Wystup, Uwe

Working Paper
Vanna-volga pricing

CPQF Working Paper Series, No. 11

Provided in Cooperation with:
Frankfurt School of Finance and Management

Suggested Citation: Wystup, Uwe (2008) : Vanna-volga pricing, CPQF Working Paper Series, No. 11, Frankfurt School of Finance & Management, Frankfurt, M.

This Version is available at:
http://hdl.handle.net/10419/40192

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
No. 11

Vanna-Volga Pricing

Uwe Wystup

July 2008

Author: Prof. Dr. Uwe Wystup
Frankfurt School of
Finance & Management
Frankfurt/Main
u.wystup@frankfurt-school.de

Publisher: Frankfurt School of Finance & Management
Phone: +49 (0) 69 154 008-0  •  Fax: +49 (0) 69 154 008-728
Sonnemannstr. 9-11  •  D-60314 Frankfurt/M.  •  Germany
Abstract

The vanna-volga method, also called the traders’ rule of thumb is an empirical procedure that can be used to infer an implied-volatility smile from three available quotes for a given maturity. It is based on the construction of locally replicating portfolios whose associated hedging costs are added to corresponding Black-Scholes prices to produce smile-consistent values. Besides being intuitive and easy to implement, this procedure has a clear financial interpretation, which further supports its use in practice.

Contents

1 Introduction 3
2 Cost of Vanna and Volga 4
3 Observations 7
4 Consistency Check 9
5 Abbreviations for First Generation Exotics 10
6 Adjustment Factor 10
7 Volatility for Risk Reversals, Butterflies and Theoretical Value 12
8 Pricing Barrier Options 12
9 Pricing Double Barrier Options 13
10 Pricing Double-No-Touch Options 13
11 Pricing European Style Options 13
11.1 Digital Options ................................................................. 13
11.2 European Barrier Options .................................................. 13
12 No-Touch Probability 14
13 The Cost of Trading and its Implication on the Market Price of One-touch Options 14
14 Example 16
15 Further Applications 16
1 Introduction

The vanna-volga (VV) method, also called the Traders’ Rule of Thumb is commonly used in foreign exchange options markets, where three main volatility quotes are typically available for a given market maturity: the delta-neutral straddle, referred to as at-the-money (ATM); the risk reversal for 25 delta call and put; and the (vega-weighted) butterfly with 25 delta wings. The application of vanna-volga pricing allows us to derive implied volatilities for any options delta, in particular for those outside the basic range set by the 25 delta put and call quotes. The notion of risk reversals and butterflies is explained in [6].

In the financial literature, the vanna-volga approach was introduced by Lipton and McGhee (2002) in [2], who compare different approaches to the pricing of double-no-touch options, and by Wystup (2003) in [5], who describes its application to the valuation of one-touch options. The vanna-volga procedure is reviewed in more detail and some important results concerning the tractability of the method and its robustness are derived by Castagna and Mercurio (2007) in [1].

The traders’ rule of thumb is a method of traders to determine the cost of risk managing the volatility risk of exotic options with vanilla options. This cost is then added to the theoretical value in the Black-Scholes model and is called the overhedge. In fact, SuperDerivatives\textsuperscript{2} has implemented a type of this method in their pricing platform, as one can read in the patent that SuperDerivatives has filed.

We explain the rule and then consider an example of a one-touch.

Delta and vega are the most relevant sensitivity parameters for foreign exchange options maturing within one year. A delta-neutral position can be achieved by trading the spot. Changes in the spot are explicitly allowed in the Black-Scholes model. Therefore, model and practical trading have very good control over spot change risk. The more sensitive part is the vega position. This is not taken care of in the Black-Scholes model. Market participants need to trade other options to obtain a vega-neutral position. However, even a vega-neutral position is subject to changes of spot and volatility. For this reason, the sensitivity parameters \textit{vanna} (change of vega due to change of spot) and \textit{volga} (change of vega due to change of volatility) are of special interest. Vanna is also called \(\frac{d\ vega}{d\ spot}\), volga is also called \(\frac{d\ vega}{d\ vol}\). The plots for vanna and volga for a vanilla are displayed in Figures 1 and 2. In this section we outline how the cost of such a vanna- and volga- exposure can be used to obtain prices for options that are closer to the market than their theoretical Black-Scholes value.

\textsuperscript{2}SuperDerivatives is an internet-based pricing tool for exotic options, see \url{www.superderivatives.com}
Figure 1: Vanna of a vanilla option as a function of spot and time to expiration, showing the skew symmetry about the at-the-money line

2 Cost of Vanna and Volga

We fix the rates $r_d$ and $r_f$, the time to maturity $T$ and the spot $x$ and define

\[
\text{cost of vanna} \triangleq \text{Exotic Vanna Ratio} \times \text{value of RR}, \quad (1)
\]

\[
\text{cost of volga} \triangleq \text{Exotic Volga Ratio} \times \text{value of BF}, \quad (2)
\]

\[
\text{Exotic Vanna Ratio} \triangleq B_{\sigma x}/RR_{\sigma x}, \quad (3)
\]

\[
\text{Exotic Volga Ratio} \triangleq B_{\sigma \sigma}/BF_{\sigma \sigma}, \quad (4)
\]

\[
\text{value of RR} \triangleq [RR(\sigma_\Delta) - RR(\sigma_0)], \quad (5)
\]

\[
\text{value of BF} \triangleq [BF(\sigma_\Delta) - BF(\sigma_0)], \quad (6)
\]

where $\sigma_0$ denotes the at-the-money (forward) volatility and $\sigma_\Delta$ denotes the wing volatility at the delta pillar $\Delta$. $B$ denotes the value function of a given exotic option. The values
Figure 2: Volga of a vanilla option as a function of spot and time to expiration, showing the symmetry about the at-the-money line

