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Capital Controls, Exchange Rate Volatility and Risk Premium

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Capital Controls, Exchange Rate Volatility, and Risk Premium

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July 2002

Abstract
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Keywords: Capital Controls, Capital Flows, Risk Premium

JEL–Classification: F32, F41

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1 Motivation of the Paper

Recent turbulence in world financial markets has reanimated the interest in capital controls as a means to stabilize the foreign exchange system of a country. Among such controls, the Tobin tax represents the most prominent form. In his Janeway Lectures at Princeton Tobin suggested in 1972 to levy a uniform international tax on all spot transactions involving the conversion of one currency into the other (published in Tobin, 1974). There are several other forms of capital controls that ultimately work like a tax, too. For example, the government may impose a special minimum reserve requirement on capital inflows or a tax on the return of investments associated with capital inflows. Since all these controls add to the cost of currency trading or reduce the net return of capital inflows, proponents of such measures argue that they can reduce the volume of destabilizing short-term capital flows and can, thereby, decrease volatility in foreign exchange markets and augment exchange rate stability. Partly, such measures are also discussed in the context of the "new financial architecture" aiming at more stability of world financial flows and the global economy (e.g. Blecker, 1999).

While other forms of capital controls are generally not considered totally unrealistic to be implemented, the verdict on the Tobin tax is fairly negative. Two main types of problems are associated with this tax. First, there are operational difficulties. These point to the questions of political and technical feasibility and the possibilities to evade the tax by shifting transactions to other countries where such restrictions are not imposed. Second, there are problems directly connected with the effectiveness of this policy tool: does it really reduce the volatility of the exchange rate and does it cause negative side-effects like reduced growth prospects? These questions do not just apply to the Tobin tax but are of interest in the context of other capital controls as well. However, the theoretical literature examining these effects is fairly limited. This paper addresses these effects by examining how capital controls that aim at reducing short-term

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1Since the Tobin Tax is an international tax, the result of the discussion about the operational problems of this type of capital control is basically that the costs for finding a worldwide consensus on all the operational problems would be prohibitively high (Spahn, 1996, p. 25). For an overview of the discussion, see also Haq (1996), and Raffer (1998).

2One reason for this is that such capital controls are often directly associated with a Tobin tax and was rejected by many from the beginning. "In fact, one might say it sank like a rock. The community of professional economists simply ignored it. The interest that occasionally arose came from journalists and financial pundits. It was usually triggered by currency crises and died out when the crisis passed from the headlines." (Tobin, 1996).
speculation in foreign exchange markets and that can be modeled as a tax on international capital flows affect exchange rate volatility, the risk premium of domestic assets, and the domestic interest rate.\(^3\)

Jeffrey Frankel (1996, p. 71 – 72) uses a simple static monetary model where the exchange rate is determined by money supply and demand. Like in our model the expectations of the investors and the speculators are modeled such that investors reduce and speculators extend the volatility of the exchange rate. Even though Frankel does not explicitly introduce a Tobin Tax into the model he comes to the conclusion that such a tax reduces the variability of the exchange rate.

Buch, Heinrich, and Pierdzioch (1999) use a Dornbusch style framework and assume that the relative importance of technical traders (or chartists) in the foreign exchange market depends on the magnitude of the Tobin Tax. In this context, the chartists are the "bad guys" who trade in a way that destabilizes the exchange rate. The "good guys" belong to a group of rational agents who know the equilibrium exchange rate, consider the destabilizing expectations and actions of the "bad guys", and also know the dynamics of the system. Rational agents stabilize the exchange rate.

The implementation of a transaction tax increases the domestic interest rate. Therefore, foreign investors who have to pay the tax burden get compensated by a higher domestic interest rate. This could result as for example Frenkel et al. (2001) show in a lower growth potential for the domestic economy. The benefits of the Tobin Tax can be seen in the change of the slope of the saddle path which gets flatter. Hence, overshooting of the exchange rate and thereby the exchange rate volatility due to future monetary shocks can be reduced.

