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## Evaluation frequency and forgetful principals

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**Julia Angerhausen**

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# Evaluation Frequency and Forgetful Principals\*

Julia Angerhausen<sup>†</sup>

## Abstract

This paper analyzes the behavior of a principal with bounded memory who can offer a two-period performance-based contract to an agent. In the model he can choose whether to evaluate the agent after each period or only at the end of the second period. If the agent is wealth-constrained, the option to evaluate him twice can be profitable. But without the constraint on the part of the agent, the principal will always prefer to evaluate only once in order to reduce evaluation costs. Finally we consider a modification of the profit function that can be interpreted as depreciation. Again, this leads us to a trade-off between more and less frequent evaluations.

Keywords: Performance evaluations, bounded memory, incentive contracts  
JEL classification code: M12, J33, D86

## 1 Introduction

Motivating employees by linking pay to performance is a popular topic for research as well as practice and the number of employees who receive a share of their compensation depending on how well they did their job rises continually. In certain categories of jobs performance can be directly linked to a quantitative measure. Well-known examples are salesmen or executives whose compensation varies largely with the amount of products sold and the performance of the firm respectively. But numerous stories about detrimental effects of pay for performance show that employee evaluations based on purely objective performance measures often come with problems. In order to avoid counterproductive

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behavior objective measures are frequently combined with or even replaced by subjective measures, which depend on the unverifiable opinion of a supervisor. However, an inherent problem of subjective evaluation procedures is that they are not contractible and therefore the employment relationship must be subject to a certain amount of trust.

This paper investigates the role of the frequency of performance evaluations in incentive contracting. We are interested in categories of jobs which we believe to be the most relevant for subjective performance evaluations. These jobs are many middle-management positions, such as business analysts or accountants whose performance is mainly reflected by qualitative attributes. The latter can only be correctly assessed by someone who has sufficient insight into the profession and the specific task. Similar characteristics also hold for many jobs in engineering. So the generated output is unverifiable by third parties and therefore a bonus must be determined according to a subjective performance evaluation process. While there is a strong focus on reputation issues in the literature for subjective performance assessments, we will follow an idea developed by Sarafidis [2004] and consider another facet of subjectivity: supervisor bias and particularly bias induced by imperfect memory. In our model supervisors, even if aiming at truthful and correct evaluations, only have a bounded capacity to handle the information necessary for the evaluation process. This is plausible as the main task of superiors for middle-management and engineering staff is rarely to supervise and evaluate employees, but this is done additionally to their day-to-day work. When furthermore the observed information on the output first needs to be analyzed to be useful for an evaluation, we expect to find a bias in performance evaluations due to imperfect memory.

In his survey of the literature on incentive contracts within firms Canice (Prendergast [1999]) emphasizes that in research there is a strong focus either on the implementation of piece-rates for workers (e.g. Lazear [2000]) or on CEO compensation (e.g. Jensen and Murphy [1990]). For both types of incentive contracts compensation can be tied to objective output measures, such as the number of produced goods or shareholder wealth.

Motivating middle-level managers and alike seems to be somewhat more subtle, more complex and a less popular issue for research. Performance for such categories of employees is generally assessed in (semi-)annual evaluations which are potentially biased by the subjectivity of the evaluator (Prendergast and Topel [1993], MacLeod [2003]). An important issue concerning subjective performance evaluations is the resulting lack of verifiability of the observed effort level, which implies that contracts based on subjective evaluations must be self-enforcing (Bull [1987], MacLeod and Malcomson [1989]).

Prendergast and Topel [1993] and MacLeod [2003] investigate bias in subjective performance evaluations induced by circumstances such as fairness or equality considerations, personal relationships between supervisors and subordinates, or discrimination due to gender or race. We on the other hand focus on bias caused by bounded rationality, assuming that the principal has imperfect memory. Therefore, the present paper is closely related to the broad literature on bounded memory as a specific form of bounded rationality and its effects on economic decision making. An early publication addressing issues of bounded memory in a game theoretic context is an analysis by Piccione and Rubinstein [1997] who consider implications of imperfect recall in extensive decision problems with a single player. Mullainathan [2002] provides a theoretical model that analyzes the imperfect treatment of information about a stochastic process. Both papers emphasize situations where the loss of information due to imperfect memory results in inefficiencies.<sup>1</sup>

The situation we analyze here is quite close to work by Yianis Sarafidis (Sarafidis [2004]) which is strongly influenced by Mullainathan's approach. Sarafidis characterizes the behavior of an individual that is rewarded on the basis of an assessment of his past performance on a fixed date. The assessment is provided by a supervisor who must rely on his imperfect memory to evaluate past informative events. The supervisor in Sarafidis' model is somewhat a receptor who processes information according to a sophisticated model of memory, but without

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<sup>1</sup>The opposite case is stressed in Frey [2004] where the focus is rather on information individuals would like to forget, but they are unable to do so.

any possibility to act. In the following model however he can decide how often he wishes evaluations to take place, so he has a means to work against his own bounded memory. In Sarafidis' paper memory is governed by two effects: information which is received more recently and information which is received more frequently is stored with a higher probability. We model only the former effect which consists in more recent information being available from memory with a higher probability.

Frequency is an important issue in the monitoring literature (e.g. Ichino and Muehlheusser [2004]). The frequency of monitoring is usually modeled via the probability that an individual's actions are controlled. For evaluations we believe that they do not take place probabilistically, so the frequency in our model is associated with deterministic events. There is related work from the public economics literature which corresponds to a methodologically similar setting: Akemann and Kanczuk [2003] study the effects of term lengths on welfare to gain insights on election frequency.

