_y} \quad (27)$$

### B.3 Frictional credit markets -flexible wage

Given the wage rule  $w_t = \alpha [y_t + \Gamma_t \theta_t] + (1-\alpha)z$  derived above, the job creation condition can be written as

$$\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t \left[ (1-\alpha)(y_{t+1} - z) - \alpha \Gamma_{t+1} \theta_{t+1} + (1-s) \frac{\Gamma_{t+1}}{q_{t+1}} \right].$$

The following preparatory steps are useful in deriving the elasticity of labor market tightness to productivity shocks in the model with credit frictions and a flexible wage. First, recall that  $\Gamma_t = \gamma(r) + \frac{K}{1+r}q(\theta_t)$  such that the job creation condition is expressed as a function of labor market tightness and productivity. Then, we take the log-linear deviations of the job creation condition around a stationary steady state:

$$\begin{aligned} \frac{\eta(1+r)}{q(\theta)} \gamma(r) \hat{\theta}_t &= (1-\alpha) \mathbb{E}_t \hat{y}_{t+1} - \alpha \left( \gamma(r)\theta + \frac{K}{1+r}(1-\eta)f(\theta) \right) \mathbb{E}_t \hat{\theta}_{t+1} \\ &\quad + \frac{\eta(1+s)}{q(\theta)} \gamma(r) \mathbb{E}_t \hat{\theta}_{t+1} \\ \hat{\theta}_t &= \frac{(1-\alpha)q(\theta)}{\eta\gamma(r) \times (1+r)} \mathbb{E}_t \hat{y}_{t+1} \\ &\quad + \left[ \frac{(1+s)}{(1+r)} - \frac{\alpha q(\theta)}{\eta\gamma(r) \times (1+r)} \left( \gamma(r)\theta + \frac{K}{1+r}(1-\eta)f(\theta) \right) \right] \mathbb{E}_t \hat{\theta}_{t+1} \end{aligned}$$

Denote  $\Phi \equiv \left[ \frac{(1+s)}{(1+r)} - \frac{\alpha q(\theta)}{\eta\gamma(r) \times (1+r)} \left( \gamma(r)\theta + \frac{K}{1+r}(1-\eta)f(\theta) \right) \right]$  for the moment, we then follow similar steps by first obtaining deviation of labor market tightness as a discounted sum of expected future deviations of productivity:

$$\hat{\theta}_t = \frac{(1-\alpha)q(\theta)}{\eta\gamma(r) \times (1+r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Phi^i \hat{y}_{t+1+i}$$

and, making use of the specification for labor productivity,  $\hat{\theta}_t = \frac{(1-\alpha)q(\theta)}{\eta\gamma(r) \times (1+r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Phi^i \rho_y^{i+1} v_t$ . Finally  $\hat{\theta}_t = \frac{(1-\alpha)q(\theta)}{\eta\gamma(r) \times (1+r)} \left( \frac{\rho_y}{1-\Phi\rho_y} \right) v_t$  and the elasticity of labor market tightness to productivity shocks is:

$$\frac{\partial \hat{\theta}_t}{\partial v_t} = \frac{(1-\alpha)q(\theta)\rho_y}{\eta\gamma(r) \times (1+r) - [\eta\gamma(r) \times (1+s) - \alpha f(\theta) (\gamma(r) + (1-\eta)\tilde{\kappa})] \rho_y}$$

where  $\tilde{\kappa} \equiv K \frac{q(\theta)}{1+r}$ .

# On-line Technical Appendix : Convergence through the trembling hand calibration method

not for publication

## C Convergence with endogenous wages

### C.1 Step 1 - getting the right value for the share of the financial sector $\Sigma = 3.0\%$

We detail the results of the first step, matching the share of credit markets in aggregate value added of  $\Sigma = 3.0\%$ , starting with the value of the objective and the credit market parameters  $\beta$  and  $\varepsilon$  at each iteration. As during the first step when the wage was fixed, matching the size of financial markets occurs by increasing the value of the the bank's bargaining weight  $\beta$ , while the elasticity of the credit matching function  $\varepsilon$  remains relatively constant.

