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Over- and Underbidding in Central Bank Open Market Operations Conducted as Fixed Rate Tender
Over- and underbidding in central bank open market operations conducted as fixed rate tender

Ulrich Bindseil

This version: February 2003

Abstract

Open market operations play a key role in allocating central bank funds to the banking system and thereby to steer short-term interest rates in line with the stance of monetary policy. Many central banks apply so-called “fixed rate tender” auctions in their open market operations. This note presents, on the basis of a survey of central bank experience, a model of bidding in such tenders. In their conduct of fixed rate tenders, many central banks faced specifically an “under”- and an “overbidding” problem. These phenomena are revisited in the light of the proposed model and the more general question of the optimal tender procedure and allotment policy of central banks is addressed.

JEL classifications: D84, E43, E52
Keywords: open market operations, tender procedures, central bank liquidity management

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1. Introduction

Open market operations play a key role in allocating central bank funds to banks in modern financial systems. To ensure an efficient allocation mechanism and an equal treatment of banks, central bank open market operations have been normally conducted at least since the late 1980s as tenders. However, these tenders are and were not always specified as genuine auctions in which banks could bid at different rates, but, even by a majority of central banks, as so called fixed rate tenders. In fixed rate tenders, the interest rate at which credit is provided is set by the central bank, and banks simply bid for the amount they wish to obtain at that rate. However, central banks then normally do not allot 100% of the bids, but only a certain quota (“allotment ratio”) applicable equally to all bids submitted. The use of fixed rate tenders instead of genuine auctions was occasionally motivated by central banks as allowing to send a stronger signal with regard to the stance of monetary policy (i.e. with regard to the intended short term interest rate). For instance according to Deutsche Bundesbank [1995, 111]: “Through fixed rate tenders, the Bundesbank can give the market a clear interest rate signal and, in periods of uncertainty, exercise a stabilising influence on interest rate movements”. Economists, which are less familiar with the implementation of monetary policy, are often surprised by the fact that in fixed rate tenders, central banks fix both the quantity and the price of the good they offer. This note is basically about this consistency issue and the circumstances under which it can be solved. On the basis of a survey of central bank experience with fixed rate tenders, a model of the aggregate bidding behavior of banks is developed which allows a comparative assessment of fixed rate tenders and genuine auctions under various circumstances. Special focus is given to the European Central Bank (ECB). The ECB has, since January 1999, allocated per year funds worth more than EUR 3 Trillion through its regular open market operations, the main type of which are for collateralised loans with 2-week maturity. The open market operations of the ECB are by far the largest tenders in the world ever conducted in terms of yearly total volumes, suggesting that their efficiency should be of highest interest. During the first 18 months of the euro, they were specified as fixed rate tenders, afterwards, they were defined as variable rate tenders (i.e. genuine auctions) with a minimum bid rate, which, as will be shown, implies properties similar to fixed rate tenders if markets expect declining central bank interest rates. The tendering procedures for fixed and variable rate tenders of the Eurosystem are described precisely in ECB [2002a]. Even though the implementation of monetary policy by the ECB has so far worked rather smoothly, the design of its open market operations has also been questioned by financial market participants and academic economists in relation to the phenomena of “overbidding” and “underbidding”. “Overbidding” refers to extremely high bid volumes submitted to fixed rate tenders, implying, ceteris paribus, extremely low allotment ratios. This phenomenon was observed in the case of the ECB in the second half
of 1999 and the first half of 2000, with bids surpassing the allotment amount by up to a factor of 100. “Underbidding” refers to the lack of bids in a fixed rate tender, such that the central bank cannot allot the liquidity actually needed by banks to fulfill smoothly their reserve requirements. It was experienced once in April 1999, four times in 2001, and twice in 2002.

Overbidding and underbidding in the ECB’s operations have found noticeable academic interest. The first authors assessing the overbidding experience of the ECB were Nautz and Oechsler, who came in October 1999 to the conclusion that (p.18-19) “the auction rules are flawed since they encourage banks to increasingly exaggerate their demand for reserves… Considering the vanishing quota the ECB’s repo auctions are about to become a farce… in view of these problems our suggestion for the ECB would be to employ price discriminating variable rate tenders…” Bindseil and Mercier [1999] provide a basic theoretical model on how central bank liquidity management affects bidding behavior in fixed rate tenders and propose a “golden rule” ensuring that overbidding does not occur, according to which allotment decisions should be such that short term market rates are kept close to the fixed tender rate. Erhard [2001] provides experimental evidence which is supposed to show the contrary, namely that the ECB’s fixed rate tender system unavoidably leads to overbidding and that “even accommodate policy cannot prevent increasing exaggeration in the bids”, hence confirming the result of Nautz and Oechsler [1999]. Ayuso and Repullo [2000], [2001] concentrate on demonstrating that the ECB had an asymmetric objective function, which made it provide systematically too little liquidity, hence creating the overbidding problem. Ayuso and Repullo [2001] and Nautz and Oechsler [1999] both share the view that allotment ratios in fixed rate tenders will either be indeterminate (or drawn from a continuum of equilibria) or will tend to infinity (or to a limit). Nautz and Oechsler [1999] make use of adaptive expectations building to reconcile the indeterminacy with the evidence. It is noteworthy that the evidence exposed in section 2 (and also its interpretation in the light of the proposed model) suggests a rather different conclusion with regard to the nature of overbidding in the case of the Eurosystem, namely that it was linked to rate hike expectations. Valimaki [2001] proposes an elaborated model of bidding in fixed rate tenders which however does not contain two elements which appear important according to our survey of experience, namely the cost of bidding and the coordinating role of the central bank when taking an allotment decision. Finally, Ewerhart [2002] provides a theoretical model of underbidding specifically in the case of an environment as the one set up by the ECB, providing several interesting insights, but again, without these two model elements.

The rest of this paper is organized as follows. Section 2 provides the survey of “overbidding” and “underbidding” experienced by central banks applying fixed rate tenders. The survey shows that both under- and overbidding normally relate either to a biased allotment policy, or to expectations of changes of central bank rates. While a biased allotment policy can be
easily corrected by the central bank, expectations of rate changes can normally be made irrelevant only through adequate reforms of the operational framework of central banks, such that the rate changes are excluded to become relevant in the period until the tender in question matures. Starting from the evidence presented in Section 2, Section 3 introduces a model of central bank tender operations and of a money market in the case of a one-day reserve maintenance period. As will be explained, the one-day model is sufficient to work out the main equilibrium conditions in the absence of expectations of rate changes. At the same time, it allows to investigate how under- and overbidding may be triggered in such an environment by an inadequate liquidity policy of the central bank. It is argued that the central bank liquidity policy should be neutral in the sense of being compatible with a money market rate equivalent to the minimum bid rate. Section 4 focuses on how rate change expectations trigger over- or underbidding. This requires the setting up of a model with two days per reserve maintenance period. This variant of the model also allows analyzing the properties of genuine auctions under rate change expectations. It is shown that fixed rate tender have, under these circumstances, specific disadvantages relative to variable rate tenders (genuine auctions) which sometimes cannot be compensated through any allotment policy. Finally, section 5 concludes on what tender procedure and allotment policy the central bank should choose under different environments and preferences and comes back to the basic question of the nature of under- and overbidding.

2. Central banks’ experience with over- and underbidding

2.1. Introduction

Nearly all central banks have sometimes used fixed rate tenders and in fact it even seems that the majority of central banks currently prefer it to pure auctions. However, in this survey, we will focus on a few of them, which have applied the procedure frequently in the 1990, namely the Swiss National Bank (CH), the Deutsche Bundesbank (D), the Banque de France (F), the Bank of Finland (FI), the Bank of Japan (J), the ECB/Eurosystem (ECB), the South African Reserve Bank (SA), the Swedish Rijksbank (SE), and the Bank of England (UK).2 A special focus will be on the Eurosystem’s experience, since, due to the features of its operational framework, the phenomena of over- and underbidding were more pronounced here than elsewhere. Over- and underbidding are reviewed successively. Before that, however, two more issues need to be exposed.

First, the central banks’ operational framework beyond the specification of the fixed rate tender is important in understanding the extent of over- and underbidding that may be observed. Especially the following four dimensions appear to be relevant, for reasons that

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2 D, F and FI are considered as up to 1998, i.e. up to the introduction of the euro.
will appear in the next paragraph. (1) The frequency of the fixed rate tender operation varies
between at least daily (CH, J, UK) to once a week (D, ECB, F, FI, SA, SE), whereby for
instance F and SE do additional daily fine tuning operations. (2) The maturity of the
operations varies between 1 day (SE), one week (CH, F), two weeks (D, ECB) and one
month (FI, SA), while some central banks (J, UK) seem to frequently operate at various
maturity. (3) Regarding the collateralisation requirements, one needs to distinguish the case
in which only *allotments* at the moment of settlement need to be covered by collateral (J,
ECB, SA, SE, CW, UK) or the *entire bid amount* needs to be covered at the moment of
bidding (DE). France switched from the first to the second approach as a reaction to
overbidding (see below). (4) While D, ECB, F, J, SA have a reserve requirement system with
averaging and a one month reserve maintenance period, SE, CH and UK do not impose
relevant reserve requirements.

Second, the basic logic underlying under- and overbidding should be explained. As
mentioned in the introduction, both result from an imbalance between the fixed tender rate
and the corresponding market rate. To understand the determinants of the market rate,
assume for a moment that there is no uncertainty concerning liquidity factors in the
remainder of the reserve maintenance period. This being the case, and assuming a perfectly
efficient interbank market, reserves are obviously either short in relation to reserve
requirements, in which case overnight rates should rise to the emergency rate charged by
the central bank (e.g. the “marginal lending rate” in case of the Eurosystem), or long, in
which case they drop to a possible floor (zero, or, if available, the rate of an excess liquidity
absorption facility offered by the central bank). Now, consider the more interesting case in
which liquidity supply until the end of the maintenance period is uncertain. The overnight rate
on any day will then correspond to the weighted average of the two standing facilities rates,
the weights being the respective probabilities that the market will be short or long at the end
of the maintenance period. Ex ante, overnight rates are constant within the reserve
maintenance period, i.e. the expected overnight rates within the maintenance period can
never diverge from one day to the next. The reason for this property is that any anticipated
change in the overnight rate within the maintenance period would trigger attempts on the
part of banks to reschedule their fulfillment of reserve requirements within the maintenance
period, which would ultimately lead to an equalization of rates. In fact, the goods “reserves
held today” and “reserves held tomorrow” are perfect substitutes in this model, since they
both contribute equally to the fulfillment of reserve requirements. Hence, denoting by $T$ the
last day of the maintenance period and by $i_{m,T}$, $i_d,T$ the rates of the marginal lending and
the deposit facilities, respectively, at the end of the reserve maintenance period, we obtain
the following core equation explaining the market overnight rate $i_t$ (see also for instance
Bartolini et al. [2002]):
\[ i_t = E_t(i_{t+1}) = \ldots = E_t(i_T) = E_t(i_{ml,T})P("short") + E_t(i_{dl,T})P("long") \]  

(1)

Inter-bank rates with the maturity of the tender operation can be assumed to be implied from expected overnight rates using the expectations hypothesis of the term structure of interest rates. As was noted for instance by Perez-Quiros and Mendizabal [2001], a necessary theoretical condition for the martingale property to hold is that the no-overdraft constraint for reserve holdings of banks with the central bank is not binding, either because reserve requirements are so high that liquidity absorbing autonomous factor shocks (net of open market operations) never push banks into overdraft at the end of the day, or because overnight overdrafting is allowed. While the latter condition is not met for any major central bank, the former indeed seems to be verified in cases of money markets like the Japanese or the euro area ones due to the substantial reserve requirements imposed. In the case of the euro area, reserve holdings in the first four years of the euro fell from their required aggregate level of EUR 100 – 130 billion once in March 1999 to a minimum of EUR 63 billion (the second lowest figure being EUR 76 billion in January 1999; reserves have not fallen below EUR 100 billion since April 2001), which still appears to be far above working balances. Indeed, Wurtz [2003] shows that the time series properties of the euro area overnight rate (EONIA) speak clearly against the relevance of the no-overdraft constraint.³

2.2 Overbidding

Survey of central bank experience

Overbidding generally occurs if market interest rates are, for the maturity of the tender operation, above the fixed tender rate, such that all banks rush to this arbitrage opportunity, and hence submit large bids. All central banks in our sample experienced at least sometimes in their fixed rate tenders bids being considerable above intended allotment amounts of the central bank, leading hence to low allotment ratios. However, the strength of the phenomenon, the reason behind it, the factors limiting its extent, and the central bank policy towards it differed considerably. The phenomenon was probably least relevant in Japan, where an ample liquidity policy and rate cut expectations have prevailed since shortly after the introduction of tender procedures in the early 1990s. The phenomenon is also of little relevance in SA where allotment ratios are normally between 60 and 100% with a minimum of around 20%. Apparently, even under rate hike expectations and although only allotted funds need to be collateralised, scarcity of collateral is sufficient in SA to deter banks from overbidding. In SE, CH and UK, allotment ratios have not dropped below a minimum of 10%.

