Keen, Steve

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Economics: The Open-Access, Open-Assessment E-Journal

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Solving the Paradox of Monetary Profits

Steve Keen
University of Western Sydney

Abstract Bruun and Heyn-Johnsen (2009) state the paradox that economics has failed to provide a satisfactory explanation of how monetary profits are generated, even though the generation of a physical surplus is an established aspect of non-neoclassical economics. They emphasise that our ability to explain phenomena like the Global Financial Crisis (GFC) will be limited while ever we are still unable to explain this fundamental aspect of capitalism. In fact this paradox can be solved very simply, using insights from what is known as “Circuit Theory”. In this paper the author shows how monetary profits are generated, and uses a monetary circuit of production model to derive policy conclusions about how to overcome a “credit crunch” that reverse the guidance given by the standard but empirically falsified “money multiplier” model of credit money creation.

Special Issue
Managing Financial Instability in Capitalist Economies

JEL E20, E51, E12, E17
Keywords Endogenous money; circuit theory, credit crunch

Correspondence Steve Keen, University of Western Sydney, Economics and Finance, Locked Bag 1797, Penrith, Australia; e-mail: s.keen@uws.edu.au
Website: www.debtdeflation.com/blogs; www.debunkingeconomics.com

http://dx.doi.org/10.5018/economics-ejournal.ja.2010-31

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1 Preliminaries

Bruun and Heyn-Johnsen (2009) state the paradox that economics has failed to provide a satisfactory explanation of how monetary profits are generated, even though the generation of a physical surplus in production is an essential component of non-neoclassical economics. They emphasise that our ability to explain phenomena like the “Great Recession” will be limited while ever we are unable to explain this fundamental aspect of capitalism.

In fact this paradox can be solved very simply, using insights from Circuit Theory Graziani (1990). Graziani’s brilliant initial proposition was that a credit economy must be using a non-commodity as money, since the alternative of “an economy using as money a commodity coming out of a regular process of production, cannot be distinguished from a barter economy” Graziani (1995: 518). From the fact that an intrinsically valueless token is nonetheless accepted as full payment in the exchange of goods, Graziani derived the conclusion that:

any monetary payment must therefore be a triangular transaction, involving at least three agents, the payer, the payee, and the bank… Since in a monetary economy money payments go necessarily through a third agent, the third agent being one that specialises in the activity of producing means of payment (in modern times a bank), banks and firms must be considered as two distinct kinds of agents (Graziani 1995: 518–519).

Unfortunately, attempts by Graziani and subsequent Circuitist authors to develop a viable mathematical model of the creation of monetary profits in a pure credit economy have to date been a failure—a situation well expressed in Rochon’s lament "How does M become M+?" (Rochon 2005: 125). This failure was not due to any weakness in the underlying vision of a pure credit economy, but to confusions of stocks with flows emanating largely from inappropriate mathematical approaches use by these authors. A simple dynamic monetary model that uses the bank account as its fundamental unit explains how capitalists can and

---

¹ Neoclassical theory ignores monetary profits, and has its marginal productivity theory of income distribution as the basis for its explanation of real profits. However this theory is subject to all the flaws pointed out in the Cambridge Controversies. The most succinct refutation of this model was given by Bhaduri (1969). See Keen (2001: 134–137) for a simple elucidation of Bhaduri’s argument.
do make a profit. In brief, “M becomes M+” via the price mechanism, which converts the sale of the physical surplus generated in production into money.

The topic has become clouded by many other issues—from the basis for the value of money itself to the impact of debt repayment on the money stock. So that I can focus solely on this issue of how monetary profits are generated, I deliberately abstract from these important but—in this context—tangential issues, as outlined below.

There are disputes in Post Keynesian monetary theory over the logical basis for the existence and value of money—notably between Chartalists who assert that taxation is the basis of money’s value, and some Circuitists—including Graziani (1989)—who assert that its acceptance in completing obligations between buyer and seller in an exchange is sufficient. The mathematical conundrum about whether capitalists can make a monetary profit when the source of their initial capital is borrowed money exists independently of this philosophical debate. The consensus to date has been that it is mathematically impossible for capitalists in the aggregate to make profits (see for example Bellofiore et al. 2000). I abstract from these philosophical and ex origo debates in order to focus simply on the mathematical issue, to show that this consensus is false.2

This dispute, and the current consensus conclusion, also exist within the confines of models of a pure credit economy—that is, models that treat money as a non-commodity issued by a private banking system, and abstract from the existence of both the State itself, and State or fiat money. The mathematical issue is therefore best treated in a model of a pure credit economy, even if a complete model of the existing monetary system must include both fiat and credit money.3

Finally, there is a difference between modern Post Keynesian theorists and Keynes over what happens to money that is used to repay debt. The convention in Circuit literature is that money used to repay debt is destroyed:

________________________________________________________________________
2 My procedure is akin to Newton’s in ignoring the debate about what gravity is, and focusing instead on the dynamics of the movement of masses under the influence of gravity. This greatly advanced humanity’s understanding of the universe, even though it abandoned the quest to discover what gravity actually is—a quest that is still extant today.

3 A model of a mixed fiat-credit monetary system has been produced and is the subject of a subsequent paper.
To the extent that bank debts are repaid, an equal amount of money is destroyed (Graziani 2003: 29–30).

Money is created as banks lend-mainly to business-and money is destroyed as borrowers fulfill their payment commitments to banks. Money is created in response to businessmen's and bankers' views about prospective profits, and money is destroyed as profits are realized Minsky (1982: xxi).

