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The Combined Effect of Taxation and Subsidization on Human Capital Investment

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Abstract

The impact of fiscal activity on human capital formation has often been analyzed with regard to the expenditure side of the budget, i.e. the subsidization of (higher) education. Recent contributions have increasingly focused on the effect of taxation on human capital accumulation. Less attention is given to the simultaneous effect of both subsidization and taxation on human capital accumulation. Since subsidies to higher education may offset the distortionary effects of taxation, the paper aims to analyze the amount of subsidization which is necessary to counteract the discouraging effect of various kinds of taxation (i.e., proportional, direct, indirect taxation and a combination of both direct and indirect taxation) on human capital, measured as the graduation rates. Furthermore, the effect of externalities from higher education on the optimal amount shall be analyzed as well as the role of taxation if direct costs of higher education are completely born by the state. The framework we use to illustrate our point is an two period cohort model with heterogeneous agents who endogenously decide on higher education with respect to taxation and subsidization.

Keywords: tax neutrality, human capital investment, education subsidies, income tax progression

JEL Classification: H21, H22, I22

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1 Introduction

The justification of subsidies to education has been discussed for decades. Advocates of public activities in the sector of education have in particular referred to externalities, credit constraints and distributional aspects. Up to this day, no convincing arguments have occurred in the literature that would justify education subsidies on distributive grounds (cf. Barbaro (2003)). The discussion on externalities has recently gained more importance, in particular due to the paper of Haveman and Wolfe (1984) and new developments in growth theory, after earlier attempts of explanation had been dismissed with regard to the neoclassical theory of marginal productivity (cf. Blaug (1970, pp. 112ff)). The importance of credit constraints is in principle indisputable (cf. Carneiro and Heckman (2002) for new evidence on credit constraints in post-secondary schooling), but Friedman and others have rightly argued that vouchers are a better means to compensate for unwanted effects which result from credit constraints. However, even if all classical arguments in favor of public subsidization cannot be dismissed as a whole, most economists argue that these arguments cannot justify the extensive or complete subsidization of investments in education that can be observed in many, in particular in European countries.

While earlier discussions were centered around the expenditure side of the budget, recent contributions rather focus on revenues. The impact of taxes on human capital accumulation has become the central element in recent literature. Trostel (1993, 1996) has shown that taxation has a negative impact on human capital investments and that education subsidies should primarily be seen and justified as a compensation for this tax distortion. While arguing in this way, Trostel uses an econometric model with a proportional tax rate. Dupor et al. (1998) have analyzed the distorting impact of a progressive taxation based on the US tax law in 1970. Their findings show that the progressivity in 1970 leads to an approximate 5-percent decline in human capital investment. Based on data from 1990, the impact differs considerably and lies between close to zero and -22%, depending on the schooling choice. Sturn and Wohlfahrt (1999) refer to the foregone smoothing benefit. Due to tax progression combined with the annual tax assessment, graduates pay more taxes than non-graduates with the same net lifetime earnings because graduates have accumulated their income in a shorter period of time.

The question in which way fiscal activities have an impact on the graduation rates in tertiary education has become a matter of policy concern after the OECD has shown that these rates differ considerably among the OECD-countries (OECD (2002), p. 39). Governments of countries with relatively low graduation rates (such as Italy, Austria and Germany) were forced to explain how they were planning to raise the graduation rate. For instance, the German Ministry of Education aims to emphasize the subsidization of investment to higher education more strongly. On the other hand, it is unclear which subsidization rate is necessary in order to just compensate for tax distortions and the revenue side of the budget is still neglected by designing educational policy.

In this paper, it will be analyzed how strong various kinds of progressive and proportional taxation distort human capital accumulation. In spite of taxation, it is assumed that the state behaves neutrally with regard to human capital accumulation. The state only subsidizes to an extent which is necessary to compensate for the negative impact of taxation. To put it differently, this paper aims to analyze the combined impact of both sides of the budget, based on an extended model of Creedy (1995, 1990).

In section 3 and his subsections (3.1 - 3.4), the optimal relation between subsidization and taxation under different tax regimes are analyzed. A concrete numerical illustration of the graduation rates, which results from the combined effect of taxation and subsidization is supplied in section 4. The paper concludes in section 5 with a discussion of the main results. Before we start with these analysis, the general framework, as it was outlined in Creedy (1995) and in Creedy and Francois (1990), will be presented.