³ As Perez-Quiros and Mendizabal [2001] show theoretically, the no-overdraft constraint, if relevant, would lead to a spike of the overnight rate at the end of the reserve maintenance period. However, in the case of the euro area, Wurtz [2003] reveals an opposite tendency of the overnight rate to drop by a few basis points on the last days of the maintenance period.
and normally, they are above 20%. It appears that in these countries, due to the absence of reserve requirements, the central banks can, even under rate hike expectations, easily provide incentives against overbidding by allotting excess funds, such that excess liquidity pushes overnight rates below the tender rate at least until the potential rate hike. However, in case of the UK, the central bank nevertheless felt that it was necessary to have two tools in hand against overbidding: counterparties are not permitted to bid for more than the size of the liquidity shortage (an average daily shortage being in the region of £2bn) and the central bank reserves the rather discretionary right to “reject individual bids and to accept individual bids in part and at levels determined by the Bank” to ensure that “access to the liquidity provided by the Bank of England is available as smoothly as possible to a wide range of market participants.” In DE, average allotment ratios declined throughout the 1990s and ended at around 15% in 1998. Incentives to overbid originated apparently from the fact that through a somewhat tight allotment policy, market rates were kept on average in the period February 1996 to December 1998 (the last period of permanent use of the fixed rate tender) 11 basis points above the tender rate. In contrast, for instance, the average spread in the euro area in the period January 1999 to June 2000 was only 7 basis points (although this period was dominated by rate hike expectations). However, overbidding was limited in the Bundesbank’s case by the requirement to cover the entire bid at the moment of bidding by collateral. Since collateral is always scarce, this limits the scope of overbidding. The case of F was somewhat similar, whereby the spread between market and tender rates was even much higher, namely 39 basis points on average from 1994 and 1998. The spread however varied considerably over time during this period, the yearly averages amounting to 27, 140, 13, 9 and 7 basis points, respectively. Before F introduced in 1994 the same requirement regarding marketable collateral as D, namely to entirely cover bids at the time of bidding, this lead to a decline of allotment ratios to less than 1%. After the change, allotment ratios stabilized around 20-30%. It is also interesting to note that in F, there were in the 1990 always two types of auctions in parallel: auctions for which marketable paper was accepted as collateral, and those were bank loans were accepted. Allotment ratios for the latter were normally around 50%, reflecting the lower total amounts of available eligible bank-loans, and the requirement to cover bids fully. Another country that needed to react to acute overbidding was FI, which switched in 1996 from pro rata allotments to systematic 100% allotments. It continued with this system until the end of 1998. Finally, the ECB also experienced intense overbidding which also forced it to abandon the standard fixed rate tender, as described in detail in the following subsection. One may conclude that apparently, central banks with reserve requirement and averaging in a banking system with plenty collateral are the most vulnerable to overbidding. Then, sufficient conditions for overbidding to emerge are either expectations of possible rate hikes within the same reserve maintenance period (e.g. the ECB), or a tendency of the Central Bank to steer liquidity conditions in a tight way such that market rates are driven through this channel above the fixed tender rate. The only limit to
overbidding is then to require bids to be covered by collateral (as D and F did) or to switch to a 100% allotment rule (FI) or to abandon the fixed rate tender (ECB). The model presented in sections 3 and 4 will be designed in a way to fully capture these relationships.

A closer look at the Eurosystem’s experience

The ECB’s overbidding episode has been described repeatedly in the literature. The ECB itself summarized its experience after announcing the switch to the variable rate tender in a press release as follows: “...The strong rise in bids in the first half of 2000 was due to the fact that, during most of that period, there were market expectations of interest rate hikes and short term money market rates were significantly above the main refinancing rate. This made it attractive for banks to bid for large amounts of liquidity from the central bank.” To provide illustrative evidence, Chart 1 draws related time series. As explained at the beginning of this section, the inter-bank overnight rate should depend on the expected central bank rates and on the expected liquidity conditions, both at the end of the reserve maintenance period. Therefore, part (a) of chart 1 shows the total net use of the marginal lending facilities on the last three days of each of the relevant 18 reserve maintenance periods (“net” means that the use of the deposit facility is deducted from the use of the marginal lending facility; “1999-2” is the first reserve maintenance period of 1999, ending on 23 February of this year, “1999-3” ended on 23 March, etc.). It appears that the ECB was biased more to the loose side and that, with the symmetric corridor of standing facility rates set around the tender rate, liquidity management of the ECB should hence, according to equation (1), not have explained the relatively high average positive spread between the market and the tender rates. Part (b) of the chart displays changes of the ECB’s fixed tender and standing facilities rates. Rates declined once in April 1999, but were then hiked 5 times in the period November 1999 to June 2000. As Gaspar; Perez-Quiros and Sicilia [2001] have shown, the rate changes of the ECB have, at least in the first two years of the euro, been very well anticipated by the market, and indeed, this is confirmed by part (c) of Chart 1, in which the 2-week EONIA Swap spread systematically increased before rate hikes. The 2-week EONIA swap rate is generally considered to be the most liquid money market instrument at this maturity. Finally part (d) of Chart 1 provides the total bid amounts in billion of euro submitted in tenders (the allotment volume was on average EUR 69 billion during the first 18 months of the euro). The correlation of increase of bid amounts with the 2-week rate spread is impressive, as the arbitrage explanation of overbidding would suggest. Bid amounts tended to increase especially in periods of a high positive differential between the market and the tender rate. The highest bid amount was reached on 7 June 2000 with EUR 8,491 billion, implying, taking into account the allotted amount of EUR 75 billion, an allotment ratio of 0.88%.
In the quotation given above, the ECB did not further spell out why the very low allotment ratios were regarded as a problem. However, it seems clear that tendering with extreme overbidding has to be regarded as a special type of allocation of funds through queuing, instead of an allocation through a pure price mechanism. Queuing is known to be a less efficient allocation mechanism, compared to the price mechanism. Queuing always occurs when a price is kept at a level which is below the market value of the good sold, at least for the quantity that is offered. The queuing equilibrium is characterized by a marginal condition
under which the marginal cost of queuing exactly fills the gap between the fixed price at which the good is offered and the market price. The queuing costs implied by overbidding are special if compared to classical cases of queuing in two respects: first, the relevant queuing cost function seems to be unstable, i.e. over time, bidders can lower their costs of overbidding through certain investments. Secondly, the queuing costs take to a large degree the form of risk taking, which is less tangible than other costs. How do we have to imagine the nature and dynamics of overbidding costs? At the start, i.e. with moderate overbidding, costs of bidding should be negligible as long as banks own enough low-opportunity cost collateral (e.g. non-Jumbo “Pfandbriefe” for which no repo market exists, or bank loans – see e.g. ECB [2001]) to cover their bid in case they would obtain the full allotment. If they bid for more, they may envisage to use more expensive, so-called “general” collateral for which other uses exist (e.g. Bunds). If they bid even beyond that, they could envisage to get the collateral in the market after the allotment decision is made public and before the settlement of the operation, which however certainly implies further cost. Indeed, the efficiency of the collateral market is not comparable to the efficiency of e.g. the interbank money market. However, in addition, banks may start to find their bidding risky since full allotment would imply that they receive so much cash that they could have problems to place it in the market due to credit limits. Finally, if the overbidding becomes extreme, banks are well aware that they would be unable to get enough collateral in the market in time, and that they are hence taking a speculative stance of which it can even be doubted that it is legally sound. Risk managers in banks typically attach a high cost to such strategies, and it is common practice in banks that a cost is attached to risk taking, such that the incentives of risk takers are adequately influenced.

One may summarize that the marginal cost of bidding should increase with the extent of overbidding. This will be modeled in the present paper in the simplest way possible, namely by assuming that bidding is free up to a certain amount, but then exhibits increasing marginal costs. This will be sufficient to derive relatively simple and intuitive behavioral equilibria.

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4 First of all, the only standardized liquid market for obtaining collateral is the repo market for expensive collateral such as Government bonds. If a trader wants to obtain collateral in this market at very short notice for a maturity which is not too common (e.g. 2 weeks), he can be sure to pay an additional mark-up. Repoing of non-standard collateral is in principle also possible, but terms need to be negotiated individually and the settlement of the collateral leg may be more complex. It should also be noted that repos are usually settled at T+1, like the main refinancing operation, such that there is very little time for the trader to conduct the deals providing the collateral. Furthermore, central banks may require the collateral to be made available early on T+1, before the time the interbank repo operations are settled. The borrowing of collateral without a cash lag is not common. So-called securities lending transactions normally involve equity which is not accepted as collateral by most central banks (see Box 1 in ECB [2002c], which also contains a general description of the euro area repo market). Finally, it should be noted that settlement of collateral transactions is always costly, which explains why unsecured interbank lending still outweighs the repo market for maturities of less than one month, i.e. the higher transactions costs of repos outweigh the benefits in terms of reduction of credit risk for short term transactions.

5 Indeed, in the hot overbidding phase in the spring of 2000, banks bid more than the total amount of eligible collateral for Eurosystem operations.
between the central bank and the market place. A dynamic adaptation of the cost of bidding curve is not modeled explicitly, but is an obvious extension within the proposed model.

2.3. Underbidding

A survey of central bank experience

Underbidding refers to a submission of bids which is insufficient to allow the central bank an allotment decision ensuring banks to fulfil their reserve requirement (or zero current accounts, if no reserve requirement system is adopted) as wished by the central bank until the maturity of the operation. Again, first the experience of a panel of central banks is briefly reviewed. Three types of underbidding may be distinguished: (1) Underbidding relating to excess liquidity supply. This case appears however to be limited to the very special case of J in 2001/2002, in which the central bank aimed at providing large excess liquidity to the market practically at a zero rate. However, since the value of this excess liquidity was also practically zero, there were not necessarily incentives to participate sufficiently to the tenders. (2) Underbidding due to scarcity of eligible collateral: this was the case a few times in SA and SE during the last years, whereby in the case of SE it related to special settlement dates of interbank-operations involving collateral. (3) Underbidding due to some peculiarity of a tender which make it unattractive for banks. This was apparently relevant for the underbidding in the euro area of 17 December 2002 (see below). (4) Underbidding due to rate cut expectations. This was the most frequent case occurring at least one time for each central bank in the panel. A relevant parameter for the actual tendency to underbid in such an environment is the reaction of central banks to underbidding, i.e. specifically whether they tended to give banks the chance to catch up in their fulfillment of reserve requirements, either through offering an additional operation, or by providing correspondingly more liquidity in the regular operations before the end of the reserve maintenance period. The role of this tendency of central banks to “bail out” underbidding banking systems will be modeled in section 4. While SA and J tended to bail out the banks systematically in case they underbid, D, CH, ECB and UK normally did not and hence imposed some costs to banks as a consequence to underbidding. CH also tends to solve the issue by shortening the maturity of operations in such circumstances to one day, which, without reserve requirements and averaging, makes rate cut expectations irrelevant for single open market operations. The most explicit description of underbidding and of the central bank’s reaction other than the ones relating to the ECB experience is the following one by the Deutsche Bundesbank, which fits well with the ECB experience (Bundesbank [1994, 65]), especially in so far as the co-ordination issue between banks is highlighted:

“At the end of January 1994, for instance, expectations of interest rate reductions were so strong that the banks… failed to bid for the liquidity they actually needed at the rate set by the Bundesbank for the fixed rate tender. Some of the credit institutions apparently relied on others bidding at the rate, which they considered to be too high. In the event, the liquidity
available fell considerably short of what was needed for minimum reserve compliance. After this 'bidders' strike' the Bundesbank was not prepared to provide assistance by means of fine-tuning measures; the banks resorted to lombard borrowing to plug the gap at the end of the month”

**A closer look at the Eurosystem’s experience**

The ECB applied from July 2000 on variable rate tenders with minimum bid rate. Such as specified by the ECB, they were quasi-equivalent to fixed rate tenders when interest rate cut expectations prevail since in that case, the minimum bid rate became a binding constraint.

