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An oversimplified inquiry into the sources of exchange rate variability

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Abstract

Exchange rates as well as relative price level and output movements are decomposed into components associated with nominal shocks as well as shocks to aggregate supply and aggregate demand. In contrast to previous analyses of such decompositions based on statistical vector autoregression (VAR) analysis, this study takes as a starting point a simple textbook model of exchange rate determination, augments it by allowing for suitably defined random shocks and transforms it into a triangular format resembling the identification procedure of the VAR methodology. Applied to major bilateral exchange rate series, the decomposition suggests that exchange rate variability is mostly driven by shocks to aggregate demand, particularly in the longer run. Overall, the evidence is roughly in line with previous decompositions obtained from statistical VARs.

JEL Classification: F41, F47, C63

Keywords: exchange rates, vector autoregression, nominal and real shocks

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1. Introduction

The volatility of nominal and real exchange rates are a perennial issue in international monetary economics. The breakdown of the Bretton Woods system of fixed exchange rates in the early 1970s marked a watershed in the perception of large fluctuations of exchange rates as being particularly detrimental to international trade and the smooth operation of international financial markets. Suggestions to return to some kind of formal international exchange rate arrangement or even setting up monetary unions can all be traced back at least in part to the issue of whether exchange rate fluctuations are harmful to the international economy. Whether or not these suggestions have merit hinges decisively on where exactly exchange rate fluctuations originate from.

One can essentially identify two distinct sources driving exchange rates, one arising in financial markets, the other in the real economy. Shocks in financial markets can be quite diverse, ranging from different national monetary policies or money demand disturbances, which may themselves be due to currency substitution effects, to speculative short-term international capital transactions. There appears to be a strong presumption in some quarters that the latter are to blame for causing the large degree of exchange rate fluctuations witnessed in today’s currency markets. Under such circumstances movements in exchange rates can be considered disruptive to the functioning of output markets, and fixing exchange rates would be the preferred strategy of sheltering the real economy from shocks in financial markets.\footnote{There is a substantial literature arguing that fixed exchange rate are superior to floating rates when money market shocks are the dominant source of disturbance. For a review see Garber and Svensson (1995).} The second source of exchange rate volatility is usually awarded far less attention in economic policy circles. This is the real-economy explanation of equilibrium exchange rate adjustments. According to this view, exchange rates are primarily driven by real factors such as changes in technology or foreign-direct investment on the aggregate supply side or shifts in preferences.
or fiscal policy on the aggregate demand side. By requiring a response of the real exchange rate defined as the relative price of domestic relative to rest-of-the-world outputs, these shocks can best be absorbed by changes in the corresponding nominal exchange rate with less or no need for domestic or foreign price levels to bring about the requisite adjustments.

The two opposing views of the sources of exchange rate volatility are each associated with a very influential class of models in international macroeconomics. The financial markets view is a direct implication of the disequilibrium approach of Dornbusch (1976), in which money market disturbances induce excessive exchange rate volatility in an environment of sluggish price adjustment. In the Dornbusch model, money market shocks induce temporary displacements of exchange rates from their equilibrium levels. Only after price levels have had time to adjust to clear money markets do exchange rates return to their equilibrium levels. The disequilibrium view would therefore predict an autoregressive component in the time-series behavior of exchange rates.

The real economy view is reflected in the work of Stockman (1980, 1987), Lucas (1982) and Hsieh (1987). In these models, exchange rate movements are equilibrating responses to disequilibria in output markets caused by aggregate demand or supply disturbances. Assuming these shocks to be permanent, the induced exchange rate adjustments are themselves permanent, implying that exchange rate series should contain a unit root.

Both of these views do have empirical support. Mussa (1986) has made a convincing case in favor of the disequilibrium approach by demonstrating the empirical relevance of price sluggishness as documented in the observable close comovement of nominal and real exchange rates coupled with the substantial increase in real exchange rate volatility for a broad range of industrial country pairings since the collapse of the Bretton Woods system.

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2 The argument that flexible exchange rates insulate the economy more effectively than fixed rates against real shocks has a long tradition and is originally due to Friedman (1953).
3 These results have been confirmed by other studies such as Baxter and Stockman (1989) and Flood and Rose (1995).
More formally, Evans and Lothian (1993) have identified a significant role of transitory shocks in driving real exchange rates, while Lothian and Taylor (1996) and Frankel and Rose (1996) have detected mean reversion in real exchange rate movements. In contrast, the real economy view implies real exchange rates to be nonstationary. This view has been corroborated empirically by Huizinga (1987), who provided early evidence that real exchange rates possess unit roots and that most of their variation is due to permanent shocks.

Whereas the sources of real exchange rate fluctuations are generally unobservable, some authors have attempted to extract nominal and real shocks from the joint behavior of time series on real exchange rates and relative price levels. The preferred strategy of obtaining such a decomposition is to apply structural vector autoregression (VAR) analysis on nominal and real exchange rates using exclusively long-run identifying restrictions in the tradition of Blanchard and Quah (1989). The simplest version of a structural VAR model of this sort is a two-dimensional system with the nominal and real exchange rates as the endogenous variables with the sole identifying restriction being the long-run neutrality of money on the level of the real exchange rate [Lastrapes (1992)]. The model can be used to identify two different underlying exogenous sources of variability which jointly drive the two endogenous variables. These two shock can then be interpreted quite naturally as nominal and real shocks, where the nominal shock exerts only transitory effects on the level of the real exchange rate while the real shock induces a permanent shift of the (equilibrium) real exchange. Clarida and Galí (1994) employ a three-dimensional version of the VAR by incorporating relative output levels as a third endogenous variable into the system. This extended model needs two additional restrictions in order to just-identify three underlying shocks. The additional restrictions come in the form of an assumed long-run neutrality of money with respect to relative output levels and the long-run neutrality of aggregate demand shocks on the level of aggregate supply (long-run vertical Phillips curve).
Both approaches have in common that they make use of long-run identification restrictions only, leaving the short-run responses of the endogenous variables completely data-determined. Yet the corresponding impulse response functions of the estimated VARs display patterns that closely match the dynamic adjustment paths predicted by the two classes of exchange rate models described above. It turns out that real shocks displace the level of the real exchange rate permanently and in the direction predicted by the equilibrium approach to exchange rate modeling, e.g., an expansionary demand shock leads to an appreciation of the real exchange rate. In contrast, nominal shocks display adjustment paths that show a hump-shaped response of the real exchange rate, a feature reminiscent of the overshooting phenomenon of the disequilibrium approach.