2 The Creedy/Francois-Model as the general framework of analysis

Creedy and Francois developed a framework in which our analysis takes places. They regard higher education as a homogenous pure investment good. A population of heterogeneous individuals who differ with respect to individual ability characteristics (endowments), denoted by $y_i$, is assumed. These endowments are crucial for the individual productivity and for the decision in favor or against taking up a university
degree. They assume a 2-period cohort model. In the first period, each individual faces the decision whether to take up a degree or, alternatively, to start working as a non-educated. In the second period, all individuals have started working. An individual chooses higher education if her net lifetime earnings with a university degree exceed the lifetime earnings in case that she does not invest in higher education. The degree causes direct (and non tax-deductable) costs, $c$, for each individual. The total costs consist of the direct costs (e.g. teaching aids, tuition fees) and the foregone earnings. The income equals the individual endowment, $y_i$. Students have the opportunity to work even in the first period and, thus, earn the portion $h$ of the income earned without higher education. Therefore, the total costs of higher education consist of

$$(1 - h) y_i + c$$  \hspace{1cm} (1)

Individuals who have completed a degree in the first period will raise their income in the second period due to the rate of return to education. To simplify matters, it shall be assumed that the individual rate of return to education, $s_i$, is proportional to the individual endowment:

$$s_i \equiv u \cdot y_i$$ \hspace{1cm} (2)

Furthermore, it is assumed that graduates cause an externality from which both graduates and non-graduates gain. This externality raises all incomes by $g$. This externality resulting from higher education is assumed to depend on the graduation rate, $p$, with $\frac{\partial g}{\partial p} > 0$ and $\frac{\partial^2 g}{\partial p^2} < 0$.

As noted above, in the first period each individual faces the decision whether to take up a degree or, alternatively, to start working without a university degree. The share’s size of those of the cohort choosing higher education depends on the distribution of $y_i$, whereby this distribution is given exogenously. $F(y)$ denotes the distribution function of $y$, so that it measures the proportion of individuals with endowments less than or equal to $\tilde{y}$. The proportion of individuals who invest in higher education is given by:

$$p \equiv 1 - F(\tilde{y})$$ \hspace{1cm} (3)

Thus, the Educational Choice Margin, $\tilde{y}$, is crucial for the analysis. An individual $i$ makes a decision for higher education if his net lifetime earnings as graduate exceed those of being a non-graduate. This is the case if his personal endowments, $y_i$, exceed the educational choice margin (ECM), $\tilde{y}$. Since the lifetime earnings of educated agents, $V^E$, and the lifetime earnings of non-educated ones, $V^N$, are given by

$$V_i^{E[bm]} \equiv hy_i - c + \frac{y_i(1 + s_i + g)}{1 + r}$$ \hspace{1cm} (4)

and

$$V_i^{N[bm]} \equiv y_i + \frac{y_i(1 + g)}{1 + r}$$ \hspace{1cm} (5)

(where $r$ represents the discount rate), it is possible to find an ability level corresponding to that of an agent who is indifferent to investing in higher education, by setting (4)=(5). The higher is $y_i$, the higher is the probability that an individual belongs to those for who higher education is worthwhile. As a consequence of this model, only those individuals with the highest endowment go to university. Particularly two fiscal instruments, a subsidization rate, $\rho$, and a proportional tax rate, $t$, influence $\tilde{y}$ and taking into account the effects of these instruments, Creedy and Francois analyze the effect of fiscal activity in a wide rage of topics.

In the following sections, we will use this framework to illustrate our point at issue. In contrast to Creedy and Francois (1990), we do not assume that $g$ depends on the graduation rate, due to simplicity reasons. Further, we will enlarge this framework with additional tax parameters.

### 3 The Basic Model as a Benchmark

We start our analysis with a benchmark case, which occurs if there are no fiscal activities, i.e., no subsidization and taxation takes place.

Considering equation (2), $\tilde{y}^{bm}$, which results from equating (4) and (5)
\[ \hat{y}_{[bm]} \equiv \psi + \sqrt{\psi^2 + \omega} \]  

(6)

where \( \psi \equiv \frac{(1-h)(1+r)}{u} \) and \( \omega \equiv \frac{c}{(1+r)} \).

Taxes should be fair and they should promote economic efficiency. An efficient tax is neutral and imposes no excess burden. Neutrality in the field of higher education means that a tax does not alter the choice, i.e., to obtain higher education or not. Optimal tax theory states that the optimal and also neutral tax is a lump-sum tax. Such a tax will not distort the choice between the alternatives and, as a consequence, a lump-sum tax will have no effect on the educational choice margin, independently of the amount of revenue a government needs.