of risk reversals and butterflies are defined by

\[
RR(\sigma) \triangleq \frac{\text{call}(x, \Delta, \sigma, r_d, r_f, T) - \text{put}(x, \Delta, \sigma, r_d, r_f, T)}{\text{call}(x, \Delta, \sigma, r_d, r_f, T) + \text{put}(x, \Delta, \sigma, r_d, r_f, T)}, \tag{7}
\]

\[
BF(\sigma) \triangleq \frac{2}{\text{call}(x, \Delta_0, \sigma_0, r_d, r_f, T) + \text{put}(x, \Delta_0, \sigma_0, r_d, r_f, T)}, \tag{8}
\]

where \(\text{vanilla}(x, \Delta, \sigma, r_d, r_f, T)\) means \(\text{vanilla}(x, K, \sigma, r_d, r_f, T)\) for a strike \(K\) chosen to imply \(|\text{vanilla}_x(x, K, \sigma, r_d, r_f, T)| = \Delta\) and \(\Delta_0\) is the delta that produces the at-the-money strike. To summarize we abbreviate

\[
c(\sigma_+^\Delta) \triangleq \text{call}(x, \Delta^+, \sigma_+^\Delta, r_d, r_f, T), \tag{9}
\]

\[
p(\sigma_-^\Delta) \triangleq \text{put}(x, \Delta^-, \sigma_-^\Delta, r_d, r_f, T), \tag{10}
\]
and obtain

$$\text{cost of vanna} = \frac{B_{\sigma x}}{c_{\sigma x}(\sigma_+^0) - p_{\sigma x}(\sigma_-^0)} \left[ c(\sigma_+^0) - c(\sigma_0) - p(\sigma_-^0) + p(\sigma_0) \right],$$

(11)

$$\text{cost of volga} = \frac{2B_{\sigma x}}{c_{\sigma x}(\sigma_+^0) + p_{\sigma x}(\sigma_-^0)} \left[ \frac{c(\sigma_+^0) - c(\sigma_0) + p(\sigma_-^0) - p(\sigma_0)}{2} \right],$$

(12)

where we note that volga of the butterfly should actually be

$$\frac{1}{2} \left[ c_{\sigma x}(\sigma_+^0) + p_{\sigma x}(\sigma_-^0) - c_{\sigma x}(\sigma_0) - p_{\sigma x}(\sigma_0) \right],$$

(13)

but the last two summands are close to zero. The \textit{vanna-volga adjusted value} of the exotic is then

$$B(\sigma_0) + p \times [\text{cost of vanna} + \text{cost of volga}].$$

(14)

A division by the spot \(x\) converts everything into the usual quotation of the price in \(\%\) of the underlying currency. The cost of vanna and volga is often adjusted by a number \(p \in [0, 1]\), which is often taken to be the risk-neutral no-touch probability. The reason is that in the case of options that can knock out, the hedge is not needed anymore once the option has knocked out. The exact choice of \(p\) depends on the product to be priced; see Table 2. Taking \(p = 1\) as a default value would lead to overestimated overhedges for double-no-touch options as pointed out in [2].

The values of risk reversals and butterflies in Equations (11) and (12) can be approximated by a first order expansion as follows. For a risk reversal we take the difference of the call with correct implied volatility and the call with at-the-money volatility minus the difference of the put with correct implied volatility and the put with at-the-money volatility. It is easy to see that this can be well approximated by the vega of the at-the-money vanilla times the risk reversal in terms of volatility. Similarly the cost of the butterfly can be approximated by the vega of the at-the-money volatility times the butterfly in terms of volatility. In formulae this is

\[
\begin{align*}
  c(\sigma_+^0) & - c(\sigma_0) - p(\sigma_-^0) + p(\sigma_0) \\
  \approx & c_{\sigma}(\sigma_0)(\sigma_+^0 - \sigma_0) - p_{\sigma}(\sigma_0)(\sigma_-^0 - \sigma_0) \\
  = & \sigma_0[p_{\sigma}(\sigma_0) - c_{\sigma}(\sigma_0)] + c_{\sigma}(\sigma_0)[\sigma_+^0 - \sigma_-^0] \\
  = & c_{\sigma}(\sigma_0)RR
\end{align*}
\]

(15)
and similarly
\[
\frac{c(\sigma_\Delta) - c(\sigma_0) + p(\sigma_\Delta) - p(\sigma_0)}{2} \approx c_\sigma(\sigma_0)BF.
\]  
(16)

With these approximations we obtain the formulae
\[
\text{cost of vanna} \approx \frac{B_{\sigma x}}{c_{\sigma x}(\sigma_\Delta) - p_{\sigma x}(\sigma_\Delta)} c_\sigma(\sigma_0) RR,
\]  
(17)
\[
\text{cost of volga} \approx \frac{2B_{\sigma x}}{c_{\sigma x}(\sigma_\Delta) + p_{\sigma x}(\sigma_\Delta)} c_\sigma(\sigma_0) BF.
\]  
(18)

3 Observations

1. The price supplements are linear in butterflies and risk reversals. In particular, there is no cost of vanna supplement if the risk reversal is zero and no cost of volga supplement if the butterfly is zero.

2. The price supplements are linear in the at-the-money vanilla vega. This means supplements grow with growing volatility change risk of the hedge instruments.

3. The price supplements are linear in vanna and volga of the given exotic option.

4. We have not observed any relevant difference between the exact method and its first order approximation. Since the computation time for the approximation is shorter, we recommend using the approximation.

5. It is not clear up front which target delta to use for the butterflies and risk reversals. We take a delta of 25% merely on the basis of its liquidity.

