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How risky is the German Pension System?
The Volatility of the Internal Rates of Return

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Bernd Raffelhüschen

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INSTITUT FÜR FINANZWISSENSCHAFT
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How risky is the German Pension System?
The Volatility of the Internal Rates of Return

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Abstract

In this paper we analyze exemplarily the volatility of the internal rates of return of the German pension system over the life-cycle of an individual born in 1957. The outcome is compared to an alternative defined-contribution or defined-benefit policy. Based on the actual data, our results show the volatility of the internal rate of return to be significantly higher under the actual policy. We furthermore find that the sustainable internal rates of return are close to zero for the youngest male cohorts and positive for females for optimistic growth scenarios. In more realistic scenarios things turn worse.

JEL classification: H55, J18

Keywords: risk – internal rate of return – sustainability

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1 Introduction

In the past, the German pension system has been exposed to numerous policy interventions: From today’s perspective dynamic pension policy for employees and workers was firstly introduced by the pension reform of 1957. The latest considerable change was applied in 2005 aiming to cope with demographic transition in Germany.

As both the “first” and the “latest” reform have and had huge effects on the public pension level and the contribution payments, this naturally obtrudes the question to what extent all the reforms and adjustments within this period affected the pension level and the contribution rate. More precisely, what do these changes mean for an individual “exposed” to the German pension system in terms of riskiness? We develop scenarios, namely defined-contribution and defined-benefit rules, for benchmarking and thus comparing the actual policy to them.

Our paper relates to the literature of political risk aspects of social security systems. For example McHale (2001) and Borgmann and Heidler (2003) define political risk as the risk of changes in the social security benefit rule. These alterations to the benefit rule obviously constitute a source of risk for the individuals’ life-cycle resources. In contrast to this, Börsch-Supan (2005) points out a more narrow perspective of political risk. He outlines political risk conditional on the circumstances (i.e. demographic development or uncertainty in future growth rates) that lead policy makers to their decisions.

In contrast to this, the type of risk we want to take into consideration is based on the perspective of an individual, that is myopic to adjustment processes: Observing the available data of the German pension system – revenues and expenditures –, the individual projects all future payments from his point of view. The significant difference is that the above mentioned political risk defined by McHale and Börsch-Supan is exclusively induced by the policy maker. In our approach we still analyze riskiness due to legal changes but as well as due to reactions of individuals, who take these policy interventions for granted. Finally, we can regard for demography-induced volatility by benchmarking the actual policy with the defined-contribution or defined-benefit scenario.

To quantify the risk as defined here, we calculate different internal rates of return over the life-cycle of an individual. For this purpose, we use a modification
of the method of generational accounting. Using this method with present data of the German pension system, Raffelhüschen (1998) was the first to calculate sustainable rates of return. In this respect we now want to go further by introducing historical data in order to calculate sustainable rates of return as well as unadjusted internal rates of return. The first measure considers all adjustments for balancing the annual budget of the German pension system realized at the end of the individual’s life-cycle. In contrast to this the unadjusted internal rates of return depict the development over the life-cycle of a cohort.

The findings of the paper are twofold: First, we calculate sustainable internal rates of return based on the actual pension policy and show that they are close below zero for male cohorts born after the year 2000 and positive for females for optimistic growth scenarios. Second, we demonstrate that the rates of return of the German pension system – computed by the myopic individual over his life-cycle – are highly volatile for the middle-aged cohort of 1957. This volatility of internal rates of return is lower under the defined-contribution or defined-benefit scenario compared to the actual policy. Furthermore the resulting sustainable rates of return are higher are higher for all cohorts born after 1990 under the alternative adjustment rules than those resulting from the actual policy.

The paper is organized as follows: Section 2 develops the method of projecting and adjusting per capita revenues and expenditures. Furthermore, it shows how internal rates of return are calculated. It closes with a discussion of the type of riskiness introduced here. Section 3 presents the underlying data and results; first of all the sustainable rates of return for different cohorts. Secondly we depict the volatility for the internal rates of return over the life-cycle. Section 3 closes with a comparison of sustainable internal rates of return of the youngest and future cohorts under the different policies Section 4 summarizes and concludes.

2 Method

2.1 Projecting net benefits

As mentioned above we want to compute internal rates of return, which base on actual payment as well as hypothetical payment flows from and to the German pension system conditional on the status quo of the year 1957. We therefore first

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1 The method was developed by Auerbach et. al (1991), (1992), and (1994).
develop a projection method for actual net payment flows between a representative agent and the German pension system for the actual policy scenario. Only in a second step, we show how hypothetical payment flows, i.e. payments flows for the defined-benefit (db) and defined-contribution (dc) scenario, can be calculated using a slight modification of our projection method.

