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Two-sided Certification: The market for Rating Agencies

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Abstract
Certifiers contribute to the sound functioning of markets by reducing asymmetric information. They, however, have been heavily criticized during the 2008-09 financial crisis. This paper investigates on which side of the market a monopolistic profit-maximizing certifier offers his service. If the seller demands a rating, the certifier announces the product quality publicly, whereas if the buyer requests a rating it remains his private information. The model shows that the certifier offers his service to sellers and buyers to maximize his own profit with a higher share from the sellers. Overall, certifiers increase welfare in specific markets. Revenue shifts due to the financial crisis are also explained.

Keywords: Certification, Rating Agencies, Asymmetric Information, Financial Markets.

JEL Classification Numbers: G14, G24, L15, D82.

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1 Introduction

This paper analyzes the impact of third party certification on markets with asymmetrically distributed information. We examine incentives of intermediaries regarding their selling behavior and associated market outcomes. In general, different objectives for the demand of certification exist on both sides of the market: for buyers and for sellers. Sellers may profit from publicly announced certification as they can differentiate themselves from lower-quality sellers in the market. Moreover, publicly available information on the quality leads to Bertrand-like price competition between buyers. In contrast, buyers seek to obtain an informational advantage over their rivals through private certification, as it allows them to extract an information rent. In the bidding process of buyers, the informed party adjusts its offer depending on the certified quality. Facing this tradeoff, we determine the optimal selling strategy of third party certifiers and reveal the impact on gains of trade.

Certifiers are most prominently present in financial markets. Those rating agencies evaluate the creditworthiness of issuers as well as the quality of financial products. Therewith, they reduce information asymmetries and increase efficiency in capital markets. Their main objective is the independent evaluation of the quality of a firm or a sovereign regarding its debt servicing likelihood.\(^1\) The market dominating rating agencies mainly rely on two business models: on the one hand, they offer their service directly to sellers (issuer-pay model) and on the other hand, they sell to potential buyers (investor-pay model).\(^2\) Despite the important role in financial markets, demand for certification services also arises in various other product or service markets, such as the markets for industrial products and second-hand automobiles.

We develop a model of certification in an asymmetric information framework and show that a profit maximizing certifier sells its service to both sides of the market, to buyers and to sellers. Following this strategy, she generates a double margin and increases revenues compared to selling solely to one side of the market. In addition, we show that welfare increases substantially through the operation of rating agencies in specific

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\(^1\)Cantor (2004) gives a brief overview on recent research on rating agencies, mainly with an empirical focus.

\(^2\)Three rating agencies share an estimated 95% of the rating market, namely Standard & Poor’s, Moody’s Investors Services and Fitch Ratings. The markets for certification services achieved above average growth rates in the last decades, as demand hiked due to the increased complexity of financial products and the attached information asymmetries.
markets as more trades are executed, which otherwise fall victim to asymmetrically distributed information. The gains from trade thereby heavily depend on the market structure. We differentiate a lemon market as introduced by Akerlof (1970) and a honey market where trades already occur without certification. In the former, the intermediary is partly able to overcome the breakdown of markets due to information asymmetries.

In general, the credible assessment of products’ qualities allows for the differentiation of sellers of low and high quality, which in turn leads to efficient quality related pricing. Hence issuers in the lemon market gain from the introduction of rating agencies. In markets with efficient trades, ex ante, buyers and sellers do not favor the appearance of a rating agency, as she reduces their potential gains from trade in equilibrium. Nevertheless, the rating agency enters such markets and meets the demand by sellers and buyers, as it allows pricing according to the actual quality of the product.

Related to the market of financial intermediaries the models show that the financing structure of rating agencies is affected by the market structure. Revenues shift towards the investor-pay model in times of financial distress and increased risk awareness. The shift is also observable empirically in the 2008-09 financial turmoil, where the increased risk awareness is reflected in the drying-up of specific markets. In markets with efficient trades, ex ante, buyers and sellers do not favor the appearance of a rating agency, as she reduces their potential gains from trade in equilibrium. Nevertheless, the rating agency enters such markets and meets the demand by sellers and buyers, as it allows pricing according to the actual quality of the product.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 introduces the basic asymmetric information framework. Section 4 presents three variations of the model describing the market for credit ratings and implications for the operation of a monopolistic rating agency on the amount of traded products and the generated welfare. Thereafter, section 5 links the theoretical findings with empirical observations and finally section 6 concludes. Formal proofs of the results are collected in the appendix.

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3For example, the interbanking market had to be shored up by central banks after the default of Lehman Brothers Inc., since trust between banks on the ability to repay loans diminished. Similarly, trades of e.g. ABS CDOs collapsed.

4Rating agencies have been blamed to be partly responsible for the financial crisis, as various ratings had been changed substantially (Benmelech and Dlugosz, 2009).
2 Related literature

Since the seminal work of Akerlof (1970), substantial attention has been devoted to the asymmetric distribution of information between agents. Market participants can be deterred from trade through the presence of different informational states of agents. The literature proposes different market mechanisms to break this welfare destroying asymmetries. In Spence (1973) agents are able to signal their private information, Klein and Leffler (1981) allow agents to build reputation in a repeated game and Grossman (1981) examines the effects of private information disclosure by issuing warranties by privately informed parties.

Another line of economic research deals with the introduction of third parties which possess appropriate technologies to assess the quality of the goods in the market and therewith offer their expertise in the market to reduce information asymmetries. Biglaiser (1993) shows that such a third party, a “middleman”, improves welfare generation.\(^5\) While Biglaiser’s middlemen are trading physically in the market, Lizzeri (1999) concentrates on the role of the third party as an information or certification intermediary. This intermediary is not dealing the products but offers a pure certification service. Therewith, the ex-ante private information becomes partly or fully observable by the entire market.

The question of who demands certification services has not received much attention in the literature. In Strausz and Stahl (2009), certificates serve as signalling device if the seller demands a rating and as inspection device if the buyer uses certification services. They examine the sales options of intermediaries in a vertically integrated bilateral monopoly setting and find that only seller certification as signalling device maximizes certifier’s profits. In contrast, we concentrate on the motivation for information revelation. On the one hand, sellers demand certificates to become public information and therewith foster competition in the product market. On the other hand, buyers seek to exploit an informational advantage in the sales process.

\(^5\) Albano and Lizzeri (2001) show that the presence of intermediaries increases the overall product quality.
3 The Setup

We consider a model with four players: one seller (it), two buyers (he) and one certifier (she).\textsuperscript{6} The seller owns a single, indivisible product of quality $q$ known to the seller and unobservable by buyers. We assume the quality $q$ to be uniformly distributed on the interval $[0, 1]$.\textsuperscript{7} The intermediary does not value the object while the seller has a reservation utility of $\alpha q$ with $\alpha \in [0, 1]$. Parameter $\alpha$ characterizes the market and is known to all players. This natural setup can be found in various markets. Banks often choose between selling an investment product now or holding it till maturity, which exhibits the reservation utility in our model. They compare the respective market outcomes and decide depending on their risk preferences, their liquidity status and the current market values.

A buyer receives the utility $q$ out of consumption of the product, but ex ante only knows the distribution of the product’s quality and therefore builds expectations on the true quality level. The seller has no possibility to communicate the quality of his product $q$ directly and credibly to the buyers. The intermediary owns a perfect evaluation technology, which enables her to determine the true value of $q$. She can credibly communicate the product’s quality.\textsuperscript{8} If demand for an evaluation exists, by either the seller or the buyers, the intermediary can determine the quality $q$ at zero costs. If the seller demands a rating, the intermediary will communicate the quality $q$ credibly to the market, which is thereafter known to all buyers, hence public information. If one or both buyers demand an evaluation of the product, the intermediary discloses the obtained information privately to the respective buyer.

The game of the model comprises 4 stages.

(1) The intermediary determines prices $p_s$ and $p_b$ for a rating sold to the seller and to each buyer, respectively.

(2) The seller may choose to order a rating from the intermediary for the price $p_s$. If a rating is sold, the information about the true quality $q$ will become public information.

\textsuperscript{6}In the remainder of the paper we use certifier, intermediary and rating agency interchangeably.

\textsuperscript{7}Variations of the distribution assumptions reduce the traceability of the model.

\textsuperscript{8}Sobel (1985) and Benabou and Laroque (1992) show conditions for which credibility can be assumed.
(3) The buyers decide simultaneously and independently whether to order a rating for the product. Buyers, who decide to order a rating, pay price $p_b$. The decision to buy information is public. The acquired information on the quality $q$ is private information.\(^9\)

(4) The product is sold in an auction between buyers.\(^{10}\) The reservation utility of the seller serves as reservation price.

We assume that the intermediary is honest and applies a perfect information revelation technology. Furthermore, we assume that she has no competitors and exploits her full monopoly power, which is in line with recent contributions.\(^{11}\) In addition we allow the intermediary to discriminate in prices between sellers and buyers, which is plausible, as different goods are sold to both sides of the market - on seller’s side public information is revealed, while on buyer’s side private information is traded. The intermediary acts as a profit-maximizing monopolist.

The utility of the seller depends on the consumption or the sale of a single product. Depending on the highest bid in the auction, the seller either sells the product or consumes it at the given reservation utility $\alpha q$. Since the seller initially decides whether to produce or sell a product, we assume it to be the first which decides whether to order a rating or not.\(^{12}\)

The buyers bid for the product in a first-price sealed-bid auction, with an a priori unknown reservation price, namely the reservation utility of the seller.\(^{13}\) The first price auction is a natural way to model the selling stage. Initial public offerings in financial markets or sales on stock markets feature a similar structure. We assume that in the first-price auction buyers are aware of the opponent’s information holdings.\(^{14}\)

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\(^9\) The certifier cannot commit to sell exclusively to one buyer.

\(^{10}\) As buyers valuation is identical a common value auction applies.

\(^{11}\) Strausz (2005) motivates the high concentration and earnings in the industry.

\(^{12}\) Simultaneous decisions by buyers and sellers do not alter the general outcomes. The chosen timing reflects the rather realistic situation that the producer of a product initially is able to decide whether selling it in an auction format with certification or without.

