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Asset allocation for pension provision

Financial provision for old age is a serious issue. This paper therefore begins by outlining the pension situation in Germany, arriving at the conclusion that, as matters stand at present, none of the three “pillars” of pension provision – state and occupational pensions and private pension schemes – can sufficiently guarantee adequate retirement income in the future. Given increasing population ageing in Germany for one, provision must focus particularly on fully funded private provisioning and also partly on occupational pension plans. Because of the need for fully-funded systems, asset allocation for pension provision taking account of the key characteristic of retirement saving – the long investment horizon – is particularly important. Thus it is being analysed what effect a varying horizon has on the risk-return properties of the asset class stocks. The analysis leads to the conclusion that the risk entailed in stock investment is reduced relative to the yield as the investment horizon lengthens. This horizon effect can be put to use for asset allocation, as illustrated with reference to a model based on the shortfall probability (zeroth order lower partial moment, LMP0). A look is also taken at alternative horizon-dependent asset allocation models. The paper concludes with an examination of the practical applicability of the LPM0 for pension provision.

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Asset allocation for pension provision

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Abstract

Financial provision for old age is a serious issue. This paper therefore begins by outlining the pension situation in Germany, arriving at the conclusion that, as matters stand at present, none of the three "pillars" of pension provision – state and occupational pensions and private pension schemes – can sufficiently guarantee adequate retirement income in the future. Given increasing population ageing in Germany for one, provision must focus particularly on fully funded private provisioning and also partly on occupational pension plans. Because of the need for fully-funded systems, asset allocation for pension provision taking account of the key characteristic of retirement saving – the long investment horizon – is particularly important. Thus it is being analysed what effect a varying horizon has on the risk-return properties of the asset class stocks. The analysis leads to the conclusion that the risk entailed in stock investment is reduced relative to the yield as the investment horizon lengthens. This horizon effect can be put to use for asset allocation, as illustrated with reference to a model based on the shortfall probability (zeroth order lower partial moment, LMP0). A look is also taken at alternative horizon-dependent asset allocation models. The paper concludes with an examination of the practical applicability of the LPM0 for pension provision.

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Asset allocation for pension provision

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1. Introduction

Pension provision in Germany rests on feet of clay. But Germany is not the only country where, for a variety of reasons, financial provision during working life and, as a result, retirement income is not secure. The aim of this paper is therefore to point out ways of efficient pension provisioning. Chapter 2 begins with a brief outline of the institutional framework for pension provision in Germany. Chapter 3 addresses asset allocation for pension provision. An important part is played here by time horizon effects on the risk-return properties of stocks, which are explained in detail. The author shows how these horizon effects can be used for asset allocation. The paper closes by examining in Chapter 4 how asset allocation models can be used in practice as a means of financial retirement provision.

2. Status quo of pension provision in Germany

The pension system in Germany, consisting of financial provision during active working life for an old-age pension in retirement, rests on three “pillars”: statutory and occupational pension schemes and private provision for old age. None can at present guarantee comprehensive and adequate retirement income.
The **statutory pension system** – the most important “source of funding” in retirement age\(^1\) accounting for 83 percent of total retirement income – suffers in particular from the combination of pay-as-you-go (PAYG) funding\(^2\) and the demographic population development.\(^3\) Our population is ageing perceptibly, as evident for example in forecasts that the old-age dependency ratio (the proportion of at least sixty year-olds divided by the number of people between the ages of twenty and less than sixty) will soar from around 35% in 1995 to well over 70% in 2040.\(^4\) In simplified form, we can view the old-age dependency ratio as the ratio of people drawing pensions to those paying in contributions; its doubling is thus indicative of the marked decline in the viability of the statutory pension scheme. Other pointers to the negative development in this pension pillar are a contribution rate which, in fact, works out at 27.7 rather than 19.1 percent if we include the government subsidies\(^5\) and would soar above 30 percent without further alterations in the parameters\(^6\) or various interventions to reduce benefits, which have already cut pensions by almost 33 percent in comparison to the original endowments under the statutory pension system.\(^7\)

**Occupational pensions** in Germany are of the least importance, as this pillar makes up a mere five percent of total German retirement income and presently posts a