We will take the feasibility of implicit contracts based on subjective performance as given and rather explore the specific impact of bounded rationality, i.e. a systematic bias in evaluations due to a *recency effect*<sup>2</sup>. Like Akemann and Kanczuk [2003], the following model subdivides the entire time span in two periods to allow for comparison of different choices concerning evaluation frequency. In this two-period setting a forgetful employer (*principal*) must decide how often he should evaluate an employee (*agent*) who works for him. At a date which has been fixed in advance, the employer attaches a subjective performance measure to the information that he has gathered over the preceding evaluation period. We argue that, depending on the length of this period, the quality of the information he has stored varies due to bounded memory, so he probably has a rather precise idea on how well his subordinate performed in recent time, but events that are more remote are also more likely to be forgotten. More frequent evaluations come at a higher cost, but at the same time they also lead to more

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<sup>2</sup>This is the component of imperfect memory in Sarafidis [2004] which implies that more recent information is stored with a higher probability.

precise information on which the principal can base the bonus payment for the agent. Given the contract designed by the principal, the agent decides in each period whether to exert high or low effort. To gain additional insights, we also investigate the impact of a wealth-constrained employee. This implies that the agent receives a fixed compensation component that is bounded from below and therefore can be higher than it would be optimal for the principal. The design of the underlying principal-agent model is very close to Laffont and Martimort [2002] who also introduce a wealth constraint in a discrete model. They refer to the constraint as *limited liability* and interpret this restriction as a first step towards risk aversion.

In the unrestricted model, we find that the principal never decides to evaluate the agent in each period. His forgetfulness implies a higher variable payment to the agent compared to perfect memory but he reincorporates this loss via a negative fixed payment and therefore he can extract the entire surplus from the employment relationship. For a wealth-constrained agent the fixed compensation component must be larger than the one optimally chosen by the principal. We show that in this case the formerly derived results will be affected in favor of more frequent evaluations. Unlike before, part of the surplus possibly goes to the agent.

In Section 2 we describe the setup of the basic model and how we will solve it in the following sections. Section 3.1 derives the optimal choices of the principal and the agent for the case of a principal who evaluates the agent's performance at the end of the last period. We repeat this analysis for the case of two evaluations in Section 3.2. In Section 3.3 we investigate how the choice of evaluation frequency changes if the agent is assumed to be wealth-constrained. Section 4 sums up the results of the previous sections. Section 5 is an digression and deals with the impact of a modified profit function for which we provide a distinct intuition. Section 6 concludes with the main insights on evaluation frequency we can extract from the different model versions when an agent faces a forgetful principal.

## 2 The Model Setup

We develop a model that shall capture a relatively long time span, e.g. one year, in an ongoing employment relationship between a principal and an agent. The principal who faces imperfect memory must decide on the frequency of performance evaluations over the entire time span when he designs a contract. In the model this time span is subdivided into two periods to allow a comparison of different choices concerning evaluation frequency. Even though we consider a two-period setting, we exclude discounting from the analysis to focus on the effect of imperfect memory on evaluation practices.

The principal, who disposes over a production technology, has employed an agent to be able to realize a surplus. But employment alone is not sufficient for the surplus to materialize, additionally the employed agent needs to exert a certain level of effort. Therefore he has to be compensated for the disutility of the effort necessary for the creation of the surplus. Besides fixed and variable compensation in our model the principal disposes over another instrument he can use to design an optimal contract: by investing in more frequent evaluations he can raise the quality of the available performance information and thus reduce variable compensation. Or to put it another way, he can decide to evaluate the agent less frequently but the additional risk that emerges from worse information about the agent's effort provision has to be outweighed by higher incentive payments.

The principal can offer a contract that determines the payments the agent receives for the output he generates on behalf of his employer. The contract is valid for the entire time span and in each period the agent decides how hard he works (high or low effort). It is common knowledge between the agent and the principal that if the agent exerts high effort ( $e_H$ ), he produces high output ( $y_H$ ) with certainty, that is  $e_H \Leftrightarrow y_H$ . Analogously we have  $e_L \Leftrightarrow y_L$  for low effort ( $e_L$ ) and low output ( $y_L$ ). We define  $y_H, e_H > 0$  and for simplicity, we set low effort and low output to zero,  $e_L = y_L = 0$ . To express these properties in the model, we choose the linear function  $y_i = f \cdot e_i$ , with  $i = L, H$ , to describe the

relationship between the agent's effort  $e_i$  and the output  $y_i$ . The parameter  $f$  stands for the productivity of the agent, which is assumed to be larger than one. As the output the agent generates with low effort is zero, the principal always wants the agent to work hard. An incentive compatible contract therefore implies the choice of  $e_H$  on part of the agent in both periods.

Both contracting parties are risk neutral and compensation is paid via a fixed component  $\alpha$  as well as a variable component depending on a bonus factor  $\beta$ . Thus, we analyze linear contracts only. Compensation based on  $\beta$  is tied to the agent's performance, which is the output he generates during the two periods.<sup>3</sup> We assume that the principal is able to observe the unverifiable output in each period, but he cannot immediately translate it into a quantifiable measure that enters the bonus function. First, the employee's achievements need to be reviewed, quantified and made explicit through a performance evaluation. So a costly thinking and classifying process must be undertaken to translate the observed performance into explicit quantitative terms that reflect the impact of performance on the principal's profit. This process is what we call the evaluation of the agent's performance. An important assumption is that the employer cannot remember information perfectly over time as long as the information he has observed has not been made explicit through an evaluation. So a principal who decides to save the cost of evaluating his agent immediately might forget the performance he observed. In the model this is represented by the principal remembering the observed output only with some probability  $\rho$  in the following period, if not both, production of output *and* an evaluation, have occurred previously.

