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Countercyclical Taxation and Price Dispersion

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Countercyclical Taxation and Price Dispersion*

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Abstract

In this paper, we explore the benefits from a supply-side oriented fiscal tax policy within the framework of a New Keynesian DSGE model. We show that countercyclical tax rules, which are contingent on the observed welfare gap or on the cost-push shock and levied on value added, remarkably reduce the adverse impact of cost-push shocks on welfare. We state that the tax rule establishes a path for the evolution of marginal cost at the firm level that largely prevents built up of price dispersion. We highlight that this tax policy is also effective under a balanced-budget regime. Hence, fiscal policy can disencumber monetary policy in the light of cost-push shocks.

JEL-classification: E32, E61, E62

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1 Introduction

A large strand of literature has focused on welfare costs of exogenous shocks within the New Keynesian framework with monopolistic distortions and nominal rigidities. In this respect, it is well-known that demand shocks can be stabilized with no welfare costs by means of monetary policy: an adjustment of the real interest rate pushes both, output and inflation, to the desired levels of society (Clarida, Gali and Gertler, 1999; and Woodford, 2003).

This is not the case for supply shocks. Although a sufficiently strong reaction of the real interest rate to inflation is argued to be the best response to limit the adverse effects of cost-push shocks on the lifetime utility of a representative agent, a trade-off between inflation and output stabilization emerges. While monetary policy steers the inflation rate closer to the welfare-optimal level, it has to accept costs stemming from larger output variability. The overall welfare costs of nominal rigidities are estimated up to three percent in consumption equivalents (Canzoneri, Cumby and Diba, 2007; and Gertler and Lopez-Salido, 2007). This highlights that monetary policy does not have a direct leverage on the supply side.

In this paper we emphasize that a discretionary countercyclical tax policy can almost completely stabilize cost-push shocks. In particular, we propose a simple tax rule that builds on value-added taxes as an instrument. Our key finding is that fiscal authorities can set up a path for value-added taxes that evolves countercyclically to cost-push shocks, and thus reduces any cost pressure at the firm level which improves welfare substantially.\(^1\) Those firms that are called upon to reset prices will then built on the promise of fiscal authorities to smooth away cost-push shocks and set prices in the neighborhood of those price setters that have to leave prices unchanged. When fiscal policy is allowed to cushion changes in tax rates by debt rather than government expenditures we state that debt adopts a near random walk behavior in the presence of cost-push shocks. The levels of tax rates and a sufficiently strong feedback from tax rates to changes in the level of existing debt are determined by long-run solvency considerations such that in steady state the budget is balanced (Canzoneri, Cumby and Diba, 2003; Linnemann and Schabert, 2003).

Although the general idea of simple fiscal rules has not been new, authors so far have mainly focused on the idea of classical demand management, where government ex-

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\(^1\)This seems in particular important as Schmidt-Grohe and Uribe (2006) report evidence from a medium-scale model which comprises a number of real and nominal frictions that price stickiness emerges at the most important distortion.
penditures are conditioned on the output gap such as J.B. Taylor (2000). In the New Keynesian framework, fiscal policy has in the majority of cases been treated exogenously so far. A notable exception is Leith and Wren-Lewis (2007), who explore the role of countercyclical fiscal policy in a full-fledged DSGE model and analyze commitment solutions for the case of a small open economy. They report evidence that price dispersion can be completely wiped out by commitment solutions when fiscal authorities employ four instruments, namely debt, government expenditures and taxes on labor and value added. Our paper is complementary to the work of Leith and Wren-Lewis: (i) We show that optimal fiscal rules under discretion and simple rules substantially improve welfare. (ii) We report analytical evidence that such rules are also effective under a balanced-budget regime by means of MSV-solutions. (iii) We present robustness results from a sensitivity analysis with respect to deep parameters. (iv) We simulate the behavior of the economy with the occurrence of markup shocks.

The paper is structured as follows: In Section 2, the basic model is introduced. Section 3 presents analytical results on fiscal rules and price dispersion. In Section 4 we compare active fiscal policy, where the fiscal policy maker pursues the countercyclical tax rule, to a passive stance of fiscal policy by using a numerical approach. In Section 5 we conduct robustness analysis. Section 6 summarizes the main findings and concludes.

2 The Model

In this section we present a New Keynesian DSGE model with firms, households, the central bank and fiscal authorities. As standard, firms are categorized into the final good sector and a continuum of intermediate good producers. Intermediate good producers have some monopoly power over prices that are set in a staggered way following Calvo (1983). Households obtain utility from consumption, public goods, leisure and invest in state contingent securities. Monetary authorities are guided by a simple Taylor rule. Government expenditures are financed by distortionary taxes levied on value added or debt. Fiscal policy is implemented by tax and spending rules.

The model is built on the framework of Gali, Lopez-Salido and Valles (2007), Leith and Wren-Lewis (2007), and Linnemann and Schabert (2003) by sharing the same kind of features such as debt financed expenditures, state contingent tax rules and staggered price setting. In particular we highlight the role of an active fiscal policy compared to a neutral stance to fight the welfare costs of price dispersion.
2.1 Final Good Producers

The final good is bundled by a representative firm which operates under perfect competition. The technology available to the firm is:

\[ Y_t = \left[ \int_0^1 Q_t(i) \frac{\varepsilon_t - 1}{\varepsilon_t} di \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}} , \]  

(1)

where \( Y_t \) is the final good, \( Q_t(i) \) are the quantities of the intermediate goods, indexed by \( i \in (0, 1) \) and \( \varepsilon_t > 1 \) is the time-varying elasticity of substitution in period \( t \). Profit maximization implies the following demand schedules for all \( i \in (0, 1) \):

\[ Q_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_t} Y_t. \]  

(2)

The zero-profit theorem implies \( P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon_t} \right]^{\frac{1}{1-\varepsilon_t}} \), where \( P_t(i) \) is the price of the intermediate good \( i \in (0, 1) \). In a similar way to Smets and Wouters (2003), we assume that \( \varepsilon_t \) is a stochastic parameter. In this context, we define \( \Phi_t = \frac{\varepsilon_t}{\varepsilon_t - 1} \) reflecting the time-varying markup in the goods market and assume that \( \Phi_t = \Phi + \hat{\Phi}_t \). Thereby, \( \hat{\Phi}_t \) is i.i.d. normal distributed, and \( \Phi = \frac{\varepsilon}{\varepsilon - 1} \) is the deterministic markup which holds in the long-run flexible price steady state.

