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Industrial water demand analysis in the Middle Olifants sub-basin of South Africa: The case of mining

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Teresa Linz,  
Daniel W. Tsegai

Industrial Water Demand analysis in the Middle Olifants sub-basin of South Africa: The case of Mining

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Abstract

This paper seeks to determine water demand of the mining sector in the Middle Olifants sub-basin of South Africa. Despite the growing economic importance of mining in the area, only little is known about its water demand and the role of water in the mines’ extraction process. By means of econometric estimation water price elasticities as well as substitution possibilities between water and other inputs are derived to analyze the response of mines to changes in water tariffs. Using primary data, a translog cost function is estimated for five mines operating in the area. Cost share equations of each input are specified and estimated using Seemingly Unrelated Regression (SUR) method. The mean cost share of water for all five mines, with around 1%, is relatively small, reflecting the low water tariff and results show industrial water demand to be inelastic. Nevertheless, with water price elasticity values ranging from -0.77 to -0.95 for the five mines, there is a potential to influence water use patterns through higher tariffs. As mines are water intensive industries, possible water savings owing to raised water tariffs should not be neglected. Water intake is found to be a substitute for labor and capital for most of the mines, implying that capital investments in water saving technologies might be an alternative means to reduce water intake of the mining sector.
Kurzfassung


Eine translogarithmische Kostenfunktion wird mit primären monatlichen Zeitreihendaten für fünf verschiedene Minen geschätzt. Für die einzelnen Produktionsfaktoren werden Gleichungen mit den jeweiligen Kostenanteilen (n=5) als abhängige Variablen mit der „Seemingly-Unrelated Regression“ (SUR) Methode geschätzt. Die durchschnittlichen Wasserkosten der Datenreihen sind mit einem Anteil von circa 1% der Gesamtkosten sehr gering und Ergebnisse zeigen eine relativ unelastische Wassernachfrage. Mit Wasserpreiselastizitäten zwischen -0.77 und -0.95 bestehen dennoch Möglichkeiten die Wassernachfrage mit Hilfe höherer Wasserpreise zu beeinflussen. Da der Bergwerksektor eine sehr wasserintensive Industrie darstellt, sollten resultierende mögliche Wassereinsparungen nicht vernachlässigt werden. Bei einem Großteil der untersuchten Bergwerke ist Wasser ein Substitut für Arbeit und Kapital, was bedeutet, dass Investitionen in wassersparenden Technologien helfen würden, die Wassernachfrage im Bergbau zu reduzieren.
1 Introduction

Water is one of the most indispensable natural resources. It is essential for human beings, economic development and biological biodiversity. However, countries have to face the challenge of rapidly growing water demands driven by an increased population and economic growth linked to rising urbanization, industrialization and mechanization (King 2004). The resulting water scarcity is one of the most pervasive natural resource allocation problems faced by development planners. Thus water resource management is an important political, social and economic issue of the present century and economists are faced by new challenges of growing and changing social demands for available water, changing technologies, laws and institutions (Louw 2002). However, it is also recognized that water scarcity not only results from insufficient supply, but it rather originates from inefficient use and poor water management (Dinar 2003). Consequently scientists started paying increased attention to the research of water management, including the examination of new strategies that involve better water allocation requiring the consideration of economic aspects.

South Africa is one of the many countries in the world experiencing water shortages. The increasing competition between users, inelastic supply of water and its high and increasing demand are some of the major problems of the country (Hassan and Crafford 2006). Limiting factors on the supply side include low and erratic rainfall patterns, limited groundwater resources, exhaustive development of available water storage as well as unequal spatial distribution. The greater part of the country is semi-arid and the average rainfall is only 450 mm (DWAF 2002) with high evaporation rates. Consequently, the requirements for water far exceed its natural availability in several river basins. Realizing the negative impacts of poor water quantity and quality, the South African Government replaced the emphasis on increasing supply by a strategy of demand management (Conradie 2002), reflected in the National Water Act (NWA), which was implemented in 1998. It promotes integrated and decentralized water resources management and emphasizes economic efficiency, environmental protection, equity and empowerment of people (Hassan and Crafford 2006). This requires better knowledge of the economic value and contribution of water in various uses, the potential of water conservation through demand management as well as powerful information systems that integrate hydrological, economic and social dimensions of water supply and demand.

This study focuses on the industrial water demand in the Middle Olifants sub-basin; a relatively poor area where different users are competing for the scarce water resources. Although water is an essential input to industrial production, few studies have dealt with industrial water use. One reason for that is little data availability and difficult data collection in the industrial sector especially in developing countries where data records are missing. However, facing growing water demand and a higher competition between water users it becomes increasingly
important to address the issue of water demand, substitution possibilities between water and other production inputs as well as water pricing. In the Middle Olifants sub-basin of South Africa, which is located in the north east of the country, the mining sector represents a fast growing industry competing with other water users, like households and the agricultural sector. Unfortunately, little is known about the actual water use, its cost and the way it enters into the production process. This study contributes to present literature by determining industrial water demand as well as corresponding price elasticities through the econometric estimation of cost functions for five mines in the Middle Olifants.

The paper is organized as follows: section two describes the study area and political and institutional settings, section three gives some theoretical background and presents the methodology, section four discusses empirical results while section five concludes.
Industrial Water Demand analysis in the Middle Olifants sub-basin of South Africa: The case of Mining

2 The Study Area

2.1 Institutional and Political Settings

With the political changes in South Africa in the early 1990’s it became possible to introduce reform processes in the water sector, which were a necessary step towards better and equal water supply especially for the poor rural population. New water regulations established the prerequisites for profound changes, both in institutional as well as in organizational settings (Hedden-Dunkhorst 2005). The main objective was the improvement of living conditions for the Historically Disadvantaged Individuals (HDIs). The access to water changed through the replacement of property rights by user rights and the decentralization of water management gave more responsibility and decision taking to local authorities.

