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Delegating budgets when agents care about autonomy

by

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Delegating budgets when agents care about autonomy

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Abstract

We consider resource allocation within an organisation and show how delegation bears on moral hazard and adverse selection when agents have a preference for autonomy. Agents may care about autonomy for reasons of job-satisfaction, status or greater reputation when performing well under autonomy. Separating allocations (overall budget and degree of delegation) are characterised depending on the preference for autonomy. As the latter increases, the degree of delegation assigned to productive and unproductive agents converges. If agents’ preferences for monetary rewards are weak, the principal will not employ financial transfers. Pooling then arises under a strong preference for autonomy.

Keywords: adverse selection, capital budgeting, delegation, intrinsic motivation, moral hazard.

JEL-classification: D 82, G31

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1 Introduction

When devising resource (or budget) allocations, principals regularly face a trade-off between delegating to an agent tasks, for which he has a comparative advantage, and retaining sufficient control of the agent’s actions. This control is warranted to contain the potential problems of adverse selection – the agent over-stating the value of the project or his own productivity in order to attract additional budget – and moral hazard – the agent providing insufficient effort in using the budget, leading to under-achievement. In a perfect world, the principal could address these agency problems either by retaining full control; or by delegating as fully as possible while at the same time exposing the agent to the strongest possible incentives, e.g. by selling the firm. In reality, monitoring costs on the one hand and wealth constraints or risk aversion on the other render these solutions impractical. Usually the principal will have to strike a balance between delegating and retaining control. Harris and Raviv (1998) explain delegation of budgeting decisions as a response to costly auditing. The cost of communication, the incompleteness of agency contracts and the ‘value of flexibility’ also tend to favour delegation (Melumad et al. 1997). Holmstrom and Roberts (1994) demonstrate that delegation - in the sense of transferring assets to the agent and/or of relaxing controls on the returns of assets accruing to the agent - paired with strong outcome related incentives, is an optimal response to a lower cost of outcome measurement or a lower risk. In contrast, more control is favoured if spill-overs between agents require central co-ordination or if agents hold lower bargaining power than the principal vis-à-vis third parties (Caillaud et al. 1996).

One common aspect of these models is that agents are concerned about central control only insofar as it limits their scope to manipulate information or engage in slacking. In this regard, the models are firmly rooted in the neo-classical paradigm in which agents are motivated by extrinsic incentives, i.e. by performance-related rewards or punishments. A more recent line of literature acknowledges that agents may be motivated by non-financial-aspects of their jobs. Frey (1993, 1997) argues that agents are driven by intrinsic motivation. This means that agents derive satisfaction from performing a task well and provide effort even in the absence of extrinsic incentives. However, intrinsic motivation is not independent of the working environment. Specifically, it may be crowded out by extrinsic incentives such as performance standards and the associated punishments or rewards. The latter tend to destroy the agents’

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1 Hence, agents not only care about outcomes but also about the process at which these outcomes are arrived at (Sen 1997). Frey and Benz (2002) argue that procedural utility derived from the mode of production (self-
self-evaluation of doing ‘something decent’ over and above what is expected or even enforced anyway. The control associated with extrinsic rewards also leads to a loss of self-determinedness that further undermines motivation. The possible crowding-out of intrinsic motivation constrains the principal’s scope in providing external performance incentives.\(^2\) Even in the absence of intrinsic motivation, agents may have a preference for autonomy. This is the case if the agent’s performance and the degree of control are – at least to some extent – observable. In this case, a good performance under a greater degree of autonomy is clearly a better signal of an agent’s ability than the same performance achieved under tight supervision.\(^3\) Thus, the reputation and, with it, the prospective earnings of an agent tend to increase in the degree of autonomy.

Applying these ideas to budgeting one should expect the optimal mix of delegation and control to be determined not only by technological and informational considerations but also by its impact on motivation. Ignoring the latter could lead to distortions in the agent’s effort and/or revelation of private information, outweighing the benefits from greater control. More positively, the principal can use autonomy as an incentive device. The present analysis addresses the implications for the allocation of budgets of an agent’s preference for autonomy under moral hazard and adverse selection. We consider a model in which a principal allocates to an agent a budget for the purpose of production and determines the agent’s autonomy in the use of it. She may also use a financial transfer to motivate the agent. The agent’s utility increases both in the transfer and in an intrinsic benefit (job satisfaction), the latter depending on the degree of autonomy. Agents differ in their efficiency in using a delegated budget either because they differ in ability or they face projects of varying profitability. Here, we assume that an agent’s type may be unknown to the principal. The agent’s preferences over autonomy determine his effort but also his choice between different budgetary allocations. Thus, the

\(^2\) Barkema (1995) provides evidence for this by showing that the effect of external intervention on work performance in Dutch firms is significantly positive (negative) in the case of impersonal (personal) control. Since intrinsic motivation tends to be more sensitive in personal relationships, the evidence lends some support to the crowding out hypothesis. Roomkin and Weisbrod (1999) provide evidence that managerial incentives are significantly weaker in non-profit as opposed to for-profit hospitals. This may be due to a number of explanations. First, it may reflect the consumption of rents as slack within non-profit organisations. Second, it may reflect that non-profit hospitals perform additional tasks related to equity objectives or the provision of public goods. In as far as these tasks are difficult to monitor, the multi-task nature of the problem may require weak incentives on all tasks. Finally, the absence of strong incentives is consistent with the presence of intrinsically motivated staff. This is particularly likely when intrinsically motivated managers self-select into non-profit organisations.

\(^3\) There is an extensive literature on the incentive effects of career concerns (e.g., Holmstrom 1999, Dewatripont et al. 1999), but to my knowledge it does not address the signal’s dependence on the degree of autonomy.
level of autonomy granted to an agent is determined technologically by the agent’s efficiency in handling the budget, motivationally by the agent’s propensity to provide effort in return to autonomy, and by the agent’s self-selection incentives.

If agents value autonomy, the principal uses delegation as a stimulus for the provision of effort. Under full information about an agent’s productivity but not about effort an efficient type receives both the greater budget and the greater degree of autonomy. Under asymmetric information, the principal distorts the overall budgets and the degree of delegation from their efficient levels in order to guarantee self-selection. In this, the agent’s preference for autonomy turns out to play an important role. Unexpected allocations arise in the presence of a strong preference for autonomy. If the preference for autonomy is sufficiently strong but not too strong, the inefficient agent receives the greater budget and the degree of delegation is distorted downwards for the efficient agent and upwards for the inefficient agent. If the preference for autonomy is very strong the budget allocation depends on whether the principal employs a financial transfer as an additional instrument to generate self-selection. If the agent values job satisfaction sufficiently more than the monetary benefit, the principal abstains from the use of financial incentives. Budgets are then pooled if the preference for autonomy is strong. Otherwise, in a separating equilibrium a transfer is paid to the inefficient type, who receives the lower budget but, perversely, a greater degree of delegation. Our findings illustrate the potentially important role of preferences for autonomy – or preferences over the mode of production more generally – for intra-organisational resource allocation. This role becomes particularly pertinent when the agent’s preferences differ significantly from the principal’s.

We can think of at least three environments in which intrinsic motivation is important and leads to a potential for conflict over autonomy between the principal and agent. Intrinsic motivation and a concern for autonomy are likely to matter to agents performing creative tasks such as R&D, the drawing up of advertising campaigns or strategic planning in general business environments, but also the development of projects in the artistic or entertainment sector. In these sectors agents are usually directly motivated by the quality of their output rather than by financial incentives alone. Another area of application for our model is an environment in which the agent enters a (possibly long-term) relationship with a client and is concerned about providing a good service to this client. This includes many of the professions such as physicians, teachers, lawyers and consultants. The third environment is one of a public or non-profit organisation, where a public service spirit is the source of incentives.
A recent literature deals with the effects on agency relationships of intrinsic motivation or, similarly, a public service spirit. Francois (2000) compares the incentives within public or non-profit as opposed to private providers when agents care in an altruistic way about the service provided. Delfgaauw and Dur (2002) derive optimal incentive contracts for intrinsically motivated workers and consider the selection of workers into the firm when motivation is unobservable but may be signalled. While asymmetric information about agent type pertains in our model as well, we consider screening rather than signalling. Glazer (2004) analyses how the principal chooses an input jointly with an intrinsically motivated agent’s choice of effort. In all of these models agents care about output but in contrast to our model their motivation does not depend on the mode of production.

Besley and Ghatak (2003) also model public service motivation but take into account that the organisation’s mission influences incentives, where a mission is defined as the attributes of a project that make people value its success over and above the monetary reward. Their analysis focuses on principal-agent matching rather than on optimal incentive schemes. Aghion and Tirole (1997) show that it pays principals to grant ‘real’ authority, even at the expense of control, if this induces agents to exert additional effort and relaxes their participation constraint. Their analysis then focuses on how organisational overload or institutional arrangements commit principals to transfer real authority to agents even when retaining formal authority. Murdock (2002) analyses how agents can be motivated by allowing them to establish a loss-making project with high intrinsic value in exchange for their effort towards a profit-making project. Due to the principal’s lack of commitment these contracts will usually have to be relational rather than explicit. While these models share with ours the idea that the principal can enhance (intrinsic) motivation by giving up some control to the agent, they focus on the moral hazard problem alone and do not consider adverse selection, the key issue of our analysis.

Bénabou and Tirole (2003: section 3.1) consider the scope for crowding in intrinsic motivation by way of granting autonomy to an agent. They assume that the agent rather than the principal is imperfectly informed about his ability. By granting autonomy the principal

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4 See also Dixit (2002). Heckman et al. (1996) provide empirical evidence supporting the presence of a public service spirit. They find that US social workers systematically select the least employable cases into job training programme in spite of performance incentives encouraging cream-skimming in favour of the most employable.

5 Within different set-ups, Bénabou and Tirole (2003) and Grepperud and Pedersen (2004) derive optimal performance pay when intrinsic motivation may be crowded out.

6 A similar spirit underlies the models by Gertner et al. (1994) and Mitusch (2000), where the agent’s effort decreases in the degree of control as this delimits a real resource (rather than an intrinsic) rent.
signals that the agent’s ability is high. By raising the agent’s confidence this raises the willingness to provide effort. Similar to our model, asymmetric information about the agent’s type bears on the degree of delegation, but in a different way, as Bénabou and Tirole assume the principal rather than the agent to be informed.

Our model is also related to the principal-agent literature as applied to budgeting problems and is closest in spirit to Harris and Raviv (1998) and Bernardo et al. (2001). Harris and Raviv (1998) consider an adverse selection setting without moral hazard in which the principal can use a costly audit as an instrument besides a budget assignment. They show that the optimal capital allocation generally implies over- (under-) investment for projects with low (high) productivity. The extent of this distortion increases in the audit cost. Harris and Raviv explain the scope for delegation – in the sense of the manager having a choice over capital allocations across two projects that cannot be predicted by the principal – as an increasing function of the auditing cost. The focus of our analysis lies not so much with an explanation of delegation, the necessity of which we take for granted, but rather with the incentive role of delegation with regard to moral hazard and adverse selection. Similar to us, Bernardo et al. (2001) consider both moral hazard and adverse selection, where in both cases agents have an incentive to over-report profitability or level of ability in order to attract high budgets. As agents are not directly concerned about output in Bernardo et al., they can only be motivated by performance pay. While we also consider the scope for performance pay, one key instrument to stimulate effort is the budget allocation, as modified by the degree of autonomy.