2.2 Intermediate Good Producers

Firms indexed by \( i \in (0, 1) \) operate in an environment of monopolistic competition. The typical production technology is given by:

\[ Y_t(i) = N_t(i) , \]  

(3)

where \( N_t(i) \) denotes labor services. Nominal profits by firm \( i \) are given by:

\[ \Pi_t(i) = \left( 1 - \tau_{t}^{VAT} \right) P_t(i) Y_t(i) - W_t N_t(i) , \]  

(4)

with \( Y_t(i) = Q_t(i) \) and \( \tau_{t}^{VAT} \) denotes a value-added tax with \( \tau_{t}^{VAT} \in (0, 1) \). As cost minimization implies that real marginal costs are equal to real wages with \( \varphi_t = \omega_t \) the profit function can be rewritten as follows:

\[ \Pi_t(i) = \left[ \left( 1 - \tau_{t}^{VAT} \right) P_t(i) - P_t \varphi_t \right] Y_t(i) . \]  

(5)

The representative firm is assumed to set prices as in Calvo (1983), which implies that the price level is determined in each period as a weighted average of a fraction of firms
\((1 - \theta_p)\) which resets prices and a fraction of firms \(\theta_p\) that leaves prices unchanged:

\[
P_t = \left[ (1 - \theta_p)(\hat{P}_t)^{1-\epsilon_t} + \theta_p P_{t-1}^{1-\epsilon_t} \right]^{\frac{1}{1-\epsilon_t}},
\]

where \(\hat{P}_t\) is the optimal reset price in period \(t\).

Each firm \(i\) that is called upon to reset prices solves the following intertemporal profit maximization problem subject to its demand function for \(Y_t(i)\):

\[
\max_{\hat{P}_t(i), N_t(i), Y_t(i)} \left\{ E_t \left( \sum_{k=0}^{\infty} (\theta_p \beta)^k \Delta_{t,t+k}[\hat{P}_t(i)(1 - \tau_{t+k}^VAT) - P_{t+k} \varphi_{t+k}]Y_{t+k}(i) - \varrho_{t+k} \left[ Y_{t+k}(i) - \left( \frac{\hat{P}_{t+k}(i)}{P_{t+k}} \right)^{-\epsilon_t} \right] Y_{t+k} \right) \right\},
\]

where \(\varrho_{t+k}\) denotes the Lagrangian multiplier in period \(t + k\), and \(\Delta_{t,t+k}\) denotes the stochastic discount factor of shareholders, to whom profits are redeemed. It is defined as \(\Delta_{t,t+k} = (U_C(C_{t+k})/U_C(C_t))\). Combining the first-order conditions, we obtain:

\[
E_t \left\{ \sum_{k=0}^{\infty} (\theta_p \beta)^k \Delta_{t,t+k}Y_{t+k}(i) \left[ \hat{P}_t(i)(1 - \tau_{t+k}^VAT) - \Phi_{t+k} P_{t+k} \varphi_{t+k}(i) \right] \right\}.
\]

### 2.3 Households

We assume a continuum of households indexed by \(j \in (0, 1)\). A typical household seeks to maximize lifetime utility:

\[
E_0 \sum_{k=0}^{\infty} \beta^k U_{t+k}(j),
\]

where \(\beta\) denotes a discount factor with \(\beta \in (0, 1)\), and period utility is given by:

\[
U_t(j) = (1 - \chi) \left( \frac{1}{1 - \sigma} C_t(j)^{1-\sigma} \right) + \chi G_t - \frac{1}{1 + \eta} N_t(j)^{1+\eta}.
\]

\(\sigma\) is a coefficient of risk aversion, \(\eta\) is the inverse of the Frisch elasticity of labor supply, and \(\chi \in (0, 1)\) measures the relative weight of public consumption \(G_t\). \(C_t(j)\) are the real consumption expenditures of household \(j\). The sequence of budget constraints reads:

\[
C_t(j) + \frac{B_{t+1}(j)}{R_t P_t} \leq \frac{W_t N_t(j)}{P_t} + \frac{\Pi_t(j)}{P_t} + \frac{B_t(j)}{P_t}.
\]

Each household decides on consumption expenditures \(C_t(j)\) and bond holdings \(B_{t+1}(j)\) and receives labor income \(W_t N_t(j)\), dividends from profits \(\Pi_t(j)/P_t\) and the gross return on bonds purchased \(B_t(j)\).
Maximizing the objective function subject to the intertemporal budget constraint with respect to consumption and bond holdings delivers the following first-order conditions:

\[(1 - \chi)C_t^{-\sigma} = \lambda_t, \quad (12)\]
\[N_t^\eta(j) = \lambda w_t, \quad (13)\]
\[\frac{1}{P_t}\lambda_t = \beta E_t \left[ \lambda_{t+1} \frac{1}{P_{t+1}} R_t \right], \quad (14)\]

where \(\lambda_t\) denotes the Lagrangian of the budget constraint (11). Combining the first order conditions yields the consumption Euler equation and the labor supply schedule:

\[C_t^{-\sigma} = \beta R_t E_t \left[ C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \right], \quad (15)\]
\[\frac{N_t^\eta(j)}{C_t^{-\sigma}} = \frac{W_t}{P_t} (1 - \chi). \quad (16)\]

Note that we can drop the index \(j\) for consumption \(C_t\) due to the existence of contingent claims markets, which equalize wealth across households at each point in time.

2.4 Fiscal Authorities

The government issues bonds and collects value-added taxes. It uses its receipts either to finance government expenditures or interest on outstanding debt. The real government budget constraint reads:

\[R_t - 1 b_{t+1} + \tau_t V A T Y_t = \frac{B_t}{P_t} - G_t. \quad (17)\]

Letting \(\bar{X}\) denote the deterministic steady state of a variable \(X_t\), and \(b_t = \frac{1}{\bar{P}} [(B_t/P_{t-1}) - (\bar{B}/\bar{P})]\) with \(\bar{B} = 0\), the budget constraint can be rewritten as:

\[R_t - 1 b_{t+1} + \tau_t V A T \frac{Y_t}{\bar{Y}} = b_t \frac{P_{t-1}}{P_t} + G_t. \quad (18)\]

The government imposes taxes according to the rule

\[\tau_t V A T = \Phi_t \chi^1 b_t \chi^2, \quad (19)\]

which is conditioned on the predetermined state variables \(\Phi_t\) and \(b_t\). In principle a sufficient strong response to the change of the level of outstanding debt \(\chi_2 > 0\) assures uniqueness and determinacy. A parameter \(\chi_1 < 0\) denotes a countercyclical fiscal tax policy. Additionally, we consider a simple tax rule. Note that in literature simple rules are
predominantly interpreted as rules where the instrument responds to observable macroeconomic variables, e.g., to the inflation rate or for instance to the welfare gap (e.g., Schmitt-Grohe and Uribe, 2007). Therefore, we opted to consider also an alternative tax rule which is conditioned on the outstanding debt and the welfare gap $X_t$:

$$\tau_t^{\text{AT}} = X_t^{\chi_1}b_t^{\chi_2}. \quad (20)$$

The welfare gap is defined as $X_t \equiv Y_t/Y_t^f$, i.e., as the ratio between the actual output and output which would occur under flexible prices. The superscript $f$ denotes flexible prices. We determine the respective parameter $\chi_1$ for both types of fiscal tax rules such that the rules are optimal from the perspective of a discretionary fiscal policy. As $\Phi_t$ and $b_t$ are predetermined state variables equation (19) describes the optimal feedback rule from a discretionary perspective.

