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A Simple Model of Outsourcing with Cournot Competition

by

Michael Hübler

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A Simple Model of Outsourcing with Cournot Competition*

Michael Hübler

Abstract:
The paper analyses a partial equilibrium outsourcing model with Cournot competition in intermediate good production. Final production is located in western Europe, whereas the intermediate good can be manufactured by a western (outsourcing) or eastern European supplier (offshore outsourcing). Interregional production (factor) allocation depending on factor prices and productivity levels is investigated analytically and graphically. The main results are: Higher production costs in one region reduce intermediate good production in both regions leading to a substitution effect between high- and low-skilled labour intensive inputs rather than between eastern and western low-skilled labour intensive inputs. The sensitivity of outsourcing activities to production cost changes is highest when the interregional cost differential is smallest.

Keywords: Offshoring, outsourcing, Cournot competition, intermediate good

JEL classification: D24, D43, F20, J31

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1 Introduction

Recently citizens, politicians and economists recognized outsourcing in the sense of moving economic activities abroad as a topic of highest importance in the context of the European Union Eastern Enlargement. People fear that production and jobs permanently escape towards East, increasing wage pressure and unemployment in the West. When unions ask for higher wages or when political decision makers claim stricter standards, say in the environmental field, the opposed argument is straight forward: Any deterioration of the western location characteristics will move production and jobs away to the East and must therefore be avoided.

Thus, this paper deals with the question, how sensitive outsourcing behaviour from West to East really is when production costs in West and East change relative to each other. This aspect has not yet been covered in the literature from a theoretical point of view. The paper assumes a special case, though, analysing a partial equilibrium outsourcing model of one western European and one eastern European intermediate good supplier in Cournot competition, i.e. interregional competition in quantities of intermediate goods. This means, the paper examines within country outsourcing versus offshore outsourcing to abroad, since the intermediate good is manufactured by independent rivalling firms located within and outside the home country. The distribution of intermediate good production to the East and West depending on exogenous wages, returns on investment and technology levels is derived analytically. The ratio of labour costs between western and eastern Europe varies between more than 20 (for example Denmark compared to Bulgaria) and about one (Portugal compared to Slovenia). In the future the wage differential between East and West is supposed to decrease due to the proceeding integration process.

The kind of outsourcing that is relevant here is offshore outsourcing in the spirit of Feenstra and Hanson (1996a and 1996b) and Sinn (2005). In contrast to other theoretical models, this approach consists of a two-stage production process with Cobb-Douglas technologies and intermediate goods as perfect substitutes. This makes the model analytically tractable. Other differences are that factor prices are exogenous and that production factor allocation responds to changes in factor prices and productivity. The model provides a useful analytic framework for modelling outsourcing and highlights the sensitivity of outsourcing activities for wage and productivity changes given an interregional production cost differential.

Does the initial wage differential between two countries or regions have an influence on the reaction of outsourcing activities to production cost changes? Will the outsourcing behaviour change when cost levels converge? Are western low-skilled labour intensive inputs proportionally replaced by corresponding eastern inputs?

The analysis shows that the elasticity of relative interregional production is largest when marginal production costs in the West and East are equal. Moreover, higher production costs in one region reduce intermediate good production in both regions leading to a substitution effect between high- and low-skilled labour intensive inputs rather than between eastern and western low-skilled labour intensive inputs. Of course, intermediate goods production falls in region A relative to B, when production costs in A rise relative to B. But intermediate goods output in B does not rise in absolute terms, since both regions interact via Cournot competition.

The paper is organized as follows: Section 2 summarizes theoretical and empirical literature on international outsourcing. Section 3 describes the model structure, section 4 introduces Cournot competition of intermediate goods suppliers, section 5 derives the allocation of intermediate good production and the elasticity of relative interregional production, section 6 analyses the resulting allocation of production factors, section 7 introduces transport costs, section 8 provides graphical representations, section 9 discusses implications and caveats and chapter 10 concludes.
2 Literature on International Outsourcing

Referring to the USA Feenstra and Hanson (1996a and 1996b) show that rising imports, reflecting the outsourcing of production activities, contributed to the decline of relative employment and wages of unskilled workers during the 1980s. When firms outsource low-skilled labour intensive activities to low-wage countries and import the produced intermediate goods, this will shift employment towards skilled labour within industries (Feenstra and Hanson (1996)). Bhagwati, Panagariya and Srinivasan (2004) refer to US outsourcing at the beginning of the the 21st century as a different phenomenon: Trade in services at arm's length that does not require geographical proximity of the buyer and the seller. This view emphasizes the role of information and communication technology opening the possibility of outsourcing call centres, software programming and data analysis to Asia connected via fast and cheap internet and telephone connections.

In Europe especially the kind of outsourcing described by Feenstra and Hanson has achieved high attention with respect to the European Union eastern enlargement. Real wages in the central and eastern European countries average around one-fifth of the respective wage levels in the former EU-15\(^1\) and create an incentive for western companies to benefit from the low eastern labour costs by outsourcing labour-intensive production abroad. Sinn (2005) calls Germany a Bazaar Economy importing and exporting large amounts of goods but adding a low production value. The value added divided by the output value in the German industry declined from 40.2 % to 34 % between 1970 and 2003 (Sinn (2005), Appendix A, figure 5). For example the high quality German Porsche Cayenne is actually produced in Bratislava to a large extent as pointed out by Sinn (2005). Marin (2002) indicates that the most dynamic and innovative segments of the German economy invest in eastern Europe and that exploiting low eastern wages is one motive for the outsourcing activities. Marin (2006) finds empirically that falling trade costs, lower levels of corruption and improvements in the contracting environment in eastern Europe influence the level of intrafirm imports from the East to Austria and Germany, which points to increased outsourcing to the East. Marin (2004) goes one step further, stating that Germany and Austria carry out outsourcing activities towards the East in order to take advantage of the abundant high-skilled labour there. She finds high educational levels among employees and more workers engaged in R&D and engineering in eastern affiliates compared to firms in Germany and Austria. Braconier and Ekholm (2001) find opposite results. Marin (2004) shows small job losses in the West due to outsourcing, because outsourcing helps western firms to stay competitive in accordance with Konings and Murphy (2001). In contrast to these studies Becker et al. (2005) find a more substantial replacement of western jobs by jobs in eastern affiliates. Kirkegaard (2005) identifies European companies supplying or receiving outsourced or offshored goods and services and consumers of the resulting final commodities as winners. On the other hand companies being unable to adapt to the outsourcing boom and workers laid off due to outsourcing and offshoring are potential losers.

In summary the literature points to some negative redistribution and employment effects for low-skilled employees. However, there is no reason for any outsourcing hysteria concerning production and jobs moving to low wage countries rapidly, considering that outsourcing improves efficiency and competitiveness and therefore reduces commodity prices under competitive markets making consumers of theses products better off.

Due to the major public awareness of outsourcing and its consequences various models have been developed recently. The model by Egger and Egger (2004a) includes multinationals' competition in quantities and price-cost margins. Egger and Egger (2004b) analyse the trade-off between transport costs and lower production costs induced by outsourcing. Kohler (2004) models the reaction of a multi-stage industry with outsourcing to changes of the final good price and fragmentation costs.