**Proposition 1** The educational choice margin remains unchanged if a lump-sum-tax system is imposed. Therefore, this benchmark can otherwise be interpreted as a case with non-distortionary taxation.

**Proof.** Add to equations (4) and (5) the lump-sum tax, \( -\tau \), for each period,

\[ \hat{y}_{[p]} - \tau \frac{1+r}{(1+r)} \]  

Since \( \hat{y}_{[bm]} \) results from (4) = (5), the tax liability would simply be cancelled out from this equality.

Due to the fact that

\[ \frac{\partial \hat{y}_{[bm]}}{\partial g} = 0 \]

the external effect \( g \) does not have an impact on \( p \).

As noted above, we assume that the government’s aim is to behave neutrally with regard to the graduation rate. This means that fiscal policy shall not influence \( p \). If taxation has a negative influence on \( p \), subsidization shall offset this effect. The following subsections analyze the amount of subsidization which is necessary to counteract the distortionary effect of the various kinds of taxation. Subsidization offsets the distortionary effects of taxation, if the resulting \( \hat{y} \) equals \( \hat{y}_{[bm]} \).

### 3.1 Proportional Taxation

The first extension of the basic model takes a proportional income tax into account. All incomes are subject to the homogenous tax rate \( t \). At the same time, the state subsidizes a part of the direct costs by the rate of subsidization, \( \rho \). By including the two fiscal instruments, equations (4) and (5) are extended to

\[ V_{E[p]} = hy_i (1-t) - c (1-\rho) + \frac{y_i (1 + s_i + g) (1-t)}{1+r} \]  

(7)

and

\[ V_{N[p]} = y_i (1-t) + \frac{y_i (1 + g) (1-t)}{1+r} \]  

(8)

Equating (7) and (8) leads to the ECM under proportional taxation:

\[ \hat{y}_{[p]} = \psi + \sqrt{\psi^2 + \omega (1-\rho)} \]  

(9)

Figure 1 shows the ECMs which result from various \( \rho \)- and \( t \)-values.

The activities of the public sector are entirely neutral if \( \hat{y}_{[p]} = \hat{y}_{[bm]} \).

**Proposition 2** If the direct cost of the degree are completely born by the state \((\rho = 1)\), the size of the tax rate has no effect on \( p \).

**Proof.** If \( \rho = 1 \), it follows that \( \hat{y}_{[p]} = 2\psi = \frac{(1-h)(1+r)}{u} \) and, thus, is independent of \( t \). ■

**Proposition 3** Under a proportional tax rate, fiscal policy which consists of the combination of revenue and spending policy, is neutral if the rate of subsidization equals the tax rate. In case the rate of subsidization exceeds the tax rate, the educational choice margin falls and \( p \) rises. In the opposite case, \( p \) falls if \( \frac{\rho}{t} < 1 \).

**Proof.** If \( \rho/t = 1 \), it follows that the term \( \frac{(1-\rho)}{(1-t)} = 1 \), and, hence, \( \hat{y}_{[p]} = \psi + \sqrt{\psi^2 + \omega} = \hat{y}_{[bm]} \). ■

It is also interesting to note that the optimal relation between subsidization and taxation does not depend on externalities. Therefore, it does not matter whether an external effect arises from higher education for policy purposes.
3.2 Indirect Income Tax Progression

In this subsection, the optimal relation of $\rho$ and $t$ shall be inferred if the concept of a flat tax (i.e. a proportional tax rate after a tax-free threshold $\kappa$ has been implemented) applies. Hence, the individual tax burden related to the basic income equals $t(y_i - \kappa)$. Since a tax-free threshold has the same effect as a lump sum transfer of $t\kappa$, the net income amounts to $t\kappa + y_i(1 - t)$.

In contrast to the analysis above, there are not only two theoretically possible groups (those above the ECM and those below), but five: two groups of individuals who invest in higher education, and three groups of those who are below the ECM. One subgroup of those investing in human capital pays no taxes in the first period since the $y_i$ of its members is below the threshold. Members of the second subgroup pay taxes in the first period as their basic income exceeds the threshold. Of those not investing in higher education, two subgroups receive a basic income which is below the threshold. Hence, these individuals pay no taxes in the first period. The first subgroup also pays no taxes in the second period $[y_i(1 + g) \leq \kappa]$. However, those who receive an income above the threshold due to $g [y_i(1 + g) > \kappa]$ in the second period have to pay taxes in the second period. The third group of those not investing in higher education pays taxes in both periods as $y_i > \kappa$. Considering all these cases would certainly complicate the analysis. Therefore, it shall be assumed that the income of students during their qualification period does not exceed the threshold. Hence, we assume that

\[ \kappa > h y_{\text{max}} \]  