In Section 3, we derive analytical results for the optimal tax rule (19), and we use both types of tax rules in sections 4 and 5, where we consider the welfare implications of both rules and check the robustness by using a numerical approach.

### 2.5 Market Clearance

In clearing of factor and good markets the following conditions are satisfied:

$$Y_t = C_t + G_t,$$

$$Y_t(j) = Q_t(j),$$

$$N_t = \int_0^1 N_t(j) \, dj.$$

### 2.6 Linearized Equilibrium Conditions

In this section we summarize the model by taking a log-linear approximation of the key equations around a symmetric equilibrium steady state with zero inflation and zero debt. In the following, a variable $\hat{X}_t$ denotes the log-linear deviation from the steady state value: $\hat{X}_t = \log(X_t) - \log(\bar{X})$.

**Households** The consumption Euler equation reads:

$$\dot{C}_t = E_t \dot{C}_{t+1} - \sigma^{-1}(\tilde{R}_t - E_t \tilde{\pi}_{t+1}) , \quad (21)$$

where $\tilde{\pi}_t$ is defined as $\tilde{\pi}_t \equiv \hat{P}_t - \hat{P}_{t-1}$, and we used that in the steady state $\bar{R} = \beta^{-1}$ which follows directly from the consumption Euler equation. Under perfectly competitive labor
markets the labor supply schedule is equal to:

\[ \hat{w}_t = \eta \hat{N}_t + \sigma \hat{C}_t. \]  

(22)

**Firms** Log-linearization of (6) and (8) around a zero inflation steady state yields the dynamics of inflation as a function of the wage \( \hat{w}_t \), a stochastic markup \( \hat{\Phi}_t \) and tax rates \( \hat{\tau}^{VAT} \):

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa [\hat{w}_t + \nu \hat{\tau}^{VAT} + \hat{\Phi}_t], \]  

(23)

with \( \kappa \equiv (1 - \theta_p)(1 - \beta \theta_p)/\theta_p \), and \( \nu \equiv \hat{\pi}^{VAT}/(1 - \hat{\pi}^{VAT}) \).

**Fiscal authorities** Log-linearizing the budget constraint around a zero steady state debt yields the following approximation up to first order:

\[ b_{t+1} + \gamma_G (\hat{\tau}_t^{VAT} + \hat{Y}_t) = \beta^{-1} b_t + \gamma_G \hat{G}_t, \]  

(24)

where for the case of a balanced budget (24) simplifies to \( \hat{G}_t = \hat{\tau}_t^{VAT} + \hat{Y}_t \). The parameter \( \gamma_G \) denotes the steady state government share which is equal to \( \hat{\tau}^{VAT} \) implied by a balanced budget in steady state.

A log-linearized fiscal spending rule is given by:

\[ \hat{G}_t = -o_Y \hat{Y}_t, \]  

(25)

where \( o_Y > 0 \) denotes the sensitivity of government expenditures with respect to output movements.\(^2\)

The simple tax rule is the log-linearized complement to (19):

\[ \hat{\tau}_t^{VAT} = \chi_1 \hat{\Phi}_t + \chi_2 b_t. \]  

(26)

Correspondingly, the log-linearization of the alternative tax rule (20) based on the welfare gap \( x_t \) is given by:

\[ \hat{\tau}_t^{VAT} = \chi_1 x_t + \chi_2 b_t. \]  

(27)

The welfare gap is defined as \( x_t \equiv \hat{Y}_t - \hat{Y}_t^f \). In the following we will refer to a passive fiscal policy if \( \chi_1 = 0 \) such that fiscal policy abstinets from following a countercyclical path for

\(^2\)Note that the welfare criterion (see section 4) is derived for the linear case: \( o_Y = 0 \) and \( o_Y = 1 \).
Monetary Policy  Monetary policy is assumed to follow the Taylor rule:

\[
\tilde{R}_t = (1 - \phi_p)\tilde{R}_{t-1} + \phi_p[\phi_\pi\hat{\pi}_t + \phi_x x_t],
\]

where \(\phi_\pi\) and \(\phi_x\) capture the reaction coefficients with respect to the inflation rate and the output gap \(x_t\); \((1 - \phi_p)\) with \(0 \leq \phi_p \leq 1\) denotes the degree of interest rate smoothing on part of the central bank.

Market Clearing  Market clearing requires that the following relation holds:

\[
\hat{Y}_t = \gamma_C \hat{C}_t + \gamma_G \hat{G}_t,
\]

where \(\gamma_C\) denotes the consumption share, which is equal to \((1 - \bar{\tau}_{VAT})\). Using (25) and (29) we can rewrite the consumption Euler equation as follows:

\[
\hat{Y}_t = E_t \hat{Y}_{t+1} - \frac{\gamma_C}{\sigma(1 + \gamma_G\gamma_Y)}(\tilde{R}_t - E_t \hat{\pi}_{t+1}).
\]

Flex-price equilibrium  The flex-price equilibrium is obtained by equating \(\hat{w}_t = \eta \hat{N}_t + \sigma \hat{C}_t\) and \(\hat{\phi}_t = \hat{w}_t\) which combines the real marginal product of labor to the marginal rate of substitution between consumption and leisure:

\[
\hat{\phi}_f^f = \Gamma \hat{\phi}_f^f, \quad \text{with} \quad \Gamma \equiv [\eta + \sigma \gamma_C^{-1}(1 + \gamma_G\gamma_Y)],
\]

where we additionally used the fiscal spending rule (25) and market clearance condition (29). From the optimal price-setting behavior of firms operating in the intermediate good sector under flexible-prices we know that:

\[
\hat{\phi}_f^f = \Phi \frac{1}{\gamma_C}(1 - \bar{\tau}_{VAT}),
\]

where we assumed that fiscal policy sets \(\chi_1 = 0\) if prices are flexible as no price dispersion prevails in the flex-price equilibrium such that \(\bar{\tau}_{VAT}^f = \chi_2 b_f^f\). Accordingly the log-deviation of real marginal cost from its deterministic counterpart \((e - 1)/e\) can then be written in log-linearized terms as: \(\hat{\phi}_f^f = -(\Phi + \tau_{VAT})\). Using the output gap \(x_t\) the log-deviation of marginal cost can be written as:

\[
\hat{\phi}_f = \Gamma \hat{\phi}_f(x_t + \hat{Y}_f^f), \quad \text{with} \quad \hat{Y}_f^f = -\frac{1}{\gamma_C}(\Phi + \tau_{VAT}^f).
\]
We can rewrite the Phillips curve in terms of \( x_t \) as:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa [\Gamma_{\varphi} x_t + \iota (\hat{\tau}_t^{\text{VAT}} - \hat{\tau}_t^{\text{VAT},f})],
\]

