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## An overreaction implementation of the coherent market hypothesis and option pricing

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Wirtschaftswissenschaftliche Fakultät  
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**An Overreaction Implementation of the  
Coherent Market Hypothesis  
and Option Pricing**

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# An Overreaction Implementation of the Coherent Market Hypothesis and Option Pricing

Rainer Schöbel and Jochen Veith\*

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## Abstract

Inspired by the theory of social imitation (Weidlich 1970) and its adaptation to financial markets by the Coherent Market Hypothesis (Vaga 1990), we present a behavioral model of stock prices that supports the overreaction hypothesis. Using our dynamic stock price model, we develop a two factor general equilibrium model for pricing derivative securities. The two factors of our model are the stock price and a market polarization variable which determines the level of overreaction. We consider three kinds of market scenarios: Risk-neutral investors, representative Bernoulli investors and myopic Bernoulli investors. In case of the latter two, risk premia provide that herding as well as contrarian investor behaviour may be rationally explained and justified in equilibrium. Applying Monte Carlo methods, we examine the pricing of European call options. We show that option prices depend significantly on the level of overreaction, regardless of prevailing risk preferences: Downward overreaction leads to high option prices and upward overreaction results in low option prices.

**JEL Classification:** G12, G13

**Keywords:** behavioral finance, coherent market hypothesis, market polarization, option pricing, overreaction, chaotic market, repelling market

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# 1 Introduction

Since more than thirty years, the widely accepted hypothesis about the behavior of asset prices in perfect markets is the Efficient Market Hypothesis (EMH) or, in its continuous-time form, the "geometric Brownian motion" hypothesis. This implies that asset prices are log-normally distributed. However, in the last twenty years, new studies of security prices have reversed some of the earlier evidence favoring the EMH. Behavioral finance has emerged as an alternative view of the financial markets suggesting, that investing in speculative assets is a social activity. It thus seems plausible that investors' behavior and hence prices of speculative assets would be influenced by social movements. A historically important study in this context is Shiller's (1981) work on stock market volatility. Shiller found, that even though the price of the aggregate stock market is importantly linked to its dividends, stock prices seem to show far too much volatility to be in accordance with a simple dividend discount model. He offered the following explanation: "This behavior of stock prices may be consistent with some psychological models. Psychologists have shown in experiments that individuals may continually overreact to superficially plausible evidence even when there is no statistical basis for their reaction". Other studies by DeBondt and Thaler (1985, 1987), Campbell and Shiller (1988) or Poterba and Summers (1988) provided some evidence that stock returns are mean reverting. De Bondt and Thaler also propose an overreaction hypothesis, whereby financial markets weigh recent information more heavily than prior information. Markets eventually realize that they formed biased expectations and prices revert back to their fundamental value.

In this paper, we follow the intensions of Shiller and DeBondt et al. and suggest a two-factor stock price model that supports the overreaction hypothesis whereby overreaction is due to herd behavior. The first factor of our model may be regarded as the present value of future dividends as a proxy of the fundamental value of the stock market. The second one is of psychological nature describing the behavior of investors and how they esteem the fundamental value in their current psychological environment. To model the behavior of investors, we use a theory of social imitation originally developed by Weidlich (1970). The theory was adopted to the financial markets by Vaga (1990) called the coherent market hypothesis (CMH). The CMH is a nonlinear statistical model using terms such as order parameters to describe the behavior of investors at a macroscopic level. Introducing two key parameters, the level of crowd behavior and the prevailing economic fundamentals, the model allows for a variety of associated market states ranging from efficient over coherent to chaotic. However, Vaga assumed a direct proportional relationship between market polarization and stock returns. The distributions of the polarization are therefore simply viewed as steady state return distributions, which is in contradiction with empirical evidence.

In this paper, we take a step further and formulate a continuous-time version of the CMH. This opens the door to apply dynamical financial theory, especially op-

tion pricing. Starting from the Fokker Planck equation describing investor sentiment in terms of polarization levels, we are able to extract and analyze the underlying stochastic process. We also find that the process exhibits a special kind of mean reversion, which can be used to model overreaction in stock prices when superimposed on a geometric Brownian motion. The resulting return distributions offer the same risk-return characteristics associated to the various states of investor sentiment as proposed by Vaga, but overcome the deficiency of a steady state perspective. Based on our stock price process, we develop a general equilibrium model for valuing derivative securities using the CIR (1985a) framework and apply it to the valuation of European call options.

The remaining part of the paper is organized as follows: In the second Section, we present the main ideas and properties of Weidlich's theory of social imitation and Vaga's adoption to financial markets, the coherent market hypothesis. In section 3, we introduce a continuous-time version of the CMH and formulate a stock price process in accordance with the overreaction hypothesis. Furthermore, we take this stock price process and develop three different equilibrium models for valuing derivative securities. Section 4 illustrates the characteristics of these models and reports some results of a comparative static analysis. In section 5 we summarize and provide some concluding remarks.

## 2 The Coherent Market Hypothesis

### 2.1 A Theory of Social Imitation

The starting point of our work is a theory of social imitation, originally provided by W. Weidlich (1970). The general idea of Weidlich was, that there is a structural similarity between a physical ensemble of interacting individual systems in thermal equilibrium and the interactions in a society of human individuals. In particular, Weidlich extends the well known Ising model (1925) of ferromagnetism to the phenomenon of polarization of opinion in social groups.

Two important characteristics determine the behavior of an Ising system: The interactions (e.g. attracting forces) between fluctuating individual subsystems and a collective parameter determining the degree of the fluctuations. As a consequence, the ensemble as a whole behaves macroscopically in a complete different way above and below a certain critical phase transition parameter.

To transfer this basic approach to human behavior, Weidlich applies the following framework: There is a group of  $n$  individuals who may influence each other with respect to their decisions and each individual can decide between two attitudes: Up (+) or down (-). The probability of finding, at time  $t$ ,  $n_+$  individuals with attitude +, and  $n_-$  with attitude - is  $f[n_+, n_-; t]$  and the transition probabilities per unit time for an individual of changing from attitude + to - or vice versa are  $p_{+-}(n_+, n_-)$  and  $p_{-+}(n_+, n_-)$ , respectively. Then, the rate of change of the probability density

function  $f[n_+, n_-; t]$  can be expressed by a master equation of the form<sup>1</sup>

$$\begin{aligned} \frac{\Delta f(n_+, n_-; t)}{\Delta t} = & (n_- + 1) p_{-+}(n_+ - 1, n_- + 1) f[n_+ - 1, n_- + 1; t] \\ & + (n_+ + 1) p_{+-}(n_+ + 1, n_- - 1) f[n_+ + 1, n_- - 1; t] \\ & - n_+ p_{+-}(n_+, n_-) f[n_+, n_-; t] - n_- p_{-+}(n_+, n_-) f[n_+, n_-; t]. \end{aligned} \quad (1)$$

which simply sums up all probability currents to and from the considered state  $(n_+, n_-)$  within the short time interval  $\Delta t$ . To describe the market's excess opinion (positive or negative), we introduce a new state variable which we call the market polarization  $q$  defined as

$$q = (n_+ - n_-)/2n; \quad q \in \left[-\frac{1}{2}, \frac{1}{2}\right] \quad (2)$$

and normalised to  $2n$ .

After some algebraic manipulation, equation (1) can be expressed in terms of  $q$  and a Taylor expansion up to the second order finally yields the Fokker Planck equation<sup>2</sup>

$$f_t(q, t) = -\frac{\partial}{\partial q} [K(q)f(q, t)] + \frac{1}{2} \frac{\partial^2}{\partial q^2} [Q(q)f(q, t)]. \quad (3)$$

Note, that the drift  $K(q)$  and the diffusion coefficient  $Q(q)$  still depend on the yet unspecified transition probabilities  $p_{+-}$  and  $p_{-+}$ . The stationary solution of equation (3) can be found by standard methods with the result

$$f_{st}(q) = \frac{c}{Q(q)} \exp \left[ 2 \int_{-\frac{1}{2}}^q \frac{K(q)}{Q(q)} dq \right] \quad (4)$$

where  $c$  is a normalization constant.

The general time-dependent solution of (3) describes the evolution of the probability density from its initial condition (e.g. Dirac's delta function at  $t = 0$ ) to the stationary distribution. Since there exists no closed expression, a series expansion of the form

$$f(q; t) = f_{st}(q) + \sum_{i=1}^{\infty} c_i \exp[-\lambda_i(t - t_0)] \phi_i(x) \quad (5)$$

may be applied. Here, the  $\lambda_i$  form a denumerable sequence of eigenvalues and  $\phi_i(x)$  are an orthonormal system of eigenfunctions to the Fokker Planck operator  $-\frac{1}{2} \frac{\partial^2}{\partial q^2} Q(q) + \frac{\partial}{\partial q} K(q)$ .

In order to further evaluate  $f_{st}(q)$  we need explicit assumptions about the transition probabilities  $p_{+-}(q)$  and  $p_{-+}(q)$ . At this point, Weidlich employes the intuitively

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<sup>1</sup>For an illustration see figure 8 in appendix A.1.

<sup>2</sup>For a detailed derivation see appendix A.1.

appealing analogy between the behavior of individuals in a social group and the elementary magnets within a ferromagnet. As in the Ising model, he assumes that the individuals are subject to two forces: An internal field created by the individuals themselves and proportional to the excess opinion of the group and an external field independent of the behavior of the individuals. The transition probabilities then depend exponentially on these force fields and adapted to our case, they are<sup>3</sup>

$$\begin{aligned} p_{+-}(q) &= \alpha \exp[-(kq + h)] \\ p_{-+}(q) &= \alpha \exp[+(kq + h)]. \end{aligned} \tag{6}$$

The adaption parameter  $k$  describes the willingness of the individuals to align to the group and preference parameter  $h$  measures the influence of the external force. Applying these transition probabilities, explicit expressions for the drift  $K(q)$  and diffusion coefficient  $Q(q)$  can be derived as

$$\begin{aligned} K(q) &= \alpha [\sinh(kq + h) - 2q \cosh(kq + h)] \\ Q(q) &= \frac{\alpha}{n} [\cosh(kq + h) - 2q \sinh(kq + h)]. \end{aligned} \tag{7}$$

Substituting (7) into (4), the solution for the stationary case can be evaluated easily using numerical methods. The resulting distributions are shown in figure 1. There are two regimes of behavior, ordered and disordered. For low adaption of the individuals to the group  $k \ll k_{crit}$ , we find almost balanced states ( $q$  around zero) with a centered distribution. If the adaption parameter  $k$  approaches its critical value, a phase transition to more ordered states occurs and the distribution undergoes a bifurcation. For an increasing preference parameter  $h$  the distributions are distorted additionally whereby the response to a change in  $h$  is largest near the transition.

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[Insert figure 1]

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## 2.2 Five Market States

The coherent market hypothesis was suggested by Vaga (1990) in contrast to the efficient market hypothesis. Vaga carried over Weidlich's concepts to the financial markets. He recognized, that the theory was able to drop the premise of rational investors and therefore relaxes the assumption of approximately normally distributed stock returns. Vaga re-interpreted the control parameters  $k$  and  $h$  and proposed to use  $k$  as a measure of the level of crowd behavior or market sentiment and  $h$  as a measure of the prevailing economic fundamentals. Each set of parameters  $(k, h)$  corresponds to a different market state. He assumed a direct proportional relationship between market polarization and stock returns. The distributions of the polarization derived by Weidlich are therefore simply viewed as return distributions.

According to Vaga, there are four types of market states which can be described by the model under reasonable assumptions about the control parameters:

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<sup>3</sup>For a detailed explanation see appendix A.2.

1. Efficient market ( $0 \leq k \ll k_{crit}, h = 0$ ): This market state can be obtained in choosing neutral fundamentals and market sentiment well below the critical threshold. In the limiting case  $k = 0$ , the return distribution is similar to the normal distribution (fig. 2.2 a).
2. Coherent market ( $k \approx k_{crit}, h \ll 0$  or  $h \gg 0$ ): If we presume crowd behavior in conjunction with strong bullish or bearish fundamentals, the model allows to create coherent market states. As we have seen in the last section, the return distribution will be skewed to the left or to the right. The expected return is different from zero, accompanied with an unusual small standard deviation. In this state, the traditional risk-return tradeoff is inverted and investors can earn above-average returns with below-average risk (fig. 2.4 a).
3. Chaotic market ( $k \approx k_{crit}, h \lesssim 0$  or  $h \gtrsim 0$ ): This market state can be modeled in supposing crowd behavior with only slightly bearish or bullish fundamentals which leads to a bimodal return distribution. This is the worst of all worlds with low return for above-average risk (fig. 2.5 a and fig. 2.6 a).
4. Unstable transitions: These are all intermediate market states that cannot be assigned to one of the former states (fig. 2.3 a).