As the VAR models seem to be compatible with both views of exchange rate determination, the variance decompositions obtained from these VARs can provide information as to the relative importance of the two strands of theory in explaining real-world exchange rate movements. Applied to monthly and quarterly data for major bilateral exchange rates post-Bretton Woods, nominal shocks account for roughly one third to one half of overall exchange rate variability at short horizons of up to one year with their importance diminishing quickly as the forecast horizon is extended. The results suggest that at these frequencies, exchange rates fluctuations appear to be predominantly equilibrium responses to real shocks. Subsequent studies have confirmed these results. Modeling a higher-dimensional structural VAR system in the spirit of Clarida and Galí, Weber (1997) finds nominal shocks to be relatively unimportant in accounting for real exchange rate variability. More recently, Astley and Garratt (2000), using the Clarida and Galí procedure, show nominal shocks to be of little
relevance as a source of real exchange rate variability in U.K. data, accounting for less than 10% of real sterling fluctuations.

The structural VAR approach has recently been criticized by Faust and Leeper (1997) as well as Faust and Rogers (2000). The first paper demonstrates that the use of infinite-order restrictions on a finite-order VAR yields reliable estimates only if the reduced-form VAR is the correct representation. The second paper points out that a curious yet general result of structural VARs shows that nominal shocks induce the exchange rate to overshoot its long-run level, but that the peak of the overshooting response occurs only four to twelve quarters after the shock has occurred. This evidence does not square with the notion of the exchange rate being an asset price which should react immediately to any kind of new information in the market (such as a nominal shock).

From an economic point of view, the structural VARs are mostly statistical decompositions with just a few extraneous economic identification restrictions imposed upon them. As a consequence, the resulting dynamics are largely a black box phenomenon. This paper aims to provide an alternative route to a VAR decomposition of exchange rate fluctuations into their underlying nominal and real shocks by starting with a simple textbook model of exchange rate determination, which is yet general enough to be compatible with both the equilibrium and disequilibrium approaches to real exchange rate modeling. The model is then triangularized to resemble the identification procedure of the VAR methodology. By invoking the contemporaneous identification restrictions of the exchange rate model, the decomposition procedure presented here bypasses the black box problem of the VAR approach and provides a robustness check on the quantitative importance of nominal and real shocks in driving real exchange rates. To this end, the transformed model is calibrated to allow for a decomposition

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4 One notable exception to this VAR evidence is Rogers (1999) who finds nominal shocks to be an important source of real exchange rate fluctuations in a long sample of the pound-dollar rate.
of actual time series on exchange rates and relative price and output levels into components associated with shocks to aggregate demand and aggregate supply as well as financial markets.

The remainder of this paper is organized as follows: Section 2 lays out the exchange rate model, Section 3 presents its triangularization, and Section 4 applies the transformed model to obtain a decomposition of exchange rate, output and price level data of the British-U.S., German-U.S. and Japanese-U.S. bilaterals. The results of this model-based decomposition can then be directly compared with those generated by earlier studies using structural VARs. A final section concludes.

2. A simple exchange rate model

There is no generally agreed-upon exchange rate model one could resort to. Standard monetary models of exchange rate determination, including the basic Dornbusch model, have long been discredited by their dismal failure in predicting exchange rates, as forcefully documented by Meese and Rogoff (1983). In contrast, the real-economy model variety neglects entirely the potentially important influence of shocks in financial markets on the dynamic behavior of exchange rates. In what follows, the characteristic elements of the two views are combined in a suitably extended rational-expectations Dornbusch-type framework by explicitly allowing for random shocks to aggregate demand and aggregate supply as well as the money market. This modeling strategy has the advantage of yielding a familiar set of equations which can easily be manipulated and solved in order to arrive at the desired exchange rate decomposition. The general modeling approach adopted here is in the spirit of earlier work by Mussa (1982) and follows in its exposition the textbook version of the
The log-linear model is laid out in Eqs. (1) through (4):

\[ m_t - p_t = \phi y_t^d - \eta i_t, \quad \phi, \eta > 0, \]  
\[ i_t - i^* = E_t (e_{t+1} - e_t), \]  
\[ y_t^d = \gamma y_t^s + \delta (e_t - p_t) + g_t, \quad \delta > 0, \]  
\[ E_t (p_{t+1} - p_t) = \psi (y_t^d - y_t^s) + E_t (\tilde{p}_{t+1} - \tilde{p}_t), \quad \psi > 0. \]

Eq. (1) is the equilibrium condition for the domestic money market, with \( m_t \) and \( p_t \) denoting the nominal money supply and the domestic price level. Money demand is modeled as a function of both the domestic nominal interest rate, \( i_t \), prevailing between dates \( t \) and \( t+1 \), and aggregate demand, \( y_t^d \), with \( \phi \) and \( \eta \) denoting the appropriate nonnegative elasticities of money demand. Eq. (2) is the uncovered interest rate parity (UIP) condition, stipulating that any differential between domestic and foreign interest rates, \( (i_t - i^*) \), be compensated by corresponding conditional exchange rate expectations, \( E_t (e_{t+1} - e_t) \). In Eq. (3) the level of aggregate demand is related to aggregate supply, \( y_t^s \), by a term capturing domestic absorption, \( \gamma y_t^s \). Normalizing the log of the foreign price level at zero, the log real exchange rate at date \( t \) is defined as \( q_t = e_t - p_t \). The parameter \( \delta \) thus reflects the elasticity of the current account to changes in the relative price of domestic-to-foreign outputs. The shift parameter \( g \) can be interpreted quite generally as any shock to aggregate demand such as autonomous shifts in consumption and investment or fiscal policy. Finally, Eq. (4) introduces a price adjustment mechanism in the form of an inflation-expectations-augmented Phillips-