\(^1\) According to World Bank purchasing power parity estimates; Boeri and Brücker (2000).
Markusen (2005) uses modules from the existing trade theory for numerical analyses. Overall, the simulations suggest welfare gains for the South and the global economy, while the outsourcing northern country may lose if it is large. A key issue in the research of Grossman and Helpman (2005) (related to Grossman and Helpman (2002) and (2003)) is the view of outsourcing as an activity requiring a costly search for a partner (in the home country or in a foreign country). Bandyopadhyay and Wall (2005a) derive the optimal amount of outsourcing for a given immigration level; the model by Bandyopadhyay and Wall (2005b) includes an oligopolistic export sector and a competitive import-competing sector and shows that an outsourcing tax can be justified under a minimum wage but not under flexible wages. Mitra and Ranjan (2005) extend the outsourcing and FDI research to dynamic behaviour with externalities and firm heterogeneity. According to their results temporary shocks can have permanent effects, and most productive firms move abroad first. Bartel, Lach and Sicherman (2005) show how technological progress lowers firms’ adjustment costs of outsourcing. Munch and Skaksen (2005) point out that outsourcing to abroad worsens the wages for unskilled-workers, whereas this effects cannot be expected from outsourcing within the country. They confirm their result by using Danish panel data. Furthermore, Senses (2006) illustrates theoretically and empirically how the elasticity of low-skilled labour demand increases in heavily outsourcing industries and how a decline in the share of unskilled labour at home lowers the elasticity on the other hand. Wang (2006) develops a model of choosing between vertical integration and outsourcing depending on cost differentials, transport costs and costs of searching for intermediate good trade partners in the tradition of Grossman and Helpman. Grossman and Rossi-Hansberg (2006) propose a new conceptual framework of the global production process focussing on tradable tasks. They show that in contrast to neoclassical trade theories (under certain conditions) all domestic parties can share in the gains from improved offshoring opportunities. A reformulation of the four basic theorems of Heckscher-Ohlin theory allowing for offshoring (fragmentation) is provided by Baldwin and Robert-Nicoud (2007).

Summing up, a large variety of models has been developed to examine different aspects related to outsourcing, such as transport costs versus lower production costs, searching for an outsourcing partner, the relationship of outsourcing and migration and the role of technological progress. Accordingly, outsourcing is a phenomenon with many facets, some of them investigated and understood, many of them not yet understood leaving room for further research in different directions. This paper deals with a facet of high public interest, not sufficiently covered theoretically: The reaction of (offshore) outsourcing to changes in production costs.

3 Model Structure

The partial equilibrium model is composed of a final good $Y$ producer, located in the western European country, and two intermediate good $X$ producers, located in the western country and the eastern country, respectively. $Y$ is produced with a Cobb-Douglas technology using the inputs $H$ and $X$:

$$Y = H^\alpha (X_W + X_E)^{1-\alpha} \quad 0.5 < \alpha < 1$$

(1)

The final good $Y$ could be an automobile produced in Germany via a complex process with a sophisticated technology.² The final good producer takes demand, e.g. for his car type, as given by the market. (The final good market form is not of importance in this case.) $H$ is a high-skilled labour and high-technology intensive input, which is available in the western area only and cannot be outsourced. This is a conservative assumption contrasting with Marin

² The model could also describe any other suitable industry, where outsourcing of intermediate goods production occurs and where a small number of intermediate goods producers compete.
(2004), but not crucial for the interpretation of the outcomes. For simplicity the production factor $H$ encompasses all processes involving highly educated employees such as design, engineering and management, the necessary high technology capital and firm specific knowledge.

The intermediate good $X$ production process includes all activities demanding low-skilled labour like manual work plus usual capital input. $X$ represents for instance interior automotive parts like dashboards and seats. $X$ can be manufactured in western ($X_W$) or eastern Europe ($X_E$). Low-skilled labour is supplied in both regions. That means there is no offshore outsourcing with the purpose of getting access to well educated workers in the East as described by Marin (2004) nor service outsourcing according to Bhagwati, Panagariya and Srinivasan (2004), but it is offshore outsourcing in the spirit of Feenstra and Hanson (1996a and 1996b). Intermediate good production can be moved to the East when production costs or wages are cheaper in the East, afterwards the manufactured intermediates are imported to the West. $X_W$ and $X_E$ are homogeneous goods and perfect substitutes so that they can be summed up to $X = X_W + X_E$. The assumption $\alpha > 0.5$ implies a higher income share for the high-skilled labour and high-technology intensive input located in the West (under perfect competition). This means that the share of the production process which is mobile via outsourcing is less than half of whole production referring to factor revenues. The intuition is that the main part of production is not outsourced but kept within the company in contrast to the example by Sinn (2005).

The Cobb-Douglas function implies the possibility to replace part of input $X$, produced with a high amount of low-skilled labour and standard technology by input $H$, produced by high-skilled workers and modern technologies, and vice versa. The $Y$ producer minimizes production costs for a given output $Y$:

$$\begin{align*}
\text{Min. } C &= w_H H + p_W X_W + p_E X_E \\
\text{s. t. } Y &= H^\alpha (X_W + X_E)^{1-\alpha}
\end{align*}$$

(2)

$w_H$ is the price of high-skilled labour intensive input $H$, $p_W$ and $p_E$ are the prices of $X$, manufactured in the western or eastern region. The final good $Y$ producer is a price taker, he sets the input factor amounts according to the factor prices $w_H$, $p_W$ and $p_E$.

The intermediate good is manufactured by independent rivalling firms located in western Europe or Eastern Europe, respectively. We call the former case outsourcing, the latter case offshore outsourcing. \(^4\)

Intermediate good $X$ production is represented by using Cobb-Douglas functions with the inputs

\(^3\) The Bazaar Economy hypothesis has been rejected by other authors, for instance Horn and Behncke (2004).

\(^4\) Outsourcing means removing part of the production process from the own company and buying it from an external company instead, where the outsourced part can be a product or a service, but not raw material (Kirkegaard (2005)). Offshore outsourcing in the current discussion is a certain type of outsourcing, namely outsourcing to a foreign country.

FDI is distinguished from outsourcing and pure portfolio investment by the ownership criterion: The investor has a certain degree of influence on the foreign investment receiving country (by holding a company share of at least 10 %). (The OLI paradigm by Dunning (1981) distinguishes three motives for FDI: ownership advantage, location advantage and internalization advantage.) In that respect the expression multinational firm is closely related to FDI. (see for instance Markusen and Maskus (2001)). We distinguish two types of FDI and multinational enterprises: vertical and horizontal. Vertical FDI is similar to offshore outsourcing, since part of the production process is done by the affiliate and the resulting intermediate good is transferred back. An important factor driving vertical FDI is the benefit from lower production costs abroad. Horizontal FDI and horizontal multinationals have little in common with offshore outsourcing, since the main purpose is direct access to foreign market instead of exporting to that market.

Offshoring in general encompasses according to the definition by Kirkegaard (2005) offshore outsourcing as well as vertical FDI and refers to imports of intermediate goods across the boarder.
capital $K$ and low-skilled labour $L$ with constant returns to scale:

$$X_i = A_i K_i^{\beta_i} L_i^{1-\beta_i}, \quad 0 < \beta_i < 1, \quad i = [W; E]$$

(3)

The production processes in West and East differ in technologies $A_i$ and in the real wages $w_i$. Furthermore, differences in the returns to capital investment $r_i$ and in the elasticities of production $\beta_i$ and $1-\beta_i$ are possible.