When there are gains from contracting the principal must decide how often to evaluate the agent: each period (twice) or at the end of the last period (once). Each evaluation comes at a fixed cost  $c_E$ , so on the one hand more frequent evaluations are more expensive, but on the other hand they lead to more precise information on which the principal can base the bonus payment for the agent.

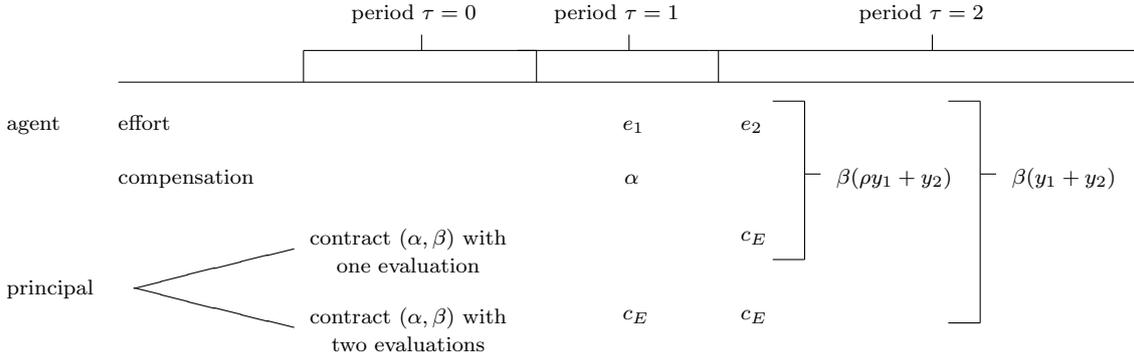
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<sup>3</sup>Note that the terms *performance*, *effort* and *output* are used as synonyms as they can be considered as equivalent in a model a deterministic production process.

If one of the parties refuses to accept the contract, there will be no employment relationship.

The chronology of events is the following: first, the principal designs a contract which contains the number of times the agent is evaluated, the amount of a fixed wage component and the bonus depending on the evaluated performance. The agent accepts the contract if his expected utility from the incentive contract is at least as high as his outside option, otherwise he rejects the offer. Once accepted, the contract lasts for two periods, and in each period the agent decides if he works hard. The principal observes and receives the output the agent produces and evaluates him truthfully on the basis of what he observed. But due to imperfect memory, the principal remembers the observed output only with some probability if there has been no evaluation in the same period. To summarize, we give a brief overview of the events:

- *Period  $\tau = 0$* : if there is a positive surplus the principal designs a work contract which contains the number of times the agent is evaluated, a fixed wage component  $\alpha$  and a factor  $\beta$  that, when multiplied with the agent's evaluated performance, determines variable compensation; the agent accepts or rejects the contract
- *Period  $\tau = 1$* : the agent exerts effort  $e_1 \in \{e_L, e_H\}$ ,  $e_L = 0, e_H > 0$  to produce the first-period output  $y_1$ ; the fixed wage  $\alpha$  is paid out
- *Period  $\tau = 2$* : the agent exerts effort  $e_2 \in \{e_L, e_H\}$ ,  $e_L = 0, e_H > 0$  to produce the output  $y_2$ ; the bonus payment is calculated according to alternative 1 or 2 and paid out to the agent
- *Alternative 1: One Evaluation*
  - when it comes to the evaluation of the agent's work at the end of the second period the principal remembers the observed output in period one with probability  $\rho \in ]0, 1[$
  - fixed evaluation costs  $c_E$  arise at the end of the second period



**Figure 1:** The Chronological Order of the Game

– the agent is paid his bonus  $\beta \cdot (\rho y_1 + y_2)$  at the end of the second period

• *Alternative 2: Two Evaluations*

- the principal evaluates at the end of each period and therefore correctly remembers the agent's output he observed over the two periods
- his total evaluation cost increases to  $2 \cdot c_E$
- the agent receives the bonus  $\beta(y_1 + y_2)$  for his output during the two periods

With this information at hand, we can write down the expected profit function of the principal in the case of one evaluation (superscript:  $o$ ) and two evaluations (superscript:  $t$ ):

$$\begin{aligned}\Pi^o(y_1, y_2) &= y_1 + y_2 - \alpha^o - \beta^o(\rho y_1 + y_2) - c_E, \\ \Pi^t(y_1, y_2) &= y_1 + y_2 - \alpha^t - \beta^t(y_1 + y_2) - 2c_E.\end{aligned}$$

The utility function of the agent takes into account compensation as well as the disutility from the provision of effort:

$$\begin{aligned}U^o(e_1, e_2) &= \alpha^o + \beta^o(\rho y_1 + y_2) - (e_1 + e_2), \\ U^t(e_1, e_2) &= \alpha^t + \beta^t(y_1 + y_2) - (e_1 + e_2).\end{aligned}$$

The principal and the agent receive zero utility if they choose the outside option, i.e.  $U(0) = \Pi(0) = 0$ . For notational convenience we have set  $E(\Pi) = \Pi$  and  $E(U) = U$  throughout the paper.