6. The prices for vanilla options are consistent with the input volatilities as shown in Figures 3, 4 and 5.

7. The method assumes a zero volga of risk reversals and a zero vanna of butterflies. This way the two sources of risk can be decomposed and hedged with risk reversals and butterflies. However, the assumption is actually not exact. For this reason, the method should be used with a lot of care. It causes traders and financial engineers to keep adding exceptions to the standard method.
Figure 3: Consistency check of vanna-volga-pricing. Vanilla option smile for a one month maturity EUR/USD call, spot = 0.9060, \( r_d = 5.07\% \), \( r_f = 4.70\% \), \( \sigma_0 = 13.35\% \), \( \sigma^+ = 13.475\% \), \( \sigma^- = 13.825\% \)

Figure 4: Consistency check of vanna-volga-pricing. Vanilla option smile for a one year maturity EUR/USD call, spot = 0.9060, \( r_d = 5.07\% \), \( r_f = 4.70\% \), \( \sigma_0 = 13.20\% \), \( \sigma^+ = 13.425\% \), \( \sigma^- = 13.575\% \)
Figure 5: Consistency check of vanna-volga-pricing. Vanilla option smile for a one year maturity EUR/USD call, spot = 0.9060, \( r_d = 5.07\% \), \( r_f = 4.70\% \), \( \sigma_0 = 13.20\% \), \( \sigma^+ = 13.425\% \), \( \sigma^- = 13.00\% \)

4 Consistency Check

A minimum requirement for the vanna-volga pricing to be correct is the consistency of the method with vanilla options. We show in Figure 3, Figure 4 and Figure 5 that the method does in fact yield a typical foreign exchange smile shape and produces the correct input volatilities at-the-money and at the delta pillars. We will now prove the consistency in the following way. Since the input consists only of three volatilities (at-the-money and two delta pillars), it would be too much to expect that the method produces correct representation of the entire volatility matrix. We can only check if the values for at-the-money and target-\( \Delta \) puts and calls are reproduced correctly. In order to verify this, we check if the values for an at-the-money call, a risk reversal and a butterfly are priced correctly. Of course, we only expect approximately correct results. Note that the number \( p \) is taken to be one, which agrees with the risk-neutral no-touch probability for vanilla options.

For an at-the-money call vanna and volga are approximately zero, whence there are no supplements due to vanna cost or volga cost.

For a target-\( \Delta \) risk reversal

\[
C(\sigma^+) - P(\sigma^-) \tag{19}
\]
we obtain

\[
\text{cost of vanna} = \frac{c_{\sigma x}(\sigma^+_\Delta) - p_{\sigma x}(\sigma^-_\Delta)}{c_{\sigma x}(\sigma^+_\Delta) - p_{\sigma x}(\sigma^-_\Delta)} \left[ c(\sigma^+_\Delta) - c(\sigma_0) - p(\sigma^-_\Delta) + p(\sigma_0) \right]
\]

\[
= c(\sigma^+_\Delta) - c(\sigma_0) - p(\sigma^-_\Delta) + p(\sigma_0), \quad (20)
\]

\[
\text{cost of volga} = \frac{2[c_{\sigma \sigma}(\sigma^+_\Delta) - p_{\sigma \sigma}(\sigma^-_\Delta)]}{c_{\sigma \sigma}(\sigma^+_\Delta) + p_{\sigma \sigma}(\sigma^-_\Delta)} \left[ \frac{c(\sigma^+_\Delta) - c(\sigma_0) + p(\sigma^-_\Delta) - p(\sigma_0)}{2} \right], \quad (21)
\]

and observe that the cost of vanna yields a perfect fit and the cost of volga is small, because in the first fraction we divide the difference of two quantities by the sum of the quantities, which are all of the same order.

For a target-$\Delta$ butterfly

\[
\frac{c(\sigma^+_\Delta) + p(\sigma^-_\Delta)}{2} - \frac{c(\sigma_0) + p(\sigma_0)}{2} \quad (22)
\]

we analogously obtain a perfect fit for the cost of volga and

\[
\text{cost of vanna} = \frac{c_{\sigma x}(\sigma^+_\Delta) - p_{\sigma x}(\sigma_0) - [c_{\sigma x}(\sigma_0) - p_{\sigma x}(\sigma^-_\Delta)]}{c_{\sigma x}(\sigma^+_\Delta) - p_{\sigma x}(\sigma_0) + [c_{\sigma x}(\sigma_0) - p_{\sigma x}(\sigma^-_\Delta)]} \left[ \frac{c(\sigma^+_\Delta) - c(\sigma_0) - p(\sigma^-_\Delta) + p(\sigma_0)}{2} \right], \quad (23)
\]

which is again small.

The consistency can actually fail for certain parameter scenarios. This is one of the reasons, why the traders’ rule of thumb has been criticized repeatedly by a number of traders and researchers.

5 Abbreviations for First Generation Exotics

We introduce the abbreviations for first generation exotics listed in Table 1.

6 Adjustment Factor

The factor $p$ has to be chosen in a suitable fashion. Since there is no mathematical justification or indication, there is a lot of dispute in the market about this choice. Moreover,
KO  knock-out
KI  knock-in
RKO reverse knock-out
RKI reverse knock-in
DKO double knock-out
OT one-touch
NT no-touch
DOT double one-touch
DNT double no-touch

Table 1: Abbreviations for first generation exotics

<table>
<thead>
<tr>
<th>product</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KO</td>
<td>no-touch probability</td>
</tr>
<tr>
<td>RKO</td>
<td>no-touch probability</td>
</tr>
<tr>
<td>DKO</td>
<td>no-touch probability</td>
</tr>
<tr>
<td>OT</td>
<td>$0.9 \times \text{no-touch probability} - 0.5 \times \text{bid-offer-spread } * (TV - 33%) / 66%$</td>
</tr>
<tr>
<td>DNT</td>
<td>$0.5$</td>
</tr>
</tbody>
</table>

Table 2: Adjustment factors for the overhedge for first generation exotics

the choices may also vary over time. An example for one of many possible choices of $p$ is presented in Table 2.