Cost minimizing $X$ manufacturing leads to the following marginal costs $c_i$:

$$c_i = \beta_i^{-\beta_i} (1-\beta_i)^{\beta_i-1} A_i^{-1} r_i^\beta_i w_i^{1-\beta_i}$$

(4)

Marginal costs $c_i$ are assumed constant and equal to the cost per unit of output. Marginal costs are derived from the exogenous parameters technology $A_i$, real wage $w_i$, real return on investment $r_i$ and the Cobb-Douglas function exponents $\beta_i$ and $1-\beta_i$. If the returns on investment and the exponents are similar in the East and West, a cost advantage can be achieved via a more efficient technology or a lower wage level.

$X$ producers maximize their profits and have oligopolistic (monopolistic) power reflected in the price for $X$ depending on the quantity of $X$, where $X = X_W + X_E$:

$$Max. \quad \Pi_i = p_i(X) \cdot X_i - c_i(X_i) \cdot X_i$$

(5)

Without any market power of $X$ producers prices would be equal to marginal costs. In general a productivity gap between East and West exists, and hence intermediate good production would take place in the area with lower marginal costs only.

In case of Bertrand competition (price competition) intermediate good manufacturing occurs only in the cheaper region, too. Now the cheaper producer can increase the price for $X$ and reduce the production quantity of $X$, but if he increases the price for $X$ more than to the marginal costs of the rival $X$ producer, he will loose all the demand for his product.

Cournot competition (competition in quantities) is the interesting case that needs further analysis. Even though intermediate goods are perfect substitutes, Cournot competition ensures that intermediate good production is distributed to both regions depending on regional production costs.

4 Cournot Competition

Production of automotive components, e.g. a certain dashboard containing distinct parts or seats, requires low-skilled manual work and to a lower extent high-skilled engineering and design. Nevertheless specific knowledge and a sufficient firm size are necessary to produce automobile parts according to the final producer's needs, in large amounts and fulfilling the quality restrictions.

The intermediate good and the final good company agree to a long term contract, so that other suppliers can hardly enter the market. For that reason it is plausible to suppose a small number of companies being able to provide the specific components needed in final production, in this model in particular two suppliers. Under the assumption of pure Cournot competition the eastern and western company offer the intermediate good $X$ at the same price $p_X = p_W = p_0$. The firms optimize their supply of $X$ taking into account the rivals reaction and the demand function for $X$ given by the western final good $Y$ producer. The conditional factor demand function can be derived from (2):
Total demand for $X$ falls with the price $p_X$ and increases with $w_H$, the price of the skilled labour intensive good $H$.

Similarly the input quantity of $H$ is expressed:

$$H(Y, w_H, p_X) = \left( \frac{\alpha p_X}{(1-\alpha)w_H} \right)^{1-\alpha} Y$$

A change in the exogenous quantity $Y$ leads to proportional shifts of the input factors $X$ and $H$. The absolute value of $Y$ is unimportant because the analysis looks at relative values.

Solving (6) for $p_X$ yields the inverse factor demand function for $X$:

$$p_X = \frac{1-\alpha}{\alpha} w_H X \cdot \frac{1}{\alpha} Y^{\frac{1}{\alpha}} X$$

Obviously the intermediate good suppliers face a non-linear and downward-sloping inverse factor demand function with respect to the total production of $X$. Since $\alpha < 1$ expanding the supply of $X$ leads to a more than proportional fall in the price $p_X$.

In market equilibrium supply equals demand for $X$ at the price $p_X$, so that we can substitute (8) into (5):

$$\text{Max. } \Pi_i = \frac{1-\alpha}{\alpha} w_H X \cdot \frac{1}{\alpha} Y^{\frac{1}{\alpha}} X - X_i = c_i X_i$$

The oligopolists maximize their profits $\Pi_i$ by choosing their production quantities $X_i$ and taking into account the total amount $X$ that includes their own and their rival’s quantity.

This results in the following first order conditions for profit maximization representing the oligopolists reaction functions with $i = [W; E]$:

$$\frac{d \Pi_i}{dX_i} = \frac{1-\alpha}{\alpha} w_H Y^{\frac{1}{\alpha}} X \left( -\frac{1}{\alpha} \frac{1}{X^{\frac{1}{\alpha}}} - 1 \right) - c_i = 0$$

$$\Leftrightarrow \frac{1-\alpha}{\alpha} w_H Y^{\frac{1}{\alpha}} X \cdot \frac{1}{\alpha} \left( 1 - \frac{X_w}{X} \right) = c_W$$

$$\land \frac{1-\alpha}{\alpha} w_H Y^{\frac{1}{\alpha}} X \cdot \frac{1}{\alpha} \left( 1 - \frac{X_E}{X} \right) = c_E$$

Appendix B, (a) shows that the second order condition for a profit maximum is fulfilled.

Dividing the latter first order conditions for East and West yields:
1 - \frac{\frac{X_W}{\alpha}}{\frac{X}{\alpha}} = \frac{c_W}{c_E} \\
\Rightarrow \frac{\alpha X - X_W}{c_E} = \frac{c_W}{c_E} \Leftrightarrow \frac{\alpha(X + X_E) - X_W}{c_E} = \frac{c_W}{c_E} \\
\Rightarrow \frac{X_W}{X_E} = \frac{(1-\alpha)c_W + \alpha c_E}{(1-\alpha)c_E + \alpha c_W} \equiv V_{WE} (12)

V_{WE}, a key variable in this model, is the ratio of production in the West relative to the East. \( c_{WE} \) means \( c_W / c_E \), i.e. marginal production costs in the West divided by marginal costs in the East, in other words the relative western marginal costs.

5 Allocation of Intermediate Good Production

This section examines how the allocation of intermediate good production to eastern and western Europe reacts to changes in the production cost differential between those regions. For this purpose the elasticity of relative interregional production is derived.

The first derivative of expression (12) with respect to marginal costs in the West relative to the East shows the reaction of relative \( X \) production \( V_{WE} \) to changes in relative marginal costs \( c_{WE} \) (more detailed in Appendix B, (b))

\[
\frac{d V_{WE}}{d c_{WE}} = \frac{(1-\alpha)[(1-\alpha) + \alpha c_{WE}] - [(1-\alpha)c_{WE} + \alpha]}{(1-\alpha) + \alpha c_{WE}} < 0 \\
(13)
\]

\( \alpha \) is larger than 0.5 per assumption and hence the term above is negative. That means, increasing marginal production costs in the West relative to the East lower the relative western production \( V_{WE} \) as expected. Facing higher production costs in the West the oligopolist reduces the profit maximizing output \( X_W \), so that the relative western \( X \) output falls.

Now the following elasticity \( \varepsilon \) can be derived from (12) and (13) in order to analyse the sensitivity of relative \( X \) production to changes in relative production costs \( c_{WE} \):

\[
\varepsilon = \frac{d V_{WE}}{d c_{WE}} \cdot \frac{c_{WE}}{V_{WE}} = \frac{1-2\alpha}{[(1-\alpha) + \alpha c_{WE}]^2} \cdot \frac{c_{WE}}{(1-\alpha)c_{WE} + \alpha} \\
\frac{(1-\alpha)}{(1-\alpha)^2 + \alpha^2 + \alpha(1-\alpha)\frac{1}{c_{WE}} + \alpha(1-\alpha)c_{WE}} < 0 \\
(14)
\]
This elasticity of relative interregional production is a measure for the sensitivity of offshore outsourcing to interregional production cost changes. When it is high, cost deviations at home lead to a large relative production shift to abroad. Term (14) and the example in figure 1 show that the elasticity of relative intermediate good production is a function of the relative production costs. We now analyse the first derivative of the denominator of equation (14) in order to find an extremum of $\varepsilon$:

$$\frac{d}{dc_{WE}}[(1-\alpha)^2 + \alpha^2 + \alpha(1-\alpha)\frac{1}{c_{WE}} + \alpha(1-\alpha)c_{WE}] = 0$$

$$\Leftrightarrow -\alpha(1-\alpha)\frac{1}{c_{WE}^2} + \alpha(1-\alpha) = 0 \Leftrightarrow c_{WE} = 1$$

The second order derivative yields:

$$2\alpha(1-\alpha)\frac{1}{c_{WE}^3} > 0$$

This is larger than zero because all terms are positive. Consequently the denominator has a minimum when relative costs are equal to one. What does this mean for expression (14)? A minimum in the denominator leads to a maximum magnitude of the complete expression, but since the numerator is negative for $\alpha > 0.5$ overall equation (14) shows a minimum lower than zero for $c_{WE} = 1$. For $c_{WE}$ towards infinity or to minus infinity the elasticity moves towards zero.