As summarized in the following table, underbidding in MROs has occurred so far seven times in the euro area, each time but once in an environment of rate cut expectations.

<table>
<thead>
<tr>
<th>Date of settlement of the MRO</th>
<th>(a) Rate cut expectation when bidding: three months EURIBOR rate – (minimum bid) tender rate</th>
<th>(b) Bid volume (= actual allotment)</th>
<th>(c) Allotment volume that would have allowed a smooth fulfillment of reserve requirements</th>
<th>(d) Reserve fulfillment deficit that accumulated until next MRO</th>
<th>(e) Net recourse to the marginal lending facility before end of reserve maintenance period</th>
</tr>
</thead>
<tbody>
<tr>
<td>07/04/99</td>
<td>-10 bp</td>
<td>67</td>
<td>84</td>
<td>113</td>
<td>11</td>
</tr>
<tr>
<td>14/02/01</td>
<td>-18 bp</td>
<td>65</td>
<td>88</td>
<td>145</td>
<td>72</td>
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<tr>
<td>11/04/01</td>
<td>-20 bp</td>
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<td>53</td>
<td>232</td>
<td>61</td>
</tr>
<tr>
<td>10/10/01</td>
<td>-27 bp</td>
<td>60</td>
<td>79</td>
<td>118</td>
<td>25</td>
</tr>
<tr>
<td>07/11/01</td>
<td>-28 bp</td>
<td>38</td>
<td>66</td>
<td>168</td>
<td>-3</td>
</tr>
<tr>
<td>04/12/02</td>
<td>-22 bp</td>
<td>112</td>
<td>116</td>
<td>25</td>
<td>-*</td>
</tr>
<tr>
<td>18/12/02</td>
<td>+19 bp</td>
<td>104</td>
<td>117</td>
<td>82**</td>
<td>22</td>
</tr>
</tbody>
</table>

*Since there was a second occurrence of underbidding within the same reserve maintenance period, the end of maintenance period recourse to standing facilities is only attributed to the last operation. **EUR 82 billion would have accumulated until the end of the reserve maintenance period. Actually, a fine tuning operation was conducted to inject in accumulated terms EUR 60 billion, which is the difference between columns (d) and (e) for the MRO of 18 December.

Disregarding for a moment the very special underbidding on 18 December 2002, column (a) reveals that expectations of rate cuts appeared in the case of the Eurosystem to be a necessary condition for underbidding to occur. The shortfall of bids relative to the neutral allotment amount (the difference between column b and c) varied between EUR 28 billion (11 April and 7 November 2001) and EUR 4 billion (4 December 2002). These shortfalls implied the accumulation of a deficit in the fulfillment of reserve requirements of up to EUR 232 billion (column d; assuming that no recourse to standing facility would have taken place).

The costs in terms of net recourse to the marginal lending facility before the end of the reserve maintenance period (column e) varied substantially according to whether or not the ECB decided to increase the allotment amounts in the subsequent tenders to allow banks to catch up with the fulfillment of their required reserves before the end of the reserve maintenance period on the 23rd of each month (i.e. to bail out the market). While the ECB rescued (at least from an ex post perspective) the market in November 2001, the bail out

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6 The spread as defined here has the advantage to go sufficiently beyond the current reserve maintenance period, such that the importance of underbidding on market rates of this maturity is limited.
was especially limited in February and April 2001, when the “bail out coefficient”, which may be defined as $1 - \frac{e}{d}$, was only 50% and 74%, respectively.

The underbid tender of 18 December 2002 was completely different from all other underbid MROs since it took place in an environment in which the minimum bid rate (of 2.75%) appeared very cheap compared to market rates (for 2 week maturity at around 2.90%). The main reason for this underbidding was, according to the financial market press and wire services, the fact that commercial bank staff in some euro area countries wanted to avoid participating to this tender and refinance instead through interbank operations since the tender matured on 31 December 2002, a bank holiday for instance in Germany.\(^7\) Of course, if banks would have known that they could have obtained MRO funds at 2.75%, and not only at the expected marginal tender rate of around 2.90%\(^8\), they would have participated, instead of relying erroneously on other bidders. In so far, the underbidding on 17 December 2002 illustrates that if banks assign a cost to participating in a tender (in this case, the cost of coming to the office on the maturity date, a bank holiday), then it can never be excluded that co-ordination failure can exceptionally lead to underbidding.

To better understand the entire nature of underbidding, the following table displays for 2001 and 2002\(^9\) information on the news content of the announcement of allotment decisions for operations that were underbid and for the operations that followed the underbid tenders within the same reserve maintenance period. With regard to the latter, it should be noted that in the first three cases in 2001, there was only one further operation within the maintenance period, while there were two in the fourth case. The news content was defined as the impact of the announcement of the allotment decision at 11:20 on overnight rate quotations made by money market brokers, whereby the average mid point of quotations between 8:00 and 11:10 is subtracted from the same figure for the period 12:00-18:00. The table suggests that both the announcement of underbid tenders and of the allotments in subsequent tenders had extraordinary news content. The weakest reactions to underbidding were observed in the tender of 14/02/01, probably since money market players did not yet anticipate the tough stance the ECB would take with regard to the bail out, and in the tender of 18/12/02, since

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\(^7\) According to Dow Jones Newswires/Elena Logoutenkova of 17 December 2002: “Euro-zone overnight money-market rates are sharply up midday Tuesday after the allocation announcement for the main refinancing tender revealed underbidding. Around 1100 GMT, overnight money was around 3.05%-3.10%, up from 2.88%-2.89% Monday. The ECB managed to allocate only EUR103.501 billion at the main refinancing operation this week, draining EUR8.29 billion from the market. Money-market traders were surprised by the outcome because normally banks prefer to secure ample liquidity ahead and over the year-end. ‘Underbidding probably has to do with the unfortunate timing of refi flows as this week’s tender will mature on Dec. 31 when many small banks will be closed’, a trader from southern Germany said. ‘The small banks wanted to save costs of running operations on Dec. 31, hoping to get necessary liquidity from the market, but now they have to pay a dear price’, another trader from Germany said. But participants are hoping for a quick additional refinancing operation from the ECB to make up for some EUR15 billion in missing liquidity.”

\(^8\) The weighted average rate of successful bids in this tender corresponded to 2.88%, i.e. 13 basis points above the minimum bid rate of 2.75%. In all other underbid tenders, the weighted average rate corresponded to the minimum bid rate.
the ECB here issued together with the announcement of the allotment a message saying that it would supply additional liquidity, which it had never done before. Also, the signs of the reactions to the publication of the allotment decision in the underbid and following tenders were mixed. Among the underbid tenders, rates always increased after underbidding became known, with the exception of the tender of 3 December 2002, where they fell because strong underbidding was expected but occurred only to a limited extent. \(^{10}\) Signs are even more mixed for the tenders following underbid tenders: apparently, counterparties were in the period under consideration unable to infer a consistent bail out policy of the ECB, such that they were surprised each time by the policy reaction. \(^{11}\) The average absolute value of the news content of underbid tenders (18.5 basis points) is similar to the news content of subsequent allotment decisions (18.7 basis points). In contrast, the average absolute news content of the other Eurosystem main refinancing operations in the period June 2000 to December 2002 has been negligible (1.7 basis points), suggesting that, as long as underbidding did not occur, the allotment decisions of the ECB were well anticipated.

### Table 2: News content of allotment decisions as measured through the difference between average overnight quotation during 12:00-18:00 and during 8:00-11:10, in basis points

<table>
<thead>
<tr>
<th>Date of settlement of the MRO</th>
<th>Underbid tender</th>
<th>Subsequent tenders in the same reserve maintenance period</th>
</tr>
</thead>
<tbody>
<tr>
<td>14/02/01</td>
<td>5.3</td>
<td>37.3</td>
</tr>
<tr>
<td>11/04/01</td>
<td>24.8</td>
<td>-16.3</td>
</tr>
<tr>
<td>10/10/01</td>
<td>19.4</td>
<td>18.1</td>
</tr>
<tr>
<td>07/11/01</td>
<td>10.8</td>
<td>-29.5 / 18.2</td>
</tr>
<tr>
<td>04/12/02</td>
<td>-38.7</td>
<td>0.1 / 11.7</td>
</tr>
<tr>
<td>18/12/02</td>
<td>11.7</td>
<td>-</td>
</tr>
<tr>
<td>Average absolute value for all tenders of respective category</td>
<td>18.5</td>
<td>18.7</td>
</tr>
<tr>
<td>Averages absolute value for all other tenders June 2000-December 2002</td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

Unsurprisingly, statistical tests fully confirm that underbid and subsequent tenders are very different from normal tenders with regard to their average absolute news content. The underbidding episodes also have been discussed by the financial market press. For instance

\(^{9}\) For 1999, the data was not available.

\(^{10}\) Indeed, most market participants interpreted the allotment as revealing that there was no underbidding. According to Dow Jones Newswires/Elena Logoutenkovka of 3 December 2002: “Traders in the euro-zone money market were caught wrong-footed Tuesday when it became clear, following the European Central Bank’s weekly refinancing auction, that there was no underbidding... The shock was all the greater because there has been strong rate-cut speculation ahead of an ECB meeting Thursday. Traders had figured the auction would therefore be underbid by banks opting to wait until after the meeting in order to get their money more cheaply. ‘I am speechless’ said a Frankfurt-based trader... In a knee-jerk reaction to the allocation announcement overnight rates slumped to 3.25%-3.26% and kept falling throughout the day, ending up below 3% by Tuesday afternoon.”

\(^{11}\) No news content was assigned to the tender following the one of 4 December 2002. This is consistent with the fact that banks did not really consider that underbidding had happened on 4 December, such that they did not feel that anything special was to be expected from the subsequent tender. Indeed, the allotment in the following tender was neutral.
the Boersenzeitung of 11 April 2001, p.2, commenting on the second case of underbidding as follows, insists also on the surprise element of underbidding (own translation):

„The publication of the tender result [of 10 April 2001] came as a bad surprise for money market participants. Despite the fact that traders had expected underbidding beforehand, the extent of it triggered growing astonishment, and later on panic demand for funds.... Traders of banks who had submitted bids to the tender suggested that the ECB should remain tough, as in February, and should not rescue the market through a quick tender. This would be the only way to teach speculators an orderly bidding behavior. Traders that had remained absent from the tender expressed their dissatisfaction regarding the current regime in the money market. Since the minimum bid rate would avoid the possibility to submit bids at low rates, similar incidents would happen again and again in the coming weeks."

For the model to be developed in sections 3 and 4, two points need to be retained from this article (which is very similar to articles written after the other underbidding cases) and the above-described evidence regarding the news content of underbid tenders: First, an important feature of the model to be developed will be the problem of bidders to make effective use of the information on liquidity needs and to coordinate their bidding behavior, such that in the case of underbidding and hence of impossibility for the central bank to take the role of co-ordinator of the total liquidity injection, the publication of the allotment has substantial news content. Secondly, the model should allow to analyze and ideally to provide an answer to the question whether underbidding can be stopped through a tight allotment policy in subsequent tenders (i.e. a low bail out co-efficient). Apparently, there are two schools among money market participants with that regard: one that feels that the central bank can cope with the phenomenon of underbidding by choosing the appropriate liquidity management strategy (i.e. mainly not to bail-out the market after underbidding occurred), while the others feel that a minimum bid rate, or, equivalently a fixed rate tender, does not allow any reasonable equilibrium in an environment of rate cut expectations.