We solve two versions of the model - a version with one evaluation and one with two evaluations - for subgame perfect Nash Equilibria via backward induction. In the course of the backward induction of each model version we derive under which circumstances the agent chooses to work hard for given compensation parameters  $\alpha$  and  $\beta$ . Moreover, we determine the parameter combinations in which he will be willing to work at all. If the principal's expected profit is positive, he will choose those parameters  $\alpha$  and  $\beta$  which yield the highest return. Otherwise he will refrain from offering a contract. After deriving the parameters for the model with one and with two evaluations, we will consider the principal's decision on the evaluation frequency that he fixes in the contract in period  $\tau = 0$ . We repeat the analysis for a wealth-constrained agent - this implies that  $\alpha$  must be larger than some lower bound - and examine how such an assumption changes the preceding results.

### 3 Analysis of the Model

#### 3.1 One Evaluation

We now investigate for which payment schemes the agent is willing to exert high effort, given the fact that the principal evaluates his performance only at the end of the second period and therefore has imperfect recall, represented by the parameter  $\rho$ . We then derive the corresponding profit for the principal and deduce his optimal choice of the contract parameters for one evaluation. Independent of the actual effort levels he chooses, the agent's expected utility over the two periods can be expressed by:

$$\begin{aligned}
 U^o(e_1, e_2) &= \alpha^o + \beta^o(\rho y_1 + y_2) - (e_1 + e_2) \\
 &= \alpha^o + \beta^o(\rho f e_1 + f e_2) - (e_1 + e_2) \\
 &= \alpha^o + (\beta^o f \rho - 1)e_1 + (\beta^o f - 1)e_2.
 \end{aligned} \tag{1}$$

And the principal's expected profit is:

$$\begin{aligned}
 \Pi^o(e_1, e_2) &= y_1 + y_2 - \alpha^o - \beta^o(\rho y_1 + y_2) - c_E \\
 &= f(e_1 + e_2) - \alpha^o - \beta^o f(\rho e_1 + e_2) - c_E \\
 &= (1 - \beta^o \rho) f e_1 + (1 - \beta^o) f e_2 - \alpha^o - c_E.
 \end{aligned} \tag{2}$$

$e_\tau$ : effort in period  $\tau = 1, 2$

$f$ : productivity of the agent, with  $f > 1$

$y_\tau$ : output in period  $\tau$ , with  $y_\tau = f \cdot e_\tau$

$\alpha^o$ : fixed compensation

$\beta^o$ : variable compensation factor

$\rho$ : probability of the principal to remember  $y_1$ ; with  $\rho \in ]0, 1[$

$c_E$ : fixed evaluation cost, with  $c_E < 0$

Now we calculate the critical parameter values for which high effort is chosen. The incentive constraints for the corresponding first period effort level (*IC1*) and for the second period effort level (*IC2*) need to be fulfilled. Furthermore the participation constraint (*PC*) must hold. So the agent will choose high effort in both periods, i.e.  $e_1 = e_2 = e_H$ , if the following conditions are satisfied:

$$\begin{aligned} (IC1) : U^o(e_H, e_2) &\geq U^o(e_L, e_2) \\ \Leftrightarrow \alpha^o + \beta^o(\rho f e_H + f e_2) - (e_H + e_2) &\geq \alpha^o + \beta^o(\rho f e_L + f e_2) - (e_L + e_2) \quad (3) \\ \Leftrightarrow \beta^o &\geq \frac{1}{f \cdot \rho}, \end{aligned}$$

$$\begin{aligned} (IC2) : U^o(e_1, e_H) &\geq U^o(e_1, e_L) \\ \Leftrightarrow \alpha^o + \beta^o(\rho f e_1 + f e_H) - (e_1 + e_H) &\geq \alpha^o + \beta^o(\rho f e_1 + f e_L) - (e_1 + e_L) \quad (4) \\ \Leftrightarrow \beta^o &\geq \frac{1}{f}, \end{aligned}$$

$$\begin{aligned} (PC) : U^o(e_H) &\geq U(0) = 0 \\ \Leftrightarrow \alpha^o + \beta^o(\rho f e_H + f e_H) - (e_H + e_H) &\geq 0 \quad (5) \\ \Leftrightarrow \alpha^o &\geq [2 - \beta^o f(1 + \rho)]e_H. \end{aligned}$$

Because of  $\rho \in ]0, 1[$  we get  $\frac{1}{f \rho} > \frac{1}{f}$ , hence only (*IC1*) in equation (3) is binding. A profit maximizing principal will implement the desired effort level at the lowest cost possible. If we set  $\beta^o = \frac{1}{f \rho}$ , we can plug this into equation (5) to calculate the lowest  $\alpha^o$  in an incentive compatible contract that implements high effort:

$$\alpha^o = [2 - \frac{1}{f \rho} f(1 + \rho)]e_H = [\frac{\rho - 1}{\rho}]e_H. \quad (6)$$

Since the principal is expected to forget part of the agent's first-period performance it is more costly to require high effort in the first period than motivating the agent in the second period. The optimal fixed component  $\alpha^o$  in the contract is negative, so the agent has to pay the principal to be allowed to work for him.

We will analyze restrictions to this possibility using a more general framework in Section 3.3.

Given these contract parameters, we can calculate the principal's profit when he proposes an incentive compatible contract:

$$\begin{aligned}\Pi^o(e_H) &= (1 - \beta^o \rho) f e_H + (1 - \beta^o) f e_H - \alpha^o - c_E \\ &= \left[2 - \frac{1}{f\rho}(1 + \rho)\right] f e_H - \frac{\rho-1}{\rho} e_H - c_E \\ &= 2(f - 1)e_H - c_E.\end{aligned}\tag{7}$$

We have just derived the profit of the principal and an optimal pair of contract parameters. Additionally, offering a contract has to be at least as good as refraining from it, i.e. choosing the outside option:

$$\Pi^o(e_H) \geq 0 \Leftrightarrow 2(f - 1)e_H - c_E \geq 0 \Leftrightarrow f \geq 1 + \frac{c_E}{2e_H}.\tag{8}$$

Condition (8) expresses the extend to which the agent's productivity  $f$  must be larger than one for the employment relationship to be profitable.

