Otto, Philipp E.; Bolle, Friedel

Working Paper
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Discussion Paper, No. 270

Provided in Cooperation with:
European University Viadrina Frankfurt (Oder), Department of Business Administration and Economics

Suggested Citation: Otto, Philipp E.; Bolle, Friedel (2009) : Small numbers matching markets: Unstable and inefficient due to over-competition?, Discussion Paper, No. 270, European University Viadrina, Department of Business Administration and Economics, Frankfurt (Oder)

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Small Numbers Matching Markets: Unstable and Inefficient Due to Over-competition?

Philipp E. Otto
Friedel Bolle

European University Viadrina Frankfurt (Oder)
Department of Business Administration and Economics

Discussion Paper No. 270
January 2009
ISSN 1860 0921

KEYWORDS: Matching market, price negotiation, optimal allocation, altruism

This work has been funded by the Deutsche Forschungsgemeinschaft under the contract BO 747/11-1.

* We would like to thank Yves Breitmoser and Werner Güth for their very helpful comments.

Email: friedel.bolle@euv-frankfurt-o.de
Postal: 11961 Berlin, Germany
Institute: Department of Economics (Wirtschaftswissenschaften)
Institutional affiliation: European University Viadrina (Europa-Universität Viadrina)

Small Numbers Matching Markets:

Unstable and Inefficient Due to Over-competition?

Small Numbers Matching Markets:

Friedel Bolle

Philipp E. Otto
Abstract

The extant literature on matching markets assumes ordinal preferences for matches, while bargaining within matches is mostly excluded. Central for this paper, however, is the bargaining over joint profits from potential matches. We investigate, both theoretically and experimentally, a seemingly simple allocation task in a 2x2 market with repeated negotiations. More than 75% of the experimental allocations are unstable, and 40% of the matches are inefficient (in cases where inefficiency is possible). By defining the novel concept "altruistic core", we can explain the occurrence of inefficient matches as well as the significant behavioral differences among our six treatments.

1. Introduction

Imagine two small firms $F_1$ and $F_2$, both urgently in need of an accountant. The two accountants $W_1$ and $W_2$ available on the regional market both promise to do a good job but their productivities in terms of saved costs/additional income depend on the specific matches. Matches are defined as efficient if they maximize the sum of productivities. Will the market allocation be efficient? What kind of wages will the players negotiate? Will the matches be "stable", i.e. will no unmatched pair have the ability to make Pareto improvements by matching (the core condition)? In our experimental 2x2 market we observe that more than 75% of all matches are unstable. Theories such as Nash Bargaining or Shapley Value which are capable of making predictions outside of the core do not perform any better. They also predict efficient matches, although 40% of our observed matches are inefficient (in the cases where inefficient matches are possible), and in most cases of efficient matches their predictions lie around the edge of the cloud of points which describes the experimental results.

We attribute the weak performance of all these concepts to an overly-competitive attitude of the market participants. We develop the concept of an "altruistic core" which offers a satisfactory explanation of our results under the assumption that, on average, market participants are spiteful. Negative altruism (spite) fosters the goal of receiving more income than one's partner does. Because these preferences may be restricted to market behavior we prefer to describe them as overly-competitive.

In the generalized problem there are $F_i$, $i = 1, \ldots, m$ and $W_i$, $i = 1, \ldots, n$ players on the two sides of the market. In many cases, these are small number markets, i.e. $m$ and $n$ are rather small. Internal job markets in firms is a good example, but also the assignment of jobs in academia, sports, or show business. Markets for top managers or for marriage partners in a remote village and also the competition for a small number of available locations in a shopping mall are further examples. In large markets, we may assume that the assignment problem is adequately described by a general search model. In small markets, however, we have to assume that the assignment problem is more complicated and that the competition for a small number of market locations in a region is more likely to make the market participants aware of the presence of other market participants. We will develop the concept of an "altruistic core", which offers a satisfactory explanation of our results under the assumption that, on average, market participants are overly-competitive.
Gale and Shapley, 1962; Roth, 1984; Sasaki & Toda, 1992; Wolfstetter, 1996; Nosaka, 2007; Lundberg and Pollak, 2008; college admission problem: Roth, 1985; house or roommate allocations: Abdulkadiroglu & Sonmez, 1999; Kamecke, 1992; hospitals – new physicians: Roth, 1990; organ transplantation: Roth et al., 2004; law clerk matching: Haruvy et al., 2006). Many suggestions for matching mechanisms try to substitute “misleading” incentives which promote, for example, premature matches between hospitals and medical graduates, or between law clerks and Federal appellate judges. Optimal mechanism design by a centralized clearing house or similar measures is, however, not the focus of this paper.

For the assignment of workers or sites to firms it is usually assumed that every match has a certain productivity (in terms of money) which the partners have to split among themselves. For our investigation we will adopt this assumption of transferable utility. We want to find out which matches are formed in the market process and how productivities are split, i.e. in our experiments the partners in a match have to bargain about the distribution of their joint profit.