Kölck and Rübesamen (2000) assume that the exchange rate path is influenced by the arrival of news about fundamentals. If new information occurs, market participants react to these news and change their exchange rate expectations. They assume that the news process consists of a common component \(\phi_t\) equal for all traders and a trader specific ‘disagreement’ component \(\Psi_{it}\). The change of the spot exchange rate \((s_t)\) is equal to

\[
\Delta s_t = \phi_t + \Psi_t
\]  

\(^3\)Capital controls that outright prohibit certain international capital flows are not examined in this paper.
with
\[ \Psi_t = \frac{1}{n} \sum_{i=1}^{n} \Psi_{it}, \] (2)
where \( n \) is the number of traders in the foreign exchange market. As they point out, a Tobin Tax does not influence the occurrence of new information but it reduces the speculative activity of the trader due to higher transaction costs. Traders act on the same news with a lower expected exchange rate change. They model this effect by introducing a factor \( \tau \) into equation (1) that reduces the change in the spot rate. The variable \( \tau \) indirectly represents the Tobin Tax. The higher the tax rate (\( \tau \)) is, the lower the change in the exchange rate due to the occurrence of new information.

\[ \Delta s_t = \tau (\phi_t + \Psi_t) \quad \text{with} \quad 0 < \tau < 1. \] (3)

The variance of the expected exchange rate is equal to

\[ \text{Var}[\Delta s_t] = \tau^2 \left( \sigma_{\phi}^2 + \frac{\sigma_{\Psi}^2}{n} \right), \] (4)

where \( \sigma_{\phi}^2 \) (\( \sigma_{\Psi}^2 \)) is the variance of \( \phi \) (\( \Psi \)). Since \( 0 < \tau < 1 \), the expected exchange rate volatility is reduced by the Tobin Tax. Nevertheless, this finding cannot be regarded as the end of the story. As Kolck and Rubesamen point out verbally, their model neglects the influence of the Tobin Tax on the overall number of foreign exchange traders. If the number of foreign exchange traders (\( n \)) is reduced by the introduction of the Tobin Tax a further factor influencing the variance of the exchange rate has to be considered in equation (4) which counteracts the primary effect.

The rest of the paper is organized as follows. The second part explains the structure of the model. The third part analyzes the effects on the risk premium and the interest rate induced by capital controls that can be modeled as a tax on international capital flows. The last chapter presents the summary and the main conclusions.

2 The Model

Our model extends the framework suggested by Jeanne and Rose (2002) who examine the effects of noise trading on exchange rate volatility under different exchange rate regimes. The approach combines a macroeconomic
monetary model with a standard microeconomic noise trader model. The actors in this model can be divided into two different groups of traders. One group consists of informed investors who have an accurate knowledge of the mechanism which determines the exchange rate and, therefore, base their decisions on rational expectations. The other group of investors consists of noise traders who misperceive the first moment of the return on domestic bonds. This assumption regarding the defect of a noise trader can be considered as the standard assumption in the noise trader literature which was introduced by DeLong et al. (1990).

We extend the model of Jeanne and Rose (2002) by introducing a restriction on international capital flows that takes the form of a price control and can, thus, be modeled as a tax on the return of domestic assets. Again, an example is the Tobin tax but our analysis is not restricted to this specific measure. We assume that the government, in implementing the capital control, is able to discriminate between rational acting fundamentalists and somewhat irrational acting noise traders. This is in line with Buch, Heinrich, and Pierdzioch (1999) as well as with Frenkel et al. (2001). Our analysis focuses on the effect of the capital control on the average risk premium. Through the interest parity condition, the risk premium on domestic assets affects the domestic interest rate and can, thus, have a significant effect on real capital stock formation and growth of the economy. Although this argument was made by Frenkel et al. (2001), it is still an open question in which direction capital controls move the risk premium given that capital controls might reduce the access of the economy to international financial markets, which increases the risk premium, and, at the same time, might reduce volatility which dampens the risk premium.

The monetary macroeconomic framework consists of an equilibrium condition for the domestic (5) and the foreign (6) money market and a purchasing power parity condition (7):

\[ m_t - p_t = -\beta r_t. \]  (5)

\[ m^* - p^* = -\beta r^*. \]  (6)

\[ s_t = p_t - p^* + \epsilon_t. \]  (7)

The variable \( m \) denotes the domestic money supply, \( p \) the price level and \( s \) the spot exchange rate. While the coefficient \( \beta \) represents the semi-interest elasticity of money demand, \( r \) stands for the domestic interest
rate. Foreign variables are labeled by an asterisk. All variables except the interest rates are in logarithms. The level of full employment income is normalized to unity so that the log value of the income drops out of the money demand function. The purchasing power party is satisfied on average, so that the exchange rate \( s \) is simply the ratio of the price levels plus an \textit{i.i.d.} normal shock \( \epsilon \). We postulate that the home country is a small open economy and the foreign country is large. Furthermore, we assume that the foreign economy is in a steady state with constant values of the money supply, the price level and the interest rate. Therefore, the time index for the foreign variables is neglected in the following analysis.