Keynes, on the other hand, spoke of a “revolving fund of credit” which was continuously replenished by the repayment of debt, which implies that money used to repay debt may be temporarily taken out of circulation, but is not destroyed:

If investment is proceeding at a steady rate, the finance (or the commitments to finance) required can be supplied from a revolving fund of a more or less constant amount, one entrepreneur having his finance replenished for the purpose of a projected investment as another exhausts his on paying for his completed investment (Keynes 1937: 247).

I side with Keynes on this issue, but to avoid complications resulting from this difference of interpretation, I first consider the historically relevant example of a private bank using paper notes that it itself creates—see Figure 1 for an example of such a note issued during the “Free Banking” period in the USA (Dwyer 1996).

A paper note model is also consistent with Graziani’s original paper on the monetary circuit, where he observed that “A true monetary economy must therefore be using a token money, which is nowadays a paper currency” (Graziani 1989: 3). These banks did not destroy their notes when debts were repaid, but treated their specie as a “revolving fund”, with notes stored until they could be recirculated in new loans:

Free banks were rarely able to keep all of their allowable note issues in circulation at all times. Ratios of idle notes to total legal circulation in New York ranged from a low of 4 % in 1852 to a high of 21.6 % during the panic of 1857. The proportion of idle notes dipped below 10 % in only three years and hovered around 15 % throughout the 1850s (Bodenhorn and Haupert 1996: 688).
Though the historical stability of this period is disputed, a private banking system of this type is not intrinsically unstable, and as I show below, capitalists can make a profit in such a system, even if their ventures are 100% debt-financed.

*Figure 1: Bank of Florence (Nebraska) Dollar Note (Smithsonian Institution 2010)*

2 The Basic Model: A Set Quantity of Notes

Consider a private bank which, having fulfilled the legal requirements for Free Banking (see Bodenhorn 2008: 183–184), creates a stock $N$ of dollar notes like those in Figure 1. These notes are initially held by the new bank in its vault. The bank then issues loans to firms, which enables the firm to hire workers, who then produce output which is sold to workers, capitalists and bankers.

A minimum of 5 classes of accounts are needed to model this system:

---

4 This period did not last and the history of this period is generally seen as chaotic (but see Rockoff 1974, Dwyer 1996 for contrary views).

5 Private note images in the Smithsonian’s National Numismatic Collection can be found at the urls [http://americanhistory.si.edu/collections/numismatics/survivin/103.htm](http://americanhistory.si.edu/collections/numismatics/survivin/103.htm) to ../119...

See [http://americanhistory.si.edu/collections/numismatics/survivin/danatext.htm](http://americanhistory.si.edu/collections/numismatics/survivin/danatext.htm) for an Art and Social History oriented presentation of these notes.

6 Following a suggestion from a referee/editor, I use capital letters for stock variables, lowercase letters for flows, and (except in the case of interest rates) greek letters for the time constants linking stocks to flows.
1. The bank vault ($B_v$), into which the newly-minted notes are first placed.
2. Firm deposit accounts ($F_D$), into which actual transfers of loaned dollars are made.
3. Workers deposit accounts ($W_D$), into which wages are paid by firms.
4. A bank transactions account ($B_T$), into and out of which interest payments are made.
5. Firm loan accounts ($F_L$), where ledger entries that record the quantity of notes that have been lent to firms.

The first four of these are physical repositories of notes. The fifth is not a repository for notes, but a ledger recording the legal claim that the bank has upon those to whom it has lent. Operations on it therefore do not involve monetary transfers, but record the impact of those transfers on the indebtedness of borrowers.

The basic transactions that occur in this model are detailed in Table 1. Seven of these steps involve the physical transfer of money:

1. Lending of money from the bank vault to the firms’ deposit accounts (row 1).
2. Payment of interest by firms to the bank’s transactions account (row 4).
3. Payment of interest by the bank to firms’ deposit accounts (row 6).
4. Payment of wages (row 7).
5. Payment of interest on workers’ account balances (row 8).
6. Payment for consumption of the output of firms by bank and workers (row 9).
7. Repayment of loans by firms (row 10).

Four steps are ledger entries only, involving the recording of a money transfer related to the level of debt:

1. Recording the loans to firms (row 2).
2. Compounding the debt at the rate of interest on loans (row 3).
3. Recording the payment of interest on loans (row 5).
4. Recording the repayment of loans (row 11).
Table 1: Basic Financial Transactions in a Free Banking Economy

<table>
<thead>
<tr>
<th>Row</th>
<th>Transaction</th>
<th>Type</th>
<th>Bank vault ((B_V))</th>
<th>Bank transaction ((B_T))</th>
<th>Firm loan ((F_L))</th>
<th>Firm deposit ((F_D))</th>
<th>Worker deposit ((W_D))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lend money</td>
<td>Money transfer</td>
<td>–a</td>
<td></td>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Record loan</td>
<td>Ledger entry</td>
<td>a</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Compound debt</td>
<td>Ledger entry</td>
<td>b</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Pay interest</td>
<td>Money transfer</td>
<td>c</td>
<td>–c</td>
<td>–c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Record payment</td>
<td>Ledger entry</td>
<td>–c</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Deposit interest</td>
<td>Money transfer</td>
<td>–d</td>
<td>d</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Wages</td>
<td>Money transfer</td>
<td>–e</td>
<td>e</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Deposit interest</td>
<td>Money transfer</td>
<td>–f</td>
<td>f</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Consumption</td>
<td>Money transfer</td>
<td>–g</td>
<td>g+h</td>
<td>–h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Repay loan</td>
<td>Money transfer</td>
<td>i</td>
<td>–i</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Record repayment</td>
<td>Ledger entry</td>
<td>–i</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sum of flows</td>
<td></td>
<td>i–a</td>
<td>c–d–f–g</td>
<td>a+b–e+g+h–i</td>
<td>e+f–h</td>
<td></td>
</tr>
</tbody>
</table>