We would like to extend this list of market states by the following case:

5. Repelling market ( $k < 0, h = 0$ ): In this market state, we would observe just the opposite of crowd behavior, namely a repelling behavior among investors. Any investor would try to avoid to have the same opinion than the majority. As a result, the return distribution becomes concentrated around  $q = 0$  and eventually a Dirac's delta function as  $k$  approaches minus infinity. (fig. 2.1 a).

All things considered, the CMH offers a framework to evaluate the risk-reward potential for a given market. However, this is only achieved in a qualitative sense. Our main point of criticism is the rigorous re-interpretation of Weidlich's polarization distributions as return distributions. It is obvious, that Vaga's distributions  $f(q, t)$  are unable to fulfill the most basic demands on financial return distributions since they end up in a steady state.



## 3 The Continuous-Time CMH

### 3.1 Dynamics of Market Polarization

The Fokker Planck equation (3) describes a diffusion process for the polarization  $q(t)$  with diffusion coefficients  $K(q)$  and  $Q(q)$ . There are two basically different approaches to the class of diffusion processes. On the one hand, one can define them in terms of the conditions on the transition probabilities  $f(q, t)$ , which is what we did in the previous sections. On the other hand, one can study the variable  $q(t)$  itself and its variation with respect to time. This leads to a stochastic differential equation for  $q$ . In fact, under the usual regularity conditions, it can be shown that there exists a Wiener process  $w(t)$  such that the polarization  $q(t)$  follows the stochastic differential equation

$$dq = K(q) dt + \sqrt{Q(q)} dw(t). \quad (8)$$

To gain further insight into the behavior of the stochastic process  $dq$ , we have to analyze the drift  $K(q)$ , the diffusion coefficient  $Q(q)$  and the associated sample paths in more detail. In figure 2 we plot the graphs of the functions  $K(q)$  and  $Q(q)$  for different market states together with the associated distributions  $f(q, t)$ . The drift exhibits kind of mean reversion towards the stable nodes which in turn leads to the maxima of the distributions  $f(q, t)$ . The diffusion coefficient has its maximum at  $q = 0$  and diminishes for growing  $|q|$ . This assures that the distribution is bounded on the interval  $[-\frac{1}{2}, \frac{1}{2}]$ . Figure 3 shows some sample paths, which are stationary. As is apparent from figure 3 b), the sample path for the "efficient" market state ( $k = 0, h = 0$ ) has nothing in common with a random walk (e.g. no growing variance over time), except that it has the same type of distribution at a given point in time.

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[Insert figure 2]

[Insert figure 3]

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### 3.2 Stock Price Dynamics

In this section, we formulate a stock price process in accordance with the overreaction hypothesis using the properties of the coherent market hypothesis. Two empirical discoveries are the guidelines of our stock price model: The observed variation in expected discounted dividends is too low to justify the volatility in stock price movements and stock prices tend to mean revert. To adress these characteristics, we assume that stock price changes are driven by two factors: The first one describes the fundamental value  $D$  of the company or market index and can be regarded as the value of all future dividends discounted to the present. The second factor is the market polarization  $q$ , which we use to describe how investors esteem the

fundamental value in their current psychological environment. Since the fundamental value is not a stationary target, we model its dynamics by a geometric Brownian motion

$$dD = \mu_D D dt + \sigma_D D dw_2 \quad (9)$$

a commonly used assumption in asset pricing. The return of the asset is then modeled by superimposing the process  $dq$  in the following way<sup>4</sup>:

$$d(\ln S) = d(\ln D) + \kappa dq \quad (10)$$

Here,  $S$  denotes the market price of the asset and  $\kappa$  describes the impact of the subjective perception of the investors. From Itô's lemma, we derive the joint Markov process for polarization  $q$  and stock price  $S$  as follows:

$$\begin{aligned} dq &= K(q) dt + \sqrt{Q(q)} dw_1 \\ dS &= [\mu_D + \kappa K(q) + \frac{1}{2} \kappa^2 Q(q)] S dt + \kappa \sqrt{Q(q)} S dw_1 + \sigma_D S dw_2. \end{aligned} \quad (11)$$

As can be seen in figure 4 (solid lines), the return distributions assigned to the process  $dS$  retain the characteristics of the density distributions of the process  $dq$ . Furthermore,  $dS$  shows the desired growing variance over time and therefore overcomes the deficiencies of Vaga's return distributions without losing its main attractions. We can see this from the contour plots in figure 5 which illustrate the evolution of the return distributions over a time period of two years.

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[Insert figure 4]

[Insert figure 5]

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### 3.3 Pricing Derivative Securities

In the following, we develop several equilibrium models for valuing derivative securities based on our stock price process. We consider an economy with a fixed number of identical investors which receive only capital income and can choose between immediate consumption or investment. The investors trade in perfectly competitive, continuous markets with no transaction costs. Trading takes place only at

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<sup>4</sup>In terms of absolute values, equation (10) can be written as

$$S = D \cdot \exp(\kappa q) \quad (10a)$$

so we can see that a high level of positive coherence leads to high stock valuations and vice versa. A loose interpretation may be that the stock price is most overvalued when everybody is too optimistic about it ( $q$  at high levels). Eventually, no more buyers can be found and it is more probable for the stock price to decrease than to increase. One major advantage of this assumption about the stock price is that  $D$  drops out of the sde for  $S$ .

equilibrium prices and the representative investor is assumed to have time-additive preferences of the form

$$E_t \left[ \int_t^\infty U(C(s), s) ds \right]. \quad (12)$$

$E_t$  is an expectation operator conditional on current wealth  $W$  at state  $(q, S)$  of the economy and  $C(s)$  is the consumption flow at time  $s$ . There are three investment assets: A stock which underlies the speculative forces described above, a risk free asset and a derivative security written on the stock price  $S$ .

The representative investor's decision problem is equivalent to maximizing (12) subject to the budget constraint

$$dW = [(1 - \omega_P - \omega_F) rW dt + \omega_P W \frac{dS}{S} + \omega_F W \frac{dF}{F} - C dt] \quad (13)$$

by selecting an optimal level of consumption and fractions  $\omega_P$  and  $\omega_F$  of wealth invested in stock and the derivative instrument respectively.

Assuming, that the value of the derivative security will not depend on wealth  $W$ , it can be shown, that the price of the derivative security  $F$  must satisfy the fundamental valuation equation

$$\frac{1}{2} tr(F_{\underline{x}\underline{x}'} \underline{\sigma} \underline{\sigma}') + F_{\underline{x}} [\underline{\mu} - \underline{\sigma} \underline{\lambda}] + F_t - rF = 0 \quad (14)$$

with  $\underline{x} = \begin{bmatrix} q \\ S \end{bmatrix}$ , drift vector  $\underline{\mu}(\underline{x}, t) = \begin{bmatrix} K(q) \\ [\mu_D + \kappa K(q) + \frac{1}{2} \kappa^2 Q(q)] S \end{bmatrix}$  and correlation matrix  $\underline{\sigma}(\underline{x}, t) = \begin{bmatrix} \sqrt{Q(q)} & 0 \\ \kappa \sqrt{Q(q)} S & \sigma_D S \end{bmatrix}$ .  $\underline{\lambda}$  denotes the market price of risk and  $r$  is the riskless rate of return.

Since the market polarization  $q$  is not a traded asset, we cannot use arbitrage arguments to fully eliminate investors' risk preferences from the derivative pricing problem. To close the model, we make additional assumptions regarding risk preferences by considering the following three types of investors:

**Case 1** *General equilibrium and risk neutrality.*

Setting  $\underline{\lambda} = 0$ , equation (14) does no longer depend on risk preferences explicitly. Thus we can calculate the price of the derivative security by assuming that risk neutrality prevails. The riskless rate of return then simply equals the drift of the stock price process

$$r(q) = \mu_D + \kappa K(q) + \frac{1}{2} \kappa^2 Q(q) \quad (15)$$

and the fundamental partial differential equation (PDE) for valuing derivative securities reduces to

$$0 = \frac{1}{2} Q(q) F_{qq} + \frac{1}{2} (\kappa^2 Q(q) + \sigma_D^2) S^2 F_{SS} + K(q) F_q + r(q) S F_S + F_t - r(q) F. \quad (16)$$

**Case 2** *General equilibrium and a representative Bernoulli investor with log utility.*

This case corresponds to the general equilibrium case of CIR (1985b) with  $r$  derived endogenously. We assume, that investors are endowed with a logarithmic utility function over consumption

$$U(C(t), t) = \exp(-\rho t) \ln C(t) \quad (17)$$

where  $\rho$  is the parameter of time preference. Since in macroeconomic equilibrium all investments must be held with a positive fraction in the representative investors portfolio, the market clearing condition is  $\omega_P = 1$ . That is, the aggregated wealth is always fully invested in stock. The risk free asset as well as the derivative security are both in zero net supply and serve for risk allocation purposes only. Solving the corresponding Bellman equation for this problem leads to the risk free interest rate

$$r(q) = \mu_D + \kappa K(q) - \frac{1}{2} \kappa^2 Q(q) - \sigma_D^2 \quad (18)$$

and the vector of factor risk prices

$$\underline{\lambda}(q) = \begin{bmatrix} \kappa \sqrt{Q(q)} \\ \sigma_D \end{bmatrix}. \quad (19)$$

The required risk premium  $\underline{\lambda} \sigma_S(q)$  regarding stock price risk becomes the lower, the larger  $|q|$  and the higher the prevailing crowd behavior  $k$  (see figure 6a). Thus, optimal guidance forces investors to reduce their risk aversion in times of high coherence and strong crowd behavior. The fundamental PDE is now given by

$$\begin{aligned} 0 = & \frac{1}{2} Q(q) F_{qq} + \frac{1}{2} (\kappa^2 Q(q) + \sigma_D^2) S^2 F_{SS} \\ & + [K(q) - \kappa Q(q)] F_q + r(q) S F_S + F_t - r(q) F. \end{aligned} \quad (20)$$

Note that in both cases so far,  $r$  is endogenous and represents the instantaneously riskless real interest rate in a pure barter economy, which can become negative in a recession or a depression.

**Case 3** *Partial equilibrium and a myopic Bernoulli investor with log utility.*

In this case, there is a small individual investor who seeks to optimize the composition of his entire portfolio. His transactions exert no influence on the formation of prices and he acts as a price taker. As above, this investor has logarithmic utility but the risk free interest rate is exogenously given and elastically supplied:

$$r = \text{const} \quad (21)$$

Accordingly, the market clearing condition is now given by  $\omega_P + \omega_r = 1$ . On the macroeconomic level, only the derivative security is in zero net supply ( $\omega_F = 0$ ). Solving the Bellman equation under this clearing condition, we get

$$\underline{\lambda}(q) = \frac{(\mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q) - r)}{\kappa^2 Q(q) + \sigma_D^2} \begin{bmatrix} \kappa \sqrt{Q(q)} \\ \sigma_D \end{bmatrix} \quad (22)$$

for the vector of factor risk prices and the fundamental PDE becomes

$$\begin{aligned} 0 = & \frac{1}{2}Q(q) F_{qq} + \frac{1}{2}(\kappa^2 Q(q) + \sigma_D^2) S^2 F_{SS} \\ & + \left[ K(q) - \frac{\kappa Q(q)}{\kappa^2 Q(q) + \sigma_D^2} \left( \mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q) - r \right) \right] F_q \\ & + r S F_S + F_t - r F. \end{aligned} \quad (23)$$

We can see from (22) that the vector of factor risk prices (as well as optimal demand  $\omega_P^*$ ) is independent of the instantaneous covariance between realized returns and state variables. Therefore, investors decisions do not take into account the effect of economic state variables on future realized returns and their behavior is described as myopic<sup>5</sup>.

The risk premium  $\underline{\lambda}\sigma_S(q)$  is shown in figure 6b. We can see that below the critical threshold  $k_{crit}$ , investors require high risk premia at negative polarization levels  $q$ , indicating high risk aversion. By contrast, investors would even pay to take risk (negative risk premia), if  $q$  is at positive levels. Above the critical threshold  $k_{crit}$ , investors get more courageous in pessimistic polarization states by lowering their required risk premia and they become cautious in optimistic ones by raising the risk premia.

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[Insert figure 6]

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## 4 Model Analysis

In this section, we solve the various fundamental valuation equations for the case of European call options using Monte Carlo methods.

### 4.1 Computational considerations

According to Feynman and Kac, the solution of (14) is equivalent to the discounted expectation of the boundary condition under the equivalent martingale measure:

$$F(S, q, t) = e^{-r(T-t)} \widehat{E}_t [g(S(T))] \quad (24)$$

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<sup>5</sup>See Feldman (1992). Albeit, we should keep in mind that investors still use this covariance to calculate the risk-adjusted drift through the risk premium  $\underline{\sigma}\lambda$ .