Solving (5) and (6) for the price levels and inserting them in equation (7) leads to the exchange rate equation

\[
 s_t = (m_t - m^*) + \beta(r_t - r^*) + \epsilon_t. \tag{8}
\]

This equation can be regarded as the standard exchange rate function of the monetary model. The microeconomic framework is modeled as an overlapping generations model. All traders live for two periods. In the first period, both types of investors have to balance their wealth between domestic and foreign bonds. In the second period, investors convert the proceeds from their investment in the domestic economy to the foreign currency in order to use it for consumption. Both types of investors are endowed with the same amount of initial wealth in foreign currency and possess the same utility function.

An investor \((j)\) maximizes the following CARA utility function with respect to the quantity of money \((b^j_t)\) invested in the domestic bond market,

\[
 \max_{b^j_t} U^j_t = E_t^j \left( -\frac{1}{e^{\alpha W_{t+1}^j}} \right). \tag{9}
\]

where \( W_{t+1}^j \) denotes the end-of-life wealth of investor \(j\) and the parameter \(\alpha\) is the coefficient of absolute risk aversion. The wealth at time \(t+1\) is equal to:

\[
 W_{t+1}^j = (1 + r^*)W_t + b^j_t \rho_{t+1}. \tag{10}
\]

Trader’s \(j\) end of life wealth is influenced by two factors reflected by the two terms on the right-hand side of equation (10). The first term is equal

\footnote{The Constant Average Risk Aversion utility function is frequently used in the noise trader literature. See for example Osler (1998).}
to the wealth at the beginning of life times the foreign interest rate. The second term is equal to the wealth generated from a domestic investment which depends on the amount of money \( (b) \) invested in domestic bonds and the (ex-post) excess return per unit of a domestic bond \((\rho_{t+1})\) which can also be interpreted as the risk premium on domestic bonds and which is given by:

\[
\rho_{t+1} = r_t - r^* - (s_{t+1} - s_t) - \tau; \quad \tau \geq 0.
\]  

(11)

The excess return per unit of domestic bonds is equal to the interest rate differential and the gain or loss from the change in the exchange rate. The new element of the model is the introduction of the capital control for which we assume that it imposes additional costs to capital flows and, thus, exerts similar effects as a tax. We model the control as a tax \((\tau > 0)\) on the return on domestic financial investments. As capital controls often target short-term capital flows because of their assumed higher volatility, it could well be possible that they can be designed in such way that they affect the return on a short-term investment significantly more than the return on a long-term investment. Thus, one may expect that the tax matters more for short-term investment decisions than for long-term investment decisions. However, we do not make this distinction in equation (11) but rather focus on a general formulation of the effects of the capital control on the return of capital inflows. The formulation of equation (11) also implies that, in this paper, we do not have a tax equivalent. Thus we exclude, for example, outright prohibitions of certain capital flows.

We assume that the two types of traders whom we take into account can be distinguished in the following way. Informed investors have rational expectations and are homogeneous within their group. Therefore, all informed investors possess the same expectations regarding the risk premium and the variance of the risk premium.

\[
E^j_t(\rho_{t+1}) = E_t(\rho_{t+1}). 
\]  

(12)

\[
Var^j_t(\rho_{t+1}) = Var_t(\rho_{t+1}). 
\]  

(13)

Noise traders have imperfect knowledge. Although they perceive the second moment of returns correctly, they misperceive the first-moment:

\[
E^j_t(\rho_{t+1}) = (1 - \tau)\gamma(\bar{\rho} + \nu_t). 
\]  

(14)

The noise in the market is common across the group of noise traders and there is no private information in the model as for example in Tauchen/Pitts
The variable $\bar{\rho}$ denotes the unconditional mean of the excess return which can also be viewed as an average risk premium. Equation (14) reflects the belief that the government is able to identify the noise traders by special characteristics and that the capital control can reduce the influence of these traders on the exchange rate. The higher the value of the parameter $\gamma$, the higher is the decline in the noise traders’ risk perception due to capital controls. The noise term $\nu_t$ is a stochastic $i.i.d.$ normal shock common across all noise traders. Hence the group of noise traders is homogeneous in itself, too. We assume that the size of noise traders’ misperception is proportional to the true unconditional variance of the exchange rate:\(^5\)

$$\text{Var}(\nu) = \lambda \text{Var}(s) \quad \text{with} \quad \lambda > 0.$$ (15)

