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Optimal Capital Income Taxation with Tax Evasion

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Abstract The paper discusses the applicability of optimal taxation theory to source-based capital incomes when significant tax evasion is observed. Without tax evasion a modified Ramsey Rule may reduce distortions brought by international capital mobility, leading to levying differentiated tax rates in domestic sectors inversely proportioned to observed elasticities in terms of capital mobility. The introduction of tax evasion brings additional complexity. The viability of optimal tax rates à la Ramsey is explored, and additional requirement (namely that tax evasion is either very low or very homogeneous) are shown to be necessary in order to allow policy-makers to obtain the tax rates minimizing total excess burden. Results are also provided to solve the optimal taxation objective when tax evasion is a relevant phenomenon and is not homogeneous throughout domestic sectors.

JEL H21, H26

Keywords Optimal taxation; capital income taxation; tax evasion

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Optimal capital income taxation with tax evasion

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1. Introduction

Within research on direct capital taxation, the dominant opinion appears against, or at best not very well-disposed to, the use of differentiated taxes. This stance is observed both in normative theory (see for example the fundamental work of Diamond and Mirrlees, 1971), and to some extent in applications to real economies. The rationale to prefer a uniform and therefore “neutral” tax with regards to economic agents’ choices, is immediately self-evident if one considers the distortions induced in resource allocation by fiscal burden differentials. Such distortions may act on more than one dimension: between various forms of employment of capitals; between investments in distinct territorial areas; on the distribution of production factors among industrial sectors; on technologies; between financing tools. Moreover, technical difficulties are often met by policy-makers when attempting to gather up-to-date statistics, to timely calibrate the differentiated tax rates to meet their policy targets.

In a paper published in 2007, Peter Birch Sørensen proposed an application of the Ramsey Rule to source-based capital income taxation, thus filling a hole in optimal taxation theory which dedicated ample discussion to discriminating indirect taxation\(^1\) and non-linear labor income taxation\(^2\), but less attention to the possibility of a discriminating tax on capital incomes, considering to a good extent self-evident the priority need to obtain neutrality in taxation. With regards to an open economy where productive sectors show different trans-national capital mobility, Sørensen

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1 In particular the contribution of CORLETT, HAGUE (1953) who extend Ramsey's framework and better define its application when cross elasticities of demands are not null and leisure time is not directly taxable.

2 Optimal taxation research is really too large to represent it here with an arbitrary number of references. I therefore forward the reader to a review of the main policy results of this tradition of study presented in MANKIW ET AL. (2009).
theoretically illustrated to what extent a differentiated tax may bring benefits (in terms of efficiency) higher than inter-sectoral distortions caused domestically.

This paper contribution is to test the applicability of Sørensen's framework of optimal capital income taxation in an open economy with constrained ability to tax non-capital incomes, to countries where a significant level of income tax evasion is detected. The introduction of tax evasion may notably complicate optimal taxation analysis since it brings an additional layer of choice along which taxpayers may move. The choice for a taxpayer whether to evade or not is conditioned, among other things, by tax rates, and that choice affects expected net returns to capital. So, unless the level of tax evasion is exactly the same in every domestic sector, in a way to modify expected returns in the same proportions when differentiated tax rates vary (an hypothesis which is hardly defendable under theoretical and empirical knowledge we have at our disposal on fiscal evasion), tax evasion must be carefully evaluated. Moreover, an additional complexity is given by the fact than one of the channels through which tax evasion is employed, is by changing the juridical form of capitals, simulating they belong to foreign individuals while in reality the latter are only men of straw or trustee of domestic owners. In this way, the policy-maker may be fooled by statistics that do not manage to distinguish “real” international mobility of capital from fictional mobility which is only instrumentally employed by taxpayers for their illicit practices, and that should not be computed in capital elasticities used to apply the Ramsey Rule.

This paper starts to discuss tax evasion in Section 2 by providing some general empirical estimates and specifically looking at the distribution of evasion among different domestic sectors of the economy. Evidence is provided about its significant magnitude, about heterogeneity of its distribution among income sources and production sectors, and on the (positive) relationship detected in literature between tax rates and the general level of evasion.

Section 3 reviews the most successful theoretical formulations in literature and relevant criticism and extensions, to provide a conceptual basis of a taxpayer's evasion choices to elaborate the subsequent analysis. Both classical stochastic models and more recent contributions from behavioral studies are presented to obtain some general indications about taxpayer's expected reactions to modifications of tax rates and enforcement policies.

Section 4 analyzes the general applicability of differentiated tax rates and levels of enforcement activity, based on observed tax evasions in domestic sectors without transnational capital mobility. First, tax rate differentiation in a Ramsey-like framework is analyzed both under exogenous tax evasion shares and in a general model with endogenous evasion. Then, other means of differentiation through audits and enforcement policies are discussed.

Section 5 finally reintroduces international capital mobility and discusses the limits of the Ramsey Rule applied to open economies in presence of significant and heterogeneous tax evasion. Conclusions are brought arguing that some additional complexity makes the application of the Ramsey Rule to capital incomes hard to enact in practice, when tax evasion is both relevant and heterogeneous.

2. Empirical estimates of tax evasion around the world

Tax evasion is a widespread fact. Nevertheless, few dedicated programs exist to address its statistical measurement, also because of evident difficulties in measuring behaviors that by definition are meant to stay hidden and are not disclosed on a voluntary basis. With regards to developed and under-developed countries, the empirical work of Schneider and Klinglmair (2004) offers a dependable and complete overview of the levels of “shadow economy” estimated with available data. Even if the definition of shadow economy does not coincide with illicitly subtracted taxable bases and so with a definition of “evaded taxable base”, the wider definition of shadow economy (which includes those economic activities not observable by the public, that are also not subject to any tax, as for example some criminal activities) makes a fairly good proxy for tax
evasion at aggregate national level. As suggested by the authors, “the shadow economy includes unreported income from the production of legal goods and services, either from monetary or barter transactions – and so includes all economic activities that would generally be taxable were they reported to the state (tax) authorities. A more precise definition seems quite difficult, if not impossible as the shadow economy evolves over time adjusting to taxes, enforcement changes, and general societal attitudes”. Estimates for the years 1999-2000 tell of average values which appear particularly high in developing countries, with a mean value for African countries as a percentage of GNP of 41%, of 26% for Asian, of 41% for Latin American. OECD countries in the same time bracket are reported a (unweighted) mean value of 16.8% of GDP, with a notable deviation between the highest extreme of 28.7% in Greece and the lowest 8.6% in Switzerland.

More recent estimates by Schneider (2007) still display significant levels of unreported economies. Expressing shadow economy as a share of GDP for the years 2002-03, the unweighted averages are: over 40% in African, Southern American, and Eastern and Central European countries; 30.4 in Asian countries; and 16.3% in 21 OECD countries, with Italy leading the way with an estimated 25.7%, and the U.S. to the bottom line with 8.4%. The unweighted average for the 145 countries included in the analysis is 35.2%. Schneider concludes that “for all countries investigated the shadow economy has reached a remarkably large size”, and “shadow economies are a complex phenomenon present to an important extent in all type of economies (developing, transition and highly developed).”

Researches like Schneider's measure shadow economy as an aggregate share of national or domestic product, but few works obtain data about the classification of tax evasion between sources, or income classes. A notable exception are the U.S. Department of the Treasury - Internal Revenue Service programs called “TCMP” and “NRP”. According to such estimates for the 2001, noncompliance is highly heterogeneous, varying between an evasion rate of 1% for wages and salaries, to 4% for interests and dividends, 12% for net capital gains, and up to an average evasion for personal incomes from commercial activities of 43%, of 29% for the corporate income tax on small enterprises (defined as having less than 10 million dollars of assets), and of 14% for the corporate income tax on large enterprises. These percentages cannot be extended outside the U.S., anyway they are still an indication, in the country with the lowest share of shadow economy in comparison with other OECD countries, of a possible strong variability in tax evasion rates. That such variability is (also) function of international mobility of tax bases, is something that reported empirical data do not allow to evaluate.

Considering instead sectoral data in a high tax evasion OECD country like Italy, the national institute of statistics ISTAT provides some clues about the distribution of shadow economy. With regards to the black market of labor during 1992-2003, an “irregularity rate” (expressing the percentage of irregular labor units on total labor units) is reported varying on average from around 26-32% in agriculture, to 5-6% in industry. Geographically, Italian regions show highly differentiated levels of tax evasion in terms of the “irregularity rate”, varying (in 2003) from about 6% in some Northern regions, to over 30% in Southern regions and islands, with a national average of 13.4%. More recent estimates (up to 2006) do not change these results much, even though a significant general reduction of tax evasion starting from 2003 is observed. Therefore, conclusions may be brought suggesting not only a relevant difference in tax evasion rates observed between production sectors and between classes of workers and income sources, but eventually also a territorial heterogeneity.

Theoretical analysis of taxpayer's behavior in regards to the choice about reporting a part or all taxable incomes (which is defined “fiscal evasion”, and is distinct from “avoidance” defined as

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3 See the review on this topic and data reported in SLEMROD (2007).
4 See ISTAT (2003, 2006, 2008). Note that the methodology adopted by ISTAT is statistical, and differs from results obtained with “macroeconomic” methodologies used by Schneider and Klinglmair.
licit subtraction of taxable bases without violating the Law) is founded on the representation at individual level of incentives given by expected monetary benefit, net of: a) tax liability; b) the tax saving obtained evading the tax; and c) costs of various nature (even not monetary) required and generated by the activity of tax evasion. The following section 3 offers a brief review of the main formulations and of recent critical contributions.