Starting from the fact that central banks today define their operational target in terms of the short term interest rate and from the observation that no other money market event, not even 11 September 2001 or the euro cash changeover, created in total so much overnight rate volatility, it appears that underbidding was the most important issue in the monetary policy implementation of the ECB since the launch of the euro in 1999. It is therefore not surprising that the ECB on 23 January 2003 eventually addressed the issue (together with overbidding) through a fundamental reform of its operational framework. According to the press release published on that day, the following two measures were decided by the ECB for implementation in spring 2004 “to improve the operational framework for monetary policy”:

First, the timing of the reserve maintenance period will be changed so that it will always start on the settlement day of the main refinancing operation following the Governing Council meeting at which the monthly assessment of the monetary policy stance is pre-scheduled. Furthermore, the implementation of changes to the standing facility rates will be aligned with
the start of the new reserve maintenance period. Second, the maturity of the MROs will be shortened from two weeks to one week. As the ECB explains in its press release, “the combination of the two measures will help remove expectations of interest rate changes during any particular maintenance period, given that changes in the ECB's key interest rates will only apply, in general, to the forthcoming reserve maintenance period and that liquidity conditions will no longer spill over from one reserve maintenance period to the next. Hence, the measures will contribute to stabilizing the conditions in which bidding in the MROs takes place.”

3. A stylized model of aggregate bidding behavior: a one-day maintenance period without rate change expectations

3.1 The model

The model developed in this and the following section focuses on the bidding equilibrium under rational expectations, i.e. under the assumption of a full understanding by market participants of the central bank's allotment strategy. The model is furthermore built in a way to capture all aspects appearing relevant from the short survey of central bank experience and a more detailed look of the case of the Eurosystem. The modeled reserve maintenance period consists in this section of only one day. Six events are distinguished: (1) The reserve maintenance period begins with the opening of the settlement accounts of banks with the central bank. At the moment of the opening of the accounts, the funds held on the current accounts are still determined by the previous maintenance period's open market operation. However, all outstanding open market operations mature on the same day. (2) The new open market operation takes place. The banks submit their bids, and the central bank takes its allotment decision on the basis of its forecast of liquidity needs and possibly its liquidity target. The allotment amount may be restricted by the available bids. The allotment decision is made public. At the same time, the central bank's private information on liquidity needs is revealed. The operation is settled. (3) The interbank market session takes place and a market clearing overnight rate $i$ is determined. (4) The realization of the autonomous liquidity factor shock takes place (autonomous liquidity factors are all those factors affecting the banking system's reserve holdings which are not monetary policy operations, such as the circulation of banknotes – see e.g. Bindseil and Seitz [2001]). (5) Finally, the banks take recourse to standing facilities to cover any liquidity imbalance. This recourse is purely mechanic, i.e. it fills the gap between the counterparties reserves and reserve requirements (set to be zero). The model assumes a perfect interbank market and homogenous banks, such that either all banks will have recourse to the marginal lending facility, or all will have recourse to the deposit facility, but there is no simultaneous recourse to both facilities by
different banks. (6) The reserve maintenance period ends. The sequence of events is summarized in the following chart.

Chart 2: the sequence of events in the one day maintenance period case:

It appears that this simple setting allows capturing all major aspects of bidding behavior, also within a reserve maintenance period and averaging, as long as rate change expectations do not come into play. First, the proposed one-day period obviously captures well the period from the last main refinancing operation of the reserve maintenance period until its end. Of course, for instance in the euro area setting, several days and several market sessions are normally included in this period, but the value added for understanding the bidding behavior in the last MRO of modeling each of these market sessions separately is limited. Secondly, one may argue that due to the martingale hypothesis as represented in equation (1), all tenders preceding the last one in the maintenance period are of little specific interest. In contrast, to model the effect of rate change expectations, at least a two-days reserve maintenance period is required to allow modeling one tender before and one tender after the decision of the central bank regarding interest rates, to obtain a meaningful model.

Consider now the key elements of the assumed framework one by one. i) **Reserve requirements** and the demand for excess reserves are zero, such that the banks will target zero balances on their account on the end of the day. ii) The reserve drain implied by autonomous liquidity factors is assumed, for the sake of simplicity, to be white noise with a structural constant, i.e. $a = A + \varepsilon + \eta$, with $A \in \mathbb{R}_{>0}$ a constant and $\varepsilon, \eta$ being normal distributed random variables with expected value zero and variances $\sigma_{\varepsilon}^2, \sigma_{\eta}^2$. It is assumed that the structural liquidity deficit of the banking system, $A$, is large, such that the probability that $a < 0$ is negligible. An autonomous factor shock e.g. $\varepsilon > 0$ means that total autonomous factors increase, i.e. ceteris paribus that the reserves of banks with the central bank decline. The central bank is assumed to have, as the market, no prior information on $\varepsilon$. However, the central bank perfectly anticipates $\varepsilon$, which is also revealed to the market at the moment of the publication of the allotment decision. Indeed, under the reasonable assumption that the allotment rule of the central bank depends on the central bank’s knowledge on liquidity needs, and that the central bank’s allotment function does not contain other terms, which are unobservable to the market, the market can fully extract $\varepsilon$ when
observing the allotment amount. Note that $\varepsilon$ is designed in a way that it could also be interpreted as capturing shocks stemming from the imperfect co-ordination of bidding. iii) Apart from the open market operation, there is one other type of monetary policy instrument available, namely **standing facilities** to which banks can have unlimited access, however at a penalty rate. For the sake of simplicity, the rate of the deposit facility is set to zero and the rate of the marginal lending facility is set to one, such that overnight rates always fluctuate in the unity space. iv) The **tender procedures** considered are defined here simply by the range of rates at which banks can submit bids. The pure variable rate tender does not restrict bid rates at all, i.e. bid rates $k \in [\underline{k}, \overline{k}]$ with $\underline{k} = 0, \overline{k} = 1$. Under the fixed rate tender, $\underline{k} = \overline{k}$, and the fixed tender rate is then written $\overline{k}$. Without loss of generality, it will be assumed here that $\overline{k} = 0.5$. Finally, a central bank may apply variable rate tenders with minimum or maximum bid rates. For instance, the ECB has been applying since July 2000 a restriction which, translated into the model, would read $\underline{k} = 0.5, \overline{k} = 1$. Note that normally, bid rates in central bank tenders need to correspond to full basis points, such that in fact the set of possible bid rates is limited, $K = \{k, k + 0.01, k + 0.02, \ldots, \overline{k}\}$. v) The **bidders** are assumed to be homogenous in all respects. Specifically, it is assumed that information available to the bidders is homogenous, as suggested by the findings of Nyborg, Bindseil and Streiblauerv [2002]. It is further taken for granted, in line with the case of the euro area, that there is a high number of bidders, such that bidders operate under full competition. In section 2, positive bidding costs was motivated. Denote $B$ the total amount of bids and by $\Omega : \{C(B) : [0, \infty[ \rightarrow [0, \infty[\}$ the set of possible cost of bidding functions that the banking system faces when intending to submit a total bid of $B$. It is assumed that for all $C \in \Omega$, $C(B) = 0$ for $B \leq \tilde{B} \in \mathbb{R}_{\geq 0}$ and $C(B) > 0$ for $B > \tilde{B}$ with $\partial C / \partial B > 0; \partial^2 C / \partial^2 B > 0$.

Hence, the marginal cost of bidding are zero until a bid volume of $\tilde{B}$ is reached, afterwards, they are positive and increasing. Note that the latter property should follow from the decline of allotment ratios if the bidding costs are for instance related to the potential costs of obtaining collateral. We will refer to $\tilde{B}$ as the free bidding potential. In the case of the Eurosystem which does not require counterparties to cover bids with collateral and which accepts a wide range of eligible collateral (needed at the moment of settlement), $\tilde{B}$ should be clearly above refinancing needs. The set of bid functions submitted to a given tender is defined as $Q : \{\beta : \{k, k + 0.01, \ldots, \overline{k}\} \rightarrow [0, \infty]\}$. Furthermore, denote $\overline{B} = \sum_{k = \underline{k}}^{\overline{k}} \beta(k)$ the total bid amount submitted to a tender. vi) An **allotment function** is understood here as a mapping from the set of bid functions and the set of all other possibly relevant variables,
denoted \( Y \), into an allotment amount, i.e. \( M = \{ m : Q \times Y \to \mathbb{R}^+ \} \). It is assumed here that the set of further variables \( Y \) is limited to information on liquidity conditions, namely that when deciding on the allotment amount \( m \) in its open market operation, the central bank takes into account, besides the bids, its autonomous factor forecasts and the liquidity surplus or deficit it would like to see at the end of the maintenance period. More specifically, two different allotment rules are considered, all of which in fact consider only the aggregate bid amount \( B \): First, the central bank may apply the “100% rule”, according to which it always provides the entire amount for which it has received bids, hence \( m = B \). Second, it may aim at a certain allotment volume, which is in principle independent of the bid amount (“discretionary fixed rate tender”) and which aims at an end of maintenance period liquidity target denoted in the following by \( \gamma \in \mathbb{R} \). The intended open market operation volume is hence \( A + \varepsilon + \gamma \). The actual allotment volume corresponds to the intended one if the total volume of bids, \( B \), allows this, i.e. \( m = \min(A + \varepsilon + \gamma, B) \). For the case this rule is combined with the variable rate tender, it is assumed here that there will always be sufficient bids, at least at low rates, such as to avoid the underbidding problem. In fact, this is the major property associated to variable rate tenders in the present paper.

In variable rate tenders, marginal tender rates are assumed to follow market rates, whereby the central bank is assumed to care only about quantities in its allotment decisions, in the same way as it does in its fixed rate tender. vii) The welfare analysis will focus exclusively on the following two types of welfare costs. Firstly, deviation of the overnight interest rate from their expected (and targeted) value is regarded as costly: \( L_1 = E((i - E(i))^2) \). Note that the interest rate related loss function focuses only on the volatility of overnight rates, and not on the bias of the overnight rate relative to the tender rate \( r \). If the central bank would dislike any bias, it could simply correct for the bias by shifting the corridor set by standing facilities and the tender rate correspondingly. Secondly, the total bidding costs are of course relevant, i.e. \( L_2 = C(B) \).

### 3.2 The fixed rate tender with 100% allotment

It seems to reflect basic economic intuition to allow the allotment amount being demand-driven if the central bank fixes the tender rate. Why, however, such a 100% allotment policy

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12 This rule has been rarely used, and if so, it was combined with fixed rate tenders (for instance by the Finnish central bank)

13 Note that any intended allotment volume can be translated in an intended market interest rate. In so far, the allotment function could also be expressed in terms of an interest rate steering strategy.

14 There is ample evidence from central banks that underbidding is practically non-existent in variable rate tenders. For instance, the Bundesbank conducted weekly open market operations since 1988 more than 300 times as variable rate tenders. No case of underbidding ever occurred. However, the underbidding in the euro area of 17 December 2002 would probably also have occurred under a pure variable rate tender, since the minimum bid rate was not binding on that day.
has been applied very rarely by central banks? The following proposition, which also provides a benchmark for the following cases, distinguishes between two cases regarding the size of the free bidding potential.

**Proposition 1:** Under the 100% fixed rate tender and a large free bidding potential, banks will exactly bid their expected liquidity needs, and the market interest rate expected by the market at the moment of the allotment decision will correspond to the tender rate. If in contrast bidding is costly even for bids below expected liquidity needs, banks will tend to bid less than their liquidity needs, and the expected market interest rate will be above the mid point of the corridor. In both cases, overnight rates will be volatile due to the component \( \varepsilon \) of liquidity needs, which cannot be considered in the banks' bid amount.