The literature on matching with transferable utility is rather limited. For the general case, Koopmans and Beckmann (1957) show that market prices exist which support the efficient matches. Under these prices no other matches can be formed without making at least one of the partners of a potential other match worse off. The set of such prices is equivalent to the core (which is never empty in this problem). Becker (1974) investigates the marriage market under this and further simplifying assumptions (men and/or women are homogeneous). There are also some macro or intermediate approaches investigating the market efficiency under different labor market conditions such as the unemployment-vacancies structure and an information technology (i.e., Crawford and Knoer, 1981; Bolle, 1985; Hosios, 1990; Fujita & Ramey, 2006; Petrongolo & Pissarides, 2001, 2006; Fahr & Sunde, 2004; Sunde 2007).

There are only few experimental studies investigating matching markets. One example is Kagel and Roth (2000) who, in contrast to our study, do not allow their subjects to negotiate because they model a situation with ordinal rankings of partners. Their experiment reproduces a phenomenon found in many examples in the field, namely premature matches. An interesting feature of their experiment is a sequence of matching rounds in which the order of the matches is predetermined. This prevents the subjects from influencing the sequence of matches. An interesting result is reported by Haruvy and Ünver (2007), though again in a worker-firm environment with ordinal rankings. They find no significant differences between high information environments (all players are completely informed) and low information environments (only their own ordering is known to them). Most similar to our experiment is Tenbrunsel et al. (1999) who, however, fully concentrate their investigation on the influence of personal relationships and the efficiency of resulting matches. General coalition experiments have been conducted by Albers (1986) and by Uhlich and Selten (1986), although with completely different payoff structures and explicit general bargaining among all group members (while our problem requires only pairwise bargaining). Many other “matching” experiments consider buyer-seller relationships with homogenous goods where, in principle, all information about preferences can be comprised in one market price (see Cason and Noussair, 2007).

The next section derives some theoretical bargaining results for the 2x2 market concerning the core (= Neumann-Morgenstern solution in this case), Nash Bargaining, Shapley Value, and a concept which we call “Nash Bargaining with implicit threats”. Section 3 describes the 2x2 matching experiment with repeated negotiations. In Section 4, the results are presented and compared to the theoretical predictions. It will turn out that none of the above concepts are capable of explaining our experimental results. Section 5 investigates fairness and altruism considerations and proposes a new theory, the “altruistic core”, which explains why and when inefficient and unstable matches occur and how joint profits are distributed among the partners.
What is the informational status of the market participants? The "natural" assumption about information is that a worker knows the productivity of matches in which she may be involved (Worker 1 knows $\alpha$ and $\beta$) but not the productivities of matches of her competitor (Worker 2 knows $\gamma$ and $\delta$).

The same applies for firms. In our experiments, we provided the subjects with exactly this type of information.

For the following two solution concepts "core" and "Nash Bargaining with the outside option no match", the information about one's partner's income is sufficient. The "Shapley Value" and "Nash Bargaining with implicit threats (NBIT)" appear to require better information. It may be an interesting question whether the bargaining process makes the necessary information available, but we will not deal with this question directly. The weak performance of the Shapley Value and of NBIT may be due to several reasons, one of these being the lack of necessary information.

Core

Koopmans and Beckmann (1957) showed that there is a set of vectors (allocations) $C = \{(w_1, \ldots, w_m; f_1, \ldots, f_n) | w_i + f_j \geq a_{ij} \text{ for all } i,j \text{ and } w_i + f_j = a_{ij} \text{ if (i, j) belongs to the optimal matches}\}$. $C$ is equal to the core, i.e. Koopmans and Beckmann (1957) showed that the core (defined later by Gillies, 1959) of the Matching Game is not empty. $C$ is also equal to the unique Stable Set = Neumann-Morgenstern solution.

When matches $A = \{(W_1, F_1), (W_2, F_2)\}$ are efficient, i.e. $\alpha + \delta > \beta + \gamma$, the core solutions are called Core A and are described by

\begin{align}
 w_1 + \delta - \beta &> w_2 > w_1 + \gamma - \alpha \\
 w_1 + f_1 = \alpha, w_2 + r_2 = \delta, w_i, f_i &\geq 0.
\end{align}

Kagel and Roth (2000) have conducted their experiments under the same information structure.

Shapley Value

The Shapley Value (Shapley, 1953) is often understood as a measure of "power", in this case of bargaining power. For ordinal preferences for partners there may be several Stable Sets, all containing the core (Ehlers, 2007).
Note that the "asymmetry" of these values results from the relation \( \delta \alpha \leq \gamma \beta \leq \). In Figure 1, the Shapley Values of Workers 1 and 2 are indicated as SV. They do not need to be in the core. If \( \alpha + \delta = \gamma + \beta \) then (4) and (6) as well as (5) and (7) coincide.