The remainder of the paper is organised as follows. Section 2 introduces the model. Section 3 derives the full information optimum and discusses the role of delegation in resolving moral hazard when output is non-contractible. In section 4, we analyse in detail the allocation under adverse selection maintaining the assumption of non-contractible output. Section 5 extends the findings to the case where financial transfers are used (section 5.1) and to the case of contractible output (section 5.2). Section 6 concludes. The proofs are relegated to an appendix.

2 The model

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7 For a more detailed review of this literature see Harris and Raviv (1998).
An agent produces an output or service, the value of which is given by a homogeneous function
\[ h(e, b, r, \beta) = ef(b, r, \beta), \]
where \( e \) is a non-contractible effort, \( b \) is the delegated budget and \( r \) is the budget, over which the principal retains control. The productivity of the budgetary input \( b \) is measured by \( \beta \). In order to simplify the analysis, we adopt a Cobb-Douglas specification
\[
f(b, r, \beta) = b^\beta r^\rho, \quad \beta \in (0,1); \quad \rho \in (0,1) \quad 2(\beta + \rho) < 1.8
\]
Thus, each budget exhibits non-negative but decreasing returns. A few words are warranted as to our interpretation of the delegated and controlled budgets. Suppose the principal assigns an overall budget \( B = b + r \) to the agent. The delegated budget \( b \) is the part of the overall allocation over the use of which the agent can dispose freely in the course of production. The controlled budget \( r \) may be spent by the principal on the purchase of inputs, which are then transferred to the agent for further use. Alternatively, the agent may have to obtain the principal’s approval on the use of \( r \), or only use this budget according to strict guidelines.

Let us now introduce the concept of delegation and autonomy we have in mind. Define \( D \equiv \frac{r}{r + b} \), \( D \in [0, \infty) \) as the ‘degree of delegation’. It appeals to us to use the degree of delegation as a proxy measure of autonomy, understood as the absence of central intervention.9 Using \( D = \frac{r}{r + b} \) together with \( B = b + r \) to rewrite \( r = \frac{rb}{1+D} \) and \( b = \frac{rb}{1+D} \), we obtain the relationship
\[
F(B, D, \beta) = f\left(\frac{rb}{1+D}, \frac{rb}{1+D}, \beta\right) = D^\beta \left(\frac{rb}{1+D}\right)^{\beta + \rho}
\]
describing production as a function of the degree of delegation and the overall budget. Subsequently, we will make use of both specifications, \( f(b, r, \cdot) \) and \( F(B, D, \cdot) \) according to analytical and presentational convenience.

The model remains general about the particular use in production of the budgetary inputs \( b \) and \( r \). Likewise we side-step the issue, as to which activities should be delegated in order to focus on the incentive role of delegation. In this regard, the function \( F(\cdot) \) is a ‘black-box’, reflecting that production can be organised in a variety of ways, involving a greater or lesser extent of delegation. The production elasticity \( \beta \) captures the efficiency of delegation. It depends on the agent’s technology and information as well as on the transaction costs

\[8\] The last constraint on the parameters guarantees concavity of the objective function.

\[9\] We define the degree of delegation as a factor-intensity for analytical convenience. A more intuitive measure of the degree of delegation would be the share of the delegated budget in the total budget \( \frac{b}{r+b} \). It is easy to verify that for any two pairs \((b, r)\) and \((b', r')\) it is true that \( \frac{b}{r+b} > \frac{b'}{r'+b'} \iff D = \frac{r}{r+b} > \frac{r'}{r'+b'} \Rightarrow \). Thus, a greater degree of delegation implies and is implied by a greater share of the delegated budget in the total budget.
involved in delegation. For example, $\beta$ may be low if the agent uses an inferior technology, if he operates under poorer information, if he wields little bargaining power vis-à-vis local suppliers, or if he is simply less effective in managing a budget. Alternatively, $\beta$ may characterise the specific project to be carried out by the agent, where projects with a high $\beta$ allow greater gains to delegation perhaps because they are of a non-standard nature so that there is little experience at the centre. The elasticity $\rho$ captures the effectiveness of controlling budgets. It is determined by the cost of communication between the principal and agent and the cost of ensuring the agents’ compliance.\textsuperscript{10} It is readily verified that $F(\cdot)$ is maximised at $\tilde{D} = \frac{\beta}{\rho}$ for any budget $B$. Indeed, if agents are unconcerned about autonomy, $\tilde{D}$ is the optimal degree of delegation. In the following, we will occasionally refer to it as the ‘technologically’ efficient degree of delegation.\textsuperscript{11}

The agent receives utility $v = u + at$, where $u$ is the agent’s non-monetary benefit of production and $at$ is the value the agent places on a monetary transfer received from the principal, e.g. a salary. The agent’s non-monetary benefit of production is given by

$$u = w(D, \alpha)h(e, b, r\beta) - \frac{e^2}{2}; \quad D = \frac{\beta}{\rho}$$

(1)

It increases in output $h(\cdot)$ and decreases in the quadratic cost of effort. The extent to which an agent benefits from his production is governed by the weight $w(D)$, embracing the agent’s preferences over autonomy and control. Specifically, let

$$w(D, \alpha) = kD^\alpha, \quad \alpha \in [0, \rho], \quad k > 0,$$

where the parameter $\alpha$ reflects the agent’s preference for autonomy.\textsuperscript{12} A greater degree of delegation, i.e. greater autonomy, raises the utility weight if and only if $\alpha > 0$. The agent’s concern about production could be explained by any of the following reasons. First,

\textsuperscript{10}In this regard, we distinguish the actions that can be controlled for by the principal from those for which the unverifiable effort $e$ is relevant. The former could relate to spending decisions, whereas the latter would relate to the effort taken by the agent in drawing up the project and generating options that enhance its value.

\textsuperscript{11}\$\tilde{D}$ can be understood as the outcome of a more complete model of $F(\cdot)$. For instance, $F(B, D, \beta)$ captures in a nutshell the model by Harris and Raviv (1998) which explains delegation $D$ (although defined differently) and the level of an overall budget $B$ as a function of auditing cost $\rho$. When agents use budgets for purchasing, there may be a trade-off between the centre’s greater bargaining power and the agents’ superior information about producers (Caillaud et al. 1996). If the production set is convex with respect to this trade-off, again this implies an optimal degree of delegation.

\textsuperscript{12}$k$ is a scaling parameter.
intrinsically motivated agents care about $h(\cdot)$, with the motivation increasing in the degree of autonomy. Second, agents may derive a ‘warm glow’ benefit à la Andreoni (1990) from providing to their customers a service $h(\cdot)$, a benefit they derive only to the extent of their ‘personal’ contribution towards it. Third, agents may be driven by professional status (e.g. Encinosa et al. 1997). As status usually rises with the degree of responsibility, it is plausible to assume that the agent’s benefit from status increases not only with output but also with the extent to which this has been produced autonomously. Finally, $w(\cdot)h(\cdot)$ may be a measure of the agent’s (discounted) future earnings, as determined by the reputation from having carried out the present task. It is reasonable to assume that the value of the reputation increases not only in the outcome but also in the degree of autonomy, as this measures the agent’s individual as opposed to the organisation’s contribution towards production.13

In the following, we assume that there are two types of agents/projects, efficient (E) and inefficient (I), where efficient agents/projects are characterised by $\beta_E > \beta_I$. Let $\lambda \in [0,1]$ denote the probability of an agent/project being an E type, or alternatively the share of E types in the population. We assume that the budgets are levied at a constant unit cost of $\phi$. A risk-neutral principal then maximises the expected net value of production

$$
\pi(b_E,r_E,t_E,b_I,r_I,t_I) = \left\{ \lambda[h(e_E,b_E,r_E,\beta_E) - \phi(b_E + r_E + t_E)] 
+ (1 - \lambda)[h(e_I,b_I,r_I,\beta_I) - \phi(b_I + r_I + t_I)] \right\} 
$$

With regard to timing, we follow the standard contracting framework under adverse selection as summarised in figure 1.

Figure 1: Timing of contract.

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13 In some instances agents may dislike responsibility and prefer central control. An analysis for the case $\alpha < 0$ can be found in Kuhn (2004).
The agent privately observes his type $\beta \in \{\beta_I, \beta_E\}$. Then, the principal offers a contract $\{b_i, r_i, t_i\}$, which the agent accepts or rejects. Upon acceptance the agent self-selects. The contract is then executed with the principal providing the budgets and the agent choosing effort. Finally, output is realised and transfers take place. We derive our main result for a setting in which neither effort, $e$, nor output, $h$, are contractible. We have in mind a situation in which the agent does not produce a marketable output but rather an intermediate input into a more general production function. This embraces activities such as R&D, advertising, designing or strategic planning, the contributions of which towards a company’s profit are difficult to verify. Output verification is also difficult in case of a bureaucrat producing a non-market good or service. In this case, the transfer $t_i$ from the principal to the agent cannot be made contingent on output but only on the budgetary allocation $\{b_i, r_i\}$. Note that the aforementioned activities often feature a strong role for motivation and autonomy. In order to gauge the robustness of our results, we present in section 5.2 some results for the case of contractible output.

