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Welfare Effects of Outsourcing in Duopolistic Markets

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Welfare Effects of Strategic Outsourcing in Duopolistic Markets

Jan König*

Abstract
This paper shows the strategic aspects of international outsourcing in a duopolistic market. Due to different costs of integrated production and outsourcing, the choice of a firm influences the strategy of the competitor via the output price. Therefore, the resulting market constellation depends on the fixed costs and the difference between marginal costs. We show that the three market constellations, both firms produce integrated, both use outsourcing and the firms operate with different strategies are possible. Also the welfare effects of the different outcomes are analysed. If the optimal firms decision is characterized by different strategies, this constellations for given costs is pareto superior to a constellation with equal strategies. On the other hand, for given costs, a resulting constellation of equal strategies can be pareto inferior or pareto superior to a constellation with different strategies.

JEL classification: D43, L13, L22, L23, L24
Keywords: strategic outsourcing, oligopoly, welfare effects

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1. Introduction

Outsourcing, i.e. the acquisition of formerly self-produced inputs from a foreign independent specialized supplier, is often viewed as a possibility to produce in a cheaper way, to cope with increasing competition due to globalization.\(^1\) The important role of outsourcing can be exemplified by the mobile communication and automobile industry. Nokia, as the leading mobile communications provider outsources 20% of its mobile production (Economist, 2002). For the automobile industry the Fraunhofer Institute and Mercer (2004) estimate that by the year 2015 automobile sub-contractors will be handling up to 80% of the development and production, i.e. the production stages with the highest fixed costs, whereas the manufacturers will focus on the post-production stage, e.g. sales, since investments at that stage mean higher profits with less capital input.

As we mentioned, the main reason for outsourcing is the realization of lower costs. However, this can be done by two ways, lower marginal costs or lower fixed costs. While the first advantages can be set off against transaction costs\(^2\), for the second motive, also higher marginal costs are possible. In this paper we assume that outsourcing becomes attractive because of fixed cost saving, but is also associated with higher marginal costs than the domestic production. Thus we see the organizational choice as an investment choice, where outsourcing stands for a long-term externalization of certain production parts. This argument plays an important role in high-investment sectors such as the automobile or aircraft industries, since autonomous input suppliers can divide their fixed costs among various buyers, but an in-house producing company cannot. Since the decision concerning the production mode influences the costs and thus the market price, other participants in an industry are affected. The other firms will react on this effect by adapting the production mode and thus, the organizational choice becomes an instrument of strategic interaction between the participants in an industry.

This paper analyses these strategic interactions between companies in an industry and the resulting production structures and their welfare implications. The starting point is a Cournot-duopoly with simultaneous organization choices. The following questions will be answered: First, how are production choices affected by the costs? Second, what effects do these choices have on welfare? As outsourcing prevents capital intensive fixed costs but also entails higher marginal costs than in-house production, the company is faced with a trade-off between investment costs saving and additional marginal cost payments. We find that relative to the costs, symmetric or asymmetric forms of production organization can emerge. When the marginal cost disadvantage of external procurement is sufficiently low (high), outsourcing (integration) becomes the dominant production structure. A medium level of the marginal costs disadvantage constitutes an asymmetrical constellation. Regarding the second question, we demonstrate via comparative statics,

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\(^{1}\) The increasing tendency towards external procurement in recent years is well documented by e.g. Hummels et al. (1998, 2001) and Yeats (2001).

\(^{2}\) The production choice is therefore made by comparing the in-house production costs with external procurement costs. In this case, outsourcing is explained by the transaction cost thesis (Williamson, 1975 and 1986).
whether the resulting market constellation for given costs is to be considered pareto superior or inferior to other constellations. We find that by decentralized choices, a resulting constellation in different strategies for given costs is always pareto superior to a strategy with equal strategies. On the other hand a resulting constellation of equal strategies can be pareto inferior or superior to a constellation with different strategies. Therefore, profit maximization behaviour by the firms does not lead in any case to the preferred market constellation from the welfare point of view.

The analysis is structured as follows. Section 2 integrates the analysis with the existing literature. In Section 3 we introduce the basic model, in which the conditions for the production organization are derived. The welfare analysis of the different production structures is undertaken in Section 4. Finally, we sum up the results in Section 5.

2. Related Literature

The literature deals with many different strands of international outsourcing because there are various types (vertical or horizontal) and different definitions (make-or-buy or fragmentation/input trade). Despite the growing significance of outsourcing, the strategic aspect, as a reason for outsourcing has been long ignored. Only in more recent analysis this gap has been closed.

To our knowledge Nickerson und Vanden Bergh (1999) are the first who discuss the strategic implications of organizational choices. Within a Cournot-duopoly in the output market, they derive the conditions for the production structure in the different Nash-Cournot-equilibria from the trade-off of fixed cost savings against higher marginal costs in the presence of outsourcing. Using a Hotelling model with differentiating goods and simultaneous production choice procedures, Shy and Stenbacka (2003) also depict the organizational choices. Here, also, the structure is determined by the trade-off between capital intensive fixed costs and the difference in marginal costs. Thus, there are threshold values of the marginal cost difference against the fixed costs, which denote the production organization. Both papers conclude that in the case of relatively high (low) fixed costs and/or low (high) marginal cost differences, the firms will outsource (produce integrated). In the case of a medium fixed cost level and/or a medium marginal cost difference, the market constellation is characterized by different production structures.

In contrast to these papers, Buehler and Haukap (2006) assume in their duopoly model a sequential production decision process. Other than in the above mentioned papers, the external procurement price is not constant, but rises with increased outsourcing. Thus, the choice of the first firm is strategic since it can – via the costs – influence the second

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3 Vertical outsourcing is characterized by the fact that an input producer specializes on intermediate good production. In contrast, horizontal outsourcing describes the fact that firms compete in the output market, but produce also parts for the rival firm. In the case of the make-or-buy choice, transaction costs, as well as non-completed contracts and their effects on a firm’s choice, are being considered, see Grossman and Helpmann (2003) and McLaren (2000). However, regarding outsourcing as fragmentation, its effects with regard to trade models are discussed (see Jones, 2000, Jones and Kierzkowski, 2001 and Kohler, 2004).
participant’s behaviour and the competition. As these companies are also faced with a trade-off between lower fixed costs and higher variable costs when deciding on outsourcing, the three constellations i) both firms use outsourcing, ii) both firms produce integrated or iii) different market structures occur subject to cost relations.4

A direct influence on the competitor can also occur through horizontal outsourcing. Kamien et al. (1989) analyses the case of a Bertrand-duopoly, where both final good producers determine via price bidding, which of them can subcontract the production. Since only the party with the lowest bid can realize outsourcing, there is a direct effect on the output price, the bids and on the price competition in the final goods market. In a Cournot-competition with convex and asymmetrical output producer costs, Spiegel (1993) demonstrates that through horizontal outsourcing, production can be efficiently divided among the companies. Outsourcing increases the subcontractor’s costs, who thus offer less output, whereas the other company has fewer costs and offers a higher amount of output. However, the effect on the total output and the consumer price is ambiguous, so that when comparing the positive increase in efficiency with the effect on the consumer surplus, a clear welfare statement can only be derived in the case of a rising total output. Using a duopoly with horizontal outsourcing, Arya et al. (2008) compare the equilibria in Bertrand- and Cournot-competition. Since the input producer can set a high price, the outsourcing firm is met with higher costs and loses some of its aggressiveness on the Bertrand-market, which may result in a higher output price and consequently, less welfare than in the Cournot-competition.

In addition, the special case of bi-sourcing (make-and-buy) and its strategic effects is implemented in the literature. This strand (see Du et al., 2005, 2006 as well as Beladi and Mukherjee, 2008) shows that the strategic effects of this type of production organization reduces the price for external procurement and minimize the hold-up problem between input supply and demand.5

In this paper we discuss the strategic effects of vertical outsourcing of a duopoly in a Cournot-competition, relating to Nickerson and Vanden Bergh (1999). We will demonstrate the point at which a market constellation is realized. Furthermore, through comparative statics, we compare the welfare effects in the different market constellations.