---

5 A textbook version of a Dornbusch-type model with supply shocks similar to the one used here can be found in Gaertner (1993).
curve where the expected rate of change in the price level is determined by sluggish responses to disequilibria in the output market plus a term capturing expected equilibrium inflation, \( E_t(\tilde{p}_{t+1} - \tilde{p}_t) \). The latter term reflects trend inflation in the face of a constant real exchange rate, so that \( E_t(\tilde{p}_{t+1} - \tilde{p}_t) = E_t(e_{t+1} - e_t) \). The three forcing variables of the system are \( m, g \) and \( y^* \). At this point it suffices to view these variables as exogenously determined random variables.

The model can easily be solved by collapsing the equations into two difference equations, one each for the dynamics of the nominal and the real exchange rates. These are:

\[
E_t(e_{t+1} - e_t) = \frac{1}{\eta} \left[ e_t - (1 - \phi \delta) q_t + \phi \gamma y_t^* + \phi g_t - m_t \right]
\]  

(5)

and

\[
E_t(q_{t+1} - q_t) = \psi (1 - \gamma) y_t^* - \psi \delta g_t - \psi g_t
\]

(6)

where the foreign interest rate has been normalized at zero. In order to simplify the subsequent derivations, define the long-run equilibrium real exchange rate as of date \( t \) as

\[
\bar{q}_t = \frac{(1 - \gamma) y_t^* - g_t}{\delta}.
\]

The deterministic saddlepath of the system can be derived by freezing the forcing variables at constant levels such that \( m_t = m, \ y_t^* = y'^*, \ g_t = g \), and thus \( \bar{q}_t = \bar{q} \). Eq. (6) can then be rewritten as

\[
E_t(q_{t+1} - q_t) = -\psi \delta (q_t - \bar{q}).
\]

(7)
Starting from any date-\(t\) deviation of the real exchange rate from its long-run level, the real exchange rate reverts to its equilibrium value as long as \(0 < \psi \delta < 2\). This stability condition is assumed to hold throughout the subsequent analysis. Solving Eq. (7) for any date \(s \geq t\) traces out the dynamic adjustment of the real exchange rate:

\[
E_t(q_s - \bar{q}) = (1 - \psi \delta)^{s-t} (q_t - \bar{q}), \quad s \geq t.
\] (8)

In order to pin down the saddlepath, Eq. (5) can be written in the equivalent form

\[
e_t - \bar{q} = \frac{\eta}{1 + \eta} E_t(e_{t+1} - \bar{q}) + \frac{1 - \phi \delta}{1 + \eta} (q_t - \bar{q}) + \frac{1}{1 + \eta} m - \phi y^s.
\]

(9)

After imposing the transversality condition

\[
\lim_{t \to \infty} \left( \frac{\eta}{1 + \eta} \right)^T E_t e_{t+T} = 0,
\]

Eq. (9) can be solved in terms of its infinite backward sum

\[
e_t - \bar{q} = m - \phi y^s + \frac{1 - \phi \delta}{1 + \eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^{s-t} E_t(q_s - \bar{q}).
\]

(10)

Finally, substitution of Eq. (8) into Eq. (10) yields the saddlepath

\[
e_t = m - \frac{1}{\delta} g + \frac{1 - \gamma - \phi \delta}{\delta} y^s + \frac{1 - \phi \delta}{1 + \eta \psi \delta} (q_t - \bar{q}).
\]

(11)
A graphical representation of the model is depicted in Fig. 1. Here the two difference equations (5) and (6) appear respectively as the upward-sloping and vertical lines after setting their LHS equal to zero. The saddlepath is marked by arrows and shows all combinations of nominal and real exchange rates from which the system converges to equilibrium. All other combinations away from the saddlepath are on divergent paths as implied by the indicated directions of motion of the system.

Fig. 1 about here

The analysis now turns to a description of the dynamic adjustment of the system in the wake of shocks to the forcing variables. Let us first consider an expansionary money supply shock or a contractionary money demand shock, both of which can be modeled as an exogenous and once-and-for-all increase in the forcing variable \( \Delta m > 0 \). The implied reactions in the nominal and real exchange rates can be computed formally by setting \( \Delta e = \Delta q \) to identify the short run impact in which the price level remains constant, and by setting \( q = \bar{q} \) to extract the long run effect. To derive the impact effect, use the definition of the equilibrium real exchange rate to rewrite the saddlepath as

\[
e_r = m - \frac{\phi \delta + \eta \psi \delta}{1 + \eta \psi \delta} \frac{1}{\delta} g + \left[ \frac{\phi \delta + \eta \psi \delta}{1 + \eta \psi \delta} - \frac{\phi \delta}{1 - \gamma} \right] \frac{1 - \gamma}{\delta} y^* + \frac{1 - \phi \delta}{1 + \eta \psi \delta} q_r.
\]  

(11')

First differencing of Eq. (11’) and setting \( \Delta e = \Delta q \) uncovers the impact reaction as

\[
\Delta e = \Delta q = \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \Delta m.
\]
The long-run effect follows after setting $q = \bar{q}$ in Eq. (11). First differencing gives $\Delta e = \Delta m$ and $\Delta q = \Delta \bar{q}$. Comparing the impacts with the steady state effects shows that the real exchange rate necessarily overshoots in the short-run as its steady-state level is unaffected by the money-market disturbance. In contrast, the nominal exchange rate only displays an overshooting response if $\phi \bar{\delta} < 1$. Fig. 2 displays the dynamic adjustment of the system for the nominal overshooting scenario in which the saddlepath is sloping upward. The shock shifts the $\Delta e = 0$ locus to the north, tracing out the dynamic adjustment path indicated by the arrows. Here the movement along the 45° line from A to B is instantaneous due to the assumption of price stickiness, which in turn requires the nominal and real exchange rates to move one-for-one in the instant following the occurrence of the shock. The system then travels along the upper saddlepath towards the new equilibrium in C.