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2. Unlike fully funded systems, under the PAYG system the capital accumulated as provision for retirement is not invested at long term; instead it is paid out again directly to the people already drawing pensions.
3. For further information on this and the following see Lahusen, R. “Asset Allocation für die Alterssicherung”, 2002, p. 8 ff.
4. See also the publication by the Federal Ministry of the Interior “Bevölkerungsentwicklung”, 1995, p. 11 and 34. Other, differing estimates exist on the development in the old-age dependency ratio: These say that in 2035 we can expect a ratio of 66.7 to 81.5 percent. For further information see Finke, R. / Stanowsky, J. “Alterssicherung”, 1998, p. 8; the Deutsche Bundesbank publication “Kapitaldeckung”, 1999, p. 16; Schmid, J. “Dilemma”, 2000, p. 8.
6. See also Börsch-Supan, A. H. “Rentenversicherungssystem”, 1999, p 35. This also quotes the result of official estimates at between 26 and 29 percent; these (rather too positive) figures come about as a result of a presumed change in the working environment, such as higher labour market participation by women or an improved employment situation. Cf. ibid. Even more pessimistic prognoses assume that the current contribution rate will have to double in order to maintain the present pension level; for further information see the publication by the Deutsches Institut für Altersvorsorge “Gesetzliche Rentenversicherung”, 2000, p. 20.
7. The different surveys conducted on this subject arrive at different conclusions. Börsch-Supan puts the benefit reductions at between 10 and 15 percent, others arrive at pension cuts of up to 33 percent. See also Börsch-Supan, A. H. “Rentenreform”, 2000, p. 10; publication by the Deutsches Institut für Altersvorsorge “Gesetzliche Rentenversicherung”, 2000, p. 9 ff.
negative trend. The previous types of pension plan – book reserve schemes (Direktzusage), direct insurance (Direktversicherung), the Pensionskasse or the support fund (Unterstützungskasse) were joined at the beginning of 2002 by Pensionsfonds, which feature advantages such as higher returns or generally greater flexibility, and are thus designed to create incentives to offer company pension schemes. These incentives are enhanced by the method of funding through Entgeltumwandlung, according to which company employees forgo future claims to payment of part of their earnings and invest the funds instead in a pension plan. These two new forms of pension scheme in Germany, Pensionsfonds and Entgeltumwandlung, are expected at least to stem the decline in importance of company pensions, particularly since employees have since January 1, 2002 been legally entitled to Entgeltumwandlung as a means of pension provision. Strictly speaking, though, Entgeltumwandlung is strengthening not company pensions, but private provision for old age, because one of the criteria determining eligibility of the occupational pension plan for state incentives – differentiation between the source of funds – stipulates that it is not the employer but the employee who finances his pension by forgoing part of his salary. The employer acts merely as organiser and not – as with “traditional” company pension plans – as the provider of their funding. In future we can expect to see the “traditional” type of occupational pension scheme becoming even less important relative to pension provision through Entgeltumwandlung.

The third pillar – private pension provision – makes up about 12 percent of total retirement income. Previously, the only inducement to save for old age lay in personal recognition of the urgent need to do so, but on January 1, 2002 regulations were introduced to encourage private provisioning. These incentives take the form of tax deductibility of contributions and state grants towards private retirement provi-

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9 For further information on this and the following see Lahusen, R. “Asset Allocation für die Alterssicherung”, 2002, p. 20 ff.; on the innovations Pensionsfonds and Entgeltumwandlung and how they can be put to use for occupational pension schemes see Lahusen, R. “Altersversorgung”, 2000, p. 499 ff.

10 Cf. Section 1a paragraph 1 of the Law on the Improvement of Company Pensions – BetrAVG.

11 Cf. Section 10a of the Income Tax Law – ESTG.
sion\textsuperscript{12} on a legally determined scale\textsuperscript{13} and are tied to various requirements of the type and manner of retirement saving. The most important are the demands made of a pension contract, which consist for example in current contributions, current disbursements, certain asset categories and – a crucial stipulation – in the guarantee that the nominal value of the contributions paid in will be available at the beginning of the disbursement period.\textsuperscript{14} Given the financial support for private pension schemes, at first sight it seems as though fresh encouragement to make personal provision has been given. But on closer consideration, the raft of regulations and restrictions hedging in the vehicles for accumulating private pension capital emerge as an impediment to their broad acceptance among the public. Moreover, the stipulation that the nominal value of contributions must be guaranteed may tend to result in lower returns, possibly offsetting or even more than counterbalancing the advantages of state subsidisation. A solution to this dilemma is to be found in the models described in Chapter 3.2 which combine high performance with a de facto guarantee for the nominal value of contributions through practicable, customised asset allocation.

3. Asset allocation for pension provision

3.1 Time horizon effects of the asset class stocks

3.1.1 Fundamental thoughts on horizon effects

Retirement provisioning and retirement benefits are characterised by their disproportionately long investment horizon. We should therefore consider whether this long time horizon influences the allocation of assets for pension provision. Controversy has raged for years on whether the trade-off between risk and return in the asset class of stocks changes with varying investment or time horizons. In practice the view is widely held that the longer the investment horizon the lower the risk of a commitment in stocks becomes (positive horizon effect), whereas theoreticians mostly insist that the risk mounts as the time horizon is extended (negative horizon effect) or maintain that the risks of stock investment are generally independent of the time horizon (neutral horizon effect). From the respective time horizons inferences are made for asset allocation. For example, on the assumption of positive horizon effects, in a

\textsuperscript{12} Cf. Sections 83 – 85 EStG.
\textsuperscript{13} Cf. Section 86 EStG.
\textsuperscript{14} Cf. Section 1 Pension Contract Certification Law – AltZertG.
portfolio with a longer time frame stocks are weighted more heavily as an asset class relative to riskless asset classes. The following gives a brief overview of the reasoning behind each of the viewpoints, a distinction being made between possible theoretical explanations and the actual horizon effects observed in the past.

3.1.2 Theoretical explanations of horizon effects

Dynamic stochastic programming is one way of attempting to document neutral horizon effects. In the determination of optimum asset allocation the allocations for individual intervals of time and, together, over longer investment periods are optimal. If we follow this reasoning, there are no differences in asset allocation for short-term and longer-range investment horizons. But the fault with this approach is that the method prevents a non-neutral horizon effect from emerging in the first place, as asset allocation for longer investment horizons must always also be optimal for all time intervals, that is to say for individual years as well. This means that in effect the time horizon is not actually varied; instead, combinations of multiples of individual single-interval time horizons are considered. The result – a neutral horizon effect – is therefore hardly surprising.