A positive causal relation between high marginal tax rates and shadow economy is detected in many empirical researches\(^5\). But, the exact correlation between tax rates and behavioral responses of work effort and capital mobility is trickier to measure. A positive correlation is found for example in Clotfelter (1983) and Frey, Feld (2002). But, it must be considered the opposite result presented by Feinstein (1991). Also, in Schneider (2007) the positive correlation between tax rates and the share of shadow economy is asymmetrical, and a reduction of tax rates is not as powerful as a means to reduce shadow economy, as it appears to be a strong causal factor for its rise: “even major tax reforms with major tax rate deductions will not lead to a substantial decrease of the shadow economy. Such reforms will only be able to stabilize the size of the shadow economy and avoid a further increase.”

From this round-up on empirical research on tax evasion, we can draw some important conclusions for the analysis of optimal capital income taxation. The first is: tax evasion is a relevant phenomenon in developed, and far more relevant in developing, countries around the world. Hence, since tax evasion modifies net-of-tax return-to-factors, behavioral responses to an increase of tax rates may significantly depend on the elasticity of taxable incomes to causal factors determining tax evasion.

A second result is the possible existence of relevant differences in tax evasion levels between distinct sectors, income sources, or territorial areas. These differentials may affect behavioral responses to fiscal policy and alter the intended distribution of the tax burden under optimal policy targets.

**3. Economic theory of tax evasion**

The following two paragraphs discuss major theoretical contributions to the economics of tax evasion. The provided conceptual grid is needed to understand how tax evasion is affected by tax rates, audit probability and sanctions. This is then instrumental to discuss in section 4 possible ways to discriminate between sectors showing heterogeneous responses to such causal factors.

**3.1 Models based on the expected net benefit**

The standard academic reference adopted to describe stylized taxpayer behavior in terms of tax evasion is represented by the “A-S Model” (from the initial letters of its original proposers Allingham and Sandmo, 1972). In order to synthetically expose the A-S Model, consider: the (linear) income tax rate \( \tau \); a sanction expressed as a share \( y \) of evaded income \( s \); labor income earned before reporting tax liability, equal to \( z \); the constraint \( 0 \leq s \leq z \).

The taxpayer, who is supposed to be rational and risk-averse, maximizes net expected utility \( V \). The value for \( V \) is obtained from a concave and increasing utility function \( U() \), assuming as input variable the earned income net of taxes and sanctions. Therefore, if \( p \) is the probability that an audit from fiscal authorities detects the evasion and inflicts sanction \( y s \), net expected benefit is given by the following expected total utility:

\[
(1) \quad V = (1-p)U(z-\tau(z-s)) + pU((1-\tau)z-(y-\tau)s)
\]

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5 See SCHNEIDER (2007), pp. 6-14, and references cited within.
The first member on the right of the equivalency sign expresses utility provided to the taxpayer by earned income net-of-tax. The last member expresses utility obtained in case the taxpayer is caught evading the tax, and forced to pay both the evaded tax and the sanction. It follows that a honest taxpayer who does not evade any amount of his taxable income chooses a value \( s=0 \), and so obtains a net utility equal to \( V=U((1-\tau)z) \).

The A-S Model was modified by Yitzhaki (1974) who proposed to set the sanction \( y \) as a share of the evaded tax rather than of the evaded income. In this way, assuming risk aversion decreasing with income for the average taxpayer, the substitution effect pushing tax evasion upward as a reaction to an increase of the tax rate is totally compensated, and only an income effect remains for which taxpayers get poorer and reduce their chosen level of evasion.

Models like the one proposed by Allingham and Sandmo, or the generalization proposed by Gary Becker (1968) who applies to the whole range of illicit behaviors an approach based on expected net benefit, constitute an useful starting point from which to build more complex theoretical models. A question is immediately required to be answered: is it realistic that all taxpayers are, to some extent, tax evaders? Realism would suggest a negative answer, and would lead to think the number of expected evaders provided by pure A-S Model to be overestimated. Estimates based on the total level of tax evasion in terms of evaded taxable base, detect that the A-S Model obtains from levels of sanction \( y \) and \( p \) probability applied in real economies, a forecast of compliance which is significantly lower than the one observed\(^6\). Nevertheless, the A-S Model can be modified and extended to include a “disutility” from tax evasion, in order to account for effects of social stigmata\(^7\) and obtain more realistic outcomes.

A different direction toward which to develop the A-S Model should therefore consider social effects, and analyze taxpayers' behavior within groups. As noted by Sandmo\(^8\), there exist various channels through which social context may affect the perception of variables affecting the \( V \) value. One of these channels is due to limited information: if the probability \( p \) is made endogenous and dependent not only from a taxpayer's level of evasion, but also from the perception that the taxpayer has of the tax evasion of his neighbors, then multiple equilibria are possible even with the same objective probability value of \( p \) and sanction \( y \). This remark may be extended not only to the observation of other taxpayers' behavior, but to a larger set of factors influencing the perception of the \( p \) probability, for example different ways tax evasion is represented by mass media.

Stochastic models like the A-S present some additional complexity when included in formulations addressing wider scopes. This is true within optimal income taxation research, where the taxpayer reacts to a modification of the income tax varying his amount of work effort together with his reported income. In a paper published in 2001, Joel Slemrod proposed therefore a general scheme where the stochastic and risk-driven approach to tax evasion is discarded. In such formulation, the taxpayer maximizes an utility function which is positively affected by the level of individual consumption (obtained from labor income net of tax, and considering the tax saving obtained through tax evasion) and by the level of income obtained from different sources (including leisure), and is negatively affected by the amount of hours worked and by a “cost” of evasion \( C \), increasing function of the amount of evaded labor income and on the amount of avoidance itself.

Leaving the more detailed scheme of stochastic models, Slemrod's approach while not as much analytical, allows to include tax evasion inside other trends of study, without necessarily requiring to manipulate a potentially large number of variables, and at the same time not excluding \textit{a priori} the chance to define cost function \( C \) (which Slemrod elaborates in implicit form to comply with the general aim of his work) in terms of dependency from the \( p \) probability and the level of

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\(^6\) See FREY, FELD (2002), pp. 3-6; and TORGLER (2003). See also the remark discussed in SLEMROD (2007), pp. 38-39, according to which this gap would be mainly due to under-measuring of actual tax evasion.

\(^7\) See SANDMO (2005), pp. 10-12; and the model presented in MYLES, NAYLOR (1995).

\(^8\) See SANDMO (2005), pp. 21-22.
sanction $y$, exogenously included into the model\(^9\). The generic definition of a function $C$, coupled with the tax saving obtained from the choice to evade a quantity $s$ of income, opens the way to the inclusion in analytical terms of some elements which are not strictly tied to expected monetary benefits, but instead tied to psychological and emotional factors like the ones described in the following paragraph.

3.2 Elements of ethics and psychology

The introduction of ethical or psychological variables in taxpayers' behavior enriches, and sometimes radically modifies, results proposed by models based on the expected benefit\(^10\). The importance of these variables is strengthened by results provided by behavioral economics studies\(^11\).

Bruno Frey\(^12\) proposed to introduce the concept of “crowding out”, which is both an ethical factor linked to a need for justice, and a psychological factor tied to emotions and to a set of features of human personality that we may consider purely individual. When analyzing the behavior of groups of taxpayers, if these factors' outcomes are not zero-sum, and on average for the considered group a tendency prevails to react to taxation with a common trend scheme, such crowding out effect may significantly affect observed taxpayer behaviors.

The fittest example for the sake of the present discussion relates to audit and punishment activities by fiscal authorities. If the average taxpayer perceives an increased level of audit enforcement as an unfair coercion, he may react by increasing, instead of reducing, his share of evaded income. In this case, the crowding out effect might reduce, entirely compensate, or reverse the expected reaction based on models built on expected net benefit, leading to a sterilization of enforcement policies or even to an opposite reaction. With crowding out effects the reported income elasticity to an increase of the level of enforcement depends from a sum of two independent effects: (a) a reduction of evaded income due to the increase of the expected cost of evasion, reducing net expected benefit according to traditional Becker (1968) scheme; (b) an additional, upward effect on tax evasion deriving from a moral reaction following a perception of having suffered an unfair coercion. The second effect (b) can be also correlated to the attitude and manner shown by fiscal authorities when contacting and dealing with a taxpayer, since these may contribute to generate a sense of higher or lower helplessness, and a perception of a more or less arbitrary treatment\(^13\). In this case, the intrinsic motivation of the taxpayer not to evade depends from his opinion about the fairness of the general public, which is (also) represented in person of the policy-maker and fiscal authorities\(^14\).

The crowding out effect, interpreted this way (as an “ethical” need), presents two interesting aspects:

1) It is probable that such sense of unfairness will be stronger in those who have never evaded taxes, than among habitual evaders. Therefore, it may be observed a rise in the number of tax evaders after an increase of taxation, more intense than what predicted by expected net benefit schemes.

2) In a dynamic context, it is possible that the crowding out effect depends on previous states of the world, and so it may generate an accumulation of “negative social capital” over time.