The financial flows in each column of Table 1 can be summed to describe the dynamics of the bank accounts in this model:
To model this system, we need to provide values for the operations $a$ to $i$. Table 2 specifies these, with each operation being related to the current level of the relevant account—lending from the vault, for example, is assumed to occur at a

\[ \frac{d}{dt} F_L(t) = a + b - c - i \]  
\[ \frac{d}{dt} F_D(t) = a - c + d - e + g + h - i \]  
\[ \frac{d}{dt} W_D(t) = e + f - h \]  

To model this system, we need to provide values for the operations $a$ to $i$. Table 2 specifies these, with each operation being related to the current level of the relevant account—lending from the vault, for example, is assumed to occur at a

<table>
<thead>
<tr>
<th>Flow</th>
<th>Description</th>
<th>Flow</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Loans to firms at the rate $\beta_V$ times the balance in the vault at time $t B_V(t)$</td>
<td>$\beta_V B_V(t)$</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>The rate of interest on loans $r_L$ times the level of loans at time $t F_L(t)$</td>
<td>$r_L F_L(t)$</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>Payment of interest on loans</td>
<td>$r_L F_L(t)$</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Payment of interest on firm deposits $F_D(t)$ at the rate $r_D$</td>
<td>$r_D F_D(t)$</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>Payment of wages by firms at the rate $\phi_D$ times firm deposits at time $t F_D(t)$</td>
<td>$\phi_D F_D(t)$</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>Payment of interest on deposits at the rate $r_D$</td>
<td>$r_D W_D(t)$</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>Payment for goods by banks at the rate $\beta_T$ times the level of the bank transaction account at time $t B_T(t)$</td>
<td>$\beta_T B_T(t)$</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>Payment for goods by workers at the rate $\omega_D$ times the level of the bank transaction account at time $t W_D(t)$</td>
<td>$\omega_D W_D(t)$</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>Repayment of loans at the rate $\phi_L$ times the outstanding loan balance at time $t F_L(t)$</td>
<td>$\phi_L F_L(t)$</td>
<td></td>
</tr>
</tbody>
</table>
constant rate $\beta_v$ related to the current amount of money in the vault at time $t$, $B_v(t)$.\footnote{Constants are used here simply so that the viability of the system can be established. Later these constants are replaced by variables.}

The full dynamic system is given by Equation (2):

$$\frac{d}{dt} B_v(t) = \phi_l \cdot F_l(t) - \beta_v \cdot B_v(t)$$

$$\frac{d}{dt} B_f(t) = r_l \cdot F_l(t) - r_d \cdot F_d(t) - r_d \cdot W_d(t) - \beta_f \cdot B_f(t)$$

$$\frac{d}{dt} F_l(t) = \beta_v \cdot B_v(t) + r_l \cdot F_l(t) - r_l \cdot F_l(t) - \phi_l \cdot F_l(t)$$

$$\frac{d}{dt} F_d(t) = \beta_v \cdot B_v(t) - r_l \cdot F_l(t) + r_d \cdot F_d(t) - \phi_d \cdot F_d(t) + \beta_f \cdot B_f(t) + \omega_d \cdot W_d(t) - \phi_l \cdot F_l(t)$$

$$\frac{d}{dt} W_d(t) = \phi_d \cdot F_d(t) + r_d \cdot W_d(t) - \omega_d \cdot W_d(t)$$

(2)

As is easily shown, with realistic parameter values (see Table 3; the values are explained later in the text prior to Table 5, and Table 5 itself) this describes a self-sustaining system in which all accounts settle down to equilibrium values, and in which capitalists earn a monetary profit.

### Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_v$</td>
<td>¾ p.a.</td>
<td>Rate of outflow of notes from the vault $B_v$</td>
</tr>
<tr>
<td>$r_l$</td>
<td>5% p.a.</td>
<td>Rate of interest on loans</td>
</tr>
<tr>
<td>$r_d$</td>
<td>2% p.a.</td>
<td>Rate of interest on deposits</td>
</tr>
<tr>
<td>$\phi_d$</td>
<td>2 p.a.</td>
<td>Rate of outflow of notes from $F_d$ to pay wages</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>1 p.a.</td>
<td>Rate of outflow of notes from $B_f$ to pay for bankers consumption</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>26 p.a.</td>
<td>Rate of outflow of notes from $W_d$ to pay for workers consumption</td>
</tr>
<tr>
<td>$\phi_l$</td>
<td>1/7 p.a.</td>
<td>Rate of repayment of loans</td>
</tr>
</tbody>
</table>
Figure 2 shows the dynamics of this system with an initial stock of N=100 million dollar notes.
The equilibrium values of the accounts can be solved for symbolically in this constant money stock model:

\[
\begin{bmatrix}
    B_{r_e} \\
    B_{r_t} \\
    F_{l_e} \\
    F_{D} \\
    W_{D}
\end{bmatrix}
= \begin{bmatrix}
    \phi_L \\
    \beta_v \frac{r_L - r_D}{\beta_T - r_D} \\
    \beta_v \\
    \beta_v \frac{(\beta_T - r_D) \cdot (\phi_D - r_D + \omega_D)}{(\beta_T - r_D) \cdot (\phi_D - r_D + \omega_D)} \\
    \beta_v \frac{(\beta_T - r_D) \cdot (\phi_D - r_D + \omega_D)}{(\beta_T - r_D) \cdot (\phi_D - r_D + \omega_D)}
\end{bmatrix}
\cdot
\begin{bmatrix}
    N \\
    16 \\
    2.571 \\
    84 \\
    75.608 \\
    5.82
\end{bmatrix}
\]