The average risk premium is equal to the average interest rate differential minus the tax rate

$$\bar{\rho} = \bar{r} - r^* - \tau.$$ (16)

Together with equation (8) this implies that the average exchange rate is equal to the average money supply differential, adjusted for the average risk premium and the tax rate.

$$\bar{s} = \bar{m} - m^* + \beta \bar{\rho} + \beta \tau.$$ (17)

Maximizing (9) is equal to maximizing the mean variance objective function\(^6\)

$$\Omega = E_t^j(W_{t+1}^j) - \frac{\alpha}{2} \text{Var}_t^j(W_{t+1}^j).$$ (18)

with respect to $b_t^j$, i.e. the amount invested by investor $j$ in domestic bonds. The demand of an individual trader ($j$) for bonds in the domestic currency is given by (see appendix A2):

$$b_t^j = \frac{E_t^j(\rho_{t+1})}{\alpha \text{Var}_t^j(\rho_{t+1})}. \quad \text{(19)}$$

Taking into consideration equations (12), (13), and (19) the utility maximizing demand of a rational investor for domestic bonds is given by

---

\(^5\)As Jeanne and Rose (2002), p. 545 point out, assuming that $\text{Var}(\nu)$ is constant is implausible, because it would imply that noise traders expect the exchange rate to be stochastic when it is in fact constant.

\(^6\)See appendix A1 for the derivation.
\[ E_t(\rho_{t+1})/[\alpha Var_t(\rho_{t+1})] \] while the demand of a noise trader can be derived from equation (14) and (19) as \((1 - \tau)^\gamma(\bar{\rho} + \nu_t)/[\alpha Var_t(\rho_{t+1})]\). To generate an equilibrium in the domestic bond market, demand has to equal a fixed supply \((\bar{B})\). The demand for domestic bonds consists of the demand of the group of rational agents and the demand of the group of noise traders, where \(N (n)\) is the number of rational agents (noise traders) entering the domestic bond market. Equilibrium in the bonds market is given by

\[
\bar{B} = \frac{NE_t(\rho_{t+1}) + n(1 - \tau)^\gamma(\bar{\rho} + \nu_t)}{\alpha Var(s)}. \tag{20}
\]

Taking the expectations of (20) at time \(t - 1\) leads to

\[
\bar{B} = \frac{N\bar{\rho} + n(1 - \tau)^\gamma\bar{\rho}}{\alpha Var(s)}. \tag{21}
\]

Solving for the average risk premium \((\bar{\rho})\) gives

\[
\bar{\rho} = \frac{\alpha \bar{B}}{N + n(1 - \tau)^\gamma} Var(s). \tag{22}
\]

The average risk premium varies positively with the risk aversion parameter \((\alpha)\), the supply of domestic bonds \((B)\) and the variance of the exchange rate \((Var(s))\). As shown in appendix A3 the deviation between the exchange rate at time \(t\) from its mean value is equal to

\[
(s_t - \bar{s}) = \frac{1}{1 + \beta} \left[ m_t - \bar{m} + \epsilon_t - \frac{n}{N} \beta(1 - \tau)^\gamma \nu_t \right]. \tag{23}
\]

The deviation of the exchange rate at time \(t\) from its mean value depends on the money supply shock, the purchasing power parity shock \((\epsilon_t)\), as well as on the size of the noise trader specific shock \((\nu_t)\). The distortion effect of the noise trader specific shock on the exchange rate will be the larger the larger the number of noise traders \((n)\). A positive shock will according to equation (14) increase the expected excess return of the noise trader group. Therefore, they will make a higher investment in the domestic bond than on average, which results in an appreciation of the domestic currency. Taking the variance of (23) leads to (see appendix A4)

\[
Var(s) = \frac{Var(m + \epsilon)}{(1 + \beta)^2 - [\beta \frac{n}{N}(1 - \tau)^\gamma]^2 \lambda}. \tag{24}
\]
The variance of the exchange rate increases with the fundamental variance of the economy ($Var(m + \epsilon)$). The introduction of capital controls ($\tau > 0$) induces two effects which influence the risk premium in opposite directions. First, it leads to a larger denominator in (24), which reduces the variance of the exchange rate and tends to decrease the risk premium in equation (22). Second, capital controls reduce the denominator in equation (22) due to the fact that they reduce the number of noise traders in the market. Given the fixed supply of domestic assets, the remaining group of rational agents needs an incentive to invest a higher share of their wealth in domestic bonds. The risk premium has to increase so that the rational agents are willing to hold a larger share of domestic bonds. As there are two effects of a capital control on the risk premium, the next section examines the optimum tax level of the capital control.