The proof is in the annex. Note here however the equilibrium condition on which the proof is based. Bidding is in equilibrium if the marginal cost of the funds obtained through bidding is equal to the expected marginal value of the funds. As long as bid costs are zero, the cost of bidding correspond to the tender rate \( \tilde{k} = 0.5 \). If bids go beyond the free bidding potential, increasing marginal bidding costs have to be added to the tender rate. Assuming the allotment rule \( m = B \), the expected value of funds decreases with an increasing bid amount since an increasing bid means an increasing allotment means a higher likelihood of a liquidity surplus at the end of the day. Indeed, starting from equation (1), the market rate for funds will correspond, under the assumptions made regarding standing facility interest rates and the distribution of autonomous factor shocks to:

\[
i = 1 - \Phi\left(\frac{B - A - \varepsilon}{\sigma_\eta}\right)
\]  

(2)

The function \( \Phi(\cdot) \) refers to the cumulative standard normal distribution. Due to the liquidity shock \( \varepsilon \), at the moment of bidding, the market value of funds is uncertain, and expectations are relevant, such that the following equilibrium condition is obtained, whereby \( f_\varepsilon(x) \) is the density function of \( \varepsilon \):

\[
\tilde{k} + \frac{\partial C}{\partial B} = E\left[\left(1 - \Phi\left(\frac{B - A - \varepsilon}{\sigma_\eta}\right)\right)\right] = \int_{-\infty}^{\infty} \left(1 - \Phi\left(\frac{B - A - x}{\sigma_\eta}\right)\right) f_\varepsilon(x)dx
\]

(3)

The case of the 100% allotment rule \( (m=\overline{B}) \) is the simplest case of equilibrium bidding. The proposition shows how in the simplest case costs of bidding are pervasive in the sense of affecting not only bid amounts but also market interest rates. The answer the proposition provides to the question why this allotment rule, although being economically intuitive, has rarely been used, is simple: it makes the central bank lose its superior knowledge and coordinating role which, under discretionary fixed rate tenders, allows it to avoid interest rate
volatility relating to the $\varepsilon$-shocks. This argument was relevant enough to make most central banks reject this approach, although first economic intuition is on its side.

3.3 Discretionary fixed rate tender
This is the type of allotment rule currently applied by all central banks using the fixed rate tender for open market operations. As the following propositions will show, it allows the central bank to make full use of its coordinating role and/or its superior knowledge of liquidity needs. The $\varepsilon$-shocks, together with the possibility of the central bank to allot less than the banks have bid for, will introduce an asymmetry. If a liquidity-absorbing $\varepsilon$-shock is experienced, then the central bank will tend to compensate it through a correspondingly lower allotment. If there is in contrast a positive shock such that the effective bid is below the liquidity needs, then the central bank cannot help. This asymmetry will tend to make banks bid more than they would under the 100\% fixed rate tender. The crucial parameter of the central bank under this procedure will be its liquidity target $\gamma$ in its allotment function $m = \min(A + \varepsilon + \gamma, B)$. We again distinguish between different cases regarding the free bidding potential. In the following, the term “underbidding” will be used precisely to describe a situation in which $B < A + \varepsilon + \gamma$, i.e. the bid is below the amount that the central bank would like to allot according to its allotment function. Again, two cases will have to be distinguished regarding the free bidding potential. First, assume a very large free bidding potential, “$\tilde{B} \gg A$”, defined such that $P(\tilde{B} - A < \varepsilon) = 0$. Leaving aside intermediate cases, it will alternatively be assumed that $\tilde{B} = 0$, i.e. that bidding is costly from the first unit of bids on. Furthermore, depending on the choice of the liquidity target $\gamma$, three cases can be distinguished. Those are reviewed in the following one by one.

The central bank aims at neutral liquidity conditions
The liquidity policy of the ECB aims at neutral liquidity conditions (see ECB [2002b]), which is rather intuitive for a central bank with a corridor set by standing facilities being symmetric around the tender rate, considering equation (1). This is also a feature of the model, where neutrality means $\gamma = 0$ i.e. $m = \min(A + \varepsilon, B)$.

Proposition 2: In the discretionary fixed rate tender with neutral liquidity policy and large free bidding potential, banks will submit exactly their free bidding potential, and the market interest rate will exactly correspond to the tender rate without any volatility. If in contrast there is no free bidding potential, banks will either bid more, at, or less than their actual liquidity needs depending on the variance of liquidity shocks and the cost of bidding function. Market interest rates will in any case be volatile.
The proof is in the annex. Part one of the proposition tells us that in case of a large free bidding potential, the banks will tend to “overbid” naturally in the domain \( \bar{B} \leq B \) in order to make the likelihood of underbidding close to zero. This allows eliminating the volatility of market overnight rates, while still avoiding any bidding costs. Hence, a perfect situation is obtained. It would be unreasonable if the central bank would in this case have an aversion against the relatively low average allotment ratio \( \bar{B} / A \) resulting in this case since this low allotment ratio does not reflect any inefficiencies while at the same time ensuring that the central bank can, through its discretionary choice of the allotment ratio, avoid interest rate volatility. In contrast, in the case of no free bidding potential, a perfect solution cannot be reached since there will not only, by definition, be bidding cost, but also interest rate volatility. Again, the pervasive role of bidding costs is confirmed. Probably, a large amount of available collateral, together with requesting that collateral only needs to cover allotted amounts at the time of settling, are sufficient pre-conditions for obtaining a large free bidding potential.

**The central bank tends to provide surplus liquidity**

This is basically the case of the Bank of Japan in 2001 and 2002, but also for the UK where overnight rates often tend to be at or slightly below tender rates. Generally, it means that the central bank tends to push market rates below fixed tender rates, if a sufficient amount of bids is submitted. The Bank of Japan set both the tender rate to zero and injected huge excess amounts of funds such that market rates were also zero. Then, participating to the tender constitutes a loss if one assumes that participation to a tender creates a minimal administrative cost for banks. In the model, the allotment function now is \( m = \min(A + \varepsilon + \gamma, B) \) with \( \gamma > 0 \), whereby only the case with a large free bidding potential \( \bar{B} \gg A \) is considered. If the central bank is able to allot the excess funds, market rates would be pushed below the tender rate, such as to make successful bidders loose money. This loss will have to be compensated by occurrence, at least from time to time, of underbidding, in the case of which the value of funds will be higher than the tender rate. The higher \( \gamma \), the lower the intended bid \( B \) will hence be, and the bid will finally tend to the 100% allotment solution with \( B = A \), when \( \gamma \) becomes very large.

**Proposition 3.** In the case of discretionary tenders and a large free bidding potential, the central bank can achieve any average allotment ratio it wishes between \( \bar{B} / A \) and 1 (i.e. it can achieve any bid volume between the expected liquidity needs and the free bidding potential) by adequately choosing a target liquidity surplus \( \gamma \geq 0 \). For \( \gamma > 0 \), it will however create interest rate volatility.

The proof is in the annex. The probability of underbidding, \( P(B < A + \varepsilon + \gamma) \), will increase monotonously with the liquidity target \( \gamma \), implying also an increase of volatility of market
rates (since market rates are always $i = 1 - \Phi(\gamma / \sigma_n)$ in case of no underbidding). One may conclude that a loose liquidity policy is normally inferior to neutrality in discretionary fixed rate tenders, since it mainly creates interest rate volatility, similarly to the 100% fixed rate tender. Only in the very special case of the Bank of Japan, such volatility was not created since market interest rates were in any case 0%.

If the central bank is systematically tight

Both the Banque de France and the Deutsche Bundesbank tended to steer short term market rates slightly above their fixed tender rate. In the model used here, this means

$$m = \min(A + \varepsilon + \gamma, B)$$

with $\gamma < 0$. The gap between market and tender rates can be reconciled with the idea of a bidding equilibrium only with the help of overbidding and its implied costs. The following proposition however highlights that tightness may be useful in case of a limited free bidding potential.

**Proposition 4.** In the case of a large free bidding potential, tightness is systematically inferior to neutrality in the sense that both lead to a zero volatility of market rates while tightness leads to bidding costs. In case of a limited free bidding potential, (e.g. $B = 0$), tightness can be superior to neutrality if the central bank attaches a high weight to the stability of interest rates.

The proof is in the annex. If bidding costs are relevant and the central bank is neutral, banks will not submit excess bids, and hence the likelihood of underbidding is positive. Obviously, by tightening its allotment policy, the central bank drives up market rates and hence the spread to the tender rate, and thereby the bid amounts submitted despite bidding costs. Thereby, the likelihood of underbidding, and also interest rate volatility decreases. Hence, a central bank which cares more about the stability of short-term rates and less about bidding costs may prefer a tight policy. Probably, both D and F belonged to that category of central banks. Of course, both contributed themselves to make bidding costly by requiring bids to be fully covered by collateral.

3.4 The variable rate tender

As mentioned beforehand, it is assumed here, also on the basis of clear empirical evidence, that variable rate tenders ensure that there will always be sufficient bids since bidders can react to low expected market rates by submitting bids at lower rates, instead of reducing their bid volume. Of course, it is not claimed here that variable rate tenders do not diverge from fixed rate tenders in many other aspects. For instance, central banks have argued that fixed rate tenders (or variable rate tenders with minimum bid rate) are superior in terms of signalling the stance of monetary policy. It is assumed that the central bank continues to follow a quantity oriented allotment policy, $m = A + \varepsilon + \gamma$, i.e. its allotment function does not
depend on the rates at which bids are submitted. If the central bank tends to be tight, bid rates will move upward with the implied higher value of funds, and the reverse in case the central bank is loose. Generally, marginal tender rates will move with expected market rates in equilibrium. Hence, the central bank can focus on achieving stable market rates around the mid point of the corridor by choosing the adequate (i.e. neutral) liquidity supply. In so far, the variable rate tender will allow to bring both types of assumed loss functions to zero.

4. Introducing rate change expectations: the two days reserve maintenance period case

4.1 Extension of the model to the 2 days case

The previous section demonstrated that in case of stable central bank interest rates, central banks of which the counterparties have a large free bidding potential should be able to achieve, for instance with the discretionary fixed rate tender procedure and a neutral liquidity policy, major operational goals, namely stable overnight rates and an efficient tender procedure without excessive bidding costs. Unfortunately, these results will no longer necessarily hold under rate change expectations. To be able to model those, the one-day maintenance period needs to be replaced by one with two days. Regarding the bidding costs, it will now be generally assumed that $\tilde{B} \gg 2A$, i.e. that the free bidding potential is so large that if banks intend to bid the free bidding potential in the first tender, then the likelihood that the bids are not sufficient to cover the liquidity needs on both days of the reserve maintenance period are negligible. This assumption will allow us to encounter bidding costs only in the case of acute overbidding. The analysis can be restricted to the case of a reserve maintenance period with only two market sessions, two allotment decisions, two autonomous factor shocks of $\varepsilon$ type, whereby $\varepsilon_1, \varepsilon_2$ are not correlated and both have variance $\sigma^2_{\varepsilon}$, and one autonomous liquidity factor shock $\eta$ being revealed at the very end of the reserve maintenance period. This case provides for a sufficient micro-cosmos of the major phenomena of relevance.

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Chart 3: sequence of events in the two days maintenance period case

<table>
<thead>
<tr>
<th>MRO1 Market Session 1</th>
<th>Central bank Council meeting</th>
<th>End of day 1</th>
<th>MRO2 Market Session 2</th>
<th>Aut factor shock</th>
<th>Standing facilities</th>
<th>End of day 2 and of period</th>
</tr>
</thead>
</table>
The following **sequence of 9 events** is assumed. (1) The reserve maintenance period begins similarly to the one-day model. However, interest rate change expectations are now present in the market. Money market players have homogeneous expectations. The size of a possible interest rate change, $\lambda$, is given, whereby it can be either positive or negative. The probabilities of a rate change are mutually exclusive in the sense that there are either rate cut, or rate hike expectations, but never both at once. For the rate cut and rate hike expectations, we introduce a series of dummy variables for the sake of a simple notation, namely $d^{hc}, d^{ce}, d^c, d^h$ which take the value 1 in case that hike expectations are present in the morning of the first day; that rate cut expectations are present in the morning of the first day, that a cut has effectively taken place at the end of the first day, that a hike has effectively taken place at the end of the first day, respectively. Note that $d^c = 1 \Rightarrow d^{ce} = 1$, $d^h = 1 \Rightarrow d^{hc} = 1$, $d^{ce} = 1 \Rightarrow d^{hc} = 0$, etc. Hence, for instance $E(\lambda d^h)$ is the expected increase of central bank interest rates in an environment of rate hike expectations. (2) The first open market operation takes place. (3) The first market session takes place and a market clearing overnight rate $i_1$ is determined. (4) The Central Bank Council takes a decision with regard to the tender rates and rates of standing facilities at the end of the reserve maintenance period. In the case of a fixed rate tender, the fixed tender rate is always shifted in parallel to the standing facility rates. (5) The first day of the reserve maintenance period ends, which means that reserve holdings at that moment in time are relevant for the fulfillment of reserve requirements over the reserve maintenance period. (6) The second open market operation takes place. (7) The second market session takes place and a market clearing overnight rate $i_2$ is determined. (8) The realization of the autonomous liquidity factor shock takes place. (9) Finally, the banking system takes recourse to standing facilities to cover the liquidity imbalance. The reserve maintenance period ends with the end of the second day.