(10)

On the contrary, the earnings of non-graduates are so high that they pay taxes in both periods. Hence, by rearranging equation (4) the net lifetime earnings of graduates amount to
and, by rewriting (5), the net lifetime earnings of non-graduates to

\[ V_{i}^{E[i][p]} = h y - c (1 - \rho) + \frac{(1 + s_i + g) y_i (1 - t) + t \kappa}{1 + r} \]  

and for various \( \rho \)- and \( t \)-values

The net lifetime earnings of non-graduates differ from those of graduates with earnings below the threshold only with regard to the double relief of the threshold (which, of course, has to be discounted in the second period). Introducing an indirect income tax progression changes the net lifetime earnings of graduates in two ways. First, no taxes on income are paid in the first period. Second, the income increases in the second period by \( t \kappa / (1 + r) \).

As the relief due to the basic allowance in the second period is the same for both graduates and non-graduates, the effect in the first period is crucial. In this case, the easing of \( t \kappa \) for non-graduates is opposed by an easing of \( h y_i \) for graduates. Since we have assumed above that the income of students during their qualification period does not exceed the threshold, \( h y_i < \kappa \) applies. By comparing the reliefs in the first period (\( h y_i \) versus \( t \kappa \)), it becomes clear that the relief for non-graduates is larger than for graduates. Therefore, it can be expected that - compared with a proportional taxation - introducing a tax-free threshold will lead to a higher educational choice margin. Thus, the share of students in the cohort, \( p \), will be reduced. Indeed, equating (11) and (12) leads to an ECM of
\[\tilde{g}^{(p)} = \frac{\psi}{(1-t)} - \frac{t(1+r)}{2u(1-t)}\]

\[+ \sqrt{\left[\frac{\psi}{(1-t)} - \frac{t(1+r)}{2u(1-t)}\right]^2 + \frac{(1-\rho)}{u}(1-t) + \frac{tk(1+r)}{u(1-t)}}\]  

(13)

**Proposition 4** Under a flat tax regime, the rate of subsidization has to exceed the tax rate if fiscal policy is neutral towards human capital accumulation in the case of indirect income tax progression.

**Proof.** If (13) and (6) are equated and \(\rho\) is isolated (see Appendix), \(\rho = t + \frac{(\kappa - \tilde{y}^{(km)})}{e}\). Dividing by \(t\) leads to

\[\frac{\rho}{t} = 1 + \frac{(\kappa - h\tilde{y}^{(km)})}{e}\]  

(14)

Since \(\tilde{y}^{(km)} < y_{max}\), the rate of subsidization has - with regard to equation (10) - to exceed the tax rate in order to ensure neutrality.

In order to see the impact of a change in the threshold on the relation of \(\rho\) and \(t\), we differentiate (14) with respect to \(\kappa\) and obtain

\[\frac{\partial \left(\frac{\rho}{t}\right)}{\partial \kappa} = \frac{1}{c}\]  

(15)

i.e. a linear slope.

It shall be stressed that the rate of subsidization, \(\rho\), has to rise more sharply in the case of indirect income tax progression, compared to the situation without a tax-free threshold, than indicated by the relation \(\rho/t\) at first sight. In order to obtain the same tax revenue, the tax rate must rise due to the tax-free threshold. Therefore, \(\rho/t = 1.2\) does not imply that the rate of subsidization has to rise by just 20 percent, but it has to increase by more than 20 percent. If we assume that the tax rate has to increase by 20 percent in order to compensate for the shortfall in revenues caused by the tax-free threshold, the tax rate rises for instance from 0.3 to 0.36. In this case, a relation of \(\rho/t = 1.2\) means that the rate of subsidization has to rise by 44 percent. Therefore, we can conclude that the rate of subsidization has to rise sharply at the transition point from proportional taxation to indirect progression if the fiscal policy is neutral with respect to human capital accumulation.

**Proposition 5** In contrast to the discussion of proportional taxation, the tax rate matters even if the direct costs of education are fully subsidized. On the other hand, if a proportional tax is imposed, the amount of externalities has no impact on the optimal \(\rho \rightarrow t\) relation.

**Proof.** Equation (13) only contains \(\rho\) in the expression \(\omega \frac{\tilde{y}}{1-t}\), but the tax rate itself is part of some other elements of this expression for the ECM. Furthermore, (14) does not depend on \(g\).