3 Policy Implications: Computing the Optimal Tax Rate

In order to derive the value of that minimizes the risk premium, we combine equations (22) and (24). We define $\xi = (1 - \tau)$ and get:

$$\bar{\rho} = \frac{\alpha B}{N + n\xi^\gamma} \frac{Var(m + \epsilon)}{(1 + \beta)^2 - \left[\frac{\beta n^\gamma}{N^\gamma}\right]^2 \lambda}.$$  \hspace{1cm} (25)

Computing the derivative of $\bar{\rho}$ with respect to $\xi$, we get for the decisive part of the first derivative:

$$\frac{\partial \bar{\rho}}{\partial \xi} = - \left[\gamma n(1 + \beta)^2 \xi^{\gamma-1} - 2\gamma \lambda \beta^2 \frac{n^2}{N^\gamma} \xi^{2\gamma-1} - 3\gamma \lambda \beta^2 \frac{n^3}{N^2} \xi^{3\gamma-1}\right] = 0.$$  \hspace{1cm} (26)

Solving for the optimal tax rate leads to (see appendix A5)

$$\hat{\tau} = 1 - \left[-\frac{N}{3n} + \frac{N}{3n} \sqrt{1 + \frac{3(1 + \beta)^2}{\lambda \beta^2}}\right]^{1/\gamma}.$$  \hspace{1cm} (27)

The optimal capital control or tax rate depends on the relative size of the group of noise traders, on the interest rate elasticity of money demand and on the ratio $\lambda$ of the noise term variance and the exchange rate variance. For example, the larger the noise trader group, the larger is the distortion due to these traders’ misperception of asset returns. Hence, the capital control has to be higher in order to limit the noise trading activity to
an optimal level. At lower levels of the capital control, the dampening
effect on exchange rate volatility more than offsets the effect of a reduction
in the risk bearing capacity of the market. In case that equation (27)
generates a negative value, the optimal control is zero meaning that even
at a low level of $\tau$, the beneficial effects cannot outweigh the negative effects.

We now examine the optimal capital control level for a specific set of
parameter values.\footnote{The parameter values for the simulation are as follows: $\alpha = 0.9$, $B = 300$, $Var(m + \epsilon) = 1$, $\beta = 0.4$, $\lambda = 1$ $\gamma = 1$, $n = 80$, $N = 100$.} Figure 1 illustrates the effect of a capital control on the
average risk premium ($\bar{\rho}$) and the variance of the exchange rate ($Var(s)$).
This serves to illustrate the effects of the number of noise traders on the
optimal capital control and the linkage between the interest rate induced
by the capital control and the resulting risk premium. Figure 1 illustrates
the effect of different capital control levels on exchange rate volatility, the
risk bearing aspect described above and the risk premium. In each case,
we illustrate the effect of the size of the noise trade group by assuming two
alternative number of noise traders in the market. Diagram (a) of Figure
1 highlights three features. First, in the model developed in the previous
section and with no capital controls in place, exchange rate volatility is
higher with a larger noise-trader group. Second, capital controls are indeed
able to reduce exchange rate volatility. Third, the decreasing effect is the
stronger, the larger the group of noise traders is relative to the group of
fundamentalists.

Diagram (b) illustrates the disadvantage of reducing the activities of noise
traders for the economy. As their importance declines, other market partic-
ipants have to hold domestic bonds. This affects how market participants
perceive the risk of these assets and, in our simulations, is reflected in what
could be called a risk-bearing factor. Formally, it is the first term on the
right hand side of equation (22). As the capital control level rises, this risk
increasing factor rises.
Figure 1: The Effect of a Capital Control on the Average Risk Premium and the Exchange Rate Volatility

Solid (dashed) line: Number of Noise Trader $n=60$ ($n=80$)
Diagram (c) shows the combined effect of lower exchange rate volatility induced by capital controls and increased risk bearing by other markets participants on the risk premium. The dashed line (relatively large number of noise traders) shows that the implementation of relatively low capital control levels reduces the risk premium but relatively high capital control levels lead to an increase in the risk premium. Thus, there is a positive optimal capital control level. However, with a lower number of noise traders (indicated by the solid line), the combined effect leads to a monotonous increase of the risk premium as capital control levels rise. Hence, the optimal capital control is zero.