The following reasons are always given for ascribing negative horizon effects, that is to say mounting risk over an increasing time frame, to the asset class stocks.

1. Traditional measures of risk such as the standard deviation of stock returns normally rise with a varying time horizon – when the entire period of time is considered – which is taken as confirmation of above-average risk. For asset allocation, the result would be falling equity weightings in the portfolio as the time horizon lengthens.

2. The cost of hedging certain returns can be calculated using the option price theory. We see that the absolute hedging costs rise with the length of the investment horizon, which is taken as proof of a negative horizon effect.

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3. When stock price movements are described using binomial models, we see from the “branching” of the price movements that the potential maximum loss rises with the increase in the number of periods.\[18\]

On closer examination, none of the three explanations for a negative horizon effect is convincing:

Re 1.: The standard deviation for stock returns over longer time horizons does indeed usually exhibit higher readings than for shorter periods – even if the annual standard deviation generally falls. It must, however, be said that there is little point in examining risk alone, because it is always performance, in other words the risk-adjusted return, that is crucial to the assessment of asset allocation, and the obtainable return must therefore also be taken into consideration. Consequently, for several reasons mentioned in the following descriptions of positive horizon effects, stock risk exposure rises relatively less steeply than returns when considering overall periods of time or, when considering time intervals, the annual risk declines while the average annual returns tend to remain the same irrespective of the time horizon. This bears out not a negative, but a positive horizon effect, which can be used for horizon-dependent asset allocation as described in Chapter 3.2.

Re 2.: Discussion on the costs of hedging portfolio returns with options likewise considers risk entirely in isolation, ignoring the opportunities ensuing from stock returns. This approach is correct for the consideration of risk as such, but it is not appropriate if conclusions are to be drawn for asset allocation, because asset allocation – as mentioned above – should pursue the aim of high performance rather than aiming exclusively for low risk. The hedging costs for the overall period do indeed climb, but at a disproportionately slower rate over an increasing time horizon, whereas returns increase in proportion to the growing investment horizon. As the time horizon is extended, so annual hedging costs fall, while annual returns remain constant.\[19\] The corollary to this argumentation is therefore a clearly positive, not a negative horizon effect.

\[18\] See e. g. Albrecht, T. “Zeithorizont”, 1999, p. 43 f.

\[19\] According to Bodie’s and our own calculations, the hedging costs for a time horizon of one year equal 7.98 percent of the amount hedged. With a ten-year time horizon the absolute hedging costs amount to 24.84 percent, but this is equivalent to annualised costs of 2.48 percent. Over a time horizon of 30 years, which is by all means customary for pension provision, the absolute hedging costs work out at 41.63 percent, which is equivalent to annual costs of 1.39 percent. Returns, on
Re 3.: The consideration of possible price movements with reference to binomial models shows maximum potential loss rising with the increasing number of periods, since the possible loss per period is multiplied over a longer investment horizon. This procedure implies a focus on the highest possible loss. But as a strategic basis for asset allocation for pension provision, such an approach is not advisable, given that – apart from its failure to consider returns – it completely disregards the fact that the likelihood of materialisation of maximum loss is significantly reduced as the number of periods increases, becoming almost negligible.

Positive horizon effects can be documented in a variety of ways. From criticism of the argumentation in support of negative horizon effects we have already seen that in fact the reasoning advanced to substantiate a negative horizon effect partly delivers evidence of a positive effect. It has already been mentioned that a standard deviation rising at a disproportionately slower rate over a longer time horizon, in combination with proportionate growth in returns, constitutes a positive horizon effect. This is underpinned theoretically by describing the movement of constant stock returns \( (r) \) by means of Brownian motion with the conditionality of normal distribution, stationarity and independence. The return for total periods of time \( (T) \) is calculated by adding the continuous returns of the time intervals:

\[
    r_{0,T} = r_{0,1} + r_{1,2} + \ldots + r_{T-1,T}
\]

Assuming equal returns for all intervals, the aggregate return is calculated simply by multiplying an individual return by the number of years:

\[
    R_{\text{total}} = T \cdot r
\]

The variance \( (\text{Var}) \) in the sum of the returns is calculated as follows:

\[
    \text{Var} (r_{0,T}) = \text{Var} (r_{0,1}+r_{1,2}+\ldots+r_{T-1,T}) = \text{Var} (r_{0,1}) + \text{Var} (r_{1,2}) + \ldots + \text{Var} (r_{T-1,T}) = T \cdot \text{Var} (r)
\]

the other hand, remain constant in annual terms over an increasing time horizon. For discussion of the hedging costs for the total periods see Bodie, Z. “Risk of Stocks”, 1995, p. 20.

20 See e.g. Albrecht, T. “Zeithorizont”, 1999, p. 43 f.


22 For details on this and the following see Spremann, K. “Portfoliomanagement”, 2000, p. 394 ff., or Winhart, S. “Asset Allocation”, 1999, p. 163 ff. Spremann and Winhart do not, however, deduce clearly positive time effects from their arguments.

23 On the assumption of yield independence the co-variances between the individual returns equal zero and are therefore discounted.
Standard deviation \( (\sigma) \) of the returns is thus calculated as

\[
\sigma_{\text{total}} = \sqrt{T} \cdot \sigma_{\text{time interval}}.
\]

This makes it immediately evident that, on the assumptions stated, the standard deviation of stock returns rises only by the root of the number of years, whereas returns increase by the number of years, providing proof of a positive horizon effect.