Figure 1: Elasticity $\varepsilon$ of intermediate good production in the West relative to the East ($V_{WE}$) with respect to relative production cost changes ($c_{WE}$), example with half-logarithmic scale, $\alpha = 0.66$
Therefore, changes in the relative marginal costs have little effect on the production distribution, when the cost differential between the two regions is large. The relative production elasticity has the highest magnitude (lowest negative value) when production costs in the regions are equal (the relative production cost is then $10^0 = 1$ in the logarithmic scale in figure 1). This effect stems from the non-linearity of the $X$ demand and supply functions or decreasing marginal returns to factor inputs, respectively. It corresponds to the intuition, that starting in a situation with equal countries, lowering costs in one country will create a high incentive to engage in offshore outsourcing. However, this requires absence of any rigidities or sluggishness in adjustment. Beginning in a situation with a large interregional cost difference, major offshore outsourcing activities have already taken place, and further cost changes will only cause slight adjustments in the outsourcing magnitude.

The next step yields the absolute quantities of $X_W$ and $X_E$, total $X$ production and the price $p_x$. We first substitute $X_E V_{WE} (= X_E \ast X_W / X_E)$ for $X_W$ in equation (11) and solve for $X_E$:

\[
\frac{1 - \alpha}{\alpha} w_H Y^{\alpha} \left[ X_E \left(1 + V_{WE}\right) \right]^{\frac{1}{\alpha}} \left( 1 - \frac{1}{\alpha} \frac{X_E}{X_E \left(1 + V_{WE}\right)} \right) = c_E
\]

\[
\Leftrightarrow X_E = \left( \frac{1 - \alpha}{\alpha} \frac{w_H}{c_W + c_E} \right)^{\alpha} \frac{1}{1 + V_{WE}} Y
\]

\[
\Leftrightarrow X_E = \left( \alpha' \frac{w_H}{c_W + c_E} \right)^{\alpha} \frac{1}{1 + V_{WE}} Y
\]

With the definition of $V_{WE}$ according to (12) in the middle bracket and some algebra (Appendix B, (c)) $X_E$ can be expressed as:

\[
X_E = \left( \frac{1 - \alpha}{\alpha} \left( 2 - \frac{1}{\alpha} \right) \frac{w_H}{c_W + c_E} \right)^{\alpha} \frac{1}{1 + V_{WE}} Y
\]

\[
\Leftrightarrow X_E = \left( \alpha' \frac{w_H}{c_W + c_E} \right)^{\alpha} \frac{1}{1 + V_{WE}} Y
\]

We write $\alpha'$ for simplicity. $1 / (1 + V_{WE})$ is the eastern European share of $X$ production. The eastern supply of $X$ increases proportionally with the exogenous final output $Y$. The higher the sum of marginal costs in the West and East compared to the price for high-skilled labour $w_H$, the lower the input of $X_E$ in final production.

Western $X$ production can be derived in an analogue way using (10) and replacing $X_E$ by $X_W / V_{WE}$:

\[
\Rightarrow X_W = \left( \frac{1 - \alpha}{\alpha} w_H \right)^{\alpha} \left( 1 - \frac{1}{\alpha} \frac{1}{1 + \frac{1}{V_{WE}}} \right)^{\alpha} \frac{1}{1 + \frac{1}{V_{WE}}} c_W^{-\alpha} Y
\]

\[
\Rightarrow X_W = \left( \alpha' \frac{w_H}{c_W + c_E} \right)^{\alpha} \frac{1}{1 + \frac{1}{V_{WE}}} Y
\]
Now $1 / (1 + 1/V_{WE})$ is the western share of total $X$ production.

Equations (18) and (20) represent a Nash equilibrium, because both oligopolists maximize their profits considering the rival's behaviour, and the firm that deviates from this optimal production will suffer a lower profit.

Adding (20) and (18) leads to total supply of $X$:

$$X = X_W + X_E = \left( \alpha' \frac{w_H}{c_W + c_E} \right)^\alpha \left( \frac{1}{1 + 1/V_{WE}} \right) Y$$

$$= \left( \alpha' \frac{w_H}{c_W + c_E} \right)^\alpha Y = \left[ \frac{(1-\alpha)}{\alpha} \left( 2 - \frac{1}{\alpha} \right) \frac{w_H}{c_W + c_E} \right]^\alpha Y$$

(21)

The price $p_X$, which is identical for $X_W$ and $X_E$, becomes obvious by comparison of the above term with expression (6):

$$X(Y, w_H, p_x) = \left( \frac{\alpha p_x}{(1-\alpha)w_H} \right)^{-\alpha} Y = \left( \frac{1-\alpha}{\alpha} \frac{w_H}{p_x} \right)^\alpha Y$$

It follows:

$$p_X = \frac{c_W + c_E}{2 - \frac{1}{\alpha}}$$

(22)

The price for $X$ is a linear function of the sum of the marginal costs in the regions. The input of $X$ as a production factor in final production decreases when $p_X$ is raised. Consequently higher costs in one region not only reduce production in this region, but also total supply of the intermediate good $X$ by increasing $p_X$. Hence it is not immediately clear, if higher production costs in the East increases or decreases absolute $X$ production in the West. This depends on whether the production shift from East to West is lower or higher than the negative effect of the total $X$ production decline. The magnitude of $p_X$ movements due to marginal cost changes is determined by the coefficient $1 / (2 - 1/\alpha)$. For $\alpha$ approaching 1, $p_X$ is approximately the sum of marginal costs. This would imply a very high exponent in the Cobb-Douglas function for $H$ and a very low exponent for the input $X$. The magnitude becomes higher and higher, when $\alpha$ falls towards 0.5, keeping in mind, that 0.5 is a lower bound for $\alpha$ in this model.

6 Allocation of Production Factors

This section describes the profit maximizing allocation of production factors depending on production costs or factor costs. After deriving factor demand ratios, absolute factor demands are calculated in order to analyse the relative factor allocation between the two regions. The analytic relationships are used to carry out simulations in section 7.