We assume that the agent faces a wealth constraint $t \geq t_0$ and that the outside benefit $v_o$ is sufficiently low such that participation at $t \geq t_0$ is guaranteed. A slack participation constraint is not unlikely when the agent’s non-monetary benefit is substantial but cannot be extracted due to a wealth constraint. We could think of at least two reasons for why the intrinsic benefit may outweigh the outside benefit such that $u \geq v_0$. First, many of the professional, ‘creative’, charitable or public services relying on intrinsic motivation also involve quasi-rents due to the accumulation of specific knowledge or due to long-term provider-client relationships. Researchers or advertising designers working for a specific company acquire company specific knowledge that enhances both productivity and the intrinsic benefit from working for this particular company or on this particular project. The same applies to sales personnel or professionals such as physicians, lawyers or consultants who have accumulated intimate knowledge on their clients. Furthermore, it is plausible that a provider’s intrinsic benefit from servicing a client increases in the duration of the relationship as stronger personal ties develop. All of this suggests that the agent’s non-monetary benefit from performing this particular activity may well lie above his outside utility. Second, the agent may face a principal who monopolises resources that are crucial for the realisation of a project with intrinsic value to the agent. For example, the agent may be a researcher who has to rely on a highly specific research lab/technology that is monopolised by a single institution. Another
example would be the realisation of a project by an artist or a movie/theatre director that requires funding or distribution channels provided only by a monopolistic large-scale producer. The intrinsic value of the project to the agent (when carried out) is then likely to exceed the outside utility (of not carrying out the project). In all of the above cases, rent equalisation would require a possibly substantive payment from the agent to the principal. Under our assumption that such payments are ruled out by wealth constraints, the agent’s non-monetary rents cannot be (fully) extracted. The agent’s ex-post utility then satisfies \( v = u + \alpha \), where the agent could always choose a non-verifiable effort \( e = 0 \). Without further loss of generality, we normalise \( v_0 = 0 \) and \( t_0 = 0 \) so that participation is guaranteed irrespective of the agent’s type.\(^{14}\)

### 3 Allocation when output is not contractible: Motivation by delegation

For the moment we assume that the agent’s productivity \( \beta \) is observable. When neither effort nor output are contractible the principal cannot use the transfers to stimulate effort on the part of the agent. In this case, it is optimal to set \( t_e = t_i = 0 \), where the wealth constraint binds. As the principal can identify the type of individual agents/projects, she allocates type specific budgets so as to 

\[
\max_{b_e, r_e, b_i, r_i} \pi \bigg|_{t_e = t_i = 0}
\]

as given in (2). In so doing, she will take into account the effect of delegation on the agent’s motivation and provision of effort. Solving the problem backwards, we begin by considering the agent’s choice of effort. Given the budget, the agent chooses effort so as to maximise utility (1). From the first-order condition

\[
e^* = \hat{e}(b, r, \beta) = w(D)f(b, r, \beta) = kb^{\beta+\alpha}r^{\rho-\alpha}
\]

we obtain output and utility as reduced form functions of the budgets

\[
\hat{h}(b, r, \beta) = \hat{e}(b, r, \beta)f(b, r, \beta) = w(D)f(b, r, \beta) = kb^{2\beta+\alpha}r^{2\rho-\alpha}
\]

\[
\hat{u}(b, r, \beta) = \hat{e}(b, r, \beta)[w(D)f(b, r, \beta) - \frac{1}{2}\hat{e}(b, r, \beta)] = \frac{1}{2}[w(D)f(b, r, \beta)]^2 = \frac{1}{2} kb^{2(\beta+\alpha)}r^{2(\rho-\alpha)}
\]

\(^{14}\) Generally, the interaction of the wealth and participation constraints defines a range of different regimes similar to those featured in Laffont and Martimort (2002: section 3.5). With the main interest of the present analysis on the regime with a slack participation constraint, we do not provide a complete characterisation.
Placing the restriction $b \geq 1$ we obtain $\hat{h}_\beta(b,r,\beta) > 0$ and $\hat{a}_\beta(b,r,\beta) > 0$ as well as $\hat{h}_{s\beta}(b,r,\beta) > 0$ and $\hat{a}_{s\beta}(b,r,\beta) > 0$, $x = b,r,\beta$.\(^{15}\) Hence, total and marginal output as well as total and marginal utility are greater for the E type, reflecting greater productivity. Again, for convenience we will sometimes express effort, output and utility as functions $\hat{E}(B,D,\beta) = \hat{h}(\frac{B_0}{1+D}, \frac{B_r}{1+D}, \beta)$, $\hat{H}(B,D,\beta) = \hat{h}(\frac{B_0}{1+D}, \frac{B_r}{1+D}, \beta)$ and $\hat{U}(B,D,\beta) = \hat{u}(\frac{B_0}{1+D}, \frac{B_r}{1+D}, \beta)$. One can easily check that equilibrium effort and utility are concave in $D$, with $\text{sgn}\frac{\partial \hat{D}}{\partial D} = \text{sgn}\frac{\partial \hat{U}}{\partial D} = \text{sgn}[\alpha + \beta - (\rho - \alpha)D]$. The principal can then stimulate extra effort by increasing the degree of delegation up to the level

$$\hat{D}_i = \frac{\beta_i + \alpha}{\rho - \alpha} = \arg \max \hat{U}(B_i,D_i,\beta), \; i = E,I$$

(6)

that maximises the agent’s utility. The concavity of $\hat{E}(B,D,\beta)$ in $D$ reflects the conventional wisdom that granting some responsibility tends to make people work harder, while too much delegation encourages slack.

Consider now the principal’s choice of budgets. The optimum in the presence of moral hazard (with non-contractible output) $\{b_E^*, r_E^*, b_i^*, r_i^*\}$ can be characterised as follows.

Proposition 1. (i) The efficient agent receives both a greater delegated and a greater controlled budget, i.e. $b_E^* > b_i^*$ and $r_E^* > r_i^*$. (ii) Moral hazard leads to an upward distortion for both types in the degree of delegation over and above the technologically efficient level $\hat{D}_i$ if and only if agents have a preference for autonomy, i.e. if and only if $\alpha > 0$. (iii) The principal delegates to greater extent to the efficient agent, i.e. $D_E^* > D_i^*$.

Proof: See Appendix.

The optimal degree of delegation under moral hazard is given by

$$D_i^* = \frac{(2\beta_i + \alpha)}{2\rho - \alpha}; \; i = E,I$$

(7)

\(^{15}\) As is readily checked from the first-order conditions for the optimal choice of budgets as described in the Appendix to Proposition 1, $b \geq 1$ is satisfied if $\phi$ is sufficiently small.
It is readily checked that $\alpha > 0 \iff \hat{D}_i > D_i^* > \tilde{D}_i$. If and only if the agent prefers a degree of delegation $\hat{D}_i$ over and above the technologically efficient level $\tilde{D}_i$, the principal can stimulate additional effort by over-delegating.

As the agent’s utility increases both in $b$ and $r$, we have $\hat{u}(b_E^*, r_E^*, \beta_i) > \check{u}(b_i^*, r_i^*, \beta_i)$ and both types prefer the budget allocated to the E type. Inefficient agents then have an incentive to misrepresent their type, thereby causing a problem of adverse selection.

4 Allocation when productivity is unobservable: Screening by delegation

From now on, we consider the level of efficiency $\beta$ to be the agent’s private information, the principal only being informed about the distribution of types. Expressing for expositional convenience the analysis in terms of the degree of delegation $D_i$ and total budget $B_i$, we can write the principal’s problem as

$$\max_{B_i, R_i, D_i, i = E, I} \pi = \lambda \left[ \hat{H}(B_E, D_E, \beta_i) - \phi(B_i + t_E) \right] + (1 - \lambda) \left[ \hat{H}(B_i, D_i, \beta_i) - \phi(B_i + t_i) \right]$$

subject to the self-selection constraints

$$\hat{U}(B_i, D_i, \beta_i) + at_i \geq \hat{U}(B_E, D_E, \beta_E) + at_E$$  \hspace{1cm} (ICI),

$$\check{U}(B_E, D_E, \beta_E) + at_E \geq \check{U}(B_i, D_i, \beta_i) + at_i.$$  \hspace{1cm} (ICE),

and the wealth constraints $t_E \geq 0$ and $t_i \geq 0$. (ICI) and (ICE) imply the monotonicity condition

$$\check{U}_{\beta_i}(\cdot, \beta_i) \Delta B + \check{U}_{D_i}(\cdot, \beta_i) \Delta D \geq 0$$  \hspace{1cm} (M),

with $\Delta B = B_E - B_i$ and $\Delta D = D_E - D_i$. This is a necessary condition for the existence of a separating allocation.16

Let $\left\{ B_E^{**}, D_E^{**}, t_E^{**}, B_i^{**}, D_i^{**}, t_i^{**} \right\}$ denote the optimal allocation. For the moment, suppose that the principal does not use financial transfers, such that $t_{E}^{**} = t_{i}^{**} = 0$. We will characterise in Lemma 5 at the end of this section the precise circumstances under which this

---

16 The appropriate derivatives of $\hat{U}(B, D, \beta)$ are easily determined from (5) when setting $b = \frac{D_i}{t_i + B_i}$ and $r = \frac{B_i}{t_i + B_i}$.
is the case. Intuitively, the principal abstains from transfers whenever the agent has a weak preference for monetary rewards as opposed to job-satisfaction. In this case motivation by way of financial transfers becomes too expensive for the principal and she abstains.

The following Lemma narrows down the monotonicity condition for the case in which transfers are not used.

**Lemma 1.** For $t_E^{**} = t_I^{**} = 0$, it must be true that $\Delta D \geq 0$.

**Proof:** See Appendix.

Thus, as expected, it is unfeasible in the absence of transfer payments to reverse the degree of delegation. We can now characterise as follows the degree of delegation assigned to each type in a separating equilibrium. Define

$$
\kappa_i = \mu_i \frac{\rho - \alpha}{\beta_j + \rho} \left(1 + D_i^{**}\right)^\alpha \left(1 + B_j^{**, D_i^{**}, \beta_j}\right) > 0; \ i, j = E, I,
$$

where $\mu_i$ is the shadow price of the (ICI) constraint. Here, $\mu_i > 0$ is shown as part of Lemma A1 in the Appendix. The following holds.

**Lemma 2.** The optimal levels of delegation $D_i^{**}$ and $D_E^{**}$ are given by

$$
D_i^{**} = \min \left\{ D_i^* + \frac{\kappa_i}{\phi(1 - \lambda)} (\hat{D}_i - D_i^*); D_E^{**} \right\};
$$

(8a);

$$
D_E^{**} = \max \left\{ D_E^* + \frac{\kappa_E}{\phi\lambda} (D_i^* - \hat{D}_i); D_E^{**} \right\};
$$

(8b).

**Proof:** See Appendix.

Generally, the optimal degree of delegation in the presence of adverse selection, $D_i^{**}$, deviates from the level, $D_i^*$, that would be realised in the presence of moral hazard alone. According to (8a), incentive compatibility for the I type requires an increase (decrease) in the I type’s degree of delegation if this type prefers a degree of delegation, $\hat{D}_i$, that exceeds (falls short of) $D_i^*$. Likewise, we see from (8b) that (ICI) requires an increase (decrease) in the E
type’s degree of delegation if \( \hat{D}_i \) falls short of (exceeds) the degree of delegation \( D^*_E \) that is optimal for the E type in the absence of adverse selection.

Define

\[
\frac{2(\beta_E - \beta_I)}{\rho + 2\beta_E - \beta_I} \quad (9a); \quad \hat{\alpha} := \frac{2(\beta_E - \beta_I)}{(\rho + 2\beta_E - \beta_I)p^{2\beta_E} - (\rho + \beta_I)p^{2\beta_I}} \quad (9b);
\]

where \( P := \frac{\beta^p D^p}{1 + \beta^p} \) with \( B^p \) and \( D^p \) denoting the pooling levels of the budget and degree of delegation, respectively. Note that \( \hat{\alpha} \in (\overline{\alpha}, \rho) \). We can now distinguish three regimes.

**Lemma 3.** (i) \( \alpha \in [0, \overline{\alpha}) \Leftrightarrow D^*_I \leq D^*_I \leq D^*_E < D^*_E \); (ii) \( \alpha \in [\overline{\alpha}, \hat{\alpha}) \Leftrightarrow D^*_I < D^*_I \leq D^*_E < D^*_E \); (iii) \( \alpha \geq \hat{\alpha} \Leftrightarrow D^*_E = D^*_I = D^*_E \).