Now let the system be disturbed by an expansionary shock to aggregate demand such that $\Delta g > 0$. Here the impact and steady-state reactions turn out to be of identical size given by

$$\Delta e = \Delta q = -\frac{1}{\delta} \Delta g.$$ 

Fig. 3 provides a graphical exposition of this scenario. The shock shifts both the $\Delta q = 0$ and the $\Delta e = 0$ lines in the directions indicated by the arrows. It turns out that the old and the new steady states in A and B lie on the same 45° line, so that the nominal and real exchange rates immediately jump to their new steady-state levels once the shock has hit the system. The rationale for this result lies in the fact that the level of aggregate supply will not be affected by
the aggregate demand shock in the long run. Goods market equilibrium then requires aggregate demand to return to its pre-shock level. However, this implies that the level of money demand has to be identical across the two steady states. As the level of money supply has not been altered, the price level cannot have changed between the two steady states either. As the price level solely accounts for the stickiness in the model, the adjustment in the wake of an aggregate demand shock therefore must be instantaneous with no dynamics involved. The implied appreciation of both the nominal and the real exchange rates simply crowds out the entire aggregate demand impulse by inducing an expenditure switching effect of opposite and identical size.

Finally, consider an expansionary shock to aggregate supply, $\Delta y^s > 0$. In this case, the impact reactions of the nominal and real exchange rates are given by

$$\Delta e = \Delta q = \left[\frac{1 - \gamma - \phi}{\delta} - \phi \left(1 + \frac{1}{\phi \delta + \eta \psi \phi}\right)\right] \Delta y^s$$

which can either be positive or negative depending on the parameterization of the model. The steady-state effects are

$$\Delta e = \frac{1 - \gamma - \phi \delta}{\delta} \Delta y^s \quad \text{and} \quad \Delta q = \frac{1 - \gamma}{\delta} \Delta y^s$$

implying that the nominal exchange rate shift continues to be ambiguous even in the long run. In contrast, the real exchange rate effect is unambiguously positive, implying a steady-state
real depreciation in the wake of an expansionary supply shock. This result appears intuitive insofar as an increase in domestic production necessitates a decline in the relative price of domestic output to clear world goods markets. Fig. 4 displays the dynamics of the supply shock. Again both the $\Delta q = 0$ and the $\Delta e = 0$ lines shift in the directions indicated by the arrows. Starting from a position in A, the system moves along the 45° line to the south-west on impact and then travels on the saddlepath from B to to the new equilibrium in C. The figure depicts an impact appreciation followed by a gradual depreciation of both the nominal and real exchange rates in the process of convergence to the new steady state. This is of course only one of various scenarios that may materialize. A different scenario would have the $\Delta e = 0$ schedule shift only marginally to the south, implying that the new steady state in C would lie to the north-east of the initial steady state in A. In this case the impact reaction would follow along the 45° line to the north-east while the saddlepath would shift upward. Under such circumstances both the nominal and real exchange rates would depreciate both on impact and in the long run.

This completes the discussion of the effects of the various shocks on the adjustments of nominal and real exchange rates. The following section is concerned with manipulating the model in such a way as to reverse the identification of the exchange rate effects in response to the shocks analyzed above towards an identification scheme allowing the various shock components to be extracted from actual time series of nominal and real exchange rates. To this end, the model is transformed into a triangular structure reminiscent of the identification strategy used in the statistical VAR approaches. As will become clear in the next section, this triangularization is rendered possible because there is a crucial dichotomy in this model in the form of the neutrality of aggregate demand shocks on the money market and the consequent

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6 This happens whenever $\phi \delta$ is very small.
absence of any price dynamics associated with such shocks. While this dichotomy may look very particular to the model specification adopted here, it is in fact a fairly general feature encountered across a broad range of exchange rate models encompassing both the disequilibrium and the real economy views mentioned in the introduction.

3. A model transformation

The forcing variables of the model have so far been assumed to be exogenously determined random variables. In what follows, these variables are modeled in the form of explicit stochastic processes. In particular, let the variables $m_t$, $g_t$ and $y^t$ follow simple random walk processes

$$m_t = m_{t-1} + n_t,$$

$$g_t = g_{t-1} + d_t,$$

$$y^t_t = y^t_{t-1} + s_t,$$

with the error terms $n_t$, $d_t$ and $s_t$ assumed to be normally distributed with standard deviations $\sigma_n$, $\sigma_d$ and $\sigma_s$, respectively. Here $n_t$ can be interpreted as either a money supply shock or a negative money demand shock, $d_t$ is an aggregate demand shock and $s_t$ denotes a shock to aggregate supply. Rearranging the saddlepath of Eq. (11) and using subscripts for the exogenous variables yields:

$$q_t = \tilde{q}_t + \frac{1 + \eta \psi \delta}{1 - \phi \delta} \left[ e_t - m_t + \frac{1}{\delta} g_t - \frac{1 - \gamma - \phi \delta}{\delta} y^t_t \right].$$
Upon invoking the definition of the equilibrium real exchange rate and substituting $e_t = q_t + p_t$, Eq. (12) can be written as

$$q_t = \bar{q}_t + \frac{1+\eta\psi\delta}{\phi\delta + \eta\psi\delta} \left[ m_t - p_t - \frac{\phi\delta}{\gamma} y_t^r \right],$$