(34)

From the Euler-equation we know that the natural rate of interest is equal to:

\[
r^n_t - \rho = \sigma E_t (\Delta \hat{Y}_t^{f} - \Delta \hat{G}_t^{f}),
\]

(35)

where \( \rho \equiv -\log \beta \). Inserting \( \Delta \hat{Y}_t^{f} \) and \( \Delta \hat{G}_t^{f} \) the natural rate can be expressed in terms of the exogenous shock \( \hat{\Phi}_{t+1} \) and the tax rule \( \hat{\tau}_{t+1}^{\text{VAT},f} \):

\[
\hat{r}^n_t = -\sigma (1 + o_Y) \Gamma^{-1}_{\varphi} E_t [\Delta \hat{\Phi}_{t+1} + \iota \Delta \hat{\tau}_{t+1}^{\text{VAT},f}].
\]

(36)

Using the definitions of the welfare gap \( x_t \) it holds that:

\[
x_t = E_t x_{t+1} - \gamma_C (\sigma (1 + \gamma_G o_Y))^{-1} [\hat{R}_t - E_t \hat{\pi}_{t+1} - \hat{r}^n_t],
\]

(37)

and

\[
b_{t+1} - \gamma_G t \Gamma^{-1}_{\varphi} (1 + o_Y) \hat{\tau}_t^{\text{VAT},f} = \beta^{-1} b_t - \gamma_G (o_Y + 1) x_t + \gamma_G \Gamma^{-1}_{\varphi} (1 + o_Y) \hat{\Phi}_t - \gamma_G \hat{\tau}_t^{\text{VAT}}.
\]

(38)

**Discussion**

Notwithstanding that most of the features in the model are standard in particular the value-added tax augmented Phillips curve is worth stressing. First, notice that the inflation rate is a weighted average of the expected path of wage costs, the markup shock and the evolution of the value-added taxes. As we will show below this enables the government to design a path for value-added taxes which almost completely offsets any movement in cost pressure such that price dispersion across firms can be reduced. Secondly, as we formulate state contingent tax and spending rules government debt necessarily works as a buffer to accommodate movements in the spending rule and movements of the tax rate. For the case of a balanced budget regime movements in the tax rate call for adjustments in fiscal spending.

### 2.7 Graphical Illustration of the Model

Throughout Section 2, we have shown the optimization problems of households, intermediate good producers and final good firms, and we have introduced the rules of monetary and fiscal policies. To help the reader to capture how all agents interact with each other, figure 1 illustrates the sequence of the actions for a certain period \( t \) and adumbrates the intertemporal links.
3 Simple Rules and Price Dispersion

In this Section we analytically examine the role of simple tax rules on the equilibrium allocation of inflation, output, consumption, interest rates and government expenditures. To keep the calculations analytically tractable, we assume that the budget is balanced such that (26) reduces to $\tau^{VAT}_t = \chi_1 \hat{\Phi}_t$ and government expenditures are adjusted passively so that the budget equation (24) holds. Additionally, we reduce the system by inserting the natural rate of interest $\hat{r}^n_t$ and the tax rule (26) into the Phillips curve (34) and the Euler equation (37). Then the model can be written as the following set of expectational difference equations:

$$\begin{align*}
x_t &= E_t x_{t+1} - \sigma^{-1}(\hat{R}_t - E_t \hat{\pi}_{t+1}) + (\gamma G \gamma C^{-1} \chi_1 + (\sigma + \eta)^{-1}) \hat{\Phi}_t, \quad (39) \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa[(\sigma + \eta)x_t + (\tau - \sigma)\gamma G \gamma C^{-1} \chi_1 \hat{\Phi}_t], \quad (40) \\
\hat{R}_t &= \phi_x \hat{\pi}_t, \quad (41)
\end{align*}$$

where the coefficient $\chi_1$ serves as a parameter which can be freely chosen by fiscal authorities. The following propositions summarize the main results.\(^3\)

\(^3\)For the MSV-solutions, see appendix B.
Proposition 3.1  Suppose that a social planer is only concerned about price dispersion and, hence, inflation variability. Then choosing a coefficient $\chi_1 = -\gamma C \gamma^{-1}_G (\iota + \eta)^{-1}$ completely eliminates any price dispersion across firms at any date $t$.

Proof  Since the simplified model with $b_{t+1} = b_t = 0$ exhibits no endogenous state variables the fundamental solution takes the form: $\hat{\pi}_t = \delta_\pi \hat{\Phi}_t$. Applying the methods of undetermined coefficients leads to the following solution: $\delta_\pi = [1 + \kappa (\sigma + \eta) \sigma^{-1} \phi_x]^{-1} \kappa [1 + \gamma G \gamma^{-1}_G \chi_1 (\iota + \eta)]$. Inflation is completely stabilized if $\delta_\pi = 0$ which holds for $\chi_1 = -\gamma C \gamma^{-1}_G (\iota + \eta)^{-1}$.

Thus according to Proposition 3.1 fiscal authorities can completely stabilize the inflation rate by choosing $\chi_1$ appropriately. For the applied calibration, $\chi_1$ would take a numerical value of $\chi_1 = -3.20 \ (\gamma G = 0.2; \ \eta = 1; \ \iota = 0.25)$. Interestingly the coefficient $\chi_1$ only depends on two deep parameters, namely $\tau^{VAT}$ and $\eta$. In line with intuition an increasing steady state government share $\gamma G$ increases the leverage of fiscal authorities on real marginal costs and on prices such that the same equilibrium allocations can be achieved by smaller movements of the instrument $\tau^{VAT}_t$. The same holds true for $\iota$ which is defined as $\iota \equiv \tau^{VAT}_t \bar{VAT} \delta_\pi$ and is increasing in $\tau^{VAT}$. Additionally, the modulus of $\chi_1$ decreases in the inverse Frisch elasticity $\eta$ of labor supply. Thus, if economic cycles evolve less pronounced due to a more inelastic labor supply, smaller tax incentives are sufficient to yield the same effects on the evolution of marginal cost, and hence prices.

Proposition 3.2  Suppose that we compare two economies which are identical except that in one economy fiscal policy implements the simple rule $\hat{\tau}^{VAT}_t = \chi_1 \hat{\Phi}_t$ whereas in the other economy fiscal policy remains passive with $\bar{\tau}^{VAT} = \tau^{VAT}_t$ and $\bar{G} = G_t \ \forall t$. Then, for any policy choice with $\chi_1 < 0$ the evolution of the inflation rate $\hat{\pi}_t$, the welfare gap $x_t$ and nominal interest rates $\hat{R}_t$ evolve smoother than in an economy where $\chi_1 = 0$.