Basically two new laws, the Water Services Act (WSA) and the National Water Act (NWA), brought essential transformations in the water sector. The WSA from 1997 guarantees the right for a basic provision of drinking water and sewage disposal (Republic of South Africa 1997), whereas the National Water Act (NWA) from 1998 provides the possibility for a better integration of groundwater and surface water as well as a better water quantity and quality management. The NWA constitutes sustainability and equity as the central principles (Republic of South Africa 1998). It identifies the Department of Water Affairs and Forestry (DWAF) as the custody of the country’s water resources and requires a decentralized water resource management be developed to the catchment level (DWAF 2002). The NWA abolished the commitment of water rights to land property and introduced water licenses instead, which guarantee user rights up to 40 years. Furthermore, a Raw Water Pricing Strategy for Water Use Charges is included in the NWA.

The changes in the water sector are also accompanied by the process of decentralization. The objective is a sustainable and cost efficient water resource management through the transfer of responsibility from national to regional and local authorities. According to the NWA, water management areas (WMAs) must be established, which conduct the protection, use, development, conservation, management and control of water resources. Depending on watercourse catchment boundaries, social and economic development patterns, efficiency and communal interest considerations 19 water management areas were established in the country.
2.2 The Olifants Basin

The study area lies in the Olifants WMA (see figure 2-1), which is the third most water stressed basin in South Africa. It falls within three provinces, with a small part to the west within Gauteng, with the southern part mainly in Mpumalanga and the northern part in Limpopo Province, north east of the country.

Figure 2-1: Olifants Catchment

A strong influence of mineral deposits on development is particularly evident in the catchment area. Wide variations in social and economic development, climate, water availability and population density occur over the Olifants WMA. To enable improved representation of the water resource situation in the WMA under such varied conditions, and to facilitate the applicability and better use of information for strategic management purposes, the WMA was divided into four sub-areas (see Figure 2-2).
The Olifants river basin covers an area of about 54 000 km² with the Middle Olifants representing approximately 38% of the total area. The Middle Olifants sub-basin (yellow area) was chosen as research area in accordance with DWAF and the Water Research Commission (WRC) due to its high water scarcity, high poverty rate, high population as well as unequal water distribution. The competing use of water is more apparent in this part of the basin because there is high concentration of HDIs (who lived in the former homeland areas), white large scale farmers and mining industry emphasizing the need for an Integrated Water Resources Management.

Due to an imbalance of water availability and water requirement, however, not all water demands can be satisfied and different water users are competing for the scarce resource. Surface- and groundwater are major water sources. Thereby, groundwater is an important source of water to supply the current and future domestic and irrigation requirements of the extensive rural population, where water services are missing. Indeed over abstraction has already led to lowering of groundwater levels in the whole study area (DWAF 2004). Several large dams have been constructed on the Olifants River and its tributaries, and the surface water resources are already highly developed. The main storage dams in the Middle Olifants include Rhenosterkop, Rust de Winter and Flag Boshielo. Although the Loskop Dam is situated in the Upper Olifants it is of high importance for irrigation activities in the Middle Olifants (DWAF 2003). Main water users in the area comprise households, agriculture and the mining industry.
3 Theoretical Framework

3.1 Duality and Cost Functions

Duality theory allows one to represent a firm’s technology by the production function, profit function, revenue function or the cost function (Chambers 1994, McFadden 1978). This implies that all functions contain essentially the same information, meaning that they all have a dual representation (Varian 1992). The principle of duality has several important implications for economists and makes it possible to reconstruct well behaved technologies by using cost functions. It is more convenient for the researcher to have two different ways to describe technological properties since some arguments might be easier to demonstrate in just one of the functions. Thereby production and cost functions are most widely used. Compared to the production function, the cost function can be characterized by three relatively simple mathematical conditions in representing technologies (Binswanger 1974, Varian 1992).

The general practical advantage of the cost function lies in its computational simple relation to the cost minimizing input demand functions (McFadden 1978). Furthermore under the assumption of perfectly competitive markets the production function represents input quantities as endogenous variables, whereas in the cost function prices are assumed to be exogenous to the firm. Since it is more likely that input prices faced by the industry are exogenous rather than input quantities (Azzalini, Bloch and Haslehurst 2004, Bottasso and Conti 2003) a cost function will be employed in the following analysis. “The definition of the cost function as a result of an optimization yields strong mathematical properties and establishes the cost function as a sufficient statistic for all the economically relevant characteristics of the underlying technology” (McFadden 1978, 4).

The cost function represents the minimum cost of inputs of producing a given output level. It is expressed as a function of input prices and outputs (Silberberg and Suen 2001).

Through its partial derivatives with respect to each of the input prices, the cost function is related to the cost minimizing demand functions through its unique vector of cost-minimizing demands that are equal to the gradient of \( C(p_1, y) \) in \( p_i \) (McFadden 1978). This property is known as Shephard’s Lemma (McFadden 1978).

\[
x_i(p_1, y) = \frac{\partial C(p_1, \ldots, p_n, y)}{\partial p_i} \quad (3.1)
\]

where, \( p \) = input prices, \( y \) = output, \( C \) = sum of all expenditures, \( x \) = quantity of inputs, indices \( i = 1, \ldots, n \) inputs.
The theory of production can be formulated in terms of the following properties of cost functions (McFadden 1978, Samuelson 1983, Shephard 1970):

1. Non-negativity: $c(p, y) > 0$ for $p > 0$ and $y > 0$
   This property simply implies that it is not possible to produce an output $y$ at no cost. Thus if all input prices or at least one input price are strictly positive the cost of producing a positive output must also be positive.

2. Monotonicity in $p$ and $y$: if $p' \geq p$, then $c(p', y) \geq c(p, y)$ and if $y' \geq y$, then $c(p, y') \geq c(p, y)$
   The cost function is increasing in the input prices and in the level of output, implying that costs must not decrease when input prices or the levels of output are increasing.