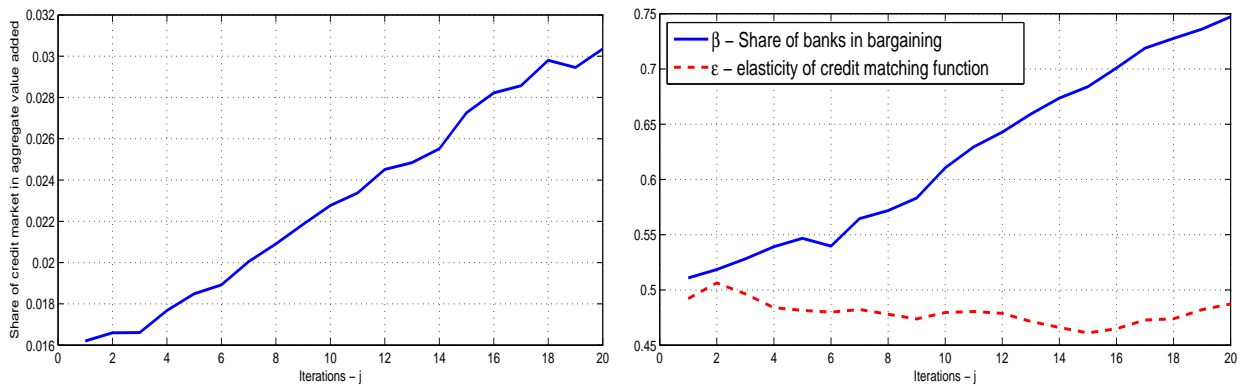


Figure 1: Step 1 - Credit market share of value added and parameters  $\beta$  and  $\varepsilon$  over  $j$  iterations

The next figure reports the progress of the remaining parameters, the first panel plotting the level parameter in the labor matching function which stabilizes around 0.4. In the second panel shows that the remaining credit market parameters remain virtually unchanged from their initial values, nor does the unit cost of job vacancies. The most noticeable change takes place in the relative bargaining weight of the worker in wage negotiations.

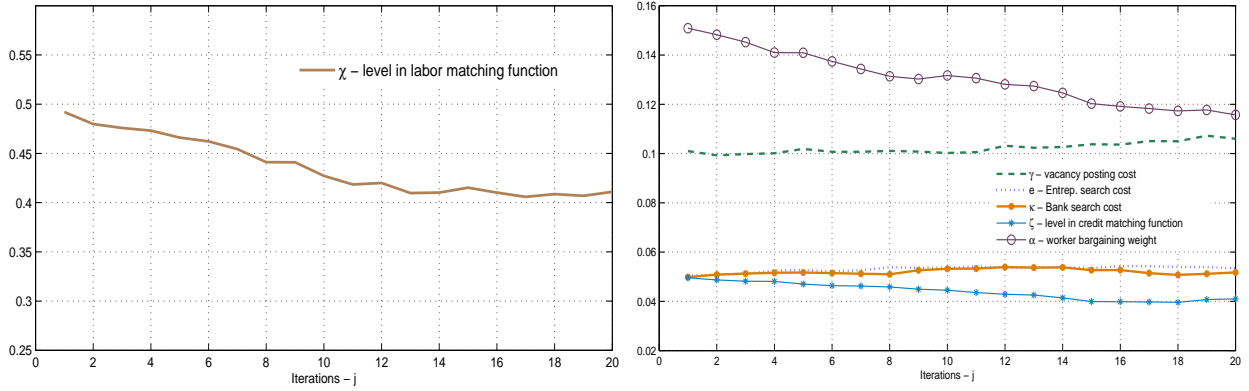


Figure 2: Step 1 - Progression of remaining parameters during process

## C.2 Step 2 - getting the right value for the elasticity of tightness with respect to productivity shocks

The second step begins with the parameter values obtained and the end of Step 1, adding perturbations to the value of non-employment,  $z$ . At each iteration we choose the set of parameters that obtain the highest elasticity of labor market tightness to productivity shocks, while satisfying our set of constraints. The first panel of the next figure presents the progress of the objective at each iteration, and the second the progression of the parameters  $\beta$ ,  $\varepsilon$  and the measure of credit market tightness  $\phi$ .