Figure 2 illustrates the relationship between the number of noise traders \((n)\), the tax equivalent of a capital control \((\tau)\) and the risk premium \((\bar{\rho})\). As the diagram reveals, the optimal capital control level, i.e., the one that minimizes the risk premium, varies with the size of the noise trader group. The larger the noise trader group is, the larger is the distortion due to the misperception of the returns. Hence, the taxing unit has to increase the tax rate with an increase of the size of the group to limit the noise trading activity to an optimal level.

One may argue that the minimal risk premium is not the objective of the policy maker. Since investment decisions depend more on the interest rate than on the risk premium, one may argue that this should also be the focus of the policy maker. A slight rearrangement of equation (16) reviews, that

\[
\bar{r} = \bar{r}^* + \bar{\rho} + \tau.
\]  (28)
Figure 2: The Effect of the Size of the Noise Trader Group and the Capital Control Level on the Average Risk Premium

The size of the domestic interest rate not only depends on the size of the risk premium but also on the size of the tax rate. Therefore, it is not guaranteed that a minimal risk premium will also result in a minimal domestic interest rate. Although it is possible to derive the first derivative of the interest rate with respect to $\tau$ it is not possible to isolate the optimal size of the tax rate afterwards. Therefore, we simulate the relationship of equation (28) to show the relationship between the domestic interest rate and the tax rate.

Figure 3 shows the same qualitative characteristics as the diagram in Figure 2. This means that, in general, the optimal capital control varies positively with the size of the noise trader group. However, Figure 3 reveals that a relatively small size of the noise trader group can lead to a situation in
which the optimal capital control level is zero. In this case, the risk and interest rate reducing effect of lower exchange rate volatility generated by even a small capital control is more than offset by the negative effects of a reduction in the risk bearing capacity of the market. As a result, combining the risk-minimizing combinations of the number of noise traders and the capital control level yields a line that, if projected to the bottom plane, runs from the front left area to the back right corner.

4 Conclusion

The combination of a monetary macroeconomic model and a microeconomic noise trader model gives insight to the effect of a capital control that can be modeled as a tax for a small open economy. While Buch et al. (1999)
as well as Frenkel et al. (2001) postulate that the risk premium increases with the implementation of a Tobin Tax, Kolck and Rübesamen (2000) show that this kind of tax is able to reduce the exchange rate variability, leading to a decrease of the risk premium.

We are able to identify two effects which work in opposite directions: On the one hand, a capital control lowers the variability of the exchange rate and reduces the risk premium as well as the interest rate of a small open economy. This effect seems to be the vulnerable effect of the tax, because it amplifies the growth potential of the economy under consideration. On the other hand, a capital control reduces the number of noise traders and thereby the risk bearing capacity of the market. As a consequence, the group of rational investors as well as the remaining noise traders have to take a larger stake of the risky asset. As their risk exposure increases, they demand a higher risk premium which increases the interest rate and therefore, lowers the growth potential of a small open economy. These two effects which work in opposite direction is at the heart of our model and can be regarded as the major contribution of this paper.

As shown in the last section, we are able to derive a formula for the optimal tax rate, which minimizes the average risk premium. This optimal tax rate varies with the size of the noise trader group: The more noise traders act on the market the higher the optimal tax rate, cause the dampening effect of exchange rate volatility will more than offset the effect of a reduction in the risk bearing capacity of the market.
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Appendix

Appendix A1: The equivalence between maximizing the expected utility and the risk equivalent

The following computations are performed to show the equivalence between maximizing the expected utility and the corresponding risk equivalent.

\[
E[e^{tx}] = \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx
\]

(29)

Combining the exponents we get

\[
E[e^{tx}] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2 + tx}{2\sigma^2}} dx.
\]

(30)

Factoring out \((-1/2\sigma^2)\) yields

\[
E[e^{tx}] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}[x-(\mu+\sigma^2t)]^2 - \frac{t^2}{2}} dx.
\]