3. Concavity
   Concavity in factor prices implies the Hessian matrix $\Delta^2_{pp}C(p, y)$ to be negative semi-definite (Diewert and Wales 1987). Hence, for the second order partial derivatives of the cost function with respect to prices, the first order principal minors must be negative. “The logarithmic second order derivative of the cost function are related to ordinary first and second order partial derivatives of a cost function, $C_i = \frac{\partial C(p, y)}{\partial p_i}$ and $C_{ij} = \frac{\partial^2 C(p, y)}{\partial p_i \partial p_j}$” (Diewert and Wales 1987), where $i \neq j$.

4. Homogeneity: $c(tp, y) = tc(p, y)$, for $t > 0$
   The cost function is homogeneous of degree one in input prices. This property implies that only relative prices matter to economically optimizing agents, meaning that as long as prices only vary proportionally the cost-minimizing choice of inputs will not vary.

5. No fixed costs: $c(p, 0) = 0$
   This property implies that no fixed costs are considered in the cost function and that all inputs are assumed to be perfectly variable. However when dealing with cost functions in the long run it will be important to face that problem of not having included all inputs whether they are fixed or variable.

Based on these properties an appropriate flexible functional form must be chosen. Since the researcher never knows the true functional form representing a firm’s production technology, different functional forms must be compared according to some a priori selection criteria, which refer to mathematical, statistical and economic properties (Zoric 2006). According to Lau (1986) the chosen functional form should exhibit five criteria: (1) Theoretical Consistency, (2) Domain of Applicability, (3) Flexibility, (4) Computational facility and (5) Factual conformity.

The quality of estimation results highly depends on the choice of functional form, which should follow the above listed selection criteria. Generally not all criteria can be satisfied at a time (Lau 1986) and one has to decide according to their importance. Since at least local theoretical consistency, computational facility and flexibility requirements should be met; Lau

---

1 The two most commonly used flexible functional forms are the Generalized Leontief introduced by Diewert (1971) and the translog introduced by Christensen, Jorgenson and Lau (1973), Diewert and Wales (1987).
(1986) suggests the domain of applicability as the only area left for compromises. Accordingly compromises concerning the domain of applicability will be accepted in this analysis and theoretical consistency will be regarded locally, in order not to lose flexibility of the functional form. Different functional forms include the Generalized Leontief cost function introduced by Diewert (1971), the Translog cost function, the Quadratic cost function, the Fourier Flexible Function and the Symmetric Generalized McFadden function. Due to their flexibility and possibility of imposing homogeneity without a loss of flexibility the Translog, Generalized Leontief, Fourier and the Symmetric McFadden cost functions are possible candidates. However, the Fourier is very complex to estimate wherefore it will not be taken into consideration when choosing an appropriate functional form. Finally the Translog cost function was chosen as the functional form to model due to its convenience to estimate and its good representation of producer behavior (Feger 2000).

**The Translog Cost Function**

The locally flexible functional form following the Generalized Leontief is the Transcendental Logarithmic or Translog, introduced by Christensen, Jorgenson and Lau (1992). It is one of the best investigated second order flexible functional forms in applied literature (Friedlander and Spady 1981, 207, Zoric 2006, 70). For water services analysis it was identified as the best functional form by Fabbri and Fraquelli (2000) and it was used by many other authors (Kim 1987, 1995). It is approximated by a second-order Taylor series of an arbitrary function and is specified as follows:

\[
\ln C(p, y) = a_0 + \sum_{i=1}^{n} \alpha_i \ln P_i + \sum_{i=1}^{m} \beta_i \ln y_i + \\
\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \beta_{ij} \ln y_i \ln y_j + \sum_{i=1}^{m} \sum_{j=1}^{m} \delta_{ij} \ln p_i \ln y_j
\]  

(3.2)

Setting \( \beta, \gamma \) and \( \delta \) to null matrices reveals that the Translog function is a generalization of the Cobb-Douglas functional form (Feger 2000, Christensen and Caves 1980). The Taylor series of the Translog function is always evaluated at a certain point, what makes it a locally flexible form. Hence, there is convergence towards the true function in an open region being usually the mean or median (Kuenzle 2005). Due to this characteristic, there are no a priori restrictions on the underlying technology, but theoretical properties can be easily imposed by linear restrictions. Symmetry restrictions are required by continuity\(^2\). Homogeneity can be imposed globally whereas concavity can only be implemented locally if local flexibility shall be preserved. Monotonicity is impossible to maintain globally without losing second order flexibility.

\(^2\) As long as the function \( C \) is twice continuously differentiable, symmetry must be met since \( \beta_s = \frac{\partial^2 C}{\partial P_i \partial P_j} = \frac{\partial^2 C}{\partial P_j \partial P_i} = \beta_s \)
3.2 Price Elasticities

After estimation of the model and verification of well-behaviour according to economic theory, estimated coefficients can be used to compute own \((\eta_{ii})\) and cross \((\eta_{ij})\) price elasticities. If the own-price elasticity is \(<|1|\) it is said to be inelastic and if it is \(>|1|\) it is elastic and the demand can be relatively good influenced by adequate pricing strategies (Féres and Reynaud 2005).

The price elasticity of demand for the factors of production \(\eta_{ij}\) is defined as (Berndt and Wood 1975):

\[
\eta_{ij} = \frac{\partial \ln x_i}{\partial \ln P_j}
\]  

(3.3)

where output quantity and all other input prices are fixed.

For the translog cost function own and cross-price elasticities can be computed with the following formulas holding output quantity and all other input prices fixed:

\[
\eta_{ii} = \gamma_{ii} + S_i^2 - S_i
\]

\[
\eta_{ij} = \gamma_{ij} + S_i S_j
\]

(3.4)

where, the \(\gamma_s\) represent the estimated coefficients of the model and \(S_i\) are the respective cost shares of inputs\(^3\).