---

4 The mentioned papers look on strategic effects of integration or separation of the input production for a final goods producer. However, this question can also be considered as a decision for the input producer. This forward integration looks on the independence of an input firm. The strategic effects of the integration-separation decision of an input producer in oligopolistic markets is analysed by, e.g. Gal-Or (1990) and Jansen (2003), which are different in the assumption about the competition in the final goods market and in the results they obtain. Gal-Or (1990) assumes a Bertrand-competition in the final market and found that all or no input producer is independent. Thus, there is, from the final goods producer's point of view, no outsourcing or only outsourcing. Jansen (2003), however, assumes a Cournot-competition in the final goods market and showed that integrated and separated input producers exist at the same time.

5 Oladi et al. (2007) analyse the effects of bi-sourcing in an international context with a rising production volume in each country. Thus, welfare can be positively influenced through trade liberalization, which is aimed at reducing outsourcing costs. Chen et al. (2004) describe the effects that trade liberalization has on horizontal outsourcing, i.e. a price increase for input and output goods.
3. Basic Model

Two identical firms – $A$ and $B$ – compete on the national market. Their competition equals a Cournot-duopoly in homogeneous goods, where the market demand is described by $p = a - b \cdot (y_A + y_B)$, where $y_i$ with $i = A; B$ characterizes the output of one of the players. The following model can be viewed as a simple description of the decision problem in the aircraft industry. Starting from a point up to which the component production is integrated, we model the organization decision for a new product with new components that cannot be manufactured on the existing production line.

In both companies, the production of the output good involves an input component, where one unit of the input good produces one unit of the output good. Due to market integration, the companies can choose between in-house production or outsourcing of the input component to a specialized external supplier. The price for the external procurement of one unit of input is fixed and exogenously given by $q$. Alternatively, this component may be produced in-house and requires an investment $F$, which is interpreted as set-up costs. The marginal costs $m$ of the integrated production are constant for each unit of the produced input. Therefore, outsourcing is beneficial, as investment costs $F$ can be saved. To avoid external procurement being the dominant strategy, $q > m$ must be hold. Thus, if a domestic company chooses outsourcing, it pays a bonus to the external supplier for the procurement and bearing of fixed costs. Consequently, the total costs of a company $i = A; B$ are

$$TC_i(y_i) = \begin{cases} m \cdot y_i + F & \text{in-house} \\ q \cdot y_i & \text{outsourcing}. \end{cases}$$

(1)

The model structure is a two-stage decision problem:

(I) Each company $i$ ($i = A; B$) chooses external procurement respectively in-house production, given the competitor's choice.6

(II) Given its own and the competitor's production structure choice, the company chooses its profit maximizing output.

Table 1: production scenarios

<table>
<thead>
<tr>
<th>firm B</th>
<th>outsourcing</th>
<th>in-house</th>
</tr>
</thead>
<tbody>
<tr>
<td>outsourcing</td>
<td>scenario 1</td>
<td>scenario 2</td>
</tr>
<tr>
<td>in-house</td>
<td>scenario 3</td>
<td>scenario 4</td>
</tr>
</tbody>
</table>

6 Thus, Nash-behaviour is assumed regarding the outsourcing decision.
By modelling the company’s decision, the problem is solved via backwards induction. Here, the individual production structure is illustrated by the superscript indices \textit{in} for in-house, \textit{out} for outsourcing and \textit{in/out} for different strategies.

3.1 Stage II: Output Decision

Given the output decision and the organizational choice of the competitor, from a company’s profit maximization

\[
\max_{y_i} \Pi_i^{\text{in}} = \left[ p(y_i + y_j) - m \right] \cdot y_i - F \quad \text{or} \\
\max_{y_i} \Pi_i^{\text{out}} = \left[ p(y_i + y_j) - q \right] \cdot y_i
\]

with \( i = A; B \) and \( i \neq j \), we derive for each scenario the individual reaction functions at the second stage.

Scenario 1: both companies choose external input procurement

\[
y_i^{\text{out}} = \frac{1}{2b} \left[ a - q - by_j^{\text{out}} \right],
\]

Scenario 4: both companies choose in-house input production

\[
y_i^{\text{in}} = \frac{1}{2b} \left[ a - m - by_j^{\text{in}} \right],
\]

Scenario 2 and 3: companies choose different strategies

\[
y_i^{\text{in}} = \frac{1}{2b} \left[ a - m - by_j^{\text{out}} \right] \\
y_j^{\text{out}} = \frac{1}{2b} \left[ a - q - by_i^{\text{in}} \right],
\]

with \( i, j = A; B \) and \( i \neq j \). Using the reaction functions, for each case the individual output and the total output can be determined.

Scenario 1: both companies choose external input procurement

\[
y_i^{\text{out/out}} = \frac{1}{3b} \left[ a - q \right] \\
Y^{\text{out/out}} = \frac{2}{3b} \left[ a - q \right],
\]
Scenario 4: both companies choose in-house input production

\[ y_{in/in}^{in} = \frac{1}{3b} [a - m] \]
\[ Y_{in/in}^{in} = \frac{2}{3b} [a - m], \]  

(4)

Scenario 2 and 3: companies choose different strategies

\[ y_{in/out}^{in} = \frac{1}{3b} [a + q - 2m] \]
\[ y_{out/in}^{in} = \frac{1}{3b} [a + m - 2q] \]
\[ Y_{in/out}^{in} = \frac{2}{3b} \left[ a - \frac{q + m}{2} \right]. \]

(5)

with \( i, j = A; B \) and \( i \neq j \), while the subscript in (5) characterizes the production mode of the specific firm.

To make sure that both participants stay in the market, negative output levels must be avoided in each market constellation. Thus, we have to calculate the requirements for positive output levels. Since the marginal outsourcing costs are higher than the domestic marginal costs, i.e. \( q > m \), the conditions for realizing positive total and individual output for identical production strategies are \( a > q \) and \( a > m \). When these requirements are met, the in-house producing participant will in the case of different strategies, also offer a positive output level. The outsourcing firm will offer a positive output if \( a - q > q - m \).

Inserting the total output into the market demand, gives the market price for each situation as illustrated in the following table.

Table 2: output prices

<table>
<thead>
<tr>
<th>firm A</th>
<th>outsourcing</th>
<th>in-house</th>
</tr>
</thead>
<tbody>
<tr>
<td>outsourcing</td>
<td>( p_{out/out}^{out} = \frac{1}{3} [a + 2q] )</td>
<td>( p_{in/out}^{in} = \frac{1}{3} [a + (q + m)] )</td>
</tr>
<tr>
<td>in-house</td>
<td>( p_{in/out}^{in} = \frac{1}{3} [a + (q + m)] )</td>
<td>( p_{in/in}^{in} = \frac{1}{3} [a + 2m] )</td>
</tr>
</tbody>
</table>

Table 2 shows that the resulting market price is positive in each constellation. Since, due to the linearity of demand, the parameter \( a \) represents the maximum willingness-to-pay, in what follows that \( 0 < p < a \) must apply. Comparing the different price levels with this requirement, it becomes clear that the market price under bilateral outsourcing always
stays below the maximum willingness-to-pay if \( a > q \), and thereby, positive output for both participants under bilateral outsourcing is guaranteed. Therefore, in that market constellation, both firms will realize positive output and the resulting output price will stay below the maximum price the consumer is willing to pay.

The same applies to a constellation with bilateral in-house production. The requirement for positive output, \( a > m \), is met, since \( q > m \) and \( a > q \) and thus \( P^{in/in} < a \). Consequently, in this scenario, both players will offer positive outputs and the output price will stay below the maximum price the consumer is willing to pay. In the case of different production structures, \( P^{in/out} < a \) applies, given that \( a > (q + m)/2 \) holds. As the conditions \( a > q \) and \( q > m \) are defined, this requirement is always met so that also under different production strategies, the price stays below the maximum price the consumer is prepared to pay.

**Assumption 1: non-negative output**

\[ \text{We assume that } a > q > m \text{ and } a - q > m. \]

Comparing the price levels in the different scenarios shows that in the presence of bilateral outsourcing the price is higher than the price in the case of bilateral in-house input production. The reason is that the external procurement price is made up of the domestic marginal costs plus a positive bonus payment. If different production structures are chosen, a medium price level is realized, since the price level is subject to the average marginal production costs. Thus, we have \( P^{out/out} > P^{in/out} > P^{in/in} \).