## 3.2 Two Evaluations

In the preceding analysis, the principal was not able to remember the agent's first period performance with certainty when he evaluated him at the end of the second period. In this section he makes an evaluation at the end of each period, so there will be no loss of information and therefore the parameter  $\rho$  does no longer appear. As each evaluation comes at fixed cost  $c_E$  the total evaluation cost increases to  $2c_E$ . We will now calculate the conditions under which the agent chooses high effort. Furthermore we will again derive the principal's optimal behavior and the corresponding profit. In a setting with two evaluations the agent's utility over the two periods is:

$$U^t(e_1, e_2) = \alpha^t + \beta^t f(e_1 + e_2) - (e_1 + e_2) = \alpha^t + (\beta^t f - 1)(e_1 + e_2).\tag{9}$$

The principal's profit function now also takes into account the doubled evaluation cost:

$$\Pi^t(e_1, e_2) = (1 - \beta^t) f(e_1 + e_2) - \alpha^t - 2c_E.\tag{10}$$

To derive the incentive compatible contract parameters in the case of two evaluations, we can use the results from Section 3.1 and set  $\rho = 1$  as this corresponds to perfect memory. Using (3), (4), and with  $\rho = 1$  we get  $\beta^t = \frac{1}{f}$ . Plugging this into equation (5) and again accounting for  $\rho = 1$ , we find the lowest  $\alpha^t$  for this incentive compatible contract:

$$\alpha^t = [2 - 2\frac{f}{f}]e_H = 0. \quad (11)$$

These contract parameters allow us to calculate the principal's profit when he proposes a contract that implements high effort:

$$\begin{aligned} \Pi^t(e_H) &= 2(1 - \beta^o)fe_H - \alpha^t - 2c_E \\ &= 2(1 - \frac{1}{f})fe_H - 2c_E \\ &= 2(f - 1)e_H - 2c_E. \end{aligned} \quad (12)$$

It now remains to be determined under which parameter values the principal will prefer an incentive compatible contract to refraining from a contractual relationship. The condition for a contract to be profitable is:

$$\Pi^t(e_H) \geq 0 \Leftrightarrow 2(f - 1)e_H \geq 2c_E \Leftrightarrow f \geq 1 + \frac{c_E}{e_H}. \quad (13)$$

If (13) does not hold the principal will choose his outside option or evaluate only once.

### 3.3 Evaluating a Wealth-Constrained Agent

In the preceding analysis we have implicitly assumed that the agent disposes over infinite wealth. No matter how high the fee the principal fixes in a contract with one evaluation, the agent will be able to pay this sum. In this subsection, we extend the analysis by allowing for a wealth-constrained agent. We do this by introducing a wealth bound  $\bar{\alpha}$ , which is the maximal amount the agent is able to pay in the first period. As long as  $\bar{\alpha}$  is in absolute terms larger than or equal to the optimal fee  $\alpha^o$  from Section 3.1 the analysis remains unaffected. But a restriction in the range  $0 \geq \bar{\alpha} > \alpha^o$  has an impact on the contracts the principal can propose with one evaluation.<sup>4</sup> The analysis with two evaluations remains

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<sup>4</sup>One could also introduce a minimum wage as constraint for  $\alpha$ . So the restriction  $\bar{\alpha} \leq 0$  is not crucial for the results, but has rather been chosen for reasons of consistency with the interpretation of the bound as a wealth-constraint. Of course this also spares us from recalculating the profitability constraint for two evaluations with a minimum wage.

unchanged, as then the optimal alpha is zero, and therefore compatible with any wealth restriction.

To observe what happens in the case of a wealth restriction, we simply have to repeat the analysis for one evaluation, replacing the optimal  $\alpha^o$  by  $\bar{\alpha}$ . This implies that the principal can only charge the agent an amount  $\bar{\alpha}$  which is lower than the optimal one but on the other hand he cannot lower incentives, as he already chooses the lowest incentive compatible  $\beta$ . So a wealth constraint lowers the principal's profits as the participation constraint is tightened while the incentive constraint remains unchanged. The profit for high effort in both periods with one evaluation becomes:

$$\begin{aligned}\Pi^o(e_H|\bar{\alpha}) &= (1 - \beta^o \rho) f e_H + (1 - \beta^o) f e_H - \bar{\alpha} - c_E \\ &= [2 - \frac{1}{f\rho}(1 + \rho)] f e_H - \bar{\alpha} - c_E \\ &= (2f - \frac{1+\rho}{\rho}) e_H - \bar{\alpha} - c_E.\end{aligned}\tag{14}$$

Motivating the agent for high effort is profitable, if:

$$\begin{aligned}\Pi^o(e_H|\bar{\alpha}) \geq 0 &\Leftrightarrow (2f - \frac{1+\rho}{\rho}) e_H - \bar{\alpha} - c_E \geq 0 \\ &\Leftrightarrow 2f - \frac{1+\rho}{\rho} \geq \frac{\bar{\alpha} + c_E}{e_H} \Leftrightarrow f \geq \frac{1}{2} (\frac{1+\rho}{\rho} + \frac{\bar{\alpha} + c_E}{e_H}).\end{aligned}\tag{15}$$

This condition corresponds to the condition in (8), but due to the introduction of the wealth constraint  $\bar{\alpha}$  the memory parameter  $\rho$  now also determines profitability. Due to  $\bar{\alpha} \leq 0$  the profitability condition for two evaluations in (13) remains valid.