3.3 Water Demand

Water demands are determined by several interrelated factors. Amongst others these are production technology, product mix, price of raw materials, price of purchased water, costs of intake facilities, treatment and recirculation for self-supplied firms, national regulations and climate (Stone and Whittington 2002). All these factors can be included in the firm’s production or cost function, which can be essentially derived from the demand for the firm’s output (Berndt and Wood 1975). However, different inputs enter the production process and the firm tends to choose that bundle of inputs which minimizes the total cost of production for a given level of

\[\frac{\partial \ln C}{\partial \ln P_i} = S_i = \frac{P_i X_i}{TC}\]

\(^3\) Cost shares of the different inputs can be calculated as
output. For a translog function water demand can be derived from Shephard’s lemma (see equation (3.1) and looks as follows:

$$x_i = (\alpha_i + \sum_{j}^{n} \gamma_{ij} \ln P_j + \beta_{iy} \ln y) \cdot \left( \frac{TC}{p_i} \right)$$

(3.5)

where, $x_i$ represents the demand for water and TC represents total costs. The total costs can be divided into costs for water and other inputs (j), while the costs for water change again with the price of the input. Since the costs for water can be defined by $p_w * x_w$ the formula can be derived as follows:

$$x_w = a \left( \frac{TC_j + p_w x_w}{p_w} \right)$$

(3.6)

where, $a$ is $\alpha_i + \sum_{j}^{n} \gamma_{ij} \ln P_j + \delta_{iy} \ln y$ and $TC_j$ represents the costs for all inputs except water. Solving (3.6) for $x_w$ results in:

$$x_w = a \frac{TC_j}{p_w} \left( \frac{1}{1 - a} \right)$$

(3.7)

By varying the price of water in equation (3.7) while holding other variables constant at their means, a price-quantity schedule of water can be calculated.
4 Econometric Estimation of Industrial Water Demand

4.1 Data, variables and descriptive statistics

Data was collected during field research from August – November 2007 and validated and complemented in September 2008. The number of existing mines and their location in the Middle Olifants sub-basin had to be identified, through different expert interviews with the regional and national Departments of Mineral and Energy (DME), the Departments of Water Affairs and Forestry (DWAF) and the Water Research Commission (WRC). 12 mines were identified to be in operation (see Figure 4-1). However, for only 5 mines historical time series data was available. These include Havercroft Mine, the three platinum mines Marula, Lebowa and Lonmin and the Fluorspar Mine Vergenoeg.

Figure 4–1: Mines in the Middle Olifants

South Africa is not only the leader in the export of gold, ferrochrome and manganese but also the world’s largest exporter of andalusite. The dominating producer has three operating mines in South Africa with Havercroft Mine in the MO (Lanham 2005). Andalusite is an aluminum nesosilicate mineral used in the manufacture of refractory finished products such as
bricks, castables and monolithic shapes mainly for the iron and steel industry (Andalusite Resources 2009).

The South African platinum group metals (PGMs) industry is a key global supplier of PGMs and is the largest component of the South African mining sector (Chamber of Mines of South Africa 2006). The six platinum group metals are ruthenium, rhodium, palladium, osmium, iridium, and platinum. Platinum is the most important of the PGMs and some 61% of the world's platinum is used in autocatalysts, designed to reduce noxious emissions from vehicles (Anglo Platinum 2008).

The Fluorspar Mine Vergenoeg is the world’s largest known fluorspar (CaF2) resource with a lifespan in excess of 100 years at current production rates (Metorex Limited 2009). Fluorspar is used for fluorocarbon gases for refrigerators and air-conditioning, in the metallurgical industry as well as the ceramic, optical-glass and welding industries. Mining is by opencast methods (Metorex Limited 2009).

Data collected is based on historical monthly time series, whereas due to differing time spans and variables included, each mine will be analyzed separately. Variables used in the analysis are described in the following. With the Producer Price Index (PPI) (Statistics of South Africa n.d.) collected data was corrected for inflation by changing nominal data to real values to be able to make sensible comparisons across time despite of price changes.

**Dependent Variable**

The dependent variable is the sum of monthly expenditures on the variable inputs entering the production functions (Dupont and Renzetti 2001). For this analysis it includes monthly expenditures on water intake, electricity, labor, capital and diesel. This information could be mostly gained from the questionnaire and through calculations otherwise. Dividing monthly expenditures of input $x_i$ by total costs results in the % cost share of each input.

**Independent Variables**

As independent variables those variables were chosen to be included that are variable in the long-run, depicting a long-run function. Chosen variables are entering the variable cost function as input prices.

*Price of Water*

The monthly water price could be calculated with data obtained from the mines by dividing monthly expenditures on water intake by monthly quantity of water intake.

*Price of Electricity*

The unit price of electricity is calculated the same way and also here enough data on monthly electricity expenditures and quantities was available from the mines to calculate the unit price of electricity Rand/kwh.
Price of Labor

The price of labor is counted in Rand/man hour. However, if mines did not have the number of man hours but rather monthly numbers of workers, monthly man hours were obtained by multiplying the number of workers by 45 times $4^4$. Hence unit price of labor could be obtained.

Capital

As a long-run cost function is considered in the analysis, capital enters as a variable input as capital stock can adjust to price and output level changes in the long-run. Since capital costs particularly include interests for external finance (Rürup 2002), capital costs for mines are defined as monthly costs for interest paid. If data on capital costs was available mines could only give it per year. Hence yearly costs were divided by twelve to arrive at the monthly capital costs. However if no costs could be obtained at all other sources were consulted. Information could be drawn from the financial annual reviews obtained from the mine manager (Anglo Platinum 2006). Unfortunately mines could not give more information on depreciation rates, live expectancy and capital stock. Following Garcia and Thomas (2001), who calculated an index for input materials as a unit cost per output, the price of capital is calculated accordingly by dividing total monthly capital costs by the output in tones extracted.