**Proof:** See Appendix.

We thus know how the degree of delegation for each type evolves with the preference for autonomy, \( \alpha \). Before we discuss the separating and the pooling allocation more fully let us establish how the total budgets \( B^*_E \) and \( B^*_I \) depend on \( \alpha \).

**Lemma 4.** (i) \( \alpha \in [0, \overline{\alpha}) \Rightarrow B^*_I > B^*_I; B^*_E > B^*_E \), where \( B^*_I < B^*_E \) if and only if \( \alpha \) sufficiently small; (ii) \( \alpha \in [\overline{\alpha}, \hat{\alpha}) \Rightarrow \max\{B^*_I, B^*_E\} < \min\{B^*_I, B^*_E\} \); (iii) \( \alpha \geq \hat{\alpha} \Leftrightarrow B^*_I < B^*_I = B^*_E = B^*_E < B^*_E \).

**Proof:** See Appendix.

Under adverse selection, the principal always pays a budget to the efficient (inefficient) type that falls short (exceeds) the optimal budget, under moral hazard alone. Self-selection requires that a rent is paid to the informed agent, in our case the I type. Where agents care far less about financial transfers than about their job satisfaction, the principal does not use a transfer but rather pays the rent in form of an excess budget \( B^*_I - B^*_I > 0 \). Rental payments in form of excessive budgets are associated with an efficiency loss, as the marginal product of \( B^*_I \) falls short of the marginal cost of funds. By reducing the E type’s budget from its level \( B^*_E \) the principal lowers the attractiveness of mimicking this type and is, thus, able to reduce the excess budget paid to the I type and the associated inefficiency. This resembles the finding by
Harris and Raviv (1998) where, in the presence of an imperfect monitoring technology, the principal reacts to a situation of asymmetric information by reducing the gap in the budgets aimed at the efficient and inefficient type. Bearing this in mind, we can now proceed with a fuller characterisation of the allocation under asymmetric information. The findings in Lemmas 3 and 4 can be combined as follows.

**Proposition 2.** (a) If agents have a weak preference for autonomy \((\alpha \in [0, \bar{\alpha}])\), the principal delegates to both types more than she would in the absence of adverse selection. She pays a greater total budget to the inefficient type (as compared to the efficient type) if and only if the degree of delegation is sufficiently high. (b) If agents have a strong preference for autonomy \((\alpha \in [\bar{\alpha}, \hat{\alpha}])\), the principal delegates less (more) to the efficient (inefficient) type than she would in the absence of adverse selection. She pays a greater total budget to the inefficient type. (c) If agents have a very strong preference for autonomy \((\alpha \in [\hat{\alpha}, \rho])\), the principal pools the degree of delegation and the budgets.

According to the agent’s preference for autonomy, we can distinguish three regimes. Regimes (a) and (b) involve separation; regime (c) involves pooling. When screening the agents, the principal uses the degree of delegation in addition to the budget. This is best illustrated with reference to the benchmark case in which agents do not have preferences about the mode of production, i.e., about the degree of autonomy. In this case, \(\alpha = 0\) which from (8a) and (8b) implies \(D_{E}^{**} > D_{E}^{*} > \hat{D}_{I}^{*}\) and \(D_{I}^{**} = D_{I}^{*} = \hat{D}_{I}^{*}\). Since the agent prefers the same degree of delegation as the principal, any deviation from the optimal level leads to a reduction in job satisfaction. Recall that the information rent paid to the I type (in terms of an excess budget \(B_{I}^{**} - B_{I}^{*} > 0\)) is set so as to equalise the job satisfaction the I type receives on his own and on the E type’s contract. But then any deviation from the optimal degree of delegation \(D_{I}^{*} = \hat{D}_{I}\) must lead to a decrease in satisfaction on the ‘own’ job calling for an increase in the budget/rent. The principal thus refrains from a distortion. In contrast, an increase in the E type’s degree of delegation renders this allocation less attractive for the I type and, thereby, helps to contain the information rent and the associated distortions in the budgets.

If \(\alpha > 0\) the principal and agent no longer agree on a preferred degree of delegation.

Specifically, the agent prefers a greater degree than the principal, \(\hat{D}_{I} > D_{I}^{*}\). There is now scope for the principal to reduce the information rent (in terms of excess budget) by granting the I type a more preferred degree of delegation. As long as the preference \(\alpha\) is sufficiently
low relative to the productivity spread $\beta_E - \beta_I$ [regime (a)], the I type’s preferred degree of delegation falls short of the one the principal would assign to the E type, i.e. $\hat{D}_I < D_E^*$. The principal can then enhance the I type’s satisfaction on the own job by setting the degree of delegation in the interval $D_I^{**} \in \left[D_I^*, \hat{D}_I\right]$, and reduce the I type’s satisfaction on the E type’s job by setting $D_E^{**} > D_E^*$. This allows the principal to contain the rent and the associated distortions in the budget levels. Indeed, as long as $\alpha$ is sufficiently close to zero, the E type will receive the greater total budget.

If $\alpha$ is large relative to the productivity spread [regime (b)], gaining greater autonomy becomes a strong incentive for I when seeking to select E’s allocation. In this case $\hat{D}_I > D_E^*$, so that the I type prefers a degree of delegation in excess of what the principal would grant to the E-type. While continuing to delegate to I in excess of $D_I^*$, the principal now reduces the degree of delegation to the E type below $D_E^*$ in order to make this allocation less attractive.

While the principal still delegates to the E type in excess of I type, she now assigns a greater total budget to the I type (as opposed to the E type). This case is illustrated in figure 2a, where the E type receives a budgetary assignment E and the I type an assignment E’. The dashed line $B_E B_{E'}$ gives the iso-budget curve corresponding to $B_{E}^{**}$, where obviously $B_{E}^{**} < B_{I}^{**}$. With both E and E’ lying on the same indifference curve $\hat{U}_I$, incentive compatibility is satisfied for the I type. Being more efficient under a greater degree of delegation, the E type strictly prefers the allocation at E despite the lower overall budget. As total budgets are reversed, greater autonomy is now effectively traded against a lower total budget.

Figure 2a: Separation in regime (b). Figure 2b: Non-existence of separation in regime (c).
Finally, for a very high preference for autonomy only a pooling allocation is feasible, whereby both agents receive the same budget and the same degree of delegation [regime (c)]. Here, the agent’s preference for autonomy is so strong that the principal would effectively have to delegate more to the I type than to the E type. Figure 2b illustrates that this is impossible. Suppose E and E’ are assigned to the E and I type, respectively, such that $D_E^{**} < D_I^{**}$. While the I type is indifferent, this allocation clearly violates the E type’s incentive constraint (ICE). While a reduction in the overall budget to the level at E’’ would restore incentive compatibility for the E type, it is now violated for the I type. The best the principal can then attain is an allocation in which both budgets and the degree of delegation are pooled.

This is reflected in the fact that an allocation with $\Delta D = D_E^{**} - D_I^{**} < 0$, as is required for $\alpha > \hat{\alpha}$ in regime (c), violates the monotonicity condition (see Lemma 1). The underlying reason is a direct conflict between the incentive compatibility constraints (ICI) and (ICE) that cannot be resolved in the absence of transfers. Guesnerie and Laffont (1984) call an environment in which separation becomes unfeasible ‘non-responsive’. In their set-up and, similarly, in Laffont and Martimort (2002: section 2.10.2) non-responsiveness results from a conflict between the allocation that maximises total surplus and the monotonicity condition. In our case, this is different. It is easily checked that the maximisation of total surplus

$$\lambda \left[ H\left(\cdot, \beta_e\right) + U\left(\cdot, \beta_e\right) - \phi B_e \right] + (1 - \lambda) \left[ H\left(\cdot, \beta_i\right) + U\left(\cdot, \beta_i\right) - \phi B_i \right]$$

implies $\Delta D > 0$ and is, therefore, compatible with monotonicity. In our case, non-responsiveness occurs due to a conflict between rent extraction (from the I type), requiring $\Delta D < 0$ and the monotonicity condition. Separation with $\Delta D > 0$ would increase the efficiency loss due to the rental payment in form of an excess budget $B_I^{**} - B_I^* > 0$ by more than it would increase the value of production.\(^{17}\)

We conclude this section by providing the condition under which the principal abstains from financial transfers.

\(^{17}\) Morand and Thomas (2003) provide, within a different set-up, conditions for such a clash between rent extraction and monotonicity.
Lemma 5. (i) \( t^*_E = 0 \) for all \( (a, \alpha) \). (ii) There exists a function \( a(\alpha) \in (0, \infty) \) such that \( t^*_I = 0 \iff a \leq a(\alpha) \).\(^{18}\)

**Proof:** See Appendix.

Recall that the principal cannot extract the full job-satisfaction of agents due to wealth constraints. As in our case the inefficient agents seeks to mimic the efficient one, it is then optimal to always set the transfer \( t^*_E = 0 \). However, one would expect that a transfer paid to the inefficient type would help to relax the (ICI) constraint. While this is generally true, the use of a transfer becomes too costly when the agent has a weak preference for the transfer relative to his job-satisfaction, i.e. if \( a \) is sufficiently low. In this case, a substantial transfer would be required to motivate truth-telling on the part of the I type. Note that the expected cost of the transfer (in real resource terms), \( \frac{(1 - \lambda)}{a} \), becomes large when the agent’s preference \( a \) is low. The principal then abstains from transfers, whenever \( \frac{(1 - \lambda)}{a} \) exceeds at \( t_I = 0 \) the marginal efficiency loss from the distorted budget allocation.\(^{19}\)

5 Extensions of the model

We now turn to an overview of the allocation when the agent’s job-satisfaction is sufficiently low for the principal to motivate him by use of financial transfers. In section 5.1 we consider a situation where output is still non-contractible. Nonetheless the principal can facilitate the separation of agents by linking the transfers to the budgetary allocation. In section 5.2 we allow output (but not effort) to be contractible.

5.1 Use of financial transfers when output is non-contractible:

In this section we extend our analysis and consider the full parameter space \( (a, \alpha) \). We can identify five regimes for the budgetary allocation corresponding to the areas I-V in figure 3. We do not provide here a formal derivation of the various boundaries which is tedious without providing further insights. The interested reader is referred to Kuhn (2004: Lemma 5).

\(^{18}\) While we have derived this finding for linear preferences in money, the result easily extends to more general utility functions \( z(t) \). Here, pooling is sustained if the marginal utility from money \( z'(t) \) is sufficiently low when evaluated at the wealth constraint \( t = t_0 \) (in our case normalised to zero).

\(^{19}\) It can be shown that the \( a(\alpha) \) schedule has a negative slope for \( \alpha \geq 0 \). This implies that financial transfers are the less likely to be used the lower the agents’ preference for autonomy. This is because the inefficiency in the budget allocation falls as \( \alpha \) decreases towards zero, where the agent’s and principal’s preferences with regard to autonomy converge.
We have discussed already areas I and V, corresponding to the separating and pooling allocation in the absence of transfers. Recall that the neglect of financial stimuli is not down to an ad-hoc assumption but rather the outcome of an optimisation on the part of the principal, where transfers are too costly for $a < a(\alpha)$.