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\(^9\) On the possibility to convert stochastic expected net benefits into certain equivalents, see for example COWELL (1990). A similar strategy will be adopted in section 5 to deal with capital income taxation, in order to keep discussion general and not tied to a specific taxpayer's behavioral model.


\(^12\) See FREY (1997); and specifically for the application of the crowding out effect to tax evasion: FREY, FELD (op. cit.).

\(^13\) See FREY, FELD (op. cit.), pp. 8-11.

\(^14\) See also TORGLER (2002).
when fiscal pressure, or the level of enforcement, remain high for some time.\textsuperscript{15}

3) Considering taxation as a social act, taxpayers' willingness to evade may be conditioned by the observed behavior of other taxpayers.\textsuperscript{16} This factor of “tax morale” supports the idea of a link between individual taxpayers decisions about tax evasion and other taxpayers' which is not based on informational constraints (an hypothesis discussed under par. 3.1 as a possible extension of expected benefit models), but on social and ethical needs.

The crowding out effect may happen not only through the ethical channel discussed before, but as a consequence of a transfer of self-controlling individual functions of the taxpayer.\textsuperscript{17} When the expected sanction for a violation of a binding law is increased, the intrinsic motivation bringing the individual to limit some behaviors within self-imposed boundaries may be reduced, because the “controlling” function is moved by the individual from his interior and personal sphere, to external institutions to which the power-right to control and punish such socially despicable or dangerous behaviors is demanded. This psychological channel is based on the idea that individuals are able to build, when punishments are not severe, a spontaneous motivation to obey the law, or if no binding law exists, to obey social norms.

A distinct trend of study completely departs from the expected net benefit framework and states the possibility that tax evasion choices, or more generally any choice on the violation of civil rules, or even in any situation where human beings stand before a choice, strongly depend from emotional factors. Feelings like: embarrassment, guilt, fear, remorse, are emotions that may associate with being afraid of being caught while performing a censurable act, like evading taxes, and so they may amplify (or dampen) deterrent effects imposed by a higher probability of being caught. This emotional factor may be exploited by policies aimed at amplifying the effect of enforcement, for example introducing by law the publication of detected evaders' names.\textsuperscript{18}

While the theory of crowding out adds to, and does not intend to substitute neoclassical economics, some authors deny such formulation when dealing with tax evasion, for which elements of emotional and cognitive nature (probably together with information constraints, and a fundamental inability of the human being to manage large number of data at the same time) would be so much prevailing to generate a gap between taxpayers choices in real economies and results provided by traditional theoretical literature\textsuperscript{19}, given the supposed inability of the individual to maximize total utility expected from his choices, in the sense defined by rational expectations and market efficiency theory.

In the analysis that follows, taxpayer's behavior is not modeled explicitly in order to keep discussion general and allow for different microeconomic foundations. Social and behavioral variables are allowed throughout the text as possible modifying factors of the outcomes of basic A-S Model (or of the Allingham-Sandmo-Yitzhaki Model), but taxpayers are always considered as rational, utility-maximizing decision-makers.

\section*{4. Tax discrimination based on tax evasion}

In a recent contribution, Peter Sørensen (2007) argued in favor of the introduction of differentiated capital income taxes, inversely proportional to the degree of international mobility of

\textsuperscript{15} An empirical test of such observation would constitute an important argument against \textit{una tantum} taxation.\textsuperscript{16} See the results presented by FREY and TORGLER (2007).\textsuperscript{17} See AKERLOF, DICKENS (1982); and FREY (op. cit.), chapter 9.\textsuperscript{18} See CORICEILLI ET AL. (2007).\textsuperscript{19} Specifically some authors argue that rational choices and time consistency of preferences for well-informed agents are often too unrealistic assumptions to describe individual behaviors observed in real markets. For a critical interpretation of some of the main results obtained by Behavioural Economics research, see LANTERI, CARABELLI (2007).
capitals. The ratio of this theoretical position directly recalls results from the tradition of optimal taxation: if different forms of employment of capital, or different production sectors to invest into, present a differentiated tax base elasticity, it seems reasonable given the exogenous taxation revenue objective of the policy-maker, to burden more the less mobile capitals, and to burden with lighter taxes (and aligned with average tax rates applied internationally\(^{20}\)) highly mobile capitals. In an open economy, where it is not feasible or it is very costly to monitor capitals moved to foreign countries and to tax them uniformly with a neutral tax equal to the tax rate applied on domestic capitals, if for any reason the government cannot burden the entire tax on immobile factors (a reasonable assumption under distributional needs, if we consider that such immobile factors include labor, and properties which include housing), then it appears optimal to apply a Ramsey Rule adapted to capital taxation. This modified Ramsey Rule states that the optimal\(^{21}\) differentiated tax on domestic capital incomes is the one which, at the margin, causes the same proportional (quantitative) loss of investment in each sector\(^{22}\).

In real countries this approach is observed in property investment taxation, where often the general income tax is accompanied by property taxes, additional income tax rates, or taxes on “normal” incomes, designed in a way that makes tax burden on property higher than on employments of capital in mobile assets. Moreover, some formulations of Scandinavian *Dual Income Taxation* include a lower tax on capital invested by foreigners, and in some countries the tax burden on financial assets, which are usually particularly mobile, is kept lower than other kinds of investments, or is subtracted from personal income tax progressivity. Sørensen's arguments, therefore, offer a motivation to sustain such policy approach observed in real countries.

Transnational mobility of capitals, and the elasticity of taxable bases in terms of tax evasion, are two phenomenons often connected, but not necessarily nor always so. To provide some examples where mobility and evasion are dissociated:

1) capitals invested in regulated financial markets are very mobile internationally, but tax evasion is relatively easy to cope with in practice thanks to the ability to levy income taxes (and eventually, taxes on capital gains) on financial intermediaries managing operations on electronic trading systems.

2) Invested capitals in property are by definition “immobile”, and are also easily detectable by fiscal authorities, being not concealable.

3) Earnings obtained by professionals and other self-employed workers are often easier to evade, but their international mobility depends on anti-money laundering and customs controls, especially as far as cash or digital currency movements are concerned.

4) Incorporated firms are subject to accounting duties that may make tax evasion costlier, but capital mobility is facilitated by the ability to register separated branches abroad, which thanks to non-extensibility of domestic fiscal audits in other countries (with some notable exceptions) and thanks to the shield of anonymity provided by some countries, allow to transfer and hide capitals from the eyes of domestic fiscal authorities, through methodologies that are difficult to detect without thorough investigations.

Therefore, mobility has to be separately studied from tax evasion. My approach to this topic is then to analyze the behavioral response in terms of tax evasion to differentiated tax rates or audit probabilities, when no behavioral response in terms of capital mobility or work effort is present. In

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\(^{20}\) In the cited model, Sørensen imposes an arbitrage constraint, for which net-of-tax returns obtained from domestic investments of capital must be equal to the expected return (net of any source tax) obtainable abroad. See SØRENSEN (op. cit.), p. 17. When gross returns between countries are equal, such constraint translates into the need to have equal average effective tax rates. When considering countries with similar economies and economic development, as OECD countries are, the latter situation fairly approximates reality, and allows to discuss in terms of a parity of tax rates instead of a parity of expected net returns, as usually it is observed in political debates.

\(^{21}\) “Optimal” in the meaning used by Ramsey is, of course, the tax policy which brings the least total excess burden to taxpayers.

\(^{22}\) See SØRENSEN (op. cit.), pp. 16-20.
section 5, these findings are then used to build a general framework to jointly address the two elasticities of reported incomes and of capital mobility.

Considering just (domestic, and transnational) tax evasion excluding licit phenomenons of tax avoidance, we may note how some production sectors show a higher presence of evaders, or of average evaded tax per evader. Following these considerations, a question arises if, to what extent, and how, is acceptable and beneficial to levy a differentiated tax, designed to burden more (or less) sectors denoting higher evasion. From this point of view, the word “sector” may refer to a product sector\textsuperscript{23}, but also to a class of income earners, discriminated by their level of income or wealth, or by income source (self-employed labor, capital, employed labor). Moreover, if a geographically differentiated tax evasion is observed, “sectors” may be constituted by territorial aggregates. In this case it is required a coordination with local and decentralized taxation to guarantee not only fiscal equity (individual equity if taxpayers' preferences are considered geographically homogeneous, or equity between territorial groups if heterogeneous preferences are accepted), but also to minimize distorsive effects on the allocation of production factors between local jurisdictions.

The following three paragraphs discuss how tax rates and audits may be differentiated between two domestic sectors based on differences in tax evasion. Results show that while theoretically it is indeed possible to design a pair of tax rates minimizing total excess burden, a number of relevant practical issues have to be addressed. Some of these difficulties (and a few more) are met in case the policy-maker uses the tool of differentiated enforcement activities and audits instead of tax rates.

4.1 On tax rates

Following an approach à la Ramsey\textsuperscript{24} applied to tax evasion of capital income taxation, the effective tax rate levied on domestic sectors where tax evasion is higher (from now on marked as a single sector $A$) should be differentiated from the tax rate levied in those sectors (marked as $B$) where taxpayers evade (or are in a position to evade) less.