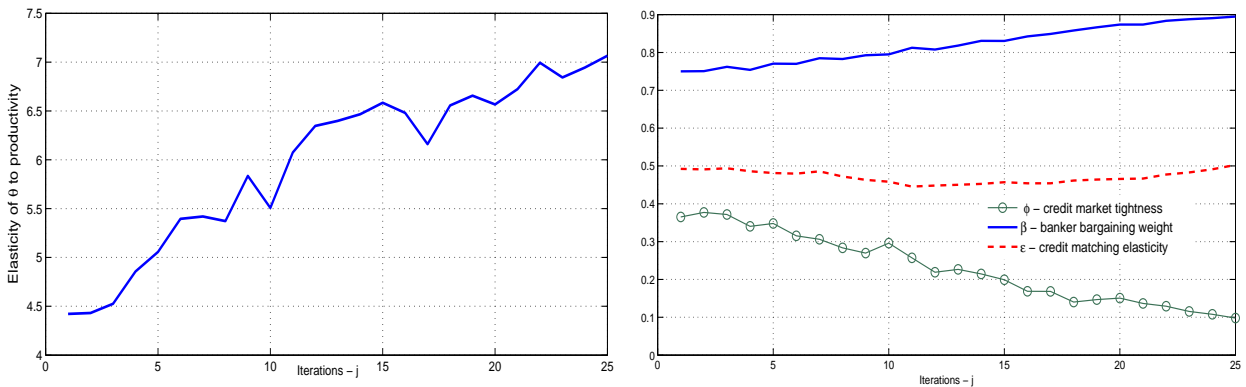


Figure 3: Step 2 - Progression of target and credit market parameters

The next figure plots the progress of the remaining parameter, in particular  $z$ . In this second step the level parameter in the matching function increases in step with the value of non-employment in order to respect the constraint of an equilibrium rate of unemployment below 8%. The unit cost of job vacancies

remains relatively constant. The cost of prospecting on the credit market declines somewhat but remain symmetrical for banker and entrepreneur. Lastly, there is a slight rise in the level parameter of the credit matching function. Finally, we constraint the bargaining weight to  $\alpha \geq 0.1$ .

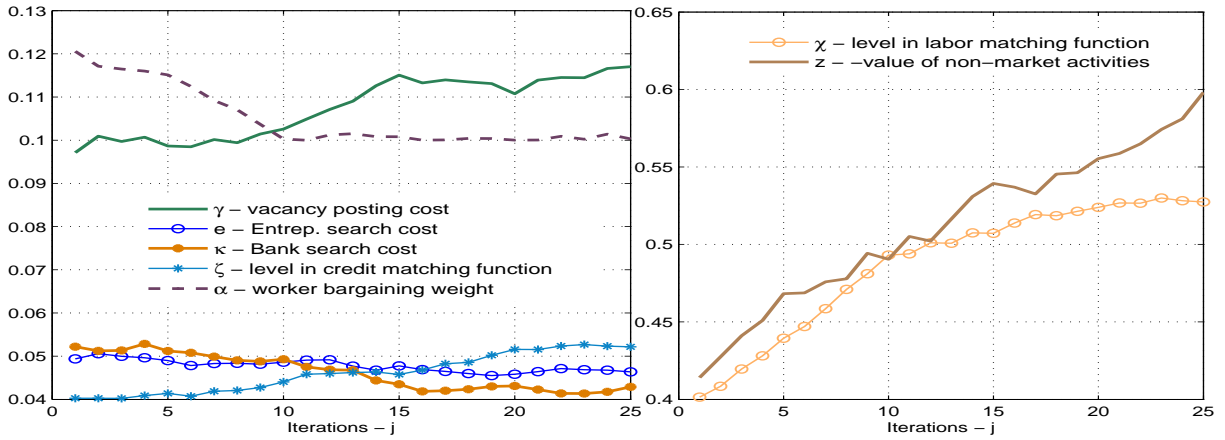


Figure 4: Step 2 - remaining parameters over process