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A Note on the Computation of the Equity Premium and the Market Value of Firm Equity

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A Note on the Computation of the Equity Premium and the Market Value of Firm Equity

Abstract

Turnovsky (1995) derives in a continuous-time model of a decentralized economy that the correct specification of the firm’s objective function is to maximize the initial value of its outstanding securities. The firm value is the discounted flow of real earnings. For the discrete-time version of the model, we show that the correct computation of the firm value needs to be modified. Depending on the specific formula employed, different values of the equity premium result.

JEL-Code: G12, C63, E22, E32.
Keywords: asset prices, firm value, equity premium.

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1 Introduction

Asset pricing in production economies has been extensively studied in the literature, e.g. in Jermann (1998). In these models, the asset value of the firm $\tilde{V}_t$ in period $t$ is typically computed as the expected discounted sum of future cash flows:

$$\tilde{V}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} CF_t,$$

(1.1)

where $\beta$, $\lambda_t$, and $CF_t$ denote the discount factor of the household, the marginal utility of consumption, and the cash flow in period $t$. The operator $\mathbb{E}_0$ denotes mathematical expectations with respect to information as of period 0.

We instead derive the firm value as:

$$V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \frac{\lambda_t}{\lambda_{t-1}} CF_t.$$

(1.2)

In the following, we will briefly describe the model of Jermann (1998) that is often used as a benchmark for the computation of asset prices and solve it for the firm value. In addition, we show that the value of the equity premium depends on the specification of the firm value.

2 The Economy

We consider the asset pricing model of Jerman (1998) and follow the description of this model in Herr and Maußner (2009). The set-up is the standard real business cycle model with a household and production sector. Productivity is subject to a random shock. There is a single good that is used in investment and consumption. Time is discrete and denoted by $t$.

1For example, see equation (2.1) of Jermann (1998).

2We could have also derived these results in a much simpler version of a production economy without habit formation and adjustment cost of capital. We, however, have chosen the more complex model as it features so prominently in the asset pricing literature, e.g. in Boldrin et al. (2001) or Lettau and Uhlig (2000).
2.1 Households

A representative household supplies labor in a fixed amount of $N = 1$ at the real wage $w_t$. Besides labor income he receives dividends $d_t$ per unit of share $S_t$ he holds of the representative firm. The current price of shares in units of the consumption good is $v_t$. His current period utility function $u(.)$ depends on current and past consumption, $C_t$ and $C_{t-1}$, respectively. Given his initial stock of shares $S_t$ the households maximizes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{(C_{t+s} - bC_{t+s-1})^{1-\eta} - 1}{1-\eta} \right\}, \quad \eta \geq 0, \beta \in (0,1)$$

subject to the sequence of budget constraints

$$v_t(S_{t+1} - S_t) \leq w_t + d_tS_t - C_t. \quad (2.1)$$

The first-order conditions of this problem are:

$$\lambda_t = (C_t - bC_{t-1})^{-\eta} - \beta b \mathbb{E}_t (C_{t+1} - bC_t)^{-\eta}, \quad (2.2a)$$

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} R_{t+1}, \quad (2.2b)$$

$$R_t := \frac{d_t + v_t}{v_{t-1}}, \quad (2.2c)$$

where $\lambda_t$ is the Lagrange multiplier of the budget constraint and $R_t$ denotes the current period gross return on equity.

2.2 Firms

The representative firm uses labor $N_t$ and capital $K_t$ to produce output $Y_t$ according to the production function

$$Y_t = Z_t N_t^{1-\alpha} K_t^\alpha, \quad \alpha \in (0,1). \quad (2.3)$$

The level of total factor productivity $Z_t$ is governed by the AR(1)-Process

$$\ln Z_t = \rho Z_{t-1} + \epsilon^Z_t, \quad \epsilon^Z_t \sim N \left(0, (\sigma^Z)^2\right). \quad (2.4)$$

The firm finances part of its investment $I_t$ from retained earnings $RE_t$ and issues new shares to cover the remaining part:

$$I_t = v_t(S_{t+1} - S_t) + RE_t. \quad (2.5)$$
It distributes the excess of its profits over retained earnings to the household sector:

\[ d_t S_t = Y_t - w_t N_t - RE_t. \]  
(2.6)

Investment increases the firm’s future stock of capital according to:

\[ K_{t+1} = \Phi(I_t/K_t)K_t + (1 - \delta)K_t, \quad \delta \in [0, 1], \]  
(2.7)

where \( \delta \) denotes the rate of depreciation and adjustment costs \( \Phi(.) \) are a positive concave function.