If:

- $r_0$ is the gross unit return to capital made equal in sectors $A$ and $B$;
- $s_A > s_B$ express the evaded unit returns;
- $Q(r)$ is the function (equal in both sectors) expressing supplied quantity of capital in equilibrium at a given level of unit net-of-tax return $r(t)=r_0-t(r_0-s)$, and demand is considered perfectly inelastic at return $r_0$;
- the policy-maker is benevolent and utilitarian, and wants to minimize total excess burden, given an exogenous revenue constraint;
- $t_A$ and $t_B$ are ad valorem taxes levied on reported incomes from invested capitals (in the form of interests, dividends, or rents);

then optimal differentiated taxation is given by\textsuperscript{25}:

$$
(2) \quad t_B = \frac{r_0 - s_A}{r_0 - s_B} \frac{Q^2(r) e_A}{Q^2(r) e_A},
$$

where $e_A$ and $e_B$ are the point elasticities of supply to the tax rates $e = \frac{\partial Q(r)}{\partial t} \frac{t}{Q}$, given net-

\textsuperscript{23} The football sector is a particularly well-fitting example of a production sector where tax evasion practices are commonly observed. See the OECD-FATF (2009) report.

\textsuperscript{24} See the fundamental work of RAMSEY (1927).

\textsuperscript{25} The following results are formally obtained in the Appendix, at point (A).
of-tax and net-of-evasion unit returns in the two sectors:

\[
\begin{align*}
(3.a) \quad r_A(t_A) &= r_0 - t_A(r_0 - s_A) \\
(3.b) \quad r_B(t_B) &= r_0 - t_B(r_0 - s_B)
\end{align*}
\]

Equation (2) may be thought as an adaptation of classical Ramsey Rule\(^{26}\) to two domestic sectors where tax evasion levels are different, elasticity of supply to net unit return is constant, and demand and supply follow the same descriptive functions in both sectors: if no tax evasion is observed at all, optimal tax rates are to be the same in \(A\) and \(B\) since it will be verified that \(e_A = e_B\) and \(Q(r_A) = Q(r_B)\). Tax rates have to be equal also if tax evasion is not null and equal in both sectors, that is if: \(s_A = s_B > 0\).

This result holds if tax evasion is invariant to the tax rate. The invariance of tax evasion to the tax rate is a special case which may be theoretically considered by adopting the A-S Model modified by Yitzhaki (see previous paragraph 3.1), and taking income effects as negligible. Or, the share of evaded income may be considered constant for small variations of the tax rate. Also, tax evasion may be locally invariant to the tax rate in case crowding-out effects (see par. 3.2) are relevant and able to nullify taxpayer's positive reaction to a tax rate decrease. Therefore if the assumption that \(\frac{\partial S}{\partial t} = 0\) is acceptable, total distortions caused by taxation for a given taxation revenue objective set by the policy-maker, measured as excess burdens, are minimized if the policy-maker is able to obtain aggregate estimates for elasticities and tax evasion rates in sectors \(A\) and \(B\), and levies different tax rates according to equation (2).

As discussed in Section 2, tax evasion is normally sensitive to the tax rate, therefore equation (2) does not provide a general solution for the optimizing policy-maker (even though it may provide an approximation for small variations of the tax rate). Let \(s_A(t_A)\) and \(s_B(t_B)\) express such variable evasion rates, and imagine \(s_A(t_A)\) to have steeper derivative than \(s_B(t_B)\), and consider both to be increasing with \(t\).\(^{27}\) Initially when no tax is levied, tax evasion is null in both sectors. A progressive introduction of a neutral tax in \(A\) and \(B\) brings taxpayers in sector \(A\) to evade more than taxpayers in \(B\). Taxpayers in \(A\) enjoy an expected net-of-tax return higher than taxpayers in \(B\), so the quantity corresponding to the point where demand and supply meet in equilibrium is higher in \(A\) than in \(B\).

To express the optimizing rule with tax evasion sensitive to tax rates, equation (2) must be substituted by the following\(^{28}\):

\[
\begin{align*}
Q_A(r_A) - \frac{t_A}{(r_0 - s_A(t_A))} \frac{\partial s_A(t_A)}{\partial t_A} Q_A(r_A) + t_A \frac{\partial Q_A(r_A)}{\partial t_A} = e_A Q_A^0 [ -t_A \frac{\partial s_A(t_A)}{\partial t_A} + r_0 - s_A(t_A) ]
\end{align*}
\]

\[
\begin{align*}
Q_B(r_B) - \frac{t_B}{(r_0 - s_B(t_B))} \frac{\partial s_B(t_B)}{\partial t_B} Q_B(r_B) + t_B \frac{\partial Q_B(r_B)}{\partial t_B} = e_B Q_B^0 [ -t_B \frac{\partial s_B(t_B)}{\partial t_B} + r_0 - s_B(t_B) ]
\end{align*}
\]

Equation (3) allows both for differentiated supply functions and tax evasion. Under general assumptions, no simple rule as the one expressed in (2) is possible. Therefore, practical utility of simple optimal taxation rules meet significant limits in the complexity brought by tax evasion. A

---

26 Note that \(e_A\) and \(e_B\) are the elasticities of supplied quantity to the tax rates, and not as in original Ramsey's analysis, the elasticities to price.

27 In the present discussion it is assumed an elasticity of tax evasion positively correlated to tax rates.

28 Equation (3) is obtained with the same procedure used for equation (2) and illustrated in the Appendix under point B). Constant exogenous shares \(s_A\) and \(s_B\) therefore become \(s(t_A)\) and \(s(t_B)\), both having non-null first derivatives with respect to tax rates.
policy-maker willing to follow (3) would need a significant number of data: a supply function \( Q(r) \), sufficiently stable in time to be empirically estimated; a formula to derive evaded income \( s \) as a function of the tax rate \( t \), given exogenously fixed audit probabilities and sanctions for each domestic sector; the elasticities of supply to net return-to-factor.

Some additional arguments may be brought against formulations (2) and (3) of optimal taxation:

1) Departing from the hypothesis of a benevolent policy-maker, if sector \( A \) includes more evaders (or, taxpayers who evade taxes for higher average amounts in comparison with sector \( B \), and equation (3) leads to \( t_B > t_A \), the burden of a lower tax rate on \( A \) could be possibly perceived as an ethical violation, and thus be rejected by the electoral system (as long as the voting majority is not a group formed by evaders in sector \( A \)). An ethical problem could then rise, tied to the acceptance by taxpayers-voters of that body of written and unwritten norms, all-together constituting the “social capital” of a community.

2) A second issue relates to efficiency. When talking about tax evasion without international capital mobility where reported income elasticity to tax rate is negative, the tax rate differential here discussed may bring a domestic inter-sectoral mobility, of honest investors attracted by the fiscal advantage in terms of expected net returns in the less taxed sector. Such capital movements induce an evident distortion in factor allocation in comparison with a situation where tax revenues are financed with neutral taxation, with consequent negative impact on national productivity. Also, if capital elasticity in terms of domestic inter-sectoral mobility is higher than the negative elasticity of reported incomes to tax rates, this may induce a transfer of capitals owned by habitual evaders who intend to exploit the more favorable fiscal treatment in the less taxed domestic sector, without reducing their planned tax evasion rate, in such a way that the final effect is not easily forecast.

3) Lastly, an issue may arise in terms of equity within each sector, even in complete absence of inter-sectoral mobility of investments. Within the most taxed sector honest taxpayers will see their tax burden grow, while evading taxpayers (under hypothesis of positive elasticity of tax evasion to tax rates) will possibly increase their share of tax evasion. The effective tax burden after fiscal reporting (given a fixed probability of detection and sanction) increases for the honest taxpayer more than for the tax evader, and so not only equity is violated in such sector, but it is done to the detriment of non-evaders thus rising ethical issues. If inter-sectoral mobility exists, in absence of information constraints and price viscosity, the increased tax burden will be capitalized in the market value of assets in the highest taxation sector, therefore (honest) taxpayers will not be able to avoid the increased burden by adopting an “exit” strategy.

These objections, together with the complexity of (3), constitute a significant obstacle to the adoption of effective differentiated tax rates designed with the aim to minimize total excess burden in presence of heterogeneous and significant tax evasion. But, one could argue that such differentiation is not necessarily to be designed using tax rates, or the definitions of the taxable bases (i.e. with differentiated deductions). This topic is discussed in the following paragraphs 4.2 and 4.3.

### 4.2 On automatic tax audits

Some countries deal with tax evasion by adopting presumptions by law about “normal” or “minimum” levels of taxable incomes, computed on the basis of fixed ratio indexes or statistical data obtained analyzing prices, incomes and assets on private markets in fixed time intervals.

Perhaps one of the most complete mechanism of this kind are so-called “studi di settore” adopted in Italy and similarly in some other countries with the aim to fight tax evasion among small

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29 On the role of tax evasion with regards to equity needs of taxation, see SCOTCHMER (1987).
enterprises and self-employed workers.

“Studi di settore” and other similar tools that find, in sort of an “automatic” way, a level of taxable income under which taxpayers are not allowed to report, or under which the proof burden is reversed on the taxpayers and in favor of fiscal authorities, provide a hard limit (of produced incomes with respect to owned assets) under which taxpayers are forced to pay the tax (or to provide proof of their “abnormally low” reported income) also on unearned incomes. The earning of higher incomes in comparison with said minimum limits, does not suffer for any additional burden: taxpayers do not face a higher risk of sanctioning (in comparison with a situation where automatic audits are not enforced) if they report just the minimum income obtained by automatic tax audits, and evade (even totally) incomes over such minimum.