Since we want to price European call options, the boundary condition is given by  $g = \max(S(T) - X, 0)$ . Using equation (24), we are able to derive a solution of (14) doing Monte Carlo simulations. For this purpose, we employ the Euler approximation of the stochastic processes  $dq$  and  $dS$ :

$$\begin{aligned} q_t &= q_{t-1} + \left[ K(q_{t-1}) - \lambda_q(q_{t-1})\sqrt{Q(q_{t-1})} \right] \Delta t + \sqrt{Q(q_{t-1})} \Delta w_{1,t} \\ S_t &= S_{t-1} + r(q_{t-1})S_{t-1}\Delta t + \kappa\sqrt{Q(q_{t-1})}S_{t-1}\Delta w_{1,t} + \sigma_D S_{t-1}\Delta w_{2,t} \end{aligned} \quad (25)$$

To improve the efficiency of the simulation procedure, we use the antithetic variates technique. For every pair of random variables  $(\Delta w_{1,t}, \Delta w_{2,t})$  the four stock prices  $S_{t1}(\Delta w_{1,t}, \Delta w_{2,t})$ ,  $S_{t2}(\Delta w_{1,t}, -\Delta w_{2,t})$ ,  $S_{t3}(-\Delta w_{1,t}, \Delta w_{2,t})$  and  $S_{t4}(-\Delta w_{1,t}, -\Delta w_{2,t})$  are calculated to get the antithetic variates estimate

$$F(S_t) = \frac{1}{4} \{F(S_{t1}) + F(S_{t2}) + F(S_{t3}) + F(S_{t4})\}.$$

For our nonlinear problem, it proved to be efficient to increase the number of sample paths at the expense of time step  $\Delta t$ .<sup>6</sup> Therefore we use a time step of  $\Delta t = 1/252$  corresponding to one trading day and to ran 100'000 sample paths for each option series .

## 4.2 Sample option prices

For each of the three considered types of investors, sample option prices based on the stock price distributions  $S(q, t)$  are presented in tables 1-3. The maturity of the options is six month, but general results appear to be true for all maturities. The tables are divided into panels, labeled A-E, covering a broad range of parameter values including most of the scenarios introduced in chapter 2. Panel A investigates the influence of the degree of crowd behavior  $k$  on option prices. Panels B and C contrast option prices in different economic climates  $h$ . Panels D and E examine the effect of varying initial overreaction. For each option, three numbers are calculated: A "new" price corresponding to our simulated "true" distribution, a Black-Scholes price equivalent and the Black-Scholes implied volatility associated with the new price. The latter serves as a proxy for the skewness of the implied return distribution. To calculate the Black-Scholes price, we use for all three models the volatility of the simulated terminal distribution. In case of the risk neutral investor and representative Bernoulli investor, we use the risk free rate that is consistent with put-call parity. In case of the myopic Bernoulli investor, we fix  $r$  at  $\ln(1.1) = 0.0953$ . Independent of risk preferences, we set  $S = 100$ ,  $\mu_D = 0.1$ ,  $\sigma_D = 0.1$ ,  $\rho = 0$ ,  $\alpha = 50$ .

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<sup>6</sup>For example, the standard error of  $F$  for an at the money option (myopic Bernoulli investor, efficient market state) using 100'000 sample paths at time step  $\Delta t = \frac{1}{252}$  is 0.291%. The standard error of  $F$  using only 10'000 sample paths at the finer time step  $\Delta t_2 = \frac{1}{10}\Delta t$  is 0.710%. Since the number of calculated increments is the same in both settings, they take the same amount of computing time.

### Case 1 *General equilibrium and risk neutrality.*

Here, our aim is to find out, how options would be priced in the original sense of the coherent market hypothesis. By assuming that risk neutrality prevails, no risk-adjustment is needed and we can simply use the original return distributions to price options. On panel A of table 1, we present option prices based on repelling market, efficient market and chaotic market environments. Our first observation is, that rising crowd behavior leads to higher implied volatilities but also higher interest rates. As a result, option prices are the higher, the higher crowd behavior  $k$ . Second, option prices in repelling and efficient markets don't differ much from their Black-Scholes equivalents, while in chaotic markets, deviations are significant. Third, implied volatilities exhibit a strong volatility frown for positive  $k$  and a slight volatility smile for negative  $k$ , indicating the presence of short and fat tails respectively. In figure 7a, we have plotted the differences between the new option price and the Black-Scholes equivalent at different levels of crowd behavior. The more positive  $k$ , the more overprices Black-Scholes far-in- and far-out-of-the-money options while at-the-money options are underpriced. For negative  $k$ , effects of mean reversion and stochastic volatility are mostly offsetting each other.

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[Insert Figure 7]

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Panels B and C encompass the values that appear to characterize options in coherent market states, allowing  $h$  to range from -0.1 to 0.1 while crowd behavior is either below ( $k = 1.8$ ) or above ( $k = 2.1$ ) the critical threshold. As we can see, coherent bear markets are associated with low interest rates and low option prices while coherent bull markets come along with high interest rates and high option prices<sup>7</sup>. In weak coherent markets, deviations from the Black-Scholes equivalent are rather small. In strong coherent bear markets, out of-the-money options are underpriced by Black-Scholes, whereas in strong coherent bull markets, out of-the-money options are overpriced.

The most interesting results are obtained by varying initial overreaction by the choice of polarization  $q_0$ . As indicated in panels D and E, we find, that a downside overreaction ( $q < 0$ ) exerts an upward influence on option prices while an upside overreaction ( $q > 0$ ) has the opposite effect. Figure 7b shows the difference between the new option price and the Black-Scholes price in a downward versus an upward overreaction. Upward overreaction leads to positive differences in-the-money and negative differences out-of-the-money, indicating the presence of a left skewed implied return distribution. In a downward overreaction, the reverse is true.

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<sup>7</sup>This is not an unusual quality. Our model suggests, that positive fundamentals should come along with high interest rates and vice versa. Thus, our model is in accordance with the commonly observable behaviour of a central bank, which regulates interest rates by switching to expansive fiscal policy in poor economic climates and to restrictive fiscal policy in boom times.

**Case 2** *General equilibrium and a representative Bernoulli investor with log utility.*

The difference between the representative Bernoulli investor and the risk neutral investor lies in the risk adjustment of expected returns. Since the representative Bernoulli's risk premium is always positive, risk adjusted expected returns are lower (see dotted lines in figure 4). This in turn leads to a reduction of all option prices (compare option prices of table 2 with those of table 1). Surprisingly, risk adjustment has no influence on the qualitative pricing features already discussed above (figures 7c and 7d). Implied volatility patterns as well as general pricing behavior regarding variations in  $k$ ,  $h$  and  $q$  remain the same as for the risk neutral investor.

**Case 3** *Partial equilibrium and a myopic Bernoulli investor with log utility*

The myopic Bernoulli investor accounts for overreaction by expanding the risk-adjusted return distribution beyond the borders of the original return distribution, both, to the left and to the right (see dashed lines in figure 4). Risk preferences provide, that drift uncertainty is fully transferred into volatility uncertainty. The drift of the adjusted stock price process becomes a constant and we have to deal with a purely stochastic volatility model.

As can be seen from panel A of table 3, volatility and hence option prices are decreasing with rising crowd behavior  $k$ . Thus, options are priced lower in chaotic markets than in efficient markets. Second, observations regarding implied volatility remain the same as discussed above, except that the volatility smile for negative  $k$  is more pronounced while the volatility frown for positive  $k$  is reduced. If we have a look at figure 7e, we can see how this translates into differences between the new option price and equivalent Black-Scholes price. Since there is no mean reversion, deviations between the model price and Black-Scholes price extend over a much larger range of strike prices.

Looking at Panels B and C, we find that the influence of economic bias  $h$  on option prices is rather small, independent of prevailing crowd behavior. Thus, coherent markets don't matter much in the risk adjusted world of the myopic Bernoulli investor.

Regarding overreaction, we have similar pricing patterns as for the other two types of investors: Downward (upward) overreaction is associated with right (left) skewed return distributions and high (low) option prices (see panels D and E). Here, the intuition behind can be explained isolated from effects due to mean reversion: Consider the case in which  $q_0$  is below zero. Higher  $q$  (still negative) is associated with higher volatilities  $\sqrt{Q(q)}$  but also with higher stock prices. As  $q$  rises,  $S$  rises and the probability of large positive changes in  $S$  increases. This means, that very high stock prices become more probable. On the other hand, lower  $q$  is associated with lower stock price volatilities and stock prices. If  $q$  and hence  $S$  falls, it becomes less likely that large changes take place and terminal stock price



will be low. The net effect is, that the terminal return distribution is positively skewed. When  $q_0$  is positive, the reverse is true. Thus, the underlying mechanism is very similar to traditional stochastic volatility models where positive or negative correlation between volatility and stock price leads to right or left skewed return distributions respectively<sup>8</sup>.

## 5 Conclusion

Motivated by the theory of social imitation originally developed by Weidlich, we present a behavioral model of stock prices that supports the overreaction hypothesis. According to Weidlich's theory, group polarization can be described in a probabilistic manner. Vaga was the first who recognized the theory's usefulness to describe return distributions of financial markets. He re-interpreted Weidlich's control parameters as level of crowd behavior and prevailing economic fundamentals respectively and formulated the so called coherent market hypothesis. This hypothesis offers a rich variety of market states where the relationship between risk and return is solely explained by investor sentiment.

In this paper, we take a step further and formulate the continuous-time version of the CMH. This opens the door to established formalism of financial theory, especially option pricing. Starting from the Fokker Planck equation describing investor sentiment in terms of polarization levels, we are able to extract and analyze the underlying stochastic process. We find that the associated distributions of this diffusion end in a steady state and therefore are unable to represent return distributions. We also find that the process exhibits a special kind of mean reversion, which can be used to model overreaction in stock prices when superimposed on a geometric Brownian motion. The resulting return distributions offer the same risk-return characteristics associated to the various states of investor sentiment as proposed by Vaga, but overcome the deficiency of a steady state perspective.

Using our overreaction stock price model, we develop a two factor general equilibrium model for pricing derivative securities. The two factors of our model are the stock price and market polarization which determines the level of overreaction. We examine three competing hypothesis about investor preferences under which either the market price of risk is zero or derived endogenously and option prices are consistent with the absence of arbitrage: Risk-neutral investor, representative Bernoulli investor and myopic Bernoulli investor. In case of the latter two types of investors, we are able to draw conclusions about their pattern of behavior from their required risk premia. The representative Bernoulli investor's risk premium diminishes as market polarization and hence overreaction increases (in either direction). This implicates that he acts contrarian in pessimistic but contagious in

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<sup>8</sup>In traditional stochastic volatility models, skewness is introduced by assuming an instantaneous correlation between the driving Wiener processes:  $dw_S dw_\sigma = \rho dt$ . For an analytical approach, where volatility follows an Ornstein-Uhlenbeck process, see Schöbel and Zhu (1999).

optimistic market environments. Optimal behavior in case of the myopic Bernoulli investor is quite different. Below the critical level of crowd behavior, this investor exhibits strong herding behavior: When market polarization is negative, he wants to be compensated for risk, whereas when it is positive, he even pays for taking risk. By contrast, above the critical threshold he mostly takes contrarian positions by reducing the risk premium at positive polarization levels and increasing it at negative levels. Overall, we could show that herding as well as contrarian investor behavior may be the optimal course of action in response to an apparently irrational stock price process driven by investor sentiment.

Applying Monte Carlo simulations, we examine European call options. Although our three model specifications behave quite multi-faceted regarding the various pricing parameters, they exhibit one common feature: Option prices depend significantly on the level of overreaction. Upward overreaction leads to low option prices and downward overreaction leads to high option prices. The main reason behind this lies in the model's intrinsic assumption about stochastic volatility: Upward overreaction is accompanied by a negative correlation between stock price and volatility, whereas in a downward overreaction, this correlation is positive. As a result, the skewness of the terminal return distribution can be negative or positive, providing the observed pricing biases.