In the same way, the total output and the individual company’s output can be compared for the different scenarios. In the case of bilateral outsourcing, due to the higher output price and the decreasing market demand, the total output is smaller than when both companies produce in-house. When both companies use the same strategy, the firms share total demand in equal parts and thus also the individual output is lower in case of bilateral outsourcing compared to the case of bilateral in-house production. Under different production structures, a medium price level is achieved, which also entails a medium total output level. However, other than in scenario 1 and 4, the individual market shares differ due to the different marginal costs incurred by the organization choice. The market share \( s \) of the outsourcing company is

\[ s^{in/out}_{out} = \frac{Y^{in/out}_{out}}{Y^{in/out}} = \frac{(a-q)-(q-m)}{(a-q)+(a-m)} \]

and the share of the integrated producing firm is

\[ s^{in/out}_{in} = \frac{Y^{in/out}_{in}}{Y^{in/out}} = \frac{(a-m)+(q-m)}{(a-q)+(a-m)}. \]

Thus, the participant who uses in-house production will have a larger market share since he benefits from the marginal cost advantage and is able to offer a higher output at a given market price. Since the market is divided up between the firms, in the case of different production strategies it follows that \( s^{in/out}_{out} < \frac{1}{2} < s^{in/out}_{in} \). When the external procurement price rises, the marginal cost difference increases in favour of the in-house producing company, which leads to an
increase in its output and market share, while the output and market share of the outsourcing company decreases, i.e. \( \hat{c}_{s_{in}}^{in/ou} / \hat{c}_q > 0 \) and \( \hat{c}_{s_{out}}^{in/ou} / \hat{c}_q < 0 \).

**Proposition 1:**

a) For the prices, \( p^{out/out} > p^{in/out} > p^{in/in} \) applies and resulting in \( Y^{in/in} > Y^{in/out} > Y^{out/out} \) for the total output.

b) For the individual output, we have \( y_{in}^{in/out} > y_{in}^{in/in} > y_{out}^{out/out} > y_{out}^{in/out} \).

### 3.2 Stage I: Outsourcing Decision

The strategy, which is chosen by a company depends on the profit to be gained and on the difference between the fixed cost savings and the additional marginal costs through the bonus payment to the external supplier. The individual profits in the various scenarios are shown in Table 3.

**Table 3: profits**

<table>
<thead>
<tr>
<th></th>
<th>firm B</th>
<th>outsourcing</th>
<th>in-house</th>
</tr>
</thead>
<tbody>
<tr>
<td>outsourcing</td>
<td>[ \Pi^{out/out}_{in} = \frac{1}{9b} (a-q)^2 ]</td>
<td>[ \Pi^{in/in}_{in} = \frac{1}{9b} (a-q-2m)^2 - F ]</td>
<td>[ \Pi^{in/out}_{out} = \frac{1}{9b} (a+m-2q)^2 ]</td>
</tr>
<tr>
<td>in-house</td>
<td>[ \Pi^{in/out}_{in} = \frac{1}{9b} (a+q-2m)^2 - F ]</td>
<td>[ \Pi^{in/out}_{out} = \frac{1}{9b} (a-m)^2 - F ]</td>
<td></td>
</tr>
</tbody>
</table>

Comparing the profits of the different scenarios allows us to derive equilibrium conditions, which indicate what market constellations at which point arrive at a Nash-equilibrium. The relations derived indicate the relation between fixed costs \( F \) and marginal cost disadvantage \( q - m \), i.e. the advantages and disadvantages of an external procurement subject to the demand \( p \).

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7 For details see Appendix.
Lemma 1:

a) If the fixed costs are high, respectively, the marginal cost disadvantage is small, so that \(\frac{4}{9b}(a-m)(q-m)<F\), in a Nash-equilibrium, both companies will perform via outsourcing.

b) If the fixed costs are low, respectively, the marginal cost disadvantage is high, so that \(\frac{4}{9b}(a-q)(q-m)>F\)\), in a Nash-equilibrium, both companies will produce in an integrated production mode.

c) If the fixed costs, respectively, the marginal cost disadvantage is of medium value, so that \(\frac{4}{9b}(a-m)(q-m)>F>\frac{4}{9b}(a-q)(q-m)\), then we have a Nash-equilibrium with an asymmetrical production structure in which one company has an integrated input production while the other company outsource the input production.

Solving the first paragraph of Lemma 1, we find that both firms use outsourcing if

\[ q < q_{\text{crit, out}}^{\text{out}} = \frac{9b}{4} \frac{F}{a-m} + m. \]  

(6)

The critical value for bilateral in-house production is obtained by the solution of the second paragraph of Lemma 1. Here, due to the quadratic structure, we obtain two solutions

\[ q_{\text{crit, in}} = \frac{a+m}{2} \pm \sqrt{\left(\frac{a-m}{4}\right)^2 - \frac{9b}{4} F}. \]

Both critical values have to fulfilled Assumption 1, which means that they have to lie in the interval \((m;a)\) and have to be smaller than \(\frac{a+m}{2}\). Comparing our critical values with Assumption 1, we see that the bilateral integration can be observed for

\[ q_{\text{crit, in}} > q > q_{\text{crit, in}}^{\text{in}}. \]

(7)

Thus, a constellation with different production modes, which is characterized by

\[ q_{\text{crit, in}}^{\text{in}} < q < q_{\text{crit, in}}. \]

(8)

So far, we have ensured that the individually produced output is positive and that the final market price stays below the maximum price the consumer is willing to pay. In addition,
however, these two requirements also have to ensure that the firm’s profit is strictly positive, as this is the criteria for staying in the market.

Considering Table 3 and Assumption 1, we can see that in an equilibrium with bilateral outsourcing both participants make a positive profit for \( a > q \). In the case of different strategies, Assumption 1, \( (a - q) > (q - m) \), is sufficient to provide the outsourcing participant with positive profits. For the in-house producing company \( [(a - m) + (q - m)]^2 > 9bF \) must apply. In a market constellation where both companies produce integrated, in addition to Assumption 1, \( (a - m)^2 > 9bF \) must hold. By comparing these restrictions, it can be seen that since \( q > m \) applies, \( (a - m)^2 > 9bF \) suffices.

\textit{Assumption 2: non-negative profits}

\[ \frac{(a - m)^2}{9b} \geq F \text{ applies.} \]

Assumptions 1 and 2 ensure that the three market constellations i) both use outsourcing, ii) both produce integrated or iii) firms use different production structures occur.

For given domestic marginal costs, the resulting market constellation depending on the relation between domestic fixed costs and outsourcing price can be graphically illustrated. Figure 1 illustrates the possible constellations under Assumptions 1 and 2 as well as under Lemma 1. Also the zero profits conditions, which determine the boundary conditions for the validity of the model, are incorporated.

\textit{Figure 1: market constellations}

Here, since the domestic marginal costs are given, only the relation between the outsourcing price and fixed costs determines the constellation. By using Lemma 1, the fixed cost/outsourcing cost combinations characterizing the different equilibria can be obtained.

A Nash-equilibrium with bilateral outsourcing, i.e. \( \text{(out; out)} \), occurs when, given the outsourcing choice by firm \( B \), firm \( A \) also chooses external procurement. Therefore
\( \Pi_{\text{out}} > \Pi_{\text{in}} \) has to apply from what \( \frac{4}{9b} (a - m) (q - m) < F \) follows.\(^8\) For the case where \( \frac{(a - m)^2}{9b} = F \), which is illustrated in Figure 1, the outsourcing price \( q = (a + 3m)/4 \) can be calculated, where there is an indifference for firm A between choosing outsourcing or in-house production, given the competitor’s outsourcing choice.

For given fixed costs of \( F = \frac{(a - m)^2}{9b} \), due to symmetry, for all \( q < (a + 3m)/4 \), outsourcing becomes more profitable than in-house production for both participants. The reason is that the bonus payment to the external supplier is rather small, which means that the external procurement price is only slightly higher than the marginal costs of in-house production. The fixed cost savings are in that case more significant than the outsourcing disadvantage. Considering Assumption 2, in Figure 1 the light shaded triangle above the \( \Pi_{\text{out}} = \Pi_{\text{in}} \) -line and below \( \frac{(a - m)^2}{9b} = F \) depicts all fixed costs and outsourcing price combinations of a Nash-equilibrium in which both firms choose outsourcing.