(3)

3 From Account Balances to Incomes

The equilibrium yearly wages of workers (and gross interest earnings by bankers) can be calculated from Equation (3), and they in part explain why, in contrast to the conventional belief amongst Circuitist writers, capitalists can borrow money, pay interest, and still make a profit. Though only $100 million worth of notes were created, the circulation of those notes generates workers’ wages of $151 million per annum (given the parameter values used in this simulation), 1.5 times the size of the value of the notes in the economy (see Figure 3):

\[
\phi_D \cdot F_D = \frac{N}{\beta_v + \phi_L} \beta_v \frac{(\beta_T - r_L) \cdot (\omega_D - r_D)}{(\beta_T - r_D) \cdot (\phi_D - r_D + \omega_D)} = 2 \cdot \$75.608 = \$151.216 \text{p.a.}
\]

(4)

This indicates the source of the Circuitist conundrums: the stock of money has been confused with the flow of economic activity that money can finance over time. A stock—the initial amount of notes created in this model—has been confused

---

8 The equilibrium gross banker’s income (the rate of interest on loans times the equilibrium level of \(F_D\)) is also easily calculated—it is 5% of the equilibrium level of debt of $84 million, or $4.2 million per annum.
with a flow—the economic turnover in notes per year. In fact, for a wide range of values for the parameter $\phi_D$, the flows initiated by the money borrowed by the firms over a year exceed the size of the loan itself.

This is possible because the stock of money can circulate several times in one year—something that Marx accurately enunciated over a century ago in Volume II of *Capital* (though his numerical example is extremely large):

“Let the period of turnover be 5 weeks, the working period 4 weeks... In a year of 50 weeks ... Capital I of £2,000, constantly employed in the working period, is therefore turned over 12½ times. 12½ times 2,000 makes £25,000” (Marx and Engels 1885, Chapter 16: The Turnover of Variable Capital).

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9 This statement from Graziani (1989) is indicative of the error of confusing the initial loan with the volume of transactions that can be generated by such a loan over a year: “If on the other hand, wage-earners decide to keep part of their savings in the form of liquid balances (that is, banking deposits), firms will get back from the market less money than they have initially injected in it” (Graziani 1989: 520).
Aggregate wages and aggregate profits therefore depend in part upon the turnover period between the outlay of money to finance production and the sale of that production. This turnover period can be substantially shorter than a year, in which case \( \phi_D \) will be substantially larger than 1, as I explain below.

4 The Making of Monetary Profits

A second fundamental insight from Marx lets us explain what \( \phi_D \) is, and simultaneously derive an expression for profits: the annual wages bill reflects both the turnover period, and the way in which the surplus value generated in production is apportioned between capitalists and workers. The value of \( \phi_D \) therefore reflects two factors: the share of surplus (in Sraffa's sense) that accrues to workers;\(^{10}\) and the turnover period measured in years—the time between \( M \) and \( M+ \). Labelling the share going to capitalists as \( \sigma \) and the share to workers as \( (1– \sigma) \), and labelling the turnover period as \( \tau_S \) and expressing it as a fraction of a year, I can perform the substitution shown in Equation (5):

\[
\phi_D = \frac{1– \sigma}{\tau_S}
\]  

Money wages are therefore:

\[
\phi_D \cdot F_D (t) = \frac{1– \sigma}{\tau_S} \cdot F_D (t)
\]  

Since national income resolves itself into wages and profits (interest income is a transfer between classes, and sums to zero across all classes), we have also identified gross profit:\(^{11}\)

---

\(^{10}\) I depart from Marx and follow Sraffa here, by specifying the division of surplus between capitalists and workers in such a way that the sum is 1. Thus if capitalists get \( s\% \) of the surplus, workers get \( [1–s]\% \).

\(^{11}\) If this seems like a Milton Friedman magic trick ("Putting a rabbit into a hat in full view of the audience, and then expecting applause when he later pulls it out again", to quote Joan Robinson from a talk she gave to Sydney University students in 1974), bear with me—later I show that profits can also be derived from the production system.
Using a value of $\sigma = 40\%$—which corresponds to historical norm of 60% of pre-interest income going to workers (see Figure 4)—this implies a value for $\tau_S$ of 0.3.

This means that the turnover period in Marx’s terminology is roughly 16 weeks. This is much longer than in Marx’s numerical illustration above, but still sufficient to give capitalists profits that are substantially greater than the servicing costs of debt. Figure 5 shows the annual incomes for each class in society over time; all are positive and the equilibrium levels (once account levels stabilise) are $151$ million, $98$ million and $2.5$ million for workers, capitalists and bankers respectively out of a national income of $252$ million (see Equation (8)).
Figure 5: Class Incomes after Interest Payments