## 6 Appendices

### 6.1 Derivation of the Fokker-Planck equation and its stationary solution

Let  $p_{+-}(n_+, n_-)$  and  $p_{-+}(n_+, n_-)$  be the transition probabilities per unit time for an individual of changing from attitude + to - and vice versa. The temporal change of the probability  $f(n_+, n_-; t)$  to find the group in the state  $(n_+, n_-)$  is the difference between gains and losses of probability per unit time. As we see from figure 8, this is kind of master equation and can be written as

$$\begin{aligned} \frac{\Delta f(n_+, n_-; t)}{\Delta t} &= \frac{\text{gains} - \text{losses}}{\Delta t} \\ &= (n_- + 1) p_{-+}(n_+ - 1, n_- + 1) f[n_+ - 1, n_- + 1; t] \\ &\quad + (n_+ + 1) p_{+-}(n_+ + 1, n_- - 1) f[n_+ + 1, n_- - 1; t] \\ &\quad - n_+ p_{+-}(n_+, n_-) f[n_+, n_-; t] - n_- p_{-+}(n_+, n_-) f[n_+, n_-; t]. \end{aligned} \quad (26)$$

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[Insert Figure 8]

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If we introduce

$$q = (n_+ - n_-)/2; \quad q \in \left[ -\frac{1}{2}, \frac{1}{2} \right]$$

as a measure of the polarization of the group, we can rewrite the transition probabilities  $p$  in terms of  $q$ :

$$w_{+-}(q) := n_+ p_{+-}(n_+, n_-) = n \left( \frac{1}{2} + q \right) p_{+-}(q)$$

$$w_{-+}(q) := n_- p_{-+}(n_+, n_-) = n \left( \frac{1}{2} - q \right) p_{-+}(q)$$

With these definitions and  $\Delta q = \frac{1}{n}$ , equation (26) simplifies to

$$\begin{aligned} \frac{\Delta f(q; t)}{\Delta t} &= w_{+-}(q + \Delta q) f(q + \Delta q, t) + w_{-+}(q - \Delta q) f(q - \Delta q, t) \\ &\quad - w_{+-}(q) f(q, t) - w_{-+}(q) f(q, t). \end{aligned}$$

A series expansion of the first two terms on the right hand side up to the second order in  $\Delta q$  leads to

$$\begin{aligned} \frac{d}{dt} f(q, t) &= w_{+-}(q) f(q, t) + \frac{\partial}{\partial q} w_{+-}(q) f(q, t) \Delta q + \frac{1}{2} \frac{\partial^2}{\partial q^2} w_{+-}(q) f(q, t) \Delta q^2 \\ &\quad + w_{-+}(q) f(q, t) - \frac{\partial}{\partial q} w_{-+}(q) f(q, t) \Delta q + \frac{1}{2} \frac{\partial^2}{\partial q^2} w_{-+}(q) f(q, t) \Delta q^2 \\ &\quad - w_{+-}(q) f(q, t) - w_{-+}(q) f(q, t) \\ &= \frac{\partial}{\partial q} [w_{+-}(q) - w_{-+}(q)] f(q, t) \Delta q + \frac{1}{2} \frac{\partial^2}{\partial q^2} [w_{+-}(q) + w_{-+}(q)] f(q, t) \Delta q^2 \end{aligned}$$

If we define the drift  $K(q)$  and the diffusion coefficient  $Q(q)$  as follows

$$\begin{aligned} K(q) &= [w_{-+}(q) - w_{+-}(q)] \Delta q \\ Q(q) &= [w_{+-}(q) + w_{-+}(q)] \Delta q^2 \end{aligned}$$

we get in the limit  $\Delta q \rightarrow 0$  a Fokker Planck equation for the probability density:

$$\frac{d}{dt} f(q, t) = -\frac{\partial}{\partial q} [K(q) f(q, t)] + \frac{1}{2} \frac{\partial^2}{\partial q^2} [Q(q) f(q, t)] \quad (27)$$

To solve this partial differential equation, it is convenient to introduce the probability 'current'

$$j(q; t) := K(q) f(q, t) + \frac{1}{2} \frac{\partial}{\partial q} [Q(q) f(q, t)].$$

Using  $j(q; t)$ , equation (27) becomes a continuity equation for the probability density function  $f(q, t)$

$$\frac{d}{dt} f(q, t) + \frac{\partial}{\partial q} j(q; t) = 0.$$

Since there is no probability current across the boundaries at  $q = \pm \frac{1}{2}$ , the boundary conditions are

$$j(q = \pm \frac{1}{2}; t) = 0. \quad (28)$$

The condition for the steady state solution is

$$\frac{d}{dt} f_{st}(q, t) = 0.$$

But this means that  $\frac{\partial}{\partial q} j_{st}(q; t) = 0$  and therefore  $j_{st}(q; t) = \text{const.}$  . Together with the boundary conditions (28) it follows that

$$j_{st}(q; t) = 0 = K(q) f_{st}(q, t) + \frac{1}{2} \frac{\partial}{\partial q} [Q(q) f_{st}(q, t)].$$

This is an ordinary differential equation and the solution can be obtained easily by integration:

$$f_{st}(q) = \frac{c}{Q(q)} \exp \left[ 2 \int_{-\frac{1}{2}}^q \frac{K(q)}{Q(q)} dq \right]$$

with the normalization constant  $c$ .

## 6.2 Derivation of the transition probabilities $p_{+-}$ and $p_{-+}$

Ising introduced an idealized model of the ferromagnet that consists of a lattice of elementary magnets (more precise magnetic moments, so-called spins  $\sigma_i$ ), which can adopt only two orientations: Parallel ( $\sigma_i = 1$ ) to an external field  $H$  or anti-parallel ( $\sigma_i = -1$ ). The total energie  $E_\lambda$  of a magnet configuration  $\lambda$  is assumed to depend on the external field and on an internal field created by the magnet-magnet interaction in the following way:

$$E_\lambda = \sum_i E_{\lambda i} = \sum_i \left( -\mu H \sigma_i - \sum_{j \neq i} I_{ij} \sigma_i \sigma_j \right)$$

$I_{ij}$  are the spin-spin interaction parameters and  $\mu$  is the magnitude of a magnetic moment. As in every thermodynamic system, the total energie  $E_\lambda$  is the central item that determines various other thermodynamic variables. For example, the probability of finding the system in the spin-configuration  $\lambda$  is given by the Boltzmann distribution

$$w(E_\lambda) = c \exp(-E_\lambda/k_B T)$$

with the Boltzmann constant  $k_B$  and temperature  $T$ . Even though the system is in thermal equilibrium it is not in a static state. Spins may flip from one orientation to the other, changing the total energy. The probability per unit time of such a flipping process is governed by the equation

$$\text{Pr}(\sigma_i \rightarrow -\sigma_i) = \frac{A}{\tau} \exp \left[ \frac{-W + E_{\lambda i}}{k_B T} \right] = \frac{A}{\tau} \exp \left[ \frac{-(W + \mu H \sigma_i + \sum_{j \neq i} I_{ij} \sigma_i \sigma_j)}{k_B T} \right] \quad (29)$$

where  $A$ ,  $\tau$  and  $W$  are appropriately chosen constants.

The characteristics of the Ising model are well documented<sup>9</sup>: There exists a phase transition temperature  $T_c$  defined by the formula

$$\sinh(2I/k_B T_c) = 1.$$

Therefore, three phases can be distinguished: For high temperatures ( $T > T_c$ ), the spins fluctuate almost independently of their neighbours entailing a mean polarization of zero. For temperatures around the phase transition temperature ( $T \approx T_c$ ) there are big clusters of aligned spins, while the mean polarization is still zero. In the case of very low temperatures ( $T < T_c$ ), one of the clusters grows at the cost of the oppositely orientated spins until all spins point in the same direction. The mean polarisation is then close to one.

The basic idea of Weidlich was to formulate the transition probabilities  $p_{+-}$  and  $p_{-+}$  in complete analogy to the Ising model. Setting  $\alpha = \frac{A}{\tau} \exp(-\frac{W}{k_B T})$ ,  $\pm\mu H = \mu H \sigma_i$  and  $\pm I(n_+ - n_-) = \sum_{j \neq i} I_{ij} \sigma_i \sigma_j$  we get from (29)

$$\begin{aligned} p_{+-}(q) &= \alpha \exp\left(\frac{-I(n_+ - n_-) - \mu H}{k_B T}\right) \\ &= \alpha \exp\left(\frac{-\frac{I}{2n}q - \mu H}{k_B T}\right) = \alpha \exp[-(kq + h)] \end{aligned}$$

$$\begin{aligned} p_{-+}(q) &= \alpha \exp\left(\frac{+I(n_+ - n_-) + \mu H}{k_B T}\right) \\ &= \alpha \exp\left(\frac{+\frac{I}{2n}q + \mu H}{k_B T}\right) = \alpha \exp[+(kq + h)]. \end{aligned}$$

For further simplification we introduced the adaption parameter  $k = \frac{I}{2nk_B T}$  and the preference parameter  $h = \frac{\mu H}{k_B T}$ .

### 6.3 Derivation of the Fundamental Valuation Equation

According to (11), the returns of the risky asset are governed by the stochastic differential equation

$$\frac{dS(\underline{x}, t)}{S} = \alpha(\underline{x}, t)dt + \underline{\eta}(\underline{x}, t)d\underline{w}(t) \quad (30)$$

with drift

$$\alpha = \mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q)$$

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<sup>9</sup>For an analytic investigation see Cohen (1968) and for a computer simulation see Ogita et al. (1969).

and diffusion vector

$$\underline{\eta} = [ \kappa\sqrt{Q(q)} \quad \sigma_D ] .$$

Let  $\underline{x} = \begin{bmatrix} q \\ S \end{bmatrix}$  be the state vector which follows the two-dimensional Markov process

$$d\underline{x} = \underline{\mu}(\underline{x}, t)dt + \underline{\sigma}(\underline{x}, t)d\underline{w}(t) \quad (32)$$

where

$$\underline{\mu}(\underline{x}, t) = \begin{bmatrix} K(q) \\ [\mu_D + \kappa K(q) + \frac{1}{2}\kappa^2 Q(q)] S \end{bmatrix}$$

is the drift vector and

$$\underline{\sigma}(\underline{x}, t) = \begin{bmatrix} \sqrt{Q(q)} & 0 \\ \kappa\sqrt{Q(q)}S & \sigma_D S \end{bmatrix}$$

is the correlation matrix of the subordinated processes  $dS$  and  $dq$ . Let further

$$\frac{dF(\underline{x}, t)}{F} = \beta(\underline{x}, t)dt + \underline{\psi}(\underline{x}, t)d\underline{w}(t) \quad (33)$$

be the stochastic process of the returns of the derivative security which may depend on all state variables, but not on wealth  $W$ .

We want to maximize

$$J(W, \underline{x}, t) = E_t \left[ \int_t^\infty U(C)ds \right] \quad (34)$$

with the utility function

$$U(C) = \exp(-\rho s) \ln C(s) \quad (35)$$

subject to the budget constraint

$$dW = [(1 - \omega_P - \omega_F) rW dt + \omega_P W \frac{dS}{S} + \omega_F W \frac{dF}{F} - C dt]. \quad (36)$$

The last equation can be rewritten using equations (30) and (33), so we get

$$dW = [r + \omega_P(\alpha - r) + \omega_F(\beta - r)] W dt + [\omega_P \underline{\eta} + \omega_F \underline{\psi}] W d\underline{w} - C dt. \quad (37)$$

The necessary optimality condition for (34) is the Bellman equation

$$\max_{C, \omega_P, \omega_F} \left[ U(C) + \frac{E_t[dJ]}{dt} \right] = 0. \quad (38)$$

We can write  $E_t[dJ]$  as follows:

$$\begin{aligned} E_t[dJ] &= J_t dt + J_W E_t[dW] + J_{\underline{x}} E_t[d\underline{x}] + \frac{1}{2} \text{tr}(J_{\underline{x}\underline{x}'} E_t[d\underline{x}d\underline{x}']) \\ &\quad + J_{\underline{x}W} E_t[d\underline{x}dW] + \frac{1}{2} J_{WW} E_t[dW^2] + o(dt) \end{aligned}$$

with the expectations

$$\begin{aligned}
E[d\underline{x}] &= \underline{\mu}dt \\
E_t[d\underline{x}d\underline{x}'] &= \underline{\sigma}\underline{\sigma}'dt \\
E_t[d\underline{x}dW] &= (\omega_P\underline{\sigma}\underline{\eta}' + \omega_F\underline{\sigma}\underline{\psi}')Wdt \\
E_t[dW] &= [r + \omega_P(\alpha - r) + \omega_F(\beta - r)]Wdt - Cdt \\
E_t[dW^2] &= (\omega_P^2\underline{\eta}\underline{\eta}' + 2\omega_P\omega_F\underline{\psi}\underline{\eta}' + \omega_F^2\underline{\psi}\underline{\psi}')W^2dt.
\end{aligned}$$