Nash Bargaining Solution

We now determine the Nash Bargaining Solution and a variant of it which is adapted to the matching market. Let \( t_{W1}, t_{W2}, t_{F1}, t_{F2} \) be the threat values or "outside options" of workers and firms (which are to be determined later). The Nash Bargaining Solution results from the maximization of the Nash product

\[
P = (w_1 - t_{W1})(w_2 - t_{W2})(f_1 - t_{F1})(f_2 - t_{F2}),
\]

here under the restriction that transfers are only possible within matches, i.e.

\[
(w_1 + f_1) = \alpha, \quad (w_2 + f_2) = \delta, \quad (w_1 + f_2) = \beta, \quad (w_2 + f_1) = \gamma.
\]

For \( t_{W1} = t_{W2} = t_{F1} = t_{F2} = 0 \) (threat = "no match") the result is

\[
(f_1 = w_1 = \alpha / 2, \quad f_2 = w_2 = \delta / 2) \quad \text{or} \quad (w_1 = f_2 = \beta / 2, \quad w_2 = f_1 = \gamma / 2).
\]

In Figure 1 and Table 1 the combination with the larger Nash product is indicated as NB. For general threat values we find

\[
\begin{align*}
(t_{W1} - t_{W1})^* & = m \quad \text{or} \quad (t_{W1} - t_{W1})^* = n,
\end{align*}
\]

and respective payoffs for the other players (see Appendix B).

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Productivities</th>
<th>Nash Bargaining Solution</th>
<th>Shapley Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>280 400</td>
<td>(140, 320)</td>
<td>(160, 300)</td>
</tr>
<tr>
<td>T2</td>
<td>280 280</td>
<td>(140, 360)</td>
<td>(160, 300)</td>
</tr>
<tr>
<td>T3</td>
<td>280 460</td>
<td>(140, 320)</td>
<td>(170, 290)</td>
</tr>
<tr>
<td>T4</td>
<td>160 400</td>
<td>(200, 260)</td>
<td>(150, 330)</td>
</tr>
<tr>
<td>T5</td>
<td>160 460</td>
<td>(230, 230)</td>
<td>(160, 300)</td>
</tr>
<tr>
<td>T6</td>
<td>280 400</td>
<td>(200, 260)</td>
<td>(160, 320)</td>
</tr>
</tbody>
</table>

Table 1: Productivities of matches (rows = workers, columns = firms, efficient matches are underlined). Nash Bargaining solution (NB), and Shapley Value (SV) of the six treatments as applied in our experiment. Note that in T4 and T6 both matching possibilities are different.
(13) and (12) indicate all possible Nash Bargaining Solutions with implicit threats. In Figure 1, this line is indicated as NBIT. The matches A can only be formed if they are efficient. Otherwise, with implicit threats, the Nash product is maximized with efficient matches (see Appendix B). Condition (13) remains unchanged if the matches B are formed and are efficient. In both cases it describes the middle of the restrictions (1). Condition (12) is substituted by

\[ \frac{2}{g - \lambda + \delta + \varnothing} \leq \frac{2}{g - \lambda - \delta + \varnothing}. \]

which is empty under the parameters of our treatment T4, but not so in treatments T5, T6.

Equal Split

As a very simple behavioral alternative we introduce ES = equal split of productivities in the matches chosen. I.e. (\(w_1 = f_1 = \alpha/2, w_2 = f_2 = \delta/2\)) for matches A and (\(w_1 = f_2 = \beta/2, w_2 = f_1 = \gamma/2\)) for matches B. In the case of efficient allocations in T1, T2, T4, and T5, NB and ES coincide.

Inefficiency

The above theories support only efficient results. Inefficient matches can result from boundedly rational or irrational behavior, from social preferences or, involuntarily, because other players stick to inefficient matches. Boundedly rational behavior may be detected by investigating the bargaining process in detail. In this paper, however, we want to concentrate on boundedly rational behavior. In Section 5, we will explain inefficient matches by social preferences, but here we want to focus on investigating the bargaining process in detail. In this paper, however, we want to concentrate on boundedly rational behavior. In Section 5, we will explain inefficient matches by social preferences, but here we want to focus on investigating the bargaining process in detail. In this paper, however, we want to concentrate on boundedly rational behavior. In Section 5, we will explain inefficient matches by social preferences, but here we want to focus on investigating the bargaining process in detail.
The laboratory experiment (Lab) was run on z-Tree (Fischbacher, 2007). The individual matching with a partner who is in a similar strategic situation. In T4 and T5 efficiency requires matching between a strategically advantaged and a disadvantaged partner which should result in large income differentials within matches. In T3 and T6, where all matches are efficient, most allocation to a role was displayed on the start screen. On the next screen the matching process was informed about at least once. They took part either in the laboratory experiment (Lab) or in the classroom experiment (Class). Every session provided us with one independent observation.

In the classroom experiment (Class), the firms of the two groups were seated at separate tables and were not allowed to move. They also wrote a protocol which contained every observed interaction. In an analysis of the 160 observations, 81% were coded as "no change" and 19% were coded as "change". The code included any change in the treatment, any change in the behavior of the subjects, any change in the allocation, or any other deviation from the expected behavior. The code was then checked for consistency by two coders. In 90% of the cases, the coders agreed on the classification. The remaining cases were discussed and a consensus was reached.