If the external supplier bonus (and thus the difference between in-house marginal and outsourcing price) is sufficiently high so that the fixed cost savings achieved through outsourcing cannot compensate the higher marginal costs, both participants will choose in-house production. A Nash-equilibrium with bilateral in-house production, i.e. \((\text{in}; \text{in})\), must fulfill \( \Pi_{\text{in}} > \Pi_{\text{in}} \) for firm A, given that firm B chooses the integrated production. This requirement leads to the condition \( \frac{4}{9b} (a - q) (q - m) > F \). The graphic illustrates by the \( \Pi_{\text{in}} = \Pi_{\text{in}} \) -curve the indifference between in-house and outsourcing choice for firm A, given the in-house choice by firm B. However, outsourcing can only be an option as long as firm A realises non-negative profits. Consequently, the \( \Pi_{\text{out}} = \Pi_{\text{in}} \) -curve is only defined up to \( \Pi_{\text{out}} = 0 \), which corresponds with an outsourcing price of \( q = (a + m)/2 \). In the case of higher external procurement prices, firm A will definitively choose in-house production.\(^9\) Since the \( \Pi_{\text{out}} = \Pi_{\text{in}} \) -curve stands for all fixed cost/outsourcing price combinations, where firm A, for the given in-house choice by firm B, is indifferent between integrated production and outsourcing. Thus, the area below this curve illustrates all combinations where firm A (and thus both participants) produce in-house. The reason is that, based on the combinations on the \( \Pi_{\text{out}} = \Pi_{\text{in}} \) -curve, at each outsourcing price \( q \in (m; (a + m)/2) \)

\(^8\) For symmetry, the same calculus applies for firm B, given the outsourcing choice by firm A. Thus, the derived conditions apply to both participants.

\(^9\) Since the profits are identical on the \( \Pi_{\text{out}} = \Pi_{\text{in}} \) -curve, \( \Pi_{\text{in}} = 0 \) also has to apply at \( \Pi_{\text{out}} = 0 \). However, this is only the case if \( \frac{(a - m)^2}{9b} = F \) and \( q = (a + m)/2 \). Consequently, the \( \Pi_{\text{out}} = \Pi_{\text{in}} \) -curve ends on the intersection of the zero profit conditions \( \Pi_{\text{out}} = 0 \) and \( \Pi_{\text{in}} = 0 \), with a maximum at \( q = (a + m)/2 \).
and given the competitor’s in-house choice, firm $A$ will choose the in-house production if fixed costs are decreasing, as this promises higher profits than outsourcing, where the lower fixed costs doesn’t affect the profit.

For an external procurement price $q \in ((a + m)/2 ; a)$, firm $A$ does not choose outsourcing, given the competitor’s in-house choice, since here a loss is realized. This results, since firm $A$ chooses the external procurement only if $q \leq (a + m)/2$ respectively

$$\Pi_{\text{out}}^{\text{in}} \geq 0.$$  

Thus, for $F \geq \frac{(a - m)^2}{9b}$ and $q \in ((a + m)/2 ; a)$, both participants definitively produce in an integrated way. Therefore, the grey area below the $\Pi_{\text{out}}^{\text{in}} / \Pi_{\text{out}}^{\text{in}}$ -curve and the $\Pi_{\text{in}}^{\text{in}} = 0$ -line indicates all combinations of fixed costs and outsourcing price for which a Nash-equilibrium with bilateral integrated production exists.

If $\frac{4}{9b} (a - m)(q - m) > F > \frac{4}{9b} (a - q)(q - m)$ holds, an equilibrium in different strategies, i.e. (in / out) or (out / in), exists. To explain this fact, we can use the former mentioned curves of equal profits as the basis. Since we know, that for $q > (a + m)/2$ both firms choose the integrated production, a constellation with different strategies can only occur in the interval $q \in (m; (a + m)/2)$. If the fixed costs for any external procurement price in that interval is so high that a combination of both lies above the $\Pi_{\text{out}}^{\text{in}} = \Pi_{\text{out}}^{\text{in}}$ -curve, an equilibrium in differing strategies exists. Here, firm $A$ prefers outsourcing to in-house production, given firm $B$’s in-house choice, due to sufficiently high fixed cost savings and thus $\Pi_{\text{out}}^{\text{in}} > \Pi_{\text{in}}^{\text{in}}$ holds. The same occurs when using the $\Pi_{\text{in}}^{\text{out}} = \Pi_{\text{out}}^{\text{out}}$ -curve as a basis, where firm $B$’s decision to outsource the input production is given. Firm $A$ prefers in-house, if the fixed costs for any external procurement price are sufficiently low. Therefore, all fixed cost/external procurement price combinations with an equilibrium in differing strategies are shown by the white area between the $\Pi_{\text{out}}^{\text{in}} = \Pi_{\text{out}}^{\text{in}}$ -curve and the $\Pi_{\text{out}}^{\text{in}} = \Pi_{\text{in}}^{\text{in}}$ -curve, limited by $F = \frac{(a - m)^2}{9b}$.

4. Production Choice and Welfare

We know the effects of the production structure on firm’s profit, i.e. on the supply side. However, the production choice also affects consumers via the price and the resulting output. To evaluate the effects on all participants in an economy, an indicator must be found that includes supply and demand. In this context, the welfare criterion is used.

Referring to the previous analysis, the question we focus in this section is: How does the organizational choice affect the economy’s welfare, i.e. both sides of the market? To answer this, we will compare the welfare under the different market constellations to derive whether one production structure will be pareto superior or inferior to another if firms, for given costs, behave rational, i.e. are profit orientated.
4.1 Welfare Indicator

The welfare indicator used here consists of the sum of the rent, i.e. the producer profits and the consumer rent, where \( W \) indicates the welfare and \( CS \) the consumer surplus. For a better differentiation, again, the superscripts indicate the different constellation of the used production modes: i.e. \( in / in \), \( out / out \) or \( in / out \) standing for bilateral in-house production, bilateral outsourcing or different production structures.

Using the known results for the price and output in each scenario, as well as the market demand, we have

Scenario 1: both companies use outsourcing

\[
CS_{out / out}^{out / out} = \frac{2}{9b} [a-q]^2 \\
W_{out / out}^{out / out} = \frac{4}{9b} [a-q]^2.
\] (9)

Scenario 4: both companies produce via in-house

\[
CS_{in / in}^{in / in} = \frac{2}{9b} [a-m]^2 \\
W_{in / in}^{in / in} = \frac{4}{9b} [a-m]^2 - 2F,
\] (10)

Scenario 2 and 3: the companies use different strategies

\[
CS_{in / out}^{in / out} = \frac{2}{9b} \left[ a - \frac{q+m}{2} \right]^2 \\
W_{in / out}^{in / out} = \frac{1}{9b} \left[ 4(a-m)(a-q) + \frac{11}{2}(q-m)^2 \right] - F.
\] (11)

Knowing the welfare levels for all constellations, we can compare them to determine whether for given costs another market constellation, other the existing one, is pareto superior and preferable, from the welfare theory point of view.

4.2 Welfare Comparison

Similarly, as in Figure 1, using equations (9) to (11), all fixed cost and outsourcing price combinations can be illustrated, which achieve equal welfare levels in the different constellations. By comparing the welfare levels, we determine the three curves \( W_{in / in}^{out / out} = W_{out / out}^{out / out} \) and \( W_{in / out}^{out / out} = W_{in / out}^{in / out} \), but also the threshold values at which changing the existing choice increases welfare. These values have to meet Assumptions 1 and 2 and thus have to lie in the interval \((m; a)\). A comparison of these
threshold values with the critical values (6) to (8) shows whether the resulting equilibrium is pareto superior or pareto inferior to other market constellations.

**Bilateral Outsourcing Characterizes the Market Constellation**

According to (6), \( q < q_{\text{crit}}^{\text{out/out}} = \frac{9b}{4} \frac{F}{a-m} + m \) defines, for given domestic marginal cost, the upper bound of the external procurement price, in relation to the fixed costs, at which a market constellation with bilateral outsourcing occurs.