(13)

while first differencing and again using the definition of the equilibrium real exchange rate results in

$$\Delta q_t = \frac{1+\eta\psi\delta}{\phi\delta + \eta\psi\delta} (n_t - \pi_{t-1}) - \frac{1}{\delta} d_t \left[ \frac{1-\gamma}{\delta} - \frac{1+\eta\psi\delta}{\phi\delta + \eta\psi\delta} \frac{\phi\delta}{\gamma} \right] s_t,$$

(14)

where $\Delta q_t = q_t - q_{t-1}$ and $\pi_{t-1} = p_t - p_{t-1}$. Using Eq. (3) together with the definition of the expected equilibrium inflation rate, $E_t(\bar{p}_{t+1} - \bar{p}_t) = E_t(e_{t+1} - e_t)$, Eq. (4) can be rewritten as

$$E_t(p_{t+1}) = E_t(e_{t+1}) - (1-\psi\delta) q_t - \psi\delta \bar{q}_t,$$

(16)

while solving Eq. (13) for $e_t$ yields

$$e_t = \bar{q}_t + \frac{1+\eta\psi\delta}{\phi\delta + \eta\psi\delta} \left[ m_t - \frac{1-\phi\delta}{\phi\delta + \eta\psi\delta} p_t - \frac{1+\eta\psi\delta}{\phi\delta + \eta\psi\delta} \frac{\phi\delta}{\gamma} y_t^r \right].$$

(17)

Leading Eq. (17) by one period and substituting for $E_t(e_{t+1})$ in Eq. (16) gives
\[
E_t(p_{t+1}) = -(1 - \psi \delta) \frac{\phi \delta + \eta \psi \delta}{1 + \eta \psi \delta} q_t - \psi \delta \frac{\phi \delta + \eta \psi \delta}{1 + \eta \psi \delta} \bar{q}_t \\
\quad + \frac{\phi \delta + \eta \psi \delta}{1 + \eta \psi \delta} E_t(\bar{q}_{t+1}) + E_t(m_{t+1}) - \frac{\phi \delta}{\gamma} E_t(y^*_t)
\]  

(18)

Making use of the random walk properties \( E_t(\bar{q}_{t+1}) = \bar{q}_t \) and \( E_t(y^*_t) = y^*_t \), Eq. (18) implies upon first differencing:

\[
\pi_t = -(1 - \psi \delta) \frac{\phi \delta + \eta \psi \delta}{1 + \eta \psi \delta} (\Delta q_t - \Delta \bar{q}_t) + n_t - \frac{\phi \delta}{\gamma} s_t.
\]  

(19)

Finally, substituting Eq. (14) into Eq. (19) and collecting terms gives

\[
\pi_t = (1 - \psi \delta) \pi_{t-1} + \psi \delta n_t - \psi \delta \frac{\phi \delta}{\gamma} s_t.
\]  

(20)

Eq. (20) has a straightforward interpretation in terms of the model dynamics. Recall from the qualitative analysis of section 2 that the money market is affected by nominal shocks and aggregate supply shocks, but not by shocks to aggregate demand. As any disequilibrium in the money market is eventually resolved by suitable adjustments in the price level, shocks affecting the money market will also affect the instantaneous rate of inflation. Whereas any excess supply in the money market \((n>0)\) raises the instantaneous inflation rate, an expansionary shock to aggregate supply \((s>0)\) lowers inflation because the real exchange rate \(q\) rises faster than the nominal rate \(e\) in the process of the dynamic adjustment towards the new steady state along the saddlepath (compare Fig. 4). As the price level is just the difference
between the nominal and real exchange rates, \( p = e - q \), inflation must be negatively affected throughout the adjustment process in the wake of an expansionary supply shock.

Eqs. (14) and (20) constitute the transformed model. Note that Eq. (14) expresses any movement in the level of the real exchange rate as a function of all three kinds of shocks while Eq. (20) identifies inflation variability as being caused by shocks to either the money market or to aggregate supply. These two equations can thus be used to recursively pin down two of the three shocks hitting the system. The third shock has to be fed into the model from outside. In what follows, this third shock is chosen to be the supply disturbance which is assumed to be directly related to the level of output in the economy.

In order to facilitate a comparison of the importance of the various shocks identified below with those generated by the statistical VAR analyses cited in the introduction, a final step in the current analysis is to derive the appropriate variance decompositions and impulse response functions characteristic of the VAR approach. These can now be obtained from Eqs. (14) and (20). First solve Eq. (20) by expressing current inflation as the backward sum of shocks to the money market and aggregate supply:

\[
\pi_t = \psi \phi \sum_{j=0}^{\infty} (1 - \psi \phi)^j \left[ n_{t-j} - \frac{\phi \delta}{\gamma} s_{t-j} \right],
\]

then substitute the resulting expression into Eq. (14) to obtain:

\[
\Delta q_t = \frac{1 + \eta \psi \phi}{\phi \delta + \eta \psi \phi} n_t - \frac{1 + \eta \psi \phi}{\phi \delta + \eta \psi \phi} \psi \delta \sum_{j=0}^{\infty} (1 - \psi \phi)^j n_{t-j}
\]

\[
+ \left[ \frac{1 - \gamma}{\delta} - \frac{1 + \eta \psi \phi}{\phi \delta + \eta \psi \phi} \frac{\phi \delta}{\gamma} \right] s_t + \frac{1 + \eta \psi \phi}{\phi \delta + \eta \psi \phi} \frac{\phi \delta}{\gamma} \psi \delta \sum_{j=0}^{\infty} (1 - \psi \phi)^j s_{t-j} - \frac{1}{\delta} d_t
\]