The actual net payments are developed using the projection method generally applied in generational accounting studies.\(^2\) The utilization of this method is repeated several times for a variety of projection years: Beginning with the year 1957 and continuing with the annual reapplication of the projection for all possible projection years over the life-cycle of the individual born in 1957.

For this purpose we assume that an agent of cohort 1957 possess all the necessary information to project his “expected” payments to the pension system as well as his “expected” transfers from the pension scheme on the basis of the status quo in 1957. He assumes that these payments and receipts are constant for the rest of his life – except for a growth adjustment, that bases on the actual real growth rates for the years before 2004 and on a constant real growth rate of 1.5 percent thereafter. In the following new individual (projection) year 1958 the agent receives new information about the actual values of 1958. With this new information he adapts his expectation for future payment flows and thus calculates a new expected rate of return with the information of the bygone year 1957 and the new information of the new projection year 1958. Again, he assumes that the (growth-adjusted) payments and receipts are constant over the rest of his life. In doing so, it is obvious that volatility arises for different projections due to varying net payments based on the respective new legal information. For all years of a cohort (following their birth-year 1957) and their assumed maximal life-time of one hundred years this procedure yields 100 different rates of return each containing information of the corresponding projection year.

A fundamental restriction serves as a starting point for the projection payments of type \(i\) – in our case \(i\) covers the contributions (\(con\)) and federal subsidies (\(feds\)) to the pension system as well as the benefits paid by the pension scheme to existing (\(exir\)) and entering retirees (\(entr\)) – in the projection year \(py\) and in all other years over the life-cycle \(s\) prior to the (individual’s) respective projection year \(s \leq py\): The sum of all age-specific average individual payments \(h_{a,i,s}\)

\(^2\)For a detailed depiction of this projecting method, see Bonin (2001).
weighted with the cohort size \( P_{a,s} \), equals the corresponding macroeconomic aggregate \( H_{i,s} \) – where \( H_{i,s} \) is measured in real value terms of 1957 and the minimum (maximum) age \( a \) is defined to be 0 (100).

\[
H_{i,s} = \sum_{a=0}^{100} h_{a,i,s} \cdot P_{a,s},
\]

\( \forall s \leq py \).

As it is impossible or rather too extensive to gather exactly all the payments between individuals and the pension system satisfying equation (1). Generational accountants rescale a micro profile \( \eta_{a,i,s} \), which captures the relative payment position of different age groups, to the corresponding macroeconomic aggregate with the help of a rescaling factor

\[
\theta_{i,s} = \frac{H_{i,s}}{\sum_{a=0}^{100} \eta_{a,i,s} \cdot P_{a,s}},
\]

\( \forall s \leq py \).

The rescaled age-specific average individual payments \( h_{a,i,s} \) can then easily be computed by:

\[
h_{a,i,s} = \theta_{i,s} \cdot \eta_{a,i,s},
\]

\( \forall s \leq py \).

To project the age specific individual payments for all years after the respective projection year \( (s > py) \), we use the actual annual growth rates \( g_j \) for all years \( j \).

\[
h_{a,i,s} = h_{a,i,py} \prod_{j=py}^{s-1} (1 + g_j),
\]

\( \forall s > py \land i = \text{con, feds, entr} \).

A slightly different approach has to be used to project age-specific individual payment flows for benefits paid to existing retirees \( h_{a,exir,s} \). The projection of the age specific benefits received by the group of the future existing retirees is based on the following idea, namely adding up the flow of entering retirees and phasing out the stock of (then) existing retirees. Thus, the average age-specific benefits

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for retirees \( h_{a,\text{ben},s} \) of age \( a \) for all years over the life-cycle \( s \) after the respective projection year \( py \) are given by:

\[
 h_{a,\text{ben},s} = h_{a,\text{exir},s} = h_{a-1,\text{exir},s-1} \cdot (1 + g_{j-1}) + h_{a,\text{entr},s}. \tag{5}
\]

In a next step, we compute the annual net benefits which a representative cohort \( c \) receives from the German pension system in all years of remaining life time. For all the years \( (s) \) between the birth year of the cohort \( (c) \) and the respective projection year \( (py) \) the annual net benefits \( nb_{a,s} \) can easily be derived with equations (1) to (3). For all years \( (s) \) after the respective projection year \( (py) \) and the last (potential) year the agent is alive \( (c+100) \), the age-specific payment flows are given by equations (4) and (5). Hence the net benefit which a cohort receives from the German pension system is equivalent to the pension benefit received minus the contribution payments to the pension system, and minus the share of the public subsidies that are allocated to him:

\[
 nb_{s-c,s} = h_{s-c,\text{ben},s} - h_{s-c,\text{contr},s} - h_{s-c,\text{feds},s}, \tag{6}
\]

\[ \forall \ c \leq s \leq c + 100 \land s \geq 1957. \]

Note that every recalculation of equations (2) to (6) for a new projection year changes the new underlying data and thus the calculation for the net benefits \( (nb_{s-c,s}) \) in the future \( (s \geq py) \).