The model will be solved as follows: For a given tender procedure and allotment strategy of the central bank, the banking sector solves collectively its optimization problem under perfect competition to arrive at some bid volume under which the expected cost of obtaining funds equals the expected value of funds in the inter-bank market. Then, the analysis turns to the choice of the central bank regarding the tender procedure and allotment rule, and works out the optimal parameter values of the allotment rules, as far as relevant, assuming different possible loss functions of the central bank. Finally, the outcomes of the different procedures,  

15 Note that since we have set the rate of the deposit facility to zero, any reduction of this rate implies that it becomes negative. Even though somewhat counterintuitive, this does not pose any problem in the model. The model could of course also be restated with a corridor set by standing facilities at a higher level.
for their respective optimal specification, are compared. This is done separately for rate hike and loss expectations, since the central bank can of course switch from one procedure to the other when expectations change. The following two alternative procedures and tender rules will be analyzed.

(1) The discretionary fixed rate tender may in principle contain as many additional functional terms as the central bank can imagine. For the sake of shortness, the analysis here is limited to two intuitive functional forms. For the case of rate hike expectations, a contingent positive liquidity target is foreseen, reflecting the idea that one might counteract overbidding in these circumstances through excess liquidity:

\[ m_1 = \min(A + \varepsilon_1 + \pi_{he} d_{he}, B_1), \quad m_2 = \min(A + \varepsilon_2, B_2) \]

The contingent liquidity target could in principle also appear in the second allotment, but the proposed assignment is probably more intuitive. For the case of rate cut expectations, the allotment function

\[ m_1 = \min(A + \varepsilon_1, B_1), \quad m_2 = \min(A + \varepsilon_2 + \alpha(A + \varepsilon_1 - m_1), B_2) \]

is proposed. It foresees that, in principle, the central bank always allots the expected same day liquidity need. However, in case of underbidding on day one, it is willing to partially bail-out the banks on the second day, i.e. to provide them more liquidity to allow them to catch up at least partially with their fulfillment of reserve requirements. The allotment decisions on the second day may compensate hence a share \(0 < \alpha < 1\) of the liquidity shortage that occurred on the previous day in relation to underbidding. A central bank with \(\alpha = 0\) has “no mercy” with the banking system that has underbid, while a central bank with \(\alpha = 1\) goes for “full bail-out”.

(2) The variable rate tender, which we assume to be purely quantity oriented, may also contain some contingent terms in the allotment rule, as for instance, including several possible terms at once,

\[ m_1 = A + \varepsilon_1 + \pi_{he} d_{he}, \quad m_2 = A + \varepsilon_2 + \pi_{ce} d_{ce} + \pi_{ed} d_{ed} + \pi_{h} d_{h} \]

Specifically, contingent liquidity components relating to rate hike/cut expectations or actual rate hikes/cuts are included, whereby the analysis is limited to either one or the other of those (considering both at once represents a simple extension). The coefficients \(\pi_{he} > 0, \pi_{ce} < 0, \pi_{h} > 0, \pi_{e} < 0\) are constants that can be chosen by the central bank as part of their choice of an allotment policy.

Note that the 100% allotment rule \((m_1 = B_1; m_2 = B_2)\), will not be analyzed in the case of rate change expectations since the assumptions underlying our model are not exactly suitable for this case. Indeed, applying the model leads to the result that under rate hike expectations, banks tend to cover their need for reserves for the entire reserve maintenance period in the first tender, and to hold no reserves on the second day, while the opposite result is obtained under rate cut expectations. However, in that case, the demand for working
balances would become the binding constraint on the days on which banks would tend to hold very little reserves, but this demand is not modeled here. Also, aggregate liquidity shocks which would push the system on aggregate into an immediate use of the marginal lending facility would matter in practice. Both issues could be ignored in all other cases at least for the euro area with the high very high reserve requirements since in none of these cases a very irregular path of reserve requirements is obtained (see also the discussion of the martingale property in section 2).

The tender procedures and allotment strategies will be analyzed in relation to the following four possible components of loss functions:

- $L_0 = E(i_i - \bar{k})^2 + E(i_2 - \bar{k} + \lambda)^2$ could also be entitled as the “operational target loss function”, assuming that decisions of rate changes are at the same time decisions to change the operational target accordingly. However, it is easy to show that no allotment rule or tender procedure exists that can bring this loss function down to zero. Indeed, in an environment where e.g. $E(\lambda d^b) \neq 0$, achieving the aim $i_i = \bar{k}$ and $i_2 = \bar{k} + \lambda d^b$ would imply $E(i_2) \neq i_1$ which contradicts the martingale property. A similar argument holds under rate cut expectations. One may shift the level of rates on both days up or down, but one can never engineer an expected change of market rates within the maintenance period. Hence, the loss function $L_0$ will not be pursued further.

- $L_1 = E(i_i - i_1)^2$ reflects the idea that within the reserve maintenance period, interest rates should be stable.

- $L_2 = E(m_1 - A)^2 + E(m_2 - A)^2$ focuses on the smoothness of total bank reserves over time. Reserves play an important role as buffers against all kinds of aggregate and individual liquidity shocks. Hence, an allotment rule that leads to very low reserves on some days may be viewed as detrimental to the stability and smoothness of the interbank market and the payment system. Note that an externality may be at stake here: while it could be beneficial for single banks to profit from arbitrage opportunities by front- or back-loading strongly their reserve fulfillment, this strategy may, if practiced by all, be welfare-reducing for the banking system as a whole.

- $L_3 = E(C(B_1)) + E(C(B_2))$ finally focuses on bidding costs, which will only be relevant in the case of overbidding, since it is assumed that $\widetilde{B} >> 2A$.

It will become clear in the course of this section that $\varepsilon$-shocks will not constitute a problem in the discretionary fixed rate tender under rate hike expectations. However, $\varepsilon$-shocks leading to underbidding will constitute a general fixed rate tender problem in the case of rate cut expectations. Hence, one of the major questions to be investigated will be how the implied
underbidding problems can be controlled. Now, the different allotment rules and their implications on the possible loss functions of the central bank will be considered one by one, starting from the simplest case.

4.2 Discretionary fixed rate tenders under rate hike expectations
This is exactly the case of the ECB in the second half of 1999 and first half of 2000. According to ECB [2002, 48], “the ECB often engineered relatively loose liquidity conditions during the intensive phase of overbidding in the second half of 1999 and the first half of 2000”, however without being successful in stopping overbidding and avoiding the perceived need to switch to the variable rate tender. The following proposition assesses this economically intuitive but apparently unsuccessful policy within the proposed model.

**Proposition 5:** In the absence of a contingent liquidity target for rate hike expectations, the central bank may allot the liquidity needs on each day of the maintenance period, i.e. there is no risk of underbidding. Market interest rates will be free of noise on each day. A gap corresponding to the expected rate hike between the tender and the market rate on the first day, will have to be counterbalanced by bidding costs. Any positive contingent liquidity target provided on day 1 will imply a positive likelihood of underbidding on the second day and will not reduce overbidding on the first day unless it exceeds A, i.e. unless on day 1, the allotment amount is at least \( m_1 = 2A + \varepsilon_1 \).

The proof is in the annex. The proposition explains why limited excess liquidity does not help to reduce underbidding in the tenders before the relevant meeting of the central bank’s interest rate setting council. The intuition is that it is just impossible for the central bank to maintain the excess liquidity until the end of the maintenance period, since the banking system will not be willing to bid in the last tender such as to obtain an interbank market rate below the second day’s tender rate. This implied underbidding in the second tender ensures that expected market rates on day 1 are still at the level of the expected end of maintenance period tender rate. The obtained outcome is hence worse relative to a neutral policy, since overbidding is unchanged while interest rate volatility on day 2 is added. Positive bidding costs can be avoided only if the central bank is so aggressive in providing excess liquidity in tender one that actually, the outcome of the 100% allotment rule is obtained. However, central banks also appear to dislike the associated non-smooth reserve fulfillment path. The ECB was therefore right to discontinue such a policy after a while.

4.3 Discretionary fixed rate tender under rate cut expectations
Now, banks will be tempted to underbid in the first tender, and the central bank has to choose on the second day the “bail out” coefficient \( \alpha \). Again, as regard the bidding in the first tender, this is exactly the case of the ECB in 2001 since, as argued before, a variable rate tender with minimum bid rate in the middle of the interest rate corridor is under rate cut expectations like a fixed rate tender under rate cut expectations.
Proposition 6: Under rate cut expectations and discretionary fixed rate tenders, banks will sometimes underbid even in case of a zero bail out coefficient. On the second day, they will either bid their free bidding potential (in case of no underbidding on the first day or in case of a full bail out) or more (in case of a less than full bail out co-efficient and underbidding in the first tender). Furthermore, there exists a critical value of the bail-out coefficient such that a lower bail out implies expected market rates on the first day being at the level of tender rates on the first tender and positive bidding in the first tender, while a higher bail out pushes expected market rates in the first tender below tender rates in this tender and hence leads to zero bidding. Market rates will be noisy on both days of the maintenance period whenever the bail out co-efficient is not one.

The proof is in the annex. The proposition has a series of important policy implications. First, there is no way to avoid systematically underbidding even if the central bank does not bail out the market at all. This was in principle also the experience of the ECB, which experienced underbidding again and again in 2001 despite a relatively tight policy towards it. Therefore, if one aims at eliminating underbidding and its related interest rate volatility, other changes need to be made. Second, as long as the bail-out is not complete, whenever there is underbidding in the first tender, then there will be overbidding in the second tender with associated bidding costs. This was of course not the case for the ECB with its variable rate tender with minimum bid rate: the post underbidding tenders were indeed genuine variable rate tenders with banks competing for funds through higher bid rates, avoiding thereby the costs of overbidding. Finally, there exists a partition of the possible values of the bail out parameter, which decides on the properties of the bidding behavior and interest rates. Note that the effect of lowering the bail-out-coefficient on the volatility of market rates is ambiguous: on one side, it reduces the likelihood of underbidding, on the other, it increases its effect on rates if it happens. Hence, the choice of this parameter should not be viewed as a simple mean to reduce interest rate volatility associated to underbidding. The same holds for the overbidding and implied bidding costs on the second day in case of a pure fixed rate tender. In contrast, lowering the bail out coefficient unambiguously smoothens the reserve fulfillment path.

4.4 The variable rate tender

Consider the case of rate hike expectations. The case of rate cut expectations will not be considered separately, since it is entirely symmetrical. Instead, two case of contingent liquidity components of the allotment function will be considered separately, namely one contingent on expectations, and one contingent on actual changes of central bank rates. In contrast to the discretionary fixed rate tender, it is now assumed, as mentioned before, that there will always be enough bids, such that the central bank can in both tenders allot any amount it wishes to. This is in fact the pre-condition for adding contingent liquidity targets, which were apparently bound to fail under fixed rate tenders.

i) A liquidity target contingent on hike expectations
For instance in the second half of 2000, rate hike expectations in the euro area led to a considerable increase of market rates and hence bid rates above the minimum bid rate. Therefore, the variable rate tender with minimum bid rate in fact became a genuine variable rate tender. Some voices were raised at that time that the departure of bidding rates and hence of marginal allotment rates from the minimum bid rate was questioning the signaling of the monetary policy stance of the ECB. It is hence of interest to see whether under a pure variable rate tender, the central bank can counteract this tendency of market and bidding rates to anticipate a rate hike through an adequate allotment policy. The allotment rule here is \( m_1 = A + \varepsilon_1 + \pi^{hr} d^{hr} \); \( m_2 = A + \varepsilon_2 \). The following proposition summarizes the outcome in this case.

**Proposition 7**: Under variable rate tenders and rate hike expectations, a central bank following a quantity oriented allotment policy with a term contingent on rate hike expectations may fully or partially prevent the interest rate from anticipating the rate hike. However, it cannot prevent that the rate decision will have an effect on market rates (i.e. that the potential rate decision implies intra-maintenance period volatility of interest rates). Apart from that, there will be no noise in market interest rates.