\(^{13}\) For further types of common value auctions with asymmetric informed bidders, the academic research is quite silent about picking the “right” equilibrium. We follow therefore the findings by Wilson (1967), Weverbergh (1979), Milgrom and Weber (1982), Rob (1985), Hendricks et al. (1994); Kagel and Levin (1999), Campbell and Levin (2000) and Kim (2008). Second-price common value auctions feature multiple equilibria. Sequential bargaining with a Stackelberg leader yields similar results. For modelling the first price auction we refer to recent findings by e.g. Larson (2009).

\(^{14}\) Relaxing this assumption does not fundamentally alter the solution, but reduces the value of private information in the game and diminishes the profit of the privately informed party.
By applying a market parameter $\alpha$ we partly embed a basic adverse selection framework. The market parameter $\alpha$ determines the difference in valuation $(1-\alpha)q$ between buyers and sellers, which generates the possible gains from trade. Ex ante expected welfare $W_{\text{max}}$ generated by one particular trade yields:

$$W_{\text{max}} = \int_{0}^{1} (1-\alpha)qdq = \frac{1-\alpha}{2}.$$  

As we primarily focus on the market outcomes and the welfare implications, we take an ex-ante viewpoint and study different quality levels of the seller. This is equivalent to a model where each seller of the quality interval $[0, 1]$ faces two buyers once, with $W_{\text{max}}$ being the maximum realizable welfare.

4 Optimal behavior of a monopolistic certifier

The following section contains the results for different selling strategies. After analyzing the market without a certifier, we investigate certification solely on one side of the market, either on the seller side or the buyer side. Thereafter, we examine the optimal strategies of two-sided certification. To end this section we give some intuition complementary to the calculations which clarifies the economic findings of the model.

4.1 The market without the certifier

It is known since Akerlof (1970), that in specific markets trade may collapse due to asymmetrically distributed information. Sellers cannot be differentiated according to their quality level and buyers are only willing to pay a uniform price reflecting the average quality in the market. Facing the relatively low average price, high-quality sellers do not accept the price, and consequently leave the market. This affects the buyers’ beliefs on the average quality offered by the remaining sellers. This dynamic may lead to the collapse of the entire market.

In a market with $\alpha > \frac{1}{2}$ the only equilibrium with rational expectations about quality is

\footnote{For high values of $\alpha$ the problem of asymmetric information becomes exuberant in the setup as markets may collapse.}
the pair of bidding strategies \((0, 0)\) and no product with positive quality being sold. In the remainder of the paper we will refer to this market setting as the 'Lemon Market'.

With a deviating bid of \(b\) a buyer wins the auction if the bid exceeds the reservation utility of the seller. The expected quality of such a product is \(E[q|\alpha q \leq b]\). As \(q\) is uniformly distributed the expected quality is \(q^e = \frac{b}{2\alpha}\). The parameter \(\alpha\) is greater than \(\frac{1}{2}\) and thus \(q^e < b\) holds. Consequently, a deviation does not pay off and the equilibrium bids are unique and the market collapses.\(^{16}\)

Contrarily, in the market with \(\alpha \leq \frac{1}{2}\) the equilibrium bidding strategy for each buyer is to bid his own valuation for a product of unknown quality which is \(q^e = \frac{1}{2}\). In the remainder of the paper we refer to this market setting as the 'Honey Market'.\(^{17}\) Every seller accepts a bid \(b = \frac{1}{2}\) as \(\frac{1}{2} \geq \alpha q\) for all \(q \in [0, 1]\). All products are traded and the maximum welfare is realized. The whole gains from trade are earned by the sellers, as buyers bid in expectations and compete in prices for the product, and ultimately realize no profits. Proposition 1 states the results for both markets without certification.

**Proposition 1**

(a) In the Lemon Market \((\alpha > \frac{1}{2})\) no trades occur without certification.

(b) In the Honey Market all goods are traded for price \(q^e = E\{q\} = \frac{1}{2}\) and the entire welfare of \(W_{\text{max}} = \frac{1-\alpha}{2}\) is exploited without certification.

4.2 One-sided certification

The information asymmetries might be overcome by an intermediary, who credibly provides the quality level of the seller for the buyers. Thereby, it is important to distinguish the different roles of information provision when selling the service to the buyers or the seller. On the one hand, the intermediary might announce the rating result publicly. Therewith the intermediary maximizes the amount of potential buyers for the product. This is desirable for the seller, as public information revelation results in a Bertrand-like pricing competition on buyer-side which increases seller’s profit. On the other hand, the information might be privately owned by one or both potential buyers. The intermediary reveals in this case relevant information solely privately, which allows the informed buyer to use the informal advantage in the selling process. As a result, buyers are willing to pay for the certification service to generate an extra profit. The

\(^{16}\)The applied auction format mirrors exactly the well known asymmetric information dynamics of the Akerlof model (Akerlof, 1970), since the reservation price is unknown.

\(^{17}\)Contrary to lemons, that rot from inside, honey is one of the most durable groceries.
intermediary therefore limits the distribution of information in order to maximize her own profits. The seller faces a limited number of buyers, which potentially shifts the bargaining power partly to the buyers. In our model the intermediary cannot credibly commit to sell the certification solely to one of the buyers, as she has an incentive to deviate in accepting an offer from the second buyer. The following section discusses alternatives and equilibrium outcomes of the model with one-sided certification.

4.2.1 One-sided seller-certification

To study the alternative channels, assume first that an intermediary offers her service exclusively to the seller for a profit maximizing price $p_s$. To solve the model, we determine the perfect bayesian equilibrium. In the last stage of the game the buyers are symmetrically informed: either both are informed about the quality of the product, or both are uninformed and can solely build quality expectations. Each price $p_s$ at which there is demand for certification induces a quality threshold. The threshold emerges as the seller’s profit from certification is increasing in quality in both markets, the Lemon and the Honey Market. Hence, all sellers with a quality above a certain level $\bar{q} \in [0, 1]$ order a rating and make a profit. Figure 1 shows the continuum of quality levels and the interval on which sellers order a rating.

![Figure 1: Quality threshold $\bar{q}$](image)

The seller assures that the product is traded for the price $q$, if it ordered a rating and the information on the quality is publicly announced, since buyers share a common valuation for the product and therefore compete in prices. The critical quality level $\bar{q}$ is determined by the seller, who is indifferent between receiving $\bar{q}$ and paying $p_s$ or either being traded for the expected quality in the Honey Market or consuming its reservation utility in the Lemon Market. All uncertified products exhibit quality in the interval
[0, \bar{q}]. Buyers build beliefs on the quality which we denote by \( q^e \) in the Honey Market, whereas the Lemon Market collapses. Low quality sellers are still able to pool with superior quality sellers up to \( \bar{q} \), since buyers are unable to distinguish between sellers, as the remaining quality is unknown.

If no rating is demanded by the seller, buyers will be uninformed about the true quality of the product and will not bid in the Lemon Market; in the Honey Market they will bid their expected valuation \( q^e \). For this case, the same intuition holds as without certification. If the seller demands a rating, the only equilibrium in the first price sealed-bid common value auction is to bid the own valuation, which is the publicly announced true quality \( q \). The following Lemma 1 illustrates the buyers’ bidding behavior.

**Lemma 1** (a) In the Lemon Market uninformed buyers bid 0 and informed buyers bid their valuation \( q \). (b) In the Honey Market uninformed buyers bid \( q^e \) and informed buyers bid \( q \).

The seller has to value the different options in the specific markets. It might either order a rating for a given price \( p_s \) in order to receive the price for the true valuation, or it faces the outcome for non-rated sellers in the respective market without paying the certification fee and in turn either sells the product for the expected average price in the Honey Market or sustains from selling in the Lemon Market. Therefore, the profit maximizing price of the certifier enables the sellers with the highest quality products to generate an extra rent by ordering a rating. The following Lemma 2 illustrates the induced quality threshold, that depends on the certification price.

**Lemma 2** (a) In the Lemon Market a seller orders a rating for certification price \( p_s \) iff \( q > \bar{q}(p_s) = \frac{p_s}{1-\alpha} \). (b) In the Honey Market the seller orders a rating for certification price \( p_s \) iff \( q > \bar{q}(p_s) = 2p_s \).

The certifier maximizes her revenues \( \Pi_C(p_s) = p_s(1-\bar{q}(p_s)) \) by either selling to few high-quality sellers or by increasing the number of certificates and simultaneously lowering the respective price \( p_s \). Higher certification prices induce higher \( \bar{q} \). A high market parameter \( \alpha \) will c.p. increase the threshold value \( \bar{q} \) in the Lemon Market, since a higher reservation value decreases the potential gains from trade.

Proposition 2 states the optimal pricing strategy of the intermediary and the equilibrium results for the relevant market measures; \( \Pi_S \) denotes the expected profits of the seller,
\( \Pi_C \) denotes the expected profits of the certifier and \( W \) is the realized welfare in the respective market.

**Proposition 2**  
(a) In the Lemon Market with one-sided seller-certification the profit maximizing price for the certifier is \( p_s = \frac{1-\alpha}{2} \). A seller with quality \( q \geq \bar{q} = \frac{1}{2} \) orders a rating. The profit of the certifier is \( \Pi_C = \frac{1-\alpha}{4} \) and the seller’s profit sums up to \( \Pi_S = \frac{1-\alpha}{2} \). Buyers do not make any profits and overall welfare is \( W = \frac{3}{8}(1-\alpha) \).
(b) In the Honey Market with one-sided seller-certification the profit maximizing price for the certifier is \( p_s = \frac{1}{4} \). A seller with quality \( q \geq \bar{q} = \frac{1}{2} \) orders a rating. The profit of the certifier is \( \Pi_C = \frac{1}{8} \) and the seller’s profit sums up to \( \Pi_S = W_{\text{max}} - \frac{1}{8} \). Buyers do not make any profits and the entire possible welfare \( W = W_{\text{max}} \) is realized.