After now having developed the actual net benefits, we can derive hypothetical net benefits on basis of 1957 in order to calculate the db- and dc-scenario. In order to do so, we need to slightly modify the previous equations (2) to (6): The aggregates \( H_{i,1957} \) and the profiles for the benefits of existing and entering retirees \( \eta_{a,i,1957} \) (with \( i = \text{exir} \) and \( \text{entr} \)) are held constant for all years over the life-cycle, independent of the respective projection year. Thus, we again use equations (2) to (6) as well as (7), however, without adjusting the data for the respective projection year. Hence, the outcome of the hypothetical net benefits under the status of 1957 are given by:

\[
 nb_{s-c,s}^{1957} = h_{s-c,\text{ben},s}^{1957} - h_{s-c,\text{contr},s}^{1957} - h_{s-c,\text{feds},s}^{1957}, \tag{7}
\]

\[ \forall \ c \leq s \leq c + 100, \ s \geq 1957, \land py \geq 1957. \]
Obviously, these net benefits under the information of 1957, $nb^{1957}_{s-c,s}$, do not change with new calculations for different projection years as the data is not adjusted.

In the next section we will adjust the net benefits for every projection year allowing for a db- and a dc-policy.

2.2 Adjusting net benefits for sustainability

In this section we want to adjust the actual net benefits (for $py > 2003$) as well as the hypothetical net benefits (for $py > 1957$) in a way that sustainability is restored gradually. The result of this is that the annual budget constraint of the pension system is balanced for the projection year. Note again that the individual has only information on historical and the current projection year. For every following projection year, he or she updates information for the new projection year and assumes the adjusted net benefits of the projection year to be constant. This update-procedure is recurring for every projection year over the remaining life-cycle.

The reason for adjusting the hypothetical net benefits is to create a db- and a dc-policy beginning with the year 1957. In order to apply a db-policy for every respective projection year e.g., we have to readjust contributions and federal subsidies for all years $s$, with $s > py > 1957$. This assumption generates volatile hypothetical adjusted net benefits, but note that it covers demography-induced volatility only. This is an important fact as we later on want to compare the volatility of the adjusted actual net benefits to the adjusted hypothetical net benefits.

Adjusting the actual net benefits for all years $s$, with $s > py > 2003$, allows us to consider the implications of the actual policy from the year 2003. We can thus compare the actual policy over the entire individual’s life-cycle to the db- (dc-) scenario. The internal rate of return of the individual’s last projection year after having completed the life-cycle will also be the individual’s ex post sustainable rate of return. This in turn corresponds to a forecasted internal rate of return of the German pension system based on the legal status of today.

First of all, we begin with the annual budget constraint of the German pension system. It quantifies the dimension of the (un)sustainable financial situation for each year. With the knowledge of what was stated above, the unconstrained budget of German pension system can be expressed as the annual aggregated net
benefits received by all retirees in a respective year. Hence, the aggregated net benefits ($NB_s$) in a certain year $s$ can be calculated as the sum of $nb_{a,s}$ weighted with the corresponding cohort size $P_{a,s}$:

$$NB_s = \sum_{a=0}^{100} nb_{a,s} \cdot P_{a,s}.$$  

(8)

In order to obtain the annual age-specific individual net benefits, $nb_{a,s}$, a general form of equation (7) must be used:

$$nb_{a,s} = h_{a,ben,s} - h_{a,contr,s} - h_{a,feds,s}.$$  

(9)

The first thing we now want to consider is the adjustment of the two alternative scenarios, namely the defined benefit and the defined contribution rule. To do so, we use the hypothetical net benefits ($nb^{1957}_{a,s}$) developed in the status quo 1957 of the previous section. Due to the assumption that the annual budget constraint has to be satisfied in every projection year, the net benefits of the respective projection year have to be adjusted. For the db-policy the payments to the expenditure gap have to be adapted. This is done by introducing a scaling parameter $\mu^{db}_{py}$ which reflects the relation of the periodical public liabilities (the numerator) to contributions and federal subsidies paid in the projection year depending on the policy scenario (the denominator). For the db-policy $k = db$ we thus get:

$$\mu^{db}_{py} = \frac{NB^{1957}_{py}}{\sum_{a=0}^{100} (h_{a,cons,py}^{1957} + h_{a,feds,py}^{1957}) \cdot P_{a,py}} + 1.$$  

(10)