Positive horizon effects are also found when lower partial moments are used to describe stock price risks.\(^{24}\) The zeroth order lower partial moment, the shortfall probability, falls, for example, for a pre-specified minimum return target of zero percent as the investment horizon increases, as illustrated in the following figure.\(^{25}\)

It is clearly evident from the figure that the risk of stock investments – in terms of the shortfall probability – decreases as the investment horizon increases.

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\(^{25}\) See Appendix for calculation of the shortfall probability. The following were used as input parameters: expected value of stock returns ten percent per annum, standard deviation of stock returns for the one-year time horizon 20 percent / rising by the root of the number of periods, minimum return zero percent. The figure of zero percent for the minimum return denotes preservation of the nominal value of capital such as is currently required in Germany for entitlement to state incentivisation of pension provision. The positive time horizon effect can also be demonstrated for positive minimum annual returns.
Since the shortfall probability disregards the amount of the shortfall suffered, it is a good idea to consider the first lower partial moment – the expected shortfall – as well. This expectation, defined as the product of the shortfall probability and the average value of the shortfall, then develops as follows relative to the time horizon:

As with the zeroth order lower partial moment, a positive horizon effect is evident. However, it kicks in only as from a two-year investment horizon, as the expected shortfall for one-year intervals is slightly lower than for two-year periods.

To sum up, the theoretical foundations for positive horizon effects are convincing. This is supported by the empirical observations described in the following.

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26 For details on the shortfall expectation see e.g. Poddig, T. / Dichtl, H. / Petersmeier, K. “Portfolio-management”, 2000, p. 137.

27 See Appendix for calculation of the shortfall expectation. The trend depicted occurs with the following input parameters: expected value of annual stock returns ten percent, standard deviation of stock returns beginning at 20 percent, minimum return target zero percent. Positive time horizon effects also result overall with positive annual minimum returns.

28 It must be borne in mind with these calculations that the shortfall expectation refers to the respective total time horizon (and not to individual years).
3.1.3 Empirical results on horizon effects

When examining real time series of stock returns we can show that the previous theoretical inferences of a positive horizon effect are not musings far removed from practice but that they do, indeed, tend to describe reality correctly. The risk-return properties of the MSCI World Index for the years 1970 to 2000 were analysed by calculating the **shortfall probabilities** of the MSCI World Index returns for time horizons of one to 20 years. 6.5 percent was taken as the pre-specified annual minimum return. The following picture emerges:

![LPM0 MSCI World Index](image)

Once again a positive horizon effect is apparent, with the shortfall probability dropping clearly as from a three-year horizon. From the 15-year horizon onward the shortfall probability equals zero percent. This means that the extremely restrictive requirement of an annual 6.5 percent return was met in each of the time horizons examined, which illustrates the growing advantageousness of an investment in stocks over an increasing horizon.

A similar picture emerges when we consider the first lower partial moment, the **shortfall expectation**. This drops – for a pre-specified minimum return of zero percent – from 3.62 percent as from three-year horizons to zero percent from ten-year horizons. These values can be interpreted as indicating that for an investment horizon of,
for example, three years an average loss of altogether 3.62 percent can be expected, falling to zero percent for horizons of ten years and more, so that a neutral or positive return can be expected at all events.  

The risk and return properties of the MSCI World Index, and also the theoretical considerations regarding horizon effects, are of fundamental importance in terms of asset allocation for pension provision. With reference to actual stock price trends, we have now also seen a stock investment in the past developing increasingly advantageously for the investor over increasingly long horizons – such as are customary when saving for old age. This confirms the assumption that consideration of the investment horizon is of elementary significance for asset allocation. If this development in the past is projected into the future, this horizon effect can be utilised for pension provision by focusing, over long horizons, primarily on investments in risky securities, which customarily yield higher returns, whereas for short periods invest-

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29 It must be borne in mind with these calculations that the minimum return refers to annual periods, while the shortfall expectation refers to the respective total period. This lends greater informative value to calculation of the shortfall expectation, as the losses are of interest primarily over total periods of time. For example, constant shortfall expectation readings for increasing total periods of time mean falling shortfall expectations for annual intervals, which per se already indicates a positive time horizon effect. In the actual return series of the MSCI World Index shortfall expectations even drop for the total periods.

ments in stocks appear risky and should therefore partly be avoided. In the following it is shown how the horizon effects discussed can be used for pension provision in the form of asset allocation models.

### 3.2 Asset allocation models for pension provision

#### 3.2.1 LPM0 model

In principle a portfolio can be composed of a raft of different asset classes, such as stocks, fixed-income securities, real estate or derivative financial instruments. Since such a variety of investment products is not practicable for retirement saving, the asset classes are reduced to riskless assets (represented by money market funds, for example) and risky assets (e.g. stocks). This reduction is sensible not only for reasons of practicability, but also from a theoretical perspective, since according to the Tobin separation theorem a portfolio comprises precisely the two asset classes riskless and risky securities. With the Tobin separation theorem, the risky asset class, and hence the risky portfolio, comprises all risky investments available on the market in their respective market weightings. This stock portfolio is structured identically for all investors. Unsystematic, or non-systematic, risk is eliminated in the risky portfolio. The investor’s individual risk preference is expressed only through different weightings of riskless and risky assets (but not through different structuring of the stock portfolio itself), which considerably simplifies asset allocation. That this simple asset allocation of risky securities – namely the depiction of entire stock market segments by tracking stock indices – also constitutes one of the most beneficial methods of portfolio structuring, is evidenced by a large number of empirical studies documenting the clear superiority of passive portfolio management (tracking indices) over active management (selection of individual securities).