In area II, the principal employs the transfer $t_i^{**} > 0$. Here, the cost of a monetary transfer to the I type is sufficiently low as to render efficient its use as an additional screening instrument. This allows a reduction in the screening-induced inefficiencies in budgeting. Specifically, for a higher $a$ the principal reduces the size of the budgetary distortions $\left|B_i^{**} - B_i^\dagger\right|$, $i = E, I$, implying a reduction in the excess budget for the I type and a mitigation in capital rationing of efficient types. A greater $a$ also allows reducing the distortions in the degree of autonomy $\left|D_i^{**} - D_i^\dagger\right|$. Nonetheless, for finite values of $a$, the degree of delegation designated to the two types converges as the boundary between areas II and III is approached. On the boundary itself, the degree of delegation is pooled but not the total budget. Here, the E type receives a greater budget, $B_E^{**} > B_i^{**}$, while incentive compatibility for the I type is maintained by the transfer $t_i^{**} > 0$. Intuitively, as the preference for autonomy grows relative to the preference for financial transfers, it becomes increasingly costly both in terms of budgetary distortions and in terms of the financial transfer for the principal to uphold a large gap in the degree of delegation/autonomy.
In areas III and IV, we have $B_E^{**} > B_I^{**}$ and $t_I^{**} > 0$ and $D_E^{**} < D_I^{**}$, implying a reversal of the degree of delegation. The E type receives the greater budget, $B_E^{**} > B_I^{**}$ which in area III is sufficient to guarantee a slack incentive constraint for the E type. Nonetheless, the budgetary allocation is perverse in that the principal assigns the greater degree of delegation to the I type despite its lower productivity in the use of the delegated budget.\footnote{This notwithstanding the efficient type still receives the greater delegated budget, i.e. $b_E^{**} > b_I^{**}$.} This distortion increases as the preference for monetary rewards $a$ falls. A lower effectiveness of monetary transfers in inducing self-selection on the part of the I type also implies that the principal has to close the gap $B_E^{**} - B_I^{**} > 0$. As the E type’s allocation is thus rendered less and less attractive, this leads to a situation in area IV where the incentive constraints bind for both types. Here, a further reduction in $a$ will now also lead to a gradual shrinking in the gap $D_I^{**} - D_E^{**} > 0$ from a maximum at the boundary between areas III and IV until pooling both in budgets and in the degree of delegation is established on the boundary $\alpha(a)$ between areas IV and V.

The feasibility of separation within regime IV (and similarly in regime III) can be understood as follows. Both incentive constraints (ICE) and (ICI) continue to bind within regime IV, as they did within the pooling regime V. In contrast to regime V, however, the efficient use of financial transfers allows the principal to reconcile the conflicting self-selection constraints. This is illustrated in figure 2b. Consider the budget allocations E to the E type and I to the I type, where $\Delta D = D_E - D_I < 0$ and $\Delta B = B_E - B_I > 0$. While allocation E is preferred by both types, the principal sets the transfer $t_I > 0$ such that (ICI) is just binding. (ICE) is then satisfied if the monotonicity condition (M) holds. Satisfaction of (ICI) requires

$$
\Delta B = \frac{\partial I}{\partial I^E} \Delta D + \frac{\partial I}{\partial I^I},
$$

Inserting this into (M) and inserting then the appropriate derivatives from (5) yields the equivalent condition

$$
\frac{\partial I}{\partial I^E} \Delta D + \frac{\partial I}{\partial I^I} \geq 0.
$$

Since $at_I > 0$ allocations involving $\Delta D < 0$ are now feasible. Here, the spread in budgets $\Delta B > 0$ can be made sufficiently large to induce the E type to self-select even a contract involving a lower degree of delegation and a lower financial transfer. By contrast, we see that pooling is not a pathological outcome but supported by a range of pairs $(\alpha, a)$ in region V. Although the principal could break a pooling equilibrium by use of financial transfers, she refrains from this when agents are disinterested in financial rewards relative to the working conditions (here: the
degree of autonomy) and inducement of self-selection would require inefficiently high payments.

5.2 Allocation when output is contractible

In this section we consider the case, where the principal is able to link the agent’s compensation to output and not merely to the budget allocation. Here, the principal pays a transfer \( t_i > 0 \) if and only if the agent produces a pre-specified output level \( h_i, i = E, I \). Note that under the wealth constraint the principal cannot fine the agent for deviations from the target. As there is no uncertainty involved in production we are dealing with a case of ‘false moral hazard’ (e.g. Laffont and Martimort 2002: section 7.1.4). Given the budget allocation \( \{b_i, r_i\} \), a production target \( h_i \) implies an effort \( e_i \). We can then write the moral hazard constraint for type \( i \) as

\[
\text{(MHi)},
\]

where \( u(e_i, b_i, r_i, \beta_i) \) is the agent’s utility when foregoing the transfer and supplying effort \( e_i^* = \hat{e}(b_i, r_i, \beta_i) = \arg \max u(e_i, b_i, r_i, \beta_i) \). In the following, we provide a brief overview of the results under full information and asymmetric information, respectively. A more detailed exposition of the analysis as well as formal proofs can be found in Kuhn (2004).

Allocation in the absence of adverse selection

When output is contractible, the principal can use financial rewards to stimulate effort beyond the level that would be volunteered even by a motivated agent.\(^{21}\) The extent to which this is possible is governed by the agent’s responsiveness to financial rewards. As before, there is over-delegation if and only if the agent has a preference for autonomy, i.e. if and only if \( \alpha > 0 \), the efficient agent receiving a greater degree of delegation. However, while in the absence of transfers, budgets were distorted with a view to increase the agent’s marginal benefit from providing effort, in the case of contractible output, the principal can stimulate effort more directly by setting an appropriate target \( h_i \). The distortion in the budgets away from their first-best levels now helps the principal to contain the payment required to

\(^{21}\) This relies on the implicit assumption that financial incentives do not replace the agent’s non-financial motivation. In reality, crowding out of intrinsic motivation may matter (Frey 1993, 1997; Bénabou and Tirole 2003, Grepperud and Pedersen 2004). This complication would not alter our main results.
implement a given effort $e_i$. The financial rewards are thus leveraged and greater levels of effort can be stimulated. Finally, the availability of performance pay allows the principal to reduce the distortion in the degree of delegation away from the technological optimum.

**Allocation in the presence of adverse selection**

Here, the principal can design an output related reward scheme, but she is unable to observe the agent’s type. To explore the optimal contract under adverse selection we introduce some additional notation. Let $e^*_i := \hat{e}(b_j, r_j, \beta_i) = \arg \max u(e_i, b_j, r_j, \beta_i)$, $i, j \in \{E, I\}$ denote the effort that maximises type $i$’s utility when facing type $j$’s allocation, and let $\bar{e}_i := \frac{j(b_j, r_j, \beta_i)}{j(b_j, r_j, \beta_i)} \bar{e}_j$, $i, j \in \{E, I\}$ denote the effort type $i$ needs to expend in order to meet type $j$’s performance target $\bar{h}_j = \bar{e}_j f(b_j, r_j, \beta_j)$. In this case, the self-selection constraints

\[
u(\bar{e}_E, b_E, r_E, \beta_E) + at_E \geq \max \left\{ u(\bar{e}_IE, b_E, r_E, \beta_i) + at_E, u(e^*_E, b_E, r_E, \beta_i) \right\} \]  

\[
u(\bar{e}_E, b_E, r_E, \beta_E) + at_E \geq \max \left\{ u(\bar{e}_EI, b_I, r_I, \beta_i) + at_I, u(e^*_E, b_I, r_I, \beta_i) \right\}, \]  

need to be taken into account besides the moral hazard constraints, (MHI) and (MHE).

However, we can ignore the wealth constraints as these are implied by (MHI) and (MHE).

Note from (ICI’) and (ICE’) that adverse selection can arise in two forms. Each type may have an incentive to mimic the other type either with a view to obtaining the possibly more attractive performance pay [the first argument in bracelets on the RHS of (ICI’) and (ICE’), respectively], or with a view to maximising job-satisfaction on the other type’s budget allocation (the second argument). As is common for models of mixed adverse selection and moral hazard, there is no clear-cut rule to determine the binding constraints. We can distinguish two cases.

**Case 1:** Neither type has an incentive to mimic the other in order to attain the performance reward, implying that the LHS in (ICI’) and (ICE’) exceeds the first element in bracelets on the RHS. This requires that the effort level assigned to the I type be sufficiently low. Otherwise, the E type would have an incentive to mimic the I type and capture the performance payment at a low effort. The I type’s performance target is sufficiently low (i.e. sufficiently close to the level in the absence of rewards) if (and only if) the preference for financial rewards, $a$, is not too large. When a misrepresentation of type is not motivated by a desire to attain a more preferred payment, performance targets (conditional on the budget
allocation) remain undistorted. The budget allocation is then analogous to the one realised in the case of non-contractible output (see figure 3 and Proposition 3).

**Case 2:** Here, the preference for financial rewards is sufficiently high so that the E type has an incentive to mimic the I type. As long as the spread in productivity $\beta_e - \beta_i$ is sufficiently large, the I type does not aspire to attain the E type’s performance reward. The following adverse selection incentives then arise for the two types. While the E type seeks to obtain the more attractive performance contract offered to the I type [the first element in bracelets in (ICE’)] binds], the I type seeks to obtain the more attractive budget allocation designated to the E type without aiming to attain the performance target [the second element in bracelets in (ICI’)] binds]. The principal responds to the compounded incentive problem as follows. Similar to the standard model with adverse selection (e.g. Laffont and Martimort 2002: section 7.4.1), the principal seeks to reduce the E type’s rent by lowering the I type’s target (and effort) as well as the degree of delegation assigned to this type. Meanwhile, the E type’s allocation remains distorted as described for case 1 with a view to reducing the I type’s rent.

6 Conclusions

We have studied a principal agent model of budget delegation to illuminate some of the implications of agents’ preferences for autonomy. The principal faces both a moral hazard and adverse selection problem to which she can react by adjusting the budget, the degree to which it is delegated as well as a payment to the agent. Under full information about the agent’s productivity the principal adjusts the budget and its delegation in order to stimulate additional effort or – in case of performance pay – in order to reduce the payment. Our key results relate to the case in which the principal is uninformed about productivity under delegation. Contracts turn out to be sensitive to the agent’s preference for autonomy and to the trade-off between job satisfaction and financial transfers. Generally, if agents strongly prefer job satisfaction to financial transfers, the latter will not be used in order to facilitate self-selection. A separating allocation can then be attained only if preferences for autonomy are not too strong. The distortion in the degree of delegation depends on the agents’ preference for autonomy. If agents are indifferent to autonomy, a separating allocation requires over-delegation to the efficient type and no distortion for the inefficient type. For an increasing preference for autonomy the gap in the degree of delegation is gradually closed, and pooling arises when the preference for autonomy is sufficiently strong. This form of non-responsiveness arises when the agent’s and the principal’s preferences regarding the
allocation of autonomy are sufficiently divergent. Separation remains feasible when the agent’s preference for financial transfers is sufficiently strong. In this case, strong preferences for autonomy lead to a reversal in the degree of delegation. These results highlight the important implications of divergent preferences with regard to the mode of production.