For the dc-policy we instead adapt the benefits according the scaling parameter $\mu^{dc}_{py}$ for every projection year $py$. For the dc-policy $k = dc$ we get:

$$\mu^{dc}_{py} = \frac{NB^{1957}_{py}}{\sum_{a=0}^{100} (h_{a,exir,py}^{1957} + h_{a,entr,py}^{1957}) P_{a,py}} + 1.$$  

(11)

These scaling parameters are used to adjust all projected age-specific contributions or benefits:

$$h_{a,i,s}^{1957} = \mu^{k}_{py} \cdot h_{a,i,s}^{1957}$$  

(12)

\[\forall s \geq py \land i = \begin{cases} 
\text{con} & \text{if } k = db \\
\text{exir,entr} & \text{if } k = dc.
\end{cases}\]
Thus, the net benefit $\text{nb}_{s-c,s}^{1957,k}$ that the individual has to pay in and after the projection years are developed according to the equations (1) to (6) and adjusted with equations (10) and (12), or (11) and (12), respectively.

We now remain with computing the adjusted actual net payments. The adjustments are similar to the standard method of contribution projections for calculations of internal rates of return. The adjustment changes for every projection year after 2003. Again we have to assure the annual budget is balanced for every year $s > py = 2003$. To adjust the actual net benefits, we have to use a modified version of equation (10) for all years $s > py$. In a first step, we have to change the benefits paid to the retirees. The projecting procedure for the age-specific individual payments of this type corresponds to equation (4) to (6), only using the legal definition of the pension adjustment formula of the year 2004 in order to conform with today’s policy. In a second step, we adjust the net benefits resulting from equations (2) to (7) with:

$$\nu_s = \frac{\text{NB}_s}{\sum_{a=0}^{100} (h_{a,\text{con},s} + h_{a,\text{fed},s}) \cdot P_{a,s}} + 1.$$  \hspace{1cm} (13)

### 2.3 Calculating internal rates of return

In this chapter we display the internal rates of return for the actual as well as the hypothetical net benefits. The variation in the rates of return for every projection year quantifies volatility. Again, the “last” rate of return is the final average ex post interest on the contributions paid.

We first compute the present value of the net benefits an individual born in $c$ receives throughout his whole life $PV(nb)^c_{py}$. It results from discounting the annual net benefits to the birth year $c$ with a discount factor $(1+r)$ and weighting them with the annual survival rate $sr_{c,s}$, i.e. the probability that an individual of cohort $c$ is alive in year $s$.

$$PV(nb)^c_{py} = \sum_{s=c}^{c+100} nb_{s-c,s} \cdot sr_{c,s} \cdot (1+r)^{c-s}. \hspace{1cm} (14)$$

The internal rate of return is the interest rate that sets equation (14) equal to zero. So far we have not indicated the underlying projection year, although we know that the projected individual payments $h_{a,i,s}$ depend on $s \geq py$ (see equation (3) and (4)). The projection year of the internal rate of return is now indicated with
We index the actual internal rate of return for the agent born in $c$, depending on the respective projection year $py$, as well as for the policy scenario $k$. The index $ac$ stands for the actual policy, and again $db$ for the db-policy, respectively $dc$ for the dc-policy under the legislation of 1957. The internal rates of return are solved with an iteration process so that

$$
\sum_{s=c}^{c+100} nb_{s-c,s} \cdot sr_{c,s} \cdot (1 + irr_{c,py}^j)^{c-s} \equiv 0.
$$

An important characteristic of this internal rate of return $irr_{c,py}^k$ is that it covers all information of the status quo in the projection year. The individual calculates the internal rate of return with the past information and only projects the observed status quo of the respective projection year. Hence, the agent assumes that he will be confronted with the same age-specific average transfers as the living entering retirees of the projection year.

### 2.4 Benchmarking riskiness

In this section we discuss the volatility of internal rates of return. It is first of all crucial to point out the underlying assumption about the individual’s expectation as we determine volatility from the individual’s point of view. Here, we assume that the individual projects his or her rate of return on basis of the actual data, thus being myopic towards all future development. The individual only observes the average expenditures for an average entering and existing retiree, as well as the average contributions and tax payments. Given this information, he finally projects them with the real per-capita growth rate. By adapting his projections to the actual data for every projection year, he is subsequently exposed to volatility in case the new average observations differ from the projected ones.

We now want to focus on the difference in volatility in the actual policy compared to the db- and dc-scenario. Since db and dc pay-as-you-go systems “automatically react” to the demographic development, we want to propose here form resulting volatility as benchmark. We are thus comparing the volatility of the actual policy with either the db- or the dc- scenario. Hence we define the riskiness of the German pension system for an individual as the difference between the actual and the db- or dc-scenario. It is now to answering the following question: How big is the risk for an individual in the actual system compared to
the risk the individual would have faced in case of the implementation of a db- or dc-system in 1957?