(21)
Eq. (21) constitutes the impulse response function of the real exchange rate. Recognizing that the rate of change of the nominal exchange rate is defined as \( \Delta e_t = \Delta q_t + \pi_{t-1} \), an analogous derivation yields the impulse response function of the nominal exchange rate as:

\[
\Delta e_i = \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} n_i - \frac{1 - \phi \delta}{\phi \delta + \eta \psi \delta} \psi \delta \sum_{j=0}^{\infty} (1 - \psi \delta)^j n_{r_{t-j}}
\]

\[
+ \left[ \frac{1 - \gamma}{\delta} - \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \psi \delta \right] s_i + \frac{1 - \phi \delta}{\phi \delta + \eta \psi \delta} \psi \delta \sum_{j=0}^{\infty} (1 - \psi \delta)^j \ s_{r_{t-j}} - \frac{1}{\delta} d_i.
\]  

(22)

A comparison of Eqs. (21) and (22) reveals that the impact reactions of the nominal and real exchange rates to any of the three shocks are identical, although the dynamic lagged responses may be different. An aggregate demand shock is instantly absorbed by identical and once-and-for-all adjustments of the nominal and real exchange rates. In contrast, both a money market disturbance and a shock to aggregate supply induce dynamic adjustments of the system. Eq. (21) implies that any shock in the money market generates an infinite dynamic response of the real exchange rate, the limit of which is exactly equal and opposite in sign to the impact displacement. This implies that money market shocks leave the steady-state level of the real exchange rate unaltered. In comparison, the dynamic adjustment of the nominal rate is absolutely smaller in sum than the impact reaction (and may even be of equal sign), implying that the nominal rate will depreciate permanently in the wake of an expansionary money supply shock. Finally, an expansionary aggregate supply shock again causes identical impact effects in the nominal and real rates, although the dynamic adjustment paths diverge. Whereas the steady-state level of the real exchange rate unambiguously depreciates in the aftermath of an expansionary aggregate supply disturbance (\( \Delta q > 0 \)), the response of the nominal rate is indeterminate a priori and depends on the parameterization of the model.
Eqs. (21) and (22) can now be used to obtain the variance decompositions of the conditional variances of $q_t$ and $e_t$. First note that the levels of the real and nominal exchange rates at horizon $T$ can be written as:

$$q_{t+T} = q_t + \sum_{s = t+1}^{t+T} \Delta q_s.$$

and

$$e_{t+T} = e_t + \sum_{s = t+1}^{t+T} \Delta e_s.$$

Substituting Eq. (21) into Eq. (23) and consolidating terms yields:

$$q_{t+T} = q_t + \frac{1}{\delta} \sum_{s = t+1}^{t+T} [(1 - \gamma) s_c - d_s] + \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} n_{t+T}$$

$$+ \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} (1 - \psi \delta) \right] n_{t+T-1} + \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} (1 - \psi \delta)^2 \right] n_{t+T-2}$$

$$+ \ldots + \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} (1 - \psi \delta)^{T-1} \right] n_{t+1} - \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \frac{\phi \delta}{\gamma} s_{t+T}$$

$$- \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \frac{\phi \delta}{\gamma} (1 - \psi \delta) \right] s_{t+T-1} - \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \frac{\phi \delta}{\gamma} (1 - \psi \delta)^2 \right] s_{t+T-2}$$

$$- \ldots - \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \frac{\phi \delta}{\gamma} (1 - \psi \delta)^{T-1} \right] s_{t+1}$$

After taking variances on both sides of Eq. (25), the following expression for the conditional variance of $q_{t+T}$ as of date $t$ obtains:
\[
\var_r(q_{t+T}) = T \left( \frac{1-Y}{\delta} \right)^2 \sigma_s^2 + T \left( \frac{1}{\delta} \right)^2 \sigma_d^2 \\
+ \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \right]^2 \left[ 1 + (1 - \psi \delta)^2 + (1 - \psi \delta)^4 + \ldots + (1 - \psi \delta)^{2T-2} \right] \sigma_n^2 \\
+ \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \right]^2 \left( \frac{\phi \delta}{\gamma} \right)^2 \left[ 1 + (1 - \psi \delta)^2 + (1 - \psi \delta)^4 + \ldots + (1 - \psi \delta)^{2T-2} \right] \sigma_y^2
\]

where \( \sigma_s^2 \), \( \sigma_d^2 \) and \( \sigma_y^2 \) denote the variances of the shocks to the three forcing variables. The expression can be simplified to read:

\[
\var_r(q_{t+T}) = \left[ T \left( \frac{1-Y}{\delta} \right)^2 + \left( \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \right)^2 \left( \frac{\phi \delta}{\gamma} \right)^2 \frac{1 - (1 - \psi \delta)^{2T}}{1 - (1 - \psi \delta)^2} \right] \sigma_s^2 \\
+ T \left( \frac{1}{\delta} \right)^2 \sigma_d^2 + \left[ \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta} \right]^2 \frac{1 - (1 - \psi \delta)^{2T}}{1 - (1 - \psi \delta)^2} \sigma_n^2.
\]

Eq. (26) shows that all three shocks contribute to the variability of the real exchange rate at short horizons. However, as the forecast horizon \( T \) approaches infinity the first two expressions on the RHS grow without bounds while the third converges to a constant, implying that ultimately the variance of the real exchange rate is exclusively determined by real shocks in the long run.

Finally, an expression for the variance decomposition of the nominal exchange rate can be obtained in close analogy to the derivations above and is given by:
\[
\text{var}_t(e_{t+T}) = \left[ T \left( \frac{1 - \gamma}{\delta} \right)^2 + T \left( \frac{\eta \bar{\phi}}{\gamma} \right)^2 + \left( \frac{1 - \phi \bar{\delta}}{\phi \bar{\delta} + \eta \psi \bar{\delta}} \right)^2 \left( \frac{\phi \bar{\delta}}{\gamma} \right)^2 \frac{1 - (1 - \psi \bar{\delta})^{2T}}{1 - (1 - \psi \bar{\delta})^2} \right] \sigma^2_s + \\
+ T \left( \frac{1}{\delta} \right)^2 \sigma^2_d + \left[ T + \left( \frac{1 - \phi \bar{\delta}}{\phi \bar{\delta} + \eta \psi \bar{\delta}} \right)^2 \frac{1 - (1 - \psi \bar{\delta})^{2T}}{1 - (1 - \psi \bar{\delta})^2} \right] \sigma^2_n.
\]  

(27)

This completes the theoretical analysis. The next section will be concerned with a calibration of the model and an assessment of the relative importance of the various shocks in accounting for the observed levels of variability in actual time series on nominal and real exchange rates.