Proof  Since in both economies the simplified model exhibits no endogenous state variable the fundamental solution takes in both cases the form $\hat{X}_t = \delta_X \hat{\Phi}_t$, with $\hat{X}_t = [\hat{\pi}_t \ x_t \ \hat{R}_t]$ and $\delta_X = [\delta_\pi \ \delta_x \ \delta_R]$. Thus a necessary and sufficient condition for a smoother evolution of the economy is $|\delta_{X}^A|_{i,1} < |\delta_{X}^P|_{i,1}$ for $i = 1, 2, 3$, where the superscripts $A$ denote active and $P$ passive. As shown in appendix B a necessary and sufficient condition for this inequality
to hold is that \( \chi_1 < 0 \).

Thus according to Proposition 3.2 it holds that any policy choice with \( \chi_1 < 0 \) accommodates a smoother evolution of the economy. Without any statement on welfare, we can already conjecture that an active fiscal stance is welfare improving if government expenditure is pure waste as the welfare function for this case only builds on inflation \( \hat{\pi}_t \) and the welfare gap \( x_t \). The nominal interest rate will be smoothed as it is just a linear transformation of the inflation rate itself according to the Taylor rule. This in turn, implies a smoother evolution of the real interest rate which fosters a more stable consumption path \((C_t/C_{t+1})\) as can be seen from the Euler-equation:

\[
\frac{1}{\beta} = E_t \left[ \left( \frac{C_t}{C_{t+1}} \right)^\sigma \left( \frac{R_t}{\pi_{t+1}} \right) \right],
\]

which states that the product of the real interest rate and the intertemporal ratio of consumption will always be equal to the inverse of the discount factor.

For the case of a balanced budget, changes in the tax rate have to be cushioned by fiscal spending \( \hat{G}_t \). Therefore, fiscal spending is more volatile than under a passive fiscal stance. Notwithstanding the output gap \( \hat{Y}_t \), defined as the weighted sum \( \gamma_C \hat{C}_t + \gamma_G \hat{G}_t \), evolves less volatile. This reflects that the additional volatility in government expenditure is overcompensated by the stable evolution of consumption itself. As the expected variability of the weighted average of private and public consumption good decreases, this is welfare enhancing given the concavity of preferences.

### 4 Welfare

Next we characterize the model if we allow for debt financed expenditures by means of numerical analysis. As shown in the appendix C the welfare criterion is derived by a second-order approximation of the average utility of a household around the deterministic long-run steady state. The welfare function can be written as follows (see Erceg, Henderson, and Levine, 2000, Gali and Monacelli, 2007, and Woodford 2003):

\[
W_0 = \sum_{t=0}^{\infty} \beta^t E_0(L_t),
\]

where

\[
L_t = \frac{\xi}{\kappa} \hat{\pi}_t^2 + (1 + \eta) \hat{Y}_t^2 + \iota (\hat{G}_t - \hat{Y}_t)^2.
\]
In the following we discuss the implementation of the proposed tax rules. We start with the optimal rule under discretion given by (26), which is based directly on the shock \( \hat{\Phi}_t \), and check afterwards whether similar results will hold for the simple tax rule (27).

4.1 Optimal Tax Rule under Discretion

Since we do not have a distinctive imagination for an appropriate numerical parameter except that \( \chi_1 < 0 \), we opt to choose the parameter such that the welfare function (43) is minimized.\(^4\)

Figure 2 portrays the dynamic responses of selected variables to a markup shock. For the baseline case fiscal policy remains passive with \( \chi_1 = 0 \) whereas for the active stance with \( \chi_1 < 0 \) fiscal policy aspires to improve welfare by controlling the evolution of marginal cost. The following remark summarizes the main findings:

**Remark:** The implementation of rule (26) largely disconnects the evolution of the inflation rate from exogenous markup shocks. If free to choose fiscal authorities prefer long debt cycles to cushion the exogenous shock.

The impulse responses portray that a sharp cut in taxes \( \hat{\tau}_t^{VAT} \) levied on the value-added prevents built up in cost pressure. The tax cut occurs in particular in the first quarter, when the geometrically decaying markup shock hits strongest. As a fraction of firms \( \theta_P \) is called upon to reset prices they foresee that any price pressure is undone by fiscal authorities by the targeted tax path that keeps the sum of wage path, markup shock and tax path flat. Due to the moderate evolution of the inflation rate monetary authorities do not need to raise the nominal interest rates sharply. This in turn detains Ricardian households to reallocate planned consumption expenditures by large into the future. As consumption accounts for 80% of output we observe a more moderate drop in production. If fiscal authorities are free to choose, they absorb the tax cut by a near-random walk behavior in debt. Note as markup shocks are symmetrically distributed a near-random walk behavior in debt implies that the persistent swings cancel out each other. On the

\(^4\)We also optimized over the parameter \( \chi_2 \) which governs the feedback from changes in debt and taxes. The algorithm preferred small values which are close to those proposed by Linnemann and Schabert (2003). As the algorithm often fall prey to indeterminacy for too small values of \( \chi_2 \), we chose a calibration of \( \chi_2 = 0.06 \).
contrary, contemporaneous government-expenditure changes are welfare reducing as they increase the expected variability in consumption of public goods. The point estimate for

Figure 2: Stabilization by the Optimal Fiscal Tax Rule

Notes: Responses of selected variables to a markup shock. Solid lines indicate a state independent passive fiscal policy with $\chi_1 = 0$. The dotted line shows the impulses of the model when fiscal policy is active with $\chi_1 < 0$ and $\chi_2 > 0$. For the applied baseline calibration see appendix A. All depicted variables are denoted in log-deviations.

the parameter $\chi_1$ and the associated standard errors are reported in Table 1. The point estimate for $\chi_1$ is equal to -3.11 with a standard error of 0.16. For the baseline scenario this implies that the implementation of the simple policy rule reduces the value of the loss function by 93 percent. A complete stabilization is not feasible, as the increase in distortionary taxes from the first quarter onwards needs to be sufficiently strong to bring back debt to its initial steady state level. Under the header “range” we report evidence that the proposed policy rule is robust with respect to deviations from the optimal reaction coefficient $\chi_1$. To illustrate this we deviate from the optimal coefficient such that the implementation of the policy rule smooths the business cycle. Therefore, as a robustness exercise we report how far we can deviate in both directions from the optimal coefficient
such that the computed distance $\left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}^{\text{Passive}} - \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}^{\text{Active, upper, lower}}$ is still greater than zero. Generally the results indicate no large asymmetries when fiscal authorities tend to choose too high or too low coefficients $\chi_1$, which indicates that the loss ratio largely behaves linearly when deviating from the baseline by altering $\chi_1$. For the case of large asymmetries we would have expected the reported values for $\chi_1^{\text{lower}}$ and $\chi_1^{\text{upper}}$ to have a substantially different distance to $-3.11$. The range from $-6.22$ to $0.00$ impressively demonstrates that for a large set of parameters $\chi_1$ the policy rule stabilizes the economy substantially. Therefore, we conclude that the proposed rule is robust with respect to variations in $\chi_1$.