The point is that the individuals now only consider the difference of volatility as riskiness. Thus, we assume that the individuals are not shocked by periodical adjustments due to demographic developments. This means that we relax the assumption of complete myopia. In contrast to this, any other developments of the system induce riskiness to the individuals.

3 Calculation and results

3.1 Data

In order to calculate the internal rates of return for the German pension system the following data is required: population, survival rates, macroeconomic aggregates, age-specific micro profiles of the German pension system, and growth rates, all of them from the years 1957 up to 2003. Furthermore, a population projection with the projected survival rates for the future is needed.

Demography

The population data for the past as well as the assumptions for the projections are taken from the “German Bureau of the Census” (Statistisches Bundesamt). The past cohort and age-specific survival rates are available as electronic data and taken from periodical life tables of the German Bureau of the Census.\(^3\) For the projection of the population and the survival rates we use the assumptions of the medium variant of the 10th coordinated population projection.\(^4\)

Aggregates of the German pension system

The time-series of aggregates for the Old Länder starting from the year 1957

\(^3\)As there are several years only with abridged period life-tables – for the years before the abridged life-table 1979/1981 the electronically published life-tables are only available in a ten-years-interval – we develop the missing period life-tables, i.e. survival rates, by adjusting the published data linearly.

\(^4\)The calculation method of the projection is based on Bonin (2001) and calculated with the cohort component method proposed by Leslie (1945).

\(^5\)The projected old-age dependency ratio – individuals aged over 60 in relation to the 20 to 59 year-olds – amounts to 0.45 in the year 2003, rises only slightly up to 0.49 in 2015 and increasing steadily to 0.77 in the year 2058. We use this ratio as approximation for the ratio of retirees to labor force.
and the New Länder beginning from the year 1991 are published by the “Association of German Retirement Insurance Organizations” (Verband Deutscher Rentenversicherungsträger) in VDR (2004). The corresponding budget is composed as follows: The revenues of the German pension system are roughly split into age-specific contributions and federal subsidies. In the year 2003, the contributions made up about 73 percent and the subsidies about 26 percent of the revenues in Germany. The share of subsidies was the same in the year 1957 but decreased to 15.4 percent in the year 1973 to increase again. The largest part of all expenditures in 2003 were the pension expenditures with 89 percent. In the past, the overall expenditures expressed as GDP ratio raised from about 6.5 percent in the 1957 to 11.1 percent in the year 2003.

In line with our method, we need to split up the aggregates and calculate the share of pension expenditures to entering retirees. We do this by multiplying the average pension expenditure for an entering retiree with the respective amount of entering retirees. The residual amount is distributed to the existing retirees. Furthermore, we proportionally add up all other expenditures of the German pension system to the pension expenditures for existing and entering retirees.

Note that there is a special treatment for the development of the federal subsidies. The legal status quo assumes that the aggregate of the federal subsidies – not considering the so called “additional federal subsidies” (zusätzlicher Bundeszuschuss) – increases with the per-capita real growth rate and rises with increasing contributions. Thus, the per-capita payment increases more than proportional to the per-capita contributions in the case of Germany, i.e. we have a decreasing tax base and increasing retirees. As a lower bound and probably as a more realistic long-term scenario for the development of the federal subsidies, we assume that the federal subsidies increase according to the development of the tax base and the contribution rate. Thus, the per-capita payments increase with

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6 In addition to this there is a small share of further revenues, namely refunding and other revenues, which are added to the subsidies as well as a small share of non age-specific wealth returns, which we are not considering.

7 This data is also published in VDR (2004).

8 The individuals also receive health- and long-term-care benefits, both ranging about seven percent, as well as a small share of child-rearing benefits. A small share of refunds for contributions as well as an amount of benefits for participation at working life remain, which however we do not consider here.
the per-capita growth rate and the increase of the contribution rate.

Determining age- and sex-specific profiles

Before projecting the rescaled per capita net payments, we need to distribute the aggregates on age-specific profiles per-capita of the population. Before doing so, the following step of empirical work is needed: Age-specific profiles per capita of the population ($\theta_{i,s}$, see equation (3)) are obtained by modifying the respective raw age-specific micro data. This is done by multiplying the raw micro data with the ratio of the relevant people (e.g. entering retirees) to the total population of the respective age. We can now simply calculate the age- and sex-specific profiles by extending the method onto males and females and their profiles, respectively.\footnote{See Bonin (2001) for a more detailed set-up.}

Thus, we rescale eight different age- and sex-specific raw profiles with the four types of aggregates.