The adoption of automatic audits differentiated between more and less evading domestic sectors encounters at least the same hardships discussed for differentiated tax rates. By imposing a minimum “normal” taxable income, higher in one sector for reasons similar to those discussed in a Ramsey-like framework for tax rates, the tax burden also falls onto honest taxpayers investing their capitals in that sector. This additional burden is unfair under equity needs.

It is to highlight that a tax hitting “normal” income or, alternatively, the effective earned income if the latter is reported to be higher than the former, is equal in practice to levying a tax on capital income exempting “normal” return-to-capital associated with a wealth tax. Under this view, the adoption of differentiated automatic audits for sectors $A$ and $B$, equals to levying a differentiated wealth tax, accompanied by a neutral tax hitting only incomes beyond the “normal” return to capital.

### 4.3 On audits and punishments

If both the adoption of differentiated tax rates, and the use of different automatic audit mechanisms are rejected, it only remains one last tool policy-maker may use: enforcement, that is the combined technology of fiscal reporting auditing, of controls of different kinds (inspections, accounting duties, etc.), and of sanctions both preemptive (i.e. in case of omitted fiscal reporting) and subsequent to the detection of an act of tax evasion.

The choice for an enforcement activity focused on the higher tax evasion sector seems, at a first glance, a good way to obtain revenue without harming ethical, efficiency and equity needs, as long as total level of public spending (which would otherwise also burden honest taxpayers) is not increased as a consequence of higher levels of enforcement.

This point of view is arguable noting that many kinds of fiscal duties charge taxpayers with additional costs (“red tape costs”). Therefore, the enforcement tool may be compared to some extent to the adoption of differentiated tax rates illustrated in paragraph 4.1, bringing again the same issues in terms of ethics, allocation distortions through capital mobility, and equity. In other words, an enforcement policy focused on some domestic sectors could be optimal if it would not cause a burden also for honest taxpayers. Moreover it may cause a crowding out effect (see par. 3.2) for which the increased level of enforcement could provoke an increase in tax evasion for those taxpayers who would have not otherwise evaded the tax, or not as much.

Going back to the initial assumption according to which the policy-maker exogenously chooses his taxation revenue objective, the increased public spending due to enforcement activities is summed to such objective, so the enforcement may increase tax revenue reducing evasion, but may also require itself an additional (distortionary) tax revenue. The question on what enforcement level is to be considered optimal is therefore a typical problem of cost-benefit analysis, that has been extensively treated in literature. Given a level of public spending devoted to enforcement,
focusing activities in sector A may increase expected revenue only if the elasticity of reported income to enforcement levels is higher than in sector B. But, this is not obvious at all, even if by assumption the elasticity of reported income to tax rates are higher in sector A.

By adopting the A-S Model, partial derivatives of the function expressing reported income with respect to sanction y and p probability (see previous paragraph 3.1), are always positive. On the contrary, partial derivative of the function expressing reported income with respect to tax rate does not always assume a value above or below zero, but strictly depends on the assumptions accepted for the taxpayer's utility function, who may or may not be risk-averse and with varying intensity. The fact that a habitual evader chooses to invest in sector A, by itself does not provide hints about the behavior of such taxpayer in regards to risk. It is equally possible that after a modification of the enforcement policy, the taxpayer chooses a tax evasion level of s, and moves his investments in sector A to minimize risk, or on the contrary that he accepts a given level of risk (maybe very high), and transfers to sector A to maximize his net expected benefit adapting his evasion level s accordingly. In other words, we cannot be sure that the taxpayer's choice happens simultaneously in regards both to risk level, whose perception as previously noted may be blurred by limited information and dependent from the observation of other taxpayers' behaviors, and to monetary benefit. Rejecting this simultaneity of choice, models based on stochastic evaluation of net benefit may result inadequate to provide practical guidelines in this ambit.

But even accepting the assumption of simultaneous rational choice with perfect information, and assuming equal expected gross returns in both sectors A and B, still we are unable to state with certainty that the higher elasticity of reported incomes to tax rate in one sector, is higher also with respect to the enforcement level. The first elasticity might be a symptom of taxpayers' perception of enforcement and audits efficacy, or of a different attitude toward risk which would lead to consider taxpayers' elasticity to enforcement lower in sector A than in B. In conclusion, in absence of a robust and dependable behavioral model expressing reported income elasticity to fiscal enforcement activities, a differentiation of the latter among domestic sectors with higher or lower observed tax evasion appears quite an hazard.

A different criterion favoring differentiated fiscal audits may be found in the idea according to which sector A showing higher tax evasion, allows to obtain higher percentage of positive audits, or higher average detected unreported incomes. If this is the case, under cost-benefit analysis each dollar spent for enforcement would generate more benefits in sector A than in sector B.

This approach, too, is arguable. The higher observed evasion in sector A will hardly be ascribable to reasons tied to single individuals investing their capitals there, but rather it seems reasonable to suppose that such behavior is incentive-compatible and due to the fact that in sector A, tax evasion is easier or less costly, or (given an equal level of enforcement) the risk p of a positive audit is lower. If this is the case, then spending more for enforcement activities in A instead of B is exactly the opposite of what cost-benefit analysis suggests.

The two domestic sectors may be defined as distinct geographical zones, where different cultural and social backgrounds bring to a diversified risk perception and evaluation. In this case sector A may include financially evolved zones where taxpayers are widely knowledgeable about the instruments they are allowed to use to hide incomes to fiscal authorities. Or, sector A might include less developed zones, like rural and suburban areas where taxpayers' perception of the enforcement activities is such that the latter has milder impact on their expected net benefit. In these cases too the effectiveness of enforcement is reduced in sector A in comparison with B, so under taxpayer who chooses to evade, and as a consequence a stochastically defined cost. The trade-off for the welfare-maximizing policy-maker is transferred on the choice among the distortions induced in private markets by a tax which is designed to be harder to evade, and a less distortionary but easier to evade tax.  


32 Differences in taxpayers' behavior associated to geographical variables may be due to a number of distinct reasons: differences in "social capital", in criminal rates, in levels of shadow economy, in the structure of territorial government and taxation. See BROSIO ET AL. (2002).
strict efficiency criterion, a discrimination of fiscal enforcement should be focused on sector B33.

5. Internationally mobile capitals with tax evasion

Now it is possible to reconcile the analysis of capital taxation when international mobility is observed, with tax evasion.

To simplify discussion, I assume the existence of a function \( a(.) \) expressing unit “cost” of evasion (for each unit of evaded income), and a function \( r_s(t, r^{L_0}, a) \) that associates to each possible tax rate \( t \) on capital income a share \( r_s \) of evaded unit income, given the expected gross return on invested capital and the unit cost \( a(.) \). The values obtained for \( r_s \) and for \( a \) may be interpreted in principle as a synthetic representation based on expected net benefit formulations, given (exogenous) values for sanctions and positive audit probability, in the spirit of Slemrod (2001). What is of interest here is how \( r_s(t, r^{L_0}, a) \) reacts to changes of \( t \), being it increasing with \( t \) or not, taken other exogenous variables as constants. Therefore, utility maximization by taxpayers is not made explicit in the following discussion, and it is considered as implicitly pursued within the choice expressed by \( r_s(t, r^{L_0}, a) \) determining the level of chosen tax evasion34.

Let us define the following:

- a) \( X \) and \( Y \) are domestic production sectors, where capital is internationally mobile but immobile between the two sectors. \( X \) presents a capital elasticity to expected net returns, in terms of trans-national mobility, higher than sector \( Y \).
- b) \( t \) is the rate of a source-based ad valorem tax on capital income, initially made equal in both sectors and marked as \( T \). The policy-maker is supposed to be unable to tax foreign capital income, therefore the source-based tax \( t \) is only levied on domestic capital.
- c) \( r^{L_0} \) is the unit return to capital, gross of tax and before tax reporting, obtained in sector \( L^0 \) (\( X \) or \( Y \)). In a static analysis setting \( r^{L_0} \) is a constant. Par. 5.2 will discuss the modifications of the gross return in subsequent times.
- d) \( K_L \) is total capital invested abroad, whose returns are not taxable by domestic policy-maker; \( K_X \) and \( K_Y \) are, respectively, total capitals invested in sectors \( X \) and \( Y \). Capitals in the short run (long-run considerations are discussed in a second stage) are imperfectly internationally mobile, therefore their elasticity to net expected return is positive but with finite values. A total stock of capital is available and constant for all sectors and equal to: \( K_{TOT} = K_K + K_X + K_Y \).
- e) \( a(.) \) is, as already stated, the unit cost of tax evasion made dependent: 1) partly from exogenous elements, like the (perceived) probability of a positive audit and sanction level, and the expenses to be undertaken to hide capitals to fiscal authorities; 2) partly and with second order effect from the level of the tax rate \( t \) and of the gross-of-tax return \( r^{L_0} \). \( a(.) \) may assume different values \( a_x \) and \( a_s \) in sectors \( X \) and \( Y \).
- f) \( r_D \) is the unit net-of-tax return to capital, considered: 1) the amount of evaded income \( r_s(t, r^{L_0}, a) \) (which differs in sectors \( X \) and \( Y \) due to different values of unit cost \( a_x \)); 2) unit cost \( a_k \) met in sector \( L^0 \) to obtain such evasion level, and therefore: \( r_D = r^{L_0} - t (r^{L_0}_0 - r_s(t, r^{L_0}_0, a_s)) - a_k r_s(t, r^{L_0}_0, a_s) \).