For options with strike $K$, barrier $B$ and type $\phi = 1$ for a call and $\phi = -1$ for a put, we use the following pricing rules which are based on no-arbitrage conditions.

KI is priced via $KI = \text{vanilla} - KO$.

RKI is priced via $RKI = \text{vanilla} - RKO$.

RKO is priced via

$$RKO(\phi, K, B) = KO(-\phi, K, B) - KO(-\phi, B, B) + \phi(B - K)NT(B).$$

DOT is priced via DNT.
NT is priced via OT.

7 Volatility for Risk Reversals, Butterflies and Theoretical Value

To determine the volatility and the vanna and volga for the risk–reversal and butterfly, the convention is the same as for the building of the smile curve. Hence the 25% delta risk–reversal retrieves the strike for 25% delta call and put with the spot delta and premium included [left-hand-side in Fenics] and calculates the vanna and volga of these options using the corresponding volatilities from the smile.

The theoretical value (TV) of the exotics is calculated using the ATM–volatility retrieving it with the same convention that was used to build the smile, i.e. delta–parity with premium included [left-hand-side in Fenics].

8 Pricing Barrier Options

Ideally one would be in a situation to hedge all barrier contracts with portfolio of vanilla options or simple barrier building blocks. In the Black-Scholes model there are exact rules how to statically hedge many barrier contracts. A state of the art reference is given by Poulsen (2006) in [3]. However, in practice most of these hedges fail, because volatility is not constant.

For regular knock-out options one can refine the method to incorporate more information about the global shape of the vega surface through time.

We chose $M$ future points in time as $0 < a_1\% < a_2\% < \ldots < a_M\%$ of the time to expiration. Using the same cost of vanna and volga we calculate the overhedge for the regular knock-out with a reduced time to expiration. The factor for the cost is the no-touch probability to touch the barrier within the remaining times to expiration $1 > 1 - a_1\% > 1 - a_2\% > \ldots > 1 - a_M\%$ of the total time to expiration. Some desks believe that for at-the-money strikes the long time should be weighted higher and for low delta strikes the short time to maturity should be weighted higher. The weighting can be chosen (rather arbitrarily) as

$$w = \tanh[\gamma(|\delta - 50\%| - 25\%)]$$

with a suitable positive $\gamma$. For $M = 3$ the total overhedge is given by

$$\text{OH} = \frac{\text{OH}(1 - a_1\%) * w + \text{OH}(1 - a_2\%) + \text{OH}(1 - a_3\%) * (1 - w)}{3}.$$  

Which values to use for $M$, $\gamma$ and the $a_i$; whether to apply a weighting and what kind varies for different trading desks.
An additional term can be used for single barrier options to account for glitches in the stop–loss of the barrier. The theoretical value of the barrier option is determined with a barrier that is moved by 4 basis points and 50% of that adjustment is added to the price if it is positive. If it is negative it is omitted altogether. The theoretical foundation for such a method is explained in [4].

9 Pricing Double Barrier Options

Double barrier options behave similar to vanilla options for a spot far away from the barrier and more like one-touch options for a spot close to the barrier. Therefore, it appears reasonable to use the traders’ rule of thumb for the corresponding regular knock-out to determine the overhedge for a spot closer to the strike and for the corresponding one-touch for a spot closer to the barrier. This adjustment is the intrinsic value of the reverse knock-out times the overhedge of the corresponding one-touch. The border is the arithmetic mean between strike and the in-the-money barrier.

10 Pricing Double-No-Touch Options

For double-no-touch options with lower barrier \( L \) and higher barrier \( H \) at spot \( S \) one can use the overhedge

\[
\text{OH} = \max\{\text{Vanna-Volga-OH}; \delta(S - L) - TV - 0.5\%; \delta(H - S) - TV - 0.5\%\}.
\]

(26)

where \( \delta \) denotes the delta of the double-no-touch.

11 Pricing European Style Options

11.1 Digital Options

Digital options are priced using the overhedge of the call– or put–spread with the corresponding volatilities.

11.2 European Barrier Options

European barrier options (EKO) are priced using the prices of European and digital options and the relationship

\[
\text{EKO}(\phi, K, B) = \text{vanilla}(\phi, K) - \text{vanilla}(\phi, B) - \text{digital}(B)\phi(B - K).
\]

(27)
12 No-Touch Probability

The no-touch probability is obviously equal to the non-discounted value of the corresponding no-touch option paying at maturity (under the risk neutral measure). Note that the price of the one-touch is calculated using an iteration for the touch probability. This means that the price of the one-touch used to compute the no-touch probability is itself based on the traders’ rule of thumb. This is an iterative process which requires an abortion criterion. One can use a standard approach that ends either after 100 iterations or as soon as the difference of two successive iteration results is less than $10^{-6}$. However, the method is so crude that it actually does not make much sense to use such precision at just this point. So in order to speed up the computation we suggest to omit this procedure and take zero iterations, which is the TV of the no-touch.

13 The Cost of Trading and its Implication on the Market Price of One-touch Options

Now let us take a look at an example of the traders’ rule of thumb in its simple version. We consider one-touch options, which hardly ever trade at TV. The tradable price is the sum of the TV and the overhedge. Typical examples are shown in Figure 6, one for an upper touch level in EUR-USD, one for a lower touch level.