Diesel

Data on monthly diesel expenditures and quantities was sufficiently available from the mines to calculate the unit price of diesel in Rand/liter. Only for one mine data on this variable could not be obtained, wherefore it was neglected in that case.

Output

The product of output differs according to the different types of mines (see Figure 4-1). Data on monthly extracted quantities was sufficiently available from the mines.

Descriptive statistics

Table 4-1 shows some descriptives of the variables used in the analysis. The total observations for all five mines sum up to 218. The mean value for all 5 mines is given as well as minimum and maximum values. Due to different cost structures of the mines these can partly

---

$4^4$ During the expert interviews it was found out that the average weekly working hours in the mining sector usually sum up to 45 and to arrive at a monthly number this is multiplied by the number of weeks per month.

$5^5$ No weighting was done, since it is not very probable that these costs differ monthly.

$6^6$ 45 observations could be obtained for three of the five mines, whereas 48 and 35 observations could be obtained for the remaining two mines.
vary strongly. Looking at the water intake it becomes obvious that platinum is water intensive relatively to other mines.

Table 4-1: Descriptive statistics of the variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Price (R/m³)</td>
<td>P₞</td>
<td>0.674</td>
<td>0.551</td>
<td>0.001</td>
<td>2.397</td>
</tr>
<tr>
<td>Electricity Price (R/kwh)</td>
<td>Pₑ</td>
<td>0.124</td>
<td>0.036</td>
<td>0.038</td>
<td>0.207</td>
</tr>
<tr>
<td>Labor Price</td>
<td>P₁</td>
<td>30.11</td>
<td>9.584</td>
<td>13.65</td>
<td>87.06</td>
</tr>
<tr>
<td>Capital Price (R/y)</td>
<td>Pₖ</td>
<td>65.190</td>
<td>48.02</td>
<td>2.8226</td>
<td>203.3</td>
</tr>
<tr>
<td>Diesel Price (R/l)</td>
<td>P₉</td>
<td>3.074</td>
<td>1.110</td>
<td>0.8435</td>
<td>8.467</td>
</tr>
<tr>
<td>Output platinum (t/month)</td>
<td>yₚ*</td>
<td>109730</td>
<td>50538</td>
<td>27827</td>
<td>2040894</td>
</tr>
<tr>
<td>Output others (t/month)</td>
<td>yₒ*</td>
<td>8491</td>
<td>4449</td>
<td>3032</td>
<td>16014</td>
</tr>
<tr>
<td>Water intake platinum (m³/month)</td>
<td>Qₚₒ</td>
<td>264726</td>
<td>278369</td>
<td>12836</td>
<td>1046008</td>
</tr>
<tr>
<td>Water intake others (m³/month)</td>
<td>Qₒₒ₂</td>
<td>58263</td>
<td>33494</td>
<td>14442</td>
<td>121160</td>
</tr>
</tbody>
</table>

*indices p and o represent platinum mining and others respectively
Source: Own survey

The low water price is also reflected in the low cost share of water costs with only 1%, shown in. Cost shares are again evaluated at the mean of all mines.
4.2 Estimation Procedure

According to choice criteria and the characteristics of mentioned functional forms, the translog function was identified to be an appropriate functional form to represent a firm’s technology (see section 3.1). For the data at hand the cost function for a nonhomothetic production structure looks like:

\[
\ln C_t = \alpha_0 + \alpha_w \ln P_{wt} + \alpha_e \ln P_{et} + \alpha_l \ln P_{lt} + \alpha_c \ln P_{ct} + \alpha_d \ln P_{dt} + \beta_y \ln y_t \\
+ \frac{1}{2} \gamma_{ww} \ln P_{wt}^2 + \frac{1}{2} \gamma_{ee} \ln P_{et}^2 + \frac{1}{2} \gamma_{ll} \ln P_{lt}^2 + \frac{1}{2} \gamma_{cc} \ln P_{ct}^2 + \frac{1}{2} \gamma_{dd} \ln P_{dt}^2 + \frac{1}{2} \beta_{yy} \ln y_t^2
\]  

(4.1)

where \( t = 1, \ldots, n \) represents the periods and the indexes correspond to variable inputs as water, electricity, labor, capital and diesel. Index \( y \) represents quantity of output.

Given differentiability of the cost function the cost shares of all inputs can be expressed as elasticities of the cost function with respect to the input prices (see equation (3.1)). Using Shephard’s lemma the cost share equations can be derived by logarithmic differentiation of the cost function:

\[
S_{it}(P, y) = \frac{\partial \ln C_t}{\partial \ln P_{it}} = \alpha_i + \sum_j^n \gamma_{ij} \ln P_{jt} + \delta_{iy} \ln y_t
\]  

(4.2)

where indexes \( i \) and \( j \) correspond to variable inputs. Hence the resulting cost shares \( S_{it} \) for the translog cost function can be written as:

\[
\frac{\partial \ln C_t}{\partial \ln P_{wt}} = S_{wt} = \alpha_w + \gamma_{ww} \ln P_{wt} + \gamma_{we} \ln P_{et} + \gamma_{wl} \ln P_{lt} \\
+ \gamma_{wc} \ln P_{ct} + \gamma_{wd} \ln P_{dt} + \delta_{wy} \ln y_t
\]

(4.3)

\[
\frac{\partial \ln C_t}{\partial \ln P_{et}} = S_{et} = \alpha_e + \gamma_{ew} \ln P_{wt} + \gamma_{ee} \ln P_{et} + \gamma_{el} \ln P_{lt}
\]

A translog function requires approximation of the underlying cost function to be made at a local point (Filippini 1999). Thus all explanatory variables are normalized at overall sample means as point of approximation (Tresch and Zlate 2007).
This partially differentiating of the cost function with respect to the price of one input yields the demand function of that input, which is dependent on the level of output and the vector of input prices (Renzetti 1992). However, equation (4.3) has one important implication that must be taken into account in further analysis; the share equations sum up to unity:

\[ \sum_{i=1}^{n} S_{it} = 1 \quad (4.4) \]

This implies that only n-1 share equations are linearly independent and consequently only n-1 share equations are estimated.