31 See e.g. Steiner, M. / Bruns, C. “Wertpapiermanagement”, 2000, p. 23.
33 Unsystematic risk comprises the company-specific risk of securities in contrast to systematic risk, which expresses market risk.
34 In the large majority of cases funds with active portfolio management exhibit lower returns – even disregarding the (higher) costs – than the index return taken as the benchmark. A comparison of actively managed funds and mutual funds subject to passive management also shows passively managed funds with a significantly more favourable return, according to various empirical studies. Moreover, funds governed by a passive investment strategy entail considerably lower management fees than funds with active portfolio management. Furthermore, the combination of more than one security reduces unsystematic risks through the diversification effect, if the returns between the respective securities exhibit a correlation of less than plus one. If an index is perfectly tracked, the index portfolio contains only market risk and no unsystematic risk at all. See, for example, Benke, H. “Blue-Chip-Index”, 2001, p. 248; Heda, K. / Heine, K. / Oltmanns, E. “Indexfonds”, 2001, p. 109 ff. on the benefits of index funds and passive portfolio management.
According to the Tobin separation theorem, optimum asset allocation is determined with reference to the capital market line, which in the $\mu$-$\sigma$ diagram ($\mu =$ expectation of stock returns, $\sigma =$ standard deviation of stock returns) connects the riskless investment ($r_f$) with the stock portfolio (M), and the investor’s individual risk-return tradeoff function. The points on the capital market line represent portfolios with equity weightings from zero percent ($r_f$) to 100 percent (M) and also above 100 percent (points on the line to the right of M). The optimum portfolio is the one in which the individual risk-return function touches the capital market line ($P^*$).

![Figure: Tobin Separation](image)

In practice, however, this approach gives rise to the problem that individual risk-return functions cannot be adequately identified. This can be avoided by applying alternative portfolio selection criteria. These alternatives are summarised in the group of shortfall criteria (Roy, Kataoka and Telser criterion) developed around the same time as portfolio selection according to Markowitz, but which in the past have mostly been disregarded. The shortfall criteria are based on the zeroth order lower partial moment (shortfall probability). Asset allocation is determined – given the existence of riskless and risky investments – by depicting in the $\mu$-$\sigma$ diagram shortfall lines (straight lines comprising $\mu$-$\sigma$ combinations connected with exactly one shortfall probability) in combination with the capital market line. The point of intersection of both straight lines denotes optimum asset allocation ($P^*$) in terms of optimum allocation into risky and riskless assets. On closer consideration of the three shortfall criteria we see that only the Telser criterion leads to meaningful asset allocation solu-

\[\text{For details on this and the following see Elton, E. J. / Gruber, M. J. “Portfolio Theory”, 1995, p. 238 ff.}\]
For asset allocation according to the Telser criterion both the maximum shortfall probability and the minimum return target are pre-specified. As illustrated in the following figure, the shortfall line in the $\mu$-$\sigma$ diagram is clearly determinable by means of these two parameters, as the incline of the shortfall lines is determined by the pre-specified shortfall probability and the ordinate section by the minimum return. The point of intersection of the shortfall line and capital market line – that is to say optimum asset allocation – has the property of being $\mu$-$\sigma$-efficient according to Markowitz on the one hand and, on the other, of not overshooting a pre-specified shortfall probability or undershooting a predetermined minimum return.

![Figure: Asset allocation on the basis of the Telser criterion](image)

This procedure represents significant progress on the Tobin separation theorem, because a pension saver is not normally in a position to quote his personal risk-return function – as required for the Tobin separation – although he can indeed pre-specify values for the maximum shortfall probability he is prepared to tolerate and for the minimum return he wishes to obtain. In this way optimum portfolios can be identified on the one hand and allowance made for investors’ individual risk preferences on the other by, for example, reducing the tolerated shortfall probability as a sign of greater risk aversion or increasing the predetermined minimum return target (and vice versa).

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38 $\mu$-$\sigma$ efficiency means that, with a given yield expectation, no portfolio exists with lower standard deviation or, with a given standard deviation, no portfolio exists with a higher return expectation.
resulting in lower (higher) equity weightings. It is therefore possible and practicable to customise optimum and \( \mu-\sigma \)-efficient asset allocation for individual investors – which, moreover, takes account of their personal risk preferences.

Once optimum asset allocation has been determined in this fundamental way, it must be examined in terms of horizon effects. Basing the argumentation on chart considerations, we can state that the position of the shortfall line does not alter with a varying time horizon, since the shortfall line is determined by the pre-specified minimum return target (ordinate section) and maximum shortfall probability tolerance (incline of the shortfall lines) and neither change with variations in the horizon.\(^{39}\) The efficiency line, however, turns to the left as the investment horizon lengthens, since – as explained in Chapter 3.1 – in this case the risk is reduced relative to the return. Crucial now is the relative position of the optimum portfolio on the continuum between complete investment in riskless assets (point \( r_f \)) and complete investment in risky securities (point \( M \)).\(^{40}\) The following figure shows that the optimum portfolio moves toward the stock portfolio as the horizon lengthens, which is tantamount to heavier equity weighting. The result of asset allocation – entirely in line with the intuitive assumption based on the positive horizon effects described earlier – therefore consists of an equity weighting in a portfolio that rises as the horizon grows longer.