The proof is in the annex. Before assessing the contingent target, note that independently from it, the outcome is rather satisfactory: there is no noise in market rates, no bidding cost, and the reserve fulfillment path is smooth (at least if, what is equivalent to the rule proposed above, the contingent allotment term is split equally between the two allotment decisions).

Regarding the contingent term, it achieves the aim to avoid the anticipation of rate hikes in market overnight rates, but it also has the implication that market and tender rates fall below the mid point of the corridor on the second day. This drop in market rates relative to official rates appears odd in an environment of rate hike expectations. Indeed, for instance the ECB appeared to have concluded that large contingent liquidity elements of the described sort are not useful (see ECB [2002b]).

**ii) A liquidity target contingent on the actual occurrence of an interest rate hike.**

Can the previous outcome be improved if the liquidity target is contingent on the rate hike itself, and not on expectations? The allotment rule takes here the form: \( m_1 = A + \varepsilon_1 \); \( m_2 = A + \varepsilon_2 + \pi^h d^h \). The following proposition suggests that under this allotment rule, the central bank can fully stabilize market interest rates within the reserve maintenance period.

**Proposition 8**: Under variable rate tenders and rate hike expectations, a central bank following a quantity oriented allotment policy with a term contingent on the occurrence of a rate hike may fully prevent the interest rate from anticipating the rate hike. In addition, it can achieve a perfect stability of interest rates within the reserve maintenance period.
The central bank may hence achieve zero losses with regard to all three loss functions considered. Still, one may argue that also this rule has a draw-back, since if the rate hike takes place, market rates are not allowed to follow it until the end of the maintenance period. Then, however, it could have been easier to simply implement the rate hike itself only in the following reserve maintenance period. This rule could indeed give the impression that the rate setting council of the central bank and the liquidity management of the central bank have different views about the timing of the implementation of an interest rate change in the market. Probably, this was also the reason why it is not applied by central banks (see e.g. ECB [2002b]).

5. Conclusions
On the basis of a simple model of bidding in central bank fixed rate tenders, the question of the optimal tender procedure and optimal allotment policy of the central bank and in particular the debates on “over”- and “under”- bidding were revisited with a focus on the bidding equilibrium under rational expectations, i.e. under a full anticipation of the central bank’s allotment strategy. The model was built in a way to capture all aspects appearing relevant from a short survey of central bank experience and a more detailed look of the case of the Eurosystem. A one-day and a two-days reserve maintenance period were distinguished, the latter allowing to incorporate rate change expectations. The one-day maintenance period case was analyzed to focus especially on the precise role of bidding costs for the bidding equilibrium. While the modeling focused mainly on the fixed rate tender (which included the case of variable rate tenders with minimum bid rate under rate cut expectations), the variable rate tender was included in the analysis in a very simple form to allow a comparative assessment. Among the fixed rate tenders, the 100% allotment and the discretionary allotment variant were distinguished, whereby the latter was further specified by the allotment function followed by the central bank. In terms of central bank preferences, three alternative or complementary aspects were considered, namely the stability of interest rates within the reserve maintenance period, the smoothness of the reserve fulfillment path, and the bidding costs. The conclusions of the paper may be summarized along the following six aspects. Firstly, the failure of banks to coordinate their bidding perfectly and the impossibility to make a perfect use even of public autonomous factor forecasts, as well as the costs attached to bidding appeared as necessary ingredients of a sensible model of the bidding behavior of banks in open market operations. Secondly, in the absence of rate change expectations, both the variable and the discretionary fixed rate tender perform well. The 100% allotment rule has the disadvantage of implying overnight interest rate volatility and noise in the reserve fulfillment path. In case
of the use of the fixed rate tender procedure and a large free bidding potential, the liquidity management of the central bank should be neutral in the sense that the central bank should target market rates equal to tender rates. In case there is no large free bidding potential, this conclusion is no longer generally valid, since a tighter liquidity management may contribute to reduce the risks of underbidding and hence the volatility of interest rates. The fact that the ECB, which does not impose bids to be covered by collateral, tends to follow a neutral allotment policy, while the Bundesbank and the Banque de France, which required such a coverage, tended to be tight, appears to be in line with this insight. Thirdly, under conditions of rate change expectations, it appears that the fixed rate tender tends to have, in the model proposed here, some specific disadvantages relative to the variable rate tender. The discretionary fixed rate tender has a weakness especially under rate hike expectations, namely overbidding and its associated costs. Under rate cut expectations, the discretionary fixed rate tender invites to underbid to some extent and hence to create some associated volatility of overnight rates. At the same time, the reserve fulfillment path tends to be destabilized to a certain extent. Fourthly, if discretionary fixed rate tenders are chosen under rate hike expectations, the model suggests that it is difficult to “fight” against overbidding through excess liquidity. Indeed, the approach of the ECB in the second half of 1999 to provide relatively systematically excess liquidity at the end of the reserve maintenance periods did not stop overbidding, but contributed to volatility of overnight rates and allotment ratios. Fifthly, under discretionary fixed rate tenders and rate cut expectations, a small bailout co-efficient will ensure that underbidding will tend to be relatively rare. Only in case the central bank has a strong aversion against interest rate volatility, it may consider a full bailout and thereby eliminate the interest rate shocks due to the imperfect coordination of bidding and use of autonomous factor forecasts, which seems however unlikely to be applicable in practice. In so far, one may conclude that the truth in the debate reported in the Boersenzeitung (see section 2) regarding whether underbidding can be stopped lies in the middle: underbidding will happen again, but the central bank has, by choosing a low bailout coefficient, the possibility to make it less frequent. Some additional noise will in any case remain relative to a switch to the variable rate tender. Six, one may come back to the academic debate presented in the introduction on whether the fixed rate tender is generally a badly specified procedure to which overbidding is inherent (e.g. Nautz and Oechsler [1999]) or whether it was just the ECB which steered liquidity in a too tight way and thereby triggered the overbidding problem (Ayuso and Repullo [2001]). In the light of the model presented in this paper and the interpretation of the evidence it suggests, both interpretations seem to be misleading. Instead, the conclusion should be that fixed rate tenders, if combined with an adequate liquidity policy, are well suited to conditions of stable interest rates, but that they may indeed cause some problems (bidding costs, interest rate noise, non-smooth reserve fulfillment path) in an environment of strong rate change expectations, whereby a central bank with a framework of reserve requirements and averaging over a period being longer
than the maturity of its operations then has only very limited, if any, possibilities of stabilization through a specific liquidity policy.

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Appendix: proof of propositions

Proof of Proposition 1: First, it has to be shown that if \( m = B \) and \( \bar{k} = 0.5 \) and \( \bar{B} > A \), then \( B = A \) and \( E(i) = \bar{k} \) and \( \var(i) > 0 \). As explained the equilibrium condition for bidding is:

\[
\frac{1}{\bar{k}} + \frac{\partial C}{\partial B} = E \left[ 1 - \Phi \left( \frac{B - A - \varepsilon}{\sigma_\eta} \right) \right] = \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{B - A - x}{\sigma_\eta} \right) \right] f_\varepsilon(x) dx \tag{A-1}
\]

Denote by \( h_1(B) \) the left-hand side and by \( h_2(B) \) the right-hand side of the equation above. Obviously, \( h_1(B) : [0, \infty[ \rightarrow [\bar{k}, \infty[ \) is monotonously increasing and \( h_2(B) : [0, \infty[ \rightarrow [0, \infty[ \) is monotonously decreasing and both are continuous. If \( \bar{B} > A \), \( B = A \Rightarrow h_1(B) = h_2(B) = 0.5 \); \( B > A \Rightarrow h_1(B) > h_2(B) < 0.5 \) and \( B < A \Rightarrow h_1(B) < h_2(B) > 0.5 \). Hence, \( B = A \) is the unique equilibrium bid and \( h_1(B) = E(i) = \bar{k} \). Now consider the case \( \bar{B} < A \). Here, \( B = A \Rightarrow h_1(B) > h_2(B) = 0.5 \); \( B > A \Rightarrow h_1(B) > h_2(B) < 0.5 \); and, \( B = 0 \Rightarrow h_1(B) = 0.5 < h_2(B) \). Hence, there is a unique equilibrium bidding volume \( B < A \) with \( h_1(B) = E(i) \). The volatility of market overnight rates follows obviously from the stochastic impact of \( \varepsilon \) on \( i \) as described in \( h_2(B) \).

Proof of Proposition 2: Now \( m = \min(A + \varepsilon, B) \) and \( \bar{k} = 0.5 \). Note that, as previously, \( h_1(B) = \bar{k} + \partial C / \partial B \). In contrast, \( h_2(B) \) will now be composed of two elements: the value of funds will be either \( \bar{k} \) (in case banks bid \( A + \varepsilon \) or more) or higher (if banks bid less). The market interest rate in case underbidding occurred will obviously depend on the extent of underbidding, which becomes immediately known after the allotment. Consider first the case \( \bar{B} \gg A \), for which it has to be shown that \( B = \bar{B} \) and \( i = \bar{k} = 0.5 \). The equilibrium condition for the optimal bidding volume is (see also Theorem 20.1 of Greene [1997, 949]):

\[
\begin{align*}
\hat{r} & = E \left[ r(\theta \leq B - A) \right] + \left[ 1 - \Phi \left( \frac{B - A - (\varepsilon \mid \varepsilon > B - A)}{\sigma_\eta} \right) \right] P(\varepsilon > B - A) \\
& \text{with} \quad h_1(B) = \bar{k} + \partial C / \partial B \end{align*}
\]
\[ k = \frac{k}{\sigma} \left( \frac{B - A}{\sigma} \right) + \int_{B - A}^{\infty} \left( 1 - \Phi \left( \frac{B - A - x}{\sigma} \right) \right) \phi \left( \frac{x}{\sigma} \right) dx \left( 1 - \Phi \left( \frac{B - A}{\sigma} \right) \right) \]

First, it has to be shown that \( B \geq \bar{B} \). The function \( h(B) : [0, \infty[ \rightarrow [1/2, 1] \) is a continuous function which falls monotonously towards 0.5 without ever reaching that value for bids below \( \bar{B} \). Hence \( B < \bar{B} \Rightarrow h_1(B) < h_2(B) \). Now, it has to be shown that \( B \leq \bar{B} \). It has been assumed that \( \bar{B} \gg A \) such that \( P[\bar{B} < A + \varepsilon] \) is negligible, i.e. zero compared to \( C / B \) for any \( B > \bar{B} \). Hence, for \( B > \bar{B} : h_1(B) > h_2(B) \). Therefore \( B = \bar{B} \) is the unique equilibrium. Consider now the case \( \bar{B} = 0 \). It is easy to show, by providing examples, that all cases \( B > A, B = A, B < A \) can indeed occur.

**Proof of Proposition 3:** It has to be shown that if \( m = \min(A + \varepsilon + \gamma, B), \bar{k} = 0.5 \) and \( \tilde{B} \gg A \), then \( \forall q \in [1, \tilde{B}/A], \exists \gamma : B/A = q \) and \( \forall \gamma > 0, \text{var}(i) > 0 \). For \( \gamma = 0 \), the allotment ratio will be \( \tilde{B}/A \), according to proposition 2. When \( \gamma \to \infty \), \( m = A \), and according to proposition 1, \( D/A = 1 \). It hence only needs to be shown that the function \( B^*(\gamma) : [0, \infty[, \to [A, \bar{B}] \), being defined as the function assigning to values of \( \gamma \) equilibrium bid amounts \( B \) according to the bidding equilibrium equation \( h_1(B) = h_2(B) \), is continuous.

While \( h_1(B) = \bar{k} \), the function \( h_2(B) \) in this case is:

\[
h_2(B) = E \left[ \left( 1 - \Phi \left( \frac{\gamma}{\sigma} \right) \right) \left( P[\varepsilon \leq B - A - \gamma] \right) \right] + \left( 1 - \Phi \left( \frac{B - A - (\varepsilon \mid \varepsilon > B - A - \gamma)}{\sigma} \right) \right) \left( P[\varepsilon > B - A - \gamma] \right)
\]

\[
= E \left[ \left( 1 - \Phi \left( \frac{\gamma}{\sigma} \right) \right) \left( P[\varepsilon \leq B - A - \gamma] \right) \right] + \left( 1 - \Phi \left( \frac{B - A - \varepsilon}{\sigma} \right) \right) \left( P[\varepsilon > B - A - \gamma] \right)
\]

Since \( h_2(B) \) is continuous in \( B \) and in \( \gamma \), \( B^*(\gamma) \) will also be continuous. Finally, the claim \( \forall \gamma > 0, \text{var}(i) > 0 \) follows directly from the fact the market equilibrium condition since stochastic underbidding has to counterbalance the interest rate effect of the expected liquidity surplus.