Annual Incomes After Interest Payments

<table>
<thead>
<tr>
<th>Year</th>
<th>Millions of dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>151.33</td>
</tr>
<tr>
<td>2</td>
<td>151.33</td>
</tr>
<tr>
<td>3</td>
<td>151.33</td>
</tr>
<tr>
<td>4</td>
<td>151.33</td>
</tr>
<tr>
<td>5</td>
<td>151.33</td>
</tr>
<tr>
<td>6</td>
<td>151.33</td>
</tr>
<tr>
<td>7</td>
<td>151.33</td>
</tr>
<tr>
<td>8</td>
<td>151.33</td>
</tr>
<tr>
<td>9</td>
<td>151.33</td>
</tr>
<tr>
<td>10</td>
<td>151.33</td>
</tr>
</tbody>
</table>

Workers: \( \frac{1-\sigma}{\tau_S} \cdot F_D(t) + r_D \cdot W_D(t) = 151.33 \) in equilibrium

Capitalists: \( \frac{\sigma}{\tau_S} \cdot F_D(t) + r_D \cdot F_D(t) - r_L \cdot F_L(t) = 98.12 \) in equilibrium

Bankers: \( r_L \cdot F_L(t) - r_D \cdot (F_D(t) + W_D(t)) = 2.57 \) in equilibrium

The value of \( \tau_S \) also determines the velocity of money: the ratio of nominal GDP to the proportion of the money stock in circulation (the equivalent of \( M_3-M_0 \) in monetary statistics, since in this pure credit model there is no fiat money), which is 3 given the parameters used in this simulation. This is within the highly volatile range suggested by historical data (see Figure 6).

Table 4 summarises the equilibrium values for account balances, gross and net incomes in this hypothetical pure credit economy.
Figure 6: US GDP to Money Supply Ratios

Table 4: Equilibrium Account Balances, Gross and Net Incomes

<table>
<thead>
<tr>
<th>Account balances</th>
<th>Class incomes</th>
<th>Net incomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank vault</td>
<td>16</td>
<td>N/A</td>
</tr>
<tr>
<td>Firm loans</td>
<td>84</td>
<td>N/A</td>
</tr>
<tr>
<td>Firms</td>
<td>75.6081</td>
<td>100.811 (profits)</td>
</tr>
<tr>
<td>Workers</td>
<td>5.8205</td>
<td>151.216 (wages)</td>
</tr>
<tr>
<td>Bankers</td>
<td>2.5714</td>
<td>4.2 (debt servicing)</td>
</tr>
<tr>
<td>Totals</td>
<td>84 (in Deposits)</td>
<td>252.027+4.2</td>
</tr>
</tbody>
</table>

We can also derive a symbolic expression for the equilibrium level of profits $\Pi_c$:

$$
\Pi_c = \frac{\sigma \cdot N \cdot \beta_T \cdot (\beta_T - r_L) \cdot (\omega_D - r_D)}{(\beta_T - r_D) \cdot (\beta_T + \phi_T) \cdot \left(\frac{1-\sigma}{\tau_s} + \omega_D - r_D\right)}
$$

(9)
This allows us to specify the general conditions under which equilibrium monetary profits will exceed zero, given the existence of a physical surplus from production. They are far from onerous: the rate at which the bank transaction account turns over each year has to exceed the rate of interest on loans \( (\beta_L > r_L) \) and the rate at which the workers’ deposit account turns over has to exceed the rate of interest on deposits \( (\omega_D > r_D) \). Reasonable values for these parameters easily meet these conditions, as detailed below.

5 Other Parameters and Time Lags

The parameters \( r_L \) and \( r_D \) are nominal interest rates, and their values are roughly in line with historical norms at times of low-inflation; that leaves the parameters \( \beta_V \), \( \phi_L \), \( \omega_D \) and \( \beta_T \) to account for.

The values for \( \beta_V \) and \( \phi_L \) respectively specify how rapidly the balance in the vault is turned over, and how rapidly loans are repaid, and were chosen so that the equilibrium value of \( B_V \) would be roughly the value noted by Bodenhorn and Haupert (1996: 688) of 15% of available notes:

\[
\frac{\phi_L}{\beta_V + \phi_L} = 0.16
\]

The parameters \( \omega_D \) and \( \beta_T \) signify how rapidly workers and bankers respectively spend their bank balances on the output produced by firms: workers are assumed to turnover their accounts 26 times a year—which corresponds to workers living from fortnightly paycheque to paycheque, with only modest savings. Bankers are assumed to turnover their account just once a year, reflecting their much higher per capita incomes.

In the remainder of the paper, all parameters are expressed using the systems engineering concept of a time constant, which gives the fundamental frequency of a process. In every case, the time constant is the inverse of the parameter used thus far; for instance, the value of 26 for \( \omega_D \) corresponds to workers’ consumption having a fundamental frequency of 1/26th of a year, or two weeks.

---

Table 5: Time Constants in the Model

<table>
<thead>
<tr>
<th>Parameter and value</th>
<th>Time constant and value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_V = \frac{3}{4}$</td>
<td>$\tau_V = \frac{4}{3}$ years</td>
<td>Banks lend their reserve holdings of notes every 15 months</td>
</tr>
<tr>
<td>$\phi_L = \frac{1}{7}$</td>
<td>$\tau_L = 7$ years</td>
<td>Firms repay their loans every 7 years</td>
</tr>
<tr>
<td>$\omega_D = 26$</td>
<td>$\tau_W = \frac{1}{26}$ years</td>
<td>Workers spend their savings every 2 weeks</td>
</tr>
<tr>
<td>$\beta_T = 1$</td>
<td>$\tau_D = 1$ year</td>
<td>Bankers spend their savings every 1 year</td>
</tr>
<tr>
<td>$\tau_P = 1$ year</td>
<td>Time constant in price setting (introduced in Equation (18))</td>
<td></td>
</tr>
<tr>
<td>$\tau_M = 15$ years</td>
<td>Banks double the money supply every 15 years (introduced in Table 7 on page 24)</td>
<td></td>
</tr>
</tbody>
</table>