## 4 Results

We complete the analysis by deriving the behavior of the agent and the principal throughout the entire game. This is done by a comparison of the principal's profits which are summarized in Table 1. The first subsection deals with the

Case:	$\alpha \gtrsim 0$	$\alpha \geq \bar{\alpha}$
$\Pi^o(e_H)$	$2(f - 1)e_H - c_E$	$(2f - \frac{1+\rho}{\rho})e_H - \bar{\alpha} - c_E$
$\Pi^t(e_H)$	$2(f - 1)e_H - 2c_E$	$2(f - 1)e_H - 2c_E$

**Table 1:** All Profits

	One Evaluation	Two Evaluations
$\Pi^{o/t}(e_H) \geq 0$	$f \geq 1 + \frac{c_E}{2e_H}$	$f \geq 1 + \frac{c_E}{e_H}$
$\Pi^{o/t}(e_H)$	$2(f-1)e_H - c_E$	$2(f-1)e_H - 2c_E$
$\alpha^{o/t}$	$\frac{\rho-1}{\rho}e_H$	0
$\beta^{o/t}$	$\frac{1}{f\rho}$	$\frac{1}{f}$
$\Pi^{o/t}(e_H) < 0$	$f < 1 + \frac{c_E}{2e_H}$	$f < 1 + \frac{c_E}{e_H}$

**Table 2:** Results for one and two evaluations

case of unrestricted wealth. Subsection 4.2 is based on the assumption that the agent's wealth lies somewhere between the optimal  $\alpha^o$  and zero. Without a binding wealth constraint, the profit with a single evaluation is always higher than with two evaluations, as the principal hands on the cost of his forgetfulness to the agent by "paying" a negative fixed compensation. In other words: switching from one to two evaluations leaves the principal's revenue unchanged - it is the agent who bears the entire cost of the principal's forgetfulness - and it only raises his fixed cost. This result changes when we assume that the agent has a binding wealth constraint. Now two evaluations can be an interesting option, as the principal is no longer able to hand on the entire cost of his forgetfulness.

#### 4.1 No Wealth Constraint

Table 2 resumes the critical values, the contract parameters and the profits in the unrestricted model. The critical productivities  $f$ , determining the principal's choice between the two contracts and the outside option, are always independent of  $\rho$ . For one evaluation, the more the principal forgets (smaller  $\rho$ ), the more he will charge the agent via  $\alpha^o = \frac{\rho-1}{\rho}e_H$ , where  $\frac{\partial\alpha^o}{\partial\rho} = \frac{\rho-(\rho-1)}{\rho^2}e_H = \frac{1}{\rho^2}e_H > 0$ . He can do so as he is the only employer in the labor market and the agent's outside option equals zero. The agent agrees as the fee he pays is outweighed by the variable compensation he receives. To put it another way, due to the principal's forgetfulness the agent can extract a rent via the variable compensation. But thanks to his market power, the principal can recover the rent by charging a fee corresponding to the amount of the rent.

We determine the principal's choice concerning the evaluation frequency by a

Case:	$\alpha \gtrless 0$	$\alpha \geq \bar{\alpha}$
$\Pi^o(e_H) \gtrless \Pi^t(e_H)$	$c_E > 0$	$\alpha^o = \frac{\rho-1}{\rho}e_H \gtrless \bar{\alpha} - c_E$
$\Pi^o(e_H) \geq 0$	$f \geq 1 + \frac{c_E}{2e_H}$	$f \geq \frac{1+\rho}{2\rho} + \frac{\bar{\alpha}+c_E}{2e_H}$
$\Pi^t(e_H) \geq 0$	$f \geq 1 + \frac{c_E}{e_H}$	$f \geq 1 + \frac{c_E}{e_H}$
Result:	one evaluation or outside option	if $\frac{\rho-1}{\rho}e_H > \bar{\alpha} - c_E$ : one evaluations or outside option if $\frac{\rho-1}{\rho}e_H < \bar{\alpha} - c_E$ : two evaluations or outside option

**Table 3:** Tradeoffs Between the Principal's Options

comparison of his possible gains with one and with two evaluations. Equation (16) illustrates a straight-forward result: as the principal can shift the cost of his forgetfulness to the agent his profit with a single evaluation is always higher than with two evaluations since the latter imply higher fixed costs:

$$\Pi^o(e_H) > \Pi^t(e_H) \Leftrightarrow 2(f-1)e_H - c_E > 2(f-1)e_H - 2c_E \Leftrightarrow c_E > 0. \quad (16)$$

## 4.2 Wealth-Constrained Agent

Comparing profits with one and with two evaluations leads to the following result:

$$\begin{aligned} \Pi^o(e_H|\bar{\alpha}) &\gtrless \Pi^t(e_H) \\ \Leftrightarrow (2f - \frac{1+\rho}{\rho})e_H - \bar{\alpha} - c_E &\gtrless 2(f-1)e_H - 2c_E \\ \Leftrightarrow \frac{\rho-1}{\rho}e_H &\gtrless \bar{\alpha} - c_E. \end{aligned} \quad (17)$$

While evaluating twice is unattractive as long as  $\alpha$  is unbounded, this changes when we assume that the agent is wealth-constrained. Due to the fact that he is now unable to pay the optimal  $\alpha^o < \bar{\alpha}$  for entering the contractual relationship the extent of the principal's memory imperfection becomes determinant for his choice between the contracts. According to the inequality in (17) we can divide our results into two cases. In a first parameter range, the probability that the principal will forget the observed performance is high relative to the difference of fixed cost of employing the agent, i.e. the evaluation cost minus the fixed compensation component. Under these circumstances, the principal will either