Table 1: The Estimated Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>St.Dev.</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaction Coefficient</td>
<td>$\chi_1$</td>
<td>-3.11</td>
<td>0.16</td>
<td>[-6.22, 0.00]</td>
</tr>
</tbody>
</table>

4.2 Alternative Simple Tax Rule

Analogously to the procedure in the previous subsection, we simulated the impulse response functions for the simple tax rule based on $x_t$. Figure 3 exhibits that compared to the previous section, the impulse response functions for the selected variables take a very similar course. Hence, we can state that the implementation of the simple tax rule is highly suitable for stabilizing the economy after the materialization of markup shocks. For the baseline calibration the loss reduction is 72 percent, which is somewhat worse than for the discretionary optimum which reduced the loss by 93 percent. This might be explained by the following trade-off. Inflation is driven by marginal cost, which can be decomposed as $\Gamma_{\varphi} \hat{Y}_t + t \hat{\tau}^{VAT} + \hat{\Phi}_t$. If fiscal authorities target a tax path which sets the linear combination of $\hat{Y}_t$, $\hat{\tau}^{VAT}$ and $\hat{\Phi}_t$ equal to null. Accordingly, fiscal authorities attaching a high weight towards inflation stability have no strong incentive for output gap smoothing, as a decline in the output gap also stabilizes the inflation rate. This in particular prevails for the case of a simple rule where the tax path is not fine tuned towards the discretionary optimum.

The corresponding point estimates for $\chi_1$ are given in Table 3: We obtain $\chi_1 = -8.10$ with a standard error of 0.22, which implies that the implementation of the simple policy rule reduces the value of the loss function by 72 percent. The loss reduction is significant at the one percent level. The results are robust over a large range for $\chi_1$ from $-15.77$ to $0.00$. 

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Notes: Responses of selected variables to a markup shock. Solid lines indicate a state independent passive fiscal policy with $\chi_1 = 0$. The dotted line shows the impulses of the model when fiscal policy is active with $\chi_1 < 0$ and $\chi_2 > 0$. For the applied baseline calibration see appendix A. All depicted variables are denoted in log-deviations.

5 Relevance of the Tax-Rule

Markup shocks are costly in terms of welfare as monetary authorities lack an instrument on the supply side of the economy to cushion the adverse effects of cost pressure. Following the Tinbergen (1959) logic we have shown that a state contingent tax can improve welfare remarkably.

In the following we discuss the implications of these issues by computing welfare gains

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>St.Dev.</th>
<th>Range</th>
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<tr>
<td>Reaction Coefficient</td>
<td>$\chi_1$</td>
<td>-8.10</td>
<td>0.22</td>
<td>[-15.77, 0.00]</td>
</tr>
</tbody>
</table>
using different parameter constellations. This exercise has two main purposes. On the one hand we want to analyze whether the proposed rule is robust to perturbations of the baseline parametrization. On the other hand we present further insights why the rule works from a micro-founded perspective.

5.1 Robustness of the Optimal Tax Rule

Precisely speaking we compute the expected value of the loss $E_0\{\sum_{t=0}^{\infty} \beta^t L_t\}$ for the active and the passive fiscal policy stance and then take the ratio of the two. If the ratio takes the value one, then the loss would be equal under the two regimes. If the value of the ratio is below (above) one, then the loss under an active fiscal policy is smaller (larger) than the loss under the passive fiscal stance. By means of computing these ratios we succeed to uncover those parameter constellations which improve or worsen the relative performance of the proposed policy rule compared to the fallback position of a passive fiscal policy.

The line indicates how the computed ratio changes when the parameter displayed at the top of the figure is altered, while the rest remains fixed at the baseline calibration. For each altered coefficient, e.g. for $\eta$, the coefficients in the fiscal policy rule $\chi_1$ and $\chi_2$ are reoptimized such that the welfare function (43) is minimized.

The labor supply elasticity $\eta$, the Taylor rule coefficients $\phi_\pi$, $\phi_x$, and $\phi_\rho$, the Calvo-parameter $\theta_p$, and the degree of correlation in the markup shock are varied within ranges typically found in the literature. The robustness analysis indicates that for all deep parameters except $\phi_x$ the welfare gains are extremely robust, irrespectively of the chosen calibration.

The robustness analysis also indicates that the relative advantage of the proposed policy rule decreases if monetary policy reacts stronger to the welfare gap. Nevertheless, the loss reduction is still round about 45 percent, even for a coefficient of $\phi_x = 0.5$, which is higher than the values typically found in literature (e.g., Smets and Wouters, 2003). This might be explained by the fact that the proposed tax rule is successful in stabilizing inflation, but does not close the output gap $x_t$. This implies that a monetary authority that takes the output gap into account reintroduces real interest rate variability, and thus business cycle fluctuations.

---

5For the reviewed literature and the applied ranges see appendix A.
Notes: Evolution of the expected loss ratio defined as the ratio of the expected loss if fiscal policy is active with $\chi_1 < 0$ compared to a passive stance $\chi_1 = 0$,

$$E_0 \left( \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}^{\text{Active}} / \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}^{\text{Passive}} \right)$$.

Appendix A summarizes the ranges of deep parameters typically found in the literature.

5.2 Robustness of the Simple Rule

Figure 5 portrays that the shape of the ratios are qualitatively almost identical under the modified rule (27) compared to figure 4. But the figure shows that the loss ratios are shifted upward for the baseline calibration around 20%. Note that there are two notable differences standing out: Based on review of the literature it seems fair to conduct the robustness analysis in a range between 1.8 to 10 quarters, which corresponds to $\theta_P$ ranging between 0.45 to 0.90.\(^6\) The figure illustrates that the performance of the rule improves if

\(^6\)With respect to the value of the Calvo parameter $\theta_P$ there exists a considerable disagreement in the literature. Del Negro et. al. (2005) for instance estimate an average price duration of three quarters for the euro-area using full information Bayesian techniques; Smets and Wouters (2003) estimate a price duration of 10 quarters. Gali, Gertler, and Lopez-Salido (2001) report a value round about four quarters using single equation GMM approach. Empirical work on price setting in the euro area using micro evidence report relatively low price durations with a median round about 3.5 quarters (see Alvarez et. al., 2006, for a summary of recent micro evidence). Comparable studies for the U.S. like Altig et. al. (2005) report much lower average price durations of just 1.6 quarters, which they claim to be more
Notes: Evolution of the expected loss ratio defined as the ratio of the expected loss if fiscal policy is active with $\chi_1 < 0$ compared to a passive stance $\chi_1 = 0$, $E_0 \left( \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}^{\text{Active}} / \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}^{\text{Passive}} \right)$. Appendix A summarizes the ranges of deep parameters typically found in the literature.

the degree of price stickiness increases. The implementation of the policy rule prevents that a wedge can be driven between the production schedules by price dispersion and thus enhances welfare as the variability of inflation decreases.