The two experiments simulate a simplified labor market interaction. They are designed to test the performance of bargaining theories in different conditions. The experiments are structured in such a way that the participants are randomly assigned to different roles in each treatment. The role assignment is randomized over the sessions. Every subject was allocated to a role exactly once per treatment. The order of the treatments and the individual allocation to roles was randomized over the sessions. Every subject was informed about the productivities of the two matches they could participate in, i.e. W1, W2, F1, and F2. The order of the treatments was randomized over the sessions. Every participant was informed about the productivities of the two matches they could participate in, i.e. W1, W2, F1, and F2. The order of the treatments was randomized over the sessions. 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In important cases, bargaining theory performs well under all these different conditions is a particularly strong test.
analyses the results of Lab and Class are pooled. Severely influence the average bargaining result in a given match. Thus, in most of the following communication between subjects increases the efficiency in every respect, but it does not weakly significant (Fisher test; p = 0.051). We conclude therefore that the direct contact and

<table>
<thead>
<tr>
<th>Table 3:</th>
<th>44%</th>
<th>28%</th>
<th>23%</th>
<th>34%</th>
<th>47%</th>
<th>71%</th>
<th>50%</th>
<th>65%</th>
</tr>
</thead>
<tbody>
<tr>
<td>No matches (amount)</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>Class</td>
<td>Lab</td>
<td>Class</td>
<td>Lab</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33%</td>
<td>39%</td>
<td>39%</td>
<td>39%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This difference is highly significant (Fisher test; p < 0.001) because in the classroom (average age 22.8, 37.5% male; 70% German; 16.3% Polish; 14.8% other nationalities) and 80 students were allocated to the classroom experiment. The reason for the efficiency advantage of face-to-face bargaining is the low number of incomplete matches (< 1%) in the classroom compared with the high number (17%) in the laboratory (see Table 3). This difference is highly significant (Fisher test; p < 0.001).

Table 2: Absolute and relative frequencies of "no matches" and absolute and relative (with respect to matches) frequencies of A matches and B matches. Frequencies of efficient matches are also more inefficient matches in the Lab (46%) than in the Class (34%). This difference is compared with the high number (17%) in the laboratory (see Table 3). This difference is highly significant (Fisher test; p < 0.001). The complete ordering w_{1} \leq w_{2} \leq f_{1} \leq f_{2} \leq \delta \leq \gamma \leq \beta \leq \alpha \leq w_{0} holds for most cases. (f for 84.3% is smaller than the respective number in the case of the workers without consideration of the cases of zero income if no match is formed. (In appendix C, different treatments can be found in Appendix C. In Table 2, average results are reported for W_{A}, W_{B}, and W_{C} differentiated according to treatment, to matches A or B, and laboratory or classroom experiment. Significant differences (on the <.05 level) between Lab and Class are indicated by an asterisk.) From the relation w_{1} \leq w_{2} \leq f_{1} \leq f_{2} \leq \delta \leq \gamma \leq \beta \leq \alpha \leq w_{0} we may derive strategic considerations do play a role because W_{1} > w_{2} > f_{1} > f_{2} > \delta > \gamma > \beta > \alpha > w_{0} is in a given position in all treatments. This analysis is based on two variables: the channel of communication and the amount of the income. W_{1} denotes the income if no match is created at all. (\bullet is the symbol for a case of zero income if no match is formed. (In appendix C, different treatments can be found in Appendix C. In Table 2, average results are reported for W_{A}, W_{B}, and W_{C} differentiated according to treatment, to matches A or B, and laboratory or classroom experiment. Significant differences (on the <.05 level) between Lab and Class are indicated by an asterisk.) From the relation w_{1} \leq w_{2} \leq f_{1} \leq f_{2} \leq \delta \leq \gamma \leq \beta \leq \alpha \leq w_{0} we may derive strategic considerations do play a role because W_{1} > w_{2} > f_{1} > f_{2} > \delta > \gamma > \beta > \alpha > w_{0} is in a given position in all treatments. This analysis is based on two variables: the channel of communication and the amount of the income. W_{1} denotes the income if no match is created at all. (\bullet is the symbol for a case of zero income if no match is formed. (In appendix C, different treatments can be found in Appendix C. In Table 2, average results are reported for W_{A}, W_{B}, and W_{C} differentiated according to treatment, to matches A or B, and laboratory or classroom experiment. Significant differences (on the <.05 level) between Lab and Class are indicated by an asterisk.) From the relation w_{1} \leq w_{2} \leq f_{1} \leq f_{2} \leq \delta \leq \gamma \leq \beta \leq \alpha \leq w_{0} we may derive strategic considerations do play a role because W_{1} > w_{2} > f_{1} > f_{2} > \delta > \gamma > \beta > \alpha > w_{0} is in a given position in all treatments. This analysis is based on two variables: the channel of communication and the amount of the income. W_{1} denotes the income if no match is created at all. (\bullet is the symbol for a case of zero income if no match is formed. (In appendix C, different treatments can be found in Appendix C. In Table 2, average results are reported for W_{A}, W_{B}, and W_{C} differentiated according to treatment, to matches A or B, and laboratory or classroom experiment. Significant differences (on the <.05 level) between Lab and Class are indicated by an asterisk.) From the relation w_{1} \leq w_{2} \leq f_{1} \leq f_{2} \leq \delta \leq \gamma \leq \beta \leq \alpha \leq w_{0} we may derive strategic considerations do play a role because W_{1} > w_{2} > f_{1} > f_{2} > \delta > \gamma > \beta > \alpha > w_{0} is in a given position in all treatments. This analysis is based on two variables: the channel of communication and the amount of the income. W_{1} denotes the income if no match is created at all.
The numerous cases of no matches and inefficient matches (see Table 3) cannot be explained by any of our strategic theories. Can these concepts explain at least the income distribution in the efficient matches? (Nash Bargaining even differentiates between efficient matches. It predicts A in T3 and B in T6.) The figures in Appendix C show, however, that in most cases the bargaining results are not centered around NB or SV. In the (efficient) matches B in T4, T5, and T6 78% of the \( w_1 \) results are smaller and 87% of the \( w_2 \) results are larger than NB predicts. If matches A are efficient then 75% of the \( w_1 \) results are smaller and 69% of the \( w_2 \) results are larger than SV predicts. Therefore, not even if we disregard the contradictions by inefficient matches do NB and SV provide us with a satisfactory description of behavior.