	One Evaluation $\frac{1-\rho}{\rho}e_H < c_E - \bar{\alpha}$	Two Evaluations $\frac{1-\rho}{\rho}e_H > c_E - \bar{\alpha}$
$\Pi^{o/t}(e_H \bar{\alpha}) \geq 0$	$f \geq \frac{1+\rho}{2\rho} + \frac{\bar{\alpha}+c_E}{2e_H}$	$f \geq 1 + \frac{c_E}{e_H}$
$\Pi^{o/t}(e_H \bar{\alpha})$	$(2f - \frac{1+\rho}{\rho})e_H - \bar{\alpha} - c_E$	$2(f-1)e_H - 2c_E$
$\alpha^{o/t}$	$\bar{\alpha}$	0
$\beta^{o/t}$	$\frac{1}{f\rho}$	$\frac{1}{f}$
$\Pi^{o/t}(e_H \bar{\alpha}) < 0$	$f < \frac{1+\rho}{2\rho} + \frac{\bar{\alpha}+c_E}{2e_H}$	$f < 1 + \frac{c_E}{e_H}$

**Table 4:** Results with a wealth-constrained agent

offer a contract with two evaluations inducing high effort in every period or he will not employ the agent.

In a second parameter range, where the effect of the principal's imperfect memory is too weak for more than one evaluation to be profitable, the principal either chooses a contract with high effort and one evaluation or he does not offer any contract. Table 3 lists the tradeoffs between all relevant options of the principal. Table 4 reviews the optimal choices, critical values and outcomes when the agent is wealth-constrained.

## 5 A Setup with Depreciation of Output

In the previous analysis we have described a forgetful principal's choice of a linear contract when he takes the produced output entirely into account but can forget part of it until the time of the evaluation. To point out the impact of the assumption that the principal benefits entirely from the output - no matter if he forgets it or not - we will now analyze how the optimal contract changes when first period output enters the profit function with probability  $\theta$  only, instead of being taken into account entirely. When  $\theta$  equals  $\rho$ , the principal is not forgetful, but only evaluates the output that has not depreciated. The case  $\theta < \rho$  could be interpreted as a combination of depreciation and forgetfulness, while with  $\theta > \rho$  we have a benevolent principal, who ignores some part of the depreciation, determined by the difference of the two parameters. The agent's incentive and participation constraints in (3), (4) and (5) remain unaffected by this modification, but the profit function for one evaluation now is represented

by:

$$\begin{aligned}
\Pi'^o(e_1, e_2) &= \theta y_1 + y_2 - \alpha^o - \beta^o(\rho y_1 + y_2) - c_E \\
&= f(\theta e_1 + e_2) - \alpha^o - \beta^o f(\rho e_1 + e_2) - c_E \\
&= f[(\theta - \beta^o \rho)e_1 + (1 - \beta^o)e_2] - \alpha^o - c_E.
\end{aligned} \tag{18}$$

Plugging in the incentive compatible values for  $\alpha^o$  and  $\beta^o$  yields:

$$\begin{aligned}
\Pi'^o(e_H) &= [\theta - \frac{1}{f\rho}\rho + 1 - \frac{1}{f\rho}]fe_H - \frac{\rho-1}{\rho}e_H - c_E \\
&= \theta fe_H - e_H + fe_H - \frac{1}{\rho}e_H - e_H + \frac{1}{\rho}e_H - c_E \\
&= (1 + \theta)fe_H - 2e_H - c_E.
\end{aligned} \tag{19}$$

In comparison with equation (7) profit with one evaluation in this setup is lowered by  $(1 - \theta)fe_H$ . As profit now depends on the depreciation parameter  $\theta$ , this parameter now influences profitability as well as the choice on evaluation frequency. Profitability is given by:

$$\Pi'^o(e_H) \geq 0 \Leftrightarrow f(1 + \theta)e_H - 2e_H - c_E \geq 0 \Leftrightarrow f \geq \frac{2e_H + c_E}{(1 + \theta)e_H}. \tag{20}$$

To ensure profitability the productivity parameter  $f$  must be larger than we had derived in (8). Furthermore, comparing profits with one and with two evaluations leads us to:

$$\Pi'^o(e_H) \geq \Pi^t(e_H) \Leftrightarrow [f(1 + \theta) - 2]e_H - c_E \geq 2[(f - 1)e_H - c_E] \Leftrightarrow f \leq \frac{c_E}{(1 - \theta)e_H}. \tag{21}$$

This result is different from the comparison in (16) as now there is a parameter range where the principal will prefer to evaluate a non-constrained agent twice. To derive the parameter range where one evaluation is chosen we combine the profitability condition and the condition for  $\Pi'^o(e_H, e_H) \geq \Pi^t(e_H, e_H)$ . Then  $f \geq \frac{2e_H + c_E}{(1 + \theta)e_H}$  and  $f < \frac{c_E}{(1 - \theta)e_H}$  must hold at the same time. Such a productivity  $f$  exists if:

$$\frac{c_E}{(1 - \theta)e_H} > \frac{2e_H + c_E}{(1 + \theta)e_H} \Leftrightarrow (1 + \theta)c_E > (1 - \theta)(2e_H + c_E) \Leftrightarrow \frac{1 - \theta}{\theta}e_H < c_E. \tag{22}$$