A seller with quality above \( \frac{1}{2} \) orders a rating in both markets, the Lemon and the Honey Market, and pays a price of \( \frac{1}{4} \) in the Honey Market and a smaller price of \( \frac{1-\alpha}{2} \) in the Lemon Market. Compared to the profits in the market without certification, sellers gain in the Lemon Market, since the intermediary enables them to trade their products and increases their rents from zero to \( \frac{1-\alpha}{4} \). In contrast, the overall gains of all sellers in the Honey Market are reduced by \( \frac{1}{8} \), because the intermediary receives parts of their potential gains from trade. As a result, the introduction of an intermediary increases welfare in the Lemon Market and does not affect welfare in the Honey Market.

Remarkably, the certification price and the profit of the certifier do not depend on market parameter \( \alpha \) in the Honey Market. This may seem astonishing at first glance, as potential gains from trade differ significantly between varying market settings characterized by the market parameter \( \alpha \). Due to the fact that the products are even traded without a certifier and thus, the reservation utility of the seller does not enter the equilibrium result\(^\text{18}\) (with all products being traded) this must be the case by construction. Hence, the certifier cannot gain from variations in \( \alpha \) and the profit of the certifier is capped to \( \frac{1}{8} \).

### 4.2.2 One-sided buyer-certification

In contrast to offering of the certification service exclusively to the seller, the intermediary might opt to serve solely the other side of the market, namely the buyers, by

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\(^{18}\)Bidding strategies are always independent of the underlying market parameter \( \alpha \).
selling her rating service for the price $p_b$. The objective for information revelation is fundamentally different: in the case of seller-certification publicly announced ratings are required to differentiate the product from the remainder in the market. In contrast, a buyer can only realize information rents if he exclusively possesses the information. Both buyers decide simultaneously whether to order a rating and build expectations on the likelihood of being the only consumer of the certification service.

In pure strategies, no symmetric equilibrium exists: if both buyers order a rating, they will accrue losses and a deviation will pay off; if both buyers do not order a rating, it will pay off to order a rating as the deviating buyer ends up being exclusively informed. Thus, the only symmetric equilibrium is a mixed-strategy equilibrium in which each buyer decides with a certain probability $\omega$ to order a rating. Since buyers are indifferent whether to order a rating, the expected profit is zero. By assumption buyers are aware of the distribution of the information in the market at the beginning of the first-price auction.

If both buyers are informed, buyers will bid their own valuation $q$, since they enter into a price competition as in the case of seller-certification. This result holds in both markets. For the remaining information structures, results differ between market structures. In the Lemon Market, the unique equilibrium, if both buyers are uninformed, is to bid zero, since the asymmetric information feature prevails as in the case of no certification. If one of the buyers is exclusively informed, the bidding strategy is $b = \alpha q$ and the uninformed bids zero. Thereby, the informed buyer extracts the entire information rent, as the seller is indifferent between accepting and rejecting the offer. The uninformed buyer cannot gain in the auction by making a positive bid, since the expected quality of the product is lower than his bid if he beats the bid of the informed buyer. In the Honey Market the results differ significantly. If both buyers do not order a certificate for the given price $p_b$, the buyers will bid their expected valuation $q^e = \frac{1}{2}$. In the case of only one exclusively informed buyer, his equilibrium bidding strategy is to bid $b = \frac{1}{2} q$. The uninformed mixes on the interval $[0, \frac{1}{2}]$ according to distribution function $F(b) = 2b$ and generates an expected profit of zero. This is the unique equilibrium in a first-price auction with asymmetrically informed bidders as shown by Weverbergh (1979). Lemma 3 states the bidding behavior of the buyers.

**Lemma 3** Buyer’s bidding behavior depending on the information structure and the type of the underlying market is given by the bidding functions in Table 1.
Table 1: Bidding behavior of buyers with seller-certification

<table>
<thead>
<tr>
<th></th>
<th>informed</th>
<th>uninformd</th>
</tr>
</thead>
<tbody>
<tr>
<td>informed</td>
<td>$(q, q)$</td>
<td>$(\alpha q, 0)$ if $\alpha &gt; \frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(\frac{1}{2}q, F(b))$ if $\alpha \leq \frac{1}{2}$</td>
</tr>
<tr>
<td>uninformed</td>
<td>$(0, \alpha q)$ if $\alpha &gt; \frac{1}{2}$</td>
<td>$(0, 0)$ if $\alpha &gt; \frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$(F(b), \frac{1}{2}q)$ if $\alpha \leq \frac{1}{2}$</td>
<td>$(q^<em>, q^</em>)$ if $\alpha \leq \frac{1}{2}$</td>
</tr>
</tbody>
</table>

The distribution function of bids for a single uninformed buyer is $F(b) = 2b$.

Lemma 3 shows that the advantage of being exclusively informed differs fundamentally between the Lemon and the Honey Market: In the Lemon Market an informational advantage leads to winning the auction with probability 1 and thereby realizing the entire gains from trade $V^L_{ib}$. In contrast, being exclusively informed in the Honey Market leads to some positive expected payoff $V^H_{ib}$ in the upcoming auction with a lower probability, as shown in Lemma 4.

**Lemma 4**  
(a) In the Lemon Market the expected payoff of a single informed bidder is $V^L_{ib} = \frac{1-\alpha}{2}$.  
(b) In the Honey Market the expected payoff of an exclusively informed bidder is $V^H_{ib} = \frac{1}{6}$.

Lemma 4 shows that the expected payoff is always positive and buyers therefore always favor the alternative of being exclusively informed. The jump at the border of $\alpha = \frac{1}{2}$ in the two markets reflects the substantially diverging equilibria of the auction formats and the attached potential gains. In general, a buyer follows a strategy to obtain an information advantage to maximize the expected profit. As the buyers randomize over the decision to order a rating using symmetric mixed strategies, their expected overall profit is zero; the buyers gamble for profits.

As Lemma 4 exhibits the expected payoffs net of the price $p_b$ for the private rating, each price $p_b$ for the certification service induces a different probability of ordering a rating. As buyers play a mixed strategy in the information acquisition game, the probability $\omega$ is determined by the indifference condition of receiving the corresponding payoffs in the respective markets $V_{ib}$ with probability $1 - \omega$ at the price $p_b$ or having an expected payoff of zero. The induced rating probabilities are shown in Lemma 5.

**Lemma 5**  
(a) In the Lemon Market a buyer orders a rating at a given price $p_b$ with a
probability $\omega(p_b) = \max\{0, 1 - \frac{2p_b}{1-\alpha}\}$. (b) In the Honey Market a buyer orders a rating at a given price $p_b$ with a probability $\omega(p_b) = \max\{0, 1 - 6p_b\}$.

Lemma 5 shows that higher rating prices induce lower probabilities $\omega$ to order a rating by buyers. The certifier maximizes her profits $\Pi_C(p_b) = (\omega(p_b))^2 2p_b + 2\omega(p_b)(1 - \omega(p_b))p_b$ by either attracting few buyers with a high price and low rating demand or decreasing the price to increase the likelihood $\omega$ that a buyer demands a rating. From the perspective of the intermediary the most profitable case is to sell her service to both investors, since she can extract a double dividend $2p_b$, as both buyers might pay the price $p_b$ for the certificate.

Intuitively, with a price higher than $V_{ib}^L$ respectively $V_{ib}^H$ the demand diminishes to zero, as the expected payoff of being exclusively informed is lower than the rating price $p_b$. The intermediary will therefore choose a price which is lower. Proposition 3 exhibits the perfect bayesian equilibrium of the game.

**Proposition 3** (a) In the Lemon Market with one-sided buyer-certification the profit maximizing price for the certifier is $p_b = \frac{1-\alpha}{4}$. The probability that a buyer orders a rating is $\omega = \frac{1}{2}$. The profit for the certifier is $\Pi_C = \frac{1-\alpha}{4}$ and the seller’s profit is $\Pi_S = \frac{1-\alpha}{8}$. Buyers do not make any profits and overall welfare is $W = \frac{3}{8}(1 - \alpha)$.

(b) In the Honey Market with one-sided buyer-certification the profit maximizing price for the certifier is $p_b = \frac{1}{12}$. The probability that a buyer orders a rating is $\omega = \frac{1}{2}$. The profit for the certifier is $\Pi_C = \frac{1}{12}$ and the seller’s profit is $\Pi_S = W_{max} - \frac{1}{12}$. Buyers do not make any profits and the entire possible welfare $W = W_{max}$ is realized.

In expectation a buyer will not make any profits, even though he generates profits out of an information advantage. The generated rent diminishes in equilibrium to zero, since buyers accrue losses when both order a rating, which offsets the gains of exclusive information. The likelihood of ordering a rating by the buyers is substantial. They seek to maximize their profits by bidding informed in half of the cases. As a result, one exclusively informed bidder evolves in 50 percent of the cases, while respectively in 25 percent investors are either equally informed or uninformed.

Similarly, to the intuition in the case of one-sided seller-certification, the rating price and the profit of the certifier do not depend on the market structure in the Honey Market. The certifier cannot exploit the additional gains from trade in a market with
a low market parameter $\alpha$. The bidding behavior is independent from the reservation price of the seller in the auction, as the market also clears without a certification service. The jump in the sellers profit function at $\alpha = \frac{1}{2}$ finally reflects the strict distinction in the participation behavior of uninformed buyers in the auction of the product. This is mainly due to the fact that uninformed buyers in the Honey Market are also willing to buy the product, while they refrain from bidding in the Lemon Market.

### 4.3 Comparison of one-sided certification

Comparing the results of the two types of one-sided certification reveals the differences between the information provisions by the certifier. Firstly, the evaluated products in both models differ. With seller-certification, the best half of the products is traded and with buyer-certification it is a random draw from all products that are evaluated and thereafter sold, since the products cannot be differentiated ex-ante. The traded products differ in both market settings, the Lemon and the Honey Market.

In every market, demand for certification service exists and the intermediary realizes profits by offering the information revelation service. Even in the market where trades occur without a certification service, the players demand a rating to maximize their profits and overall loose parts of their profits in total. The profit of the intermediary is higher in the Honey Market than in the Lemon Market. Even though the intermediary enables trades in the Lemon Market and therefore contributes to welfare generation, the overall rents are higher in the Honey Market and in turn the intermediary also extracts a higher absolute value of the rents at stake. The profit shares are smaller in the Honey Market, since the information asymmetries are overcome by the market itself even without an intermediary and thus the market power of the intermediary is smaller.