### The Core and NBIT

As Appendix C and Table 4 show, the core is a successful predictive concept only in T1 (and perhaps in T3) and only if we concentrate on matches A. In all other cases of efficient matches (A in T2, B in T4 and T5) the number of results in the core is not higher than its relative area predicts (see Table 4). Selten's (1991) measure of predictive success for area theories is impressive only for the efficient choices of T1 (75%-18.75%=56.25%). In T1 and T2 there are, in the case of inefficient matches, no results in Anticore B (in accordance with our expectations). In T4 and T5, however, strategic considerations are foiled by the large numbers of bargaining results in Anticore A. These numbers are significantly higher than the relative area of Anticore A suggests. The tests will be discussed in more detail in connection with Table 6.

### Learning

Our experiment was not established in order to investigate learning, in particular because the number of repetitions was only six and because every bargaining situation was different. In spite of this, we look for influences of the variable "period", i.e. of the position of a treatment in the sequence presented to the subjects.

For efficient matches neither \( w_1 \) nor \( w_2 \) are significantly correlated with "period". For inefficient matches, however, Table 5 shows significant developments. \( w_2 \) and \( f_2 \) learn to exploit their advantageous strategic positions. In addition, there is a trend (\( r = 0.161 \)) toward efficient matches. This correlation coefficient with \( p < 0.05 \) is strongly significant (\( p = .0013 \)).

### Table 5: Trend or learning effects in efficient/inefficient matches. * (**) indicates significant correlation coefficients with \( p < .05 \) (\( p < .01 \)).

<table>
<thead>
<tr>
<th>Period</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>T2</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>T3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Workers versus Firms

In treatments T1, T3, and T5, \( w_1 \) and \( f_1 \) as well as \( w_2 \) and \( f_2 \) are in the same strategic position. Do they earn the same amount? Yes, they do. In the laboratory as well as in the classroom, the differences are neither small nor insignificant.

### Table 6: Results compared with predictions by the core. Area = percentage of points in the grid with a width of 1. * significantly (\( p < .001 \)) higher proportion of choices within (Anti)Core than when the choices are random.

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answers (on a five point scale) could be explained by altruism or inequity theories. Two prominent models highlight the influence of inequality in outcome satisfaction. First, Fehr and Schmidt (1999, henceforth F&S) suggest the utility function where it is assumed that $0 < c_i$ and $0 \leq b_i \leq a_i$.

Second, Bolton and Ockenfels (2000, henceforth B&O) assume

For empirical purposes we specified

In addition, as a simple alternative to inequity theories we consider altruism in the form

In all models the result depends on the question of whether person $i$ considers the incomes of all four participants ("group") or only that of the person she is matched with ("match"). We computed both variants for all three models.

Empirical Results

The regression results (see Appendix F) for satisfaction scores show that there is only a small average effect (rather small influences of altruism/inequity aversion) which seems to be captured equally well by all models ($\text{AdjR}^2$ between 0.2105 and 0.2256).

The $R^2$ values for the explanation of "fairness" were between 0.01 and 0.09 for all the three models which is considerably less than for satisfaction. Nonetheless it is debatable whether parameters of inequity theories should be estimated using both evaluations. This would have required, however, a lengthy discussion about methods.

The estimation of individual parameters is difficult because we have at most six observations for each individual. Therefore, we concentrated on the simplest model, namely the altruism model (17) which incorporates the smallest number of parameters. We chose the "match" version because in this case complete information is guaranteed. If the other version is chosen, the estimation of individual parameters is difficult because we have at most six observations in the sample.

Figure 2: Percentage of participants with positive altruism coefficients depending on the number of inefficient matches they formed (max = 4). Numbers in figure = number of cases.