However, before estimating the set of equations for each mine separately the variables have to be checked for stationarity as it is time series data. Stationarity of data implies that its mean and variance do not vary systematically over time and thus do not follow a time trend (Lütkepohl und Krätzig 2004). The Augmented-Dickey-Fuller test can be used to test the null hypothesis of a unit root\(^8\) against the alternative of stationarity. The test uses an OLS regression of the first difference of the time series itself and has options to include a constant term, a time trend and lagged dependent variables to take account for serial correlation. Only some variables show the existence of a time trend. However, due to the fact that the time span was only up to maximal 4 years and that the dependent variables were all stationary no time trend was included in the analysis, what would have increased necessary restrictions and complexity of the model.

Equation (4.1) can be estimated directly by Ordinary least squares (OLS). However OLS estimates only have desirable properties if certain assumptions are met (Varian 1992) and it neglects the additional information contained in the share equations(4.3). One of these assumptions is that the right-hand side variables should not be correlated with the disturbances in

\[ \frac{\partial \ln C_t}{\partial \ln P_{it}} = S_{it} = \alpha_i + \gamma_{lw} \ln P_{wt} + \gamma_{le} \ln P_{et} + \gamma_{lt} \ln P_{lt} + \gamma_{ly} \ln y_t \]

\[ \frac{\partial \ln C_t}{\partial \ln P_{ct}} = S_{ct} = \alpha_c + \gamma_{cw} \ln P_{wt} + \gamma_{ce} \ln P_{et} + \gamma_{cl} \ln P_{lt} + \gamma_{cy} \ln y_t \]

\[ \frac{\partial \ln C_t}{\partial \ln P_{dt}} = S_{dt} = \alpha_d + \gamma_{dw} \ln P_{wt} + \gamma_{de} \ln P_{et} + \gamma_{dt} \ln P_{lt} + \gamma_{dy} \ln y_t \]

---

\(^8\) if a series has a unit root it is non-stationary
the regression. Since this is not the case here, as will be shown later another estimation method has to be applied. Furthermore there might be a problem of multicollinearity because there are a large number of regressors, which do not vary much. This results in imprecise parameters estimates (Uri 1978). In order to avoid these problems and to improve efficiency the optimal cost-minimizing input demand equations can be estimated, by transforming them into cost shares equations (Berndt 1991). Furthermore given the large number of parameters to be estimated, the effective degrees of freedom can be increased by estimating the input cost share equations (Halvorsen and Smith 1984).

The outlined model then consists of several equations, which are related due to the facts that some explanatory variables are the same and that the disturbances are correlated across equations (Sauer 2005b). Such systems of structural equations are not estimated by normal OLS but rather by more elaborated techniques such as two-stage least squares or maximum likelihood methods. Seemingly unrelated regression (SUR) estimation is used to jointly estimate the systems of equations with correlated disturbances described above (Seddighi, Lawler and Katos 2000). Symmetry and homogeneity are imposed prior to estimation whereas concavity restrictions must only be imposed if not satisfied.

### Imposing Concavity

Several economists have developed numerical methods for imposing curvature conditions, if not satisfied after estimation. For a translog cost function, matrix $A$ can be defined to be the matrix of second order factor price coefficients $\gamma_{ij}$, the $i$th share function $S_i(w, y, t) = w_i x_i(w, y, t)/C(w, y, t)$, the share vector $s = [S_1, \ldots, S_i]^T$ and $s^k$ the diagonal matrix of shares. Using these definitions the translog cost function can be rewritten in matrix form: $M = A - s^k + ss^T$. Hence, $\Delta^2 w C(w, y, t)$ will be negative semi-definite if $A - s^k + ss^T$ is a negative semi-definite matrix (Diewert and Wales 1987). Provided that the share vector $s$ is positive the matrix $-[s^k + ss^T]$ must be negative semi definite.

A method to impose negative-semi definiteness was developed by Jim Anderson (1982, 1983) and involves forcing the trace\(^9\) of a matrix involving the coefficients and shares (Tresch and Zlate 2007). The method involves selecting values of the coefficients and testing if the principal minors of the combined coefficient/share matrix $M$ have the correct signs that are preconditions for fulfilling the negative semi-definite condition.

---

\(^9\) A trace of a matrix is defined as the sum of the diagonal elements. At the same time the trace of a matrix is the sum of its eigenvalues.
It is important to note that the elements of the matrix $M(p, y)$ depend on $(p, y)$ since input shares $S_i$ depend on $(p, y)$ (Zoric 2006, Diewert and Wales 1987).

Thus bounds are defined so that the Matrix $M$ is non-positive definite. According to Anderson (1982, 1983) the system is estimated and then the trace of Matrix $M$ is forced until concavity is satisfied. The coefficients are restricted such that all the first-order principal minors are $\leq 0$. Thus for each case one has to compute bounds for which the first-order principal minors are negative and impose the corresponding constraints in the estimation. According to Zlate (2008) concavity can be imposed (a) at the mean shares, (b) at one observation point and (c) at every point in the sample.

However, imposing concavity at every point of the sample for the translog function will destroy flexibility. Thus instead of imposing concavity globally it will be imposed at a chosen reference point. According to Ryan and Wales (2000) the imposition of concavity only at one point will very often lead to satisfaction of concavity at most or all data points in the sample. Hence to assure that the estimated cost function is at least locally concave one has to firstly estimate the system with SUR and examine the definiteness of the Hessian matrix. If it is not negative-semi definite, concavity has to be imposed in one of the three cases mentioned above. The system has to be re-estimated with additional concavity constraints and verified again afterwards.