In the Honey Market, the certifier prefers to sell the service to the seller-side, since the information value for privately informed buyers is too low, and thus the certifier generates lower revenues. The uninformed buyer bids randomly in the Honey Market, which reduces the information advantage, whereas he withstands bidding in the Lemon Market. In a Lemon Market the certifier is indifferent on which side to offer her service. The intuition for this result is that in both cases the certifier sells the whole bargaining power; in the case of public information the seller can sell his product at the maximum price of $q$ and in the case of a private information advantage the respective buyer can
buy the product for the minimum price of $\alpha q$. The sellers’ profits are equal in the Lemon Market under both regimes, but are lower in the Honey Market with seller-sided certification, since the preferences of certifier and seller are reversed in the discussed cases.

All potential gains from trade are realized in the Honey Market, but the intermediary increases exploited welfare in the Lemon Market to 75 percent of potential welfare.

Figure 2: Profit shares with one-sided seller-certification

Figure 2 shows the realized potential welfare for all market parameters as well as for the different players. Remarkably, not all potential rents are realized in the Lemon Market. 25 percent are lost even with the presence of an intermediary.20

Corollary 6 states the main results of the previous section on one-sided certification.

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19 The certifier is able to correct for potential losses on buyer-side through double-certification by a lower certification price.

20 Note that the potential welfare varies significantly with the market parameter, as $W_{\text{max}} = \frac{1 - \alpha}{2}$. 
Corollary 6 (a) In the Lemon Market buyers, seller and the certifier are indifferent between one-sided buyer- and seller-certification. The welfare gains are positive and equal under both regimes compared to no gains from trade without certification.

(b) In the Honey Market the certifier prefers to offer her service to the seller side, while the seller prefers (ex ante) the certifier to operate on the buyer side. Welfare is not affected by certification.

4.4 Two-sided certification

The model of two-sided certification combines the previous models of one-sided certification. The certifier maximizes profits by selling the certification service either to the seller or to the buyers. She can discriminate in prices by offering public and private ratings. The seller either orders a rating directly and therewith differentiates its quality directly from the remaining sellers in the market and induces buyer-sided competition in prices, or remains unrated. Buyers seek to be exclusively informed by ordering private ratings to gain some informational advantage. We show that the certifier profits from the fact, that she can sequentially segment the market by discriminatory pricing for public and private information disclosure.

To start with the analysis of the game described one sees that the structure of the equilibrium outcome of the game is as follows. In equilibrium the continuum of sellers is divided into two segments: one containing the high quality sellers, \( q \in [\bar{q}_{II}, 1] \), where sellers order a rating and are traded for the price of the true quality \( q \). A second segment contains the lower-quality sellers \( q \in [0, \bar{q}_{II}] \), which do not order a rating. In this interval the higher quality part of the sellers speculate that both buyers order a rating, or trades occur without a certificate. The decision of the seller depends on the quality of his own product and on the (endogenous) prices of the certification service \( p_s \) and \( p_b \) set by the intermediary. \( ^{21} \)

Figure 3 illustrates the segmentation of the sellers with different quality levels. A shift of the quality threshold level between one-sided seller-certification (\( \bar{q} \)) and two-sided certification (\( \bar{q}_{II} \)) evolves.

In the following we solve the game for a Perfect Bayesian Equilibrium. The optimal

\(^{21}\)This basic intuition for the equilibrium structure does not exclude corner solutions, i.e. \( \omega = 0 \) or \( q_{II} = 1 \).
bidding behavior depends on the quality threshold $\bar{q}_{II}$, which is known in equilibrium, and the information structure of the buyers. If both buyers are informed about the quality $q$ the unique bidding equilibrium is $(q,q)$, which holds in both markets, and exhibits the price competition of buyers for the product. With one exclusively informed buyer, the market structure impacts the equilibrium outcomes: In the Lemon Market the informed buyer bids $\alpha q$ and the uninformed one does not bid at all. In the Honey Market with only one exclusively informed buyer, his equilibrium bidding strategy is to bid $b = \frac{1}{2} q$ and the uninformed mixes on the interval $[0,\frac{1}{2} \bar{q}_{II}]$ according to the distribution function $F_{\bar{q}_{II}}(b) = \frac{2}{\bar{q}_{II}} b$ and generates an expected profit of zero. With two uninformed buyers the market collapses in the Lemon Market. In the Honey Market the buyers bid the expected quality of an uncertified product which we denote by $q^e$. Buyers thereby enter a price competition and realize no profit.

Compared to the bidding behavior in the one-sided buyer-certification model the only differences emerge from (1) the equivalence of one seller-sided rating and two buyer-sided ratings and (2) the threshold $\bar{q}_{II}$ which determines the potential quality levels of not publicly certified products. The possible bidding equilibria for the different information structures of both buyers are illustrated in Lemma 7.

Table 2: Bidding behavior for the case of two-sided certification

<table>
<thead>
<tr>
<th>informed</th>
<th>uninformd</th>
<th>uninformed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{q}$</td>
<td>$\bar{q}_{II}$</td>
<td>$\bar{q}$</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>no rating</td>
<td>rating</td>
<td></td>
</tr>
</tbody>
</table>

The distribution function of bids for a single uniformed buyer is $F_{\bar{q}_{II}}(b) = \frac{2}{\bar{q}_{II}} b$. 

The distribution function of bids for a single uniformed buyer is $F_{\bar{q}_{II}}(b) = \frac{2}{\bar{q}_{II}} b$. 

18
Lemma 7  
Buyer’s bidding behavior depending on the information structure, the quality 
threshold and the type of the underlying market is given by the bidding functions in 
Table 2.

Lemma 7 shows, similar to the case of one-sided buyer-certification, that the advantage 
of being exclusively informed differs fundamentally between the Lemon Market and the 
Honey Market.

In the Lemon Market an informational advantage leads to winning the auction with 
probability 1 and thereby realizing the entire gains from trade $V_{ib}^L$, whereas being ex-
clusively informed in the Honey Market only leads to some positive expected payoff $V_{ib}^H$ 
in the upcoming auction with a certain probability smaller than 1 as the uninformed 
bidder still bids in the auction.

Lemma 8  (a) In the Lemon Market the expected payoff of a single informed bidder 
is $V_{ib}^L(p_s, p_b) = (1 - \alpha)\frac{\bar{q}_{II}(p_s, p_b)}{2}$.  (b) In the Honey Market the expected payoff of an 
exclusively informed bidder is $V_{ib}^H(p_s, p_b) = \frac{1}{6}\bar{q}_{II}(p_b, p_s)$.

Lemma 8 illustrates the expected payoffs of a buyer given he is exclusively informed. 
The values reflect the information value for the buyer. In the Honey Market, the 
information value does not depend on the market parameter $\alpha$, indicating, that the 
value of information is capped, as without a certification service trades would also 
 occur. As in the case of one-sided buyer-certification a jump in the payoff function $V_{ib}$ 
at $\alpha = \frac{1}{2}$ exists, indicating the different probabilities of winning the auction in the two 
markets with one informed buyer.

Depending on the certification price $p_b$ and the induced threshold $\bar{q}_{II}$ the buyers them-
 selves choose the equilibrium rate of ordering private information on a product’s quality. 
Obviously, they never order a rating if the seller already publicly revealed the information. 
As the equilibrium is again in mixed strategies the buyers have to be indifferent 
between ordering a rating by paying $p_b$ and receiving the corresponding $V_{ib}$ with a cer-
tain probability or having an expected payoff of zero. Lemma 9 states the individual 
rating probabilities in equilibrium.

Lemma 9  (a) In the Lemon Market a buyer orders a rating at given prices $(p_s, p_b)$ with 
a probability $\omega(p_s, p_b) = \max\{0, 1 - \frac{2p_b}{(1-\alpha)\bar{q}_{II}(p_s, p_b)}\}$.  (b) In the Honey Market a buyer 
orders a rating at given prices $(p_s, p_b)$ with a probability $\omega(p_s, p_b) = \max\{0, 1 - \frac{6p_b}{\bar{q}_{II}(p_s, p_b)}\}$.
The probability of ordering a rating by the buyers diminishes in \( p_b \), which is intuitive, as the certification service is a normal good. With increasing threshold values \( \bar{q}_{II} \), the probability increases, as the potential gains at stake increase. Comparing Lemma 8 and Lemma 9 states that \( \omega \) decreases to zero as the price of the certification service \( p_b \) approaches the corresponding \( V_{ib} \).

At the second stage of the game the sellers decide whether they require the certifier to publicly reveal the quality of their product \( q \) or to stay pooled with other uncertified products.\(^{22}\) The intuition for the consideration of the seller with a product of a given quality is as follows. A high-quality seller in both markets tries to publicly disclose its true quality to both buyers by ordering a rating to avoid pooling with the uncertified remainder of the market. A mid-quality seller hopes to be rated by both buyers to avoid pooling with low-quality sellers, which leads to the same information structure as if the product’s quality is disclosed publicly, while the certification costs \( p_s \) are shifted towards the buyers. Depending on the market structure a low-quality seller does not hope to be rated by buyers in the Honey Market, as it then realizes the expected price of the remaining pooled products, whereas it favours to be rated in markets where its product is otherwise not traded (Lemon Market). Hence, the quality threshold \( \bar{q}_{II} \) not only depends on the price for seller certification, but also on the price for buyer certification, as the seller might expect to be rated by the buyers. The threshold value is determined by the seller who is indifferent between requesting a rating and revealing its quality or refraining from ordering. In the latter case it either hopes to be rated by at least one buyer, as the indifferent seller is pooled with products of lower quality. Lemma 10 states the induced quality thresholds depending on the certification prices set by the intermediary.

**Lemma 10** (a) In the Lemon Market a seller orders a rating for certification prices \((p_s, p_b)\) iff \( q > \bar{q}_{II}(p_s, p_b) = \frac{4p_s^2}{(4p_b - p_s)(1 - \alpha)} \). (b) In the Honey Market a seller orders a rating for certification prices \((p_s, p_b)\) iff \( q > \bar{q}_{II}(p_s, p_b) = \frac{18p_b^2}{6p_b - p_s} \).