4. Some calibration results

The variance decompositions for the nominal and real exchange rates are now conducted on actual data. In order to facilitate a comparison with the results from the earlier statistical VAR studies mentioned in the introduction, very similar data are used here as well. These are quarterly data collected from the IMF International Financial Statistics Database. The data set comprises bilateral nominal and real exchange rates as well as bilateral inflation and industrial output series for Britain, Germany and Japan, all relative to the U.S. and extending over the post-Bretton Woods period from 1973:2 to 1998:4. Real exchange rates are constructed using national CPIs and own-currency to dollar exchange rates while relative series on industrial production are used as a proxy for aggregate supply shocks.

The robustness of the decompositions reported below have been checked with respect to modifications of all parameters in the model. This exercise revealed that some parameters have a stronger influence on the results than others. In particular, the income elasticity and

---

8 The endpoint of the sample has been chosen to coincide with the discontinuation of the DM/$ exchange rate.
interest semi-elasticity of money demand, $\phi$ and $\eta$, exert only a marginal influence on the decompositions and are therefore maintained at $\phi = 1$ and $\eta = 0.2$ throughout. In contrast, the rate of domestic absorption, $\gamma$, the degree of price flexibility, $\psi$, and the elasticity of the current account, $\delta$, play more important roles in driving the results, particularly when the parameters are altered substantially. As domestic absorption cannot realistically depart too much from values of around .8 or .9, the parameter $\gamma$ is kept at a value of .8 throughout. However, neither $\delta$ nor $\psi$ are tightly constrained to certain intervals other than being positive and satisfying the stability condition of the model, $\psi \delta < 2$. Therefore three different scenarios are presented below. The first case is a low-elasticity scenario with $\psi = 0.5$ and $\delta = 0.5$, which reflects the case of "elasticity pessimism" associated with a J-curve effect in the balance of payments. The second scenario analyzes a medium-elasticity case by setting $\psi = 0.75$ and $\delta = 1.0$, while the high-elasticity scenario is based on parameter realizations of $\psi = 1.0$ and $\delta = 1.5$, in which the Marshall-Lerner condition of the balance of payments is satisfied.

Fig. 5 graphs the impulse response functions of the nominal and real exchange rates where attention is restricted to the low-elasticity scenario for illustrative purposes. These functions are not based on actual time series but simply trace a unitary shock through the calibrated system of equations. The shapes of these functions thus simply recycle the information already contained in the phase diagrams of Figs. 1 through 4.

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9 The empirical literature on price elasticities of the demand for imports and exports is rather extensive. In their survey paper, Goldstein and Khan (1985) report the sum of the average long-run estimates of these elasticities for industrial countries to be as large as 2.0. They also find that the short-run (0-6 months) elasticities are considerably smaller than the long-run elasticities and that the Marshall-Lerner condition is frequently violated at these horizons.
Tables 1 and 2 report on the variance decompositions of the nominal and real exchange rates. Here the three scenarios of low, medium and high elasticities are distinguished. Numbers are reported for the impact reaction and for forecast horizons of 4, 8 and 20 quarters. The results differ substantially depending on which elasticity scenario is chosen. On the assumption of low price elasticities for imports and exports, monetary shocks account for shares of overall impact nominal and real exchange rate variability of 44.54%, 35.92% and 43.61% for the British, German and Japanese bilaterals, respectively. In contrast, aggregate supply shocks make up just a tiny fraction of overall instantaneous exchange rate variability, accounting for 2.16%, 6.51% and 3.03% for these three countries, respectively. Remarkably, in excess of one half of the instantaneous exchange rate variability is due to aggregate demand shocks. This share rises even further when the forecast horizon is extended as the fraction of monetary variability becomes quite small at longer horizons.

When the elasticities are scaled up, the influence of monetary variability is substantially reduced while aggregate supply shocks gain in importance. The latter explain up to 17-18% of the overall impact exchange rate variability and even account for 23-25% of nominal exchange rate fluctuations at the 4-quarter horizon. The contribution of money market shocks is reduced to 20% and below for the impact figure and reach a maximum of 25-28% at the 4-quarter nominal exchange rate horizon.

Overall, the evidence that emerges from the model calibration compares quite favorably with the results from earlier statistical VAR studies. Particularly the low-elasticity scenario corresponds rather closely with the statistical decompositions of nominal and real exchange rate fluctuations. Higher elasticities impart a more important role to supply shocks and reduce
the influence of money market shocks but leave the substantial importance of aggregate demand shocks in explaining exchange rate variability largely unaffected.

5. Conclusion

Whether real exchange rate fluctuations are caused by nominal or by real shocks has profound implications for the design of exchange rate systems. Fixing nominal exchange rates prevents excessive exchange rate volatility and helps stabilize output markets in an environment of nominal shocks. In contrast, flexible exchange rates allow the latter to adjust quickly to maintain output market equilibrium when real shocks are predominant. Whereas the sources of exchange rate fluctuations are generally unobservable, attempts have been made to infer these shocks from time series on exchange rates as well as relative price and output levels through structural vector autoregression (VAR) analysis. This paper has used a different route by starting from a structural exchange rate model general enough to incorporate nominal shocks as well as shocks to aggregate demand and aggregate supply. In order to infer the shocks from the same time series which are used in standard VAR analysis, the model is triangularized to allow for a recursive identification of the various shock components.