Ignoring terms of order  $o(dt)$ , we can now substitute into the Bellman equation to get

$$\begin{aligned}
0 &= \max_{C, \omega_P, \omega_F} [U(C) + J_t + J_W([r + \omega_P(\alpha - r) + \omega_F(\beta - r)]W - C) \\
&\quad + J_{\underline{x}}\underline{\mu} + J_{\underline{x}W}(\omega_P\underline{\sigma}\underline{\eta}' + \omega_F\underline{\sigma}\underline{\psi}')W \\
&\quad + \frac{1}{2}J_{WW}(\omega_P^2\underline{\eta}\underline{\eta}' + 2\omega_P\omega_F\underline{\psi}\underline{\eta}' + \omega_F^2\underline{\psi}\underline{\psi}')W^2 + \frac{1}{2}tr(J_{\underline{x}\underline{x}'}\underline{\sigma}\underline{\sigma}')].
\end{aligned}$$

The first order conditions are

$$U_C - J_W = 0 \quad (39)$$

$$J_W(\alpha - r) + \underline{\eta}\underline{\sigma}'J'_{\underline{x}W}W + J_{WW}(\omega_P\underline{\eta}\underline{\eta}' + \omega_F\underline{\eta}\underline{\psi}')W = 0 \quad (40)$$

$$J_W(\beta - r) + \underline{\psi}\underline{\sigma}'J'_{\underline{x}W}W + J_{WW}(\omega_P\underline{\psi}\underline{\eta}' + \omega_F\underline{\psi}\underline{\psi}')W = 0. \quad (41)$$

As shown in Merton (1971), the investor's value function  $J$  is partially separable and has the form

$$J(W, \underline{x}, t) = \frac{\exp(-\rho t)}{\rho} \ln(W) + H(\underline{x}, t). \quad (42)$$

With this value function, Equation (39) becomes the usual consumption optimality condition derived by Merton

$$C = \rho W. \quad (43)$$

**Case 1** *Market clearing condition*  $\omega_P = 1$ :

In this case,  $\omega_r = \omega_F = 0$  and we can solve equation (40) for  $r$ :

$$r = \alpha - \underline{\eta}\underline{\eta}' \quad (44)$$

Substituting this result into (41) yields

$$\beta = r + \underline{\psi}\underline{\lambda}_2 \quad (45)$$

with market price of risk

$$\underline{\lambda}_2 = \underline{\eta}'. \quad (46)$$

**Case 2** *Market clearing condition*  $\omega_P + \omega_r = 1$ :

In this case, only the derivative security is in zero net supply ( $\omega_F = 0$ ) and we obtain the optimal portfolio decision  $\omega_P^*$  from (40):

$$\omega_P^* = \frac{1}{\underline{\eta}\eta'}(\alpha - r). \quad (47)$$

If we substitute  $\omega_P^*$  into (41), the drift of the derivative security can be written as

$$\beta = r + \underline{\psi}\underline{\lambda}_3 \quad (48)$$

where

$$\underline{\lambda}_3 = \frac{\underline{\eta}'}{\underline{\eta}\eta'}(\alpha - r) \quad (49)$$

is the vektor of factor risk prices.

To derive the fundamental valuation equation we have to apply Ito's lemma to  $F(\underline{x}, t)$  with the result

$$\begin{aligned} dF &= \left( F_t + F_{\underline{x}}\underline{\mu} + \frac{1}{2}tr(F_{\underline{x}\underline{x}'}\underline{\sigma}\underline{\sigma}') \right) dt + F_{\underline{x}}\underline{\sigma}d\underline{w} \\ &= \beta(\underline{x}, t)Fdt + \underline{\psi}(\underline{x}, t)Fd\underline{w}(t). \end{aligned} \quad (50)$$

Setting the identities  $\beta(\underline{x}, t) = \frac{1}{F} \left( F_t + F_{\underline{x}}\underline{\mu} + \frac{1}{2}tr(F_{\underline{x}\underline{x}'}\underline{\sigma}\underline{\sigma}') \right)$  and  $\underline{\psi}(\underline{x}, t) = \frac{1}{F} (F_{\underline{x}}\underline{\sigma})$  into equations (45) and (48) respectively, we get

$$\frac{1}{2}tr(F_{\underline{x}\underline{x}'}\underline{\sigma}\underline{\sigma}') + F_{\underline{x}}(\underline{\mu} - \underline{\sigma}\underline{\lambda}_i) + F_t - rF = 0. \quad (51)$$

Resubstituting the expressions for  $\underline{x}$ ,  $\underline{\mu}$ ,  $\underline{\sigma}$ ,  $\alpha$ ,  $\underline{\eta}$  and  $\underline{\lambda}_i$  finally leads to our valuation equations for derivative securities.

## References

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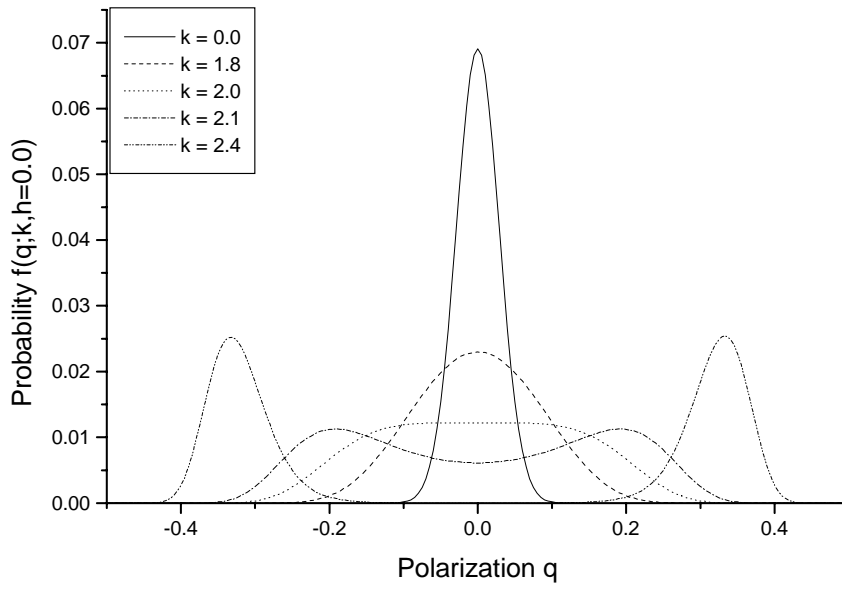


Figure 1a: Phase transition in the case of no preferences ( $h = 0.0$ ).

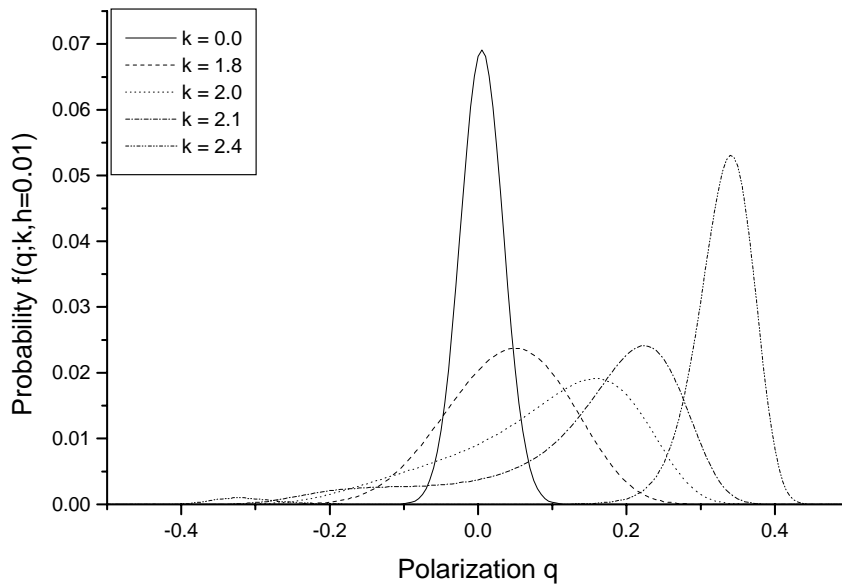


Figure 1b: Phase transition in the case of positive preferences ( $h = 0.01$ ).

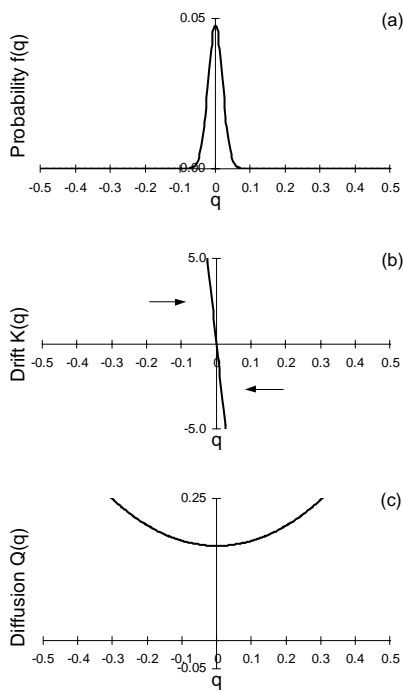


Fig. 2.1: Repelling market  
( $k = -1.8$ ,  $h = 0.0$ )

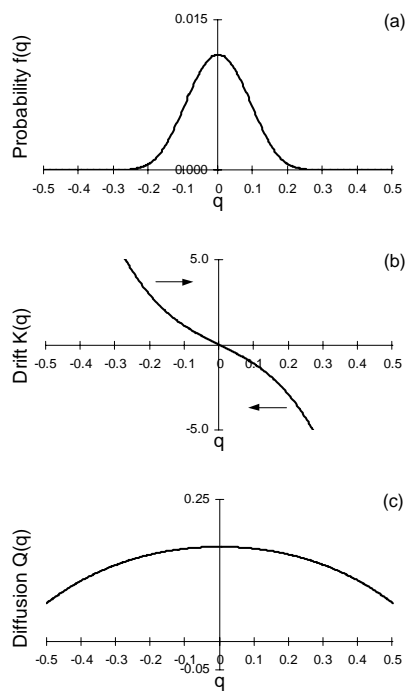


Fig. 2.2: "Efficient market"  
( $k = 1.8$ ,  $h = 0.0$ )

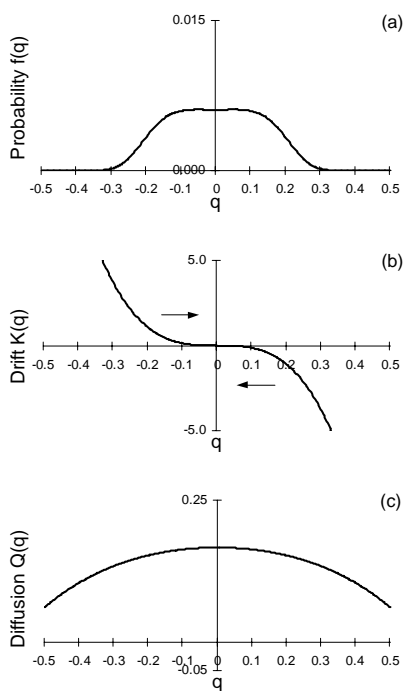


Fig. 2.3: Unstable transition  
( $k = 2.0$ ,  $h = 0.0$ )

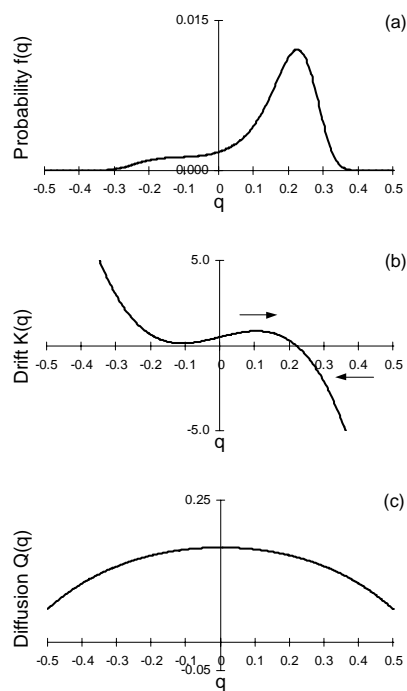


Fig. 2.4: Coherent bull market  
( $k = 2.1$ ,  $h = 0.01$ )