The raw contribution profiles and the amount of contributors are published by the Association of German Retirement Insurance Organizations since the year 1984. Contrary to the ordinary earnings profiles, the difference between the male and female contribution-profile is smaller. The reason for this is the fact that the “income threshold” (Beitragsbemessungsgrenze) in the German pension system simply caps the higher incomes of the males.\footnote{Because micro data on contributions is available only starting from the year 1984 we hold the profiles constant for the years before.}

The federal subsidies are distributed with a constant mix of profiles: half sales and half income taxes – which approximately meets the share of direct and indirect taxes. These are obtained from the German Income and Expenditure Survey (“Einkommens- und Verbrauchsstichprobe 1993”).

Last, we need raw age- and sex-specific profiles and the quantity of entering and existing retirees in order to develop the expenditure side. All are published annually by the Association of German Retirement Insurance Organizations. The volumes for the existing retirees were published by the “Federal Minister for Labor” (Bundesministerium für Arbeit) before 1984.\footnote{The profiles for existing retirees are only available in five-year age-groups before 1984. We adjust them linearly. The profiles for entering retirees are obtained with annual age groups.} These age- and sex-specific profiles consist of average pensions of both insurance classes (workers and em-

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employees) and all the pension types.\footnote{One exception is the orphan’s pension as there is no age-specific data for the earlier volumes.}

**Projection of net payments**

Finally, we need the real growth rates of the past periods as well as an assumed growth rate for the years after 2003. In line with the assumptions of the so called “Rürup-Commission” we take a real growth rate of 1.5 percent.\footnote{See Kommission für die Nachhaltigkeit der Sozialen Sicherungssysteme (2003).}

Since we do not address the risk aspect to growth development, we assume that the individual has information about all real growth rates until 2003. The time-series of the growth of the average per-capita gross payments for workers and employees are taken from VDR (2004). The consumer-price index to deflate the growth rates and aggregates are released by the “German Council of Economic Advisors” (Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung). The per-capita contributions and federal subsidies to the German pension system for the future are projected with the respective growth rate and adjusted according to section 2.2 after 2003.\footnote{This yields a contribution rate of percent 24.55 percent in 2050.}

The per-capita expenditures depend on the actual pension adjustment formula for entering and existing retirees in case of the actual policy. We thus account for the so called “modified gross-wage adjustment” and the sustainability factor both reflecting the latest reform measures introduced.\footnote{See Kommission für die Nachhaltigkeit der Sozialen Sicherungssysteme (2003) for a detailed discussion.}

### 3.2 Sustainable internal rates of return

In this chapter we depict the sustainable internal rates of return of different cohorts under the actual policy. They are to be interpreted as projected ex post internal rates of return realized at the end of the individuals’ life-cycle. We briefly want to relate our calculations to the rates of return calculated in the literature. Common scenario-based approaches in the literature often use the unrealistic scenario of a standard retiree (“Eckrentner”) with 45 years of annual average income history using the regular retirement path at the age of 65. In contrast to this we – apart from contributions – consider federal subsidies financed via taxation and all expenditures of the German pension system for an average male
and female agent. Besides additional expenditures included in the different budget positions, we especially account for all different kinds of paths into retirement. We furthermore extend the analysis to non-insurance credits (e.g. education and child rearing). We develop the per-capita payments with age- and sex-specific profiles per capita of the population and the aggregates. Thus, we do not relate the individual payments of a contributor over the life-cycle to his claims of pension benefits. This is certainly a difference to the standard scenario-based approaches. On the other hand, this method is necessary to include on average all possible cases if individual eligibility. Thus, as already indicated, we can include federal subsidies and non-insurance covered benefits.

Our calculations are done with stochastic survival rates. But we do not use stochastic probabilities for other possible events of life-risks, e.g. disability, survival of spouse, probabilities to receive different non-insurance credits. In this respect we may have a setback concerning these risk categories.

Figure 1: **Sustainable internal rates of return for different cohorts – actual policy**

(With different real growth rates)

Figure 1 shows the sustainable internal rates of return for different cohorts. The rate of return decreases from 1.4 percent (for the male cohort 1940 assuming a real growth rate of 1.5 percent) and reaches zero for all cohorts after 2000. In
contrast to the rates of return for the males, the internal rates of return for the women remain positive. The female cohort of 1940 starts off with an internal rate of return of 3.9 percent which decreases to 2.1 percent for the female cohorts of 2010.

The reason for the higher rates of return for females is that they pay a lower share of contributions and federal subsidies financed via taxation and mainly paid by men. However, they in average receive e.g. almost all the pensions for survivors and credits for child-rearing. Hence, we get a disparity between financing and receiving non-insurance covered benefits with an increasing contribution rate in the future. Furthermore, we see for both males and females that the substantial higher contribution rates in the 1990s and the following reforms in the beginning of 2000 reduce the rates of return notably for all cohorts born after 1940. The other half of the reduction in rates of return in comparison to the cohort 2010 proceeds gradually through the sustainability factor and the higher contributions and federal subsidies. It is important to see that the increase in life-expectancy of younger cohorts does not compensate the cuts. Figure 1 also shows that we have substantial negative rates of return for younger males in case of a smaller future real growth than 1.5 percent. In the (pessimistic) scenario of zero real growth we see that this lack in growth may induce the major negative impact for younger and future cohorts.