![Figure 6: Overhedge of a one-touch in EUR-USD for both an upper touch level and a lower touch level, based on the traders’ rule of thumb](image)

Clearly there is no overhedge for one-touch options with a TV of 0% or 100%, but it is worth noting that low-TV one-touch options can be twice as expensive as their TV,
sometimes even more. SuperDerivatives\textsuperscript{3} has become one of the standard references of pricing exotic FX options up to the market. The overhedge arises from the cost of risk managing the one-touch. In the Black-Scholes model, the only source of risk is the underlying exchange rate, whereas the volatility and interest rates are assumed constant. However, volatility and rates are themselves changing, whence the trader of options is exposed to instable vega and rho (change of the value with respect to volatility and rates). For short dated options, the interest rate risk is negligible compared to the volatility risk as shown in Figure 7. Hence the overhedge of a one-touch is a reflection of a trader’s cost occurring due to the risk management of his vega exposure.

Figure 7: Comparison of interest rate and volatility risk for a vanilla option. The volatility risk behaves like a square root function, whereas the interest rate risk is close to linear. Therefore, short-dated FX options have higher volatility risk than interest rate risk.

\textsuperscript{3}http://www.superderivatives.com
14 Example

We consider a one-year one-touch in USD/JPY with payoff in USD. As market parameters we assume a spot of 117.00 JPY per USD, JPY interest rate 0.10%, USD interest rate 2.10%, volatility 8.80%, 25-delta risk reversal -0.45% \(^4\); 25-delta butterfly 0.37% \(^5\).

The touch level is 127.00, and the TV is at 28.8%. If we now only hedge the vega exposure, then we need to consider two main risk factors, namely,

1. the change of vega as the spot changes, often called vanna,

2. the change of vega as the volatility changes, often called volga or volgamma or vomma.

To hedge this exposure we treat the two effects separately. The vanna of the one-touch is 0.16%, the vanna of the risk reversal is 0.04%. So we need to buy 4 (=0.16/0.04) risk reversals, and for each of them we need to pay 0.14% of the USD amount, which causes an overhedge of -0.6%. The volga of the one-touch is -0.53%, the volga of the butterfly is 0.03%. So we need to sell 18 (=0.53/0.03) butterflies, each of which pays us 0.23% of the USD amount, which causes an overhedge of -4.1%. Therefore, the overhedge is -4.7%. However, we will get to the touch level with a risk-neutral probability of 28.8%, in which case we would have to pay to unwind the hedge. Therefore the total overhedge is -71.2%*4.7%=-3.4%. This leads to a mid market price of 25.4%. Bid and offer could be 24.25% - 36.75%. There are different beliefs among market participants about the unwinding cost. Other observed prices for one-touch options can be due to different existing vega profiles of the trader’s portfolio, a marketing campaign, a hidden additional sales margin or even the overall condition of the trader in charge.

15 Further Applications

The method illustrated above shows how important the current smile of the vanilla options market is for the pricing of simple exotics. Similar types of approaches are commonly used to price other exotic options. For long-dated options the interest rate risk will take over the lead in comparison to short dated options where the volatility risk is dominant.

\(^4\)This means that a 25-delta USD call is 0.45% cheaper than a 25-delta USD put in terms of implied volatility.

\(^5\)This means that a 25-delta USD call and 25-delta USD put is on average 0.37% more expensive than an at-the-money option in terms of volatility.
References


Index

butterfly, 5

double barrier options, 13

European style options, 13

no-touch probability, 14

one-touch, 14

overhedge, 3

risk reversal, 5

SuperDerivatives, 15

traders’ rule of thumb, 2

vanna, 3

vanna-volga pricing, 2

volga, 3
<table>
<thead>
<tr>
<th>No.</th>
<th>Author/Title</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.</td>
<td>Almer, Thomas / Heidorn, Thomas / Schmaltz, Christian The Dynamics of Short- and Long-Term CDS-spreads of Banks</td>
<td>2008</td>
</tr>
<tr>
<td>92.</td>
<td>Burger, Andreas Produktivität und Effizienz in Banken – Terminologie, Methoden und Status quo</td>
<td>2008</td>
</tr>
<tr>
<td>91.</td>
<td>Löchel, Horst / Pecher, Florian The Strategic Value of Investments in Chinese Banks by Foreign Financial Institutions</td>
<td>2008</td>
</tr>
<tr>
<td>89.</td>
<td>Schalast, Christoph / Stralkowski, Ingo 10 Jahre deutsche Buyouts</td>
<td>2008</td>
</tr>
<tr>
<td>86.</td>
<td>Hölscher, Luise / Rosenthal, Johannes Leistungsmessung der Internen Revision</td>
<td>2007</td>
</tr>
<tr>
<td>85.</td>
<td>Bannier, Christina / Hansel, Dennis Determinants of banks’ engagement in loan securitization</td>
<td>2007</td>
</tr>
<tr>
<td>83.</td>
<td>Bannier, Christina Heterogeneous Multiple Bank Financing: Does it Reduce Inexcent Credit-Renegotiation Incidences?</td>
<td>2007</td>
</tr>
<tr>
<td>82.</td>
<td>Cremers, Heinz / Lohr, Andreas Deskription und Bewertung strukturierter Produkte unter besonderer Berücksichtigung verschiedener Marktszenarien</td>
<td>2007</td>
</tr>
<tr>
<td>80.</td>
<td>Cremers, Heinz / Walzner, Jens Risikosteuerung mit Kreditderivaten unter besonderer Berücksichtigung von Credit Default Swaps</td>
<td>2007</td>
</tr>
<tr>
<td>78.</td>
<td>Gerdesmeier, Dieter / Roffia, Barbara Monetary Analysis: A VAR Perspective</td>
<td>2007</td>
</tr>
<tr>
<td>77.</td>
<td>Heidorn, Thomas / Kaiser, Dieter G. / Muschiol, Andrea Portfoliooptimierung mit Hedgefonds unter Berücksichtigung höherer Momente der Verteilung</td>
<td>2007</td>
</tr>
<tr>
<td>75.</td>
<td>Abrar, Kamyr Fusionskontrolle in dynamischen Netzsektoren am Beispiel des Breitbandkabelsektors</td>
<td>2006</td>
</tr>
<tr>
<td>74.</td>
<td>Schalast, Christoph / Schanz, Kai-Michael Wertpapierprospekte: Markteinflussungspublizität nach EU-Prospektverordnung und Wertpapierprospektgesetz 2005</td>
<td>2006</td>
</tr>
</tbody>
</table>
73. Dickler, Robert A. / Schalast, Christoph