The decision of the seller as well as the decision of the buyers to order a rating depend on the rating price set by the certifier. With increasing certification prices, the amount

\(^{22}\) The timing of our setup does not influence the results. Simultaneous decisions of the seller and the buyers whether to order a rating lead to the same equilibrium profits and welfare. In this setting, high-quality sellers will also opt to order a rating and thus a quality threshold \( \bar{q}_{II} \) evolves, which equals the threshold in the standard setting, since no information rent can be extracted from a product with a public rating.
of ratings for the respective side decreases. Hence, the certifier sets revenue-maximizing prices for her service, that allow her to skim the rents in the market. She faces a trade-off by increasing the price of seller-certification $p_s$, which leads to a loss of demand by the seller, while the remaining market becomes more attractive for the buyers, as some higher quality products allow for higher potential information rents, and therewith c.p. the revenues from buyers increase. Accordingly, the certifier maximizes her profit by inducing the optimal combination of a threshold $\bar{q}_{II}$ and a buyer-sided certification in the remaining market. Proposition 4 captures the optimal pricing strategy for the certifier and the equilibrium outcomes of the model of two-sided certification.

**Proposition 4**

(a) In the Lemon Market with two-sided certification the profit maximizing price for seller-certification is $p_s = \frac{16}{27}(1 - \alpha)$ and for buyer-certification is $p_b = \frac{2}{9}(1 - \alpha)$. The probability that a buyer orders a rating is $\omega = \frac{1}{3}$ and the quality threshold value is $\bar{q}_{II} = \frac{2}{3}$. The profit for the certifier is $\Pi_C = \frac{8}{27}(1 - \alpha)$ and the seller’s profit is $\Pi_S = (1 - \alpha)\frac{17}{102}$. Buyers do not make any profits and overall welfare is $W = (1 - \alpha)\frac{65}{162} \neq W_{\text{max}}$.

(b) In the Honey Market with two-sided certification the profit maximizing price for seller-certification is $p_s = \frac{3(3 - \sqrt{5})}{2(2 + \sqrt{5})}$ and for buyer-certification is $p_b = \frac{\sqrt{5} - 1}{4(2 + \sqrt{5})}$. The probability that a buyer orders a rating is $\omega = \frac{1}{2 + \sqrt{5}}$ and the quality threshold value is $\bar{q}_{II} = \frac{3}{4}(3 - \sqrt{5})$. The profit for the certifier is $\Pi_C = \frac{9 - 3\sqrt{5}}{8 + 4\sqrt{5}}$ and the seller’s profit is $\Pi_S = \frac{1 - \alpha}{2} - \Pi_C$. Buyers do not make any profits and the entire possible welfare $W = W_{\text{max}}$ is realized.

In equilibrium, one third of the sellers order a rating compared to one half in the case of one-sided certification. In either case, the best part of the sellers order a rating and a threshold value $\bar{q}_{II}$ and $\bar{q}$, respectively, evolve. Interestingly, sellers with quality $q \in [\frac{1}{2}, \frac{2}{3}]$ choose to order a rating if there is solely one-sided seller-certification, but refrain from ordering, if the option of being subsequently rated by the buyers exist.\(^{23}\)

Two main reasons for the findings prevail. On the one hand, the intermediary slightly increases the seller price of the rating with two-sided certification and thereby reduces the demand and on the other hand, the seller gambles to be rated by both buyers. Therewith, it avoids paying the certification price and increases its own profits. In

\(^{23}\)Given the optimal seller-certification price $p_s$ of two-sided certification in the Lemon Market, and assuming no buyer certification the quality threshold is $\bar{q} = \frac{16}{27} < \frac{2}{3} = \bar{q}_{II}$. In the Honey Market, the threshold increases from $\bar{q} = \frac{3(3 - \sqrt{5})}{2 + \sqrt{5}} < \frac{3}{4}(3 - \sqrt{5}) = \bar{q}_{II}$. The shift is shown in Figure 3.
total, however, the expected profit of the seller is lower with two-sided certification than in the other models, while the profit of the certifier increases. Buyers do not make any profits in equilibrium. The profit variations hold in both markets.

The rating probability of the buyers decreases with two-sided certification compared to one-sided certification, since the available information rents are smaller, because high-quality sellers already left the market by publicly revealing their quality. Furthermore, Proposition 4 shows that the prices for buyers with two-sided certification remained fairly stable in both markets compared to the model of one-sided buyer-certification. Hence, buyers adjust their behavior by lowering the rating probability.

The effects on the overall welfare depend on the market structure. In the Lemon Market a certification service increases welfare substantially. It rises from $\frac{7}{8}(1 - \alpha)$ to $\frac{65}{162}(1 - \alpha)$, as the number of ratings increases and therewith also the number of trades in the market. In contrast, welfare is not affected in the Honey Market, as even without a certifier, no inefficiencies occur. The market is always cleared. The welfare gains are even higher in the case of two-sided certification than with one-sided certification. As a result one might conclude, that two-sided certification should be promoted to allow for an efficient allocation of resources, if the value of information asymmetries is high.

The general intuition for the result is as follows. The certifier’s strategic decision to sell on both sides of the market has two effects in opposing directions. On the one hand he becomes his own competitor which weakens his position relative to operating only on one specific side of the market. On the other hand she is able to exploit some rents on both sides by taking advantage of the special conditions on each side. Seller- and buyer-side differ fundamentally in the sense that in a seller market it will be the high-quality sellers who order a rating and in a buyer market rated products are randomly selected by speculative buyers. Hence, profits are generated in the high segment in the former market and are generated randomly in the latter. By combining these two market features in a profit-maximizing way the certifier has to determine the optimal threshold of sellers self-selecting to order public ratings. From the perspective of one-sided seller-rating by making the publicly rated market smaller the market for buyer-ratings becomes more attractive. The positive effect of having a bigger market for unrated products outweighs the negative effect of becoming her own competitor. Put from the perspective of one-sided buyer-certification by introducing seller-certification it becomes less attractive for buyers to order a rating as the high-quality segment already separates from the poor-quality products at an earlier stage. This negative effect is
outweighed by the rents the certifier can exploit on the high-quality sellers which are willing to pay a relatively high price for this separation.

5 Application to the Rating Market

This section links the theoretical results with empirical observations in the rating industry and discusses the findings. The increasing complexity of financial markets in the last decades caused a massive increase in the reliance on credit ratings by investors, issuers and regulatory bodies. Issuers, such as firms or sovereign entities share mainly two incentives to demand ratings: they expect to receive a lower premium on their financial instruments and to face a broader investment pool and therewith reduce the liquidity premia in the market. Institutional investors, such as e.g. insurers, reinsurers and pension funds require ratings of financial products before the assets can enter into their portfolios. Many of the investors follow long-term strategies and apply portfolio governance rules, consisting of buy and sell restrictions linked to rating changes, to manage their portfolios (Löffler, 2004). Therefore, retaining a strong investment rating in some or even all of their asset classes is essential. Private investors also rely on publicly available ratings to optimize their portfolios and reduce information acquisition costs.

The market for rating agencies is highly concentrated and is estimated to generate revenues of $4.5 billions per year. The two biggest rating agencies, Moody’s and Standard & Poor’s, share 80 percent of the market and together with the number three, Fitch Ratings, the market share becomes 95 percent. The operating margins of the leading rating agencies are close to 50 percent and relatively stable over the last years, even in the current turmoil of financial markets.

Several arguments for the high concentration and the high profit margins in the rating industry were stressed in the recent debate:

1. a rigorous accreditation procedure by the national regulators,

2. perpetuation of honest ratings through reputation and the high costs of deviating from reliable ratings,

\[\text{Natural and synthetic entry barriers might be the reason.}\]
3. portfolio rules that directly link the investment decision to ratings by specific rating agencies,

4. and the reliance on third party ratings within various regulatory processes.

A rigorous accreditation procedure of rating agencies by the Securities and Exchange Commission in the US can only partly explain the highly impeding competition in the market, since currently 10 Nationally Recognized Statistical Rating Organizations exist. In addition, in other regions in the world a high concentration is also observable, e.g. in Japan two players share most of the market, namely the Japan Credit Rating Agency as well as the Rating and Investment Information Inc..

Strausz (2005) underlines the importance of high profits to avoid bribing in the industry. A rating agency compares the discounted cash-flow of honest certification with a deviation strategy that includes profits from bribing. With decreasing profits from honest certification, the likelihood of incorrect ratings rises, which is welfare decreasing.

Various institutional investors limit their management in their portfolio choice. They rely on ratings to limit the risk exposure and the potential losses by specific financial products. Often they require investments to be rated above a minimum threshold value. Additionally they require multiple ratings, to avoid rating shopping by firms (Skreta and Veldkamp, 2009). As a consequence, contract clauses manifest the position of incumbents, deter entries in the rating market and establish major entry barriers.

National regulators also heavily rely on credit ratings of the major rating agencies and often request not only one rating, but up to three ratings in the regulatory process (Benmelech and Dlugosz, 2009). According to the Basel II accords, minimum capital requirements for banks are computed using different weights for specific rating groups (BIS, 2004) in order to assess the risk exposure of bank portfolios. Furthermore, the collateral which is required to obtain central bank liquidity has to meet minimum rating requirements. As a consequence, we model the certification intermediary to be a monopolistic supplier, that is free to set profit maximizing prices for the certification service.

Furthermore, the certifier in our model possesses a perfect evaluation technology. Following the tremendous shock after the default of Lehman Brothers Inc. in September 2008, rating agencies were blamed for their inaccurate ratings. Various rating changes are observable since that day. However, the market evaluated the risks similar to rat-
ing agencies, and thus they can hardly be blamed ex post for modeling the financial interlinkages and potential contagion or spillover effects inadequately.