33 With regards to enforcement activities, like with tax rates and automatic audits, there is potential for a clash with diffused ethical norms. Which political party is able to convince the voting body that it is better to concentrate audits and controls on sectors where taxpayers evade less? And what final effects (i.e. if crowding out à la Frey is relevant) may be expected if such policy is concretely enacted?

34 This approach does not exclude that a more articulated formulation (which will not be covered in this discussion as not affecting the core arguments here illustrated), should include a representation of the level of evasion \( r_s \), given constant values for \( r^{L_0} \), \( t \) and \( a \), as function also of the amount of invested capital and earned income, and a representation of unit cost \( a \) which should probably also depend from the total amount of evaded taxes other than unit value (in other terms, it is probable that unit costs sustained for each evaded dollar are not the same if one evades few dollars or large capitals). It could also include costs of non-monetary nature as the ones discussed under paragraph 3.2.
$r_{0f}(.)$ is initially considered equal in both sectors $X$ and $Y$, and may be increasing or decreasing with $t$. Moreover, $r_{0f}(.)$ varies from a minimum value of $r'_{0}(1-t)$ in case no evasion is employed (so $r_{s}(t, r'_{0}, a_{s})=0$), to a superior extreme when total evasion occurs that is dependent from the form of the functions $r_{s}(t, r'_{0}, a_{s})$ and $a_{t}$.

g) $t_{o}$, $t_{i}$ are taxes defined in the same way as the neutral tax $T$, but differentiated for sectors $X$ and $Y$.

h) $r_{T}$ is the net return defined by $r_{0f}(.)$ and obtained in both domestic sectors when the uniform tax rate $T$ is applied. $r_{o}$, $r_{s}$ are the net sectoral returns defined by function $r_{0f}(.)$, when differentiated tax rates $t_{o}$, $t_{i}$ are applied

i) $r'_{k}$ is the expected value of $r_{s}$ or $r_{o}$, that taxpayer expects to obtain investing in sector $L^{th}$ after having decided the share of income to evade.

j) $k^{L}(r'_{k})$ expresses the value for individual investment in sector $L^{th}$, chosen as function of the expected return $r'_{k}$. Total investment abroad is obtained by difference as $K_{E} = K_{TOT} - K_{X}- K_{Y}$.

k) the policy-maker is supposed to be benevolent and maximizing an utilitaristic social welfare function. The welfare maximization is pursued by minimizing total excess burden caused in the two domestic sectors, subject to an exogenous taxation revenue constraint.

### 5.1 Static analysis

The implicit assumption throughout the following discussion is that each individual, after having decided to invest in sector $L$, chooses first his level of tax evasion $r_{s}(t, r'_{0}, a_{s})$ based on perceived values of enforcement (probability of a positive audit, level of the sanctions) and on the observed value for $r'_{0}$ and for tax rates. Consequently, he decides his target net-of-tax and net-of-evasion return $r_{L}$. In a subsequent stage, the taxpayer who chose a given target value for $r_{L}$, decides about how much capital $k^{L}(r'_{k})$ to invest in sector $L$ and how much capital to invest abroad, function of the difference of net returns obtained comparing the (exogenous) net return expected abroad, with the domestic expected net return $r'_{k}$ obtainable considering his previously taken choice of tax evasion. Given a distribution function $f(r'_{k})$ which associates to each possible level of $r'_{k}$ the number of taxpayers who chose the same behavior in terms of target net return, total capital employed in that sector will be:

\[
K_{L} = \int_{r_{0}^{L}(1-t)}^{r_{0}^{L}} k^{L}(r'_{k}) f(r'_{k}) dr'_{k}
\]

In an academic framework where all taxpayers maximize expected utility as a function of the expected net return, and where it is possible to detect an abstract “representative agent” who expresses with his values for $r'_{k}$ and $k^{L}$ the “average” behavior of all taxpayers investing in that sector, it becomes possible to graphically represent his behavior thus simplifying the illustration of some interesting phenomena. I will adopt this simplifying hypothesis only for reasons of opportunity instrumental to the following graphical representation.

Given equation (4) and the taxable unit income $R_{L} = r_{0}^{L} - r_{s}(t, r'_{0}, a_{s})$ reported by the representative taxpayer, the policy-maker taxation revenue is given by: $t_{x} R_{X} K_{X} + t_{y} R_{Y} K_{Y}$.

Net return $r_{T}$ is initially made equal in both production sectors $X$ and $Y$ for the representative agent, and equal to the average expected net return obtained investing abroad. This equals to affirm that a condition of international arbitrage is respected before introducing modifications to the neutral tax $T$. The initial condition where the tax $T$ is adopted, to which it corresponds the same net-of-tax and net-of-evasion return $r_{T}$ in both domestic sectors, is represented in graph $A$ (curves are designed as straight lines only for the sake of simplicity). On the left the supply curve is drawn for sector $X$, which presents by assumption a less steep inclination in comparison with sector $Y$ (drawn
in the right box). The demand curve in both sectors is supposed to be horizontal at the level of gross return \( r^L_0 \), and such that for every considered level of net return, it absorbs the entire quantity of invested capital \( K_L \).

On the horizontal axis the aggregate invested capital in equilibrium is represented. Points \( A_X \) and \( A_Y \) correspond to net returns obtained in absence of tax evasion; points \( B_X \) and \( B_Y \) correspond to gross returns, which are visually useful as a benchmark to evaluate distortions caused by tax rates. On the vertical axis returns are represented: gross-of tax returns; net-of- tax returns; and net-of-tax and net-of-evasion returns.

Starting from the situation represented in graph \( A) \), the application of differentiated tax rates in compliance with modified Ramsey Rule for open economies without tax evasion, would need \( t_x < T < t_y \). If we set an additional assumption for the representative agent, forcing \( r_D(.) \) to be strictly decreasing with \( t \), but as the tax rate increases his level of tax evasion increases too\(^{35} \), it follows that after levying \( t_x \) and \( t_y \) it is verified: \( \frac{\partial^2 r_Y}{\partial t^2} > \frac{\partial^2 r_X}{\partial t^2} > 0 \) and \( r_x > r_T > r_y \).

\[ A) \text{ Neutral tax } T \]

\[ \begin{align*}
& r^X_0 \\
& r_X \\
& r^X_0(1-T) \\
& A_X \quad K^x \quad B_X
\end{align*} \]

\[ \begin{align*}
& r^Y_0 \\
& r_Y \\
& r^Y_0(1-T) \\
& A_Y \quad K^y \quad B_Y
\end{align*} \]

\[ B) \text{ Differentiated taxes } t_x \text{ and } t_y \]

\[ \begin{align*}
& r^X_0 \\
& r_X \\
& r^X_0(1-t_x) \\
& A'_X \quad K_X \quad B^X
\end{align*} \]

\[ \begin{align*}
& r^Y_0 \\
& r_Y \\
& r^Y_0(1-t_y) \\
& A'_Y \quad K_Y \quad B_Y
\end{align*} \]

\(^{35} \) This assumption is coherent with empirical findings presented in paragraph 2. From a theoretical point of view, it corresponds to the outcome of pure A-S model with risk-aversion decreasing with income (with or without its extensions with “social costs” of evasion), provided that income effects never become so strong to bring tax evasion elasticity to tax rates into negative values. Of course one could believe in the “Laffer curve” for tax evasion, and state that up from a given tax rate value, tax evasion grows so strongly to make net taxable income \( r_D \) decreasing with the tax rate. Empirical available evidence seems to me not supportive of the latter assumption, but even with increasing \( r_D \) to tax rates, the core argument that Ramsey Rule applied to open economies cannot disregard tax evasion, if it is relevant and differentiated between domestic sectors, still applies.
This is represented in graph B). The new net returns \( r_x \) and \( r_y \) are associated to a total level of invested capitals respectively of \( K'_X \) and \( K'_Y \). To ease visual confrontation, the value for invested capitals obtained with neutral taxation, respectively \( K^0_X \) and \( K^0_Y \), and the conditions in absence of tax evasion when net return are simply \( r^0_x(1-t_X) \) and \( r^0_y(1-t_Y) \), are also reported. Note that the proposed illustration is only one of the possible situations observable in real economies, in this case featuring the assumption that \( r_0(.) \) is strictly decreasing with \( t \).

This graphical representation allows to understand how the variation of \( K_X \) as a consequence of the switching to differentiated tax rates (which would be higher than the variation of \( K_Y \) in absence of evasion), may be significantly different from the case when no tax evasion is included, even turning the ratio between variations upside-down.

When applying the Ramsey Rule starting from a situation with no tax on capital income, the objective is to obtain an equal proportional variation in terms of quantities of invested capitals in sectors \( X \) and \( Y \). In this way total excess burden is minimized. Going back to previous graph, without tax evasion this objective is obtained if, at the margin, is:

\[
\frac{B_X - A'_X}{B_X} = \frac{B_Y - A'_Y}{B_Y}
\]

Introducing a non-null level of tax evasion in both sectors, this equivalency cannot be obtained at net returns \( r^0(1-t_X) \), because these are never reached by representative taxpayers being evasion non-null. Indeed by applying a pair of differentiated tax rates so as to satisfy (5), net-of-evasion returns may lead to very different outcomes, as represented in the lower area of the graph. Therefore, policy-maker's objective should instead be to obtain:

\[
\frac{B_X - K'_X}{B_X} = \frac{B_Y - K'_Y}{B_Y}
\]

which is only satisfied by levying a pair of tax rates \( t_X \) and \( t_Y \) obtained under equation (3) (see again paragraph 4.1). Such tax rates may be very different from the ones computed considering zero tax evasion, and possibly (under some assumptions) require that \( t_Y > t_X \), if tax evasion effects are stronger than mobility effects. Only policy target expressed by (6) allows to obtain the minimization of total excess burden, while the pursuit of (5) will lead to unknown outcomes since it does not take into account tax evasion affecting net returns and quantities in equilibrium.