(31)

\[
E[e^{tx}] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}[(x-(\mu+\sigma^2t))^2 - (\mu+\sigma^2t)^2 + \mu^2]} dx
\]

(32)

and

\[
E[e^{tx}] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}[(x-(\mu+\sigma^2t))^2 - (\mu+\sigma^2t)^2 + \mu^2]} dx
\]

(33)

Due to the fact that the first exponent is independent from \(x\), we can write

\[
E[e^{tx}] = e^{\mu t} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-(\mu+\sigma^2t))^2}{2\sigma^2}} dx.
\]

(34)

The expression under the integral is the normal distribution function and is therefore equal to unity if we redefine \(x \sim N(\mu + \sigma^2t, \sigma^2)\). Therefore, we get

\[
E[e^{tx}] = e^{\mu t + \frac{1}{2}\sigma^4 t^2}.
\]

(35)
Appendix A2: The demand for domestic bonds

Inserting equation (10) into equation (18), we get:

$$
\Omega = E_t^l \left[ (1 + r^*)W_t + b_t^l \rho_{t+1} \right] - \frac{\alpha}{2} Var_l^l \left[ (1 + r^*)W_t + b_t^l \rho_{t+1} \right].
$$

(36)

Due to the fact that only $\rho_{t+1}$ is stochastic, we can rewrite this expression as

$$
\Omega = E_t^l \left[ (1 + r^*)W_t + b_t^l \rho_{t+1} \right] - \frac{\alpha}{2} (b_t^l)^2 Var_l^l (\rho_{t+1}).
$$

(37)

Maximizing (37) with respect to $b_t^l$, we get

$$
\frac{\partial \Omega}{\partial b_t^l} = E_t^l (\rho_{t+1}) - \frac{\alpha}{2} 2 (b_t^l) Var_l^l (\rho_{t+1}) = 0.
$$

(38)

Solving for $b_t^l$, we can now derive equation (19) as used in the text of the paper:

$$
b_t^l = \frac{E_t^l (\rho_{t+1})}{\alpha Var_l^l (\rho_{t+1})}.
$$

Appendix A3: The deviation of the exchange rate from its average level

In order to derive equation (23) we slightly rearrange equation (22) and get

$$
\bar{\rho} [N + n(1 - \tau)^\gamma] = \alpha \bar{B} Var(s).
$$

(39)

Equation (20) can be reorganized as follows:

$$
\bar{B} \alpha Var(s) = NE_t(\rho_{t+1}) + n(1 - \tau)^\gamma (\bar{\rho} + \nu_t).
$$

(40)

Equating (39) and (40) leads to

$$
\bar{\rho} [N + n(1 - \tau)^\gamma] = NE_t(\rho_{t+1}) + n(1 - \tau)^\gamma (\bar{\rho} + \nu_t).
$$

(41)

Solving (41) for the average risk premium ($\bar{\rho}$) we get

$$
\bar{\rho} = E_t(\rho_{t+1}) + \frac{n}{N} (1 - \tau)^\gamma \nu_t.
$$

(42)
Substituting the interest differential of equation (8) for the expression in equation (11) yields

\[ \rho_{t+1} = \frac{1}{\beta} (s_t - m_t + m^* - \epsilon_t) - (s_{t+1} - s_t) - \tau. \]  

We now apply the expected value at time \( t \) of the expression in equation (43) at time \( t \) and take into account that \( E_t(\epsilon_t) = \epsilon_t \) and that the expected value of next periods exchange rate is equal to its average value \( E_t(s_{t+1}) = \bar{s} \) :

\[ E(\rho_{t+1}) = \frac{1}{\beta} (s_t - m_t + m^* - \epsilon_t) - (\bar{s} - s_t) - \tau. \]  

Substituting (44) into (42) yields

\[ \bar{\rho} = \frac{1}{\beta} (s_t - m_t + m^* - \epsilon_t) - (\bar{s} - s_t) - \tau + \frac{n}{N} (1 - \tau)^\gamma \nu_t. \]  

Multiplying this equation by \( \beta \) leads to

\[ \beta \bar{\rho} = (s_t - m_t + m^* - \epsilon_t) - \beta (\bar{s} - s_t) - \beta \tau + \frac{n}{N} \beta (1 - \tau)^\gamma \nu_t. \]  