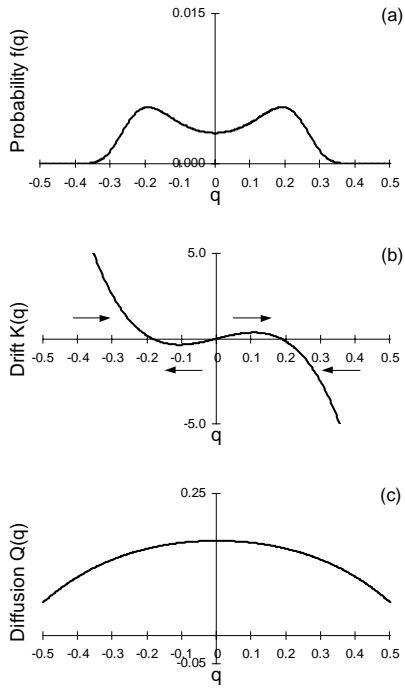


Fig. 2.5: Chaotic market (neutral)  
 $(k = 2.1, h = 0.0)$

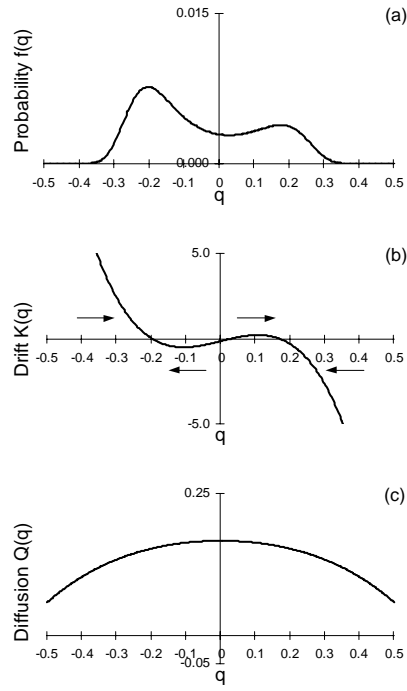
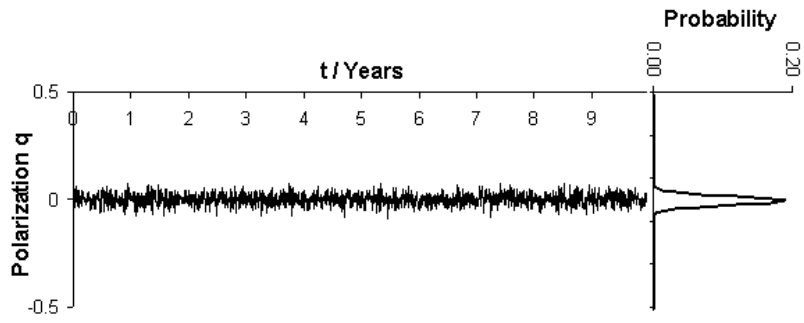
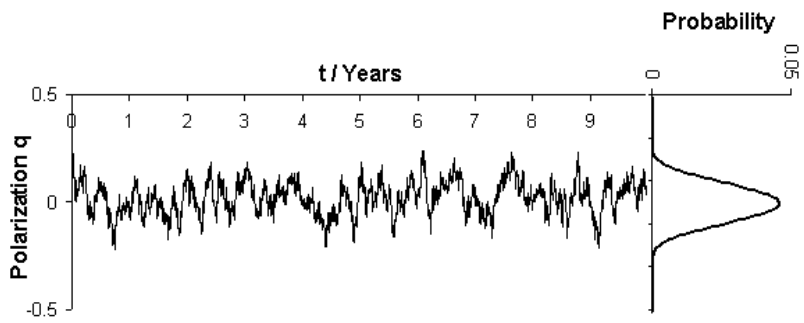


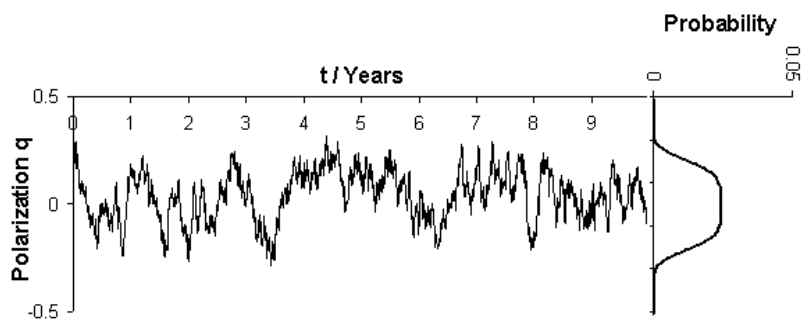
Fig. 2.6: Chaotic market (bearish)  
 $(k = 2.1, h = -0.003)$



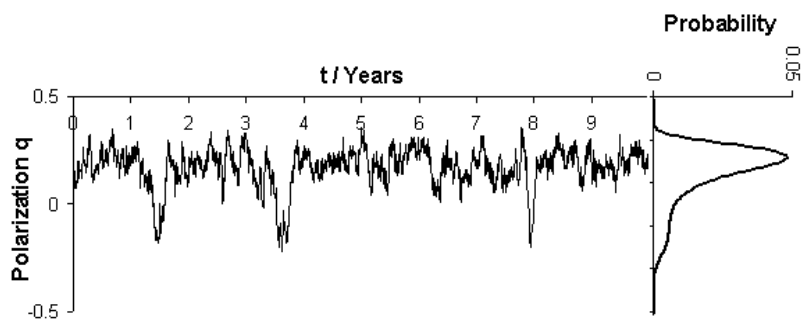
a) Repelling market



b) "Efficient market"

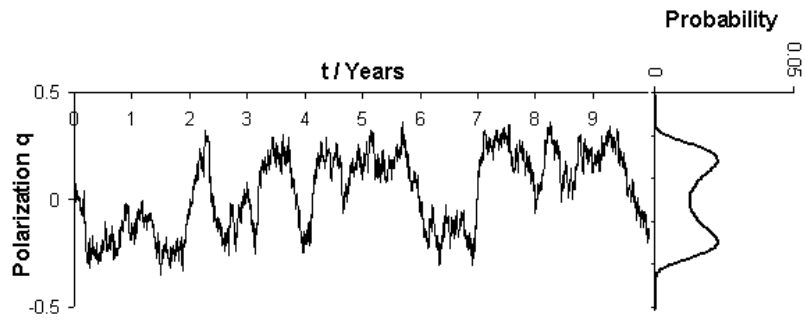


c) Unstable transition

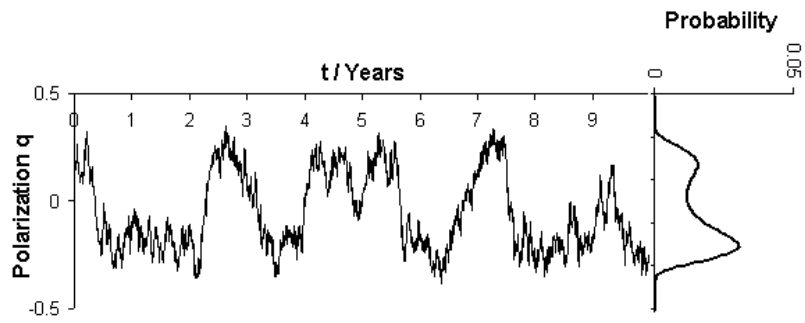


d) Coherent bull market

Figure 3: Trajectories of the stochastic process  $dq$ .



e) Chaotic market (neutral)



f) Chaotic market (bearish)

Figure 3: Trajectories of the stochastic process  $dq$  (continued).

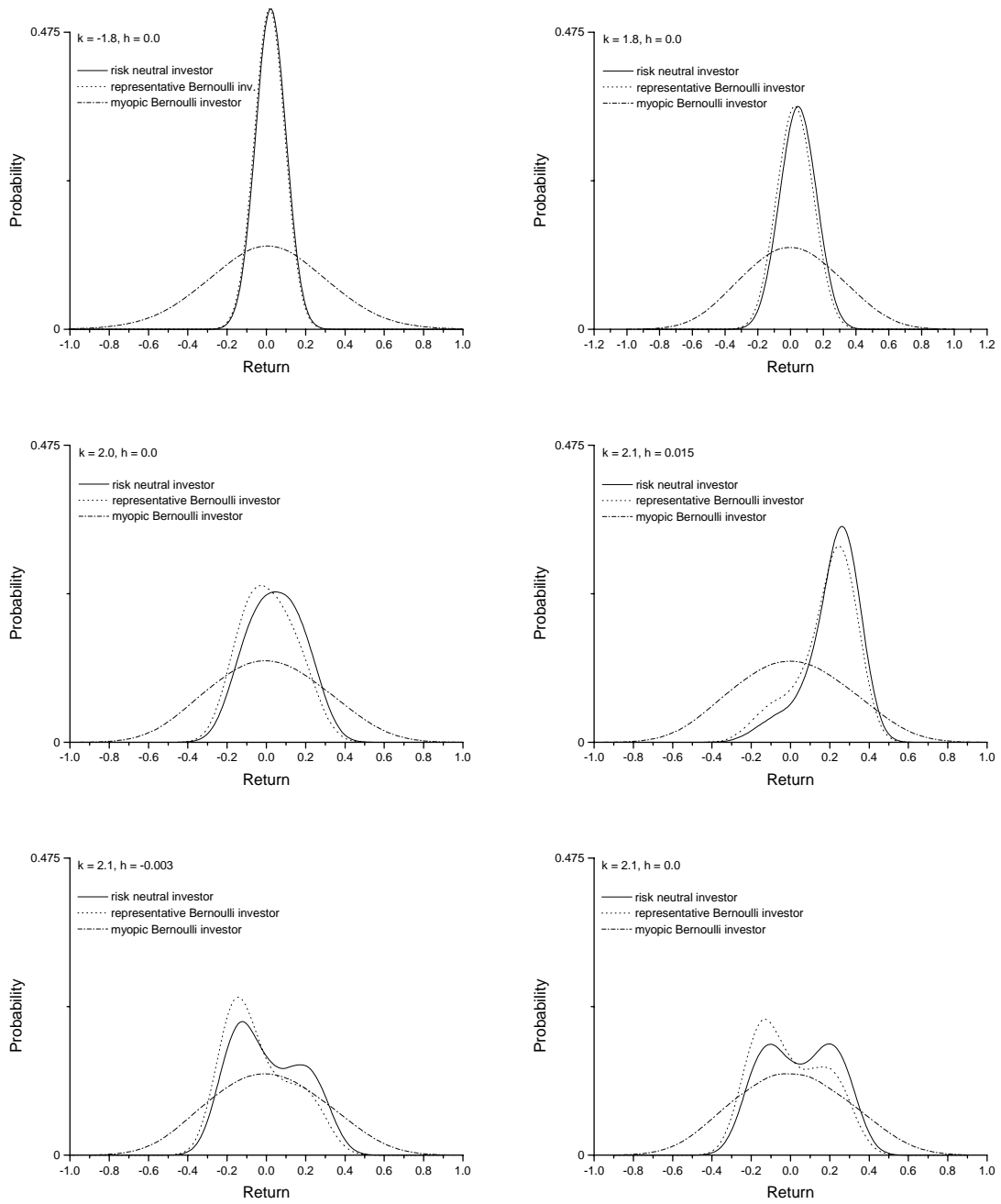


Figure 4: Implied return distributions for risk-neutral investor (original), representative Bernoulli investor and myopic Bernoulli investor in different market states.