### 2.3 Computation of the Firm Value

To derive the objective function of the firm, we follow Turnovsky (1995). Let \( V_{t+1} \) denote the value of the shares at the end of period \( t \), \( V_{t+1} = v_t S_{t+1} \). From (2.5), (2.6), and (2.2c), we get

\[
V_{t+1} = v_t S_{t+1} = (v_t + d_t)S_t - (Y_t - w_t N_t - I_t)
= \frac{v_t + d_t}{v_{t-1}}v_{t-1}S_t - CF_t
= R_t V_t - CF_t,
\]

where the cash flow in period \( t \) is defined as \( CF_t = Y_t - w_t N_t - I_t \). Iteration of these equation yields (1.2):

\[
V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \frac{\lambda_t}{\lambda - 1} CF_t.
\]  
(2.8)

### 2.4 Optimality Conditions of the Firm

The firm’s objective is to maximize \( V_0 \) given in (2.8) subject to the constraint (2.7). The first-order conditions of the firm are:\(^3\)

\[
w_t = (1 - \alpha)Z_t N_t^{-\alpha} K_t^\alpha,
\]  
(2.9a)

\[
q_t = \frac{1}{\Phi'(I_t/K_t)}.
\]  
(2.9b)

\[
q_t = \mathbb{E}_t \beta^{\lambda_t+1} \frac{\lambda_{t+1}}{\lambda_t} \left\{ \alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1} \Phi(I_{t+1}/K_{t+1}) + 1 - \delta \right\}.
\]  
(2.9c)

---

\(^3\)Note that the same set of conditions derives from maximizing (1.1).
In addition, the transversality condition

$$\lim_{t \to \infty} E_0 \beta^{t+1} \frac{A_t}{\Lambda_{t-1}} q_t K_{t+1} = 0$$

(2.9d)

must hold.

3 Computation of the Asset Prices

In order to illustrate the implications of the firm value specification for asset pricing, we consider a numerical example. We will first describe our calibration procedure before we present our details for the computation of the equity premium.

3.1 Calibration

Periods correspond to quarters. We parameterize the function $\Phi$ from (2.7) as

$$\Phi\left(\frac{I_t}{K_t}\right) := \frac{a_1}{1 - \zeta} \left(\frac{I_t}{K_t}\right)^{1 - \zeta} + a_2, \quad \zeta > 0.$$  

(3.1)

The choice of the parameters $a_1$ and $a_2$ is described in the Appendix. For the remaining parameters, we use values that are standard in the literature. In particular, we follow Heer and Maußner (2009), Section 6.3.4. Table 3.1 displays the respective values. The discount factor $\beta$ is set equal to 0.994 implying an annual risk free rate in the stationary equilibrium of 2.4%.

<table>
<thead>
<tr>
<th>Table 3.1</th>
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</thead>
<tbody>
<tr>
<td>Benchmark calibration</td>
</tr>
<tr>
<td>Preferences $\beta=0.994$ $b=0.8$ $\eta=2$</td>
</tr>
<tr>
<td>Production $\alpha=0.27$ $\delta=0.011$ $\rho^2=0.90$ $\sigma^2=0.0072$ $\zeta=1/0.23$</td>
</tr>
</tbody>
</table>

The solution of the model are functions $g^i$, $i \in \{K, Y, C, I, \lambda, q\}$, that determine $K_{t+1}$, $Y_t$, $C_t$, $I_t$, $\lambda_t$, and $q_t$ given the current period state variables $K_t$, $C_{t-1}$, and the log of the productivity shock $\ln Z_t$. We use the quadratic approximation of $g^\lambda$ at the stationary equilibrium which is derived in the Appendix.\footnote{The source code is available in the Fortran program \texttt{Model Equity 2.0 for} and can be downloaded from Alfred Maußner’s homepage 'http://www.wiwi.uni-augsburg.de/vwl/maussner/'.}

4
3.2 Asset Price Implications

In the following, we derive the risk-free rate of return and three different expressions for the return on equity, \( R_{t}^{e1} \), \( R_{t}^{e2} \), and \( R_{t}^{e3} \).

In our model the risk free rate of return \( r_{t} \) is given by

\[
    r_{t} = \frac{\lambda_{t}}{\beta E_{t} \lambda_{t+1}} - 1.
\]

Since

\[
    \lambda_{t+1} = g^{\lambda}(K_{t+1}, C_{t}, \ln Z_{t+1})
    = g^{\lambda}(g^{K}(K_{t}, C_{t-1}, \ln Z_{t}), g^{C}(K_{t}, C_{t-1}, \ln Z_{t}), 0, \ln Z_{t} + \epsilon_{t+1}^Z)
    = : \tilde{g}^{\lambda}(K_{t}, C_{t-1}, \rho \ln Z_{t} + \epsilon_{t+1}^Z),
\]

and \( \epsilon_{t+1}^Z \) is normally distributed, the expected value of the Lagrange multiplier equals

\[
    E_{t} \lambda_{t+1} = \int_{-\infty}^{\infty} \tilde{g}^{\lambda}(K_{t}, C_{t-1}, \rho \ln Z_{t} + \epsilon_{t+1}^Z) \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(\epsilon_{t+1}^Z)^2}{2\sigma^2}} d\epsilon_{t+1}^Z.
\]

We use the Gauss-Hermite 6-point quadrature formula to approximate the integral on the right-hand-side of this equation.