4.3 Parameter Estimates

The presented model was estimated for each and every mine separately, due to different outputs of the mines, different time frames of the present data as well as the special requirements of the project, in which the study is embedded. After having estimated equation (4.1) with $n-1$ share equations with SUR and having tested for the satisfaction of concavity conditions, it was found that concavity had to be imposed for every model. The point of concavity imposition was chosen arbitrarily at the last observation in most cases. Table 4-2 shows for each mine the point of observation where concavity was imposed, the number of observations and each time frame.
The parameter estimates of the translog functional form shown in Table 4-3 cannot be interpreted directly. However they can be used for further calculations. The Breusch-Pagan test confirms the assumption of a heteroskedastic correlated error vector for every mine and hence justifies the application of a seemingly unrelated estimation procedure for the model.

The adjusted R squares are relatively high except for Havercroft mine. Most of the coefficients are statistically different from zero at a significant level of 99%, implying that the model estimates and further calculations are meaningful for simulations.
The estimated cost functions are dual to well-behaved production functions as they are mostly non-negative, monotonically increasing, linear homogeneous and concave in \( p \). Non-negativity is verified by calculating the estimated value of the cost function for every data point. Monotonicity in \( p \) is mostly satisfied as the fitted input demand functions are non-negative and monotonicity in output is satisfied as the respective derivative is positive (Chambers 1994). Concavity is not satisfied for all observations since it was only imposed locally and not globally. However most of the mean values for the diagonal elements of matrix \( M \) have the expected negative sign and it can be shown that the functions still behave theoretical consistent over the range of observations when looking at elasticities and demand.
4.4 Own and Cross-Price Elasticities

Elasticities can be calculated using equations (3.4). All elasticities are computed using the estimated coefficients and mean values of the actual shares, rather than using fitted shares, since estimated elasticities are distributed asymptotically normal only when actual shares are used (Renzetti 1992).

Table 4-4 shows that all own price elasticities have the expected negative sign. Water demand is responsive to a change in its own price. With an own price elasticity between -0.77 and -0.95 result is in line with other studies. Grebenstein and Field (1979) and Babin et al. (1982) estimated a quite low elasticity of -0.33 and -0.56 respectively for US industries. The own price elasticity of water for the Canadian manufacturing sector calculated by Dupont and Renzetti (2001) is with -0.77 very similar to the above results. Kumar (2004) finds a water price elasticity of -1.11 and Féres and Reynaud (2005) estimate the water price elasticity for Brazil to be -1.085. The results for the own price elasticity for water in South Africa are within the range of other results from developed and developing countries. There is no sign that platinum mines behave differently than other mines concerning water demand. However it can be shown that Marula and Lonmin mine both being platinum mines almost react equal to water price changes. The highest water demand price elasticity can be found at the Fluorspar mine, Vergenoeg.

<table>
<thead>
<tr>
<th>Mine</th>
<th>Own Price Elasticities</th>
<th>Own Price Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>η_{ww}</td>
<td>η_{ee}</td>
</tr>
<tr>
<td>Havercroft</td>
<td>-0.7669*** (4.81e-13)</td>
<td>-0.6973*** (1.75e-13)</td>
</tr>
<tr>
<td>Marula</td>
<td>-0.8969*** (1.09e-12)</td>
<td>-0.4727*** (6.79e-13)</td>
</tr>
<tr>
<td>Lebowa</td>
<td>-0.7741*** (2.27e-10)</td>
<td>-0.6596*** (3.78e-13)</td>
</tr>
<tr>
<td>Lonmin</td>
<td>-0.8723*** (1.50e-12)</td>
<td>-0.3597*** (5.00e-13)</td>
</tr>
<tr>
<td>Vergenoeg</td>
<td>-0.9542*** (2.56e-12)</td>
<td>-0.4759*** (4.41e-13)</td>
</tr>
</tbody>
</table>

(1) standard errors are reported in parentheses; *, **, *** imply significance at the 10-, 5-, or 1 %-level

Looking at the substitutability and complementary between the inputs shown in Table 4-5 labor can be found to be a substitute for capital and diesel, although the elasticities are quite low here. However this is also in line with other studies where cross elasticities of labor and capital
behave equally. Water is found to be a complement to electricity and diesel for most mines, whereas it is a substitute for labor and capital for three of the five mines. For Havercroft mine a 1% rise in the price of electricity will lead to a 0.62% drop in the use of water showing the complementary nature of electricity and water. The more water is used the more electricity is needed for the pumping of water. Only Marula and Vegenoeg mine show the contrary relation of water and electricity. However for Marula mine the positive cross price elasticity turned out to be insignificant. For Havercroft and Lonmin mine the substitutability between water and labor is with around 2.26 and 3.21 respectively quite high. However these results can be confirmed with previous studies from Grebenstein and Field (1979) and Babin et al. (1982). Only Féres and Reynaud (2005) also find capital to be a substitute for water, however the cross price elasticity is with 0.19 much lower. This shows that water demand could still be decreased a lot in the study area through more capital investment. Only for two mines results give negative values for the water/capital cross price elasticity. However it is insignificant for one of them.