3.3 The volatility of the internal rates of return

In this section we want to quantify the volatility of the internal rates of return of the German pension system. Therefore we analyze the development of the internal rate of return from an ex ante perspective at the birth of an individual. We choose the cohort born in the year 1957 because the birth-year of this cohort is equivalent to the year of the implementation of the wage-indexed German public pension scheme. The cohort 1957 is the first one fully exposed to the “new” social security system after World War II. This volatility can be seen as upper bound for younger cohorts from today’s level of information. The riskiness may change with further changes of the status quo in the future. The older cohorts are exposed to similar risk but since they partly face the existing system before 1957, which is not examined here, we can not compare them.

In Figure 2 we depict the rates of return for the male and female cohort for
every projection year and for the different policy scenarios: actual policy, dc-policy, and db-policy. In this section we will focus on the male cohort as all the changes are similar but lower for the female cohort (see Figure 1).

![Figure 2: The volatility of the rates of return for the cohort 1957](with 1.5 percent real growth rate)

We are thus able to show that there is only a slight difference between the db- and dc-policy in the last five decades, i.e. the first 47 years of the individual’s life-cycle. The reason for this is that adjustments induced by demographics are relative small in this period or rather the rate of return responds in the same way as long as the individual is still a contributor (which was the case). However, there is a substantial difference between both scenarios and the actual policy, which directly leads to the first main result of the paper: There is a substantial difference in their internal rate of return of actual policy compared to the dc- or db-policy. This difference is due to a multitude of legislative changes and responses in retirement behavior of the individuals instead of having an indefeasible rule as in the dc- or db-scenario. Both alternative scenarios allow only the retirement facts and behavior of the year 1957. Thus the “expected” rate of return of the individual obviously changes less than in the actual scenario.
Another interesting point is that the rates of return for all three scenarios are almost similar in the beginning of the 1960s and return to be similar in the beginning of the 1990s. This means that the cohort 1957 would have been better off by assuming the status quo of the German pension policy at their birth to be constant. The difference of the actual scenario and the db- or dc-scenario in the period between the beginning of the 1960s and 1990s, as shown in Figure 1, displays a substantial risk of the system for the individuals. This is the case because the German pension policy is characterized by permanently changing degree of generosity. Periods of excessive windfalls to the retirees result in phases of massive consolidation by rising contributions and/or redemption of the benefit level. Finally, the period of the 1990s shows the impact of the German reunification and the rising demographic pressure for all three scenarios.

For illustrative purposes we want to mention some major changes in the German pension expenditure chronologically. These give an intuition for the changes of the actual internal rate of return as depicted in Figure 2. It is important to mention that there are also lagged reactions of the individuals to legal changes affecting the actual internal rate of return.

In the 1960s (i) an increasing share of credits for times spent in war, (ii) higher credits for education, as well as (iii) an increasing participation to the pension system all increased the rate of return. These benefit extensions are followed by a rising contribution rate of 14 percent in 1965 to 17 percent in 1970. In the beginning of the 1970s a major pension reform introduced more generous rules for diverse paths of early retirement and, among other things, minimum pensions. Again this reform was followed by cuts in the internal rate of return due to a changed and capped indexation of the pension adjustment formula at the end of the 1970s. In the beginning of the 1980s early retirement for pensions due to incapability to work was introduced. This was followed again by cuts of the internal rate of return due to increasing contribution rates from 18 percent in 1982 to 19.2 percent in the year 1986. In the following years the contribution rate decreased to 17.5 percent in 1993 but blew up to 19.2 percent in 1994 due to the German reunification. Furthermore the federal subsidies increased significantly.

\[\text{The German Unification imposes a financial compensation of about 2.5 percent in the year 2003 on the West German contribution and tax payer. West Germany usually runs a surplus which is partly used to finance the deficit in East Germany. Note, however, that due to a severe east-west migration, a substantial number of West-German contribution payers are in fact of East-German origin, that is the regional redistribution can not be seen as a burden posed by}\]
in the 1990s. Finally deductions for early retirement in the beginning of 2000 reduced the rate of return.