Furthermore, the effectiveness of the rule decreases with the degree of correlation in the markup shock $\zeta$. The higher the degree of correlation the larger will be the price dispersion inflicted upon the economy. Those firms that are called upon to reset prices will anticipate further shocks in the same direction which triggers a larger adjustment of prices. Therefore, the rule is welfare enhancing in an environment of correlated shocks as it promises to firms a stable evolution of prices and thus a limited degree of price dispersion for the economy. If, however, the degree of correlation in the markup shock becomes too large, fiscal authorities have to change the tax rate substantially which calls for subsequent tax increases as solvency considerations have to be fulfilled.

consistent with recent evidence drawn from US micro-data.
6 Conclusions

In this paper we addressed the question whether fiscal policy can wipe out price dispersion by implementing a countercyclical tax rule. Our motivation stems from the fact that there is a large strand of literature which stresses the role of monetary policy to enhance welfare in an environment of nominal rigidities (Woodford, 2003). However this strand of literature has paid so far little attention to the question whether fiscal policy can improve welfare with respect to nominal frictions. In the event of cost-push shocks Woodford (2003) shows that monetary policy faces a trade off between stabilizing the inflation rate and stabilizing the output gap. A sufficiently strong feedback from movements in the inflation rate is argued to be the best response to limit the adverse effects of cost-push shocks on lifetime utility of a representative consumer to generate a unique and determinate equilibrium. Notwithstanding these arguments, the costs of nominal rigidities are estimated to be up to three percent in consumption equivalents (Canzoneri, Cumby and Diba, 2007).

This highlights that monetary policy does not have a direct leverage on the supply side of the economy. Therefore, we proposed that fiscal policy should use its value-added tax, as an additional instrument in a state contingent way such that the evolution of marginal cost is stabilized around its deterministic steady state. Our findings suggest that countercyclical taxation can remarkably reduce the impact of cost-push shocks on welfare. The reduction in expected losses, when fiscal authorities switch from a passive towards an active fiscal stance are quantified around 93% for the optimal tax rule and 72% for the simple tax rule. Key to the functioning of the tax-rule is that the fraction of firms that adjusts prices anticipates the promise of fiscal authorities to target a value-added tax path that eliminates any cost pressure at the firm level. Accordingly, those firms that are called upon to reset prices will set them in the neighborhood of those firms that leave prices unchanged. This prevents any inefficient built-up in prices across firms at any date $t$.

The Keynesian tradition considers fiscal policy as operating over the aggregate demand effect. We showed that fiscal policy can use its distortionary instruments to unfold stabilizing effects on the economy upon an aggregate supply channel.
Appendices

A Calibrated Parameters

In Section 5 of the main text we conduct some sensitivity analysis to demonstrate the robustness of the proposed policy rule. While conducting this exercise we rely on ranges of the deep parameters chosen in a way to best represent the uncertainty found in the literature as reported in Table 3.

Table 3: Values and Ranges for the Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Household</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>$\sigma$</td>
<td>1.00</td>
</tr>
<tr>
<td>Inverse of the Labor Supply Elasticity</td>
<td>$\eta$</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>B. Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price Elasticity of Demand for an Intermediate Good</td>
<td>$\varepsilon$</td>
<td>11.00</td>
</tr>
<tr>
<td>Variety</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price Stickiness</td>
<td>$\theta_P$</td>
<td>0.75</td>
</tr>
<tr>
<td><strong>C. Monetary Policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor Rule: Smoothing</td>
<td>$\phi_p$</td>
<td>0.50</td>
</tr>
<tr>
<td>Taylor Rule: Inflation</td>
<td>$\phi_x$</td>
<td>1.50</td>
</tr>
<tr>
<td>Taylor Rule: Welfare Gap</td>
<td>$\phi_x$</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>D. Fiscal Authorities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiscal Rule (optimal): Markup shock</td>
<td>$\chi_1$</td>
<td>-1.60</td>
</tr>
<tr>
<td>Fiscal Rule (simple): Welfare Gap</td>
<td>$\chi_1$</td>
<td>-7.34</td>
</tr>
<tr>
<td>Fiscal Rule (both): Debt</td>
<td>$\chi_2$</td>
<td>0.06</td>
</tr>
<tr>
<td>Steady State VAT Level</td>
<td>$\bar{\tau}^{VAT}$</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>E. Exogenous Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Markup Shock: Persistence</td>
<td>$\zeta$</td>
<td>0.75</td>
</tr>
</tbody>
</table>

B Derivation of the MSV Solution

Balanced budget and active stance Substituting out the tax-rate $\hat{\tau}_t^VAT$ and the natural rate $\hat{r}_t^n$ of interest the reduced form system can be written as:

$$x_t = E_t x_{t+1} - \sigma^{-1}(\phi_x \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + \left(\frac{\gamma_G}{\gamma_C} \chi_1 + (\sigma + \eta)^{-1}\right) \hat{\Phi}_t,$$  \hfill (B.1)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \left[(\sigma + \eta) x_t + \left(\frac{\gamma_G}{\gamma_C} \chi_1\right) \hat{\Phi}_t\right],$$  \hfill (B.2)

$$\hat{R}_t = \phi_x \hat{\pi}_t.$$  \hfill (B.3)

The rest of the system is recursive and can be solved afterwards. Let us posit a fundamental (minimum state variable) solution of the following generic form (McCallum, 1983):

$$\hat{\pi}_t = \delta_\pi \hat{\Phi}_t \text{ and } x_t = \delta_x \hat{\Phi}_t,$$

where the coefficients $\delta_\pi$ and $\delta_x$ remain to be determined. With $E_t x_{t+1} = E_t \delta_x \hat{\Phi}_{t+1} = 0$ and $E_t \hat{\pi}_{t+1} = E_t \delta_\pi \hat{\Phi}_{t+1} = 0$, this leads to the following conditions for the undetermined coefficients:

$$\delta_\pi = \kappa (\sigma + \eta) \delta_x + \kappa (\sigma + \eta) \frac{\gamma G}{\gamma C} \chi_1,$$  \hfill (B.4)

$$\delta_x = -\sigma^{-1} \phi_x \delta_\pi + (\sigma + \eta)^{-1} + \frac{\gamma G}{\gamma C} \chi_1.$$  \hfill (B.5)