Table 4-5: Cross Price Elasticities

<table>
<thead>
<tr>
<th>Mine</th>
<th>$\eta_{we}$</th>
<th>$\eta_{wl}$</th>
<th>$\eta_{wc}$</th>
<th>$\eta_{wd}$</th>
<th>$\eta_{el}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Havercroft</td>
<td>-0.6248*** (0.09241)</td>
<td>2.2598*** (0.09659)</td>
<td>0.7144*** (0.04247)</td>
<td>-1.5825** (0.14636)</td>
<td>0.5575*** (0.06662)</td>
</tr>
<tr>
<td>Marula</td>
<td>0.4355 (0.53257)</td>
<td>-3.1500*** (0.71631)</td>
<td>3.8871*** (0.44177)</td>
<td>-0.2756 (0.42941)</td>
<td>0.5709*** (0.07590)</td>
</tr>
<tr>
<td>Lebowa</td>
<td>-0.5346*** (0.19763)</td>
<td>-1.4809*** (0.28589)</td>
<td>1.9081*** (0.17342)</td>
<td>0.8815*** (0.18642)</td>
<td>-0.5729*** (0.03705)</td>
</tr>
<tr>
<td>Lonmin</td>
<td>-0.1469 (0.15823)</td>
<td>3.2101*** (0.15086)</td>
<td>-2.0026*** (0.11234)</td>
<td>-0.1881 (0.13709)</td>
<td>0.1354*** (0.01878)</td>
</tr>
<tr>
<td>Vergenoeg</td>
<td>0.7587*** (0.06831)</td>
<td>0.4109*** (0.14451)</td>
<td>-0.2154 (0.13819)</td>
<td>0.1752*** (0.00716)</td>
<td></td>
</tr>
</tbody>
</table>
Table 4–5 con’t

<table>
<thead>
<tr>
<th>Mine</th>
<th>$\eta_{ec}$</th>
<th>$\eta_{ed}$</th>
<th>$\eta_{lc}$</th>
<th>$\eta_{ld}$</th>
<th>$\eta_{cd}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Havercroft</td>
<td>0.2863***</td>
<td>0.2145***</td>
<td>-0.0890***</td>
<td>0.1567***</td>
<td>2.5908***</td>
</tr>
<tr>
<td></td>
<td>(0.02746)</td>
<td>(0.06828)</td>
<td>(0.00338)</td>
<td>(0.00303)</td>
<td>(0.11326)</td>
</tr>
<tr>
<td>Marula</td>
<td>-0.2315*</td>
<td>0.08978</td>
<td>0.1227***</td>
<td>0.0280***</td>
<td>0.0069</td>
</tr>
<tr>
<td></td>
<td>(0.11049)</td>
<td>(0.07313)</td>
<td>(0.09901)</td>
<td>(0.00851)</td>
<td>(0.03048)</td>
</tr>
<tr>
<td>Lebowa</td>
<td>0.9576***</td>
<td>0.2771***</td>
<td>0.21036***</td>
<td>0.0494***</td>
<td>-0.0623***</td>
</tr>
<tr>
<td></td>
<td>(0.03903)</td>
<td>(0.02694)</td>
<td>(0.00151)</td>
<td>(0.00219)</td>
<td>(0.00276)</td>
</tr>
<tr>
<td>Lonmin</td>
<td>0.1955***</td>
<td>0.0370**</td>
<td>0.1412***</td>
<td>0.0019</td>
<td>0.0114***</td>
</tr>
<tr>
<td></td>
<td>(0.01916)</td>
<td>(0.01882)</td>
<td>(0.00212)</td>
<td>(0.00266)</td>
<td>(0.00381)</td>
</tr>
<tr>
<td>Vergenoeg</td>
<td>0.2614***</td>
<td>0.2178***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00749)</td>
<td>(0.00192)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) standard errors are reported in parentheses; *, **, *** imply significance at the 10-, 5-, or 1 %-level

Most of the estimated elasticities from Table 4-4 and 4-5 are at least statistically significant at the 5% level.

4.5 Water Demand Curves

Using equation (3.7) the demand for water intake can be derived holding all other inputs and output constant at their means. 4-3 shows the resulting water demand curve for each mine. Marula Mine is the mine that shows the highest demand curve whereas Lebowa Mine has the lowest demand curve.

The slopes of the curves represent the marginal value of water, which can be used for further water allocation decisions. It can be shown that water demand is quite inelastic from a certain price for all mines.
Figure 4-3: Water Demand Curves
5 Conclusions

In this paper, we investigated water demand for five mines in the Middle Olifants sub-basin of South Africa. The structure of water demand has been characterized by estimating a translog cost function with time series data over several months. No considerable differences of water price elasticities for the different mines could be found. With an overall mean water price elasticity of -0.852 results are similar to other studies (Dupont and Renzetti 2001). Descriptive statistics show that water charges vary widely across mines and hence, a first step of improved water management should include the adjustment of different water price levels in the same sector. The estimated parameters presented in the previous section indicate that there is a potential of influencing water use patterns by raising relative prices of water as well as non-water inputs. Hence, results show that price is an important variable in industrial water use, what should underlie a whole new approach to industrial water management. It suggests that water charges would initiate water conservation through e.g. the movement to greater levels of recirculation and over the longer run increased capital stocks. In terms of using economic or market-based instruments to achieve water conservation goals, governments can, amongst others, either adopt water use taxes or alternatively create tradable permits for direct water withdrawals.

By examining water demand in a developing country using primary data for such an intensively water using sector, this paper adds value to existing industrial water demand literature. Water price elasticities give important information on the demand structure of water and can help policy makers to design appropriate water tariffs or other water related policy instruments. The demand curves can be further used for water allocation decisions not only within the mining sector but also between the mining sector and other economic sectors. Owing to data limitation, the paper falls short of wide variations of input prices and a good representation of capital stock. Irrespective of these limitations, however, the paper serves as a pilot study in the area as well as background information for future research. This study underlines the need for improved data collection in the mining sector. In view of the growing importance of the mining industry in South Africa, there is a need for more research and a thorough understanding of water’s role in the extraction process prior to the implementation of policies aimed at altering the mines’ behavior.
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Industrial Water Demand analysis in the Middle Olifants sub-basin of South Africa: The case of Mining


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Industrial Water Demand analysis in the Middle Olifants sub-basin of South Africa: The case of Mining


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