Applied to major bilateral exchange rate series, the decomposition suggests that monetary shocks account for roughly one third to one half of overall short-run exchange rate variability when the model is parameterized with low elasticities. Most of the remaining variability is accounted for by aggregate demand shocks with shocks to aggregate supply being negligible. If higher elasticities are chosen, the share of supply shocks rises to explain up to one quarter of exchange rate variability as the fraction of nominal variability drops. Under any parameterization, most of the variability in exchange rates is explained by aggregate demand shocks, particularly so at longer forecast horizons. The results of the present study are roughly in line with those obtained from statistical VAR analysis, although here the fraction of
exchange rate variability stemming from aggregate supply sources is somewhat larger, particularly when high elasticities are chosen. Overall, the evidence from quarterly data seems to suggest that exchange rate fluctuations appear to be predominantly equilibrium responses to real shocks rather than being caused by excessively volatility financial market.
References


Fig. 1: Graphical representation of the model
Fig. 2: A money market shock
Fig. 3: An expansionary shock to aggregate demand
Fig. 4: An expansionary shock to aggregate supply
Fig. 5: Impulse response functions of nominal and real exchange rates
<table>
<thead>
<tr>
<th>Impact</th>
<th>4 quarters</th>
<th>8 quarters</th>
<th>20 quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Britain</strong></td>
<td>0.0216, 0.5330, 0.4454</td>
<td>0.0158, 0.6852, 0.2990</td>
<td>0.0116, 0.7969, 0.1915</td>
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<tr>
<td><strong>Germany</strong></td>
<td>0.0651, 0.5757, 0.3592</td>
<td>0.0464, 0.7192, 0.2344</td>
<td>0.0333, 0.8196, 0.1471</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td>0.0303, 0.5336, 0.4361</td>
<td>0.0222, 0.6853, 0.2925</td>
<td>0.0163, 0.7965, 0.1872</td>
</tr>
</tbody>
</table>

(a) low-elasticity scenario (ψ = 0.5, δ = 0.5)

<table>
<thead>
<tr>
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<th>20 quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Britain</strong></td>
<td>0.1420, 0.6262, 0.2318</td>
<td>0.0781, 0.7987, 0.1232</td>
<td>0.0458, 0.8859, 0.0683</td>
</tr>
<tr>
<td><strong>Germany</strong></td>
<td>0.1611, 0.6557, 0.1832</td>
<td>0.0867, 08181, 0.0952</td>
<td>0.0503, 0.8975, 0.0522</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td>0.1468, 0.6551, 0.1981</td>
<td>0.0790, 0.8179, 0.1031</td>
<td>0.0459, 0.8976, 0.0565</td>
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</tbody>
</table>

(b) medium-elasticity scenario (ψ = 0.75, δ = 1.0)

<table>
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<tbody>
<tr>
<td><strong>Britain</strong></td>
<td>0.1721, 0.6242, 0.2037</td>
<td>0.2340, 0.4878, 0.2782</td>
<td>0.1594, 0.6521, 0.1885</td>
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<tr>
<td><strong>Germany</strong></td>
<td>0.1836, 0.6302, 0.1862</td>
<td>0.2506, 0.4942, 0.2552</td>
<td>0.1700, 0.6579, 0.1721</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td>0.1704, 0.6506, 0.1790</td>
<td>0.2353, 0.5164, 0.2483</td>
<td>0.1573, 0.6776, 0.1651</td>
</tr>
</tbody>
</table>

(a) high-elasticity scenario (ψ = 1.0, δ = 1.5)

Table 1: Variance decompositions of the nominal exchange rate
<table>
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<tr>
<td>Britain</td>
<td>0.0216, 0.5330, 0.4454</td>
<td>0.0157, 0.6885, 0.2958</td>
<td>0.0115, 0.7995, 0.1890</td>
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<td>Germany</td>
<td>0.0651, 0.5757, 0.3592</td>
<td>0.0460, 0.7222, 0.2318</td>
<td>0.0330, 0.8219, 0.1451</td>
</tr>
<tr>
<td>Japan</td>
<td>0.0303, 0.5336, 0.4361</td>
<td>0.0220, 0.6886, 0.2894</td>
<td>0.0161, 0.7992, 0.1847</td>
</tr>
</tbody>
</table>

(a) low-elasticity scenario \((\psi = 0.5, \delta = 0.5)\)

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<th>20 quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Britain</td>
<td>0.1420, 0.6262, 0.2318</td>
<td>0.0555, 0.8596, 0.0849</td>
<td>0.0324, 0.9221, 0.0455</td>
</tr>
<tr>
<td>Germany</td>
<td>0.1611, 0.6557, 0.1832</td>
<td>0.0612, 0.8737, 0.0651</td>
<td>0.0354, 0.9299, 0.0347</td>
</tr>
<tr>
<td>Japan</td>
<td>0.1468, 0.6551, 0.1981</td>
<td>0.0558, 0.8737, 0.0705</td>
<td>0.0323, 0.9302, 0.0375</td>
</tr>
</tbody>
</table>

(b) medium-elasticity scenario \((\psi = 0.75, \delta = 1.0)\)

<table>
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<tr>
<td>Britain</td>
<td>0.1721, 0.6242, 0.2037</td>
<td>0.0776, 0.8322, 0.0902</td>
<td>0.0437, 0.9070, 0.0493</td>
</tr>
<tr>
<td>Germany</td>
<td>0.1836, 0.6302, 0.1862</td>
<td>0.0824, 0.8356, 0.0820</td>
<td>0.0463, 0.9090, 0.0447</td>
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<tr>
<td>Japan</td>
<td>0.1704, 0.6506, 0.1790</td>
<td>0.0751, 0.8475, 0.0774</td>
<td>0.0419, 0.9161, 0.0420</td>
</tr>
</tbody>
</table>

(a) high-elasticity scenario \((\psi = 1.0, \delta = 1.5)\)

Table 2: Variance decompositions of the real exchange rate