Inserting (B.5) into (B.4) yields

$$\delta_\pi = [1 + \kappa (\sigma + \eta) \sigma^{-1} \phi_x]^{-1} \cdot \kappa [1 + \gamma G \gamma C^{-1} \chi_1 (1 + \eta)],$$  \hfill (B.6)

and

$$\delta_x = \frac{\sigma \gamma C + (\sigma + \kappa (\sigma - \eta) \phi_x) (\sigma + \eta) \gamma G \chi_1}{(\sigma + \eta) (\sigma + \kappa \phi_x (\sigma + \eta)) \gamma C}.$$  \hfill (B.7)

Balanced budget and passive policy Let us define the neutral benchmark system as $\hat{G}_t = \hat{\pi}_t^VAT = 0$. Then the model can be stated as:

$$x_t = E_t x_{t+1} - \sigma^{-1}(\phi_x \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + (\sigma + \eta)^{-1} \hat{\Phi}_t,$$  \hfill (B.8)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\sigma + \eta) x_t,$$  \hfill (B.9)

where the MSV solution reads:

$$\delta_x = [1 + \sigma^{-1} \phi_x \kappa (\sigma + \eta)]^{-1} (\sigma + \eta)^{-1},$$  \hfill (B.10)

and

$$\delta_\pi = \kappa [1 + \sigma^{-1} \phi_x \kappa (\sigma + \eta)]^{-1}.$$  \hfill (B.11)
Comparison of active versus passive fiscal policy  In the following, we compare the MSV solutions for an economy where fiscal policy implements policy rule (25) versus an economy where fiscal policy remains passive with $\hat{G}_t = \hat{\tau}^{\text{VAT}} = 0$. The superscript $P$ denotes passive whereas the superscript $A$ denotes active.

Inflation:

$$\delta^P_\pi > \delta^A_\pi$$

$$\Rightarrow \kappa [1 + \kappa\sigma^{-1}(\sigma + \eta)\phi_\pi]^{-1} > \kappa [1 + \kappa\sigma^{-1}(\sigma + \eta)\phi_\pi]^{-1} [1 + \gamma G\gamma^{-1} C_1 (1 + \eta)]$$

$$\Rightarrow 1 > 1 + \gamma G\gamma^{-1} C_1 (1 + \eta)$$

$$\Rightarrow 0 > \gamma G\gamma^{-1} (1 + \eta) C_1 \Rightarrow C_1 < 0, \quad \eta, \gamma G, \gamma C > 0$$

Welfare gap:

$$\delta^P_x > \delta^A_x$$

$$\Rightarrow [1 + \kappa\sigma^{-1}(\sigma + \eta)\phi_\pi]^{-1} (\sigma + \eta)^{-1}$$

$$\frac{\sigma G + (\sigma + \kappa(\sigma - \eta)\phi_\pi)(\sigma + \eta)\gamma C_1}{(\sigma + \eta)(\sigma + \kappa\phi_\pi(\sigma + \eta))\gamma C}$$

$$\Rightarrow \sigma G > \sigma G + (\sigma + \kappa(\sigma - \eta)\phi_\pi)(\sigma + \eta)\gamma C_1$$

$$\Rightarrow 0 > (\sigma + \kappa(\sigma - \eta)\phi_\pi)(\sigma + \eta)\gamma C_1$$

$$\Rightarrow C_1 < 0, \quad \gamma G, \kappa, \sigma, \eta > 0$$

\[\square\]

C Utility-Based Welfare Function

The utility function is given by:

$$U(C, N, G) = (1 - \tau) \log C + \tau \log G - \frac{N^{1+\eta}}{1 + \eta}. \quad \text{(C.1)}$$

Note that the weight $\tau$ in the utility function is equal to the steady state share of government spending $\tau = G/Y$. Taking a second-order approximation around the consumption part of the utility function yields:

$$\log(C_t) = \log(Y_t - G_t) = \frac{1}{1 - \tau} (\hat{Y}_t - \tau \hat{G}) - \frac{1}{2} \frac{\tau}{(1 - \tau)^2} (\hat{Y}_t - \hat{G}_t)^2 + t i p + o(||a^3||) \quad \text{(C.2)}$$
Where it holds that: \( \hat{x}_t = \bar{x}_t + (\bar{x}_t - x) \). We denote the gap \( \hat{Y}_t = \log Y_t - \log \bar{Y}_t \) and the fiscal gap \( \hat{G}_t = \log G_t - \log \bar{G}_t \). Note that \( \hat{Y}_t \) comprises the sum of the deviation of output from the distorted (short term) steady state and the deviation of the distorted steady-state output from the efficient long-term steady state. Taking a second-order approximation around the disutility of labor term yields:

\[
N^1 + \frac{\eta}{1 + \eta} = \hat{N}_t + \frac{1}{2} \hat{N}_t^2 + t_i + o(||a^3||). \tag{C.3}
\]

We find the relationship \( N_t = Y_t D_t \), which is derived in the following:

\[
N_t = \int_0^1 N_t(i) di = \int_0^1 Y_t(i) di = Y_t \int_0^1 \frac{Y_t(i)}{Y_t} di = Y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} di = Y_t D_t. \tag{C.4}
\]

After log linearization, we obtain:

\[
\hat{N}_t = \hat{Y}_t + q_t. \tag{C.6}
\]

Where \( q_t = (\varepsilon/2)\sigma_t^2 \) and \( q_t \) is defined as:

\[
q_t \equiv \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} di. \tag{C.7}
\]

The intertemporal welfare function is given by the discounted sum of the approximated utility functions:

\[
W_t = \sum_{t=0}^{\infty} \beta^t U_t(C_t, G_t, N_t) = \sum_{t=0}^{\infty} \beta^t \left[ (1 + \eta)\hat{Y}_t^2 + \nu(\hat{G}_t - \hat{Y}_t)^2 + \varepsilon \sigma_t^2 \right]. \tag{C.8}
\]

Now, we aim at expressing \( \sigma_t^2 \) in terms of \( \pi_t^2 \) while following the proof given by Woodford 2003:

\[
\sum_{t=0}^{\infty} \beta^t \sigma_t^2 = \sum_{t=0}^{\infty} \beta^t \left[ t_i p + \sum_{s=0}^{t} \frac{\theta_{\pi_t}^s}{\theta_{\pi_t}^s} \tilde{\pi}_t^2 + o(||a||^3) \right] = \frac{1}{\kappa} \sum_{t=0}^{\infty} \beta^t \tilde{\pi}_t^2 + t_i + o(||a||^3). \tag{C.9}
\]

Using this result (C.8) can be rewritten as follows:

\[
W_t = \sum_{t=0}^{\infty} \beta^t \left[ \frac{\varepsilon}{\kappa} \tilde{\pi}_t^2 + (1 + \eta)\hat{Y}_t^2 + \nu(\hat{G}_t - \hat{Y}_t)^2 \right]. \tag{C.10}
\]
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