At first we look at the input of high-skilled labour intensive input $H$ by plugging $p_X$ into (7):
\[
H(Y, w_H, c_W, c_E) = \left( \frac{\alpha}{(1-\alpha) \left( 2 - \frac{1}{\alpha} \right) w_H} \right)^{1-\alpha} \frac{c_W + c_E}{Y}
\]

\[
= \left( \frac{1}{\alpha'} \frac{c_W + c_E}{w_H} \right)^{1-\alpha} Y
\]

Solving (21) for \(Y\) and replacing \(Y\) above:

\[
H(X, w_H, c_W, c_E) = \left( \frac{1}{\alpha'} \frac{c_W + c_E}{w_H} \right)^{1-\alpha+\alpha} \frac{X}{\frac{1}{\alpha'} \frac{c_W + c_E}{w_H}} X
\]

\[
\Leftrightarrow \frac{H}{X} = \frac{c_W + c_E}{1-\alpha} \frac{\alpha}{w_H} \frac{1}{2 - \frac{1}{\alpha}} = \frac{p_X}{\frac{1-\alpha}{w_H}} \frac{1}{\alpha}
\]

This cost minimizing ratio of high-skilled labour intensive input \(H\) to the low-skilled manufactured input \(X\) leads back to the standard microeconomic outcome, but with \(p_X\) expressed as shown before. A rising sum of marginal costs in \(X\) manufacturing shifts the input intensity in final production from \(X\) to \(H\). In contrast to the standard relationship the additional impact of \(\alpha\) is unclear, because \(\alpha\) also affects \(p_X\) via the factor \(1 / (2 - 1 / \alpha)\).

At second we write the conditional factor demands for low-skilled labour \(L_i\) and capital \(K_i\) according to the production function in (3) with \(i = [W; E]\) in order to produce the amounts \(X_i\) at minimal costs:

\[
K_i(X_i, r_i, w_i) = \frac{1}{A_i} \left( \frac{\beta_i w_i}{(1-\beta_i) r_i} \right)^{1-\beta_i} X_i
\]

\[
\Leftrightarrow X_i = A_i \left( \frac{(1-\beta_i) r_i}{\beta_i w_i} \right)^{1-\beta_i} K_i
\]

\[
L_i(X_i, r_i, w_i) = \frac{1}{A_i} \left( \frac{\beta_i w_i}{(1-\beta_i) r_i} \right)^{\beta_i} X_i
\]

\[
\Leftrightarrow X_i = A_i \left( \frac{\beta_i w_i}{(1-\beta_i) r_i} \right)^{\beta_i} L_i
\]
Now we first set \( i = W \) and then \( i = E \) in (26) and replace \( X_W \) and \( X_E \) in (12):

\[
A_W \left( \frac{(1-\beta_W) r_W}{\beta_W w_W} \right)^{1-\beta_W} K_W = V_{WE} A_E \left( \frac{(1-\beta_E) r_E}{\beta_E w_E} \right)^{1-\beta_E} K_E
\]

\[
\iff \frac{K_W}{K_E} = V_{WE} \frac{A_E}{A_W} \left( \frac{(1-\beta_E) r_E}{\beta_E w_E} \right)^{1-\beta_E} \frac{1-\beta_W}{\beta_W w_W}
\]

This can be simplified in case of \( \beta = \beta_W = \beta_E \):

\[
\frac{K_W}{K_E} = V_{WE} \frac{A_E}{A_W} \left( \frac{r_E}{w_E} \right)^{1-\beta} = V_{WE} \frac{A_E}{A_W} \left( \frac{w_E}{r_E} \right)^{1-\beta}
\]

where

\[
V_{WE} = f \left( \frac{-c_W}{c_E} \right) = f \left( \frac{-A_E}{A_W} \frac{w_W}{w_E} \frac{r_W}{r_E} \right)
\]

The minus signs indicate that \( V_{WE} \) is a falling function of \( A_E / A_W \) and \( w_W / w_E \) and \( r_W / r_E \). The capital input in the West compared to the East is therefore determined by the ratio of the productivities, the relative western return on investment and the relative western wage level, where relative means West compared to East, and also price of capital relative to the price of labour.

A higher relative western return on investment unambiguously lowers the relative western capital input (directly in (30) and indirectly via \( V_{WE} \)). The effect of a higher relative western wage on the capital input is twofold: On the one hand a higher western wage shifts production towards more capital input, on the other hand a higher wage increases overall production costs, which drives production and thus capital from West to East. The result of a productivity improvement in the West is ambiguous, too: It lowers production costs extending western \( X \) production, but at the same time a given output \( Y \) can be produced with less inputs.

The ratio of labour inputs is expressed similarly using (28) in (12):

\[
\frac{L_W}{L_E} = V_{WE} \frac{A_E}{A_W} \left( \frac{\beta_E w_E}{(1-\beta_E) r_E} \right)^{\beta_E} \frac{1-\beta_W}{\beta_W w_W} \left( \frac{\beta_W w_W}{(1-\beta_W) r_W} \right)^{\beta_W}
\]
When $\beta = \beta_w = \beta_E$ this can be written as:

$$\frac{L_W}{L_E} = V_{WE} \frac{A_E}{A_W} \left( \frac{w_E}{r_E} \right)^\beta = V_{WE} \frac{A_E}{A_W} \left( \frac{w_E}{w_W} \frac{r_W}{r_E} \right)^\beta$$

where $V_{WE} = f \left( \frac{c_W}{c_E} \right) = f \left( \frac{A_E}{A_W} \cdot \frac{w_W}{w_E} \cdot \frac{r_E}{r_W} \right)$

Now a higher relative western wage definitely lowers the labour input in the West in comparison to the East (directly in (32) and indirectly via $V_{WE}$), but the impacts of changes in $r_w / r_E$ and $A_w / A_E$ are per se not clear because of the mechanism explained before.⁵

Hence, we can separate two simultaneous effects:

- Distribution of production and hence capital and labour to western and eastern Europe, that is allocation between the regions.
- Substitution of capital and labour within the regions.

Summing up the factor allocation process can be explained by the following driving forces:

- The wage for the high-skilled workers $w_H$ compared to the sum of marginal production costs in the East and West determines the ratio of $H$ to $X$ in final good production.
- The marginal cost ratio $c_w / c_E$ determines the distribution of $X$ production to West and East.
- The capital intensities of $X$ production in the regions follow from the ratio $w_i / r_i$.

### 7 Transport Costs

Of course manufacturing the intermediate good $X$ in the eastern region for final production in the western region involves transportation costs for $X$. Referring to the classic approach by Samuelson, transport costs $C$ are represented by a quantity melting like ice. For that purpose an additional factor $(1 - C)$ is introduced in (3) and combined with the productivity coefficient $A_E$ for simplicity:

$$X_E = A_E' K_E^{\beta_E} L_E^{1-\beta_E} mit A_E' = (1 - C) A_E, \ 0 < C < 1$$

It is immediately obvious that transport costs lower the eastern productivity or in other words increase eastern marginal production costs and hence shift production from East to West.

---

⁵ Furthermore, interregional capital labour ratios such as $K_w / L_E$ can be derived in the same way.
8 Graphical Representation

The following graphs visualize the effects of a change in relative production costs in form of a rising western wage level as an intuitive example. The simulations are based on the equations derived in section 5. Figure 2 illustrates the change in absolute amounts of total $X$ production, $X$ production in the West and East separately and the high-skilled labour intensive input $H$, when the western wage rises relative to the eastern wage, while the eastern wage is fixed (wage West divided by wage East). A change in labour costs represents a change in marginal production costs.