Using (9) and (10) we obtain the outsourcing price, which yields an equal welfare level in a constellation with bilateral outsourcing and bilateral integrated production. Solving \( W_{\text{in/in}} = W_{\text{out/out}} \) we get as the threshold values

\[
\tilde{q}_1 = a - \sqrt{(a-m)^2 - \frac{9b}{2} F}, \\
\tilde{q}_2 = a + \sqrt{(a-m)^2 - \frac{9b}{2} F}.
\]

Starting from a point on this curve, we can deduce, that for given domestic marginal and fixed cost a lower outsourcing price leads to \( W_{\text{in/in}} < W_{\text{out/out}} \). This holds, since the outcome in a constellation with bilateral integrated production is unaffected, but in case of bilateral outsourcing lower costs of external procurement increase profits and consumer surplus due to lower market price and higher output and therefore the associated welfare level increases. Thus we can conclude that \( W_{\text{out/out}} > W_{\text{in/in}} \) for \( q < \tilde{q}_1 \) respectively \( q > \tilde{q}_2 \) and \( W_{\text{in/in}} > W_{\text{out/out}} \) for \( \tilde{q}_1 < q < \tilde{q}_2 \).

To derive the binding constraint, these values have to be compared to the Assumptions 1 and 2. It is obvious that when Assumption 2 is met, only \( \tilde{q}_1 \) lies in the interval \((m;a)\) and has to be included in our analysis. Thus, a constellation with bilateral integrated production leads to higher welfare than a constellation with bilateral outsourcing if

\[
q > \tilde{q}_1 = a - \sqrt{(a-m)^2 - \frac{9b}{2} F}. 
\tag{12}
\]

To answer the question whether abandoning the optimally bilateral outsourcing in favour of bilateral in-house production leads to higher welfare, the threshold value \( \tilde{q}_1 \) must be compared to the marginal value \( q_{\text{crit}}^{\text{out/out}} \). Under Assumption 2, the comparison of equations (6) and (12) proves that

\[
q_{\text{crit}}^{\text{out/out}} = \frac{9b}{4} \frac{F}{a-m} + m < a - \sqrt{(a-m)^2 - \frac{9b}{2} F} = \tilde{q}_1. 
\]
From this relationship follows, that in a Nash-equilibrium with bilateral outsourcing, i.e. \( q < q_{\text{crit}}^{\text{out/out}} \), welfare cannot be increased when both participants switch from outsourcing to in-house production. This result is intuitively, since in the case of switching the production mode, both firms act against their best strategies and thus their profits decrease as the fixed costs are not compensated by lower marginal costs. Of course, there is an increase in output and consumer surplus due to the lower marginal costs and resulting lower market price, however, due to the relative small difference between outsourcing costs and marginal costs of integrated production, this positive effect is not strong enough to compensate the firms’ losses. Thus, changing the production structure from bilateral outsourcing to bilateral in-house production leads to lower welfare.

The constellations of bilateral outsourcing and different strategies can be compared in a similar way. Using the equations (9) and (11) we can calculate the threshold value for which different strategies becomes advantageous from the welfare theory point of view. The threshold values for \( W_{\text{in/out}} = W_{\text{out/out}} \) are

\[
\hat{q}_1 = -\frac{4a - 7m}{3} + \frac{16}{9} \left( a - m \right)^2 + 6bF
\]

\[
\hat{q}_2 = -\frac{4a - 7m}{3} - \frac{16}{9} \left( a - m \right)^2 + 6bF.
\]

As one can see, \( \hat{q}_2 < 0 < m \) applies and thus Assumption 1 is not fulfilled, which means that this threshold value can be neglected. Therefore, a constellation with different strategies leads to a higher welfare than a situation with bilateral outsourcing, if

\[ q > \hat{q}_1 = -\frac{4a - 7m}{3} + \frac{16}{9} \left( a - m \right)^2 + 6bF. \tag{13} \]

To answer, if a change from the optimal choice of bilateral outsourcing towards a constellation with different strategies increases the welfare, we have to compare the equations (6) and (13), which yield

\[ q_{\text{crit}}^{\text{out/out}} = \frac{9b}{4} \frac{F}{(a - m)} + m > -\frac{4a - 7m}{3} + \frac{16}{9} \left( a - m \right)^2 + 6bF = \hat{q}_1, \]

and thus for \( q \in (\hat{q}_1, q_{\text{crit}}^{\text{out/out}}) \) a welfare increasing change of production strategies is possible if, starting from a constellation with bilateral outsourcing, one firm would switch to an integrated production. The marginal costs of the firm that has changed its strategy fall, thereby reducing the average marginal costs and the market price. These effects will be accompanied by a rise in the total output. Since lower market price and higher output favour the consumer, the consumer surplus increases. Here, too however, both companies suffer profit losses: the company that continued use of outsourcing as the output price falls.
at constant marginal costs so that its market share falls below 50%, and the company with integrated production, as it acts against its best strategy for a given choice of the competitor. To evaluate, if it is possible, that the firm’s losses are offset by the gain of the consumer, we have to distinguish two cases. In the interval \( \hat{q}_1 < q < q_{\text{crit}}^{\text{out}}/\text{out} \) the marginal costs difference is sufficiently high, so that the positive effect on the consumer surplus caused by a relatively large price reduction prevails and the welfare will be higher with different production structures. If the outsourcing price is sufficiently low and lies in the interval \((m; \hat{q}_1)\), due to the relative small marginal cost difference, the negative effect on profits prevails and the welfare level in an asymmetrical production structure is smaller.

We can sum up in:

**Proposition 2:**

If the market constellation is defined by bilateral outsourcing,

a) for given costs, this constellation is pareto superior to a constellation with bilateral in-house production,

b) the welfare level can be increased through an asymmetrical production organization, if the external procurement price is sufficiently high, \( \hat{q}_1 < q < q_{\text{crit}}^{\text{out}}/\text{out} \).

Figure 2 shows this for the special case \( F = \frac{(a-m)^2}{9b} \). In this situation, both participants choose external procurement, if \( q \leq q_{\text{crit}}^{\text{out}}/\text{out} = \frac{a + 3m}{4} \).

All combinations of fixed costs and external procurement price, which lead to the same welfare level in a constellation with bilateral outsourcing and different production structures, are illustrated by the \( W_{\text{out}}^{\text{out}}/\text{out} = W_{\text{in}}^{\text{in}}/\text{out} \) -curve. For any combination below this curve, \( W_{\text{out}}^{\text{out}}/\text{out} < W_{\text{in}}^{\text{in}}/\text{out} \) applies, and above the curve \( W_{\text{out}}^{\text{out}}/\text{out} > W_{\text{in}}^{\text{in}}/\text{out} \). This is true, since starting from any combination on this curve for every outsourcing price \( q \leq \frac{a + 3m}{4} \) lower fixed costs implies due to a higher profit of the integrated producing firm an increasing welfare in different production strategies, while the welfare level in a constellation with bilateral outsourcing is unchanged. The analysis shows that a constellation with bilateral outsourcing is pareto inferior to a constellation with different production strategies, i.e. \( W_{\text{out}}/\text{out} < W_{\text{in}}^{\text{in}}/\text{out} \), if \( \hat{q}_1 < q < q_{\text{crit}}^{\text{out}}/\text{out} \) holds.\(^{1}\) This can be seen in Figure 2, since, at the assumed fixed costs \( F = \frac{(a-m)^2}{9b} \) for \( q \in [\hat{q}_1; (a + 3m/4)] \), all combinations of the outsourcing price and the assumed fixed costs lie below the \( W_{\text{out}}^{\text{out}}/\text{out} = W_{\text{in}}^{\text{in}}/\text{out} \) -curve. Consequently, the equilibrium of bilateral outsourcing in this

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\(^{1}\) Given the assumption for this assumed fixed costs, we obtain for the critical welfare values \( \hat{q}_1 = \frac{a - m}{3} (\sqrt{22} - 4) + m \) and \( \check{q}_1 = a - \frac{a-m}{\sqrt{2}} \) with \( \check{q} < \hat{q}_1 \).
area leads to a lower welfare level compared to a constellation with different production structures. Graphically, this is shown by the grey area B, which illustrates all combinations of fixed costs and outsourcing prices, which lead to a constellation with bilateral outsourcing, but yields lower welfare than a constellation with different production structure. In opposite, the white area A, which is restricted by the assumed fixed costs $F = \frac{(a - m)^2}{9b}$ and the $W_{\text{out/out}}^\text{in/out} = W_{\text{in/in}}^\text{out/out}$-curve, characterizes all combinations, where the resulting constellation of bilateral outsourcing leads to a higher welfare level than a constellation of different production structures.