72. Belke, Ansgar / Polleit, Thorsten
How the ECB and the US Fed set interest rates 2006

71. Heidorn, Thomas / Hoppe, Christian / Kaiser, Dieter G.
Heterogenität von Hedgefondsindizes 2006

70. Baumann, Stefan / Loch, Horst
The Endogeneity Approach of the Theory of Optimum Currency Areas - What does it mean for ASEAN + 3? 2006

69. Heidorn, Thomas / Trautmann, Alexandra
Niederschlagsderivate 2005

68. Heidorn, Thomas / Hoppe, Christian / Kaiser, Dieter G.
Möglichkeiten der Strukturierung von Hedgefondsportfolios 2005

67. Belke, Ansgar / Polleit, Thorsten
(How) Do Stock Market Returns React to Monetary Policy? An ARDL Cointegration Analysis for Germany 2005

66. Daynes, Christian / Schalast, Christoph
Aktuelle Rechtsfragen des Bank- und Kapitalmarktrechts II: Distressed Debt - Investing in Deutschland 2005

65. Gerdemeier, Dieter / Polleit, Thorsten
Measures of excess liquidity 2005

64. Becker, Gernot M. / Harding, Perham / Hölscher, Luise
Financing the Embedded Value of Life Insurance Portfolios 2005

63. Schalast, Christoph
Modernisierung der Wasserwirtschaft im Spannungsfeld von Umweltschutz und Wettbewerb – Braucht Deutschland eine Rechtsgrundlage für die Vergabe von Wasserversorgungskonzessionen? – 2005

62. Bayer, Marcus / Cremers, Heinz / Klüß, Norbert
Wertsicherungsstrategien für das Asset Management 2005

61. Loch, Horst / Polleit, Thorsten
A case for money in the ECB monetary policy strategy 2005

60. Richard, Jörg / Schalast, Christoph / Schanz, Kay-Michael

59. Heun, Michael / Schlink, Torsten
Early Warning Systems of Financial Crises - Implementation of a currency crisis model for Uganda 2004

58. Heimer, Thomas / Kohler, Thomas
Auswirkungen des Basel II Akkords auf österreichische KMU 2004

57. Heidorn, Thomas / Meyer, Bernd / Pietrowiak, Alexander
Performanceeffekte nach Directors Dealings in Deutschland, Italien und den Niederlanden 2004

56. Gerdemeier, Dieter / Roffia, Barbara
The Relevance of real-time data in estimating reaction functions for the euro area 2004

55. Barthel, Erich / Gieri, Gaunio / Kühn, Ilmhart-Wolfram
Unterschiedliche Ansätze zur Messung des Humankapitals 2004

54. Anders, Dietmar / Binder, Andreas / Hesdahl, Ralf / Schalast, Christoph / Thöne, Thomas

53. Polleit, Thorsten
The Slowdown in German Bank Lending – Revisited 2004

52. Heidorn, Thomas / Siragusano, Tindaro
Die Anwendbarkeit der Behavioral Finance im Devisenmarkt 2004

51. Schütze, Daniel / Schalast, Christoph (Hrsg.)
Wider die Verschleuderung von Unternehmen durch Pfandversteigerung 2004

50. Gerhold, Mirko / Heidorn, Thomas
Investitionen und Emissionen von Convertible Bonds (Wandelanleihen) 2004

49. Chevalier, Pierre / Heidorn, Thomas / Krieger, Christian
Temperaturderivate zur strategischen Absicherung von Beschaffungs- und Absatzrisiken 2003