The labor market equilibrium condition (2.9a) and equation (2.7) imply that the right-hand-side of (2.9c) can be written as

\[
    1 = \beta E_{t} \frac{\lambda_{t+1} Y_{t+1} - w_{t+1} N_{t+1} - I_{t+1} + q_{t+2} K_{t+2}}{q_{t} K_{t+1}},
\]

\[
    = \frac{\lambda_{t+1} d_{t+1} + v_{t+1}}{v_{t}} = \beta E_{t} \frac{\lambda_{t+1} R_{t+1}}{\lambda_{t}}
\]

where the second equality follows from equations (2.5) and (2.6) and the observation that \( q_{t} K_{t+1} = v_{t} S_{t+1} \) (see Heer and Maußner (2009), p. 317). Therefore, the gross rate of return on the shares of the representative firm equals

\[
    R_{t+1}^{e1} = \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1} K_{t+2}}{q_{t} K_{t+1}}.
\]

Alternatively, we can use the value of the firm as computed by (1.1) and (1.2), respectively, in order to derive an expression for the return on equity. For this reason, we assume that the number of shares is constant and equal to one, \( S_{t} \equiv 1 \). Consequently,

\[
    R_{t+1}^{e2} = \frac{d_{t} + V_{t+1}}{V_{t}} \quad \text{(3.3)}
\]

\[
    R_{t+1}^{e3} = \frac{d_{t} + \tilde{V}_{t+1}}{V_{t}} \quad \text{(3.4)}
\]
We use a random number generator to compute a long artificial time series for \( R_{t+1}^i - r_t \), \( i = 1, 2, 3 \). The averages of these time series are our measures of the ex-post equity premium implied by the model. For a time series of 1,000,000 observations and the parameters from Table 3.1, the results are summarized in 3.2. We find an average annual risk-free rate of about 2.4% percent and an equity premium of 4.00%, 3.86%, and 3.94% for the three different computations considered in (3.2), (3.3), and (3.4), respectively.\(^6\)

### 4 Conclusion

In this note, we show that the correct firm value is given by (1.2) rather than (1.1). The equity premium that result from the model of the production economy depends on this specification. For our parametric example, we find equity premia in the range of 3.86% to 4.00%.

---

\(^5\)Note, \( \alpha Y_{t+1} = Y_{t+1} - w_{t+1}N_{t+1} \).

\(^6\)In particular, we used the same sequence of shocks and, therefore, the same sequence of state variables for the computation of the three different formulas of the equity premium.
5 Appendix: Deterministic Stationary Equilibrium

Our solution strategy rests on a second-order approximation of the model. Therefore, we must consider the stationary equilibrium of the deterministic counterpart of our model. To this end, we set $\sigma Z = 0$ so that $Z_t$ equals its unconditional expectation $Z = 1$ for all $t$. Stationarity implies $x_{t+1} = x_t = x$ for any variable $x_t$ in our model. As usual, we specify $\Phi$ so that adjustment costs play no role in the stationary equilibrium, i.e., $\Phi(I/K)K = \delta K$ and $q = \Phi'(\delta) = 1$. This requires that we choose

$$a_1 = \delta^\zeta,$$
$$a_2 = \frac{-\zeta\delta}{1 - \zeta}.$$

These assumptions imply the stationary solution for the stock of capital:

$$K = \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta}\right)^{\frac{1}{\alpha - 1}}. \tag{5.1a}$$

Output, investment, consumption, and the stationary solution for $\lambda$ are then given by

$$Y = K^\alpha, \tag{5.1b}$$
$$I = \delta K, \tag{5.1c}$$
$$C = Y - I. \tag{5.1d}$$
\[ \lambda = C^{-\eta}(1 - b)^{-\eta}(1 - b\beta). \]  

(5.1e)

In order to determine the firm’s value, we use a recursive formulation of (1.1) and (1.2), respectively. In particular, (1.1) gives rise to

\[ \tilde{V}_t = CF_t + \beta \frac{\Lambda_{t+1}}{\Lambda_t} \tilde{V}_{t+1}, \]  

(5.2)

whereas (1.2) implies

\[ V_t = \beta \frac{\Lambda_t}{\Lambda_{t-1}} CF_t + \beta \frac{\Lambda_t}{\Lambda_{t-1}} V_{t+1}. \]  

(5.3)
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