From an empirical perspective one would expect that budget allocations will be less sensitive to differences in the productivity of a delegated budget within those organisations or organisational units that rely on agents with a strong preference for autonomy. Indeed, pooling is the likely outcome if strong preferences for autonomy are coupled with weak interest in financial rewards relative to job-satisfaction. Cursory evidence suggests that this ties-in rather well with the prevalence of weaker performance incentives within public or non-profit organisations relying on intrinsically motivated agents (Roomkin and Weisbrod 1999, Francois 2000, Dixit 2002, Besley and Ghatak 2003). A tendency towards pooled budgets within such organisations may be viewed critically in that it implies a misallocation of funds both to productive and unproductive agents. However, it should be borne in mind that pooling is an optimal response to the agents’ strong focus on job-satisfaction, which could only be swung by excessive financial transfers. Furthermore, the productivity gains from intrinsically motivated agents may well over-compensate the lack of efficiency in the budget allocation. While we derive the main results for the case in which output (and effort) are non-contractible, we show that their substance carries over to the case of contractible output.

In Kuhn (2004), we show in addition that similar results obtain when agents differ with regard to the preference for autonomy. A number of limitations and possible extensions deserve discussion. First, we assume that the agent’s input, effort, and the principal’s input, the budget, are complements in production. This implies that the principal, acting as first-mover, can stimulate additional effort by increasing the budget. More generally, budget and effort may also be substitutes, in case of which the principal would reduce her input.22 However, as long as the efficient type receives the greater budget and greater degree of delegation, this would not substantively alter our analysis of budgeting under asymmetric information. Second, we assume that the principal uses the budgets as a screening device when productivity is the agent’s private information. Alternatively, one could follow Bénabou and Tirole (2003) and assume that the principal is informed about the productivity of the project and thus its value to the agent. In this case, the principal could signal to the agent a productive/attractive project by over-budgeting and over-delegating to an extent that would

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22 See Glazer (2004) for an analysis of both cases.
not be profitable for the non-productive project. However, separation may not be possible if a high preference for autonomy on the part of the agent and the ensuing effort incentives make it profitable to over-delegate to all types. Third, we have disregarded the effects of uncertainty. One way of introducing risk into the present framework is to model output as a random variable, where the agent’s type $\beta$ corresponds to the probability of making effective use of the delegated budget. As risk is attached to the use of a delegated budget more than to the use of a controlled budget the principal has an incentive to under-delegate relative to the first-best in order to reduce the agent’s risk premium. The presence of risk-aversion may have a positive bearing on the feasibility of separation. As the inefficient agent faces a greater probability of failure under delegation, aversion to this risk reduces the incentive to aspire for the efficient agent’s allocation. Thus, for high levels of $\alpha$, risk contributes to reducing the distortion in the separating allocation. If agents are very risk averse, the presence of risk may even reverse the incentive problem.

7 Appendix

Proof of Proposition 1: Using (4) in (2) we obtain the first-order conditions

$\hat{h}_b(b_i, r_i, \beta_i) - \phi = 0 \quad (A1a)$

$\hat{h}_i(b_i, r_i, \beta_i) - \phi = 0; \quad i = E, I \quad (A1b).$ 23

Part (i): follows from comparative static analysis of the system (A1a) and (A1b) under use of (4). Parts (ii) and (iii): Using (A1a) and (A1b) together with the appropriate derivatives from (4) we obtain

$\hat{h}_i(b_i, r_i, \beta_i) = \frac{(\alpha + \rho \beta_i)^*}{\alpha + \rho \beta_i} = 1 \Leftrightarrow D_i^* = \frac{\alpha + \rho \beta_i}{2\rho - \alpha} ; \quad i = E, I.$

Obviously then, $\frac{dD_i^*}{d\beta_i} > 0$ and $\frac{d\phi_i^*}{d\beta_i} > 0$ implying the statements in part (ii) and (iii).■

Proof of Lemma 1: Given $t^*_E = t^*_I = 0$, the satisfaction of (ICI) requires $\Delta B = \frac{\partial \phi_i (\beta_i)}{\partial \beta_i} \Delta D.$

Inserting this into the monotonicity condition (M) yields the equivalent condition

$\left[ -\frac{\partial \phi_i (\beta_i)}{\partial \beta_i} \hat{\phi}_i (\beta_i) + \hat{\phi}_i (\beta_i) \right] \Delta D \geq 0.$

Inserting the appropriate derivatives from (5) one can rewrite the condition to $\frac{\partial \phi_i (\beta_i)}{\partial \beta_i} \Delta D \geq 0 \Leftrightarrow \Delta D \geq 0.$ ■

In the following, we will make repeated use of the following Lemma.

23 The second-order condition holds if and only if $\hat{h}(\theta_i, \beta_i)^2 \hat{h}(\theta_i, \beta_i) - \left[ \hat{h}(\theta_i, \beta_i) \right]^2 > 0; \quad i = E, I.$ Using (4) and observing $2(\beta_E + \rho) < 1$ and $\beta_E > \beta_I$, it is readily checked that this is satisfied.
Lemma A1. Let $\mu_1$ and $\mu_2$ denote the multipliers associated with the (ICI) and (ICE) constraint, respectively, and let $\mu_3$ and $\mu_4$ denote the multipliers associated with the wealth constraints $t_i \geq 0$ and $t_E \geq 0$, respectively. The optimal allocation entails (i) $\mu_1 > \mu_2 \geq 0$; (ii) $\mu_4 > 0$; (iii) $\mu_2 = 0$ if $D_E > D_I$ or if $D_E = D_I$ and $B_E \neq B_I$; and (iv) $\mu_3 = 0$ if $D_E < D_I$ or if $D_E = D_I$ and $B_E \neq B_I$.

Proof: In terms of $(b_i, r_i)$ the problem is \[ \max_{b_i, r_i : i \in E, I} \pi, \] as given in (2), subject to the self-selection constraints

\[ \hat{u}(b_i, r_i, \beta_i) + at_i \geq \hat{u}(b_E, r_E, \beta_E) + at_E \] \hspace{1cm} (ICI),

\[ \hat{u}(b_E, r_E, \beta_E) + at_E \geq \hat{u}(b_i, r_i, \beta_i) + at_i, \] \hspace{1cm} (ICE),

and the wealth constraints $t_i \geq 0$ and $t_E \geq 0$, respectively. Using the designated shadow prices, we can write the first-order conditions for $t_i$, $t_E$, $b_i$, $r_i$, $b_E$ and $r_E$ as follows.

\[ -(1-\lambda) + a(\mu_1 - \mu_2) + \mu_3 = 0 \] \hspace{1cm} (A2a),

\[ -\lambda - a(\mu_1 - \mu_2) + \mu_4 = 0 \] \hspace{1cm} (A2b),

\[ (1-\lambda)[\hat{h}_x(b_i, r_i, \beta_i) - \phi] + \mu_i \hat{u}_x(b_i, r_i, \beta_i) - \mu_3 \hat{u}_x(b_i, r_i, \beta_i) = 0 ; \ x = b, r \] \hspace{1cm} (A2c/d),

\[ \lambda[\hat{h}_x(b_E, r_E, \beta_E) - \phi] - \mu_1 \hat{u}_x(b_E, r_E, \beta_E) + \mu_4 \hat{u}_x(b_E, r_E, \beta_E) = 0 ; \ x = b, r \] \hspace{1cm} (A2c/f).\(^{24}\)

Using $\{b_E^*, r_E^*, t_E^*, b_i^*, r_i^*, t_i^*\}$ to denote the optimal allocation, we can now prove the parts of the Lemma as follows.

Part (i): We prove $\mu_1 > \mu_2$ by contradiction. Thus, suppose $\mu_2 \geq \mu_1$. From (A2a) this implies $\mu_3 > 0$ and, therefore, $t_i = 0$. (ICE) and (ICI) then imply

\[ 0 \leq at_E = \hat{u}(b_i^*, r_i^*, \beta_i) - \hat{u}(b_E^*, r_E^*, \beta_E) \leq \hat{u}(b_i^*, r_i^*, \beta_i) - \hat{u}(b_E^*, r_E^*, \beta_i) \] \hspace{1cm} (A3),

\(^{24}\) The second-order conditions are satisfied for $2(\beta_E + \rho) < 1$ if the parameter $k$ in the function $\hat{u}(\cdot)$ [see RHS of (5)] is sufficiently low.
where the first inequality follows from the wealth constraint and the second inequality is strict if and only if $\mu_1 > 0$. Using (A2c) and (A2d), we obtain for $x = b, r$

$$0 = (1 - \lambda)\hat{h}_{x}(b, r, \beta) - \phi + \mu_1 \hat{u}_{x}(b, r, \beta) - \mu_2 \hat{u}_{x}(b, r, \beta_E) \leq (1 - \lambda)\hat{h}_{x}(b, r, \beta) - \phi + \mu_1 [\hat{u}_{x}(b, r, \beta) - \hat{u}_{x}(b, r, \beta_E)] \leq (1 - \lambda)\hat{h}_{x}(b, r, \beta) - \phi,$$

implying $b_1^* < b_i^*$ and $r_1^* < r_i^*$. Similarly, one obtains from (A2e) and (A2f) $b_E^* > b_E^*$ and $r_E^* > r_E^*$. Together with $\hat{u}(b_E^*, r_E^*, \beta) > \hat{u}(b_i^*, r_i^*, \beta)$ this implies $\hat{u}(b_1^*, r_1^*, \beta) - \hat{u}(b_E^*, r_E^*, \beta) < 0$, which contradicts (A3).

Part (ii): $\mu_2 > 0$ follows from (A2b) under observation of $\mu_1 > \mu_2$.

Part (iii): Assuming $D_E > D_I$ we use figure 2a to show that this contradicts $\mu_2 > 0$.\footnote{Taking the appropriate derivatives from (5) one obtains $\frac{db}{dr} |_{\beta = \beta_E} = \frac{d\hat{u}_{x}(b, \beta_E)}{d\beta} = \frac{d\hat{u}_{x}(b, \beta_E)}{d\beta} > 0$, implying that for every pair $(r, b)$ the I type’s indifference curves are steeper sloped than the E type’s.}

Consider the allocations $E = \{B_E, D_E\}$ and $I = \{B_I, D_I\}$, obviously satisfying $D_E > D_I$.