The business model with respect to the sales model of certification services changed significantly over time. Before 1970, ratings were primarily sold to investors, who subscribed to attain certification information, which were thereafter private information of subscribers. We investigate this sales scheme in Section 4.2.2. After 1970, the rating agencies decided to additionally sell their services to the other side of the market, to firms or issuers, which we investigate in Section 4.4. After the firm receives a rating, the information is immediately public and can be observed by all market participants. This sales scheme, offering certification services and at the same time consultancy, raised the question of potential conflicts of interest. Firms, especially banks, might succumb bribing in an issuer-pay model. The failure of rating agencies in the current financial crisis is said to be a consequence of the intertwined relationships.\(^{25}\) This argument is insufficient for the explanation of recent rating failures, since long-term rating evaluations concluded that they are rather accurate (Reinhart et al., 2002) and in the current setting not susceptible to bribing.\(^{26}\)

Table 3 illustrates exemplarily Moody’s revenue shares generated by selling rating services to investors and issuers, respectively. The pattern shows a relatively high revenue share of the issuer-pay model, which is decreasing with the extent of the worldwide financial crisis. The efficiency of markets reduced tremendously during the financial turmoil and at the same time, issuer-generated revenues declined, while investor-generated revenues hiked (Table 3). Some markets totally broke down and no trades were completed.\(^{27}\) Various financial institutions had to adjust their market-to-market book values, which lead to a downward spiral and even higher downward pressure on prices. Thereby two reasons prevailed: on the one hand, the degree of asymmetric information in some markets increased, and on the other hand, the expected risks of products perceived by sellers and buyers increased.

In our model the seller contributes two-third of the certifiers’ revenues in the lemon market, while the buyers contribute the remaining one third. In the Honey Market the seller contributes 86 percent, and the buyers only 14 percent. Hence, the profit shares

\(^{25}\) Review e.g. Sy (2009) for a detailed argumentation.

\(^{26}\) Reinhart et al. (2002) compare the historical performance of ratings, as the deviation of estimated from the realized default probability.

\(^{27}\) E.g. Interbanking markets, CDO markets and various other markets.
of the issuer are lower in the lemon market. As a result, our findings of the model are in line with recent observations of the financing of rating agencies (Table 3).

Table 3: Moody’s yearly revenues and revenue shares depending on sales scheme

<table>
<thead>
<tr>
<th></th>
<th>2008</th>
<th>2007</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moody’s Analytics (mainly investor-pay model)</td>
<td>550.7</td>
<td>479.1</td>
<td>397.3</td>
</tr>
<tr>
<td>Moody’s Investors Service (mainly issuer-pay model)</td>
<td>1,268.3</td>
<td>1,835.4</td>
<td>1,685.6</td>
</tr>
<tr>
<td>Total revenues</td>
<td>1,755.4</td>
<td>2,259.0</td>
<td>2,037</td>
</tr>
<tr>
<td>Issuer-pay revenue share</td>
<td>72.3%</td>
<td>81.2%</td>
<td>82.7%</td>
</tr>
</tbody>
</table>

Source: (Moody’s, 2008, p.94).
Notes: Consolidated revenues of business segments in the respective years in millions US$.

Further market patterns observed in the current financial crisis are also depicted in our model. Increasing volatility in the markets and higher risks for the valuation of the product by both parties, are reflected by higher values of $\alpha$ for a given product market. Thus the outcomes for particular product markets shift towards the lemon market, which increases the impact of a rating agency on welfare generation. Especially in times of dried up markets, as observed during the current financial crisis, rating agencies might contribute to the reestablishment of functioning markets. Besides the increase of asymmetric information and the lack of trust in the markets, a revaluation of products and an adjustment of perception of risks between investors and issuers occurred and in turn some markets broke down.

Contrary to Lizzeri’s no revelation result (Lizzeri, 1999), we show that strong incentives exist for a monopolistic rating agency to issue information to both sides of the market. This result emerges as the rating agency can sell the same product to two parties with different objectives and in turn is able to discriminate in prices. The result objects to the argument, that the rating agencies mainly changed their business model towards the issuer-pay model to succumb bribing by firms or other rated entities. The main objective is profit maximization.

In our model without a certification service two market outcomes arise: in one market the costs of asymmetric information does not hinder investors and issuers from exchanging their products. The reservation utility of the best seller is lower than the expected quality of all sellers by the buyers and consequently, all products are traded in the market; a 'Honey Market' arises.

\[ \text{A reduction of the value of the product to } (1-x)q \text{ for the buyer and } (1-x)\alpha q \text{ for the seller is equivalent to a market with a market parameter } \alpha > \alpha. \]
In a Lemon Market a financial intermediary can partly overcome the asymmetric information problem, as a high proportion of potential trades is realized. The intermediary receives a high fraction of the rents generated by the market. In the Honey Market, the total welfare is not affected by the introduction of the intermediary, as the market mechanism already generates the maximum welfare.

Our results demonstrate that a profit-maximizing certifier prefers to operate on the seller’s side in a Honey Market, if she has to decide to offer the services merely to one side of the market, while she is indifferent in a Lemon Market. Figure 4 depicts the shares of all parties involved in the market if the certification service is offered to the seller’s side and to both sides of the market. In the Lemon Market not the entire welfare can be realized through certification, but a substantial proportion of 75 percent. In both markets, the certifier extracts a high amount of the potential rents, which rise up to 50 percent of potential welfare in the lemon market. Firms gain in the lemon market by hiring the intermediary, as they extract 25 percent of potential welfare, which could not be realized in an alternative way. In a Honey Market the intermediary does not increase welfare and the seller will be unwilling to share the rents with the intermediary in the market (ex-ante).

If the intermediary decides to merely sell to the investor’s side, its revenues shrink by one third. It is important to notice, that the traded products differ between both sales schemes: if the seller orders a rating, the best half of the firms will demand a certificate, whereas if buyers order ratings, they cannot differentiate between good and bad firms and therefore will select randomly.

Comparing the outcomes of one-sided certification with the model in which the intermediary sells its services firstly to the sellers and, if they reject the offer, secondly to the buyers, the welfare in the lemon market increases even further. With two-sided certification, about 70 percent of all products are traded in equilibrium, including the third containing the highest quality. The welfare loss is down to about 20 percent compared to 100 percent in the case without certification.

Figure 4 shows the slight increase of the intermediaries’ share on welfare in the Honey Market. Compared to the 50% jump in profits from offering ratings to the firm’s side instead of operating on investor’s side solely, the increase in profits of the intermediary

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29Relaxing the assumption of a sequential game and allowing sellers and buyers to demand a rating simultaneously does not alter the solutions.
by offering the certification service on both sides in a Honey Market is only about 8%.

The profit for the intermediary is highest in the market with two-sided certification, which is rather astonishing, since the certifier might crowd out demand by sellers in the primary market through the introduction of the evaluation service on the buyer side. By offering the certification service on both sides of the market, the intermediary faces a negative second-order effect from sellers hoping to be rated by two buyers, which reduces the revenues generated on the seller side for any given price. At the same time, the average quality of non-rated sellers increases, which increases the attractiveness of being exclusively informed for the buyers, and in turn demand for investor ratings at any given price hikes. The model shows that the introduction of two-sided certification seems to outweigh the negative effect of being her own competitor. Our model further shows that a rating agency is likely to enter every asymmetric information market. Comparing the profits between a Lemon and a Honey Market in most cases she even
prefers the Honey Market, even though the volume of trades is not affected.\textsuperscript{30} The potential gains from trade overcompensate the trade enhancing role of the rating agency in the Lemon Market. Appendix A.1 gives a summary report on the equilibrium values of the main variables in the model.

6 Conclusion

This paper analyzes incentives of certifiers regarding their selling behavior and associated market outcomes. In an asymmetric information framework three options are considered: offering certification services merely to buyers or to sellers and the sale on both sides of the market. Buyers and sellers have diverging interests in ordering certification services. Sellers intend to induce a price competition for their good by ordering ratings that become public information for buyers and therewith they increase their profits. Buyers instead gain from certification since they can earn an extra information rent and are able to apply more sophisticated bidding strategies if they are privately informed.

Two opposing effects prevail for the intermediary if she decides to offer the certification service on both sides of the market, which dilute incentives for buyers and sellers. Firstly, the certifier enters into price competition with himself, since sellers might refrain from ordering a rating, as they hope to be rated by more than one buyer. Buyers suffer from price competition for high-quality products, which is initiated by sellers ordering a rating directly. Secondly, the certifier combines the different market features by inducing a high-quality segment and a low-quality segment. This is due to the fact that high-quality sellers seek to be separated from low-quality sellers and therefore demand certification services at a given price. In the remaining low-quality segment buyers try to gain an informational advantage by randomly buying private ratings. Depending on the decision of the intermediary on which side to offer the service, the market outcomes and welfare generation vary.

The model shows that an intermediary, who offers her services solely to one side of the market, enables trades in a market in the sense of Akerlof (1970) and thereby increases welfare. Depending on the sales scheme, either selling merely to the buyer or merely

\footnote{A small interval $\alpha \in [0.5; 0.54]$ exists in which the rating agency has higher profits in the Lemon than in the Honey Market.}
to the seller, the profit shares of the parties vary. The certifier maximizes its profit by selling to the seller side in a market with high potential gains from trade and is indifferent in a Akerlof-type market. Furthermore, we show that the profit maximizing strategy for the rating agencies is to sell to both sides of the market. This holds independently of the size of potential gains from trade; a business strategy observed in the rating market since the 1970s. The welfare in markets with two-sided certification increases, as more and also adequately priced products are traded. As the valuation of buyers and sellers converge, the intermediary tends to generate comparatively higher revenues from the buyers side. This revenue shift is apparent in balance sheet data as a consequence of the 2008-2009 financial crisis.