The period after 2003 is then dominated by adjustments due to the modified-gross indexation and the sustainability factor. On the one hand, it should be emphasized that the indexation – in contrast to the dc-scenario – does not burden the cohort 1957 with the full costs of demographic transition. On the other hand the actual policy and dc-policy converge towards almost similar rates of return for the male cohort of 1957. A high difference exists between the db-policy and the other policies, i.e. the actual and the dc-policy. It arises because the 1957-born individual is almost not affected to adjustments due to demographic transition in the db-policy. At this point it is important to mention that we do not consider the impact of uncertainty of different macroeconomic variables on the optimal design of social security schemes. This means that for our purpose we do not need to favor any of the two alternative adjustment rules, e.g. defined-contribution or defined-benefit. We rather specify a corridor of riskiness. Depending on the optimal choice of the dc- or db-policy, the riskiness amounts in the difference between the actual and the dc- or db-policy, respectively.

Table 1: Descriptive statistics of internal rate of returns

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<th></th>
<th>Male cohort 1957</th>
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<tr>
<td></td>
<td>actual policy</td>
<td>dc-policy</td>
<td>db-policy</td>
</tr>
<tr>
<td>Max</td>
<td>4.67</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Min</td>
<td>0.46</td>
<td>0.47</td>
<td>2.07</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.38</td>
<td>0.95</td>
<td>0.30</td>
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<thead>
<tr>
<th></th>
<th>Female cohort 1957</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>actual policy</td>
<td>dc policy</td>
<td>db-policy</td>
</tr>
<tr>
<td>Max</td>
<td>5.66</td>
<td>4.42</td>
<td>4.42</td>
</tr>
<tr>
<td>Min</td>
<td>2.63</td>
<td>2.11</td>
<td>3.64</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.96</td>
<td>0.81</td>
<td>0.19</td>
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Table 1 shows the standard deviation of the actual policy and the different scenarios. We find that policy changes in the past and future impose a substantial risk on the individual’s actual calculated rates of return compared to the db- or dc-policy. The standard deviation in the actual scenario is about 0.4 (3.5) times higher than in the dc- (db-) policy for males. The volatility of the female cohort born in 1957 is slightly lower. The standard deviation of the actual policy is about 0.15 (3.25) times higher than in the dc- (db-) policy.

East Germans to their Western counterparts.
Another important finding is the minimum value of the internal rate of return. It corresponds to the sustainable internal rate of return in the previous section since the last value of the life-cycle is the lowest. The minimum value of the actual policy is almost similar (close) to the dc-policy for males (females). The sustainable rate of return of the db-policy is higher for males and females. We take this a starting point for a calculation of sustainable rates of return of all the three policies for different cohorts of an average individual.

Figure 3: **Sustainable internal rates of return for different cohorts - actual vs. dc- and db-policy**

(average individual with real growth rate of 1.5 percent)

Figure 3 shows the results and leads to a summarization of the discussion in the last two subsections. It shows that sustainable internal rates of return are higher than the actual policy under both alternative scenarios, namely defined-contribution and defined-benefit policy, for the cohorts born after 1990. This means that the actual policy induces lower rates of return for the youngest and future cohorts and imposes a substantial volatility on individuals shown exemplarily for the cohort 1957.
4 Conclusion

In this paper we analyze the development of the German pension system in terms of internal rates of return by constructing a projection method for their calculation.

The main issue of this paper is the risk, to which individuals in the German pension system are exposed. Assuming a myopic individual born in 1957, we show that the volatility of rates of return is between 0.4 and 3.5 times higher for males and 0.15 and 3.25 times higher for females than in a defined-contribution or defined benefit scenario based on the status quo of 1957. This is a remarkable result: If the German pension system would have been designed as defined-contribution or defined-benefit rule rather than the actual discretionary policy, the risk would be significant lower. Furthermore, when discussing risk issues concerning private capital-based provisions for old-age, one should not neglect the riskiness public pension system may bring along for individuals.

An important byproduct of this study is the calculation of sustainable rates of return including federal subsidies and non-insurance covered benefits. We show that the sustainable rates of return are close to zero for the youngest and future male cohorts and positive for females for optimistic growth scenarios. This can be explained as follows: Males are burdened by a higher share of taxes which are adjusted during demographic transition. In contrast to this however, the non-insurance covered benefits do not rise during demographic transition. In other words, the neglecting of non-insurance covered benefits and federal subsidies for the calculations of net benefits simply leads to an overestimation of internal rates of return especially for male cohorts. If real growth rates are lower things will turn worse. Furthermore the calculations reveal ex post sustainable internal rates of return under the defined-contribution or defined-benefit system that are at all events higher than the actual rates of return for an average individual younger than the cohort 1990.

Summing up it can be said that a defined-contribution or defined-benefit policy induces substantially less volatility on the exemplary 1957 born individual than the actual policy does and the rates of return are higher for the youngest and future cohorts. But what it comes down to is that besides the youngest and future cohorts even older cohorts may prefer a defined-contribution or a defined-benefit policy. This is due to the fact that they would be confronted with a substantial lower volatility under an indefeasible rule.
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