![Figure 2:](image-url)
If there were average altruism with the respective equations provides

\[ \text{Substituting the utilities in (12) with (19) and (20), the altruistic core is a line and can be connected with efficient as well as altruistic core requires efficient matches. If } \alpha \rightarrow \gamma, \text{ the altruism core changes its shape in a way that it becomes a part of the altruistic core replacing efficient matches. If } \beta \rightarrow \gamma, \text{ the altruistic core replaces the efficient core and below 0.5, the values of } \gamma \text{ and the values of } \delta \text{ are affected. From (21) and above, the value of } \delta \text{ is the function of } \gamma. \]

Interestingly, the altruistic core of 2x2 markets is fundamentally different for

\[ \text{above } 5.0 \text{ and below } 0.5. \]

Let us assume that \( W \) and \( \beta \) are substituted by \( \gamma \) and \( \delta \), then the upper bound is moved by \( \alpha \) and \( \gamma \). For matches of \( B \), the upper bound is moved by \( \gamma \). The inequalities above indicate significant correlation coefficients with p < 0.05.

\[ \begin{align*}
\text{Correlation between income and own/other's (in the match) altruism coefficient.} \\
\end{align*} \]

<table>
<thead>
<tr>
<th>Income</th>
<th>Own</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>60</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>70</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>80</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>90</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\text{The Altruistic Core} \\
\end{align*} \]

<table>
<thead>
<tr>
<th>( \text{Income} )</th>
<th>( \text{Own} )</th>
<th>( \text{Other} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>60</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>70</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>80</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>90</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\end{align*} \]
The main conclusions from our experimental matching markets are:

6. Conclusion and Discussion

The bargaining results are influenced by strategic considerations, altruism (spite, an overly-
average spite), and efficiency is affected by the laboratory versus classroom situation and average as well.

In Table 8, the consequences of mild and strong spite within a group, we expect (w = w - 5.0) to lie between w = w + 0.5 and w = w + 3.6 (Table 4). On the basis of predictions of the UAC, the eight participants are newly allocated to the two markets and the roles. Therefore, in an adjusted test, we substitute, in sessions with treatments where the two negotiations resulted in the same match A or B, the two data points by their average. In cases of one incomplete matching or dependency of behaviour within a session. Within a session (at most) two results per treatment are connected with inefficient matches. In T3 and T6, 5.01 ≤ a ≤ 0.1 are connected with inefficient matches (bold types) and worker versus firm role, or learning (in efficient matches).

In this context, Selten's (1991) measure of predictive success (= hit rate minus relative area of UAC) delivers the statistical tests in Table 8 as well as Table 4 have not taken into account the possible dependency of behaviour within a session. Within a session, 9 of the 12 cases are connected with inefficient matches. In T1 and T2, 0 ≤ a ≤ 1 are connected with inefficient matches (weak types) and 0.5 ≤ a ≤ 0.1 are connected with efficient matches (bold types) and worker versus firm role, or learning (in efficient matches). In T1, T2, T3, T4, T5, T6, core with average as well as average spite.

The UAC is defined as the union of all these altruistic cores. In 9 of the 12 cases of mild and strong spite, the UAC is equal to a line (and equal to the "egoistic" core) which has a relative area of only 0.2%. The hit rate of 20% even in this degenerate case does not contain any experimental result. The consequences of

Table 8: Selten's Measure

<table>
<thead>
<tr>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
<th>T6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
<td>1.2</td>
</tr>
</tbody>
</table>

... shows that only T1 and T2 with matches B are rather unsatisfactory. The UAC of T6 with average spite is due to the prominence of one point on the line. A closer look at the three insignificant cases indicates significance only on the 5% level, and in T2 A and T4 marginally less powerful (here +). In T1, T2, T3, T4, T5, T6 all matches are efficient and altruistic cores exist for A and B matches and all

does not contradict the results shown in Appendix C. In T2, for example, the altruistic core for

The consequences of
Beyond these qualitative results we tested different bargaining theories. None of these (core=NM solution in our problem, Nash Bargaining, Shapley Value, as well as the self-developed “Nash bargaining with implicit threats”) is generally supported by our data. The merging of the core concept with altruism resulting in the “altruistic core” is successful, however. Assuming that on average spite dominates, the altruistic core explains the occurrence of inefficient and unstable matches (as a consequence of strong spite) as well as most qualitative differences among our treatments. Because these preferences may be restricted to bargaining behavior, we describe them as overly-competitive.

A possible variation of the 2x2 matching experiment is its generalization to larger markets. Similar to oligopoly experiments (Huck et al., 2004), an increasing number of competitors may reduce the deviation from the competitive equilibrium. Another question concerns the influence of complete information. In addition, a closer look into the dynamics of the bargaining process itself might provide insight. Here, we may find further explanations for non-matches and inefficiency as well as additional determinants of the resulting payoffs.

Our conclusion about the importance of social preferences is in line with most other experimental results concerning the interaction of a small number of agents. The dominating “flavor” of such preferences seems to depend, however, a lot on the situation. Bargaining seems to induce a strong competitive (spiteful) attitude in many people.

References


Appendix A

C = core = unique Stable Set (von Neumann – Morgenstern solution).