From a policy perspective, it is not necessarily the case that observing intermediaries being paid by the issuers indicates a cooperation of the two parties or even beautifying the default probability. In a functioning market we expect intermediaries to have a strong tendency to offer their services to both sides of the market, with a preference for the seller side. As a result one might argue, that the presence of intermediaries in inefficient markets, as the Lemon Market in our model, should be strengthened, as they are able to solve the inefficiencies due to asymmetrically distributed information. Therewith welfare increase substantially. In a Honey Market with less asymmetric information, the intermediary is not required for the occurrence of trades, but the prices of traded goods vary. With respect to an efficient allocation of resources the true valuation of goods, which is revealed by rating agencies, is substantial and mirrors a reasoning for a rating service.
## Appendix

### A.1 Summary results

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<th></th>
<th>Only sellers</th>
<th>Only buyers</th>
<th>Both sides</th>
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<td>price for seller rating</td>
<td>$\frac{1-\alpha}{2}$</td>
<td>-</td>
<td>$\frac{16}{27} (1-\alpha)$</td>
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<td>$\frac{1-\alpha}{4}$</td>
<td>$\frac{8}{27} (1-\alpha)$</td>
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<td>profit seller</td>
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<td>$\frac{1-\alpha}{8}$</td>
<td>$\frac{17}{162} (1-\alpha)$</td>
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<tr>
<td>welfare</td>
<td>$\frac{3}{8} (1-\alpha)$</td>
<td>$\frac{3}{8} (1-\alpha)$</td>
<td>$\frac{65}{162} (1-\alpha)$</td>
</tr>
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</table>

| $\alpha < \frac{1}{2}$ (Honey Market) |             |             |            |
| price for seller rating | $\frac{1}{4}$ | -           | $\frac{3(3-\sqrt{5})}{2(2+\sqrt{5})}$ |
| price for buyer rating  | -            | $\frac{1}{12}$ | $\frac{\sqrt{5}-1}{4(2+\sqrt{5})}$ |
| high-quality threshold  | $\frac{1-\alpha}{2}$ | -           | $\frac{3}{4} (3 - \sqrt{5})$ |
| buyer’s rating probability | -           | $\frac{1}{2}$ | $\frac{1}{2+\sqrt{5}}$ |
| profit certifier       | $\frac{1}{8}$ | $\frac{1}{12}$ | $\frac{9-3\sqrt{5}}{8+4\sqrt{5}}$ |
| profit seller          | $\frac{1-\alpha}{2}$ | $\frac{1-\alpha}{12}$ | $\frac{1-\alpha}{2} - \frac{9-3\sqrt{5}}{8+4\sqrt{5}}$ |
| welfare                | $\frac{1-\alpha}{2}$ | $\frac{1-\alpha}{2}$ | $\frac{1-\alpha}{2}$ |


B Appendix Proofs

B.1 Proof of Proposition 1

The proof follows directly from the text.

B.2 Proof of Lemma 1

The proof follows directly from the text.

B.3 Proof of Lemma 2

(a) A seller will order a rating if \((1 - \alpha)q - p_s \geq 0\). As the left-hand-side is increasing in \(q\) the threshold level \(\bar{q}\) of being indifferent of ordering a rating is determined by \((1 - \alpha)\bar{q} - p_s = 0\) which yields \(\bar{q}(p_s) = \frac{p_s}{1 - \alpha}\).

(b) A seller will order a rating if \((1 - \alpha)q - p_s \geq \frac{1}{2}\bar{q} - \alpha q\). As the left-hand-side is increasing in \(q\) the threshold level \(\bar{q}\) of being indifferent of ordering a rating is determined by solving \((1 - \alpha)\bar{q} - p_s = \frac{1}{2}\bar{q} - \alpha q\) for \(\bar{q}\) which yields \(\bar{q}(p_s) = 2p_s\).

B.4 Proof of Proposition 2

(a) The maximization problem of the certifier is given by:

\[
\max_{p_s} \Pi_C(p_s) = (1 - \bar{q}(p_s))p_s. \tag{1}
\]

Plugging the result of Lemma 2(a) into the profit function yields the profit \(\Pi_C\) depending solely on \(p_s\) as:

\[
\Pi_C(p_s) = p_s(1 - \frac{p_s}{1 - \alpha}). \tag{2}
\]

Maximizing w.r.t. \(p_s\) yields \(p_s = \frac{1 - \alpha}{2}\), and hence, \(\bar{q} = \frac{1}{2}\), with a corresponding profit of the certifier of \(\Pi_C = \frac{1 - \alpha}{4}\). The sellers in the quality interval \([\bar{q}, 1]\) order a rating and subsequently sell their product for price \(q\) in the first price sealed bid auction, hence,
their profit in this segment is:

$$\Pi_S = \int_{\frac{1}{2}}^{1} (1 - \alpha)qdq - \frac{1 - \alpha}{4} = \frac{1 - \alpha}{8}. \tag{3}$$

As the lower segment is not traded in the Lemon Market overall realized welfare adds up to
$$W = \frac{3}{8}(1 - \alpha)$$
and a rent of
$$W_{\text{max}} - W = \frac{1 - \alpha}{8}$$
is lost due to the asymmetrically distributed information.

(b) The maximization problem of the certifier is given by:

$$\max_{p_s} \Pi_C(p_s) = (1 - \tilde{q}(p_s))p_s. \tag{4}$$

Plugging the result of Lemma 2(b) into the profit function gives the profit $$\Pi_C$$ depending solely on $$p_s$$ as

$$\Pi_C(p_s) = p_s(1 - 2p_s). \tag{5}$$

Maximizing w.r.t. $$p_s$$ yields $$p_s = \frac{1}{4}$$, and hence, $$\tilde{q} = \frac{1}{2}$$ with a corresponding profit $$\Pi_C = \frac{1}{8}$$. The remainder of the market (quality interval $$[0, \tilde{q}]$$) is traded without a rating at a price of $$\frac{\tilde{q}}{2} = \frac{1}{4}$$ and as all products are traded in this market the profit of the seller is: $$\Pi_S = W_{\text{max}} - \Pi_C = \frac{1 - \alpha}{2} - \frac{1}{8} \tag{6}.$$  

\section*{B.5 Proof of Lemma 3}

The proof for unequally informed buyers in a Honey Market is shown in Weverbergh (1979). The remaining proofs follow directly from the text.

\section*{B.6 Proof of Lemma 4}

(a) A single informed buyer in a Lemon Market receives the investment object for a price of $$\alpha q$$ in the auction with a probability of 1. Hence, the payoff for a product of quality $$q$$ is $$(1 - \alpha)q$$. As expected quality in this market is $$q^e = \frac{1}{2}$$, the ex-ante expectation for the value of being exclusively informed is $$V_{ib}^e = (1 - \alpha)q^e = \frac{1 - \alpha}{2}$$.
(b) As the informed buyer bids $\frac{1}{2}q$ in the auction and the uniformed randomizes the latter sometimes wins. The probability of winning the object for the informed buyer depends on $q$ and is defined as $F(\frac{1}{2}q) = q$. Therefore, the expected payoff (ex-post) for the informed bidder is $F(\frac{1}{2}q)(1 - \frac{1}{2})q = \frac{1}{2}q^2$. Hence, the ex-ante expectation for the value of being exclusively informed is

$$V_{ib}^E = \int_0^1 \frac{1}{2}q^2 dq = \frac{1}{6}. \tag{7}$$

\[\blacksquare\]

**B.7 Proof of Lemma 5**

(a) As the buyers are indifferent between ordering a rating or staying uninformed in the unique mixed-strategy equilibrium the probability $\omega$ of ordering a rating is given by:

$$(1 - \omega)V_{ib}^L - p_b = 0. \tag{8}$$

Using Lemma 4(a) we obtain $(1 - \omega)\frac{1 - \alpha}{2} - p_b = 0$ and solving for $\omega$ gives $\omega(p_b) = 1 - \frac{2p_b}{1 - \alpha}$.

(b) Applying the same logic and using Lemma 4(b) we obtain

$$(1 - \omega)V_{ib}^E - p_b = 0 \Rightarrow (1 - \omega)\frac{1}{6} - p_b = 0. \tag{9}$$

Solving for $\omega$ yields $\omega(p_b) = 1 - 6p_b$. \[\blacksquare\]

**B.8 Proof of Proposition 3**

(a) The maximization problem of the certifier is given by:

$$\max_{p_b} \Pi_C(p_b) = (\omega(p_b))^2 2p_b + 2\omega(p_b)(1 - \omega(p_b))p_b. \tag{10}$$

The profit function can be simplified to $\Pi_C(p_b) = 2\omega(p_b)p_b$. Plugging the result of Lemma 5 (a) into the profit function yields $\Pi_C(p_b) = 2p_b - \frac{4p_b^2}{1 - \alpha}$. Maximizing w.r.t. price $p_b$ gives $p_b = \frac{1 - \alpha}{4}$. This leads to $\omega = \frac{1}{2}$ and thus, the certifier’s profit is $\Pi_C = \frac{1 - \alpha}{4}$.

In the Lemon Market sellers only realize gains if the information on their quality is
known to both buyers. In \( \omega^2 = \frac{1}{4} \) of the cases the expected profit is \( \frac{1-\alpha}{2} \) and hence the overall seller profit is \( \frac{1+\alpha}{8} \). The accumulated welfare adds up to \( W = \frac{3}{8}(1-\alpha) \).

(b) The maximization problem of the certifier is given by:

\[
\max_{p_b} \Pi_C(p_b) = (\omega(p_b))^2 p_b + 2\omega(p_b)(1-\omega(p_b))p_b.
\]  

(11)
The profit function can be simplified to \( \Pi_C(p_b) = 2\omega(p_b)p_b \). Plugging the result of Lemma 5 (b) into the profit function we obtain \( \Pi_C(p_b) = 2p_b - 12p_b^2 \). Maximizing w.r.t. \( p_b \) yields \( p_b = \frac{1}{12} \). This leads to \( \omega = \frac{1}{2} \) and thus, the certifier’s profit is \( \Pi_C = \frac{1}{12} \). As in the Honey Market all projects are realized it turns out that the profit of the seller is \( \Pi_S = \frac{1-\alpha}{2} - \frac{1}{12} \) and the realized welfare is \( W = W_{max} \).

\[\text{B.9 Proof of Corollary 6}\]
The proof follows directly from the text.

\[\text{B.10 Proof of Lemma 8}\]
The proof follows directly from the text.

\[\text{B.11 Proof of Lemma 8}\]
For the entire proof we take \( \bar{q}_{II} \) as fixed. Let \( G_{\bar{q}_{II}}(q) \) denote the uniform distribution on the interval \([0, \bar{q}_{II}]\) with corresponding density function \( g_{\bar{q}_{II}}(q) = \frac{1}{\bar{q}_{II}} \).

(a) The probability of winning the auction for the informed bidder is 1. The quality remaining un-certified in the market is distributed according to \( G_{\bar{q}_{II}}(q) \). In expectation the informed buyer wins an object of quality \( \frac{\bar{q}_{II}}{2} \) for a bid of \( \alpha \frac{\bar{q}_{II}}{2} \), and hence realizes an expected profit of \( V_{ib}^L(\bar{q}_{II}) = (1-\alpha) \frac{\bar{q}_{II}}{2} \).