The graphical illustration is additionally complicated if we consider that \( r_s(t, r^0 \; a) \) may present in one or both sectors a decreasing behavior to the tax rate, or be monotonic only for some intervals of \( t \). But, notwithstanding the specific form and causal factors of \( r_s(.) \), the general result here discussed remains valid: the application of the Ramsey Rule to real economies can be effectively pursuit only in one of two cases: 1) if tax evasion effects are even in the two domestic sectors, or 2) if they are negligible. Otherwise, the minimization of total excess burden requires to estimate tax evasion behavioral responses to tax rate modifications, and to adopt equation (3) to derive optimal rates.

### 5.2 Dynamic analysis

Focusing now on dynamic analysis, in a period \( j \) subsequent to the initial time when gross return \( r^0 \) was measured, the individual taxpayer will observe a new gross return in sector \( L \) which I mark as \( r_j^L \). Gross return \( r_j^L \) is affected by aggregate choices of all investors in sector \( L \), therefore it
will likely present a different value than initial \( r'_{ij} \) (this is true if marginal return to invested capital is not uniformly constant for every value of \( K_i \) or \( K_j \)). In the \( f^{th} \) period, the individual taxpayer again decides his preferred level of tax evasion, obtaining a new expected net return in sector \( L \) equal to \( r'_{ij} \), function to which he will adapt his choices on the quantities of capital to invest in the domestic sector and abroad. This progressive adjustment continues up to the point when in absence of modifications in exogenous variables and with invariant fiscal policy, the expected net return in the domestic sector is compatible with a condition of international arbitrage. If capital mobility is considered infinitely elastic in the long run, then such condition of dynamic equilibrium is reached only when net-of-tax return with tax evasion in the domestic sector is equal to the net return obtained abroad.

Gross unit returns in sector \( L^{th} \) for the representative agent may therefore be made dependent from the aggregate capital invested in previous period \( j-1 \): \( r_j^X(K_{j-1}) \), with \( D[r_j^X]<0 \) in case decreasing returns are observed. With an exogenous and invariant net return \( r^X \) obtained abroad, the arbitrage condition requires that expected net-of-tax and net-of-evasion returns on capital are equal in sectors \( X \) and \( Y \): \( r'_{ij} = r^X = r'_{ij} \). Once the arbitrage condition is reached and with invariant tax policy, it will be \( K^X_j = K^L_j \) in each subsequent period, so we may write:

\[
r^X_j(K^X_j) - t_s(r^Y_j(K^L_j)) - r^X_s(.)) = r^X_j(K^L_j) - t_s(r^Y_j(K^L_j)) - r^Y_s(.) - a_y r^Y_s(.)
\]

or equivalently, marking the difference between gross returns in sectors \( X \) and \( Y \) as \( \Delta r \):

\[
\Delta r = r^X_j(K^X_j) - r^X_s(.) - t_s(r^Y_j(K^L_j)) - r^Y_s(.) + a_y r^X_s(.) - a_y r^Y_s(.)
\]

The choice of the policy-maker to set a pair of Ramsey-Rule-complying tax rates \( t_i \neq t_j \), leads in the long run under perfect international capital mobility\(^{36}\), to equal net returns in every \( L^{th} \) domestic sector, but to different sectoral levels of tax evasion and taxation burden. But, it should be noted that the initial assumption of different capital mobility for the domestic sectors which justified the adoption of differentiated tax rates in the first period, becomes hard to sustain once we allow for a perfect or nearly-perfect capital mobility for all domestic sectors in the long-run\(^{37}\).

An alternative path may be instead that the policy-maker tries to adapt, in each period, differentiated tax rates to the elasticities observed in previous periods, with the aim to keep them aligned with elasticity variations in compliance with target (3). This is an interesting case to evaluate the applicability of the Ramsey Rule with tax evasion: if it is not possible to completely distinguish the elasticity of capitals due to under-reporting from the elasticity due to international mobility, the policy-maker will find it impossible to reach optimal taxation, because data used for his quantitative evaluations will be biased by variations of taxable bases of heterogeneous nature. As argued in previous paragraphs, one of the typical mechanisms used to evade taxes is indeed to conceal capital, sheltering it or changing its juridical form to simulate it is owned by foreign subjects (trustee, etc.), or employing it as working capital in fictional commercial trading with the aim to inflate deductible production costs and evade income taxes.

Where instead the policy-maker is able to completely divide the two types of elasticity and to detect with certainty, from gathered statistics, real quantities of capital invested in each domestic sector, there is still a practical issue to solve when applying equation (3). This issue lies in the fact that expected net returns in domestic sectors will vary not only with tax rates and with the

\(^{36}\) Rational expectations together with the belief that tax rates and conditions affecting tax evasion rates will not be modified in future times, may lead to an instantaneous adaptation of the quantities of invested capitals in the domestic sectors.

\(^{37}\) Assuming of course, that the definition of “long-run” encompasses a reasonable period of time, measurable in terms of years more than of decades, or centuries. In a “very long term” the assumptions of a constant taxation revenue objective, constant tax evasion functions, and constant domestic supply functions seem to lose any meaning.
intensities of capital (consequent to a more or less accentuated outflow toward foreign countries), but also with the possible tax saving obtainable through tax evasion. The latter is determined partly exogenously, partly as now illustrated in an endogenous way function to tax rates and expected gross returns, following descriptive functions that may significantly differ among domestic sectors.

6. Conclusions

The adoption of differentiated tax rates in reverse proportion to international mobile capital elasticity is based on the possibility to detect some sectors where capital mobility is, with reasonable certainty, very high or very low. Indicators for this scope are given by the observation of high relative shares of immobile and labor production factors. Non-competitive markets and rents are strengthening elements for the application of higher tax rates\(^{38}\).

This approach is surely acceptable when tax evasion is absent or not relevant. But a significant presence of evasion may induce forms of sheltering of the capital factor which is effectively used within production sectors but under-reported, thus biasing evaluations. The latter aspect may be coped with by adopting, as data function to which to discriminate sectors with higher capital intensity, statistics about production cycles at firm-level for the typical enterprise, rather than aggregate macroeconomic sectoral data. In this way it is possible to approximate the typical form of the production technology for that sector, and to deduce “real” sectoral intensities of production factors.

Moreover, in the medium and long term, modifications induced by reported income elasticity on the net return to capital factor, may act on the quantity of capital invested into the domestic sector, and also act on the sectoral production function technology modifying the relative shares of inputs between capital, labor, and rent factors. Modifications of this kind may alter in time domestic production functions, especially if it exists a significant degree of substitution between factors. In these cases, the discriminating element to adopt such policies seems to be a detailed comprehension and evaluation of tax evasion. A comparison of the expected intensity in modifications of capital elasticity in terms of pure mobility and of pure tax evasion, given a variation of the tax rate in a domestic sector, seems the only way to concretely estimate the final effects of taxation.

The results discussed in previous sections highlight some necessary cautions to be addressed for a correct application of optimal income tax rates on mobile capitals. Prof. Sørensen himself when concluding his article, wisely puts on guard against a too confident use of his theoretical results, stating that: “governments should be careful when drawing policy conclusions from the insight that the theoretically optimal policy seeks to minimize tax-induced capital flight. If governments try to pursue this rule but do not have full information on the technological parameters influencing capital mobility, firms will have a strategic incentive to label themselves as being particularly mobile in order to qualify for favorable tax treatment. However, in sectors where the tax elasticity of capital demand is known with a high degree of certainty to be either very high or very low, policy makers may want to accept some deviations from tax neutrality in order to reduce the distortionary effects of source-based capital taxation”.

The issues discussed in present paper oppose an additional argument to the applicability of theoretical results of optimal taxation to real economies. Indeed it is possible that the taxable bases

\(^{38}\) In concluding his article Sørensen suggests, using Cobb-Douglas functions to describe production technologies in domestic sectors (between which the assumption of immobility of capital factor is maintained), the possibility to observe and utilize the relative factor intensities in each domestic sector in order to design a differentiated tax on capital income. Given equal conditions, sectors showing a more labor-intensive production or requiring higher use of land rents, will see \textit{ceteris paribus} reduced the share of production factors assignable to capital. These sectors will be therefore less sensitive to a tax on capital returns, and they will be able to better sustain a higher tax rate than other domestic sectors, where capital participates to production with high shares of input and where taxation would bring more distorsive effects (in terms of international mobility). Cfr. SØRENSEN (op. cit.), pp. 21-23.
elasticity in terms of tax evasion substantially alters results obtained through the Sørensen-Ramsey framework. As illustrated before and independently from the specific behavioral model chosen to describe taxpayers' decisions in terms of evasion, modifications of the tax rate on capital incomes affect the expected net-of-tax return in a complex and non-linear way. Moreover, boundaries separating tax evasion from tax avoidance do not offer a perfectly defined and insurmountable barrier, and capital owners may adopt (and adapt) behaviors which substitute avoidance to evasion, thus impairing the ability for the policy-maker to distinguish if an observed elasticity is due to a capital outflow toward foreign countries, or to illicit behaviors not recognized as such.