48. Becker, Gernot M. / Seeger, Norbert
Internationale Cash Flow-Rechnungen aus Eigner- und Gläubigersicht 2003

47. Boenkost, Wolfram / Schmidt, Wolfgang M.
Notes on convexity and quanto adjustments for interest rates and related options 2003
<table>
<thead>
<tr>
<th>#</th>
<th>Author(s)</th>
<th>Title</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>Rolbach, Peter</td>
<td>Behavioral Finance - Eine Alternative zur vorherrschenden Kapitalmarkttheorie?</td>
<td>2001</td>
</tr>
<tr>
<td>30</td>
<td>Heidorn, Thomas / Jaster, Oliver / Willeitner, Ulrich</td>
<td>Event Risk Covenants</td>
<td>2001</td>
</tr>
<tr>
<td>29</td>
<td>Biswas, Rita / Löchel, Horst</td>
<td>Recent Trends in U.S. and German Banking: Convergence or Divergence?</td>
<td>2001</td>
</tr>
<tr>
<td>28</td>
<td>Eberle, Günter Georg / Löchel, Horst</td>
<td>Die Auswirkungen des Übergangs zum Kapitaldeckungsverfahren in der Rentenversicherung auf die Kapitalmärkte</td>
<td>2001</td>
</tr>
<tr>
<td>27</td>
<td>Heidorn, Thomas / Klein, Hans-Dieter / Siebrecht, Frank</td>
<td>Economic Value Added zur Prognose der Performance europäischer Aktien</td>
<td>2000</td>
</tr>
<tr>
<td>26</td>
<td>Cremers, Heinz</td>
<td>Konvergenz der binomialen Optionspreismodelle gegen das Modell von Black/Scholes/Merton</td>
<td>2000</td>
</tr>
<tr>
<td>25</td>
<td>Löchel, Horst</td>
<td>Die ökonomischen Dimensionen der ‚New Economy‘</td>
<td>2000</td>
</tr>
<tr>
<td>24</td>
<td>Frank, Axel / Moormann, Jürgen</td>
<td>Grenzen des Outsourcing: Eine Exploration am Beispiel von Direktbanken</td>
<td>2000</td>
</tr>
<tr>
<td>23</td>
<td>Heidorn, Thomas / Schmidt, Peter / Seiler, Stefan</td>
<td>Neue Möglichkeiten durch die Namensaktie</td>
<td>2000</td>
</tr>
<tr>
<td>22</td>
<td>Böger, Andreas / Heidorn, Thomas / Graf Waldstein, Philipp</td>
<td>Hybrides Kernkapital für Kreditinstitute</td>
<td>2000</td>
</tr>
<tr>
<td>21</td>
<td>Heidorn, Thomas</td>
<td>Entscheidungsorientierte Mindestmargenkalkulation</td>
<td>2000</td>
</tr>
<tr>
<td>20</td>
<td>Wolf, Birgit</td>
<td>Die Eigenmittelkonzeption des § 10 KWG</td>
<td>2000</td>
</tr>
<tr>
<td>19</td>
<td>Heidorn, Thomas / Kantwill, Jens</td>
<td>Eine empirische Analyse der Spreadunterschiede von Festsatzanleihen zu Floatern im Euroraum und deren Zusammenhang zum Preis eines Credit Default Swaps</td>
<td>2002</td>
</tr>
<tr>
<td>17</td>
<td>Moormann, Jürgen</td>
<td>Terminologie und Glossar der Bankinformatik</td>
<td>2002</td>
</tr>
<tr>
<td>16</td>
<td>Heidorn, Thomas</td>
<td>Bewertung von Kreditprodukten und Credit Default Swaps</td>
<td>2001</td>
</tr>
<tr>
<td>15</td>
<td>Heidorn, Thomas / Weier, Sven</td>
<td>Einführung in die fundamentale Aktienanalyse</td>
<td>2001</td>
</tr>
<tr>
<td>14</td>
<td>Seeger, Norbert</td>
<td>International Accounting Standards (IAS)</td>
<td>2001</td>
</tr>
<tr>
<td>13</td>
<td>Moormann, Jürgen / Stehling, Frank</td>
<td>Strategic Positioning of E-Commerce Business Models in the Portfolio of Corporate Banking</td>
<td>2001</td>
</tr>
<tr>
<td>11</td>
<td>Rollbach, Peter</td>
<td>Behavioral Finance - Eine Alternative zur vorherrschenden Kapitalmarkttheorie?</td>
<td>2001</td>
</tr>
<tr>
<td>10</td>
<td>Heidorn, Thomas / Jaster, Oliver / Willeitner, Ulrich</td>
<td>Event Risk Covenants</td>
<td>2001</td>
</tr>
<tr>
<td>9</td>
<td>Biswas, Rita / Löchel, Horst</td>
<td>Recent Trends in U.S. and German Banking: Convergence or Divergence?</td>
<td>2001</td>
</tr>
<tr>
<td>8</td>
<td>Eberle, Günter Georg / Löchel, Horst</td>
<td>Die Auswirkungen des Übergangs zum Kapitaldeckungsverfahren in der Rentenversicherung auf die Kapitalmärkte</td>
<td>2001</td>
</tr>
<tr>
<td>7</td>
<td>Heidorn, Thomas / Klein, Hans-Dieter / Siebrecht, Frank</td>
<td>Economic Value Added zur Prognose der Performance europäischer Aktien</td>
<td>2000</td>
</tr>
<tr>
<td>6</td>
<td>Cremers, Heinz</td>
<td>Konvergenz der binomialen Optionspreismodelle gegen das Modell von Black/Scholes/Merton</td>
<td>2000</td>
</tr>
<tr>
<td>5</td>
<td>Löchel, Horst</td>
<td>Die ökonomischen Dimensionen der ‚New Economy‘</td>
<td>2000</td>
</tr>
<tr>
<td>4</td>
<td>Frank, Axel / Moormann, Jürgen</td>
<td>Grenzen des Outsourcing: Eine Exploration am Beispiel von Direktbanken</td>
<td>2000</td>
</tr>
<tr>
<td>3</td>
<td>Heidorn, Thomas / Schmidt, Peter / Seiler, Stefan</td>
<td>Neue Möglichkeiten durch die Namensaktie</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>Böger, Andreas / Heidorn, Thomas / Graf Waldstein, Philipp</td>
<td>Hybrides Kernkapital für Kreditinstitute</td>
<td>2000</td>
</tr>
<tr>
<td>1</td>
<td>Heidorn, Thomas</td>
<td>Entscheidungsorientierte Mindestmargenkalkulation</td>
<td>2000</td>
</tr>
<tr>
<td>No.</td>
<td>Author/Title</td>
<td>Year</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>------------------------------------------------------------------------------</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Beta als Risikomaß - Eine Untersuchung am europäischen Aktienmarkt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.</td>
<td>Cremers, Heinz</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Optionspreisbestimmung</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.</td>
<td>Cremers, Heinz</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Value at Risk-Konzepte für Marktrisiken</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gründung einer deutschen Strombörse für Elektrizitätsderivate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>Deister, Daniel / Ehrlcher, Sven / Heidorn, Thomas</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CatBonds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>Jochum, Eduard</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hoshin Kanri / Management by Policy (MbP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>Heidorn, Thomas</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kreditderivate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>Heidorn, Thomas</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kreditrisiko (CreditMetrics)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>Moormann, Jurgen</td>
<td>1999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Terminologie und Glossar der Bankinformatik</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Löchel, Horst</td>
<td>1998</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The EMU and the Theory of Optimum Currency Areas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>09.</td>
<td>Löchel, Horst</td>
<td>1998</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Die Geldpolitik im Währungsraum des Euro</td>
<td></td>
<td></td>
</tr>
<tr>
<td>08.</td>
<td>Heidorn, Thomas / Hund, Jürgen</td>
<td>1998</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Die Umstellung auf die Stückaktie für deutsche Aktiengesellschaften</td>
<td></td>
<td></td>
</tr>
<tr>
<td>07.</td>
<td>Moormann, Jürgen</td>
<td>1998</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stand und Perspektiven der Informationsverarbeitung in Banken</td>
<td></td>
<td></td>
</tr>
<tr>
<td>06.</td>
<td>Heidorn, Thomas / Schmidt, Wolfgang</td>
<td>1998</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LIBOR in Arrears</td>
<td></td>
<td></td>
</tr>
<tr>
<td>04.</td>
<td>Ecker, Thomas / Moormann, Jürgen</td>
<td>1997</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Die Bank als Betreiberin einer elektronischen Shopping-Mall</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03.</td>
<td>Jahresbericht 1996</td>
<td>1997</td>
<td></td>
</tr>
<tr>
<td>02.</td>
<td>Cremers, Heinz / Schwarz, Willi</td>
<td>1996</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Interpolation of Discount Factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>01.</td>
<td>Moormann, Jurgen</td>
<td>1995</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lean Reporting und Führungsinformationssysteme bei deutschen Finanzdienstleistern</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FRANKFURT SCHOOL / HfB – WORKING PAPER SERIES**