\(^{39}\) We must remember that the annual minimum return target is identical for all horizons. The minimum return increases, when we consider the entire horizon, given a positive parameter, as do the total standard deviation and total expected stock return. The tolerated shortfall probability applies to the total period. The results of asset allocation remain identical regardless of whether we consider annual averages or values for the entire horizon.

\(^{40}\) An optimum portfolio to the right of \( M \) means an equity weighting of more than 100 percent and therefore results in borrowing.
As already explained, the minimum return, the annual riskless coupon, the shortfall probability and the annualised expectation of constant return are pre-specified exogenously and hence independent of the horizon. The only variable that alters in the horizon is the average standard deviation of the returns on the equity portfolio. Since this falls relative to the stock return as the time horizon increases, the optimum weighting of risky assets in the total portfolio logically rises.

Optimum asset allocation can be plotted using charts or – to obtain a more precise result – determined analytically. Asset allocation is made, for example, using the assumptions: It is assumed that the values of the input parameters for the model, that is the expected yield of risky and riskless securities and standard deviation of the former, are correct. While the problem of forecasting input parameters is not a focal point of this study, it can nevertheless tend to be solved. Similarly, it is assumed that the standard deviation remains constant with the passage of time – a common assumption but nonetheless a simplification. The assumption also applies that the standard deviation increases, as the horizon is extended, by the root of the number of years. It is further assumed that a correlation of zero exists between the returns on riskless and risky securities, the result being that possible correlation effects between the two asset classes are disregarded. A basic assumption of the model is that the time horizon for pension provision and pension benefits can be forecast. For the pension provision horizon, this is comparatively easy to forecast by setting, for example, a required age at which the pension is to start being drawn. More difficult to forecast is the horizon for the benefit stage, since the time of death must be known. The problem can be solved – as provided for in the relevant statutory regulations – by means of a drawdown pension up to the age of 85, followed by an annuity. It should further be noted that the assumptions apply for the Tobin separation theorem. Construction of the portfolio is based on the assumption of normal distribution. The assumption of normally distributed stock returns is customary and accepted as a simplifying assumption with the use of logarithmic returns.

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41 See the Appendix for analytical determination of optimum asset allocation. The model is based on the following assumptions: It is assumed that the values of the input parameters for the model, that is the expected yield of risky and riskless securities and standard deviation of the former, are correct. While the problem of forecasting input parameters is not a focal point of this study, it can nevertheless tend to be solved. Similarly, it is assumed that the standard deviation remains constant with the passage of time – a common assumption but nonetheless a simplification. The assumption also applies that the standard deviation increases, as the horizon is extended, by the root of the number of years. It is further assumed that a correlation of zero exists between the returns on riskless and risky securities, the result being that possible correlation effects between the two asset classes are disregarded. A basic assumption of the model is that the time horizon for pension provision and pension benefits can be forecast. For the pension provision horizon, this is comparatively easy to forecast by setting, for example, a required age at which the pension is to start being drawn. More difficult to forecast is the horizon for the benefit stage, since the time of death must be known. The problem can be solved – as provided for in the relevant statutory regulations – by means of a drawdown pension up to the age of 85, followed by an annuity. It should further be noted that the assumptions apply for the Tobin separation theorem. Construction of the portfolio is based on the assumption of normal distribution. The assumption of normally distributed stock returns is customary and accepted as a simplifying assumption with the use of logarithmic returns.
following parameters: For the **shortfall probability** values of one or five percent are plausible, by analogy with common parameters for Value at Risk. The first value used here therefore means that there is a one percent probability of the pre-specified minimum return target being undershot and a 99 percent likelihood of its being met or exceeded. For the minimum return a parameter of zero percent is to be recommended, as this takes into account the statutory requirement of preservation of the nominal value of the capital invested for what is popularly known as the “Riester pension” (named after the German labour minister responsible for the introduction of state-incentivised occupational and private pension provision). Further input parameters are a riskless return of five percent, an expected return of ten percent on risky assets and standard deviation of the risky returns by an estimated 20 percent. Furthermore, because the possibility of borrowing is ruled out, the equity weighting has been limited to 100 percent, with the positive consequence that for very long horizons (22 years and above) the shortfall probability falls to below one percent. The outcome of asset allocation is depicted in the following figure.

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42 A return of approximately ten percent is frequently quoted as the longstanding average return on equities; see e.g. Stehle, R. “Renditevergleich”, 1999, p. 1 ff.; Stehle, R. / Huber, R. / Maier, J. “DAX”, 1996, p. 277 ff. Concerning the returns resulting from equity portfolios with different investment strategies, see Helm, L. i. a. “Aktiendepots”, 1998, p. 14 ff.

43 Values around twenty percent are quoted as the average of standard deviations of past stock returns; see e. g. Gibson, R. C. “Balancing Financial Risk”, 1990, p. 83. The value of twenty percent increases by the root of the number of years.
As intuitively to be expected, the equity weighting rises for new investment in pension provision the longer the horizon, the underlying positive horizon effect being evident from the results. The proportion of riskless assets – defined as one less the equity weighting – falls as the horizon increases.