Since $\mu_1 > \mu_2$, it follows from $\mu_2 > 0$ that both (ICE) and (ICI) bind simultaneously. Noting $\mu_4 > 0 \iff t_E = 0$ and the constraint $t_I \geq 0$ it must be true that

$$a_t = \hat{U}(E, \beta_I) - \hat{U}(I, \beta_I) = \hat{U}(E, \beta_E) - \hat{U}(I, \beta_E) \geq 0.$$ By construction, the allocations $E' = \{B_E', D_I\}$ and $E'' = \{B_E'', D_I\}$ satisfy $\hat{U}(E, \beta_I) = \hat{U}(E', \beta_I)$ and $\hat{U}(E, \beta_E) = \hat{U}(E'', \beta_E)$. Consequently,

$$a_t = \hat{U}(E', \beta_I) - \hat{U}(I, \beta_I) = \hat{U}(E'', \beta_E) - \hat{U}(I, \beta_E) \geq 0 \quad (A4).$$

For not too large differences in the budgets $B_E' > B_I$ and $B_E'' > B_I$, the second equality in (A4) implies $\hat{U}_b(\cdot, \beta_E)(B_E' > B_I) = \hat{U}_b(\cdot, \beta_I)(B_E' > B_I)$. Since $\hat{U}_b(\cdot, \beta_E) > \hat{U}_b(\cdot, \beta_I)$ this implies $B_E' > B_E''$, a contradiction. Hence, $\mu_2 > 0$ is incompatible with $D_E > D_I$.

We go on to show that $\mu_2 > 0$ contradicts $D_E = D_I$ and $B_E \neq B_I$. To see this, consider the allocations E and $I' = \{B_I', D_E\}$ in figure 2a. Here, $\mu_2 > 0$ implies

$$a_t = \hat{U}(E, \beta_I) - \hat{U}(I', \beta_I) = \hat{U}(E, \beta_E) - \hat{U}(I', \beta_E) \geq 0.$$ For small differences in budgets
\( B_E - B_i' > 0 \), this implies \( \hat{U}_B(\cdot, \beta_E)(B_E - B_i') = \hat{U}_B(\cdot, \beta_i)(B_E - B_i') \). For \( \hat{U}_B(\cdot, \beta E) > \hat{U}_B(\cdot, \beta_i) \) this is satisfied if and only if \( B_E = B_i' \), a contradiction.

Part (iv): Assume by contradiction that \( \mu_3 > 0 \) and \( D_E < D_i \) are simultaneously true. Noting that \( \mu_3 > 0 \) implies \( t_i = 0 \), it follows that (ICI) requires \( \hat{U}(B_i, D_i, \beta_j) - \hat{U}(B_E, D_E, \beta_j) = 0 \). This would, indeed, be satisfied by the allocations \( E = \{B_E, D_E\} \) and \( E' = \{B_i, D_i\} \) in figure 2b. However, it is immediately established that
\[
\hat{U}(E, \beta_j) - \hat{U}(E', \beta_j) = 0 \Rightarrow \hat{U}(E, \beta E) - \hat{U}(E', \beta_E) < 0
\]
implying that (ICE) is violated. Hence, \( \mu_3 > 0 \) and \( D_E < D_i \) cannot hold at the same time. Consider now \( D_E = D_i \). Since \( \mu_3 > 0 \) implies \( t_i = t_E = 0 \), it follows from (ICI) and (ICE) that \( B_E = B_i \). Hence, \( D_E = D_i \) and \( B_E \neq B_i \) imply \( \mu_3 = 0 \).

**Proof of Lemma 2:** Equating the LHS of (A2c) and (A2d), and rearranging gives
\[
\left( \hat{h}_r(b_i, \pi, \beta) \left[ \int \frac{h_i(b_i, \pi, \beta)}{h_i(b_i, \pi, \beta)} - 1 \right] \right) + \frac{\mu_i}{\beta_i} \hat{u}_r(b_i, \pi, \beta) \left[ \int \frac{\hat{h}_i(b_i, \pi, \beta)}{\hat{h}_i(b_i, \pi, \beta)} - 1 \right] = 0.
\]
Adding \( \phi \left[ \frac{h_i(b_i, \pi, \beta)}{h_i(b_i, \pi, \beta)} \right] \) to both sides yields
\[
\phi \left[ 1 - \frac{h_i(b_i, \pi, \beta)}{h_i(b_i, \pi, \beta)} \right] \left[ \phi - \hat{h}_r(b_i, \pi, \beta) \right] \left[ \int \frac{h_i(b_i, \pi, \beta)}{h_i(b_i, \pi, \beta)} - 1 \right] + \frac{\mu_i}{\beta_i} \hat{u}_r(b_i, \pi, \beta) \left[ \int \frac{h_i(b_i, \pi, \beta)}{h_i(b_i, \pi, \beta)} - 1 \right] = 0.
\]
where the second equality follows under observation of
\( \phi \left[ \phi - \hat{h}_r(b_i, \pi, \beta) \right] = \frac{\mu_i}{\beta_i} \hat{u}_r(b_i, \pi, \beta) \left[ \phi - \hat{h}_r(b_i, \pi, \beta) \right] - \frac{\mu_i}{\beta_i} \hat{u}_r(b_i, \pi, \beta) \left[ \phi - \hat{h}_r(b_i, \pi, \beta) \right] \) as from (A2d). Inserting the relevant derivatives from (4) and (5) into (A5) and rearranging yields
\[
D_i = \frac{\mu_i}{\beta_i} \hat{u}_r(b_i, \pi, \beta) \left[ \phi - \hat{h}_r(b_i, \pi, \beta) \right] - \frac{\mu_i}{\beta_i} \hat{u}_r(b_i, \pi, \beta) \left[ \phi - \hat{h}_r(b_i, \pi, \beta) \right] + \frac{\mu_i}{\beta_i} \hat{u}_r(b_i, \pi, \beta) \left[ \phi - \hat{h}_r(b_i, \pi, \beta) \right] - \frac{\mu_i}{\beta_i} \hat{u}_r(b_i, \pi, \beta) \left[ \phi - \hat{h}_r(b_i, \pi, \beta) \right] = 0.
\]
From Lemma 1, we note that $D_{E}^{**} \geq D_{I}^{**}$ must be true. Part (iii) of Lemma A1, then implies that $\mu_{2} = 0$ if $D_{E}^{**} > D_{I}^{**}$. From part (i) of the same Lemma we know that $\mu_{1} > 0$.

Observing that $\hat{\alpha}(b_{i}, r_{i}, \beta_{j}) = \frac{\rho - \alpha}{\beta_{j}} (1 + D_{I}) \hat{\beta}(B_{i}, D_{i}, \beta_{j}) = \kappa_{i}, i, j = E, I$, and employing the definitions in (6) and (7) then allows us to establish (8a). Starting from (A2e) and (A2f), it is straightforward to derive (8b) in a similar fashion.

**Proof of Lemma 3:** Part (i): Since $\alpha \geq 0 \Leftrightarrow D_{I}^{*} \leq \hat{D}_{I}$, it follows from (8a) that $D_{I}^{*} \leq D_{I}^{**}$ for $\mu_{2} = 0$. From (6), (7) and (9a), we obtain that $\alpha < \bar{\alpha} \Leftrightarrow \hat{D}_{I} < D_{E}^{*}$. For $\mu_{2} = 0$, it then follows from (8b) that $D_{E}^{*} < D_{E}^{**}$. Hence,

$\alpha \in [0, \bar{\alpha}] \Leftrightarrow D_{I}^{*} \leq D_{I}^{**} < D_{E}^{*} < D_{E}^{**}$, where the inequalities imply $\mu_{2} = 0$ from part (iii) Lemma A1.

Parts (ii) and (iii): Since $\alpha \geq \bar{\alpha} \Leftrightarrow D_{E}^{*} \leq \hat{D}_{I}$, it follows from (8b) that $D_{E}^{**} \leq D_{E}^{*}$ for $\mu_{2} = 0$. With $D_{I}^{*} < D_{I}^{**}$, we have to prove that $\alpha < \hat{\alpha} \Leftrightarrow D_{E}^{**} > D_{I}^{**}$. Using (8a) and (8b) we obtain

$$D_{E}^{**} \geq D_{I}^{**} \Leftrightarrow \left\{ \left[ 1 + \mu_{1} (b_{E}, r_{E}, \beta_{E}) \right] \hat{D}_{E}^{*} - \left[ 1 - \mu_{1} (b_{E}, r_{E}, \beta_{E}) \right] \hat{D}_{I}^{*} \right\} \geq 0,$$

or after employing (A2d) and (A2f) and rearranging

$$D_{E}^{**} \geq D_{I}^{**} \Leftrightarrow Z + \mu_{2} \left[ \hat{\alpha}(b_{E}, r_{E}, \beta_{E}) + \hat{\beta}_{r}(b_{E}, r_{E}, \beta_{E}) \right] \left( \hat{D}_{E} - \hat{D}_{I} \right) \geq 0$$

(A6)

$$Z = \hat{h}_{i}(b_{E}, r_{E}, \beta_{E}) \left( D_{E}^{*} - \hat{D}_{I} \right) - \hat{h}_{i}(b_{i}, r_{i}, \beta_{j}) \left( D_{I}^{*} - \hat{D}_{I} \right).$$

Inserting the appropriate derivatives from (4) and rearranging terms we can rewrite

$$Z = \frac{1}{\rho - \alpha} \left[ (2b_{E} + \alpha) b_{E}^{2} + r_{E}^{2p-\alpha-1} - (2b_{I} + \alpha) b_{I}^{2} + r_{I}^{2p-\alpha-1} \right],$$

Assume for the moment $\mu_{2} = 0$ and consider a pooling allocation $\{b_{E} = b_{I} = b_{p}; r_{E} = r_{I} = r_{p}\}$ that satisfies
It is readily verified that this implies \( \alpha = \frac{2(b_\ell - b_\rho)\rho^2 \beta_\ell}{(\rho + 2b_\ell - b_\rho)\rho^3 - (\rho + \beta_\ell)\rho^3} = \hat{\alpha} \), where \( P := b^p = \frac{\rho}{1 + \rho} \).

Note that \( \hat{\alpha} \in (\overline{\alpha}, \rho) \). From comparative static analysis of the system

\( \hat{u}(b_1, r_1, \beta_1) - \hat{u}(b_E, r_E, \beta_1) = 0 \), (A2a) and (A2c)-(A2f) each with \( \mu_2 = 0 \) we obtain after tedious calculations \( \frac{d\hat{u}}{dx} \bigg|_{x=t_j=0; z=0; \mu_z=0} < 0 \). Note that for \( t_E = t_1 \), (ICI) and (ICE) imply \( D_E = D_I \Leftrightarrow B_E = B_I \). Hence, \( \frac{d\hat{u}}{dx} \bigg|_{x=t_j=0; z=0; \mu_z=0} < 0 \). But then,

\[
Z \bigg|_{t_E=t_j=0} = \begin{cases} 
\text{>} & \Leftrightarrow \alpha = \hat{\alpha} \\
\text{\langle} & \Leftrightarrow \alpha \neq \hat{\alpha}
\end{cases}
\]

(A7).

Recall from Lemma A1, part (ii) that \( D_E^{**} > D_I^{**} \Rightarrow \mu_z = 0 \). Hence, from (A6)

\( Z > 0 \Rightarrow \mu_z = 0 \). Considering \( Z = 0 \), we can prove by contradiction that this implies \( \mu_z = 0 \).