**CENTRE FOR PRACTICAL QUANTITATIVE FINANCE**

<table>
<thead>
<tr>
<th>No.</th>
<th>Author/Title</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>Wystup, Uwe</td>
<td>2008</td>
</tr>
<tr>
<td></td>
<td>Foreign Exchange Quanto Options</td>
<td></td>
</tr>
<tr>
<td>09.</td>
<td>Wystup, Uwe</td>
<td>2008</td>
</tr>
<tr>
<td></td>
<td>Foreign Exchange Symmetries</td>
<td></td>
</tr>
<tr>
<td>08.</td>
<td>Becker, Christoph / Wystup, Uwe</td>
<td>2008</td>
</tr>
<tr>
<td></td>
<td>Was kostet eine Garantie? Ein statistischer Vergleich der Rendite von langfristigen Anlagen</td>
<td></td>
</tr>
<tr>
<td>07.</td>
<td>Schmidt, Wolfgang</td>
<td>2007</td>
</tr>
<tr>
<td></td>
<td>Default Swaps and Hedging Credit Baskets</td>
<td></td>
</tr>
<tr>
<td>06.</td>
<td>Kilin, Fiodor</td>
<td>2007</td>
</tr>
<tr>
<td></td>
<td>Accelerating the Calibration of Stochastic Volatility Models</td>
<td></td>
</tr>
<tr>
<td>05.</td>
<td>Griebsch, Susanne/ Kühn, Christoph / Wystup, Uwe</td>
<td>2007</td>
</tr>
<tr>
<td></td>
<td>Instalment Options: A Closed-Form Solution and the Limiting Case</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>Author/Title</td>
<td>Year</td>
</tr>
<tr>
<td>-----</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>Interest Rate Convexity and the Volatility Smile</td>
<td></td>
</tr>
<tr>
<td>03.</td>
<td>Becker, Christoph / Wystup, Uwe</td>
<td>2005</td>
</tr>
<tr>
<td></td>
<td>On the Cost of Delayed Currency Fixing</td>
<td></td>
</tr>
<tr>
<td>02.</td>
<td>Boenkost, Wolfram / Schmidt, Wolfgang M.</td>
<td>2004</td>
</tr>
<tr>
<td></td>
<td>Cross currency swap valuation</td>
<td></td>
</tr>
<tr>
<td>01.</td>
<td>Wallner, Christian / Wystup, Uwe</td>
<td>2004</td>
</tr>
<tr>
<td></td>
<td>Efficient Computation of Option Price Sensitivities for Options of American Style</td>
<td></td>
</tr>
</tbody>
</table>

**HFB – SONDERARBEITSBERICHTE DER HFB - BUSINESS SCHOOL OF FINANCE & MANAGEMENT**

<table>
<thead>
<tr>
<th>No.</th>
<th>Author/Title</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>01.</td>
<td>Nicole Kahmer / Jürgen Moormann</td>
<td>2003</td>
</tr>
<tr>
<td></td>
<td>Studie zur Ausrichtung von Banken an Kundenprozessen am Beispiel des Internet</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Preis: € 120,--)</td>
<td></td>
</tr>
</tbody>
</table>

Printed edition: € 25.00 + € 2.50 shipping

Download: [http://www.frankfurt-school.de/content/de/research/Publications/list_of_publication](http://www.frankfurt-school.de/content/de/research/Publications/list_of_publication)

**Order address / contact**
Frankfurt School of Finance & Management
Sonnemannstr. 9 – 11 • D – 60314 Frankfurt/M. • Germany
Phone: +49(0)69 154008–734 • Fax: +49(0)69 154008–728
eMail: m.biemer@frankfurt-school.de

Further information about Frankfurt School of Finance & Management may be obtained at: [http://www.frankfurt-school.de](http://www.frankfurt-school.de)