6 Production, Prices and Monetary Profits

Consider a simple production system in which output is proportional to the labour input $L$ with constant labour productivity $a$:

$$Q = a \cdot L$$

(11)

Labour employed in turn equals the monetary flow of wages divided by the nominal wage rate $W$:

$$L = \frac{1 - \sigma}{\tau_s} \cdot F_D + W$$

(12)

Prices then link this physical output subsystem to the financial model above. In equilibrium, it must be the case that the physical flow of goods produced equals the monetary demand for them divided by the price level. We can therefore derive that in equilibrium, the price level will be a markup on the monetary wage, where the markup reflects the rate of surplus as defined in this paper.
To answer Rochon's vital question, \( M \) becomes \( M+ \) (that is, monetary profits are realised) via a price-system markup on the physical surplus produced in the factory system. This markup can be derived simply by considering demand and supply factors in equilibrium. The flow of demand is the sum of wages and profits (since interest payments are a transfer and do not contribute to the value of output—despite Wall Street's bleatings to the contrary). The monetary value of demand is thus:

\[
D_M = \frac{1 - \sigma}{\tau_s} F_D + \frac{\sigma}{\tau_s} F_D = \frac{F_D}{\tau_s} \tag{13}
\]

The physical units demanded equals this monetary demand divided by the price level:

\[
D = \frac{D_M}{P} = \frac{1}{\tau_s} \cdot \frac{F_D}{P} \tag{14}
\]

In equilibrium this physical demand will equal the physical output of the economy:

\[
Q_e = a \cdot \frac{1 - s}{\tau_s} \cdot \frac{F_{D_e}}{W} = D_e = \frac{1}{\tau_s} \cdot \frac{F_{D_e}}{P_e} \tag{15}
\]

Solving for the equilibrium price \( P_e \) yields:

\[
P_e = \frac{1}{(1 - \sigma)} \cdot \frac{W}{a} \tag{16}
\]

The markup is thus the inverse of workers' share of the surplus generated in production. Circuit theory therefore provides a monetary expression of Marx's theory of surplus value, as it was always intended to do.\(^\text{13}\)

With these physical and price variables added to the system, we are now able to confirm that profit as derived from the financial flows table corresponds to profit as the difference between the monetary value of output and the wage bill (in this simple single-sectoral model).

\(^\text{13}\) Though this is not an endorsement of the Labour Theory of Value, which I reject on other grounds: see (Keen 1993a and 1993b).
Table 6: Parameters and Variables for Physical Production Subsystem

<table>
<thead>
<tr>
<th>Variable, parameter or initial condition</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Labour productivity $a = Q/L$</td>
<td>2</td>
</tr>
<tr>
<td>$W$</td>
<td>Nominal wage</td>
<td>1</td>
</tr>
<tr>
<td>$P_e$</td>
<td>Equilibrium price $P_e = \frac{1}{1-\sigma} \cdot \frac{W}{a}$</td>
<td>0.833</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Initial price</td>
<td>1</td>
</tr>
<tr>
<td>$L_e$</td>
<td>Equilibrium employment $L_e = \frac{1-\sigma}{\tau_s} \cdot F_{De} \cdot W$</td>
<td>151.216</td>
</tr>
<tr>
<td>$Q_e$</td>
<td>Equilibrium output $Q_e = L_e \cdot a$</td>
<td>302.432</td>
</tr>
</tbody>
</table>

Using the values given in Table 6, it is easily confirmed that the equilibrium level of profits derived from the financial flows corresponds to the level derived from the physical production system:

$$\frac{1-\sigma}{\tau_s} \cdot F_{De} = 100.811$$

$$P_e \cdot Q_e - W \cdot L_e = 100.811$$

The price relation given above applies also only in equilibrium. Out of equilibrium, it is reasonable to postulate a first-order convergence to this level, where the time constant $\tau_p$ reflects the time it takes firms to revise prices. This implies the following dynamic pricing equation:

$$\frac{d}{dt} P = -\frac{1}{\tau_p} \left( P - \frac{1}{1-\sigma} \cdot \frac{W}{a} \right)$$

A simulation also confirms that the monetary flows (demand) and the monetary value of physical flows (supply) converge over time (Figure 7).

This solves the “paradox” of monetary profits: it was not a paradox at all, but a confusion of stocks with flows in previous attempts to understand the monetary circuit of production.
7 Analysing the “Great Recession”

We can now use this framework to consider one aspect of the current financial crisis: if a “credit crunch” occurs, what is the best way for government to address it?—by giving fiat money to the banks to lend, or by giving it to the debtors to spend?

Our current crisis is, of course, more than merely a “credit crunch”—a temporary breakdown in the process of circulation of credit. It is also arguably a secular turning point in debt akin to that of the Great Depression (Keen 2009), as Figure 8 illustrates. While the model developed here cannot assess this claim, it can assess the differential impact of a sudden injection of fiat money to rescue an economy that has experienced a sudden drop in the rate of circulation and creation.