Suppose \( Z = 0 \) and \( \mu_z > 0 \) hold at the same time. In this case, it follows from (A6) that

\( D_E^{**} > D_I^{**} \), contradicting \( \mu_z > 0 \). Hence, \( Z \geq 0 \Rightarrow \mu_z = 0 \) must be true.

From (A6) and (A7) it follows \( \alpha \leq \hat{\alpha} \Leftrightarrow D_E^{**} \geq D_I^{**} \), with a strict inequality on the RHS implying and being implied by one on the LHS. Finally, consider \( \alpha > \hat{\alpha} \). From (A7), this implies \( Z \bigg|_{t_E=t_j=0} < 0 \). For \( t_E = t_1 = 0 \), (ICI) and (ICE) imply

\( \hat{u}(b_1^{**}, r_1^{**}, \beta_1) - \hat{u}(b_E^{**}, r_E^{**}, \beta_1) = \hat{u}(b_1^{**}, r_1^{**}, \beta_1) - \hat{u}(b_E^{**}, r_E^{**}, \beta_1) = 0 \). However, it is readily checked from figure 2b that for the utility specification in (5), this is only satisfied for a pooling allocation, i.e. for \( \{ b_E^{**} = b_1^{**} = b_p, r_E^{**} = r_1^{**} = r_p \} \). Hence, \( \alpha > \hat{\alpha} \Rightarrow D_E^{**} = D_I^{**} \).

**Corollary.** \( \mu_z \bigg|_{t_E=t_j=0} > 0 \Leftrightarrow \alpha > \hat{\alpha} \).

**Proof:** We have obtained \( \alpha \leq \hat{\alpha} \Rightarrow \mu_z \bigg|_{t_E=t_j=0} = 0 \) as part of the proof of Lemma 3. As

\( Z \bigg|_{t_E=t_j=0} < 0 \) and \( D_E^{**} = D_I^{**} \) both hold for \( \alpha > \hat{\alpha} \), it follows from (A6) that

\( \alpha > \hat{\alpha} \Rightarrow \mu_z \bigg|_{t_E=t_j=0} > 0 \). In combination this implies the corollary.
Proof of Lemma 4: From the first-order conditions (A2e) and (A2f), respectively, it follows for $\mu_2 = 0$ that $\hat{h}_b(b_E, r_E, \beta_E) = \phi + \mu_{\beta} \hat{u}_b(b_E, r_E, \beta_1) = \xi^E b > 0$ and $\hat{h}_r(b_E, r_E, \beta_E) = \phi + \mu_{\beta} \hat{u}_r(b_E, r_E, \beta_1) = \xi^E r > 0$. As $\hat{u}_b(., \beta_1) > 0$, $\hat{u}_r(., \beta_1) > 0$ and $\mu_1 \geq 0$, we have $\xi^E b \geq 1$ and $\xi^E r \geq 1$. It follows from the second-order condition $\hat{h}_{bb}(\cdot, \beta_E) \hat{h}_{rr}(\cdot, \beta_E) - \hat{h}_{br}(\cdot, \beta_1)^2 > 0$ that the optimal values $\hat{b}_E(\xi^E b, \xi^E r)$ and $\hat{r}_E(\xi^E b, \xi^E r)$ are decreasing functions in $\xi^E b$ and $\xi^E r$. But then $b_{E}^{**} = \hat{b}_E(\xi^E b, \xi^E r) \leq \hat{b}_E(1,1) = b^*_E$ and $r_{E}^{**} = \hat{r}_E(\xi^E b, \xi^E r) \leq \hat{r}_E(1,1) = r^*_E$ is always true. By a similar proof, it can be shown that $b_{i}^{**} \geq b_i^*$ and $r_{i}^{**} \geq r_i^*$. Together this implies $B_{i}^{**} \geq B_i^*$ and $B_{E}^{**} \leq B^*_E$. This proves the relevant inequalities in parts (i)-(iii).

Note that the budget lines in $(r,b)$ space (see figure 2a) have the slope $-1$. As indifference curves are strictly convex, it follows for $D_E \geq D_I$ that $\frac{db_{i}(r_i)}{dr_i} \bigg|_{\alpha = \hat{\alpha}, \beta_i = \hat{\beta}_i} = \frac{-\rho - \rho}{\rho - \alpha} \leq -1$ is sufficient for $B_i < B_E$. Using the definition $\hat{D}_i = \frac{\alpha + \beta_i}{\rho - \alpha}$, the condition can be equally expressed as $D_I \geq \hat{D}_I$. Similarly, $\frac{db_{i}(r_i)}{dr_i} \bigg|_{\alpha = \hat{\alpha}, \beta_i = \hat{\beta}_i} = \frac{-\rho - \rho}{\rho - \alpha} \geq -1$ or equivalently $D_E \leq \hat{D}_E$ are sufficient for $B_i > B_E$.

We then have $\alpha = 0 \iff D_I^{**} \geq \hat{D}_I \iff D_I^{**} \geq \hat{D}_I \iff B_i^{**} < B_{E}^{**}$, which, evoking continuity, proves part (i). Similarly, $\alpha \in [\bar{\alpha}, \hat{\alpha}] \iff D_E^{**} \leq \hat{D}_I \iff D_E^{**} \leq \hat{D}_I \iff B_{E}^{**} < B_{i}^{**}$, which proves part (ii). Finally, for $\alpha \geq \hat{\alpha}$ a pooling allocation obtains where $D_E^{**} = D_I^{**} = D_p \iff B_{E}^{**} = B_{i}^{**} = B_p$, which proves part (iii).

Proof of Lemma 5: Part (i) follows immediately from Lemma A1, part (ii).

Part (ii): In what follows we write the equilibrium degree of delegation $D_{i}^{**} = d'(\alpha, a)$, $i = E, I$ and the shadow prices $\mu_i = \hat{\mu}(\alpha, a)$ and $\mu_2 = \hat{\mu}(\alpha, a)$ as functions of the parameters $\alpha$ and $a$. From (A2a), we obtain $\mu_1 = \max \left\{ (1 - \lambda) - a \left[ \hat{\mu}(\alpha, a) - \hat{\mu}(\alpha, a) \right], 0 \right\}$. Using this, we can now show that there exists a function $a(\alpha) \in (0, \infty)$ such that $\mu_2 > 0 \iff a \leq a(\alpha)$.

Consider a situation with $\mu_2 > 0$, or equivalently with $t_E = t_I = 0$, where $t_E = 0$ follows from Lemma A1, part (ii). From the corollary to Lemma 3, we know that
\( \mu_2 \bigg|_{r_e=r_f=0} > 0 \iff \alpha > \hat{\alpha} \). Thus, we distinguish two cases. First, consider \( \alpha \leq \hat{\alpha} \), where

\( \mu_2 = \hat{\mu}(\alpha, a) = 0 \) and therefore \( \mu_3 = \max \{ (1-\lambda) - a\hat{\mu}(\alpha, a) \} \). For \( \mu_3 > 0 \) comparative static analysis of the system \( \hat{u}(b_1, r_1, \beta_1) - \hat{u}(b_e, r_e, \beta_1) = 0, \) (A2a) and (A2c)-(A2f) with \( \mu_2 = 0 \) yields \( \hat{\mu}_s(\alpha, a) = 0 \). Hence, \( \frac{\partial [1-\lambda-a(\hat{\mu}(\alpha, a))]_{\mu_3>0}}{\partial a} = -\hat{\mu}(\alpha, a) < 0 \). Since \( \mu_3 > 0 \Rightarrow \hat{\mu}_a = 0 \), it follows that \( \mu_3 > 0 \Rightarrow \hat{\mu}(\alpha, a) = \hat{\mu}(\alpha, 0) = \hat{\mu}(\alpha) \). Furthermore, \( \hat{\mu}(\alpha) \in (0, \infty) \) is positive and finite. But then \( \lim_{a \to 0} [(1-\lambda) - a\hat{\mu}(\alpha)] = (1-\lambda) > 0 \) and \( \lim_{a \to 0} [(1-\lambda) - a\hat{\mu}(\alpha)] = -\infty \). Hence, there exists a \( a^-(\alpha) \in (0, \infty) \) such that \( \mu_3 = (1-\lambda) - a\hat{\mu}(\alpha) > 0 \iff a \leq a^-(\alpha) \).

Now consider \( \alpha > \hat{\alpha} \), where \( \mu_2 = \hat{\mu}(\alpha, a) > 0 \) and, therefore,

\( \mu_3 = \max \{ (1-\lambda) - a\hat{\mu}(\alpha, a) \} \). Comparative static analysis for the full pooling system \( \hat{u}(b_1, r_1, \beta_1) - \hat{u}(b_e, r_e, \beta_1) = 0, \hat{u}(b_1, r_1, \beta_1') - \hat{u}(b_1, r_1, \beta_1) = 0, \) (A2a) and (A2c)-(A2f) with \( \mu_2 > 0 \) yields \( \hat{\mu}_a(\alpha, a) = \hat{\mu}_a(\alpha, a) = 0 \). Hence,

\[ \frac{\partial [1-\lambda-a\hat{\mu}(\alpha, a)]_{\mu_j>0}}{\partial a} = -\hat{\mu}(\alpha, a) - \hat{\mu}(\alpha, a) < 0 \] \] Since \( \mu_3 > 0 \Rightarrow \hat{\mu}_a = \hat{\mu}_a = 0 \), we have \( \mu_3 > 0 \Rightarrow \hat{\mu}(\alpha, a) - \hat{\mu}(\alpha, a) = \hat{\mu}(\alpha, 0) - \hat{\mu}(\alpha, 0) = \hat{\mu}(\alpha) - \hat{\mu}(\alpha) \). Furthermore,

\( \hat{\mu}(\alpha) - \hat{\mu}(\alpha) \in [0, \infty] \). But then \( \lim_{a \to 0} [(1-\lambda) - a\hat{\mu}(\alpha) - \hat{\mu}(\alpha)] = (1-\lambda) > 0 \) and

\( \lim_{a \to 0} [(1-\lambda) - a\hat{\mu}(\alpha) - \hat{\mu}(\alpha)] = -\infty \). Hence, there exists a value \( a^+(\alpha) \in (0, \infty) \) such that \( \mu_3 = (1-\lambda) - a\hat{\mu}(\alpha) - \hat{\mu}(\alpha)] > 0 \Rightarrow a \leq a^+(\alpha) \).

Finally, \( \lim_{a \to \hat{a}} [(1-\lambda) - a\hat{\mu}(\alpha, a) - \hat{\mu}(\alpha, a)] = (1-\lambda) - a\hat{\mu}(\alpha, a) = \lim_{a \to \hat{a}} [(1-\lambda) - a\hat{\mu}(\alpha, a)] \) implies \( a^-(\hat{\alpha}) = a^+(\hat{\alpha}) \). The negative slope of \( a^-(\alpha) \) and \( a^+(\alpha) \) can be established from the system (A2a)-(A2f) under use of the implicit function theorem. But then this implies a continuous (and decreasing) function \( a(\hat{\alpha}) \) on the domain \([0, \rho]\).■

8 References