The following assumptions about parameters have been made for the specific production functions used in the model:\(^6\)

For any relative wage the graphs show the inputs necessary to produce one unit of final output $Y$. The Cobb-Douglas exponent for the high-skilled labour intensive input $\alpha$ equals 0.66 and that one for the $X$ input therefore 0.34. Furthermore, the price of $H$ is set 15 times higher than the payment for eastern low-skilled workers ($w_e = \text{wage East} = 1$). Accordingly there is a much higher income share for high-skilled workers such as managers, engineers and designers creating $H$ than for low-skilled workers manufacturing $X$. In the graph the western wage ($w_w$) rises exogenously from 1 up to 5. For example the labour cost ratio between Germany and the Czech Republic amounts to about 5 and might decrease in the future. Returns on capital in the East and West are set equal to one,

\(^6\) For a comparison of relative wages, productivity and labour costs between Germany and Austria and eastern European regions see Marin (2004).

Figure 2: Absolute production allocation depending on an increasing western wage
since the graph focuses on wage differentials. The exponents $\beta_i$ related to capital inputs in intermediate good production are assumed to be 0.34 in both regions. The exponents $1 - \beta_i$ for labour inputs consequently lead to higher income shares for low skilled-workers than for capital owners in every region. The western $X$ producer has an advantage in total factor productivity, for simplicity $A_E$ is set to 1 and $A_W$ to 2. The reason is access to a better production technology in the West. Finally transport costs $C$ are included amounting to 10% of the transferred good $X$ causing a further cost disadvantage for the East.

Another assumption is that for equal wages (wage $\text{West} / \text{wage East} = 1$, left hand side) the manufactured quantity of $X$ in the West is higher than in the East due to the double western productivity level compared to the East and transport costs of bringing $X_E$ form East to West. However, rising the western wage while holding the eastern wage constant reduces western $X$ production strongly. At the same time eastern $X$ production slightly decreases resulting in an even larger decrease in total $X$ supply. The reason is that higher western production costs increase $p_X$ (the common price for eastern and western good $X$) proportionally as shown before in (22). When the price for the factor $X$ increases, the final good producer replaces $X$ by $H$ leading to a (sharp) decline in the demand for $X$. This causes a rising slope of the $H$ curve in figure 2.

Furthermore, figure 3 points out the mechanisms on the $X$ supply side. The curves are the graphical representations of the reaction functions (10) and (11) of the western and the eastern firm, each representing profit maximizing outputs as a function of the rival's output. The parameter values are set as before. The curves are asymmetric because the eastern oligopolist faces transport costs and an inferior technology. Now a rise of the western wage from 1 to 5 shifts...
the western reaction curve downwards lowering western output strongly \((dX_W)\) and decreasing eastern output slightly \((dX_E)\).

Figure 4 plots the relative factor distribution, i.e. western in relation to eastern \(X\) production, western compared to eastern low-skilled labour and capital input as well as the ratio of the western high-skilled labour intensive factor \(H\) to low-skilled labour intensive \(X\) being produced in the West and East. Again the western wage is increased relative to the fixed eastern wage. All parameter values are the same as before and the results are in accordance with these of figure 2. For the case of identical wages (left hand side) the output of \(X\) in the West is higher than in the East \((X_{WE} > 1)\) because of the western technological advantage and the eastern transport costs disadvantage. Nevertheless low-skilled labour input is lower in the West than in the East, and capital input is lower in the West, too. This follows from the fixed final good \(Y\) output limiting the demand for factors \(H\) and \(X\) and from the western productivity advantage just described.

![Figure 4: Relative factor allocation depending on an increasing western wage](image)

Obviously all curves in figures 2 and 4 have a sharper (rising or falling) slope when labour costs are similar (left hand side) in accordance with the higher elasticity of relative production for equal marginal costs derived in chapter 4. This is caused by the strictly convex inverse factor demand function (8) which is steeply falling for low prices in combination with Cobb-Douglas type intermediate good production and the resulting reaction functions.7

7 Since a Cobb-Douglas function with constant returns to scale implies a decreasing marginal product of labour, the inverse factor demand function has a convex falling shape.
9 Implications and Caveats

The model analysis provides the following implications:

It reveals that the elasticity of interregional outsourced production movements as a reaction to interregional cost changes depends on the original gap in production costs between the regions. When intermediate good production occurs in two regions with similar production costs, any cost change in one region will have strong effects. So, a higher wage agreement in the western intermediate good firm or a higher wage level after bargaining with unions lowers the western intermediate good production relative to the eastern one a lot. In the same way higher additional labour costs, tax and insurance, have a strong negative influence on western production and labour input. Given a situation of a large East-West production cost differential and completed offshore outsourcing activities, additional cost changes will have little effect on outsourcing, thus contradicting outsourcing fears.

Moreover, higher marginal costs in one region not only reduce intermediate good output in this region, but also in the other region due to a rising common intermediate good price, which reduces demand for the intermediate good. Thus, the cost increase reduces intermediate good production in both regions. Of course the output decline is higher in the region, where the cost increase occurs. In order to keep final output constant low-skilled labour intensive intermediate good input is substituted by equivalent high-skilled labour and advanced technology rich input that can be found in western Europe. Consequently any rise in intermediate good production costs benefits western high-skilled workers. The outcome for low-skilled workers in the region with rising costs depends on whether the cost increase includes higher wages. In case of higher wages the entire group of low-skilled workers in that region can still lose because the production decline due to higher costs eliminates jobs. If the wage increase overcompensates the reduced labour input the workers can all be better off after redistribution. However, this is unlikely the case. Without any cost change but less production in the other region capital owners and workers involved in intermediate good production in the other region both lose, but only slightly because the output decline is small compared to the region with rising costs. Looking at absolute numbers in this model, we conclude that there is no competition between low-skilled workers in the East versus the West, but mainly between western high-skilled on the one hand and both eastern and western low-skilled workers on the other hand. Of course, referring to relative intermediate good production, any cost increase in a region lowers the relative production share of this region compared to the other region.

According to the graphical interpretation for a specific parametrization, an advantage via a higher productivity leads to a higher intermediate good output with relatively less factor inputs.

When applying and interpreting the model several caveats should be considered:
The model includes supply of final output \( Y \) only, assuming a constant demand for \( Y \) in order to concentrate on the production side and to make the model analytically tractable. The analysed changes in factor prices or technology do not affect the price for \( Y \), either. But since the development of consumer tastes and technological change during the allocation process are unknown, it is reasonable to set these variables exogenous and constant.
An oligopoly in intermediate good production is a sensible assumption for certain sectors of the economy, but not for all sectors. Production of low-technology intermediate products with low investment barriers and expected profits invites new entrants. However, an oligopoly could also occur in final rather than intermediate good production, particularly in German automobile fabrication.
Pure competition in quantities (Cournot competition) with homogeneous goods and a valid “Law of One Price” ignores any possibility for the intermediate good oligopolists to set a price different to the rival. This is suitable for small to moderate differences in production costs, whereas the incentive to set a lower price than the rival and attract all the demand becomes stronger with a
larger cost gap. On the other hand a large cost differential points to a big difference in per capita incomes and the levels of development. Hence a poor infrastructure, unsecured property rights, corruption and other disadvantages and risks dominate the outsourcing decision of investors rather than pure production costs. These factors prevent higher outsourcing into the low cost country. Therefore, it is important to keep in mind that other aspects driving outsourcing decisions like countries' infrastructure, taxes and laws are not captured in the model. Furthermore, the findings are based on convex decreasing inverse factor demand functions derived from Cobb-Douglas technologies. The intermediate good producers tend to keep the output low with the aim of holding the price high. Moreover, the exponent for the high-skilled labour intensive input in the Cobb-Douglas function is assumed to be larger than one half so that the exponent for the low-skilled labour intensive input is smaller than one half. This assumption is sensible since it implies a higher income share or return for the high-skilled labour and high technology intensive input found in the western region than for the low-skilled labour intensive input that is outsourced. It indicates in a way that production is mainly located in the western region. The analysis presented in the paper does not hold for Cobb-Douglas exponents smaller than one half for the immobile high-skilled labour and high technology input. The results show the strongest elasticity of relative interregional production when the interregional cost difference is smallest, that is the countries are similar. In reality there is a sluggishness of production movements due to home bias and costs of planning and organizing the outsourcing adjustment. This sluggishness opposes the offshore outsourcing incentive, and the outsourcing costs create a threshold, i.e. a minimum cost differential, which is necessary to cause any offshore outsourcing. Nevertheless it is sensible to assume perfect adjustment without rigidities in a theoretical model pointing out ideal economic processes. Finally, a precondition for the analysis is a situation of completed adjustment of factor allocation according to efficiency or profit maximization, respectively. When applying the model to the current situation of the European Union we need to take into account, that dynamic adjustment processes are still in progress. Consequently, adjustment processes towards the equilibrium offset the outcomes resulting from the model.