The condition in (22) allows us to split our results into two categories, one where only two evaluations or the outside option will be chosen and another category where one or two evaluations will be chosen for profitable parameter ranges. The results are summarized in Table 5. As  $\theta$  lowers the expected surplus for one

	$\frac{1-\theta}{\theta}e_H > c_E$	$\frac{1-\theta}{\theta}e_H < c_E$
$\Pi^{o'}(e_H) \geq 0$	-	$\frac{c_E}{(1-\theta)e_H} > f \geq \frac{2e_H+c_E}{(1+\theta)e_H}$
$\Pi^{o'}(e_H)$	-	$f(1+\theta)e_H - 2e_H - c_E$
$\Pi^t(e_H) \geq 0$	$f \geq 1 + \frac{c_E}{e_H}$	$f > \frac{c_E}{(1-\theta)e_H}$
$\Pi^t(e_H)$	$2(f-1)e_H - 2c_E$	$2(f-1)e_H - 2c_E$
$\Pi^{o'/t}(e_H) < 0$	$f < 1 + \frac{c_E}{e_H}$	$f < \frac{2e_H+c_E}{(1+\theta)e_H}$

**Table 5:** Results with a depreciation of the output

evaluation, of course social welfare - here that equals profit - is inferior to the one-evaluation welfare in a model without depreciation. Note that contrary to the results in Table 4 the critical condition does not split the results into a range with one evaluation and another with two evaluations, but for  $\frac{1-\theta}{\theta}e_H < c_E$  both are possible, depending on the productivity  $f$ .

After doing the calculus, what intuition can we give for the modification of the profit function? Two stories came to our mind, both related to research, one about Ph.D. students, and one about employees in R&D departments.

The classical way to obtain a Ph.D. used to be to write a thesis, consisting in a monograph resuming the entire research work of several years, and a disputation of the monograph's content. Today, most Ph.D. students' work consists of writing papers which shall be published as soon as possible. While the classical Ph.D. student's research risks to be obsolete by the date of his disputation (years after the creation of the work), the up-to-date Ph.D. student, who is evaluated with each paper he finishes, has a clear advantage as his output is not subject to devaluation until the date of a possible publication. Of course one could argue that the supervisor of a Ph.D. thesis that comes as a monograph should take into account that at the time of the research being actually done, this was state of the art. But one would at least expect that a supervisor will grade this student worse than a peer who's research output has not been devaluated, e.g. by similar but more recent publications on the same topic. Superiority of the newer practice could be suggested by its rapid propagation.

We can draw a similar scenario for an R&D department. If we suppose researchers in the department are paid a bonus for patents providing the firm with

a competitive advantage. If the evaluation of an initially successful patent occurs at a point in time where the competitive advantage has been destroyed by another more recent patent, this is likely to induce the supervisor to pay no bonus, as the patent does not provide an advantage for the firm at the time of the evaluation.

Both stories fit a model where output can lose its value over time and only the observed output at the time of the assessment enters the evaluation. With this in mind, it becomes intuitively clearer why in the basic model presented in Section 3.1 and 3.2 two evaluations must be a dominated choice. The principal has the entire market power, so he will naturally refrain from any action that diminishes welfare, i.e. his profit.

## 6 Conclusion

In this paper we analyze which role frequency plays for performance evaluations if some individuals cannot retain information perfectly over time. The main result of the unrestricted model is the following: the fact that the quality of performance information depends on evaluation frequency does not automatically create a trade-off between evaluation and incentive cost. In a contract that foresees variable and fixed compensation, the principal does not need more frequent evaluations of the agent to counterbalance his forgetfulness. The additional incentive cost caused by the loss of information due to imperfect memory will simply be shifted to the agent via a negative fixed compensation. Therefore it is also not surprising that the choice between providing high incentives and refraining from a contractual relationship remains unaffected as the level of the principal's forgetfulness varies.

But when we consider wealth-constrained agents, this outcome is weakened and now the principal's choice hinges on the degree of forgetfulness. More frequent evaluations become more profitable the higher the degree of forgetfulness, the lower the fixed cost of the evaluation and the more the agent is wealth-constrained.

Due to the principal's monopolistic position his profit in the unconstrained

model is identical to the entire surplus and the chosen contract leaves the agent with zero utility. With a binding wealth-constraint the agent's utility is strictly positive for one evaluation. While the distribution of the surplus changes in this case, its size remains unaffected by the new split. But when first period incentives for one evaluation become too expensive the principal switches to two evaluations and lowers the surplus at stake due to increased evaluation costs. Again, the agent's utility equals zero.

In our digression in Section 5 the evaluation frequency also affects the value of the first period output in the principal's profit function. This gives us more intuition why one evaluation is a dominant strategy in the basic model. Additionally we find some examples from the domain of research that fit with this modification.

The way we have set up the model also leaves space for further interpretations where supervisors face other types of bias. One example is systematical cautiousness. In addition, allowing for decreasing instead of fixed evaluation costs, will strengthen the result under the wealth constraint. We expect more frequent evaluations to gain importance with respect to the basic model when we consider risk-averse agents. That would be in line with the intuition given in Laffont and Martimort [2002] who assign similar features to situations with a wealth constraint as to a setups with risk aversion.

It seems plausible to assume that middle-managers and employees in comparable positions are wealth-constrained, though we cannot say anything about the extent of such a restriction. At least, we would expect them to be risk-averse. This suggests that in subjective review processes for middle-managers and alike, not only the measures used are important. Also the frequency with which evaluations take place does play a role. While there is a strong focus on the nature of performance measures, up to now there has been little research on the parameters determining the framework of the evaluation process. This paper sheds some first light on interdependencies at work in this context.

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