### 3.3 Direct Income Tax Progression

Application of increasing marginal tax rates to annual income discriminates against the tax-payer whose income fluctuates, and, therefore, has a negative effect on human capital investment. If net lifetime earnings are identical, the direct income tax progression results in an advantage for those individuals who can spread their net lifetime earnings consistently over a longer period of time. Thus, taxpayers with fluctuating and taxpayers with steady incomes carry different burdens. Sturn and Wohlfahrt (1999) have recently named this additional burden Foregone Smoothing Benefit.
In order to depict the discussion, the following assumptions shall be made. Non-graduates and students are liable to the tax rate $t$, according to a proportional taxation. Graduates on the other side face a higher tax rate $(t + \varepsilon)$, where $\varepsilon \in [0 < \varepsilon < (1 - t)]$ can be seen as a higher marginal bracket rate or, alternatively, as a specific graduate tax.

Taking these assumptions into account, equation (4) is transformed to

$$V_i^{EC|\rho} = hy_i(1 - t) - c(1 - \rho) + \frac{y_i(1 + s_i + g)(1 - (t + \varepsilon))}{1 + r}$$  \hspace{1cm} (16)

while equation (5) remains unchanged.

$$V_i^{N|\rho} = y_i(1 - t) + \frac{y_i(1 + g)(1 - t)}{1 + r}$$  \hspace{1cm} (17)

The ECM in this case arises from (16) = (17)

$$\tilde{g}^{[\rho]} = \left\lfloor \psi \left(1 - t\right) - \frac{\varepsilon (1 + g)}{2u(1 - t - \varepsilon)} \right\rfloor + \left\lfloor \psi \left(1 - t\right) - \frac{\varepsilon (1 + g)}{2u(1 - t - \varepsilon)} \right\rfloor^2 + \frac{\omega (1 - \rho)}{(1 - t - \varepsilon)}$$  \hspace{1cm} (18)

It becomes evident that $\tilde{g}^{[\rho]}$ for $t, \rho, \varepsilon = 0$ is equal to the ECM in the situation without public activity.

In the case of $\varepsilon = 0$, $\tilde{g}^{[\rho]}$ equals the ECM in the situation with proportional taxation. These are rather trivial partial results. However, more important for the analysis is the question concerning the neutral relation of $\rho$ and $t$. Therefore, we set (18) = (6) and receive

$$\frac{\rho}{t} = 1 + \frac{\varepsilon}{r} \left[1 + \tilde{g}^{[\rho]} \left(\frac{1 + g}{c(1 + r)} + \frac{1 - h}{c}\right)\right]$$  \hspace{1cm} (19)

**Proposition 6** The rate of subsidization has to exceed the tax rate (noticeably) in order to compensate for the distorting impact of taxation on human capital accumulation.

**Proof.** The term in brackets $\left(\frac{1 + g}{c(1 + r)} + \frac{1 - h}{c}\right)$ is positive, though small, as the costs of higher education, $c$, have a large value and the numerator, $(1 + g)$ and $(1 - h)$, is close to 1. Since $\tilde{g}^{[\rho]}$ has to be added to 1, $\rho > t$. □

**Proposition 7** While the tax rate has no effect under a proportional tax system if $\rho = 1$, it has a negative effect under an indirect progression regime (also if $\rho = 1$). An increasing (basic) tax rate, given $\varepsilon$, leads to a positive effect on the graduation rate if the direct costs are fully subsidized.

This result might be astonishing, but the intuition becomes clear if we take the following into account that. Given $\varepsilon$, the higher the basic tax rate, the lower the progressivity, measured as $\varepsilon/t$—ratio. This effect is overcompensated if $\rho < 1$. This relationship can be seen clearly in figure 3.

Another interesting result is that the optimal $\rho/t$—value depends on the size of externalities, $g$. As we have seen in the previous subsections, $g$ has no impact on the optimal $\rho/t$—relation if the marginal tax rate is constant. Now, considering an increasing marginal tax rate, the externalities have to be taken into account if we discuss the comprehensive effect of fiscal activity on the formation of higher education. This result is explainable if we note that the present kind of direct tax progression violates only graduate’s increasing incomes through $g$, as can be seen in equation (17) compared with equation (16).