First, let us derive a characteristic function. From \( \emptyset \neq I \subseteq N \), let all \( i \) follows that

\[
|I'| > |N'| \Rightarrow 1 = \begin{cases} 1 \in I' & \text{if } |I'| = 1 \Rightarrow \emptyset \subseteq I' \subseteq 1 \\
0 \text{ otherwise} 
\end{cases}
\]

For all characteristic functions \( f \),

\[
1 \in I' \Rightarrow f(i) = \begin{cases} 1 & \text{if } i \in I' \\
0 \text{ otherwise} 
\end{cases}
\]

This implies the existence of a characteristic function that is homogeneous of degree one.

\[
\forall I, J, K \in \mathcal{P}(N), \quad f(I \cup J \cup K) = f(I) + f(J) + f(K)
\]

With \( f(\emptyset) = 0 \),

\[
\sum_{i \in I} f(i) = \sum_{i \in I} \sum_{i' \in J} f(i' I') - f(I')
\]

for all injective functions (= assignments)

\[
f(\emptyset) = 0 \quad \Rightarrow \quad \forall \pi, \quad f(\emptyset) = 0
\]

with \( \pi \subseteq \mathcal{P}(N) \).

28
1. Now assume that a payoff vector \((w_i, f_k)\) from \(C\) were dominated by another vector (from inside or from outside \(C\)) via the coalition \((B_w, B_F)\). Without restriction of generality we can assume \(F_w B F \leq 1\), otherwise we exchange the roles of workers and firms. As all values stem from pair-wise assignments, the value of the coalition is described by

\[ \sum_{i \in W} B_i (i) \]

with an optimal assignment \(k^*\). (24), however, shows that this value is too low to make the coalition better off than under any core allocation. Therefore all payoff vectors in \(C\) are undominated.

2. A payoff vector is outside \(C\) if one of the inequalities \(w_i + f_k \geq a_{ik}\) is violated. Apparently this vector is dominated by every core allocation. So \(C\) is equal to the core and it is also a stable set. Is it the only stable set? 1. Shows that we cannot remove any imputation from the core because it could not be dominated. 2. Shows that we cannot add any imputation to the core because it would be dominated. Therefore the core is equal to the unique Stable Set.

Appendix B

Nash Bargaining with implicit threats.

For general threat values we find (25a)

\[
(1 - W_1(w_1, w_2)) \beta \alpha = (1 - W_1(w_1, w_2)) \gamma
\]

or (25b)

\[
(1 - W_1(w_1, w_2)) \gamma \delta = (1 - W_1(w_1, w_2)) \beta
\]

(26a)

\[
(1 - W_2(w_1, w_2)) \alpha \gamma = (1 - W_2(w_1, w_2)) \delta
\]

or (26b)

\[
(1 - W_2(w_1, w_2)) \delta \alpha = (1 - W_2(w_1, w_2)) \gamma
\]

(27a)

\[
(1 - W_2(w_1, w_2)) \beta \gamma = (1 - W_2(w_1, w_2)) \delta
\]

or (27b)

\[
(1 - W_2(w_1, w_2)) \delta \alpha = (1 - W_2(w_1, w_2)) \beta
\]

Let us first regard the case where matches \(A\) are formed. Unfortunately, the system of the eight equations (25a) to (32a) is linearly dependent (but not contradictory). So we can determine only a linear condition for \((w_1, w_2)\):

\[
\alpha \gamma - \beta \delta = 0
\]

The respective outside options are

\[
(33)
\]

\[
\alpha \gamma - \beta \delta = 0
\]

(29a) \(f_1^* - 2 w_1 = \beta\) or (29b) \(f_1^* - 2 w_1 = \alpha\)

The respective threats of Firm 1, Worker 2, and Firm 2 fulfill (30a)

\[
1 F_1 - w_2 = \gamma
\]

or (30b) \(1 F_1 - w_2 = \alpha\)

(31a) \(2 f_2^* - 1 w_1 = \gamma\) or (31b) \(2 f_2^* - 1 w_1 = \delta\)

(32a) \(2 F_2 - 1 w_1 = \beta\) or (32b) \(2 F_2 - 1 w_1 = \delta\)

Let us first regard the case where matches \(A\) are formed. Unfortunately, the system of the eight equations (25a) to (32a) is linearly dependent (but not contradictory). So we can determine only a linear condition for \((w_1, w_2)\):

\[
\alpha \gamma - \beta \delta = 0
\]

The respective outside options are

\[
(33)
\]

\[
\alpha \gamma - \beta \delta = 0
\]

(29a) \(f_1^* - 2 w_1 = \beta\) or (29b) \(f_1^* - 2 w_1 = \alpha\)

The respective threats of Firm 1, Worker 2, and Firm 2 fulfill (30a)

\[
1 F_1 - w_2 = \gamma
\]

or (30b) \(1 F_1 - w_2 = \alpha\)

(31a) \(2 f_2^* - 1 w_1 = \gamma\) or (31b) \(2 f_2^* - 1 w_1 = \delta\)

(32a) \(2 F_2 - 1 w_1 = \beta\) or (32b) \(2 F_2 - 1 w_1 = \delta\)

Let us first regard the case where matches \(A\) are formed. Unfortunately, the system of the eight equations (25a) to (32a) is linearly dependent (but not contradictory). So we can determine only a linear condition for \((w_1, w_2)\):

\[
\alpha \gamma - \beta \delta = 0
\]

The respective outside options are

\[
(33)
\]

\[
\alpha \gamma - \beta \delta = 0
\]
Results for Worker 1 (w1) and Worker 2 (w2). For illustration purposes the results are changed.