The comparison of the welfare level between the constellations of bilateral outsourcing and bilateral integration can be similarly described. The $W_{\text{in/in}}^\text{out/out}$-curve depicts all combinations of fixed costs and external procurement price with identical welfare levels in a constellation with bilateral outsourcing and bilateral integrated production. The area under this curve characterizes the combination of fixed cost and outsourcing price, where the integrated production leads to a higher welfare level than bilateral outsourcing. As we know, if $q > \tilde{q}_1$, a change towards bilateral integration would increase the welfare level for changing the production structure from bilateral outsourcing to bilateral in-house production. This result can be explained by the huge difference of marginal costs. If for a given fixed cost level the marginal cost difference is sufficiently high, a change towards bilateral in-house production increases the consumer surplus dramatically, which can offset the negative effect on profits. It is also known, that $\tilde{q}_1 > q_{\text{crit}}^\text{out/out}$ holds, which means that for given costs the welfare level cannot be increased if both firms change their production structure from bilateral outsourcing to bilateral integrated production. For the assumed fixed costs $F = \frac{(a - m)^2}{9b}$, we find that $\tilde{q}_1 = a - (a - m)/\sqrt{2} > q_{\text{crit}}^\text{out/out} = (a + 3m)/4$. Thus the constellation of bilateral integration becomes pareto inferior if the firms choose optimally a constellation of bilateral outsourcing, although the consumer surplus increases due to lower marginal costs, respectively output price. This occurs, since the firms choose for given fixed costs the integrated production if the marginal costs difference is sufficiently high. However, the starting point is a constellation with bilateral outsourcing, and thus the marginal cost difference is relative small, which means that the loss of profit due to higher fixed costs cannot be compensated by an increase of the consumer surplus. Graphically this is illustrated in Figure 2, where all combinations of outsourcing prices $q < q_{\text{crit}}^\text{out/out}$ and the assumed fixed costs, $F = \frac{(a - m)^2}{9b}$ lie above the $W_{\text{in/in}}^\text{out/out}$-curve.
Notice that Figure 2 illustrates only the special case \( F = \frac{(a - m)^2}{9b} \) and the corresponding values for the outsourcing price. For lower fixed costs these values are changing, however the derived statements are qualitatively the same. Thus, there is in any cases the possibility of a higher welfare by switching from the profit maximizing constellation of bilateral outsourcing to the constellation with different production choices, but there will be no welfare gain if both firms change their production mode.

**Bilateral Integration Characterizes the Market Constellation**

As we know from the previous analysis, i.e. equation (7), a bilateral integrated production occurs, if \( q > q_{\text{crit}}^{\text{in/in}} \).

For analysing, if another market constellation as the optimal choice of bilateral in-house production increases the welfare level, we have to compare \( q_{\text{crit}}^{\text{in/in}} \) with the threshold values, which indicate the equality of the welfare levels in bilateral integration and bilateral outsourcing, respectively the equality of the welfare levels in bilateral integration and different production strategies.

From the paragraph above, we know the threshold value \( \tilde{q}_1 \), which leads for given domestic marginal costs and fixed costs to the same welfare level in the scenarios of bilateral outsourcing and bilateral integration, i.e. \( \tilde{q}_1 \) describes the solution of \( W_{\text{out/out}}^{\text{in/in}} = W_{\text{out/out}} \). Additionally, we know that \( W_{\text{out/out}}^{\text{in/in}} > W_{\text{out/out}} \) holds for \( q > \tilde{q}_1 \). For given domestic costs a comparison of the two values \( q_{\text{crit}}^{\text{in/in}} \), presented in (7), and \( \tilde{q}_1 \), presented in (12), shows that

\[
q_{\text{crit}}^{\text{in/in}} = \frac{a + m}{2} - \sqrt{\frac{(a - m)^2}{4} - \frac{9b}{4} F} > a - \sqrt{\frac{(a - m)^2}{2} - \frac{9b}{2} F} = \tilde{q}_1.
\]
This means, that the welfare decreases if both firms use optimally the integrated production, i.e. $q_{\text{crit}}^{\text{in/in}} < q$, but both will switch to the external procurement of the input component. This result is not surprising, since a change of strategy leads to an increase in the average marginal costs and the market price, so that the market output falls. As a result, the consumer surplus falls compared to a constellation with bilateral in-house production. In addition, a change in the production structure entails profit losses for both companies as they do not pursue their best strategy. As both market sides suffer losses, welfare cannot be higher when both companies which formerly produced in-house, now procure their input goods externally.

What happens in the case of a transition to different strategies?

Using (10) and (11), by the $W_{\text{in/in}} = W_{\text{in/out}}$-curve we can illustrate all combination of marginal outsourcing costs and domestic fixed costs, which lead to the same welfare level in a situation with bilateral integrated production and a constellation in different strategies. Starting in a point on the curve, for given marginal costs, lower domestic fixed costs increases the welfare in a constellation with bilateral integrated production more than the welfare level in a constellation with different strategies. The reason is that the fixed costs affected both firms, if the constellation is characterized by bilateral integrated production, while in a constellation in different strategies only the integrated producing firm realizes this gain. Thus, if all other parameters are unchanged, the gain in the case of bilateral integration is higher. Therefore, we can conclude, that $W_{\text{in/in}} > W_{\text{in/out}}$ occurs for all combinations of fixed costs and outsourcing price below the $W_{\text{in/in}} = W_{\text{in/out}}$-curve, while for all combinations above the $W_{\text{in/in}} = W_{\text{in/out}}$-curve, we have $W_{\text{in/in}} < W_{\text{in/out}}$.

From equations (10) and (11) we obtain the threshold values

\[
\bar{q}_1 = \frac{4a + 7m}{11} - \frac{16}{121} (a - m)^2 - \frac{18}{11} b F,
\]

\[
\bar{q}_2 = \frac{4a + 7m}{11} + \frac{16}{121} (a - m)^2 - \frac{18}{11} b F,
\]

at which the welfare level is the same when either different strategies or bilateral in-house production strategies are used. When comparing the threshold values with the critical value for in-house production, $q_{\text{crit}}^{\text{in/in}} < q < \bar{q}_1$ and $q_{\text{crit}}^{\text{in/in}} < \bar{q}_2 < q$ must be met to ensure an increase in welfare when switching from a constellation with bilateral in-house production to one with different strategies.

As it can be seen, both terms only provide only a solution if $F \leq \frac{8}{11} \frac{(a - m)^2}{9b}$. In connection with Assumption 2, this means that for $\frac{8}{11} \frac{(a - m)^2}{9b} < F \leq \frac{8}{9b}$ a change to a constellation with different production structures always has a welfare increasing
effect. Starting in a constellation with bilateral integrated production, a change towards different strategies leads to a rise in the average marginal costs of production and, consequently, the market price. At the same time, output and consumer surplus are lower. This is met by an increase in the producer rent. Although the outsourcing company now suffers a profit loss, since its market share falls below 50%, the profit gain of the company that keeps on producing integrated is sufficiently high, so that there is not only a rise in the producer rent, but in welfare as well. This results since the fixed cost saving is high enough and can offset the loss of higher average marginal costs. If for the given outsourcing price \( q^{in \rightarrow in}_{crit} < q < \frac{a + m}{2} \) holds, the profit of the outsourcing firm is lower but still positive, while in the case of \( q > \frac{a + m}{2} \) the firm will realize negative profits.