In discussing the impact of an indirect progression on human capital formation, we have seen that it discourages investments in human capital more than a proportional taxation due to the fact that individuals, who choose to invest in higher education forego a benefit which amounts to $(1 - h)\tau$, which derives from the tax free threshold. In case of a direct progression, graduates pay an additional tax burden of $\varepsilon(1 + s_i + g)/(1 + r)$. The indirect progression leads to a large distortion if this additional tax burden exceeds the
forgone tax benefits, which result from a threshold. Equating both, the foregone benefit and the additional tax burden yield

$$\theta \equiv y_i \geq -\frac{(1 + g)}{2u} + \sqrt{\left[-\frac{(1 + g)}{2u}\right]^2 + \frac{2t\kappa}{\bar{\varepsilon} \psi}}$$

(20)

If the educational choice margin is high, the direct income tax progression distort more than the indirect one. But if the rate of subsidization is set to a very high value, so that the educational choice margin falls, there are circumstances there a flat tax discourages more. The intuition becomes clear if we take into account that the additional tax burden, $\varepsilon \frac{y_i(1 + s_i + g)}{(1 + r)^2}$, depends on the individual endowment, $y_i$, whereby the endowment does not matter for the foregone benefit from introducing a threshold. In section 4 we provide some numerical examples in order to illustrate this point more clearly.

3.4 Direct and Indirect Income Tax Progression

Finally, the case of a combined direct and indirect income tax progression which can frequently be observed in practice (i.e. rising marginal tax rates after a tax-free threshold) shall be regarded. Therefore, the assumptions which were made in the subsection on indirect progression shall apply. The net lifetime earnings in this case equal

$$V_i^{E[d+i\rho]} = hy_i - c(1 - \rho) + \frac{(1 + s_i + g) y_i(1 - t - \varepsilon) + t\kappa}{1 + r}$$

(21)
\[ V_i^{N[d+ip]} = y_i (1 - t) + t \kappa + \frac{(1 + g) y_i (1 - t) + t \kappa}{1 + r} \]  

Equating (21) and (22) and isolating \( y_i \) leads to the ECM of

\[ \tilde{y}[d+ip] = -\frac{(1 + r) (h - 1 + t) - \varepsilon (1 + g)}{2u (1 - t - \varepsilon)} + \sqrt{\left[ -\frac{(1 + r) (h - 1 + t) - \varepsilon (1 + g)}{2u (1 - t - \varepsilon)} \right]^2 + \omega \frac{(1 - \rho)}{(1 - t - \varepsilon)} + \frac{t \kappa (1 + r)}{u (1 - t - \varepsilon)}} \]  

The rate of subsidization which is required in order to compensate for the distorting impact of direct and indirect income tax progression on \( \tilde{y} \) equals

\[ \rho = \varepsilon \left( 1 + y_i^{[bm]} \left[ \frac{(1 + g) (1 + r) + (1 - h)}{c (1 + r)} \right] \right) + t \left[ \frac{\kappa}{c} + 1 \right] - t \frac{h \tilde{y}_i^{[bm]}}{c} \]  

Dividing by \( t \) leads to the compensating relation

\[ \frac{\rho}{t} = 1 + \frac{\kappa - h y_i^{[bm]}}{c} + \frac{\varepsilon}{t} \left( 1 + y_i^{[bm]} \left[ \frac{(1 + g) c (1 + r) + (1 - h)}{c} \right] \right) \]  

As can be seen, equation (25) consists of a combination of the optimal relation under an indirect tax regime [equation (14)] and the optimal relation under a direct tax progression [equation (19)]. As we have seen in the previous subsections, the effect of increasing tax rates is ambiguous in sign if education is fully
subsidized. Combining both kinds of taxation, it can be seen that the negative effect under the special circumstance of $\rho = 1$ resulting in the case of a flat tax overcompensates the positive one which applies under a direct tax progression. The combined effect is shown in Figure 4.

Again, as has been shown in the previous subsection, only the rise of incomes of the graduates through $g$ is taxed and, hence, the optimal $\rho/t$-value depends on $g$.

After all, it is not surprising that the educational choice margin rises considerably if both kinds of tax distortions, tax-free threshold and rising marginal tax rates, are combined.

4 Some Numerical Examples

This section uses the numerical procedure described in the previous section to provide a variety of numerical examples. The following table presents the educational choice margins which result under the tax systems discussed in the previous subsections. The parameters are set as following: $h = .1$, $u = .08$, $r = .05$, $c = 100$, $\varepsilon = .1$, $g = .1$, $\kappa = 10$ and hence, $\psi = 5.91$ and $\omega = 131.5$. These parameters are also used to plot the figures shown in the previous section.

The bottom row provides the optimal rate of subsidization for the tax rates given in the top row. According to equation (10), we set $\kappa = hy_{\text{max}}$. As a consequence, no individual will invest in higher education if the educational choice margin exceeds 100.