10 Conclusion

This paper deals with the question, how sensitive outsourcing from a high-technology area with high production costs like western Europe to a low-technology and low-costs area like eastern Europe really reacts to changes in production costs or productivity, respectively. The model presented here is a two stage outsourcing model based on Cobb-Douglas technologies and provides a detailed analysis of Cournot competition in intermediate goods as perfect substitutes. Sophisticated final good production takes place in western Europe. The intermediate activity can be located in western Europe (outsourcing) or eastern Europe (offshore outsourcing). It is shown how relative production costs, encompassing technological levels, wages and returns on investment, influence the allocation of production and of capital and labour inputs between the two regions. In the final good production stage (with fixed output) higher costs of the intermediate good, which is low-skilled labour intensive, lead to a substitution of the intermediate good input by high-skilled labour intensive input. The first key result is, that an increase of intermediate good production cost in one region reduces intermediate good production in both regions because external intermediate good input is replaced by internal high-technology and high-skilled labour input. Therefore, the model implies substitution of western high-skilled labour intensive input on the one hand and eastern and western low-skilled labour intensive input on the other hand and not interregional substitution between eastern and
western low-skilled labour. However, in relative terms higher intermediate good production costs in one region reduce the share of intermediate good production in this region compared to the other region. This means, intermediate good production falls in region A relative to B, when production costs in A rise relative to B. But intermediate goods output in B does not rise in absolute terms, since both regions interact via Cournot competition. Graphical simulations illustrate the findings.

The second key result is that the sensitivity of outsourcing behaviour to production cost changes is higher the smaller the cost difference between the regions is. Consequently the effect of changes in wage agreements or labour taxes depends on the original cost gap. Given a situation of a large East-West production cost differential and completed outsourcing activities, additional cost changes will have little effect on outsourcing behaviour. This contradicts the fear of production rapidly moving towards East once western wages or labour taxes change. Moreover, the simulation example reveals that a superior technology in one region leads to a higher relative output and lower relative factor inputs in that region compared to the other region. An increase of production costs in one region also reduces production in the other through a higher common price for the created good.

Currently labour costs differ strongly between eastern European countries. According to this simplified model and ignoring other factors driving outsourcing decisions, the sensitivity of western outsourcing activities to production cost changes in countries like Czech Republic and Hungary is higher than in Romania and Bulgaria. When eastern production cost levels converge to the western European level the outsourcing sensitivity will increase. However, the analytical model does not capture outsourcing determinants like infrastructure and civil rights predicting highest offshore outsourcing into countries with lowest labour costs.

The model serves as an easy to handle analytical tool and can be implemented in more complicated models. Particularly the final good demand side can be implemented. The analysis could also be done with CES functions instead of Cobb-Douglas technologies. Income effects for high- and low-skilled workers and capital owners in the two regions can be investigated in a more detailed way by using this model.

References


Marin, Dalia; Lorentowicz, Andzelika and Raubold, Alexander (2002), "Ownership, Capital or Outsourcing: What Drives German Investment to Eastern Europe?" Munich Economics Discussion Paper 02-03.


Appendix A

Figure 5: Value added divided by output value in German manufacturing in %, sources: Sinn (2005), Statistisches Bundesamt (2004)

Appendix B

(a) We want to show that the second order condition for a profit maximum on page 9 is indeed fulfilled. We first look at the second derivative of the maximization problem in (9):

\[
\frac{d^2\Pi_i}{dX_i^2} = \frac{1-\alpha}{\alpha}w_HY^{\frac{1}{\alpha}}\left[-\frac{1}{\alpha}\left(-\frac{1}{\alpha}-1\right)X^{\frac{-1}{\alpha}}X_i - \frac{1}{\alpha}X^{\frac{-1}{\alpha}} - \frac{1}{\alpha}\right]
\]
\[
= \frac{1-\alpha}{\alpha} w H Y^{\alpha \left(1 - \frac{1}{\alpha}\right)} \left[-\left(1 + \frac{1}{\alpha}\right) X^{\frac{1}{\alpha} - 2} \cdot X_i + 2 X^\frac{1}{\alpha} - 1\right]
\]

This term is lower than zero, i.e. we find a profit maximum, if the following condition holds (with \(X\) and \(Y\) larger than zero):

\[-\left(1 + \frac{1}{\alpha}\right) X^{\frac{1}{\alpha} - 2} \cdot X_i + 2 X^\frac{1}{\alpha} - 1 > 0\]

\[\Leftrightarrow -\left(1 + \frac{1}{\alpha}\right) \frac{X_i}{X} + 2 > 0\]

\[\Leftrightarrow 2 > \left(1 + \frac{1}{\alpha}\right) \frac{X_i}{X} \Leftrightarrow \frac{2}{1 + \frac{1}{\alpha}} > \frac{X_i}{X}\] (34)

It is necessary to prove that this is fulfilled for all possible values of \(X_i\). We first need a limit of \(V_{WE}\):

\[
\lim_{c_W \to \infty} V_{WE} = \lim_{c_E \to \infty} \left(1 - \alpha\right) \frac{c_W}{c_E} + \alpha = \frac{1 - \alpha}{\alpha} < 1
\] (35)

This limit (with a cost gap tending to infinity while the prices for \(X\) in East and West are equal) is hardly observed in a real market, but needed as a theoretical bound.

We then use (12) and the limit (35) above to show that the second order condition in (34) holds. While doing so \(X_W\) is replaced by \(X_E V_{WE}\):

\[2 > \left(1 + \frac{1}{\alpha}\right) \frac{X_i}{X} \Rightarrow 2 > \left(1 + \frac{1}{\alpha}\right) \frac{X_E}{X_E (1 + V_{WE})}\]

\[\Leftrightarrow 2 > \left(1 + \frac{1}{\alpha}\right) \frac{\frac{1}{1 - \alpha}}{1 + \frac{1 - \alpha}{\alpha}} \Leftrightarrow 2 \left(1 + \frac{1 - \alpha}{\alpha}\right) > \frac{1}{\alpha} + 1\]

We can insert the limit of \(V_{WE}\) above, because it is a lower bound for \(V_{WE}\). That means, any other possible values for \(V_{WE}\) are higher and hence lower the term on the right hand side fulfilling the inequality.

\[\Leftrightarrow 2 + \frac{2}{\alpha} - 2 > \frac{1}{\alpha} + 1 \Leftrightarrow \frac{1}{\alpha} > 1 \Leftrightarrow 1 > \alpha\]

This is true according to the definition of \(\alpha\), consequently there is a profit maximum in eastern Europe. Now \(X_E\) is expressed as \(X_W / V_{WE}\):
\[2 > \left( \frac{1}{\alpha} + 1 \right) \frac{X_i}{X} \Rightarrow 2 > \left( \frac{1}{\alpha} + 1 \right) \frac{X_W}{X_W \left(1 + \frac{1}{V_{WE}}\right)}\]

\[\Leftrightarrow 2 \left( 1 + \frac{\alpha}{1-\alpha} \right) > \frac{1}{\alpha} + 1\]

\[1 + \frac{2\alpha^2 - (1-\alpha)}{(1-\alpha)\alpha} > 0 \Leftrightarrow \frac{\alpha - \alpha^2 + 2\alpha^2 - 1 + \alpha}{(1-\alpha)\alpha} > 0\]

\[\Leftrightarrow \frac{\alpha^2 + 2\alpha - 1}{(1-\alpha)\alpha} > 0\]

This statement is true for \(1 > \alpha > 0.5\), so that a profit maximum exists in western Europe, too.