We now analyse the situation if \( F < \frac{8}{11} \frac{(a - m)^2}{9b} \). This requirement ensures that by using the marginal values (14a) and (14b), areas can be identified in which, from the welfare theory point of view it is preferable to choose a constellation with different strategies, although the firms will optimally decide for an integrated production. For analysing this, Assumption 1, i.e. \( \bar{q}_{1,2} \in \left( m; a \right) \) has to apply too, which is met by the threshold values for the case \( F < \frac{8}{11} \frac{(a - m)^2}{9b} \). Comparing (14a) and (14b) with the critical value for the in-house production, we find that

\[
q^{in \rightarrow in}_{crit} = \frac{a + m}{2} - \sqrt{\frac{(a - m)^2}{4} - \frac{9b}{4} F} < \frac{4a + 7m}{11} - \sqrt{\frac{16(a - m)^2}{121} - \frac{18}{11} bF} = \bar{q}_1,
\]

\[
q^{in \rightarrow in}_{crit} = \frac{a + m}{2} - \sqrt{\frac{(a - m)^2}{4} - \frac{9b}{4} F} < \frac{4a + 7m}{11} + \sqrt{\frac{16(a - m)^2}{121} - \frac{18}{11} bF} = \bar{q}_2,
\]

which allows us to characterize conditions, under which a change of the production mode from bilateral in-house to different strategies increases the welfare. However, we have to compare the threshold values (14a) and (14b) with the second requirement of Assumption 1, i.e. \( \bar{q}_{1,2} < (a + m)/2 \). Comparing the margin for positive profits in different strategies with the threshold values shows, that for any fixed costs with \( F < \frac{8}{11} \frac{(a - m)^2}{9b} \), \( \bar{q}_1 < (a + m)/2 \) applies as well. Thus, the firm, which switch to outsourcing, will still realize a positive profit. In contrast, \( \bar{q}_2 < (a + m)/2 \) only applies if \( 8 \frac{(a - m)^2}{9b} < F < \frac{8}{11} \frac{(a - m)^2}{9b} \). For the case of \( F < \frac{5}{8} \frac{(a - m)^2}{9b} \) and an external procurement price of \( \bar{q}_2 < q < a \), a change towards different strategies increase welfare, but then the outsourcing participant does not gain any positive profits. At this point, the fixed cost savings are too low, relative to the increase in marginal costs. However, in this
case, the positive profit effect of the still in-house producing participant outweighs the negative effects on the consumer surplus and the profit of the outsourcing company. Finally, we find that the welfare in different strategies is higher than in a constellation with bilateral integrated production if

$$q_{\text{crit}}^{\text{in/out}} < q < \tilde{q}_1 \text{ or } q_{\text{crit}}^{\text{in/in}} < \tilde{q}_2 < q < a,$$

but depending on the parameter, the firm which use outsourcing can gain positive profits or realize a loss. We can summarize as follows:

**Proposition 3:**

*If the market constellation is characterized by bilateral in-house production,*

a) *for given costs, this constellation is pareto superior to a constellation with bilateral outsourcing,*

b) *the welfare level can in any case be increased by an asymmetric production organization if the fixed costs are sufficiently high,*

$$\frac{8}{11} \frac{(a-m)^2}{9b} < F < \frac{(a-m)^2}{9b},$$

c) *the welfare level can be increased by an asymmetric production organization for* $q_{\text{crit}}^{\text{in/in}} < q < \tilde{q}_1$ *or* $\tilde{q}_2 < q < a$ *if the fixed costs are sufficiently low,*

$$F < \frac{8}{11} \frac{(a-m)^2}{9b}.$$

The statements above are illustrated in Figure 3. In the case of $F = \frac{(a-m)^2}{9b}$, there is an equilibrium with full integration if $q > q_{\text{crit}}^{\text{in/in}} = (a+m)/2$. Also, we know that bilateral integration leads to higher welfare as a constellation with bilateral outsourcing if $\tilde{q}_1 < q$. Thus $q_{\text{crit}}^{\text{in/in}} < q < \tilde{q}_1$ characterizes the points where the optimally production choice is pareto inferior to a constellation with bilateral outsourcing. Comparing the values we showed that $\tilde{q}_1 < q_{\text{crit}}^{\text{in/in}}$ holds, which means that the optimally constellation of bilateral in-house production is always pareto superior to a constellation of bilateral outsourcing.

Graphically, this is demonstrated by the fact that the combinations $F = \frac{(a-m)^2}{9b}$ and $q > q_{\text{crit}}^{\text{in/in}} = (a+m)/2 > \tilde{q}_1$ lie below the $W^{\text{in/in}} = W^{\text{out/out}}$ -curve.

The $W^{\text{in/in}} = W^{\text{in/out}}$ -curve is significant for comparing the scenario with bilateral integration to one with different strategies, where $W^{\text{in/in}} > W^{\text{in/out}}$ applies for any combinations below this curve. As we demonstrated, there will no external procurement prices, which fulfils $W^{\text{in/in}} = W^{\text{in/out}}$ if $\frac{8}{11} \frac{(a-m)^2}{9b} < F < \frac{(a-m)^2}{9b}$. Therefore, for this range of fixed costs, all combinations lie above the $W^{\text{in/in}} = W^{\text{in/out}}$ -curve, which is demonstrated by the light grey area A. In this case, a transition from bilateral integration to different structures increases the welfare, i.e. $W^{\text{in/in}} < W^{\text{in/out}}$.  

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From the above analysis, we know that for \( \frac{5}{8} \frac{(a-m)^2}{9b} < F < \frac{8}{11} \frac{(a-m)^2}{9b} \) the critical value \( \bar{q}_2 \), from which a constellation with different strategies leads to higher welfare than a constellation with bilateral integration is smaller than \( (a+m)/2 \), the point from which definitively bilateral integration, i.e. \( \bar{q}_2 < (a+m)/2 \). Figure 3 also illustrates this range of fixed costs. In \( F = \frac{5}{8} \frac{(a-m)^2}{9b} \), the threshold values for equal welfare levels are given by \( \bar{q}_1 = \frac{5a + 17m}{22} \) and \( \bar{q}_2 = \frac{(a+m)}{2} \). The critical value at which bilateral integration occurs, is \( q_{\text{crit}}^{\text{in in}} = \frac{a+m}{2} - \frac{a-m}{2} \sqrt{\frac{3}{8}} \), with \( q_{\text{crit}}^{\text{in in}} < \bar{q}_1 < \bar{q}_2 \). Thus, we definitively obtain a constellation with bilateral integrated production for \( q \in (\bar{q}_1; \bar{q}_2) \). Assuming that \( F = \frac{5}{8} \frac{(a-m)^2}{9b} \), Figure 3 shows by the light grey area B, that for any external procurement price \( q \in (\bar{q}_1; \bar{q}_2) \), the combinations are below the \( W^{\text{in in}} = W^{\text{in out}} \)-curve and thus, \( W^{\text{in in}} > W^{\text{in out}} \) applies. In what follows, area B characterizes a range of outsourcing prices, for which a change from the optimal bilateral integration towards a structure with different production modes is pareto inferior.

As Figure 3 also illustrated by the area C, for a bilateral integrated constellation with \( \frac{5}{8} \frac{(a-m)^2}{9b} < F < \frac{8}{11} \frac{(a-m)^2}{9b} \) there is always an interval of outsourcing prices \( q \in (\bar{q}_1; \bar{q}_2) \), in which range a change towards different production modes decreases the welfare level. However, this also means that for \( q \in (q_{\text{crit}}^{\text{in in}} ; \bar{q}_1) \) and \( q \in (\bar{q}_2; a) \), welfare can be increased by switching to different structures, as these combinations lie above the \( W^{\text{in in}} = W^{\text{in out}} \)-curve. Assuming \( F = \frac{5}{8} \frac{(a-m)^2}{9b} \), this is demonstrated by the areas D.

**Figure 3: welfare comparison in the case of bilateral integration**
As in the case of bilateral outsourcing, Figure 3 focuses only on the special cases 
\[ F = \frac{(a-m)^2}{9b} \quad \text{and} \quad F = \frac{5}{8} \frac{(a-m)^2}{9b}. \] 
However, similar to the paragraph above, our general conclusions are qualitatively unaffected by other fixed cost levels, since the changes in the different values do not change the order of these values. Therefore, independent of the fixed cost level, it is not possible to increase the welfare by changing the market constellation to bilateral outsourcing, if the integrated production characterizes the Nash-equilibrium. On the other hand, under certain circumstances it is possible to generate a higher welfare level, if instead of optimal bilateral integration, the firms produce by using different strategies.