To conclude with a concrete indication for policy-makers, results provided throughout the paper may be summarized in the following statement: under imperfect capital mobility where domestic sectors with a particularly high or low capital mobility are subject to limited tax evasion, or in absence of significant inter-sectoral differences in tax evasion, it is possible to risk the leaving of direct neutral taxation, because in such cases the benefits obtained in terms of lower distortions will probably surpass second-order effects due to tax evasion. But, in cases when significant differences are observed between reported income elasticities to tax rates in domestic sectors, without certainty about a number of variables and descriptive functions required to truly calculate optimal tax rates, it seems advisable to stick to traditional wisdom suggesting to pursue neutral capital taxation.

APPENDIX

This appendix demonstrates results discussed under paragraph 4.1.

A) Consider two domestic sectors called A and B, both having the same demand and supply curves, but having different levels of tax evasion. Let \( t_A \) and \( t_B \) be linear \textit{ad valorem} taxes applied respectively on the gross returns on capitals invested in sector A and sector B. Let \( r_0 \) be the unit return-to-factor of capital at the initial equilibrium of supply and demand crossing point, and \( R_0=r_0Q_0 \) be the gross-of-tax income obtained from capital, equal in both sectors. Let \( r_A \) and \( r_B \) be the net-of-tax unit return-to-factor in the two sectors; \( Q_0 \) the quantities initially exchanged in equilibrium (equal in both sectors since demand and supply functions are supposed to be the same) and before levying any tax; \( s_A \) and \( s_B \) the amount of evaded \( r_0 \), with \( s_A>s_B>0 \), and \( s_A,s_B<r_0 \), and \( s_A,s_B \) considered exogenously given and invariant to tax rates. The elasticity of unit return-to-factor of capital to quantities invested is defined as \( e=\frac{r}{Q} \frac{\Delta Q}{\Delta r} \) and is equal in both sectors.

With no tax evasion, when a tax \( t \) is levied in one of the two sectors, a variation of unit net-of-tax return is observed in such sector equal to \( tr_0 \), and an excess burden is generated equal to: \( \frac{1}{2}er_0Q_0t^2 \). In this case, the problem is the classical one presented and solved by Ramsey, to minimize the sum of excess burdens in sectors A and B, subject to the revenue constraint: \( T=r_0Qt_A+r_0Qt_B \). Since by assumption the two sectors are identical except for tax evasion, the optimal tax rate must be the same in both sectors.

Introducing tax evasion, the variation of net-of-tax unit returns in equilibrium is not equal to \( tr_0 \) anymore, but is given by some function, let it be called \( r(t) \), so that \( \Delta r=r_0-r(t) \). Since the taxpayer is now allowed to report a before-tax income lower than the "real" value of \( r_0 \), thus obtaining a tax saving, unit net returns \( r_A(t_A) \) and \( r_B(t_B) \) will reach higher values than \( r_0(1-t_A) \) and \( r_0(1-t_B) \), respectively.
Writing the variation of equilibrium quantities as: \( \Delta Q = \frac{Q^0}{r_0} \Delta r = \frac{Q^0}{r_0} \left( r_0 - r(t) \right) \), and given the definition of excess burden which is \( 1/2 \Delta Q \Delta r \), with some substitutions it obtains:

\[
(A.1) \quad \frac{1}{2} \frac{e Q_0 (r_0 - r(t))^2}{r_0}
\]

which represents the excess burden with tax evasion in each sector with constant elasticity of supply. By adopting differentiated tax rates \( t_A \) and \( t_B \), and writing supply function \( Q(r(t)) \) which expresses equilibrium quantities associated to a given net-of-tax and net-of-evasion return, the total excess burden to be minimized is obtained:

\[
(A.2) \quad \frac{1}{2} \frac{e Q_0 (r_0 - r_A(t_A))^2}{r_0} + \frac{1}{2} \frac{e Q_0 (r_0 - r_B(t_B))^2}{r_0}
\]

subject to the revenue constraint:

\[
(A.3) \quad T = (r_0 - s_A) Q(r_A) t_A + (r_0 - s_B) Q(r_B) t_B
\]

I write the function \( r(t) \) of net-of-tax and net-of-evasion unit return in the simple following form:

\[
(A.4.a) \quad r_A(t_A) = r_0 - t_A (r_0 - s_A)
\]

\[
(A.4.b) \quad r_B(t_B) = r_0 - t_B (r_0 - s_B)
\]

Substituting \( (A.4.a) \) and \( (A.4.b) \) in previous \( (A.2) \), I obtain the Lagrangian on unknown \( t_A \) and \( t_B \):

\[
(A.5) \quad V = \frac{1}{2} \frac{e Q_0 (r_0 - r_A(t_A))^2}{r_0} + \frac{1}{2} \frac{e Q_0 (r_0 - r_B(t_B))^2}{r_0} + \lambda \left[ T - (r_0 - s_A) Q(r_A) t_A - (r_0 - s_B) Q(r_B) t_B \right]
\]

I derive partial derivatives and put them equal to zero:

\[
(A.6.a) \quad \frac{\partial V}{\partial t_A} = e Q_0 \left( r_0 - s_A \right)^2 t_A \frac{Q(r_A)}{r_0} - \lambda (r_0 - s_A) Q(r_A) \frac{\partial Q(r_A)}{\partial t_A} t_A = 0
\]

\[
(A.6.b) \quad \frac{\partial V}{\partial t_B} = e Q_0 \left( r_0 - s_B \right)^2 t_B \frac{Q(r_B)}{r_0} - \lambda (r_0 - s_B) Q(r_B) \frac{\partial Q(r_B)}{\partial t_B} t_B = 0
\]

Equalizing on \( \lambda \) it obtains:

\[
(A.7) \quad \frac{e Q_0 (r_0 - s_A)^2 t_A}{r_0 (r_0 - s_A) Q(r_A) \frac{\partial Q(r_A)}{\partial t_A} t_A} = \frac{e Q_0 (r_0 - s_B)^2 t_B}{r_0 (r_0 - s_B) Q(r_B) \frac{\partial Q(r_B)}{\partial t_B} t_B}
\]
Simplifying with elementary algebra:

\[
(A.8) \quad \frac{Q(r_A) \frac{\partial Q(r_A)}{\partial t_A}}{Q(r_B) \frac{\partial Q(r_B)}{\partial t_B}} = \frac{r_0 - s_A}{r_0 - s_B}
\]

Defining point elasticities of supply to tax rates as

\[ e_A = \frac{\frac{\partial Q(r_A)}{\partial t_A}}{Q_A} \text{ and } e_B = \frac{\frac{\partial Q(r_B)}{\partial t_B}}{Q_B} \]

substituting in (A.8) we obtain:

\[
(A.9) \quad \frac{t_B}{t_A} = \frac{(r_0 - s_A)Q^2(r_B)e_B}{(r_0 - s_B)Q^2(r_A)e_A}
\]

Equation (A.9) expresses optimal capital income taxation as a function of known sectoral elasticities to tax rates, supply functions, expected gross returns and exogenous levels of tax evasion.

**B** Result (A.9) can be generalized relaxing the assumption of invariant tax evasion to tax rates, and considering different supply function for sectors A and B. Assuming \( \frac{\partial S}{\partial t} \neq 0 \), evaded unit returns are written as \( s_A(t_A) \) and \( s_B(t_B) \). Proceeding exactly as for previous equations (A.1) to (A.7), the Lagrangian is solved if \( t_A \) and \( t_B \) satisfy the following:

\[
(B.1) \quad \frac{(r_0 - s_A(t_A))Q_A(r_A) - t_A \frac{\partial s_A(t_A)}{\partial t_A} Q_A(r_A) + t_A(r_0 - s_A(t_A)) \frac{\partial Q_A(r_A)}{\partial t_A}}{e_AQ_A^0[-(r_0 - s_A(t_A))] \frac{\partial s_A(t_A)}{\partial t_A} t_A + (r_0 - s_A(t_A))^2} = ... \\
\]

\[
(B.2) \quad \frac{t_A}{(r_0 - s_A(t_A))} \frac{\partial s_A(t_A)}{\partial t_A} Q_A(r_A) + t_A \frac{\partial Q_A(r_A)}{\partial t_A} = \frac{t_B}{(r_0 - s_B(t_B))} \frac{\partial s_B(t_B)}{\partial t_B} Q_B(r_B) + t_B \frac{\partial Q_B(r_B)}{\partial t_B}
\]

Finally, with some algebraic simplifications:

\[
(B.3) \quad Q_A(r_A) - t_A \frac{\partial s_A(t_A)}{\partial t_A} Q_A(r_A) + t_A \frac{\partial Q_A(r_A)}{\partial t_A} = Q_B(r_B) - t_B \frac{\partial s_B(t_B)}{\partial t_B} Q_B(r_B) + t_B \frac{\partial Q_B(r_B)}{\partial t_B}
\]

Note that the (A.9) is a special case of the more general (B.2), obtained considering \( Q_A(r) = Q_B(r) \), and assuming \( \frac{\partial S}{\partial t} = 0 \) so that \( s_A \) and \( s_B \) become exogenous constants.
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