(b) Let the object in the auction be of a quality \( q \). By bidding \( \frac{1}{2}q \) the informed buyer wins with a probability of \( F_{\bar{q}_{II}}(\frac{1}{2}q) = \frac{q}{\bar{q}_{II}} \). If he wins his payoff is \( q - \frac{1}{2}q = \frac{1}{2}q \). Thus,
payoff ex-ante is determined by

\[ V^H_{ib}(\bar{q}_{II}) = \int_0^{\bar{q}_{II}} \frac{q}{\bar{q}^2_{II}} dq \bar{G}_{\bar{q}_{II}}(q) = \int_0^{\bar{q}_{II}} \frac{q}{\bar{q}^2_{II}} dq = \frac{1}{6} \bar{q}^3_{II}\Big|_0^{\bar{q}_{II}} = \frac{1}{6} \bar{q}_{II}. \] (12)

**B.12 Proof of Lemma 9**

(a) As the buyers are indifferent between ordering a rating or staying uninformed in the unique mixed strategy equilibrium, the probability of ordering a rating is given by:

\( (1 - \omega)V^L_{ib}(p_s, p_b) - p_b = 0 \)  \hspace{1cm} (13)

Using Lemma 8(a) we obtain \( (1 - \omega)(1 - \alpha)\frac{\bar{q}_{II}(p_s, p_b)}{2} - p_b = 0 \) and solving for \( \omega \) gives \( \omega(p_s, p_b) = 1 - \frac{2p_b}{(1 - \alpha)\bar{q}_{II}(p_s, p_b)} \).

(b) Applying the same logic and using Lemma 8(b) we obtain

\[ (1 - \omega)V^H_{ib}(p_s, p_b) - p_b = 0 \Leftrightarrow (1 - \omega)\frac{1}{6} \bar{q}_{II}(p_s, p_b) - p_b = 0. \] (14)

Solving for \( \omega \) yields \( \omega(p_s, p_b) = 1 - \frac{6p_b}{\bar{q}_{II}(p_s, p_b)} \).

**B.13 Proof of Lemma 10**

(a) A seller will order a rating if:

\[ (1 - \alpha)q - p_s \geq (\omega(p_s, p_b))^2(1 - \alpha)q. \] (15)

As the left-hand-side increases faster in \( q \) the threshold level of the seller being indifferent is determined by the condition

\[ (1 - \alpha)\bar{q}_{II} - p_s \geq (\omega(p_s, p_b))^2(1 - \alpha)\bar{q}_{II}. \] (16)
Plugging the result of Lemma 9 (a) into (16) and solving for \( \bar{q}_{II} \), gives

\[
\bar{q}_{II}(p_s, p_b) = \frac{4p_b^2}{(1 - \alpha)(4p_b - p_s)} \tag{17}
\]

(b) To determine the seller’s indifference condition we need the expected winning bid in case that only one buyer ordered a rating given quality \( q \) and upper threshold \( \bar{q}_{II} \), denoted as \( E[b_{\text{win}}|q, \bar{q}_{II}] \). With a probability of \( F_{\bar{q}_{II}}(\frac{1}{2}q) = \frac{q}{\bar{q}_{II}} \) the informed bidder wins with a bid of \( \frac{1}{2}q \). With a probability of \( 1 - \frac{q}{\bar{q}_{II}} \) the uninformed wins with an expected bid of \( \frac{\frac{1}{2}q + \bar{q}_{II}}{2} = \frac{1}{4}(q + \bar{q}_{II}) \). Thus,

\[
E[b_{\text{win}}|q, \bar{q}_{II}] = \frac{q}{\bar{q}_{II}} \cdot \frac{1}{2}q + (1 - \frac{q}{\bar{q}_{II}}) \cdot \frac{1}{4}(q + \bar{q}_{II}) = \frac{1}{4}\bar{q}_{II} + \frac{q^2}{4\bar{q}_{II}}. \tag{18}
\]

A seller will order a rating if:

\[
(1 - \alpha)q - p_s \geq (\omega(p_s, p_b))^2(1 - \alpha)q + 2\omega(p_s, p_b)(1 - \omega(p_s, p_b))(E[b_{\text{win}}|q, \bar{q}_{II}] - \alpha q) + (1 - \omega(p_s, p_b))^2(\frac{1}{2} - \alpha)\bar{q}_{II}. \tag{19}
\]

Again, the left-hand-side increases faster in \( q \). The quality threshold \( \bar{q}_{II} \) is determined by replacing all \( q \) by \( \bar{q}_{II} \) and thereby replacing \( E[b_{\text{win}}|q, \bar{q}_{II}] \) by \( E[b_{\text{win}}|\bar{q}_{II}, \bar{q}_{II}] = \frac{1}{2}\bar{q}_{II} \) yields

\[
(1 - \alpha)\bar{q}_{II} - p_s = (\omega(p_s, p_b))^2(1 - \alpha)\bar{q}_{II} + 2\omega(p_s, p_b)(1 - \omega(p_s, p_b))(\frac{1}{2} - \alpha)\bar{q}_{II} \tag{20}
\]

which can be reformulated as

\[
(1 - (\omega(p_s, p_b))^2)\frac{\bar{q}_{II}}{2} = p_s. \tag{21}
\]
Plugging the result of Lemma 9 (b) into (21) and solving for $\bar{q}_{II}$ yields:

$$\bar{q}_{II}(p_s, p_b) = \frac{18p_b^2}{6p_b - p_s}$$  \hspace{1cm} (22)

B.14 Proof of Proposition 4

(a) The maximization problem of the certifier is given by:

$$\max_{p_s, p_b} \Pi_C(p_s, p_b) = (1 - \bar{q}_{II}(p_s, p_b))p_s + \bar{q}_{II}(p_s, p_b)[(\omega(p_s, p_b))^22p_b$$

$$+ 2\omega(p_s, p_b)(1 - \omega)(p_s, p_b)p_b].$$  \hspace{1cm} (23)

The profit function can be simplified to

$$\Pi_C(p_s, p_b) = p_s + \bar{q}_{II}(p_s, p_b)[2\omega(p_s, p_b)p_b - p_s].$$

By plugging the results of Lemma 9(a) and Lemma 10(a) into the profit function of the certifier we obtain a profit function given by

$$\Pi_C(p_s, p_b) = p_s - \frac{8p_b^3}{(1 - \alpha)(4p_b - p_s)}.$$  \hspace{1cm} (24)

Maximizing the profit function w.r.t. $p_s$ and $p_b$ we finally obtain $p_s = \frac{16}{27}(1 - \alpha)$ and $p_b = \frac{2}{9}(1 - \alpha)$. The derived functions for $\omega$ and for $\bar{q}_{II}$ imply $\bar{q}_{II} = \frac{2}{3}$ and $\omega = \frac{1}{3}$. The profit for the certifier is $\Pi_C = \frac{8}{27}(1 - \alpha)$. In the market segment with a quality parameter below $\bar{q}_{II}$ a share of $1 - (\frac{2}{3})^2 = \frac{5}{9}$ of all available products is traded. Hence, the overall welfare adds up to:

$$W = \frac{5}{9} \int_0^{\frac{2}{3}} (1 - \alpha)qdq + \int_{\frac{2}{3}}^1 (1 - \alpha)qdq = (1 - \alpha)\frac{65}{162},$$  \hspace{1cm} (25)

As buyers do not make any profit in equilibrium the seller’s profit yields

$$\Pi_F = W - \Pi_C = (1 - \alpha)\frac{17}{162}.$$
(b) The maximization problem of the certifier is given by:

$$\max_{p_s,p_b} \Pi_C(p_s,p_b) = (1 - \bar{q}_{II}(p_s,p_b))p_s + \bar{q}_{II}(p_s,p_b)[(\omega(p_s,p_b))^22p_b$$

$$+ 2\omega(p_s,p_b)(1 - \omega(p_s,p_b))p_b].$$

(26)

Again, the profit function can be simplified to

$$\Pi_C = p_s + \bar{q}_{II}(p_s,p_b)[2\omega(p_s,p_b)p_b - p_s].$$

Plugging the results of Lemma 9(b) and Lemma 10(b) into the profit function we end up with the following maximization problem:

$$\max_{p_s,p_b} \Pi_C(p_s,p_b) = p_s - 6p_b^2 \frac{6p_b + p_s}{6p_b - p_s} \quad \text{s.t.} 0 \leq \omega, \bar{q}_{II} \leq 1$$

(27)

Hereby, the boundary conditions on $\omega$ and $\bar{q}_{II}$ need to be fulfilled. Using the expressions for the two parameters derived above, the constraints are equivalent to

$$3p_b \leq p_s \leq 6p_b - 18p_b^2.$$  

(28)

In the following we show that an interior optimum exists. Taking the derivative of the profit function with respect to $p_b$ gives a single non-negative root which is $p_b = \frac{1}{12}(p_s + 5\sqrt{p_s}).$ Plugging this into the first derivative of the profit function with respect to $p_s$ and solving the FOC for $p_s$ gives $p_s = \frac{3(3 - \sqrt{5})}{2(2 + \sqrt{5})}.$ Using this in the expression for $p_b$ yields $p_b = \frac{\sqrt{5} - 1}{4(2 + \sqrt{5})}.$ Calculating the certifier’s profit using optimal prices yields $\Pi_C = \frac{9 - 3\sqrt{5}}{8 + 4\sqrt{5}}.$ The profit is higher than in either case of one-sided certification (Proposition 2(b) and Proposition 3(b)), hence (28) is not binding. The induced quality threshold and the rating probability are calculated using Lemma 10(b) and Lemma 9(b), which yield $\bar{q}_{II} = \frac{3}{4}(3 - \sqrt{5})$ and $\omega = \frac{1}{2 + \sqrt{5}},$ respectively.

As buyers do not make any profit in equilibrium the seller’s profit yields

$$\Pi_S = W - \Pi_C = \frac{1 - \alpha}{2} - \frac{9 - 3\sqrt{5}}{8 + 4\sqrt{5}}.$$  

(29)
References


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