**Different Strategies Characterize the Market Constellation**

In our previous analysis, we already looked in part at the constellation with different strategies, which is given by

\[ q_{\text{out}} < q < q_{\text{in}} \]

From the previous analysis, we know that for the threshold value \( \hat{q}_1 \), for which welfare is higher with different production organizations than with bilateral outsourcing, for all fixed costs according Assumption 3 the condition

\[ \hat{q}_1 = -\frac{4a-7m}{3} + \sqrt{\frac{16}{9} (a-m)^2 + 6bf} \]

applies. Therefore, a transition to bilateral outsourcing does not increase the welfare. Here, too, the explanation is intuitive. The deviation of the in-house producing participant raises the average marginal costs and thus, the output price, which results in a reduction in the output amount and, consequently in a lower consumer surplus. Since the firm acts against its best response strategy, its profits decline. On the other hand, the outsourcing participant gets a higher market share and can increase its profits by increasing the output amount. This effect, however, does not compensate other market participants’ losses. Thus, welfare would be lower at bilateral outsourcing in comparison to a constellation in different strategies. Graphically, this was shown in Figure 2, where all outsourcing prices \( q > q_{\text{out}} \) for the assumed fixed cost level \( F = \frac{(a-m)^2}{9b} \) lie below the

\[ W_{\text{out}} < W \]

Similarly, when the outsourcing company switches to in-house production, it acts against its best response strategy and loses profit. In addition, the still in-house producing company loses profits, as its market share falls. In contrast, the consumer surplus increases. However, the positive effect is not sufficient to compensate for the negative effects. Thus, welfare decreases. Formally, this is documented for any fixed costs by
\[ q_{\text{in/in}}^{\text{crit}} = \frac{a + m}{2} - \frac{(a - m)^2}{4} \frac{9b}{4} F < \frac{4a + 7m}{11} \pm \sqrt{\frac{16(a - m)^2}{121} - \frac{18}{11} bF} = \overline{q}_{1/2}. \]

This result was pointed out in a graphical way in Figure 3. For the assumed fixed costs of \( F = \frac{5}{8} \frac{(a - m)^2}{9b} \), the optimal choice of different production structures is characterized by \( q < q_{\text{in/in}}^{\text{crit}} \). As we can see, all combination of this given fixed cost level and outsourcing prices, which lead to a constellation with different strategies, are lying above the \( W_{\text{in/in}} = W_{\text{in/out}} \)-curve.

Proposition 4:
A market constellation characterized by asymmetric production strategies is pareto superior.

The previous analysis allows for a simple and clear cut conclusion. If in a market of independent final good companies, some choose to procure their input externally while others produce their required input themselves, the companies act profit maximizing and also for the benefit of a welfare oriented institution. The reason is that, based on an equilibrium with unchanged costs, welfare cannot be increased by a change of production structure. On the other hand, in the case of identical production structures, despite the companies’ profit orientation, at given costs a change towards an asymmetric production organization may be accompanied by a gain in welfare. This may provide some leeway for market interference by influencing operational decisions concerning the production structure.

5. Concluding Remarks
The paper’s aim was to demonstrate the strategic interactions of production organizations and its welfare implications in a duopoly with homogeneous goods. Outsourcing was interpreted as a long-term investment decision whereby fixed costs could be saved. On the other hand, the marginal costs of external procurement are higher than the marginal costs of in-house production. Consequently, the trade-off between fixed cost savings and a rise in marginal costs determines the company’s production choice. Thereby, with this decision, the cost structure as well as its market position is influenced. As this is true for all companies, the choice of the production organization has a strategic component. Given the different cost parameters, the resulting strategic interactions characterize the market equilibrium. Here we find for given fixed costs, that at a relatively small marginal cost difference, outsourcing becomes the dominant strategy, whereas at a sufficiently high marginal cost difference, both companies will choose in-house production. In the case of a medium marginal cost differences, there will be different production structures.
Via the marginal costs, the choice of organization affects the output price and the consumer. Since both sides of the market, i.e. producer and consumer, are affected, we analysed the effects of the production choice from the welfare point of view. A comparison of the welfare levels of the given market structure in equal production modes and the constellation of different modes revealed that the optimally chosen production strategy is not always pareto superior. Here, we find that for a number of sufficiently big (small) marginal cost disadvantages of external procurement, welfare is higher in different strategies than in the dominant organization of bilateral outsourcing (bilateral in-house production). This means that for a constellation with bilateral outsourcing, the negative effect on firm’s profits will be offset by the increase of consumer surplus, while in the case of a constellation with bilateral in-house production, the profit increase of the still integrated producing firm will compensate the profit loss of the now outsourcing firm and the decrease of consumer surplus. Additionally, we find that if the firms’ profit orientation leads to equal production modes, for given costs, a change of the production structure by both firms never increase the welfare level. In contrast, in the case of a constellation with different production structures, the companies’ profit orientation ensures the pareto superiority.

Notice, that we assume profit maximizing behaviour for the firms. Thus, there are no incentives for the firms to change their decisions. However, given the decisions of the firms, our aim is to analyse via comparative static, if profit orientation by the firms lead to pareto superior situations or if there is scope for interactions of a welfare interested government and set incentives for changing the production mode. From our analysis, we thus come to the conclusion that in the case of identical production strategies for given costs, market interference affecting the companies’ production choice may be required in order to increase welfare, while interferences affecting the companies’ production choice decreases the welfare in case of different production modes.

Appendices

Nash-Equilibria of the Production Structure
For the Nash-equilibria, the profits of a firm in the different scenarios have to be compared.

a) bilateral outsourcing as a Nash-equilibrium
Outsourcing is the choice of firm $A$ ($B$), if for a given outsourcing decision of firm $B$ ($A$) the profit by using the external procurement is higher than by producing integrated, i.e. $\Pi^\text{out/out}_A > \Pi^\text{int/int}_B$ holds. Using the profit defined in Table 3, this is characterized by

$$\frac{1}{9b} [a - q]^2 > \frac{1}{9b} [a + q - 2m]^2 - F.$$ 

For given values of $a$, $q$ and $m$ the condition of an advantageous external procurement is
\[
\frac{4}{9b} (a - m) \cdot (q - m) < F. \tag{A.1}
\]

On the other hand, if firm \( B \) \((A)\) chooses the integrated production, the choice of firm \( A \) \((B)\) will be the external procurement if \( \Pi^{\text{in/out}}_{\text{out}} > \Pi^{\text{in/in}}_{\text{in}} \), i.e. \[
\frac{1}{9b} (a + m - 2q)^2 > \frac{1}{9b} (a - m)^2 - F. \tag{A.2}
\]

For given values of \( a \), \( q \) and \( m \) the condition of an advantageous external procurement is now
\[
\frac{4}{9b} (a - q) \cdot (q - m) < F. \tag{A.2}
\]

Comparing the conditions (A.1) and (A.2), we have, under the assumption \( q > m \), for given values of the different parameters that \((a - q) < (a - m)\). From this follows, that if (A.1) is fulfilled, also (A.2) holds, and therefore condition (A.1) describes the constraint for a dominant Nash-equilibrium with bilateral outsourcing.

b) bilateral in-house production as a Nash-equilibrium

In contrast to the comparison above, firm \( A \) \((B)\) chooses the integrated production, if for given integrated production of firm \( B \) \((A)\), the profit with bilateral in-house production is bigger than in a constellation with different strategies, i.e. \( \Pi^{\text{in/in}}_{\text{in}} > \Pi^{\text{in/out}}_{\text{out}} \). Using the profit levels defined in Table 3, this condition can be written as
\[
\frac{1}{9b} [a - m]^2 - F > \frac{1}{9b} [a + m - 2q]^2. \tag{A.3}
\]

Thus, for given values of \( a \), \( q \) and \( m \) the condition for internal production of both firms is
\[
\frac{4}{9b} (a - q) \cdot (q - m) > F. \tag{A.3}
\]

However, if firm \( B \) \((A)\) chooses outsourcing, the choice of firm \( A \) \((B)\) will be the internal production if \( \Pi^{\text{in/out}}_{\text{in}} > \Pi^{\text{out/out}}_{\text{out}} \), i.e. \[
\frac{1}{9b} (a + q - 2m)^2 - F > \frac{1}{9b} (a - q)^2. \tag{A.4}
\]

For given values of \( a \), \( q \) and \( m \) this condition is fulfilled if
\[
\frac{4}{9b} (a - m) \cdot (q - m) > F. \tag{A.4}
\]

As one can see, under the assumption \( q > m \), for given values of the different parameters \((a - q) < (a - m)\) occurs. From this follows, that if (A.3) is fulfilled, also (A.4) is met. Thus, condition (A.3) describes the constraint for a dominant Nash-equilibrium with bilateral in-house production.
c) Nash-equilibrium in different strategies

Using (A.1) and (A.3) gives the condition of a Nash-equilibrium in different strategies, where we find

\[
\frac{4}{9b} (a - q) \cdot (q - m) < F < \frac{4}{9b} (a - m) \cdot (q - m).
\]

(A.5)

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