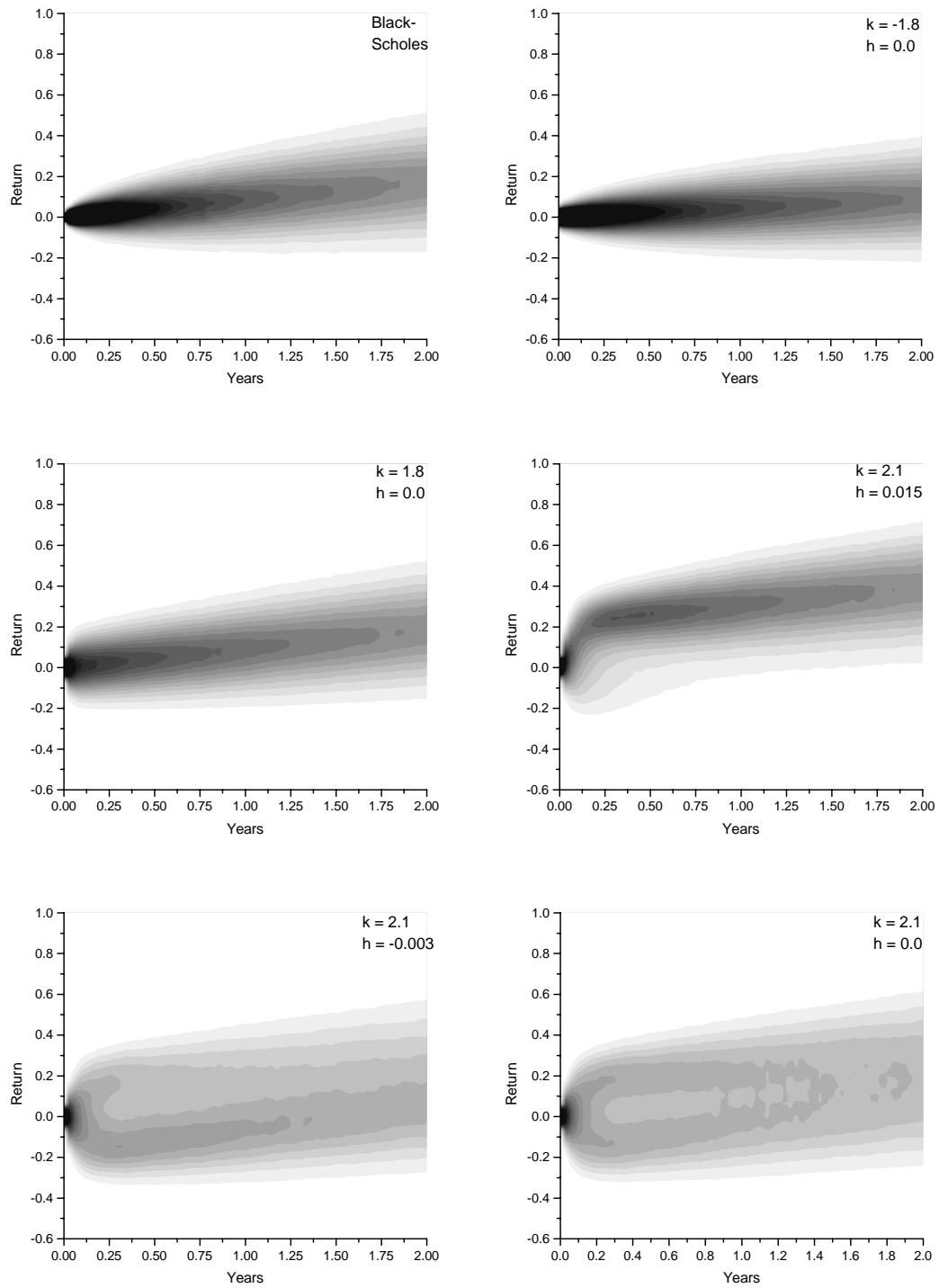


Figure 5: Contourplot of the evolution of the return distribution.



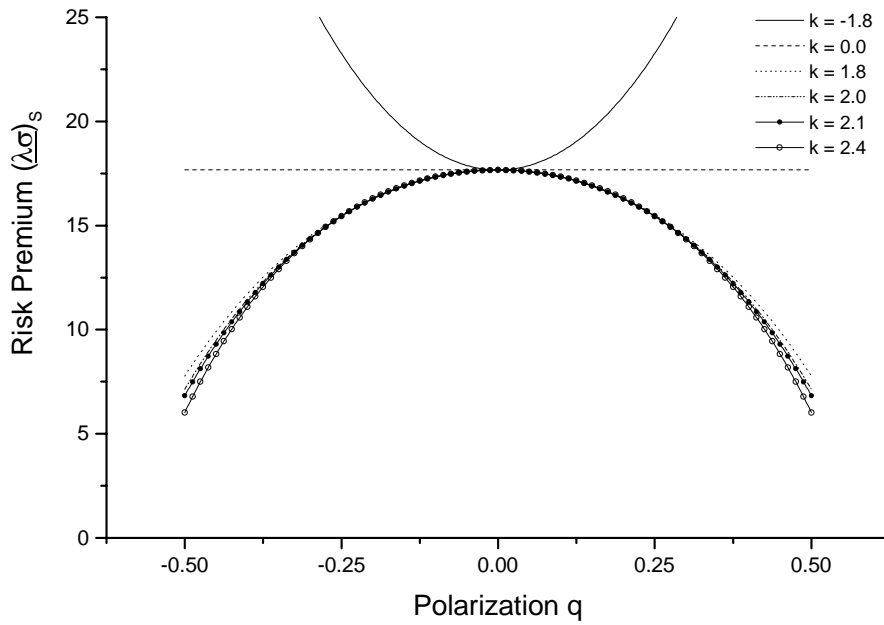


Fig. 6a: Risk premium subject to market polarization  $q$  and crowd behavior  $k$  for a representative Bernoulli investor.

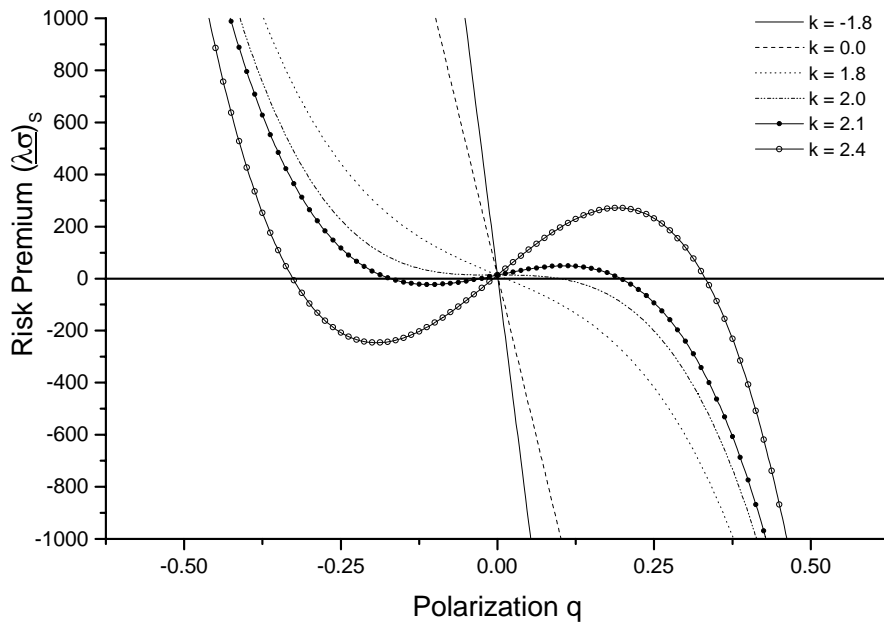


Fig. 6b: Risk premium subject to market polarization  $q$  and crowd behavior  $k$  for a myopic Bernoulli investor.

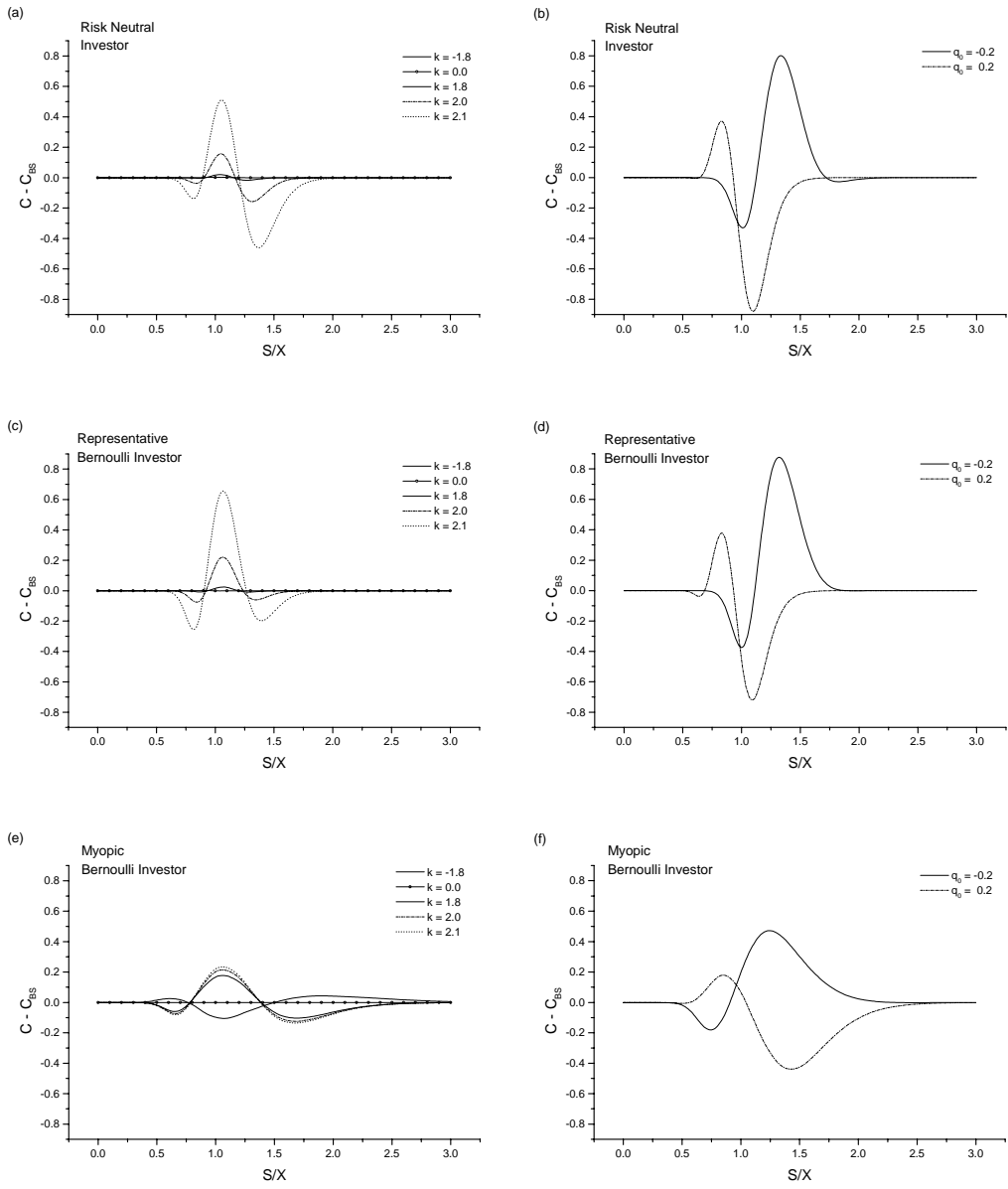


Figure 7: Differences between the model option prices and corresponding Black-Scholes prices.

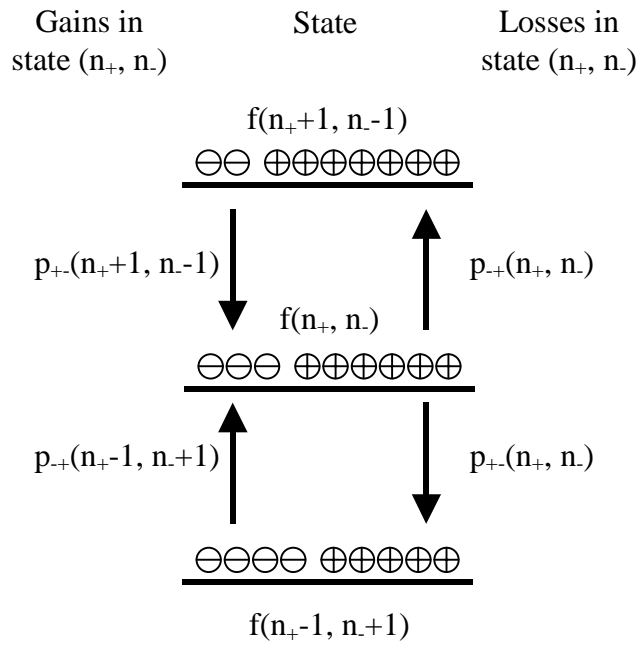


Figure 8: Probability currents to and from state  $(n_+, n_-)$ .