---

14 A model that can assess this claim will be the subject of a later paper.

15 Modeled here as a “deus ex machina” injection of money into the system—the proper modeling of a mixed private credit-fiat money economy is the subject of a subsequent paper.
of private credit. This is an important point, since although the scale of
government response to the crisis was enormous across all affected nations, the
nature of that response did vary: notably, the USA focused its attention on
boosting bank reserves in the belief—as expressed by President Obama—that the
money multiplier made refinancing the banks far more effective than rescuing the
borrowers:

And although there are a lot of Americans who understandably think that
government money would be better spent going directly to families and
businesses instead of banks – "where's our bailout?,” they ask – the truth is that
a dollar of capital in a bank can actually result in eight or ten dollars of
loans to families and businesses, a multiplier effect that can ultimately
lead to a faster pace of economic growth (Obama 2009: 3. Emphasis added).

Figure 8: Private Debt to GDP Ratios, USA & Australia

![Graph showing private debt to GDP ratios for USA and Australia from 1870 to 2020.]

Year; Sources US FRB Flow of Funds Table L1; RBA Table D02
The Australian policy response to the crisis, on the other hand, was pithily summed up in the advice given by its Treasury: “go early, go hard, go households” (Gruen 2008). Though many other factors differentiate these two countries—notably Australia’s position as a commodity producing supplier to China—the outcomes on unemployment imply that the Australian measures more successful than the American “money multiplier” approach (see Figure 9).

The next section applies this endogenous money model to consider a differential response to a credit crunch in a growing economy: an injection of funds is made into either the Banks’ Vault accounts—simulating the USA’s policy response—or into the Workers’ Deposit accounts—simulating the Australian response.

Figure 9: Unemployment Rates USA and Australia

Endogenous Money Creation and Economic Growth

To model a credit crunch in a growing economy, while otherwise maintaining the structure of the Free Banking/pure credit money model above, I move beyond...
the limitations of a pure paper money system to allow for endogenous money creation as described in Moore (1979):

"In the real world banks extend credit, creating deposits in the process, and look for the reserves later" (Holmes 1969, Moore 1979: 53); see also more recently Disyatat (2010: 7 "loans drive deposits rather than the other way around").

In the model, new credit to sustain a growing economy is created by a simultaneous increase in the loan and deposit accounts for the borrower. The financial flows in this system are given in Table 7. The two changes to Free Banking model are the addition of row 12 (and its ledger recording in row 13), with the qualitatively new operation of Money Creation being added to the previous operation of Money Transfer, and a “Deus Ex Machina” injection of fiat money into either Bank Vault or Worker Deposit accounts one year after a credit crunch.

Again, simply to illustrate that the system is viable, a constant growth parameter \( \tau_M \) has the banks doubling the stock of loans every 15 years (see Table 3):

\[
j = \frac{1}{\tau_M} \cdot F_L(t)
\]

A credit crunch is simulated by varying the three crucial financial flow parameters \( \tau_V \), \( \tau_L \), and \( \tau_M \) at an arbitrary time in the following simulation (at \( t=25 \) years): \( \tau_V \) and \( \tau_M \) are doubled and \( \tau_L \) is halved, representing banks halving their rates of circulation and creation of new money and firms trying to repay their loans twice as quickly (see Table 8). The government fiat-money rescue is modelled as a one-year long injection of a total of $100 million one year after the credit crunch.

Several extensions to the physical side of the model are required to model economic growth. In the absence of Ponzi speculation (which is the topic of a later

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16 I maintain the practice established in the Free Banking model that money is not destroyed when a loan is repaid, but is instead transferred to the bank’s capital. As noted earlier, I dispute the conventional Post Keynesian belief that money is destroyed when debt is repaid, but—as with issues such as the source of money’s value—that is a peripheral issue to the one I wish to consider in this section.
<table>
<thead>
<tr>
<th>Row</th>
<th>Transaction</th>
<th>Type</th>
<th>Bank vault ((B_V))</th>
<th>Bank transaction ((B_T))</th>
<th>Firm loan ((F_L))</th>
<th>Firm deposit ((F_D))</th>
<th>Worker deposit ((W_D))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lend money</td>
<td>Money transfer</td>
<td>–a</td>
<td></td>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Record loan</td>
<td>Ledger entry</td>
<td></td>
<td></td>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Compound debt</td>
<td>Ledger entry</td>
<td></td>
<td></td>
<td>b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Pay interest</td>
<td>Money transfer</td>
<td>c</td>
<td>–c</td>
<td></td>
<td>–c</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Record payment</td>
<td>Ledger entry</td>
<td>–c</td>
<td></td>
<td></td>
<td>–c</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Deposit interest</td>
<td>Money transfer</td>
<td>–d</td>
<td>d</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Wages</td>
<td>Money transfer</td>
<td></td>
<td></td>
<td>–e</td>
<td>e</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Deposit interest</td>
<td>Money transfer</td>
<td>–f</td>
<td>f</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Consumption</td>
<td>Money transfer</td>
<td>–g</td>
<td>g+h</td>
<td>–h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Repay loan</td>
<td>Money transfer</td>
<td>i</td>
<td>–i</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Record repayment</td>
<td>Ledger entry</td>
<td></td>
<td>–i</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>New money</td>
<td>Money creation</td>
<td></td>
<td>j</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Record loan</td>
<td>Ledger entry</td>
<td></td>
<td>j</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Government policy</td>
<td>Exogenous injection into either (B_V) or (W_D)</td>
<td>k</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Sum of flows</td>
<td></td>
<td>i–a+k</td>
<td>c–d–f–g</td>
<td>a+b–e+g+h–i+j</td>
<td>e+f–h+k</td>
<td></td>
</tr>
</tbody>
</table>