(b) The derivative on page 9 is calculated as follows:

\[\frac{d V_{WE}}{d c_{WE}} = \frac{(1-\alpha)[(1-\alpha) + \alpha c_{WE}] - [(1-\alpha)c_{WE} + \alpha] \alpha}{[(1-\alpha) + \alpha c_{WE}]^2}\]

\[= \frac{(1-\alpha)^2 + (1-\alpha) \alpha c_{WE} - (1-\alpha) \alpha c_{WE} - \alpha^2}{[(1-\alpha) + \alpha c_{WE}]^2} = \frac{(1-\alpha)^2 - \alpha^2}{[(1-\alpha) + \alpha c_{WE}]^2}\]

\[= \frac{1-2\alpha}{[(1-\alpha) + \alpha c_{WE}]^2} < 0\]

(c) It follows the transformation of (16) into (17) on page 11:

\[X_E = \left( \frac{1-\alpha}{\alpha} w_H \right)^\alpha \left( 1 - \frac{1}{\alpha} \frac{1}{1+V_{WE}} \right)^\alpha \frac{1}{1+V_{WE}} c_{E}^{-\alpha} Y\] (36)

We look at the middle bracket using equation (12):

\[1 - \frac{1}{\alpha} \frac{1}{1+V_{WE}} = 1 - \frac{1}{\alpha} \frac{1}{1 + \frac{(1-\alpha)c_{W} + \alpha c_{E}}{(1-\alpha)c_{E} + \alpha c_{W}}}\]
\[
\alpha + \frac{\alpha(1 - \alpha) c_W + \alpha^2 c_E}{(1 - \alpha) c_E + \alpha c_W} = \frac{1}{\alpha + \frac{\alpha(1 - \alpha) c_W + \alpha^2 c_E}{(1 - \alpha) c_E + \alpha c_W}}
\]

After calculating the multiplications in the numerator and denominator several terms drop out and the expression simplifies to:

\[
\frac{2 \alpha c_E - c_E}{\alpha c_E + \alpha c_W}
\]

We insert this in (36):

\[
X_E = \left( \frac{1 - \alpha}{\alpha} \right)^\alpha \left( \frac{2 \alpha c_E - c_E}{\alpha c_E + \alpha c_W} \right)^\alpha \left( \frac{1}{1 + V_{WE}} \right)^{\alpha} \frac{c_E^{-\alpha}}{Y}
\]

\[
= \left( \frac{1 - \alpha}{\alpha} \right)^\alpha \left( \frac{2 \alpha - 1}{\alpha c_E + \alpha c_W} \right)^\alpha \left( \frac{1}{1 + V_{WE}} \right)^{\alpha} Y
\]

\[
= \left( \frac{1 - \alpha}{\alpha} \right)^\alpha \left( \frac{2 - 1}{\alpha} \right)^\alpha \frac{w_H}{c_W + c_E} \left( \frac{1}{1 + V_{WE}} \right)^{\alpha} Y \quad \text{q. e. d.} \quad (37)
\]

In a similar way (19) can be transferred into (20):

\[
X_W = \left( \frac{1 - \alpha}{\alpha} \right)^\alpha \left( 1 - \frac{1}{\alpha} \frac{1}{1 + \frac{1}{V_{WE}}} \right)^\alpha \left( \frac{1}{1 + \frac{1}{V_{WE}}} \right)^{\alpha} \frac{c_W^{-\alpha}}{Y}
\]

\[
(38)
\]
We concentrate again on the bracket in the middle and use equation (12):

\[
1 - \frac{1}{\alpha} \frac{1}{1 + \frac{1}{V_{WE}}} = 1 - \frac{1}{\alpha} \frac{1}{1 + \frac{(1 - \alpha) c_E + \alpha c_w}{(1 - \alpha) c_w + \alpha c_E}}
\]

\[
= \frac{\alpha + \frac{\alpha (1 - \alpha) c_E + \alpha^2 c_w}{(1 - \alpha) c_w + \alpha c_E}}{\alpha + \frac{\alpha (1 - \alpha) c_E + \alpha^2 c_w}{(1 - \alpha) c_w + \alpha c_E}} - \frac{1}{\alpha + \frac{\alpha (1 - \alpha) c_E + \alpha^2 c_w}{(1 - \alpha) c_w + \alpha c_E}}
\]

\[
= \frac{\alpha + \frac{\alpha (1 - \alpha) c_E + \alpha^2 c_w}{(1 - \alpha) c_w + \alpha c_E}}{\alpha + \frac{\alpha (1 - \alpha) c_E + \alpha^2 c_w}{(1 - \alpha) c_w + \alpha c_E}} - 1
\]

\[
= \frac{\alpha[(1 - \alpha) c_w + \alpha c_E] + \alpha (1 - \alpha) c_E + \alpha^2 c_w - [(1 - \alpha) c_w + \alpha c_E]}{\alpha[(1 - \alpha) c_w + \alpha c_E] + \alpha (1 - \alpha) c_E + \alpha^2 c_w}
\]

Since many terms drop out we find the simple form:

\[
\frac{2 \alpha c_w - c_w}{\alpha c_w + \alpha c_E}
\]

We replace the middle term in (38):

\[
X_W = \left(\frac{1 - \alpha}{\alpha w_H}\right)^\alpha \left(\frac{2 \alpha c_w - c_w}{\alpha c_w + \alpha c_E}\right)^\alpha \frac{1}{1 + \frac{1}{V_{WE}}} c_w^{-\alpha} Y
\]

\[
= \left(\frac{1 - \alpha}{\alpha w_H}\right)^\alpha \left(\frac{2 \alpha - 1}{\alpha c_w + \alpha c_E}\right)^\alpha \frac{1}{1 + \frac{1}{V_{WE}}} Y
\]

\[
= \left(\alpha' \frac{w_H}{c_w + c_E}\right)^\alpha Y \quad q. e. d. \quad (39)
\]
with \[ \alpha' = \frac{(1 - \alpha) \left( 2 - \frac{1}{\alpha} \right)}{\alpha} \]

and divide (39) by (37) in order to prove the result:

\[
\frac{X_W}{X_E} = \frac{1 + \frac{1}{V_{WE}}}{1 + V_{WE}} = \frac{\frac{V_{WE}}{V_{WE} + 1}}{\frac{1}{1 + V_{WE}}} = V_{WE} \quad \text{q. e. d. as in (12)}
\]