<table>
<thead>
<tr>
<th>$\tilde{y}^{(bm)}$</th>
<th>$[p] = (9)$</th>
<th>$[dp] = (18)$</th>
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<td>$\rho = 0$</td>
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<td>$t = 0$</td>
<td>11.60</td>
</tr>
<tr>
<td></td>
<td>$t = .4$</td>
<td>11.43</td>
</tr>
<tr>
<td></td>
<td>$t = .8$</td>
<td>09.88</td>
</tr>
<tr>
<td>$\rho_{opt}$</td>
<td>$= 0.000$</td>
<td>0.400</td>
</tr>
<tr>
<td></td>
<td>$p = 0.800$</td>
<td>0.183</td>
</tr>
<tr>
<td></td>
<td>$dp = 0.583$</td>
<td>0.983</td>
</tr>
</tbody>
</table>

$\tilde{y}^{(bm)} = 42.61$, $[p] = (9)$, $[dp] = (18)$

| $\tilde{y}^{(bm)} = 42.61$, $[p] = (13)$, $[dp + ip] = (23)$ |
|-------------------|-------------|----------------|
| $\rho = 0$        | $t = 0$     | 42.61          |
|                   | $t = .4$    | 53.48          |
|                   | $t = .8$    | 87.53          |
|                   | $= 0$       | 46.21          |
|                   | $t = .4$    | 60.79          |
|                   | $t = .8$    | > 100          |
| $\rho = .5$       | $= 33.42$   | 42.39          |
|                   | $t = .4$    | 86.10          |
|                   | $t = .8$    | 35.31          |
|                   | $= 35.31$   | 46.41          |
|                   | $t = .8$    | > 100          |
| $\rho = 1$        | $= 11.81$   | 16.30          |
|                   | $t = 0$     | 26.43          |
|                   | $t = .4$    | 14.65          |
|                   | $t = .8$    | 20.90          |
|                   | $= 11.81$   | 48.52          |
| $\rho_{opt}$      | $= 0.000$   | 0.423          |
|                   | $p = 0.846$ | 0.183          |
|                   | $dp = 0.606$| 0.606          |

Now, the corresponding graduation rates, $p$, are reported. As noted above (confer equation 3), the graduation rate depends on the distribution of the abilities, $y$. Therefore, the form of the distribution function $F(y)$ must be specified in order to transform the educational choice margins into graduation rates. According to a large theoretical and empirical literature on income dispersion, it is supposed that $y$ is lognormally distributed with a median value of 25. The coefficient of variation was set at 0.5, which is a reasonable approximation for industrialized countries.

| $p^{(bm)} = 0.24$, $[p] = (9)$, $[dp] = (18)$ |
|-------------------|-------------|----------------|
| $\rho = 0$        | $t = 0$     | 0.1348          |
|                   | $t = .4$    | 0.0615          |
|                   | $t = .8$    | 0.0056          |
|                   | $= 0$       | 0.1348          |
|                   | $t = .4$    | 0.0595          |
|                   | $t = .8$    | 0.0000          |
| $\rho = .5$       | $= 0.2932$  | 0.1704          |
|                   | $t = .4$    | 0.0286          |
|                   | $t = .8$    | 0.2932          |
|                   | $= 0.2932$  | 0.1608          |
|                   | $t = .4$    | 0.0256          |
| $\rho = 1$        | $= 0.9281$  | 0.9281          |
|                   | $t = 0$     | 0.9281          |
|                   | $t = .4$    | 0.7932          |
|                   | $t = .8$    | 0.4408          |

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In subsection 3.3, we have compared the distortionary effects of a direct income tax progression and a flat tax. Our analysis indicates that a rising tax rate exacerbates more if the educational choice margin is high. But this effect is ambiguous in sign if, for example, the educational choice margin falls due to a large subsidization or to rising rates of return. The reason is, as has been discussed in subsection 3.3, that in this case, the foregone benefit does not depend on the individual endowment - in contrast to the direct progression. In figure 5, $\tilde{y}^{[dp]} - \tilde{y}^{[ip]} = \Delta \tilde{y}$ for various $t$-values. As can be seen, the difference is negative, i.e., $\tilde{y}^{[ip]}$ is higher than $\tilde{y}^{[dp]}$ if $\rho$ is very high so that the graduation rate rise.