\( 
\text{Sum of flows} = i-a+k + c-d-f-g + a+b-e+g+h-i+j + e+f-h+k 
\)
paper), growth in the money supply is only warranted if economic growth is occurring, which in turn requires a growing population and/or labour productivity. These variables introduce the issue of the employment rate, and this in turn raises the possibility of variable money wages in response to the rate of unemployment—a Phillips curve. These additional variables are specified in Equation (20):

\[
\frac{da}{dt} = \alpha \cdot a \\
\frac{dPop}{dt} = \beta \cdot Pop \\
\frac{dW}{dt} = P_h(\lambda) \cdot W \\
\lambda = \frac{L}{Pop}
\]

(20)

Table 8: Financial Flow Parameters before and after a Credit Crunch

<table>
<thead>
<tr>
<th>Pre-credit crunch</th>
<th>Post-credit crunch</th>
<th>Impact of credit crunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_V = 4/3 \text{ years}$</td>
<td>$\tau_V = 8/3 \text{ years}$</td>
<td>Banks lend their reserve holdings of notes every 15 months</td>
</tr>
<tr>
<td>$\tau_L = 7 \text{ years}$</td>
<td>$\tau_L = 3.5 \text{ years}$</td>
<td>Firms repay their loans every 3.5 years</td>
</tr>
<tr>
<td>$\tau_M = 15 \text{ years}$</td>
<td>$\tau_M = 30 \text{ years}$</td>
<td>Banks double the money supply every 30 years</td>
</tr>
<tr>
<td>$k = $100 \text{ million}$</td>
<td></td>
<td>Injected either into bank vault $B_E$ or worker deposit $W_D$ at year 26, one year after the credit crunch</td>
</tr>
</tbody>
</table>

17 The functional form used here is a generalised exponential function

\[ g(x) = (y_0 - m) \cdot e^{s \cdot (x - x_0)} + m \]

where $x$ is the argument (in this case, the unemployment rate), $(x_0, y_0)$ is a coordinate on the curve, $s$ the slope of the curve at that point and $m$ the minimum value of the function. In this simulation $(x_0, y_0) = (0.94, 0)$, $s = 1$ and $m = 0.04$; this means that at an unemployment rate of 6%, money wages do not change, they rise by 25% p.a. at full employment (0% unemployment), and they fall at a maximum rate of 4% p.a. at high levels of unemployment.
The parameter values and functional form for this physical growth extension are shown in Table 9.

Figure 10 shows the impact of the credit crunch upon bank accounts: loans and deposits fall while the proportion of the money supply that is lying idle in bank reserves rises dramatically.

The US empirical data to date has displayed a similar pattern, though with a much sharper increase in bank reserves as shown in Figure 11.

A very similar pattern to the empirical data is evident in the model when the US policy of increasing bank reserves is simulated (Figure 12).

The simulation of Australian household-oriented policies generates a very different dynamic: reserves still rise dramatically during the credit crunch, but their increase is not further augmented by the policy intervention. Instead, firm and worker deposits rise substantially (see Figure 13), whereas they fall in the bank-oriented rescue.

This higher level of money in circulation in the household-oriented policy intervention is the cause of the dramatic difference in the outcomes of the two policy interventions: the household-oriented approach has a far more immediate and substantial impact upon employment (Figure 14). Contrary to the expectations of President Obama and his mainstream economic advisers, there is far more “bang for your buck” out of a household rescue than out of a bank rescue.

### Table 9: Parameters and Function for Growth Model

<table>
<thead>
<tr>
<th>Variable or parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Rate of growth of labor productivity</td>
<td>1% p.a.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Rate of growth of population</td>
<td>2% p.a.</td>
</tr>
<tr>
<td>Pop</td>
<td>Population</td>
<td>Initial value = 160</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Employment rate</td>
<td>Initial value = 94.5%</td>
</tr>
</tbody>
</table>

$Ph(\lambda, \lambda_*, w_*, \text{slope}, \min) = Ph(\lambda, 94\%, 0, 1, -4\%)$

Phillips curve:

$$Ph(\lambda, \lambda_*, w_*, \text{slope}, \min) = (w_\min - \min) \cdot e^{-\frac{\text{slope}}{w_\min}} (\lambda - \lambda_*) + \min$$
Figure 10: Bank Accounts before and after a Credit Crunch

Figure 11: Drop in Business Loans and Dramatic Rise in Bank Reserves during Great Recession

Year; Sources St Louis FRED AJDRES and BUSLOANS
Figure 12: Simulating US Bank-oriented Policy towards a Credit Crunch

Bank Accounts in a Credit Crunch: Bank Rescue

Year

Millions of dollars

Figure 13: Simulating Australian Household-oriented Policy towards a Credit Crunch

Bank Accounts in a Credit Crunch: Household Stimulus

Year

Millions of dollars
Conclusion

The paradox of monetary profits is solved simply by avoiding the problem so wittily expressed by Kalecki, that economics is "the science of confusing stocks with flows" cited in Godley and Lavoie (2007). With that confusion removed by working in a framework that explicitly records the flows between bank accounts and the production and consumption they drive, it is obvious that Circuit Theory achieves what it set out to do: to provide a strictly monetary foundation for the Marx–Schumpeter–Keynes–Minsky tradition in economics. As an explicitly monetary model, it also provides an excellent foundation for explaining the processes that led to the “Great Recession”, and for testing possible policy responses to it.
Acknowledgements

This work results from a collaborative research effort between the United Nations Environment Program (UNEP) and CSIRO Sustainable Ecosystems to establish a regional report on Resource Efficiency: Economics and Outlook for Asia-Pacific. I thank 4 anonymous referees, an editor and Trond Andresen (Norwegian University of Technology) for comments that greatly improved the final paper.

References


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