Key to asset allocation is the context in which the equity percentage is considered. It is often wrongly assumed that the equity weighting must refer to the entire portfolio, as a result of which stocks are gradually switched into fixed-income investments. The intuitive reasoning behind this is that owing to the positive horizon effect the risk of stocks can be “afforded” in younger years, but that as the horizon shortens the risk grows too high, so that equities must be shifted into fixed-income assets. At first sight this argument appears convincing, but on closer consideration it is not logical. By selling stocks early and investing the proceeds in, say, money market funds, a time horizon effect is prevented from unfolding or cannot fully unfold, because the horizon is “artificially” truncated by the premature divestment. Following the model discussed in the above, it is therefore correct to hold on to stocks once purchased, pursuing a buy-and-hold strategy until the end of the investment horizon – for example until retirement. Only then is the full duration of the horizon used, so that the positive horizon effect can kick in to the full. The consequence is that the equity weightings described refer to each new investment in pension provision, whereas the assets already invested are not restructured until the end of the time horizon. This approach underscores the practicability of the model: elaborate ongoing portfolio restructuring is not necessary, the only structuring required is in each amount newly invested for pension provision subject to the recommended weightings for equities and riskless securities.

The following summarises the advantages of the model based on the zeroth order lower partial moment:

- The restriction of asset allocation to two asset classes, which can be represented for example by index funds and money market funds, can be substantiated theoretically by the Tobin separation theorem and enhances the model’s practical applicability, so that all pension provisioning can be carried out using these two products.
- Asset allocation takes account of a positive horizon effect in a sensible way.
Given plausible input parameters, asset allocation leads to realistic results.44 Investors’ individual risk preferences can be taken into account.

The possibility is also given of including the financial repercussions of inflation in asset allocation by raising the pre-specified minimum return target by the percentage rate of inflation. The result is de facto hedging against inflation effects.

The buy-and-hold strategy employed keeps the administrative work required low, so that excessively high management costs do not act as a stumbling block to practical application of the model for pension provision.

The statutory requirement of guaranteed preservation of the nominal value of the capital invested is approximately satisfied, since there is maximum likelihood that the capital employed can be disbursed again at a set point in time, given a specified return of zero percent and very low maximum shortfall probability tolerance.

3.2.2 A glance at alternative asset allocation models

An alternative to the LPM0 model described are models satisfying the assessment criteria implicitly mentioned in the above to a similarly positive extent. An appropriate enhancement is provided by a model geared not to the shortfall probability, but to the shortfall expectation, in other words to the first lower partial moment. Using this model, the amount of potential shortfalls in return is also included in the calculation.45

Another asset allocation possibility is a combined investment of zero bonds and stocks. The amount invested as retirement income is also the amount repayable on maturity that must be guaranteed under the requirements of the “Riester pension”. The present value of the amount repayable on maturity is invested in zero bonds, guaranteeing preservation of the capital originally invested. The spread between the present value and amount repayable on maturity can be invested in high-yield assets, such as stocks. This spread rises as the horizon lengthens, since the present value is lower for longer investment horizons than for shorter periods. Higher equity weightings can therefore be chosen for longer horizons; in consequence a positive

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44 If the equity weighting is not limited to 100 percent, the maximum equity weighting works out at approx. 140 percent for 30-year horizons, which can be considered comparatively realistic.

horizon effect is also evident with this model,\textsuperscript{46} which, moreover, has the advantage
that the capital originally invested can be repaid not only with a high degree of probability, as for the LPM0 model (or with a low shortfall expectation, as for the LPM1 model), but indeed with certainty.

### 3.3 Practical applicability of the LPM0 model for pension provision

In principle asset allocation models are of relevance for occupational pension plans and private provisioning in Germany, because under the pay-as-you-go (PAYG) method used to fund the statutory pension insurance scheme capital is not accumulated and cannot therefore be structured. As already evident from positive assessment of the LPM0 model, basically this is suitable for use in both occupational and private pension schemes. To take real advantage of the horizon effect, asset allocation should be tailored individually to each pension saver, and given the simple asset structuring this is quite practicable. All that is needed for practical implementation of the model are two mutual funds – for example, an equity fund investing internationally as a risky asset class and a money market fund as a risk-free asset category. For time horizon-dependent asset allocation, the two asset classes must be weighted differently for each amount newly invested in the pension plan. For example, 30 different asset structures\textsuperscript{47} must be designed for 30 age categories, as illustrated in the relevant figure in Chapter 3.2.1. These 30 asset structures can be embodied by 30 “funds of funds”, each containing the two mutual funds (equity fund, money market fund) in their respective horizon-dependent weightings. A fund of funds thus exists for each time horizon, realising the exact period-dependent asset allocation. Depending on the pension saver’s investment horizon, he will invest in one of these 30 funds of funds, pursuing a buy-and-hold strategy, as also described in Chapter 3.2.1 A 40 year-old investor intending to retire at 60 would thus have an asset weighting of 92.25 percent equity funds and 7.75 percent money market funds under the LPM0 model. In line with the recommendations given under this model, these weightings are altered up to the age of 60 only by the performance of the mutual funds, and not by active restructuring of the portfolio of assets serving to form the retirement pen-

\textsuperscript{46} For an exact description of the model cf. ibid., p. 166 ff.