From equation (17), we get \( \beta \bar{\rho} = \bar{s} - \bar{m} + m^* - \beta \tau \). Therefore, equation (46) can be rewritten as

\[ \bar{s} + m^* - \beta \tau = (s_t - m_t + m^* - \epsilon_t) - \beta (s_t - \bar{s}) - \beta \tau + \frac{n}{N} \beta (1 - \tau)^\gamma \nu_t. \]  

This shows that the foreign money supply cancels out. After collecting terms we get

\[ (\bar{s} - s_t) + \beta (\bar{s} - s_t) = (-m_t + \bar{m} - \epsilon_t) + \frac{n}{N} \beta (1 - \tau)^\gamma \nu_t. \]  

Dividing by \( -(1 + \beta) \) leads to equation (23) used in the text of the paper:

\[ (s_t - \bar{s}) = \frac{1}{1 + \beta} \left[ m_t - \bar{m} + \epsilon_t - \frac{n}{N} \beta (1 - \tau)^\gamma \nu_t \right]. \]
Appendix A4: Deriving the variance of the exchange rate

Taking into account that $\text{Var}(\nu) = \lambda \text{Var}(s)$, the variance of the expression in equation (49) can be written as

$$\text{Var}(s) = \frac{\text{Var}(m + \epsilon)}{(1 + \beta)^2} + \frac{[\beta \frac{\tau}{N}(1 - \tau)^\gamma]^2}{(1 + \beta)^2} \lambda \text{Var}(s). \quad (50)$$

Collecting terms gives

$$\frac{(1 + \beta)^2 - [\beta \frac{\tau}{N}(1 - \tau)^\gamma]^2}{(1 + \beta)^2} \lambda \text{Var}(s) = \frac{\text{Var}(m + \epsilon)}{(1 + \beta)^2}. \quad (51)$$

Solving for the variance expression gives equation (24)

$$\text{Var}(s) = \frac{\text{Var}(m + \epsilon)}{(1 + \beta)^2 - [\beta \frac{\tau}{N}(1 - \tau)^\gamma]^2 \lambda}. \quad (52)$$

Appendix A5: Computing the optimal tax rate

We start from equation (26) and factor out $\xi^{-1}$. This yields

$$\xi^{-1} \left[ -\gamma n(1 + \beta)^2 + 2\gamma \lambda \beta^2 \frac{n^2}{N} \xi^7 + 3\gamma \lambda \beta^2 \frac{n^3}{N^2} \xi^{27} \right] = 0. \quad (53)$$

Defining $x \equiv \xi^7$ we get for the expression in brackets after dividing by $3\gamma \lambda \beta^2 n^3 / N^2$:

$$x^2 + 2 \frac{N}{3} x - \frac{(1 + \beta)^2 N^2}{3 \lambda \beta^2 n^2} = 0. \quad (54)$$

This leads to

$$x = -1 \frac{N}{3} \pm \sqrt{\frac{N^2}{9 n^2} + \frac{(1 + \beta)^2 N^2}{3 \lambda \beta^2 n^2}}. \quad (55)$$

Slight rearrangements under the square root lead to

$$x = -1 \frac{N}{3} \pm \sqrt{\frac{N^2 \lambda \beta^2 + 3(1 + \beta)^2 N^2}{9 \lambda \beta^2 n^2}}. \quad (56)$$
In order to examine the capital control effect, we only have to consider the positive root. Taking into account the definition $x \equiv \xi^\gamma$ we get

$$\xi = \left[ \frac{1}{3} \frac{N}{n} - \sqrt{\frac{N^2 \lambda \beta^2 + 3(1 + \beta)^2 N^2}{9 \lambda \beta^2 n^2}} \right]^{1/\gamma}. \quad (57)$$

Since $\xi \equiv (1 - \tau)$, we get for the optimal tax rate

$$\hat{\tau} = 1 + \left[ \frac{1}{3} \frac{N}{n} - \sqrt{\frac{N^2 \lambda \beta^2 + 3(1 + \beta)^2 N^2}{9 \lambda \beta^2 n^2}} \right]^{1/\gamma}. \quad (58)$$

Rearranging of this expression yields equation (27) used in section 2 of the paper:

$$\hat{\tau} = 1 - \left[ \frac{N}{3n} + \frac{N}{3n} \sqrt{1 + \frac{3(1 + \beta)^2}{\lambda \beta^2}} \right]^{1/\gamma}. \quad (59)$$
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