X	New	B/S	I. vol.	New	B/S	I. vol.	New	B/S	I. vol.
A. $(k, h = 0.0, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$k = -1.8, r = 9.81\%$			$k = 1.8, r = 12.03\%$			$k = 2.1, r = 15.73\%$		
80	23.84	23.84	0.1055	24.69	24.69	0.1504	26.28	26.42	0.2441
90	14.36	14.36	0.1050	15.53	15.53	0.1538	18.35	18.29	0.2720
100	5.89	5.89	0.1049	7.77	7.76	0.1545	12.07	11.60	0.2850
110	1.23	1.23	0.1049	2.86	2.85	0.1542	7.18	6.74	0.2802
120	0.11	0.11	0.1050	0.75	0.76	0.1531	3.57	3.60	0.2627
130	0.01	0.00	0.1055	0.13	0.15	0.1509	1.37	1.78	0.2412
	<b>repelling market</b>			<b>efficient market</b>			<b>chaotic market</b>		
B. $(k = 1.8, h, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$h = -0.1, r = 3.93\%$			$h = 0.0, r = 12.03\%$			$h = 0.01, r = 19.98\%$		
80	21.59	21.60	0.1484	24.69	24.69	0.1504	27.62	27.62	0.1523
90	12.35	12.37	0.1515	15.53	15.53	0.1538	18.67	18.67	0.1538
100	5.32	5.29	0.1533	7.77	7.76	0.1545	10.50	10.48	0.1533
110	1.64	1.59	0.1544	2.86	2.85	0.1542	4.50	4.51	0.1517
120	0.35	0.33	0.1541	0.75	0.76	0.1531	1.38	1.43	0.1498
130	0.05	0.05	0.1523	0.13	0.15	0.1509	0.29	0.33	0.1472
	<b>weak coherent bear mkt.</b>			<b>efficient market</b>			<b>weak coherent bull mkt.</b>		
C. $(k = 2.1, h, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$h = -0.1, r = -9.48\%$			$h = 0.0, r = 15.73\%$			$h = 0.1, r = 38.94\%$		
80	16.79	17.22	0.2041	26.28	26.42	0.2441	34.21	34.19	0.2539
90	9.61	9.58	0.2372	18.35	18.29	0.2720	26.30	26.12	0.2661
100	5.39	4.57	0.2646	12.07	11.60	0.2850	18.96	18.56	0.2606
110	2.84	1.88	0.2767	7.18	6.74	0.2802	12.32	12.08	0.2410
120	1.27	0.68	0.2742	3.57	3.60	0.2627	6.74	7.15	0.2156
130	0.44	0.22	0.2622	1.37	1.78	0.2412	2.86	3.85	0.1932
	<b>strong coh. bear mkt.</b>			<b>chaotic market</b>			<b>strong coh. bull mkt.</b>		
D. $(k = 1.8, h = 0.0, q_0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$q_0 = -0.1, r = 32.00\%$			$q_0 = 0.0, r = 12.03\%$			$q_0 = 0.1, r = -7.93\%$		
80	31.84	31.84	0.1517	24.69	24.69	0.1504	16.95	16.95	0.1536
90	23.33	23.34	0.1523	15.53	15.53	0.1538	8.13	8.12	0.1544
100	15.10	15.10	0.1539	7.77	7.76	0.1545	2.70	2.69	0.1541
110	8.06	8.05	0.1545	2.86	2.85	0.1542	0.59	0.60	0.1526
120	3.36	3.34	0.1543	0.75	0.76	0.1531	0.08	0.09	0.1504
130	1.06	1.06	0.1534	0.13	0.15	0.1509	0.01	0.01	0.1484
	<b>overreaction down (efficient market)</b>			<b>fair valuation (efficient market)</b>			<b>overreaction up (efficient market)</b>		
E. $(k = 2.1, h = 0.0, q_0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$q_0 = -0.2, r = 39.57\%$			$q_0 = 0.0, r = 15.73\%$			$q_0 = 0.2, r = -9.93\%$		
80	34.37	34.40	0.1719	26.28	26.42	0.2441	17.59	17.23	0.2654
90	26.21	26.39	0.1943	18.35	18.29	0.2720	9.88	9.74	0.2467
100	18.58	18.92	0.2178	12.07	11.60	0.2850	4.19	4.79	0.2196
110	12.44	12.52	0.2423	7.18	6.74	0.2802	1.17	2.07	0.1963
120	8.09	7.61	0.2628	3.57	3.60	0.2627	0.20	0.80	0.1792
130	5.06	4.25	0.2742	1.37	1.78	0.2412	0.02	0.28	0.1680
	<b>overreaction down (chaotic market)</b>			<b>fair valuation (chaotic market)</b>			<b>overreaction up (chaotic market)</b>		

Table 1: Call option prices in case of a risk neutral investor.

X	New	B/S	I. vol.	New	B/S	I. vol.	New	B/S	I. vol.
A. $(k, h = 0.0, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$k = -1.8, r = 8.63\%$			$k = 1.8, r = 8.31\%$			$k = 2.1, r = 5.98\%$		
80	23.38	23.38	0.1063	23.28	23.28	0.1406	22.74	23.00	0.2148
90	13.86	13.86	0.1055	14.07	14.08	0.1519	15.02	14.99	0.2478
100	5.48	5.48	0.1049	6.60	6.59	0.1539	9.41	8.83	0.2750
110	1.08	1.08	0.1049	2.24	2.22	0.1545	5.34	4.71	0.2858
120	0.09	0.09	0.1049	0.54	0.54	0.1542	2.52	2.30	0.2796
130	0.00	0.00	0.1055	0.09	0.09	0.1528	0.91	1.04	0.2629
	<b>repelling market</b>			<b>efficient market</b>			<b>chaotic market</b>		
B. $(k = 1.8, h, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$h = -0.1, r = 0.3\%$			$h = 0.0, r = 8.31\%$			$h = 0.01, r = 16.37\%$		
80	20.17	20.18	0.1445	23.28	23.28	0.1406	26.30	26.30	0.1520
90	10.96	10.98	0.1484	14.07	14.08	0.1519	17.25	17.24	0.1533
100	4.37	4.33	0.1514	6.60	6.59	0.1539	9.22	9.21	0.1543
110	1.22	1.17	0.1533	2.24	2.22	0.1545	3.69	3.70	0.1534
120	0.24	0.22	0.1542	0.54	0.54	0.1542	1.05	1.08	0.1520
130	0.03	0.03	0.1538	0.09	0.09	0.1528	0.20	0.23	0.1500
	<b>weak coherent bear mkt.</b>			<b>efficient market</b>			<b>weak coherent bull mkt.</b>		
C. $(k = 2.1, h, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$h = -0.1, r = -17.47\%$			$h = 0.0, r = 5.98\%$			$h = 0.1, r = 31.06\%$		
80	13.68	14.10	0.1741	22.74	23.00	0.2148	31.60	31.60	0.2402
90	7.04	6.87	0.2023	15.02	14.99	0.2478	23.63	23.43	0.2646
100	3.65	2.73	0.2350	9.41	8.83	0.2750	16.52	16.09	0.2749
110	1.81	0.89	0.2594	5.34	4.71	0.2858	10.35	10.14	0.2676
120	0.76	0.25	0.2687	2.52	2.30	0.2796	5.42	5.85	0.2469
130	0.24	0.06	0.2654	0.91	1.04	0.2629	2.18	3.11	0.2222
	<b>strong coh. bear mkt.</b>			<b>chaotic market</b>			<b>strong coh. bull mkt.</b>		
D. $(k = 1.8, h = 0.0, q_0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$q_0 = -0.1, r = 28.26\%$			$q_0 = 0.0, r = 8.31\%$			$q_0 = 0.1, r = -11.65\%$		
80	30.55	30.55	0.1441	23.28	23.28	0.1406	15.48	15.49	0.1514
90	21.90	21.90	0.1484	14.07	14.08	0.1519	6.94	6.93	0.1538
100	13.63	13.64	0.1521	6.60	6.59	0.1539	2.11	2.09	0.1545
110	6.87	6.86	0.1539	2.24	2.22	0.1545	0.41	0.42	0.1541
120	2.66	2.64	0.1545	0.54	0.54	0.1542	0.05	0.06	0.1523
130	0.77	0.77	0.1544	0.09	0.09	0.1528	0.00	0.01	0.1504
	<b>overreaction down (efficient market)</b>			<b>fair valuation (efficient market)</b>			<b>overreaction up (efficient market)</b>		
E. $(k = 2.1, h = 0.0, q_0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$q_0 = -0.2, r = 31.25\%$			$q_0 = 0.0, r = 5.98\%$			$q_0 = 0.2, r = -18.15\%$		
80	31.58	31.62	0.1520	22.74	23.00	0.2148	15.06	14.69	0.2714
90	23.11	23.33	0.1719	15.02	14.99	0.2478	8.06	7.90	0.2702
100	15.39	15.77	0.1914	9.41	8.83	0.2750	3.22	3.73	0.2491
110	9.60	9.62	0.2145	5.34	4.71	0.2858	0.84	1.56	0.2231
120	5.87	5.26	0.2387	2.52	2.30	0.2796	0.13	0.59	0.2012
130	3.49	2.59	0.2579	0.91	1.04	0.2629	0.01	0.21	0.1846
	<b>overreaction down (chaotic market)</b>			<b>fair valuation (chaotic market)</b>			<b>overreaction up (chaotic market)</b>		

Table 2: Call option prices in case of a representative Bernoulli investor.

X	New	B/S	I. vol.	New	B/S	I. vol.	New	B/S	I. vol.
A. $(k, h = 0.0, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$k = -1.8, r = 9.53\%$			$k = 1.8, r = 9.53\%$			$k = 2.1, r = 9.53\%$		
80	26.42	26.42	0.4299	26.10	26.08	0.4105	26.05	26.03	0.4070
90	19.65	19.70	0.4277	19.34	19.22	0.4140	19.29	19.15	0.4120
100	14.19	14.27	0.4268	13.88	13.71	0.4156	13.85	13.62	0.4143
110	10.00	10.09	0.4268	9.68	9.50	0.4156	9.64	9.40	0.4143
120	6.92	6.99	0.4275	6.57	6.42	0.4145	6.52	6.33	0.4127
130	4.72	4.76	0.4286	4.33	4.25	0.4124	4.27	4.18	0.4097
	<b>repelling market</b>			<b>efficient market</b>			<b>chaotic market</b>		
B. $(k = 1.8, h, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$h = -0.1, r = 9.53\%$			$h = 0.0, r = 9.53\%$			$h = 0.01, r = 9.53\%$		
80	26.09	26.07	0.4098	26.10	26.08	0.4105	26.11	26.08	0.4113
90	19.33	19.22	0.4136	19.34	19.22	0.4140	19.34	19.23	0.4144
100	13.88	13.70	0.4155	13.88	13.71	0.4156	13.89	13.72	0.4157
110	9.69	9.49	0.4157	9.68	9.50	0.4156	9.68	9.50	0.4154
120	6.58	6.41	0.4148	6.57	6.42	0.4145	6.55	6.43	0.4141
130	4.35	4.25	0.4130	4.33	4.25	0.4124	4.32	4.26	0.4118
	<b>weak coherent bear mkt.</b>			<b>efficient market</b>			<b>weak coherent bull mkt.</b>		
C. $(k = 2.1, h, q_0 = 0.0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$h = -0.1, r = -9.53\%$			$h = 0.0, r = 9.53\%$			$h = 0.1, r = 9.53\%$		
80	26.03	26.02	0.4061	26.05	26.03	0.4070	26.06	26.03	0.4079
90	19.28	19.14	0.4115	19.29	19.15	0.4120	19.30	19.16	0.4125
100	13.84	13.61	0.4141	13.85	13.62	0.4143	13.85	13.63	0.4144
110	9.65	9.39	0.4144	9.64	9.40	0.4143	9.64	9.41	0.4141
120	6.53	6.32	0.4131	6.52	6.33	0.4127	6.50	6.34	0.4122
130	4.29	4.17	0.4105	4.27	4.18	0.4097	4.25	4.18	0.4090
	<b>strong coh. bear mkt.</b>			<b>chaotic market</b>			<b>strong coh. bull mkt.</b>		
D. $(k = 1.8, h = 0.0, q_0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$q_0 = -0.1, r = 9.53\%$			$q_0 = 0.0, r = 9.53\%$			$q_0 = 0.1, r = 9.53\%$		
80	25.97	26.01	0.4022	26.10	26.08	0.4105	26.16	26.08	0.4144
90	19.20	19.13	0.4080	19.34	19.22	0.4140	19.36	19.23	0.4151
100	13.79	13.59	0.4119	13.88	13.71	0.4156	13.84	13.71	0.4141
110	9.64	9.38	0.4142	9.68	9.50	0.4156	9.57	9.50	0.4118
120	6.59	6.31	0.4152	6.57	6.42	0.4145	6.41	6.42	0.4087
130	4.40	4.15	0.4151	4.33	4.25	0.4124	4.16	4.26	0.4049
	<b>overreaction down (efficient market)</b>			<b>fair valuation (efficient market)</b>			<b>overreaction up (efficient market)</b>		
E. $(k = 2.1, h = 0.0, q_0, \sigma_d = 0.1, \mu_d = 0.1, \rho = 0.0, \alpha = 50)$									
	$q_0 = -0.2, r = 9.53\%$			$q_0 = 0.0, r = 9.53\%$			$q_0 = 0.2, r = 9.53\%$		
80	25.63	25.78	0.3798	26.05	26.03	0.4070	26.11	25.94	0.4112
90	18.79	18.79	0.3895	19.29	19.15	0.4120	19.20	19.03	0.4080
100	13.40	13.19	0.3975	13.85	13.62	0.4143	13.55	13.47	0.4032
110	9.34	8.95	0.4034	9.64	9.40	0.4143	9.16	9.25	0.3972
120	6.38	5.91	0.4076	6.52	6.33	0.4127	5.93	6.19	0.3907
130	4.28	3.80	0.4103	4.27	4.18	0.4097	3.68	4.05	0.3841
	<b>overreaction down (chaotic market)</b>			<b>fair valuation (chaotic market)</b>			<b>overreaction up (chaotic market)</b>		

Table 3: Call option prices in case of a myopic Bernoulli investor.

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