\textsuperscript{47} These and the following comments are based on horizon-dependent asset allocation. This approach can be broadened, as described above, by adding an element of risk preference-dependency.
sion. 87.86 percent of the pension saver’s contribution paid at the age of 41 will be invested in equity funds and 12.14 percent in money market funds, and so forth. This strategy is pursued until the 59 year-old individual invests only 12.02 percent of the pension contribution in equity funds and 87.98 percent in money market funds. With this approach there is a 99 percent likelihood of being able to pay out at least the capital originally invested, under the conditions set out in Chapter 3.2.1 to the then 60 year-old saver who now wishes to draw on his retirement savings.

It is evident from the above that asset allocation can be used to accumulate retirement savings for even a large majority of people without excessive management input. This makes it fundamentally eligible for occupational pension plans, one type of which are Pensionsfonds. Key to the investment of retirement savings in Pensionsfonds is the possibility of placing up to 100 percent of the capital in equities, which the relevant regulations permit. 48

But asset allocation can be used equally as freely for the build-up of private retirement income. The pivotal criterion for state subsidisation of private retirement saving in Germany – the guarantee of preservation of the nominal value of the capital invested – can be satisfied because, depending on the choice of parameters, the capital originally employed can be repaid through the model with a degree of probability bordering on certainty. Other requirements for the Riester pension, such as current capital contributions and benefits not to begin before 60, and also the regulations governing the choice of pension product, can all be complied with. 49 The model can also be applied to a drawdown pension at retirement age; in accordance with the statutory regulations this pays out capital from the retirement savings up to the age of 85, leaving a balance that is then annuitised when the pensioner is 85 to hedge the risk of longevity.

48 Cf. Section 4 Paragraphs 2 and 3 of the Pensionsfonds Capital Investment Regulations (Pensionsfonds-Kapitalanlagenverordnung).

49 For the regulations see Section 1 Paragraph 1 AltZertG.
4. Conclusion

This paper set out by showing that adequate financial provision for old age can only be achieved with occupational and private pension plans, not through the PAYG statutory pension scheme. The main feature of provision based on the principle of full funding is the long time horizon. It has been explained here that the exceptionally long horizon for retirement savings impacts clearly on the risk-return properties of the asset class of stocks and can be turned to advantage for asset allocation. Ultimately, the performance of retirement investment is enhanced, in that the heavy weighting of stocks with long investment horizons holds out the prospect of high returns on relatively reduced risks. The model presented in this article takes account of the positive horizon effects, also caters to other retirement provisioning aims and can be used directly in practice for pension provision in Germany.
Appendix

Calculation of the shortfall probability

The shortfall probability is calculated using the following formula:

\[ \text{LPM0} = N \left( \frac{\tau_t - \mu_t}{\sigma_t} \right) \]

with

\( \text{LPM0} = \) shortfall probability
\( N(.) = \) area under the standard normal distribution curve
\( \tau_t = \) minimum return for the time horizon \( t \)
\( \mu_t = \) expectation of stock returns for the time horizon \( t \)
\( \sigma_t = \) standard deviation of stock returns for the time horizon \( t \)

Calculation of shortfall expectation

Asset allocation is based on the formula for calculation of the first lower partial moment:

\[ \text{LPM1} = \int_{-\infty}^{\tau} \left( \tau - r_p \right) f(r_p) \, dr_p \]

for \( \tau < r_f \).

with

\( r_p = \) returns on the portfolios
\( \tau = \) pre-specified minimum return target
\( \text{LPM1} = \) shortfall expectation

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50 Solution of the integral of LPM1 in the following formula is carried out by the Mathematica programme. The solution obtained has been subjected to plausibility tests. All test results were plausible.

51 This condition must be satisfied so that the pre-specified minimum return target can be obtained even with low equity weightings – in other words, in extreme cases through riskless investments only.
Analytical determination of asset allocation with the LPM0 model

The shortfall line – also described as the Telser constraint – follows the equation

\[ \mu_T = \tau - z \times \sigma_T \]

with
- \( \mu_T \) = expected return on the Telser portfolio
- \( \sigma_T \) = standard deviation of stock returns in the Telser portfolio
- \( \tau \) = pre-specified minimum return target
- \( z \) = incline of the shortfall lines

Since, at the same time, the Telser portfolio lies on the capital market line, it can also be depicted as a combination of riskless and risky investment possibilities, to which the following correlations apply:

\[ \mu_T = x \times \mu_A + (1 - x) \times r_f \]

\[ \sigma_T = x \times \sigma_A \]

with
- \( x \) = stock weighting
- \( r_f \) = risk-free return
- \( \mu_A \) = expectation of stock returns
- \( \sigma_A \) = standard deviation of stock returns

The last two equations can then be inserted into the equation of the shortfall lines, giving the following optimum stock weighting in the Telser portfolio:

\[ x = \frac{\tau - r_f}{z \times \sigma_A + \mu_A - r_f} \]

on condition that \( \tau < r_f \).

Analytical determination of the optimum stock weighting allowing for the time horizon is carried out using the formula

\[ x(t) = \frac{\tau - r_f}{z \times \sigma_A(t) + \mu_A - r_f} \]

on condition that \( \tau < r_f \).

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52 This condition is necessary because a minimum return higher than the riskless rate of return cannot be obtained with very low equity weightings.
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