5 Discussion

In this paper, we shed a light on the combined effect of both sides of the budget on human capital accumulation. While earlier contributions are particularly centered around the expenditure side of the budget, recent contributions also take into account the revenue side. The aim of this paper is to analyze how different kinds of taxation, as the common revenue policy, affect the graduation rates in post-secondary schooling. The distinction between the different kinds of income taxation unveils remarkable effects. Furthermore, extensive subsidization does not necessarily imply a promotion of human capital. The analysis indicates that subsidization should primarily be seen as a compensation for tax distortions. Moreover, it signals that there are circumstances in which the tax rate has no effect on the graduation rate or in
which the effect is ambiguous in sign. Furthermore, we have seen that, given the existence of externalities from investments in higher education, these externalities only have an impact if the tax schedule has implemented a direct progression. If the income tax progression only results from a tax-free threshold or if taxation is proportional, externalities have no effect and, therefore, the existence of externalities can be neglected for the present purposes.

Which policy implications for governments aiming at affecting graduation rates could be drawn from the present analysis? First of all, since tax policy seems to have a stronger effect on graduation rates than the subsidization rate, policy should rather focus on the tax side and less on subsidization activities.

Furthermore, international comparisons of education policy, as carried out e.g. by the OECD (2002, Chapter B), should not exclusively focus on the expenditure volume for educational institutions. They should rather take into account the comprehensive effect of public policy for human capital formation, which clearly includes the tax system.

With regard to the normative justification for educational subsidization, this study has shown an efficiency justification for subsidies to higher education besides the classical arguments. Subsidizing education was shown to be optimal, because it offsets the distortionary effects of taxation on human capital accumulation. On the other hand, the amount of subsidization, which is necessary to offset the distortion can also be interpreted as the excess burden of taxation in the field of higher-education investment. Taking into account that these subsidies may have further unwanted effects, the main advice to policy-makers is not to subsidize more and more but to take seriously the (probably) unwanted effects of non-optimal taxation and in particular of income tax progression.

Several caveats must also be highlighted. This study is based on a model that does not analyze the effect on the graduation rates if physical capital accumulation is possible. The two-period model implicitly assumes that both periods have the same time length. Considering that the first period (i.e., the period of education formation) is shorter than the second period, the distortionary effects of taxation may be larger than the present examination provides. Furthermore, the tax rates in the various subsections of section 2 are not so easy comparable, due to a missing budget constraint. It is also neglected the interdependencies of the costs for a rising subsidization and the tax rate. This caveat seems acceptable because only about 1 or 2 percent of the public budgets in most European countries are spent for subsidization.

Appendix

Proof. of equation (14)

Define \( a \equiv \frac{(1-h)c(1+x)}{2u} \) for simplicity reasons. Then, eq. (6) = (13) becomes

\[
an + \frac{\sqrt{a^2 + \omega \frac{(1-h)}{1-t} + \frac{t_n(1+r)}{1-t}}}{2} = y^{[bh]}
\]

\[\Leftrightarrow a - y^{[bh]} = -\sqrt{a^2 + \omega \frac{(1-h)}{1-t} + \frac{t_n(1+r)}{1-t}}\]

\[\Leftrightarrow a^2 - 2ay^{[bh]} + (y^{[bh]})^2 = a^2 + \omega \frac{(1-h)}{1-t} + \frac{t_n(1+r)}{u(1-t)}.\] Isolating \( \rho \) yields

\[
\rho = 1 + \frac{tk}{c} + \frac{a}{c(1+r)} (t-1) - \frac{2ua y^{[bh]}}{c(1+r)} (t-1)
\] (26)

Eq. (6) could simply be reworked to

\[
y^{[bh]} = \frac{(1+r)}{2u} \left[(1-h) + \sqrt{(1-h)^2 + \frac{4ua}{c(1+r)}}\right] \] and hence, we get

\[
\frac{u (y^{[bh]})^2}{c(1+r)} = \frac{1-h}{c} y^{[bh]} + \frac{(1-h)^2}{c} + \frac{1}{c(1+r)} \] Inserting this expression in equation (26) yields after some rearrangements

\[
t + t \left[\frac{\lambda c(1+h)-y^{[bh]}(1-h)}{c} - \frac{2ua y^{[bh]}(t-1)}{c(1+r)}\right].\] Inserting \( a \), it can be seen that

\[
\frac{y^{[bh]}}{c} - \frac{y^{[bh]}(1-h)}{c} - \frac{2au a y^{[bh]}(t-1)}{c(1+r)} = 0.\] Therefore, it remains \( \rho = t + t \left[\frac{(1-h)y^{[bh]}}{c}\right].\)
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