Appendix C

which is equal under the parameters of our example. That is not so in terms of TS, T6.

\[
\frac{2}{\gamma - \delta + \beta + \alpha} \leq w \leq \gamma - \delta - \gamma + \beta - \alpha,
\]

\[
\frac{2}{\gamma - \delta + \beta + \alpha} \leq w \leq \gamma - \delta - \gamma + \beta - \alpha,
\]

\[\alpha \] cases it describes the middle of the restrictions (1), Condition (38) is substituted by

\[\alpha + \beta + \gamma + \delta \leq 4 \] and the profitabilities \( \alpha, \beta, \gamma, \delta \) indicate all possible Nash Equilibrium Solutions (with integer values). In Figure 1 the

\[\beta \] case to set \( \gamma \) which leads to \( \alpha = \frac{1}{2} \).
Thank you for participating in this labor market experiment. The experiment will last about one and a half hours. The payment you will receive at the end depends on both the decisions you make as well as your co-players' decisions. The following provides an overview of the experiment procedure. Please read the instructions if you have any questions.

Our experimental labor market consists of two workers and two firms. Workers can be hired by (matched with) firms. Matches are only possible between one worker and one firm. Every worker-firm match earns a certain joint profit. In order to form a match, the worker and the firm have to agree on the distribution of their joint profit.

We begin with the random allocation of the eight participants to two markets with each market consisting of two workers and two firms. You will receive a sheet of paper (Lab: see your computer screen) indicating whether you are a worker or a firm, as well as information about the different joint profit distributions you would earn in a match with one of the two potential partners from the other market side. Negotiations begin after workers have chosen a firm to bargain with. After one worker has made his choice, the other worker is allowed to bargain only with the remaining firm. One-on-one negotiations are permitted. The first phase of the negotiation ends when a worker leaves the firm's table, regardless of whether a provisional agreement has been reached. If an agreement is reached, both players must record this in a protocol. If both players are sitting alone at their tables, new negotiations may begin. The second phase of the negotiation begins after the firm has been approached by a worker. If a firm player has received a provisionally agreed-upon joint profit, he may approach another worker. Once one worker has been approached by a firm player, he is allowed to bargain only with the firm.

After the first round, five additional rounds of negotiations all consisting of different market groups and different individual role allocations will ensue. At the end of the experiment, you will receive the sum of all the shares of joint profits you have agreed to during the six rounds of negotiations.
Regression results for F&S are with $x_{\text{more}} = \sum_{i \neq j} x_{ij}$ and $x_{\text{less}} = \sum_{i \neq j} x_{ij}$.

F&S (group): Satisfaction = 1.462 + 0.00916 $x_{\text{more}}$ - 0.00389 ($x_{\text{more}}$) - 0.0000508 ($x_{\text{less}}$) [AdjR$^2 = 0.2256$]

F&S (match): Satisfaction = 1.864 + 0.00622 $x_{\text{more}}$ - 0.000339 ($x_{\text{more}}$) - 0.000343 ($x_{\text{less}}$) [AdjR$^2 = 0.2105$]

Regression results for B&O are with $x_{\text{rel}} = n \cdot \sum_{i \neq j} x_{ij}$.

B&O (group): Satisfaction = 1.874 + 0.00644 $x_{\text{more}}$ - 4.761 ($x_{\text{rel}}$) [AdjR$^2 = 0.2139$]

B&O (match): Satisfaction = 1.972 + 0.00625 $x_{\text{more}}$ - 0.251 ($x_{\text{rel}}$) [AdjR$^2 = 0.2132$]

Several studies (e.g. Engelmann & Strobel, 2004) show that the separation into two different effects, in line with F&S, better describe experimental results. But the regression results here show no major differences between the two fairness models. The results for the altruism model are (with $\sum_{i \neq j} x_{ij}$):

Altruism (group): Satisfaction = 1.53453 + 0.006314 $x_{\text{more}}$ + 0.0009580 $x_{\text{other}}$ [AdjR$^2 = 0.2199$]

Altruism (match): Satisfaction = 1.85186 + 0.006193 $x_{\text{more}}$ - 0.00003834 $x_{\text{other}}$ [AdjR$^2 = 0.2108$]

The parameter of the own result $x_{\text{more}}$ is always significant ($p < .001$ for all models). In addition, $x_{\text{more}}$ in F&S (group) and $x_{\text{rel}}$ in B&O (group) as well as in B&O (match) are significant ($p = .0003; p = .0199; p = .0269$). For the Altruism model the parameter $x_{\text{other}}$ is significant in Altruism (group) with $p = .0015$. 

The exponential parameter of B&O is $\frac{\gamma}{\Gamma(\lambda + 1)} = \frac{\beta}{\Gamma(\lambda + 1)}$. The regression results for B&O are with $\frac{\gamma}{\Gamma(\lambda + 1)}$ in steps of 0.1 from 0 to 1.25 in steps of 0.1.