**Proof of proposition 4:** It first needs to be shown that if \( m = \min(A + \varepsilon + \gamma, B), \bar{k} = 0.5 \) and \( \tilde{B} \gg A, \gamma = 0 \) is superior to \( \gamma < 0 \) in the sense that \( \gamma = 0 \Rightarrow \text{var}(i) = 0; C(B) = 0 \) while \( \gamma < 0 \Rightarrow \text{var}(i) > 0; C(B) > 0 \). It can be shown, similarly as in proposition 2, that
\( \gamma < 0 \Rightarrow B > \tilde{B} \gg A \). This implies that the probability of underbidding is negligible, and hence \( m = A + \varepsilon + \gamma \). Therefore, \( h_2(B) = (1 - \Phi(\gamma / \sigma_y)) \), and interest rates have zero variance. However, since \( h_2(B) > 0.5 \), the marginal bidding costs have to be positive in \( h_1(B) = \frac{k}{h} + \partial C / \partial B \), such that \( h_1(B) > 0.5 \). Now consider the case \( \tilde{B} = 0 \). This implies for \( \gamma = 0 \) that \( B < A \) and hence \( P(B < A + \varepsilon) > 0 \). The bidding equilibrium condition obviously also implies that \( B \) increases when the negative liquidity target \( \gamma \) further decreases. When \( B \) increases, \( P(B < A + \varepsilon) \) decreases, and hence also the probability of interest rates being different from their normal level, i.e. if \( P(B > A + \varepsilon + \gamma) \). If the central bank has a strong preference for interest rate stability, it may hence gain from a negative liquidity target.

**Proof of proposition 5:** It needs to be shown that in case of absence of contingent liquidity surplus target, i.e. if \( m_1 = \min(A + \varepsilon_1, B_1), m_2 = \min(A + \varepsilon_2, B_2), \tilde{k} = 0.5, \tilde{B} \gg 2A \), \( E(\lambda d^h) > 0 \) then \( m_1 = A + \varepsilon_1, m_2 = A + \varepsilon_2 \) \( i_1 = 0.5 + E(d^h) \), \( i_2 = 0.5 + d^h \lambda \), \( B > \tilde{B} \). \( \partial C / \partial B_1 = E(d^h \lambda) \). Note that in analogy to proposition 2, \( B_2 = \tilde{B} \), if \( B_1 \geq A + \varepsilon_1 \) \( (\Rightarrow m = A + \varepsilon_1) \), since then, the situation in the morning of day 2 is equivalent to the situation in the morning of day 1 in proposition 2. However, \( B_1 \geq A + \varepsilon_1 \) is obviously the case since under the assumed allotment rule, \( i_2 = 0.5 + d^h \lambda \), implying through the martingale hypothesis that \( i_1 = 0.5 + E(d^h) \geq \tilde{k} \). Hence, there will never be underbidding in any of the two tenders and \( m_1 = A + \varepsilon_1, m_2 = A + \varepsilon_2 \). From the bidding equilibrium condition on day 1, it follows immediately that \( \partial C / \partial B_1 = E(d^h \lambda) \). Now, it needs to be shown that if instead \( m_1 = \min(A + \varepsilon_1 + d^h \pi^w, B_1) \) with \( A > \pi^w > 0 \) then \( P(B_2 < A + \varepsilon_2) > 0 \) but still \( \partial C / \partial B_1 = E(d^h \lambda) \). If \( P(B_2 < A + \varepsilon_2) = 0 \) then \( m_1 + m_2 > 2A + \varepsilon_1 + \varepsilon_2 \) and hence \( E(i_2) < 0.5 + d^h \lambda \) implying \( B_2 = 0 \), which is a contradiction and hence \( P(B_2 < A + \varepsilon_2) > 0 \). If \( \pi^w < A \) then \( B_2 > 0 \) and hence \( E(i_2) = 0.5 + d^h \lambda \) and \( i_2 = 0.5 + E(d^h \lambda) \). But then \( \partial C / \partial B_1 = E(d^h \lambda) \). Finally, if \( \pi^w > A \) then \( B_2 = 0 \) and \( E(i_2) < 0.5 + d^h \lambda \).

**Proof of proposition 6:** If \( m_1 = \min(A + \varepsilon_1, B_1), m_2 = \min(A + \varepsilon_2 + \alpha(A + \varepsilon_1 - m_1), B_2), \tilde{k} = 0.5, \tilde{B} \gg 2A, E(\lambda d^h) < 0 \) then \( \forall \alpha \in [0,1]: P(B_1 < A + \varepsilon_1) > 0 \); \( \forall \alpha \in [0.1]: B_2 \geq \tilde{B} \); \( \alpha = 1 \Rightarrow B_1 = 0 \land B_2 = \tilde{B} \); \( \alpha < 1 \land B_1 < A + \varepsilon_1 \Rightarrow B_2 > \tilde{B} \); \( \forall \alpha \in [0.1]: \text{var}(i_1) > 0; \text{var}(i_2) > 0 \). Furthermore \( \exists \tilde{\alpha} \) with \( 1 > \tilde{\alpha} > 0 \) such that \( \alpha = \tilde{\alpha} \Rightarrow B_1 = 0, E(i_1) = \tilde{k}, \alpha > \tilde{\alpha} \Rightarrow B_1 = 0, E(i_1) < \tilde{k}; \alpha < \tilde{\alpha} \Rightarrow B_2 > 0, E(i_2) = \tilde{k} \).

First, note the bidding equilibrium condition in this case and name \( h^w(B_1) \) the right hand side of this equation:
\[ f = E(d^h \lambda) + \frac{1}{2} P(e_i < B_1 - A) + E \left[ 1 - \Phi \left( \frac{-(1-\alpha)(A - B_1 + (e_1 | e_i > B_1 - A))}{\sigma} \right) \right] P(e_i > B_1 - A) \]
\[ = E(d^h \lambda) + \frac{1}{2} \left[ 1 - \Phi \left( \frac{B_1 - A}{\sigma} \right) \right] + \left[ \int_{-\infty}^{B_1 - A} \left[ 1 - \Phi \left( \frac{-(1-\alpha)(A - B_1 + x)}{\sigma} \right) \right] \phi \left( \frac{x}{\sigma} \right) dx \right] \] (A-7)

Assume for a moment that \( P(B_1 < A + e_1) = 0 \) then, always, \( m_1 = A + e_1 \). However, since there is then no reason to underbid on the second day (see proposition 2), the market rate on day 2 would be \( i_2 = \bar{k} + d^c \lambda \), and hence \( i_2 = \bar{k} + E(d^c \lambda) \leq \bar{k} \), which implies \( B_1 = 0 \), being in contradiction to \( P(B_1 < A + e_1) = 0 \), implying hence \( P(B_1 < A + e_i) > 0, \forall \alpha \in [0,1] \). Note that if \( B_1 > A + e_1 \) and hence \( m_1 = A + e_1 \), day 2 is similar to the one day case, and proposition 2 implies that \( B_2 = \bar{B} \). If \( B_1 < A + e_1 \), obviously conditions will be at least as tight at the beginning of day 2 compared to the previous case, so bids should be at least as high, implying \( B_2 \geq \bar{B} \). If \( \alpha = 1 \), one hence always obtains, independent from \( B_1 \), \( m_1 + m_2 = 2A + e_1 + e_2 \) such that \( i_2 = \bar{k} + d^c \lambda \) and hence, \( i_2 = \bar{k} + E(d^c \lambda) \) such that \( B_1 = 0 \). However, if \( m_1 = B_1 < A + e_1 \) and \( \alpha < 1 \), then \( i_2 > \bar{k} + d^c \lambda \) and hence overbidding with \( \partial C / \partial B_2 > 0 \Rightarrow B_2 > \bar{B} \) is required to close the gap between the day 2 tender rate and the day 2 (expected) market rate. The claim \( \alpha < 1 \Rightarrow \text{var}(i_1) > 0; \text{var}(i_2) > 0 \) follows from the positive likelihood of underbidding on the first day due to the \( e_1 \) shocks, which not only affect \( i_1 \), but also \( i_2 \).

Now, it still needs to be shown that \( \exists \tilde{\alpha} \) with \( 1 > \tilde{\alpha} > 0 \) such that \( \alpha = \tilde{\alpha} \Rightarrow B_1 = 0, i_1 > \bar{k}; \alpha > \tilde{\alpha} \Rightarrow B_1 = 0, i_1 < \bar{k}; \alpha < \tilde{\alpha} \Rightarrow B_1 > 0, i_1 = \bar{k} \). It was shown that \( \alpha = 1 \Rightarrow B_1 = 0, i_1 > \bar{k} \). Now consider \( \alpha = 0 \). Under the assumption that \( P(\lambda > A) = 0, \lambda < 0.5 \), i.e. that the interest rate effect of one full day of structural liquidity missing is always larger than the effect of a potential rate change, \( B_1 > 0 \), since \( B_1 = 0 \) would imply indeed that one can expect a liquidity gap \( 2A + e_1 + e_2 - (m_1 + m_2) \geq \lambda \). Since all terms in equation (A-7) are continuous, the function \( B'_1(\alpha) \) defined by the equation will also be continuous. Also, this function declines monotonously since \( \forall B_1 \in R^+, \partial h''(B_1)/\partial \alpha < 0 \) and \( \forall \alpha \in [0,1], \partial h''(B_1)/\partial B_1 < 0 \). Therefore an \( \tilde{\alpha} \in [0,1] \) exists with the described properties.

**Proof of proposition 7:** If \( m_1 = A + e_1 + \pi_{he} d^{he} \quad m_2 = A + e_2, \quad \bar{k} = 0; \bar{\lambda} = 1, \quad \bar{B} >> 2A, \quad E(\lambda d^h) > 0 \) then \( \exists \pi_{he} \) such that \( i_1 = 0.5 \) and \( \forall \pi_{he} \ i_2 = i_1 + d^h \lambda - E(d^h \lambda) \). The interest rate on the first and second day will be:

\[ i_1 = E(d^h \lambda) + \left( 1 - \Phi \left( \frac{\pi_{he}}{\sigma} \right) \right) \] (A-8)

\[ i_2 = d^h \lambda + \left( 1 - \Phi \left( \frac{\pi_{he}}{\sigma} \right) \right) \] (A-9)
Obviously, the central bank can thus, by choosing \( \pi^h \), compensate the rate change expectations since it is assumed that \( E(d^h \lambda) < 0.5 \) and achieve that \( i_1 = 0.5 \). However, (A-8) and (A-9) also imply that \( i_2 = i_1 + d^h \lambda - E(d^h \lambda) \).

**Proof of proposition 8:** If \( m_1 = A + \varepsilon_1 \); \( m_2 = A + \varepsilon_2 + \pi^h d^h \). \( k = 0; \tilde{k} = 1, \tilde{B} \gg 2 A \), \( E(\lambda d^h) > 0 \) then \( \exists \pi^h \) such that \( i_1 = 0.5 \) and \( i_2 = 0.5 \). The interest rates on the first and second day will be:

\[
\begin{align*}
i_1 &= P(d^h = 1) \left( \lambda + 1 - \Phi \left( \frac{\pi^h}{\sigma_\eta} \right) \right) + (1 - P(d^h = 1))0.5 \quad \text{(A-10)} \\
i_2 &= (1 - d^h)0.5 + d^h \left( \lambda + 1 - \Phi \left( \frac{\pi^h}{\sigma_\eta} \right) \right) \quad \text{(A-11)}
\end{align*}
\]

The central bank can thus, again, achieve that \( i_1 = 0.5 \) by choosing accurately \( \pi^h \). By the martingale property and the (A-10), (A-11), this contingent liquidity injection at the same time leads to \( i_2 = 0.5 \).
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